Trade and Industrial Policies
under International Interdependence
―Essays on Strategic Export Subsidy Policy
国際相互依存下の貿易・産業政策
―戦略的輸出補助金政策に関する研究

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Trade and Industrial Policies under International Interdependence
- Essays on Strategic Export Subsidy Policy

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Chapter 1

Introduction

1.1 Motivation

The 21st century will witness rapid globalization with the world becoming increasingly interdependent in terms of the flow of trade, direct investment, technology, and information and knowledge. The worldwide technology diffusion, economic liberalization, and regulation relaxation will accelerate the international transactions of goods and services, and deepen the strategic interactions among nations. The governments’ policy decisions on trade, investment and environment will no longer be set separately, but will be strategically dependent on the reactions of other countries. This thesis aims to explore the pervasive influence of globalization and international economic interdependence on trade policy implementation. Throughout the thesis, I would like to clarify the economic effects of the governments’ strategic decision-making on firms’ actions, industrial profits, national welfare, and world welfare when countries compete in a globally interdependent world.

Borderless economic activities make the firms’ investment and production decisions more complicated. The firms can not only move overseas, but also allow foreign shareholding of their stocks. Firms’ diversified location choices and ownership structures also affect the governments’ strategic policy decisions. Using game theoretic approach, this thesis reexamines strategic export subsidy policies from the following viewpoints:

- firm’s relocation ability across the country.
- international cross shareholding of the firm’s stocks.
- separation of ownership and management.

The above three topics concern the firm’s external investment behavior and internal structure. This thesis elucidates how the above different setups affect the governments’ subsidization incentives and what is the optimal policy in consideration of national and world benefits. Additionally, I discuss the importance of international coordination and try to clarify what kind of coordinated policy and behavioral harmonization is necessary from the viewpoint of world welfare maximization. This study is expected to be a path-breaking research endeavor with regard to the institution-building for international coordination.
1.2 Strategic Trade Policy Theory

The discussions throughout the thesis are based on the standard strategic trade policy theory that originated in the 1980s. Let me first briefly review its development. The traditional trade theory focused on comparative advantage and productive factor endowment with constant returns to scale and perfect competition. However, it did not effectively explain phenomena such as intra-industry trade among the developed countries and the trade flows in the empirical investigations. The new trade theory stressing on increasing returns, imperfect competition, and product differentiation made remarkable progress toward the end of the 1970s and explored new explanations for modern trade analyses (see Helpman and Krugman (1985)).

With rapid growth in the firms globalization activities, the international community became increasingly interdependent. Strategic interactions in oligopoly emerged as an important element in analyzing the trade policies. At the beginning of the 1980s, the development of modern industrial organization theory and game-theoretic models led to the birth of strategic trade policy theory. Its simple and clear-cut approach showed new implications in a wide array of policy considerations and provided a new perspective on the understanding of market practices and policy formulations.

In definition, strategic trade policy refers to the policy that affects the outcomes of strategic interactions between firms in an actual or potential international oligopoly (see Spencer and Brander (2008)). Research in strategic trade policy was initiated by Brander (1981), who analyzed intra-industry trade with identical commodities, and was stylized by Brander and Spencer (1985), the well-known third-market model. Some other pioneering researches include Brander and Spencer (1984a), Spencer and Brander (1983), Dixit (1984), and Eaton and Grossman (1986).

In contrast to the traditional trade theory that advocates free trade, strategic trade policy theory provided new arguments for an interventionist trade policy. The representative model, Brander and Spencer (1985) shown in the next chapter, revealed the government’s unilateral incentive to subsidize its domestic exports since strategic subsidization gives its exporter a cost advantage and thereby shifts profits from the foreign firm toward the enhancement of national welfare in the oligopolistic market. Subsidy policy is never optimal for national welfare maximization in the traditional trade theory that is based on perfect competition. However, in the oligopolistic competition, it transfers the monopoly rents of foreign firms to the domestic economy and thereby improves the domestic welfare. The intervention to alter the strategic interaction between oligopolistic firms plays a great role in trade policy determination.

Strategic trade policy theory provided new insights into the real-world mercantilist policies and complex empirical investigations. The original purpose of the study is not to encourage the implementation of strategic trade policy, since it causes income distribution and protectionist trade disputes within a more general analysis framework. However,

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studying strategic trade policy helps understand the strategic behaviors in the oligopolistic market and extends the related research to multilateral trade agreements, foreign direct investment, environmental regulation policies, etc. Although strategic trade policy analysis has been applied into a wide range of contexts, some topics such as incomplete information, dynamic games, and economic growth were not yet explored to the fullest.

1.3 Outline of the Thesis

The thesis is organized as follows. Chapter 2 demonstrates the basic Brander and Spencer (1985)’s model. The strategic rent-shifting effect is clarified by using a reaction function approach. I discuss the role of cost heterogeneity among the firms in governments subsidy decisions. More detailed results are obtained under the special linear demand function in preparation for the analyses in the proceeding chapters.

Chapter 3 deals with an international capital liberalization game in which exporting countries choose either to open or not open the domestic market for capital inflow. This chapter clarifies that if the cost difference is large enough, the less productive country is indifferent toward closing or opening for inward direct investment, but the more productive country never has an incentive to open. International coordination to open markets is not always necessary in the capital liberalization game since it may deteriorate the welfare of the more productive country and worsen world welfare.

Chapter 4 develops a mixed international cross shareholding structure, which allows the shares of the firms to be not only owned by domestic residents, but also by foreign residents and foreign firms. Four additional effects are clarified to weaken the governments strategic subsidy incentives in the presence of international cross shareholding. An increase in the weight of foreign firm’s shareholding ratio facilitates collusion between the firms and raises the government’s strategic subsidy rate in the equilibrium. Moreover, the effects of subsidy competition on national welfare and world welfare are also analyzed under two special shareholding structures.

Chapter 5 studies the implication of the separation of ownership and management, under which the owner of a firm delegates the production decision making to a manager and designs an incentive contract for the manager. I elucidate the owner’s subsidization effect, which is hidden in the managerial delegation process. Strategic subsidy competition between the governments strengthens both the owners’ subsidization incentives and leads to the over-subsidization of the firms. When the firms’ delegation decisions are endogenous, each firm has no incentive to delegate a manager under governments’ intervention commitments because unilateral delegation leads to Stackelberg follower payoff.

Chapter 6 combines the analyses in Chapters 4-5 and examines the implication of international separation of ownership and management when the shares of the exporting firms are internationally owned and the owners of both firms make delegation decisions. It is shown that cross-country shareholding of the firms weakens both countries’ subsidization incentives irrespective of owners’ managerial decisions. However, managerial delegation may strengthen or weaken governments’ subsidization incentives and the result is dependent
on the cross shareholding structure. Chapter 6 also concludes the thesis as a whole.
Chapter 2
Basic Model

In this chapter, I review the pioneer work of strategic subsidy theory by Brander and Spencer (1985) (the BS model hereafter), which is the basic model throughout the thesis. Brander and Spencer (1985) developed a third market model and studied the impact of export subsidy policy in the international duopoly market. They indicated that government’s strategic subsidy is a trade promotion policy since the domestic exporter gains a cost advantage and grabs the monopoly rents from the foreign firms to improve national welfare. The welfare enhancement effect of strategic subsidization reversed the traditional advocacy for *laissez-faire* and the optimal tariff theory for welfare-improving trade restriction policy.

To see why a subsidy policy works in the profit-shifting mechanism, let me first demonstrate the BS model in details.

