Determination of Market Structures in a Class of Homogeneous Industries

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Abstract

Industry equilibrium for an industry with one superior firm and many inferior firms is studied. There can be two types of industry equilibria and inefficient one may result depending on the initial state of the industry.

1. INTRODUCTION

The theory of free entry equilibrium, or industry equilibrium, has long been studied by many industrial economists. With the condition that no further entry is profitable for an entrant, one can see how particular structure emerges from rational behaviour of the firms. In many studies however, the assumed structure of fundamental conditions of the industry is symmetric with respect to the demand and firms. This is so too in the theory of monopolistic competition where the differences between firms play a crucial role. The reason is twofold. First, it is the aim of some studies to show that even with a large number of small symmetric firms, the resulting structure and performance of the industry need not be regarded as competitive. (1) Second, the assumption of symmetry greatly facilitates the analysis by reducing the equilibrium to the number of firms while the introduction of asymmetry may obscure the notion of "equilibrium".

In the presence of asymmetry, an industry may have two or more types of industry equilibria or it may have only one depending on the demand and the cost conditions. This situation can be easily understood by considering an industry where there are only two potential producers, one with superior, the other with inferior, production technique. If the demand is relatively small so that the market can afford only superior one, then industry equilibrium is unique. When the demand can afford only one but it can also support inferior

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(1) The author thanks seminar participants at Waseda University for helpful comments on an earlier version of this paper.

(1) See Mankiw and Whinston (1986) and Suzumura and Kyono (1987) for the welfare aspects of the free entry equilibrium.
one, the equilibria are now two. In this case incumbent firm "blockades" the entry in the sense of Bain (1956). This case suggests that the history of an industry may be strongly affected by the firms initially in it, which is of great interest in studying the structure. If the demand becomes still larger, the two firms can make positive profits by competing each other in the market. Which situation is feasible depends not only on the demand but also on the difference of the cost functions. This simple example raises a question what exactly entry barrier means. It is apparent that when the same type of firm as the entrant is incumbent in the market, the profit of the incumbent can be seen as a rent of being earlier. But when the entrant has superior technology and is still blockaded, the profit of the incumbent may be more than that.

The purpose of this paper is to see how different types of market forms emerge as industry equilibria. Here, "industry equilibrium" means the state where no entrant with the belief that each firm active in the market behaves as a Cournot quantity-setting oligopolist finds it profitable to enter the market. To get concrete results, we consider a special class of industries in which a firm, denoted as firm 1, has smaller constant marginal cost than remaining firms. The reason for considering asymmetry between marginal costs rather than fixed costs lies in the fact that the differences of fixed costs have nothing to do with strategic considerations of firms whereas the differences of marginal costs directly affect firms' choices of quantities produced. Specifying the inverse demand of the industry, we derive the conditions under which particular market structures emerge as industry equilibria. The analysis reveals that there is possibility of two types of equilibria. Having obtained the equilibria, we calculate the quantities produced under each equilibrium and compare them.

2. THE MODEL

We first consider a general Cournot quantity-setting oligopoly model under homogeneous linear demand and linear cost functions. Let the totality of the potential firms for the industry be denoted by $N=\{1,2,\cdots,n\}$. For $i \in N$, the quantity produced by firm $i$ is $x_i$, and we write $x=(x_1,\cdots,x_n)$. The inverse demand for the industry product is given by

$$
\phi(x) = \begin{cases} 
  a-b \sum_{j=1}^{n} x_j & 0 \leq \sum_{j=1}^{n} x_j \leq \frac{a}{b} \\
  \frac{a}{b} & \sum_{j=1}^{n} x_j > \frac{a}{b} 
\end{cases}
$$

(1)

for $x=(x_1,\cdots,x_n) \in \mathbb{R}_+^n$, where $a>0$ and $b>0$. The cost function of firm $i$ is given by

$$
c_i(x_i) = \alpha_i + \beta_i x_i,
$$

(2)

where $\alpha_i>0$ and $\beta_i>0$. Then we have profit function

$$
\pi_i(x) = \phi(x) x_i - (\alpha_i + \beta_i x_i).
$$

(3)

Note that $\frac{\partial^2 \pi_i(x_i)}{\partial x_i^2} < 0$ whenever $\sum_{j=1}^{n} x_j < a/b$ $(>a/b)$. Note also that if $\frac{\alpha_i - \beta_i}{b} > \sum_{i \neq j} x_j$,
firm \( i \)'s marginal profit in a right neighbourhood of 0 is positive.