2.1 Model Setup

Consider a world consisting of three countries, 1, 2 and 3. There is a firm residing in each of countries 1 and 2, producing a homogeneous product, and exporting to country 3, which does not produce but only consumes the product in question.

Let $x_i (i = 1, 2)$ denote the output produced by firm $i$ and $c_i$ its marginal cost of production. Assume $c_i$ is exogenously constant and the markets of the countries are segmented. Denote $p$ as the market price in country 3, an importing country, $X = x_1 + x_2$ the total consumption, and $p = P(X)$ its inverse demand function.

International trade is modelled as a two-stage game involving governments and firms as follows. In the first stage, each government determines its country-specific export subsidy $s_i (i = 1, 2)$ simultaneously. In the second stage, after observing the subsidy rates $(s_1, s_2)$, the firms engage in quantity competition in the third market.

Given the subsidy rate $s_i$, the profit earned by firm $i$ is expressed by

$$\pi^i(x, s_i) = \{P(x_1 + x_2) - c_i + s_i\} x_i \quad (i, j = 1, 2; j \neq i),$$

where $x = (x_1, x_2)$ denotes the output profile.
For the simplicity of explanation, I define:

\[ \theta_i \equiv \frac{x_i}{X}, \quad \eta(X) \equiv -\frac{P}{XP(X)}, \quad E(X) \equiv -\frac{XP''(X)}{P'(X)}, \]

where \( \theta_i \) denotes firm \( i \)'s market share, \( \eta(X) \) the price elasticity of demand and \( E(X) \) the elasticity of the slope of the inverse demand curve. The market clearing condition requires \( \theta_1 + \theta_2 = 1 \).

**Assumption 2.1.** Given the subsidy rates \((s_1, s_2)\) set by countries 1 and 2, the following conditions are all satisfied:

1. \((A2.1.1)\) \( P(X) \) is strictly decreasing, continuously differentiable, and \( P(0) > c_i - s_i > P(+\infty) \) for \( i = 1, 2 \), where \( P(0) = \lim_{X \to +0} P(X) \) and \( P(+\infty) = \lim_{X \to +\infty} P(X) \).
2. \((A2.1.2)\) Each firm’s profit function \( \pi_i(x, s_i) \) is strictly concave in its own output \( x_i \), i.e.,

\[
\frac{\partial^2 \pi_i(x, s_i)}{\partial x_i^2} = P'(X)(2 - \theta_i E(X)) < 0.
\]

3. \((A2.1.3)\) \( c_i - s_i < p_i^m = P(x_i^m(s_i)) \) where \( x_i^m(s_i) = \arg\max_{x_i} \pi_i(x_i, 0, s_i) \).

\((A2.1.1)\) implies that each firm, when it is a monopolist in the third country market, has an incentive to produce a strictly positive output. \((A2.1.2)\) implies that the profit-maximizing output of each firm given the rival’s, if it ever proves to be positive, is characterized by the first-order condition (the FOC hereafter). And lastly \((A2.1.3)\) ensures that neither firm can become a monopolist when the rival has an incentive to enter the market given the monopoly output and price. Note that Assumption 2.1 is sufficient to assure the existence of a Cournot-duopoly equilibrium.

Solving from the second stage, firm \( i \)'s reaction function, denoted by \( r^i(x_j, s_i) \) is a solution to the FOC for maximizing (2-1) with respect to its own output as below.

\[
0 = \frac{\partial \pi_i(r^i(x_j, s_i), x_j, s_i)}{\partial x_i} = P \left( r^i(x_j, s_i) + x_j \right) - c_i + s_i + r^i(x_j, s_i)P' \left( r^i(x_j, s_i) + x_j \right),
\]

(2-2)

where the second-order condition (the SOC hereafter) is ensured by \((A2.1.2)\).

For the following analyses, I assume Hahn’s stability condition is satisfied.

**Assumption 2.2.** Each firm’s output is mutually a strategic substitute to the other’s, i.e., \( P'(X) + x_i P''(X) < 0 \), or alternatively, \( 1 - \theta_i E(X) > 0 \) (\( i = 1, 2 \)).

Strategic substitution holds for (i) nonconvex inverse demand function \((E(X) \leq 0)\) or (ii) a sufficiently small market share in convex demand function. On the other hand, if \( 1 - \theta_i E(X) < 0 \), each firm’s output is a strategic complement to the other’s.\(^1\)

\(^1\)The discussion for strategic substitution and strategic complementary was shown in Bulow, Geanakoplos, and Klemperer (1985).
In view of Assumption A2.1.2 and A2.2, the following properties of the reaction function are shown below by using the implicit function theorem in (2-2).

\[ r_i(x_j, s_i) \overset{\text{def}}{=} \frac{\partial r_i(x_j, s_i)}{\partial x_j} = -\frac{1 - \theta_i E(X)}{2 - \theta_i E(X)} < 0, \] (2-3)

\[ r_i(x^*_i, s_i) \overset{\text{def}}{=} \frac{\partial r_i(x_j, s_i)}{\partial s_i} = -\frac{1}{P''(X)(2 - \theta_i E(X))} > 0. \] (2-4)

(2-3) implies that each firm’s reaction curve is downward sloping. (2-4) represents that an increase in the unit subsidy rate raises the optimal response output. The associated reaction curve of firm \( i \) is shown by \( r_i x^\prime \) in Figure 2.1. The intersection labeled \( N \) represents the equilibrium output.

**Lemma 2.1.** The absolute value of the slope of the reaction function is strictly smaller than unity, i.e., \(|r_x(x_j, s_i)| < 1\) under strategic substitution.\(^2\)

The above Lemma ensures that an equilibrium, if it ever exists, should be unique and globally stable under the standard Cournot output adjustment process.

Denote \( x_i^*(s) \) as firm \( i \)'s equilibrium output where \( s = (s_1, s_2) \) represents the subsidy profile. It should satisfy:

\[ x_i^*(s) = r_i \left( x_j^*(s), s_i \right) \quad \text{for } i, j = 1, 2; j \neq i. \] (2-5)

Accordingly, \( X^*(s) \overset{\text{def}}{=} x_1^*(s) + x_2^*(s) \) denotes the equilibrium total output, \( P^*(s) \overset{\text{def}}{=} P(X^*(s)) \) the associated equilibrium price, and \( \pi_i^*(s) \overset{\text{def}}{=} \pi^i \left( x_i^*(s), x_j^*(s), s_i \right) \) the equilibrium profit of firm \( i \).

---

\(^2\)By using (2-3), \( r_x(x_j, s_i) + 1 = \frac{1}{2 - \theta_i E(X)} > 0 \) is satisfied.
2.2 Standard Strategic Export Subsidy Policies

BS model clarified that each exporting country has a positive incentive to subsidize its own exports. To ascertain this result, I first undertake comparative statics on the subsidy-ridden duopoly equilibrium with respect to a change in the export subsidy rate set by either exporting country. Differentiation of (2-5) with respect to $s_i$ yields:

$$
\left(\frac{1}{-r^i_x(x^*_i, s_j)} \right) \left( \frac{\partial x^i(s)/\partial s_i}{\partial x^*_i(s)/\partial s_i} \right) = \left( \frac{r^i_x(x^*_i, s_i)}{0} \right).
$$

(2-6)

Using (2-3),

$$
\Delta^B \overset{\text{def}}{=} 1 - r^i_x(x^*_i, s_i) r^j_x(x^*_j, s_j) = \frac{3 - E}{(2 - \theta_i E)(2 - \theta_j E)} > 0,
$$

(2-7)

where the denominator is positive in view of the concavity of profit function by Assumption A2.1.2, and the numerator is also positive since the familiar stability condition holds. Then (2-6) yields:

$$
\frac{\partial x^i(s)}{\partial s_i} = \frac{r^i_x(x^*_j(s), s_i)}{\Delta^B} = -\frac{2 - \theta_j E(X)}{P'(X)(3 - E(X))} > 0,
$$

(2-8)

$$
\frac{\partial x^j(s)}{\partial s_i} = \frac{r^j_x(x^*_i(s), s_j)}{\partial x^*_i(s)/\partial s_i} = \frac{1 - \theta_j E(X)}{P'(X)(3 - E(X))} < 0,
$$

(2-9)

where use was made of (2-3), (2-4) and (2-7). An increase in $s_i$ shifts firm $i$’s reaction curve outward and the equilibrium point changes to $N^s$. Subsidization lowers the domestic marginal cost, so in the equilibrium firm $i$’s output increases and firm $j$’s output decreases shown in Fig. 2.1.

The total output and the market price change as follows.

$$
\frac{\partial X^*(s)}{\partial s_i} = (1 + r^i_x(x^*_i, s_j)) \frac{\partial x^*_i(s)}{\partial s_i} = \frac{-1}{P'(X)(3 - E(X))} > 0,
$$

$$
\frac{\partial P^*(s)}{\partial s_i} = P'(X^*(s)) \frac{\partial X^*(s)}{\partial s_i} = \frac{-1}{3 - E(X)} < 0.
$$

The equilibrium profit of each firm should change as expressed by:

$$
\frac{\partial \pi^i(s)}{\partial s_i} = x_i P'(X^*) \frac{\partial x^*_i(s)}{\partial s_i} + x_i > 0,
$$

(2-10)

$$
\frac{\partial \pi^j(s)}{\partial s_i} = x_j P'(X^*) \frac{\partial x^*_i(s)}{\partial s_i} < 0,
$$

(2-11)

where use was made of (2-2) (2-8) and (2-9).

See Dixit (1986).
In Fig. 2.1, an increase in $s_1$ shifts the equilibrium point from $N$ to $N^*$. Country 1’s unilateral export subsidization allows the domestic firm to attain a higher production level as a Stackelberg leader, forcing the foreign firm to respond as a follower. In equilibrium, firm 1’s profit (represented by the isoprofit curve) increases from $\pi^1$ to $\pi^{1s}$, while firm 2’s profit decreases from $\pi^2$ to $\pi^{2s}$. Fig. 2.1 shows the standard profit shifting effect of export subsidization in the BS model.

Fig. 2.1: Rent Shifting Effect of Export Subsidization
2.3 Welfare Effect of Export Subsidies

The government of each exporting country aims to maximize the total surplus consisting of the domestic firm’s private profit minus the subsidy expenses.

\[ W_i(s) = \pi_i^*(s) - s_i x_i^*(s) \quad (i = 1, 2). \]

The social welfare function is equivalent to the subsidy-exclusive profit function of the domestic firm, since the increased subsidy expenses are canceled by the cost reduction of the firm. Export subsidy is regarded as an income redistribution from taxpayers to firm’s shareholders. In practice, public finance for subsidy expenses incurs distortion costs on the economy. Here I rule out the domestic distortions on subsidy transfer, assuming that the opportunity cost of a dollar of public funds equals unit.\(^4\)

The concavity of the welfare function is assumed as below.

**Assumption 2.3.** The welfare function of each exporting country is strictly concave in the own export subsidy rate.

\[ \frac{\partial^2 W_i(s)}{\partial s_i^2} < 0. \]

The governments of both exporting countries independently decide the own export subsidy rates by foreseeing their resulting effects on the market performance. Each country’s reaction function is now given by:

\[ R^i(s_j) := \arg \max_{s_i} W_i(s). \]

The strategic interdependence between the two exporting countries’ governments is governed by the shape of each reaction curve. Its slope is given by:

\[ R_i^s(s_j) := \frac{\partial R^i(s_j)}{\partial s} = -\frac{\partial^2 W_i(R_i^s(s_j), s_j)}{\partial s_i \partial s_j} = \frac{\partial^2 W_i(R_i^s(s_j), s_j)}{\partial s_i^2}, \]

for \( i, j = 1, 2 (j \neq i) \). The signum of the slope is generally indeterminate, but insofar as the demand function is linear, one can show that each country’s export subsidy is a strategic substitute to the other’s, i.e. \( \frac{\partial^2 W_i(s)}{\partial s_j \partial s_i} < 0 \).

By virtue of the FOC for welfare maximization by each exporting country’s government, the following equation holds at equilibrium.

\[ 0 = \frac{\partial W_i(s)}{\partial s_i} = x_i P'(X) \frac{\partial x_i^*}{\partial s_i} - s_i \frac{\partial x_i^*}{\partial s_i} \quad \text{(2-12)} \]

\(^4\)Neary (1994) introduced a distortion cost parameter in the welfare function as \( W_i = \pi_i - \delta s_i x_i \), where \( \delta \geq 1 \) represents the subsidy transfer distortion. The equilibrium subsidy is positive only when \( 1 \leq \delta < \frac{2}{3} \). Otherwise, if \( \delta > \frac{2}{3} \), taxing the exports is optimal.
Denote the equilibrium subsidy rate as $s^B_i$ and the superscript $B$ denotes the equilibrium values in the BS model.

$$s^B_i = x^*_i P'(X) r^*_i = -x^*_i P'(X) \frac{1 - \theta_j E(X)}{2 - \theta_j E(X)} > 0,$$

where use was made of (2-8) and (2-9). Insofar as the analyses are confined in Assumption 2.2, each exporting country has a positive incentive to subsidize the domestic firm.\(^5\) Due to the rent shifting effect, domestic firm’s profit gain outweighs the subsidy expenses, so export subsidy improves the national welfare at the end. Brander-Spencer subsidy result is summarized into the following Proposition.

**Proposition 2.1** (Brander and Spencer (1985)). *At the non-cooperative export subsidy game equilibrium, each exporting country’s government sets a strictly positive rate of export subsidy at the Nash equilibrium, i.e., $s^B_i > 0 (i = 1, 2)$ when each firm’s export is a strategic substitute to the other’s in the Cournot competition.*

When each country maximizes its own welfare at $s_i = R^i(s_j)$, either country’s marginal welfare with respect to its rival country’s export subsidy rate can be evaluated as below. Differentiating $W_j(s)$ with respect to $s_i$ yields

$$\frac{\partial W_j(s)}{\partial s_i} = x^*_j P'(X) \frac{\partial x^*_i}{\partial s_i} - s_j \frac{\partial x^*_j}{\partial s_i}$$

$$= x^*_j P'(X) \frac{\partial x^*_i}{\partial s_i} - x^*_j P'(X) r^*_i \frac{\partial x^*_j}{\partial s_i}$$

$$= \Delta^B x^*_j P'(X) \frac{\partial x^*_i}{\partial s_i} < 0,$$

where use was made of (2-8), (2-9), and (2-13).