Now let \( A = \{ s_1, s_2, \ldots, s_k \} \) be a group of firms and let \( x^A = (y_1, \ldots, y_n) \), \( y_j = x_j \) for \( j \in A \) and \( y_j = 0 \) otherwise. Assuming \( \phi (x^A) > 0 \), differentiating \( \pi_{ii} \) with respect to \( x_i \) and letting them equal to zero, we have

\[
\begin{pmatrix}
2b & b & \cdots & b \\
b & 2b & \cdots & b \\
\vdots & \vdots & \ddots & \vdots \\
b & b & \cdots & 2b
\end{pmatrix}
\begin{pmatrix}
x_{s_1} \\
x_{s_2} \\
\vdots \\
x_{s_k}
\end{pmatrix}
= \begin{pmatrix}
a - \beta_{s_1} \\
a - \beta_{s_2} \\
\vdots \\
a - \beta_{s_k}
\end{pmatrix}
\]

Solving this system of equations, we have

\[
x^*_a = \frac{k(a - \beta_a) - \sum_{j \in A}(a - \beta_j)}{b(k + 1)}
\]

(4)

Assume \( x^*_a > 0 \) for \( i=1, \ldots, k \). Summing (4), we have

\[
\sum_{i=1}^{k} x^*_a = \frac{k}{b(k + 1)} \sum_{i=1}^{k} (a - \beta_a)
\]

(5)

By (5), \( \sum_j y_j x^*_j < a/b \) and thus \( \phi (x^*_A) > 0 \). By (4), and (5),

\[
x^*_a = \frac{a - \beta_a}{b} - \sum_{j=1}^{k} x^*_j > 0
\]

and thus marginal profit of firm \( s_i \) is positive near \( x_{s_i} = 0 \) given \( x^*_j, j \neq i \). Since

\[
\delta^2 \pi_{ii} \left( x^*_A \right) x_{s_i} / \delta^2 \pi_{ii} < 0 \quad (= 0) \quad \text{whenever},
\]

\[
x_a + \sum_{i \neq j} x^*_j < a/b \quad \text{and} \quad \pi_{ii} (x^*_A) > 0
\]

we conclude that \( \{ x^*_i \}_{i=1, \ldots, k} \) is actually unique Cournot equilibrium for the group of firms \( A \).

We now further specify the model as

\[
\alpha_i = \alpha, \quad i=1,2,\ldots,n
\]

and

\[
\beta_i = \beta > \beta_i, \quad i=2,\ldots,n
\]

(6)

This means that firm 1 has superior production technique to the other firms, even if its fixed cost is no larger than the others'. Here we assume:

**Assumption 1**

\[
\alpha - \beta > \beta - \beta_i
\]

(7)

This means that the difference between the maximum possible price and the higher marginal cost is larger than the difference between two marginal costs. This assumption is necessary and sufficient that for every \( \phi \neq A \in N \) and for every \( i \in A \), the Cournot equilibrium quantity of firm \( i \) will be positive, even if the profit may be negative.

Using (6), we can classify the groups of firms as

\[
\widetilde{A}_k = \{A \subset N \mid \#A = k \quad 1 \not\in A\}, k=0,1,\ldots,n-1,
\]

\[
\widetilde{B}_k = \{B \subset N \mid \#B = k \quad 1 \not\in B\}, k=1,2,\ldots,n.
\]

We call each \( \widetilde{A}_k \) and \( \widetilde{B}_k \) as a *market form*.