**Lemma 2.2.** *At the non-cooperative export subsidy game equilibrium, an increase in the subsidy rate by either country worsens the other exporting country’s welfare, i.e., $\frac{\partial W_j(s)}{\partial s_i} < 0$ for $i, j = 1, 2 (j \neq i)$.*

The third country is a consuming country without production. Its welfare is expressed by the domestic consumption surplus as follows.

$$W_3(s) = \int_0^{X^*(s)} P(z)dz - P(X^*(s))X^*(s).$$

An increase in either country’s subsidy rate expands the total exports to the third country and thereby improves its terms of trade.

$$\frac{\partial W_3(s)}{\partial s_i} = -X^*(s)P'(X) \frac{\partial X^*(s)}{\partial s_i} > 0.$$  

\(^5\)Collie and de Meza (1986) and Bandyopadhyay (1997) discussed that the positive subsidization incentive is largely dependent on the elasticities of demand.
The third country is always benefited from export subsidization policy since subsidization pushes the market price toward the competitive level.

World welfare is given by the sum of three countries’ welfare.

\[ W_T(s) = \sum_{i=1}^{3} W_i(s) = \int_0^{X^*(s)} P(z)dz - c_1x_1^*(s) - c_2x_2^*(s). \] (2-16)

Differentiating with \( s_i \) yields

\[ \frac{\partial W_T(s)}{\partial s_i} = (P - c_i)\frac{\partial x_i^*(s)}{\partial s_i} + (P - c_j)\frac{\partial x_j^*(s)}{\partial s_i}. \]

When \( c_i \) is large enough close to \( P \), the first term in the above equation can be neglected. Since the second term is negative, subsidizing firm \( i \), the inefficient firm lowers the total production efficiency and worsens world welfare. Otherwise, when \( c_j \) is large enough, world welfare improves.

Furthermore, by using (2-14) and (2-15), the above equation can be rewritten as below.

\[ \frac{\partial W_T(s)}{\partial s_i} = \frac{\partial W_j(s)}{\partial s_i} + \frac{\partial W_3(s)}{\partial s_i} = \Delta^B x_j P'(X) \frac{\partial x_i^*(s)}{\partial s_i} + X P'(X) \frac{\partial X^*}{\partial s_i} = \Delta^B x_j P'(X) \frac{\partial x_i^*(s)}{\partial s_i} - (1 + R_j)XP'(X)\frac{\partial x_i^*(s)}{\partial s_i}. \]

Since \( \frac{\partial x_i^*(s)}{\partial s_i} > 0 \) in (2-8), simple calculation yields

\[ \frac{\partial W_T(s)}{\partial s_i} > 0 \iff \theta_j < 1 - \frac{1}{\Delta^B}, \]

which follows (2-9) and the market share definition of \( \theta_i \).

In the case of linear demand function discussed in Section 2.5, it follows that subsidizing firm \( i \) improves world welfare if and only if \( \theta_j < \frac{2}{3} \), or alternatively, \( \theta_i > \frac{1}{3} \), as shown in Lahiri and Ono (1988). Put differently, subsidizing a relatively inefficient firm, whose market share is lower that \( \frac{1}{3} \) deteriorates world welfare.

When the cost conditions are symmetric that \( c_1 = c_2 = c \), (2-16) shows that an increase in either country’s subsidy rate unambiguously improves world welfare.

\[ \frac{\partial W_T(s)}{\partial s_i} = (P - c)\frac{\partial X^*}{\partial s_i} > 0. \]

The above equation shows the allocation effect only. An increase in the subsidy rate raises the total output and reduces the welfare loss in the oligopolistic industry due to the wedge between the market price and the marginal cost. Thus, the world allocation efficiency is improved as a whole.

**Proposition 2.2.** Under symmetric cost function, subsidizing the exports improves world welfare in a third-market model.
2.4 Cost Asymmetry and Subsidization Incentives

When the exporting firms exhibit cost heterogeneity, de Meza (1986) examined how the subsidization incentives relate with the cost asymmetry. The subsidy differential can be obtained from (2-13).

\[
s_i^B - s_j^B = x_i P'(X) r_i^2 - x_j P'(X) r_j^2
\]

\[
= -x_i P'(X) \frac{1 - \theta_i E(X)}{2 - \theta_i E(X)} + x_j P'(X) \frac{1 - \theta_j E(X)}{2 - \theta_j E(X)}
\]

\[
= -x_i P'(X)(1 - \theta_i E(X))(2 - \theta_i E(X)) + x_j P'(X)(1 - \theta_i E(X))(2 - \theta_j E(X))
\]

\[
= -(x_i - x_j) P'(X)(1 - \theta_i E(X)(1 - \theta_j E(X)) + \frac{x_i P'(X)(1 - \theta_i E(X)) + x_j P'(X)(1 - \theta_j E(X))}{(2 - \theta_j E(X))(2 - \theta_i E(X))}
\]

which follows (2-3). Note that

\[
x_i P'(X)(1 - \theta_j E(X)) = x_j P'(X)(1 - \theta_i E(X))
\]

is satisfied. In view of (2-2), it yields

\[
x_i - x_j = \frac{(c_i - c_j) - (s_i^B - s_j^B)}{P'(X)}.
\]

Substitution of the above equation into \(s_i^B - s_j^B\) yields

\[
s_i^B - s_j^B = -(c_i - c_j) \frac{1 - \theta_i E(X)(1 - \theta_j E(X)) + 1}{2 - E(X)}.
\]

In view of Assumption 2.2 that \(1 - \theta_k E > 0 (k = i, j)\), the subsidy differential is inversely related to the cost differential.

\[
s_i^B - s_j^B \propto -(c_i - c_j),
\]

which shows that the more efficient the domestic firm, the greater the subsidy differential. The result is summarized into the following Proposition.

**Proposition 2.3.** (de Meza (1986)) The low-cost country has the incentive to offer the higher subsidies when each firm’s export is a strategic substitute of the other’s.

In view of (2-2), given the rival’s output, the low-cost firm benefits both from the productive advantage and the larger market shares in the Cournot competition. Subsidizing the efficient firm cuts the production cost further down and thereby makes the firm secure larger market shares so as to improve national welfare. Hence, the country with the low-cost firm results in a greater equilibrium subsidy rate than the rival country. The above subsidy differential result shown in de Meza (1986) is important in explaining the economic intuition in the proceeding chapters.
2.5 Linear Demand Case

The following linear inverse demand function is assumed so as to derive explicit results throughout the ongoing analyses.

\[ p = P(X) = a - X, \]

where \( a \) is a positive constant and \( a > c_i (i = 1, 2) \).

Each firm non-cooperatively chooses its output so as to maximize its profit given by

\[ \pi_i(x, s_i) = (a - x_i - x_j - c_i + s_i) x_i. \] (2-18)

Each firm’s reaction function for profit maximization is given by

\[ r^i_B(x_j, s_i) := \text{arg max}_{x_i} \pi_i(x, s_i) = \frac{1}{2}(a - c_i + s_i - x_j). \] (2-19)

Clearly, each firm’s optimal response output is a strategic substitute to the rival’s as \( r^i = -\frac{1}{2} < 0 \) and the condition for local stability is obviously satisfied.\(^7\)

Solving for the equilibrium output yields

\[ x^*_B(s) = \frac{\beta_i + 2s_i - s_j}{3}, \] (2-20)

where \( \beta_i := a - 2c_i + c_j > 0 (i, j = 1, 2 : j \neq i) > 0 \) for positive quantities under duopoly.

Accordingly, the equilibrium total output can by expressed as below.

\[ X^*_B(s) = \frac{1}{3} \left( 2a - \sum_{i=1,2} (c_i - s_i) \right) = \hat{X} \left( \sum_{i=1,2} (c_i - s_i) \right). \]

Notice that the equilibrium total output depends only on the sum of the subsidy-inclusive unit costs over the industry.\(^8\)

\(^6\)The SOC for each firm’s profit maximization is satisfied, i.e.,

\[ \frac{\partial^2 \pi_i(x, s_i)}{\partial x_i^2} = -2 < 0. \]

\(^7\)The following condition ensures local stability of the second-stage Nash equilibrium under the adjustment process described by

\[ \dot{x}_i = \alpha_i \{ r^i (x_j, s_i) - x_i \}. \]

\(^8\)In fact, summation of the FOCs for profit maximization over the firms give rise to (See Varian (1992))

\[ 0 = 2P(X) + XP'(X) - \sum_{i=1,2} (c_i - s_i) \quad \Rightarrow \quad X = \hat{X} \left( \sum_{i=1,2} (c_i - s_i) \right). \]
The comparative statics results yield
\[
\frac{\partial x^*_{iB}(s)}{\partial s_i} = \frac{2}{3} > 0 \quad , \quad \frac{\partial x^*_{jB}(s)}{\partial s_i} = -\frac{1}{3} < 0. \tag{2-21}
\]
Substituting (2-5) into (2-18) yields each firm’s equilibrium profit.
\[
\pi^*_{iB}(s) = \pi^i(x^*_{iB}(s), x^*_{jB}(s), s_i) = \frac{(\beta_i + 2s_i - s_j)^2}{9}.
\]
Exporting country’s welfare is expressed as below.
\[
W_i(s) := \pi^*_{iB}(s) - s_i x^*_{iB}(s) = \frac{(\beta_i + 2s_i - s_j)(\beta_i - s_i - s_j)}{9}. \tag{2-22}
\]
Each country sets its subsidy rate to maximize the net surplus in (2-22). The reaction function in the first-stage is derived as\(^9\)
\[
R^B_{ij}(s_j) := \arg \max_{s_i} W_i(s) = \frac{1}{4}(\beta_i - s_j). \tag{2-23}
\]
The equilibrium subsidy rate of country \(i\), \(s^B_i\) which is given by
\[
s^B_i = \frac{4\beta_i - \beta_j}{15}. \tag{2-24}
\]
The associated equilibrium output and profit of each firm is expressed by
\[
\hat{x}^B_i = x^*_{iB}(s^B) = \frac{2(4\beta_i - \beta_j)}{15}, \tag{2-25}
\]
\[
\hat{\pi}^B_i = \pi^*_{iB}(s^B) = (x^*_{iB}(s^B))^2 = \left(\frac{2(4\beta_i - \beta_j)}{15}\right)^2.
\]
The associated equilibrium welfare of each exporting country is expressed by
\[
\hat{W}^B_i = W_i(s^B) = 2 \left(\frac{4\beta_i - \beta_j}{15}\right)^2. \tag{2-26}
\]
When the two exporting governments engage in subsidy competition, both fall into a prisoners’ dilemma with lower welfare than \textit{laissez faire}, i.e., \(W_i(s^B) < W_i(0)\) under symmetric cost conditions. World welfare, however, rises since the gain to consumers in the importing country more than offsets the loss in welfare to the exporting countries.
\(^9\)The SOC for each country’s welfare maximization is satisfied, i.e.,
\[
\frac{\partial^2 W_i(s)}{\partial s_i^2} = -\frac{4}{9} < 0.
\]
2.6 Discussion

The main contribution in the BS model lies in that export subsidization may enhance the exporting country’s welfare in imperfectly competitive market in the absence of interdependence with the other sectors in the economy. The model is characterized by the assumption that all the outputs are sold in a third country, which does not produce but only consume the product in question. The national welfare is simply constituted by the sum of producer surplus and government surplus, or alternatively, the home firm’s subsidy-exclusive profit. The simplified setup removes the consideration for the consumption effects of strategic subsidization. When moving the consuming market from the third country to the exporting country, it is straightforward to obtain that the exporting country’s government has more incentives to subsidize the domestic products due to the dual positive effects, i.e., the profit shifting effect and terms-of-trade improvement effect. However, the optimal policy for the foreign imports is not evident since taxation increases the tax revenue and causes consumer surplus loss as well. Brander and Spencer (1984a,b) showed whether the optimal policy is import tax or import subsidy is dependent on the elasticity of the slope of the inverse demand function, i.e., the value of $E(X)$.

The third market and home market analyses can be combined into the reciprocal trade model, the basic structure of which is built by Brander (1981) and elaborated by Brander and Krugman (1983). There are two countries, and the markets of these two countries are segmented. The outputs of each firm not only supply to the domestic market, but also export to the foreign market. Each firm makes strategic production decisions toward domestic and foreign markets separately. Each government is allowed to subsidize the domestic sales and domestic exports and levy tax on the foreign imports. Then the third-market and home-market analyses are integrated into one model, as shown in Dixit (1984). The reciprocal trade model is useful in synthesizing varied trade policies; however, it lacks clarity due to the complicated structure in analysis. Third market model is somewhat simple, but is widely applied when considering industrial profits and consumer surplus separately.

Relaxing the special conditions in the BS model, a number of papers examined how different frameworks lead to alternative implications for the modified results. Markusen and Venables (1988) indicated that the rent shifting effects of export subsidies become weak when Cournot markets are integrated. Under the same assumption of integrated markets, Horstman and Markusen (1986) showed that welfare enhancing export subsidies may bring inefficient entry in the presence of decreasing average cost. In the framework of a perfectly competitive third-market, Bagwell and Staiger (2001) discussed that the optimal export policy is positive subsidy if the politically motivated governments weigh heavily on the industrial profits. The result in the BS model is also challenged by Eaton and Grossman (1986), who indicated that the so-called rent extraction effects of export subsidization hinges on the market structure of quantity competition à la Cournot with zero conjectural variations. The optimal export subsidy may become negative under Bertrand
Relaxing the assumption of entry restrictions, Dixit and Grossman (1986) pointed out that due to the lack of information for the government, free trade is the best policy when there are more than two oligopolistic export industries. However, insofar as we are confined into the original BS model framework and the long-run view of competition according to Kreps and Scheinkman (1983), one cannot neglect the exporting country’s incentive to subsidize its own domestic firms.

The succeeding chapters explore the effects of governments’ strategic subsidy policies in view of the firms’ location choice, the international cross shareholding structure and the separation of ownership and management in the framework of the BS model.

\[\text{See Appendix 2.A.}\]
Appendix

2.A Product Differentiation and Conjectural Variations

Eaton and Grossman (1986) represented a third-market model incorporating a general conjectural variations model when products are differentiated. They indicated that the so-called rent extraction effects of export subsidization hinges on the market structure of quantity competition à la Cournot with zero conjectural variations. The optimal export subsidy may become negative when firms compete in a Bertrand fashion, since taxation raises the output price and leads to higher before-export-tax profit. Bilateral intervention results in a win-win situation, improving both countries’ welfare. Maggi (1996) incorporated the Bertrand and Cournot outcome in a capacity-price competition model and showed that a unilateral single-rate small capacity subsidy is a welfare-improving policy as in Brander and Spencer (1985). The governments have incentives to grant capacity subsidies regardless of the mode of competition.

Assume that each firm produces a substitute goods to each other. The inverse demand function is $P_i(x_i) (i = 1, 2)$ satisfying $\frac{\partial P_i(x_i)}{\partial x_i} < 0$ and $\frac{\partial P_i(x)}{\partial x_j} < 0$. The profit function of firm $i$ is given by

$$\pi_i(x_i, s_i) = (P_i(x) - c_i + s_i)x_i.$$  

Conjecture Variation: Denote $\lambda_i = \frac{dx_i}{dx_j}$ as firm $i$’s conjectural variation, which represents firm $i$’s conjecture about the rival firm’s output change in response to its own output.