Now let \( x^*_A \) be the Cournot equilibrium quantity vector defined by

\[
x^*_A = \left\{ y_1, \ldots, y_n \right\}, y_i = \begin{cases}
x^*_i, i \in A \\
0, \quad i \not\in A.
\end{cases}
\]

For \( i \in N \) define set function \( v_i : P (N) \rightarrow R \) as
\[ u_i(A) = \begin{cases} 
\pi_i(x^*_A) & i \in A \\
0 & i \in A 
\end{cases} \]

A group of firms \( A \subset N \) is said to be feasible if

\[ \forall i \in A \quad u_i(A) \geq 0. \] (a)

As a convention, empty set is always feasible.

Let \( A \subset N \) be a group of firms. A group of firms \( B \subset N - A \) is said to be an entry group for \( A \). An entry group \( B \subset N - A \) for \( A \) is said to be possible if for any \( i \subset B \)

\[ u_i(A \cup B) \geq 0 \]

holds.

**Definition 1** A group of firms \( A \subset N \) is said to be an industry equilibrium if it is feasible and if there exists no non-empty possible entry group for \( A \), that is, if \( A \) satisfies feasibility condition (a) and

\[ \beta \subset N - A, \quad \beta \neq \emptyset \Rightarrow \exists j \in \beta, \quad u_i(A \cup \beta) < 0. \] (b)

A market form which corresponds to an industry equilibrium is said to be an equilibrium market form.

### 3. Analysis

We first calculate Cournot equilibrium quantities for \( A \in A_k \) and \( B \in B_k \). For \( A \in A_k \), we have from (4), (5) and (6),

\[ x^A_i = \frac{a - \beta}{b(k + 1)}, \quad p^A_i = \frac{a + k \beta}{k + 1}, \quad i \in A, \]

where \( p^A_i = \phi(x^*_A) \) and we can define firm \( i \)'s equilibrium profit under \( A \), \( v_i(A) \), as

\[ v_i(A) = \frac{(a - \beta)^2}{b(k + 1)^2} - \alpha, \quad i \in A. \] (8)

We assume \( v_i(\{i\}) > 0 \) for any \( i \in N \):

**Assumption 2**

\[ a - \beta > 2\sqrt{ab}. \]

For \( B \in \widetilde{B}_k \), we have

\[ x^B_i = \frac{a + (k - 1)\beta - k \beta_i}{b(k + 1)}, \quad i \neq 1, k \geq 2, \]

\[ x^B_i = \frac{a - 2\beta + \beta_i}{b(k + 1)}, \quad i \neq 1, k \geq 2, \]

\[ p^B_i = \frac{a + (k - 1)\beta + \beta_i}{k + 1}. \]

where \( p^B_i = \phi(x^*_B) \) and thus we have equilibrium profit \( v_i(B) \) as

\[ v_i(B) = \frac{(a + (k - 1)\beta - k \beta_i)^2}{b(k + 1)^2} - \alpha, \]

\[ v_i(B) = \frac{(a - 2\beta + \beta_i)^2}{b(k + 1)^2} - \alpha, \quad i \neq 1, k \geq 2. \]

Then (8), (10) and (11) are all decreasing in \( k \). Note also that \( v_i(B) > v_i(B) \) for \( i \neq 1 \) since \( \beta > \beta_i, x^B_i > x^B_i \).

The following proposition is given without proof.