The FOC for profit maximization yields

$$0 = \frac{\partial \pi_i}{\partial x_i} + \lambda_i \frac{\partial \pi_i}{\partial x_j} = P_i(x) - c_i + s_i + x_i \left( \frac{\partial P_i(x)}{\partial x_i} + \lambda_i \frac{\partial P_i(x)}{\partial x_j} \right). \quad (2.A-1)$$

Each firm maximizes its profit on the conjecture of the rival firm’s output change. Using $\lambda_i$ can capture different kinds of equilibria in a unified model.

- Competitive conjecture (own price $P^i$ unchanged): $\lambda^i = -\frac{\partial P_i(x_i)/\partial x_i}{\partial P_i(x)/\partial x_j}$.
- Cournot conjecture (prices adjusted): $\lambda_i = 0$. 

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• Bertrand conjecture (rival firm’s price \( P^j \) unchanged): \( \lambda_i = -\frac{\partial P_i(s)}{\partial x_i} \)

• Stackelberg Leader: \( \lambda_i = r^j_x \) (in Eq. (2-3)).

In view of (2.A-1) and (2-9), welfare maximization yields

\[
0 = \frac{\partial W_i(s)}{\partial s_i} = -x_i \lambda_i \frac{\partial P_i(.)}{\partial x_j} \frac{\partial x_i^*}{\partial s_i} + x_i \frac{\partial P_i(.)}{\partial x_j} \frac{\partial x_i^*}{\partial s_i} - s_i \frac{\partial x_i^*}{\partial s_i}
\]

\[
= \left( -x_i \lambda_i \frac{\partial P_i(.)}{\partial x_j} + x_i r^j_x \frac{\partial P_i(.)}{\partial x_j} - s_i \right) \frac{\partial x_i^*}{\partial s_i},
\]

where \( r^j_x \) represents the slope of firm \( j \)'s reaction function, i.e., firm \( j \)'s actual output changes in response. The optimal subsidy is shown as below.

\[
s_i^V = -x_i (\lambda_i - r^j_x) \frac{\partial P_i(.)}{\partial x_j}.
\]

Since \( \frac{\partial P_i(.)}{\partial x_j} < 0 \), it follows that

\[
\lambda_i \gtrless r^j_x \quad \iff \quad s_i \gtrless 0.
\]

BS model clarified that government subsidization makes the own firm achieve the Stackelberg leader position in the Cournot competition. If the firm’s conjecture about rival’s output coincides with the Stackelberg result, government intervention can not increase the domestic profit further more, so free trade is optimal. This is the so-called consistent conjecture. If the firm’s conjecture about rival’s output is larger (or smaller) than the Stackelberg result, its own output under profit maximization is less (or greater) than Stackelberg equilibrium, so the government should subsidize (or taxes) the firm to attain the point.

**Cournot Solution:** Under Cournot conjecture when \( \lambda_i = 0 \), the optimal subsidy is

\[
s_i^V = x_i r^j_x \frac{\partial P_i(.)}{\partial x_j}.
\]

\( s_i^V \) is negatively proportional to \( r^j_x \), which is defined in (2-3). Hence, if the firm’s output is a strategic substitute to the rival’s, i.e., \( r^k_x < 0 (k = i, j) \), the government has a positive incentive to subsidize the exports.

\[\text{\underline{11}}\text{Under homogeneous product competition that } P^i = P^j, \text{ competitive and Bertrand conjecture take on the value } -1, \text{ which yields } p = c_i.\]
Bertrand Solution: The demand function of firm $i$’s output is denoted as $x_i(p)$, where $p = (p_1, p_2)$ is the price profile and $\frac{\partial x_i(p)}{\partial p_i} < -\frac{\partial x_i(p)}{\partial p_j} < 0$.

Each firm’s profit function is given by

$$\pi_i(p, s_i) = (p_i - c_i + s_i)x_i(p).$$

FOC for profit maximization yields

$$0 = \frac{\partial \pi_i(p, s_i)}{\partial p_i} = x_i + (p_i - c_i + s_i)\frac{\partial x_i(p)}{\partial p_i}.$$  
(2.A-3)

Bertrand conjecture yields

$$\lambda_i = -\frac{\partial P_j(x)/\partial x_i}{\partial P_j(x)/\partial x_j} = \frac{\partial x_j(p)/\partial p_i}{\partial x_i(p)/\partial p_i},$$

where each firm conjectures that its rival remains its price unchanged in response to any changes in its own price change. Putting the above $\lambda^i$ into (2.A-1) leads to the FOC of (2.A-3) in the Bertrand competition. The equilibrium price of each firm’s product is denoted as $p_i^*(s)$.

The actual respond $r_j^x$ can be shown in the following form

$$r_j^x = \frac{\partial x_j(\cdot)}{\partial p_i} \frac{\partial p_j^*(\cdot)}{\partial s_i} + \frac{\partial x_j(\cdot)}{\partial p_j} \frac{\partial p_j^*(\cdot)}{\partial s_i} = \frac{\partial x_j(\cdot)}{\partial p_i} + \frac{\partial x_j(\cdot)}{\partial p_j} \Gamma_j^p,$$

where $\Gamma_j^p = -\frac{\partial^2 \pi_j(\cdot)/\partial p_i \partial p_j}{\partial^2 \pi_i(\cdot)/\partial p_i^2}$ represents the slope of firm $j$’s reaction curve under Bertrand competition and the stability condition requires $|\Gamma_j^p| < 1$.

Thus, to examine government’s optimal policy, I obtain

$$\lambda_i - r_j^x = \left(\frac{\partial x_j(\cdot)}{\partial p_i} - \frac{\partial x_j(\cdot)}{\partial p_i} \frac{\partial x_i(\cdot)}{\partial p_j} \Gamma_j^p\right) \frac{\partial^2 \pi_i(\cdot)}{\partial p_i \partial p_j} \propto -\Gamma_j^p.$$

In view of (2.A-2), $s_i^V < 0$ if $\frac{\partial^2 \pi_i(\cdot)}{\partial p_i \partial p_j} > 0$. That is, if each firm’s product price is a strategic complementary to the rival’s, the optimal policy is export tax. $\frac{\partial^2 \pi_i(\cdot)}{\partial p_i \partial p_j} > 0$ is satisfied in the most substitute goods cases.

In summarization, firms’ competition mode affects the governments’ optimal intervention policies greatly. Quantity competition makes the optimal policy tend to be a subsidy (Brander and Spencer (1985)), while price competition tend to be a tax (Eaton and Grossman (1986)).

\[12\] Although Bertrand solution can be shown explicitly by using a direct reaction function approach, I follow Eaton and Grossman (1986) to check the relationship between the conjecture response and actual response.
Chapter 3

Capital Liberalization of Exporting Countries

3.1 Introduction

Cross-border capital flows to emerging markets have accelerated since the early 1990s. In emerging Asia, gross capital flows - both inflow and outflow, amounted to about US$830 in 2007. In sub-Saharan African countries, these have shown a dramatic fivefold increase from 2001 to 2007. Those of emerging Europe also shows a rising trend and having gained an unprecedented share of GDP in recent history.\(^1\)

Foreign direct investment (FDI) constitutes a major component of capital inflows. After the Mexican and Asian crises in the late 1990s, a number of countries imposed measures to restrict or discourage short-term capital inflows, but long-term inflows, that is, FDI, still grow steadily in most countries. Empirical papers asserted that free capital flows for FDI investment facilitate substantial economic growth and development in emerging market countries. However, in reality, FDI in some special industries may be ruled out by legislation (e.g., aircraft and steel), mandatory approvals (e.g., pharmaceutical) and administrative procedures (e.g., automobile). This chapter intends to clarify the governments’ capital liberalization incentives for FDI investment with subsidy competition between two competing exporting countries.

Given that capital liberalization is exogenous, recent voluminous works on FDI have studied firms’ strategic location choices. Without government intervention, cost difference plays an important role in firms’ location choices between domestic and foreign production. Horstman and Markusen (1992) discussed how entry-mode (export or FDI) cost difference between trade cost and fixed cost affects the endogenized market structure. Ishikawa and Komoriya (2009a) additionally focused on location-specific cost difference\(^2\) and showed the existence of multiple equilibria.

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\(^1\)See Regional Economic Outlook, IMF.

\(^2\)Ishikawa and Komoriya (2009a) assumed that for each firm, foreign production cost is always lower than domestic production cost.
However, a number of FDI studies on location choice tackle government intervention, especially tax (subsidy) competition between two asymmetric hosting countries to attract a foreign-owned multinational enterprise. Haaparanta (1996) focused on the wage difference across countries and discussed the subsidy effects on the allocation of investment level. Haufler and Wooton (1999) and Barros and Cabral (2000) considered the effect of the difference in country size and clarified that the larger country is at an advantage in attracting the firm due to agglomeration effects. The former considered the availability of multiple policy instruments (combined with import tariff or consumption tax). The latter considered domestic employment gains from FDI and showed that the small country may win the competition when the employment gain is large enough. It was shown that subsidy competition always improves the small country’s welfare and worsens the large country’s welfare independent of the firm’s location. On the basis of the above two papers, Bjørvatn and Eckel (2006) considered country size as well as the role of market structure. With a domestic firm located in the large country, the profit-shifting effect of FDI makes the small country more attractive. Fumagalli (2003) discussed the technology spillover effect through attracting FDI. The country with the less efficient firm always gains through subsidy competition since subsidy distortion is dominated by consumer gain and domestic profit gain. Hao and Lahiri (2009) examined the role of the technology level and number of firms in the host countries. Their results clarified that tax competition has no effect on a multinational firm’s location choice if trade between the firms in the host countries is possible. Besides the tax (subsidy) bidding policy, Raff (2004) considered hosting countries’ incentives to form an FTA or CU; Albornoz and Corcos (2007) compared the effects of the policies of subsidy harmonization (zero subsidy) and coordination (maximizing joint welfare) on firm’s location choice; and Davies, Egger, and Egger (2007) discussed different taxation regimes for location advantage with regard to headquarters.

The above literatures investigated the competition policies to attract MNEs to serve the hosting countries’ markets. With the exception of Janeba (1998), no study separated the host countries from domestic consumption, that is, attracting MNEs between two export-oriented host countries. Without domestic consumption, the study returns to the original implication of tax competition. Janeba (1998) took into account a third market model, in which two exporting firms can locate in either exporting country, but not a third country, the consuming country. So, the location choice is simply determined by the tax (subsidy) rates. With firms’ free mobility, each exporting country is restrained from tax (subsidy) competition because high rates of subsidies benefit the foreign firms relocating to their home country, leading to the outflow of rent. Meanwhile, taxation restrains the rent outflow but induces both firms to go abroad, leading to a loss of tax revenue. Janeba (1998) showed that the resulting equilibrium entails free trade, the well-known race-to-the-bottom result, and that mutual capital liberalization dominates mutual capital restriction.

Most studies focused on the competition policy for attracting a single MNE, and so the implications of the production technology asymmetry among the investment firms has been

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3See Dembour (2008) for a detailed survey on competition policies and firms’ location choices.
paid little attention. In the framework of the Brander and Spencer (1985) (the BS model) without firms’ mobility, de Meza (1986) and Neary (1994) showed that cost asymmetry across two exporting firms plays an important role in subsidy competition policies. Since each firm’s equilibrium output is dependent on the cost function, the subsidized low-cost firm can capture a larger market share and extract more rent in the duopolistic market. The country with relatively efficient firm results in higher equilibrium subsidy rates, ceteris paribus.\(^5\) Given a number of cost heterogenous firms in each exporting country, Long and Soubeyran (2001) showed that export tax may be optimal dependent on the Herfindahl index and demand function. With firms’ relocation ability, Janeba (1998) showed that capital liberalization nullifies the effect of cost asymmetry. Both the low-cost and high-cost countries’ subsidization incentives are dampened and laissez faire equilibrium is realized.

This chapter studies two firms’ FDI location choices between two export-oriented countries as in Janeba (1998) and focuses on the cost asymmetry between two firms. Since no study has discussed the individual government’s incentive of capital liberalization for FDI investment, this chapter endogenizes governments’ decisions on capital liberalization in the first stage. When the capitalization policy is not exogenously given, the cost-asymmetry effect is restored to play a crucial role in the exporting countries’ tax (subsidy) competition policies.

If both exporting countries can use two policy instruments, that is, capital liberalization and subsidization, the governments should take into account the strategic effect as well as the rent-shifting effect. When liberalizing capital to let the foreign firm move in, the country intends to increase the tax revenues from the foreign firm. However, they can not impose a high tax since it not only fails to attract the foreign firm, but also may lose the tax revenue from the domestic firm. Subsidization can attract the foreign firm effectively, but causes the subsidy payment flew out to the foreign country. Thus, there is no optimal trade intervention under mutual capital liberalization. However, when capital liberalization decisions are endogenized, the high-subsidization country has an incentive to restrict capital, but the low-subsidization one does not have such incentives. The key variable lies in the cost asymmetry, which determines the optimal subsidy rate. The research is interesting in that it considers the strategic relationship between the two policy instruments.

The analysis in this chapter is based on Kiyono and Wei (2007, 2008),\(^6\) which incorporated the analyses in Janeba (1998)’s analysis into an international capital liberalization model and clarified how the cost difference affects the exporting countries’ capital liberalization incentives. The rest of this chapter is organized as follows. Section 2 constructs a four-stage capital liberalization model in which the governments of the exporting countries decide on their capital liberalization at the first stage. Section 3 briefly reviews the standard strategic subsidization incentives of the BS model analyzed in the previous chapter.

\(^4\)One exception is Katayama, Lahiri, and Tomiura (2005), who examined two host countries competing for a number of cost heterogeneous firms and clarified the dispersion among the marginal costs of the firms that affects the attractiveness of a country.

\(^5\)See details in Section 2.4.

\(^6\)The earlier version is Kiyono and Wei (2002).
as the first subgame. In section 4, the effects of relocatability of the firms following Janeba (1998) are discussed as the second subgame. Section 5 examines the subgames in which one exporting country liberalizes capital and classifies the subgame equilibria into three types. Section 6 explores the subgame perfect Nash equilibria of our capital liberalization game and the implications of non-cooperative decisions of the exporting countries on the world welfare. Lastly, in section 7, the conclusions to this chapter are summarized.
3.2 Model Setup

The model is constructed under the framework of the BS model. The game, which is called the *capital liberalization game*, incorporates the following four stages of decision.

1st stage The governments of both exporting countries decide simultaneously on whether to close or open the domestic market for capital inflow from abroad.