**Proposition 1** Under Assumptions 1 and 2, there exists \( 1 < k < n \) such that \( B_k \) is an equilibrium market form.
For $\tilde{A}_k$, $1 \leq k \leq n-1$, to be an equilibrium market form, it is necessary and sufficient that
\begin{align}
v_i(A) &> 0, \quad v_i(A') < 0, \quad v_1(\beta) < 0, \\
i \in A \in \tilde{A}, j \in A' \in A_{k+1}, \beta \in \beta_{k+1},
\end{align}
(12)
holds. Similarly, if $\tilde{B}_k$, $1 \leq k \leq n$, is to be an equilibrium market form
\begin{align}
v_i(B) &> 0, \quad v_i(B') < 0, \\
i \in B \in \tilde{B}_k, 1 \neq j \in \beta \in \beta_{k+1}.
\end{align}
(13)
Since (8), (10) and (11) are all decreasing in $k$, to see that (12) or (13) is satisfied, we first have to find roots of algebraic equations of variable $k$;
\begin{align}
(a-\beta_k)^2-\alpha\beta(k+1)^2 &= 0, &\text{for } v_i(A), \\
(a+(k-1)\beta-k\beta_1)^2-\alpha\beta(k+1)^2 &= 0, &\text{for } v_i(B), \\
(a-2\beta+\beta_1)^2-\alpha\beta(k+1)^2 &= 0, &\text{for } v_i(B),
\end{align}
(14)
Economically possible roots are, respectively,
\begin{align}
k_0 &= \frac{a-\beta}{\sqrt{\alpha\beta}} - 1, &\text{for } v_i(A), \\
k_1 &= \frac{a-\beta-\sqrt{\alpha\beta}}{\sqrt{\alpha\beta} + \beta_1 - \beta}, &\text{for } v_i(B), \\
k_2 &= \frac{a-2\beta+\beta_1}{\sqrt{\alpha\beta}}, &\text{for } v_i(B)
\end{align}
(14)
Using this, we can write necessary and sufficient conditions for $\tilde{B}_k$ to be an equilibrium market form:
\begin{align}
[k_2] &= k, \quad n > k \geq 1, \\
k_2 &= n, \quad k = n,
\end{align}
(15)
where $[\ ]$ is Gauss symbol.

Proposition 1 assures the existence of such $k$. We now consider two different cases.

Case 1.
\begin{align}
\sqrt{ab} \leq \beta - \beta_1,
\end{align}
(16)
In this case, $k_i < 0$ or $k_i$ is undefined by (14). Here we can assert:

**Proposition 2** Under (16), $\tilde{B}_k$, for some $k$, can be the only equilibrium market form, as long as Assumption 1 and 2 hold.

**Proof.** (9), (10) and (16), we have,
\begin{align}
v_1(B) &= \frac{(a-\beta+k(\beta-\beta_1))^2}{b(k+1)^2} - \alpha \\
&> \frac{(a-\beta+k\sqrt{\alpha\beta})^2}{b(k+1)^2} - \alpha > \frac{(k+1)^2\alpha\beta}{b(k+1)^2} - \alpha = 0,
\end{align}
(17)
for any $k \geq 1$ and $B \in \tilde{B}_k$. This means that $\tilde{A}_k$ cannot be an equilibrium market form.

Intuitively, if the maximum possible price is
relatively large in the sense of Assumption 1and
if the fixed cost and/or the slope of the demand
curve is small enough relative to the difference
of marginal costs, asymmetric oligopoly is likely
to be the equilibrium market form.

Case 2.
\( \sqrt{ab} > \beta - \beta_1 \).

In this case \( k_i > 1 \) by (6) and (9) \(^3\), we have

\[ k_i > k_0 - k_2. \tag{19} \]

Inequality \( k_i > k_2 \) reflects \( v_1(B) > v_i(B) \) for \( B \in \tilde{B}_k \) with \( 1 < k < n \) and \( i \neq 1, i \in B \). Here, there are
two possibilities.

Case 2.1. There exists \( k \) such that \( \tilde{B}_k \) is the only
equilibrium market form.

Case 2.2. There exist \( k \) and \( l \) such that \( \tilde{B}_k \) and
\( \tilde{A}_l \) are equilibrium market forms.

Since we know from Proposition 1 that there is a
\( k \) such that \( \tilde{B}_k \) is an equilibrium market form,
Case 2.1 is merely the negation of Case 2.2 and
thus we only have to consider the latter.

Since \( k_i > k_0 \), if \( \tilde{A}_l \) with \( n-1 \leq l \leq 1 \) is to be an
equilibrium market form, it is necessary and
sufficient by (14) that \( l+1 > k_i > k_0 \geq l \) holds. This
can be written as

\[ [k_i] = [k_0] = l, \tag{20} \]

\(^3\) By (6) and we have

\[ \begin{aligned}
\left( a - \beta - \sqrt{ab} \right) - \left( \sqrt{ab} + \beta_1, \beta \right) \\
= a - \beta - 2\sqrt{ab} > a - \beta - \sqrt{ab} > 0.
\end{aligned} \]

or

\[ l+1 > \frac{a - \beta - \sqrt{ab}}{\sqrt{ab} + \beta_1 - \beta} > \frac{a - \beta - \sqrt{ab}}{\sqrt{ab}}. \]

For this to be true

\[ \frac{1}{\frac{\beta - \beta_1}{\sqrt{ab} + \beta_1 - \beta}} > 0 \]

must hold. This is equivalent to

\[ \sqrt{ab} > (\beta - \beta_1)(\alpha - \beta). \tag{21} \]

The negation of this condition is sufficient for
Case 2.1. To sum up:

**Proposition 3** Under (18),

1. The inequality

\[ \sqrt{ab} > (\beta - \beta_1)(\alpha - \beta) \]

is sufficient for Case 2.1.

2. The condition (20) is necessary and sufficient
for Case 2.2.

<table>
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<th>a</th>
<th>b</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \beta_1 )</th>
<th>n</th>
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<th>k and l</th>
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<td>k=18</td>
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<td>0.255</td>
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<td>Case 2.2</td>
<td>k=17,l=18</td>
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</table>

Table 1: Relation of Parameters to Equilibrium Market
Forms \( \tilde{B}_k \) and \( \tilde{A}_l \).
Intuitively, if the fixed cost and/or the slope of the demand curve is large enough relative to the difference of marginal costs, market form with identical firms is likely to be an equilibrium, since in that case $\sqrt{c_b} > \beta - \beta_i$ and $[k_i] = [k_b]$ are likely to hold. Note that $l$ in (20) can be 1 if $2 > k_1 > 1$. This case gives an interesting example where a firm with superior cost condition is deterred from entering the market. The reason is that the advantage of the marginal cost, $\beta - \beta_i$, is not sufficiently large relative to the fixed cost, $\alpha$, to make entry profitable, even if the incumbent firms with inferior cost conditions earn positive profits.

With large enough $n$ and (20), two types of equilibrium market forms, $\tilde{B}_k$ and $\tilde{A}_i$, coexist. What can we say about the values of $k$ and $l$? The answer is given by the following proposition.

**Proposition 4** Under Assumptions 1, if both $\tilde{B}_k$ and $\tilde{A}_i$ are equilibrium market forms, then

$$l = k \quad \text{or} \quad l = k + 1. \quad (22)$$

**Proof.** See the Appendix. □

This somewhat contradicts our intuition because it seems possible that both monopoly by a firm with low marginal cost and competition by a large number of firms with high marginal costs form industry equilibria. But reflection tells us that if $l$ firms with the same constant marginal cost form an industry equilibrium, it seems natural that a firm with some advantage can enter a market with $l-1$ small firms. Then no small firm can enter into the resulting market with $(l-1) + 1 = l$ firms, because after the entry, the number of small firms will be the equilibrium number $l$. This is the case for $l = k$.

We can now compare the equilibrium prices and quantities of the industry realized under industry equilibrium $A \in \tilde{A}_i$ and $B \in \tilde{B}_k$.

**Proposition 5**

Under Assumption 1, if both $\tilde{B}_k$ and $\tilde{A}_i$ are equilibrium market forms, then for $l = k$

$$\sum_{i \in A} x_i^A < \sum_{i \in B} x_i^B \quad \text{and} \quad \rho_i^A > \rho_i^B. \quad (23)$$

And for $l = k + 1$

$$\sum_{i \in A} x_i^A \leq \sum_{i \in B} x_i^B \quad \text{and} \quad \rho_i^A \leq \rho_i^B. \quad (24)$$

According as

$$\alpha - \beta_i^2 (k+2)(\beta - \beta_i). \quad (25)$$

**Proof.** See the Appendix. □

The result for $l = k$ is not trivial. Since by (4), the effect of a reduction of some firm's marginal cost on the other firms' equilibrium quantities is negative.

To sum up, when $l = k$, with respect to economic efficiency, market form $\tilde{B}_k$ is more efficient since the price is lower, the supply larger and the average cost lower. But if $A \in \tilde{A}_i$ is realized as an industry equilibrium, this efficiency cannot be enjoyed since the entry of firm 1 is blocked.

Indeed, the implication depends in part on the assumption that average costs are falling. The larger the number of incumbent firms, the larger
the average costs the industry must incur.

The above result means that if there are different types of producers in an industry, different types of industry equilibria may emerge depending on the initial state of the industry. This is not a novel idea when the demand is small so that only monopolies are feasible. Two different types of firms can attain monopoly equilibrium without attracting the other into the market and monopoly by a firm with inferior cost condition may persist.

4. CONCLUDING REMARKS

In this paper, we have seen the effect of asymmetry on the structure of an industry. The analysis suggests a possibility of multiple of equilibrium market forms which are related to the initial states of the market. This relationship will be an interesting issue in considering sequential entry. (4) An equilibrium market form may be "unstable" in the sense that only small number of initial states and entry sequences can attain it, whereas another one may be fairly "stable".

Our analysis was made for a special class of industries. It is inevitable to consider such special cases since by the very nature of asymmetry, there are numerous situations which require individual considerations. Thus, general results on asymmetric industries can only be obtained by classifying the industries with respect to the types of firms in the industry and the demand for the industry product(s). In this sense, the classification of industries according to the fundamental structural characteristics is far more important for asymmetric industries than for symmetric ones. This is a theme to be studied.

APPENDIX

Proof of Proposition 4. For \( u^*_i(B \cup \{i\}) > u^*_i(B \cup \{i\}, k=1 \text{ means } l=1 \). Let \( k \geq 2 \). By definition, \( k+1 > k \geq k \) for \( k \geq 2 \) and \( l+1 > k \geq l \).

Then by (19), we have

\[
l+1 > k_1 > k_0 > k_2 \geq k
\]

and thus \( l+k \) by the minimality of \( k+1 \).

But by (14) and (18),

\[
k_0 - k_2 = \frac{\beta - \beta_1}{\sqrt{ab}} < 1.
\]

Thus \( k_2 + 1 > k_0 \) and hence \( k+2 \geq l+1 \) or \( k+1 \geq l \).

Since \( l \geq k \), we have \( l=k \) or \( l=k+1 \). \( \square \)

Proof of Proposition 5. First consider the case \( k=1 \). Let \( x = (x_2, x_3, \ldots, x_{n-1}) \) be the equilibrium quantities under \( A \) and \( y = (y_1, \ldots, y_l) \) under \( B \). By (5), we have

\[
k+1 \sum_{i=2}^{k} x_i = \frac{k(a - \beta)}{b(k+1)} < \sum_{i=1}^{l} y_i
\]

\[
= \frac{(\alpha - \beta_1) + (k-1)(\alpha - \beta)}{b(k+1)}.
\]

Since the prices are positive, we have (23).

Next let \( k+1 = l \) and \( x = (x_2, \ldots, x_{n-2}) \) be the equilibrium quantities under \( A \) and \( y = (y_1, \ldots, y_l) \) under \( B \). By (5)

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(4) See, for example, Caves and Porter (1977), Bernheim (1984), Eaton and Ware (1987) and Friedman and Thisse (1994) for discussions of such relationship in the context of entry deterrence.
\[ \sum_{i=2}^{k+2} x_i = \frac{(k+1)(a-\beta)}{b(k+2)} \] and
\[ \sum_{i=1}^{k} y_i = \frac{a-\beta_1 + (k-1)(a-\beta)}{b(k+1)}. \]
Calculation yields
\[ \sum_{i=2}^{k+2} x_i - \sum_{i=1}^{k} y_i = \frac{a-\beta-(k+2)(\beta-\beta_1)}{b(k+1)(k+2)}. \]
Thus (24) as (25).

References