2nd stage After observing the decisions on capital liberalization, the governments of both exporting countries simultaneously decide on the production (=export) subsidy rate.\(^7\)

3rd stage If at least one country is ready to liberalize capital, the firms in the other countries decide simultaneously where to locate their production plants, either in country 1 or 2. If both countries have decided to refuse capital inflow, there follows the next stage.

4th stage After observing the locations of production plants, both firms simultaneously decide on how much to produce and export to country 3.

Each government has two policy instruments: (i) the capital liberalization policy \(\sigma_i (i = 1, 2) \in \{C, O\}\) where \(C\) represents the policy of closing the domestic market against capital inflow from abroad and \(O\) the policy of opening the market, and (ii) the production subsidization policy \(s_i (i = 1, 2)\). In view of the first-stage decisions for \(\sigma_i\), the present game can be divided into four subgames as shown in Table 3.1. A subgame associated with capital liberalization policy profile \((\sigma_1, \sigma_2) (\in \{C, O\} \times \{C, O\})\) is called subgame \(\sigma_1\sigma_2\). The payoff \(W_i^{\sigma_1\sigma_2} (i = 1, 2)\) in the table denotes the equilibrium welfare of country \(i\) for subgame \(\sigma_1\sigma_2\). In terms of this terminology, subgame \(CC\) is the BS model in which both countries close their markets to restrain capital mobility, while subgame \(OO\) is the one analyzed by Janeba (1998) in which both countries are ready to liberalize capital. Therefore the model incorporates all the features of the previous studies and discusses endogenous determination of each exporting country’s capital liberalization policies.

\[\begin{array}{c|c|c}
\text{Country 1} & \sigma_1 = C & \sigma_1 = O \\
\hline
\sigma_2 = C & W_1^{CC}, W_2^{CC} & W_1^{CO}, W_2^{CO} \\
\sigma_2 = O & W_1^{OC}, W_2^{OC} & W_1^{OO}, W_2^{OO} \\
\end{array}\]

For the succeeding discussion, I first summarize the results of the BS model and Janeba (1998) as well as some other derivations necessary for the analysis.

\(^7\)Each country cannot discriminate the subsidy policy between domestic-owned and foreign owned firms when they are free to locate in either country. See Haupt and Peters (2005) for analysis on discriminate subsidy policies.
3.3 The BS Model as Subgame CC

Subgame CC is nothing but the BS model exploring governments’ incentives to subsidize the own exporting firms when each firm cannot relocate abroad. The important results in Section 2.5 are shown below for the later analysis.

Each country’s reaction function for its own welfare maximization is given by

$$ R^{iB}(s_j) = \frac{1}{4}(\beta_i - s_j). \quad (2-23) $$

The associated reaction curve of country $i$ is shown by $R^{iB}$ in Figure 3.1. The intersection labeled $B$ represents the equilibrium subsidy rate of country as below.

$$ s^{CC}_i = s^B_i = \frac{4\beta_i - \beta_j}{15} \quad (i, j = 1, 2; j \neq i). \quad (2-24) $$

The associated equilibrium welfare of each exporting country is expressed by

$$ W^{CC}_i = W_i(s^{CC}) = 2\left(\frac{4\beta_i - \beta_j}{15}\right)^2 = \hat{W}^{B}_i \quad (i, j = 1, 2; j \neq i). \quad (2-26) $$

$\beta_i$ is defined in Section 2.5 as $\beta_i := 1 - 2c_i + c_j > 0 (i, j = 1, 2; j \neq i)$ for positive quantities under duopoly. Since $\beta_1 - \beta_2 = 3(c_2 - c_1)$ holds, their ratio $\beta_1/\beta_2$ serves as the indicator of firm 1’s relative productivity or efficiency to firm 2 and is very useful for the analysis in this chapter.

In the succeeding discussion, The equilibrium results are confined to the cases when the outputs of both firms are non-negative, i.e., $x^{iB}_*(s^{CC}) \geq 0$, which is equivalent to the following assumption.\(^8\)

Assumption 3.1. $\frac{1}{4} \leq \beta_1/\beta_2 \leq 4$.

In view of (2-24), Assumption 3.1 also ensures $s^{CC}_i (i = 1, 2) \geq 0$, which means that each country has a non-negative incentive to subsidize its own exports. For the later analysis, I show that there exists a unique rate of subsidy $\hat{s}_i$ in each country $i$ such that:

Lemma 3.1. For $\hat{s}_i := \frac{\beta_i}{2}$, there holds $s > R^{iB}(s)$ if and only if $s > \hat{s}_i (i = 1, 2)$.

$\hat{s}_i$ in Lemma 3.1 plays an important role to determine each government’s best-response subsidy when the countries liberalize capital. In fact, when the subsidy rate of country $j$, $s_j$ exceeds $\hat{s}_i$, country $i$ cannot attract firm $j$ with relocatability by choosing the best-response subsidy $R^{iB}(s_j)$, since $R^{iB}(s_j)$ becomes strictly lower than $s_j$. As shown in Fig. 3.1, $\hat{s}_i$ is determined by the intersection of the reaction curve $R^{iB}R^{B'}$ and $45^\circ$ Line. In subgame CC, each country has an incentive to set relatively high subsidy rates due to the policy of banning inward direct investment from abroad. However, as discussed later, when allowing capital inflow, the governments lose the incentives to choose high subsidy rates, for such high subsidy rates lead the rent run out to those moved-in foreign firms.

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\(^8\)In view of (2-25), it yields $x^{iB}_*(s^{CC}) = \hat{x}^{B}_i = \frac{2(4\beta_i - \beta_2)}{15}$. 

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Fig. 3.1: Export Subsidization Warfare Equilibrium
3.4 Subgames $OC$, $CO$ – Unilateral Capital Liberalization

Based on the above results of subgames $CC$, I next explore the two subgames in which only one exporting country liberalizes capital, i.e., subgames $OC$ and $CO$. Since the two subgames can be solved in the same logic, I only focus on the analysis for subgame $OC$.

I impose the following assumptions to examine the subsidy game.

**Assumption 3.2.** When a firm can relocate its production plant between countries 1 and 2, it must be subject to the following constraints.

(i) The firm cannot change the location of the headquarter for management.

(ii) The firm cannot undertake production simultaneously in both countries.

(iii) The same total production cost function is available whether the firm locates the plant in country 1 or 2.

(iv) The firm stays in the own country when the two countries set the same subsidy rates.

In view of the above assumptions, (i) specifies that the firm repatriates the profit to its parent country (the source country) irrespective of location choice. (ii) assumes indivisibility in production. In order to focus on the subsidy competition between the exporting countries, (iii) assumes that each firm’s cost function is independent of its own and its rival firm’s location. The tie-breaking rule in (iv) excludes more complicated mixed strategies which are beyond the scope of this chapter.

Assumption of 3.2-(iii) plays a crucial role in the analysis in this chapter since it makes the study concentrate on the subsidy competition in affecting the firms’ location decisions. The assumption can be rationalized as follows. If labour is the single factor of production and production technology has constant returns to scale, each firm’s cost function can be written as $C(x_i) = (a_i w_i + \tau_i) x_i + F$ where $a_i, w_i, \tau_i$ and $F$ are, respectively, the labour coefficient, wage rate, transport cost and sunk cost (or fixed cost) of firm $i$. Consider two similar countries in European Union with the same wage rate. If they export the products to Japan, then the transport costs do not differ each other. Also, the fixed cost to set up a plant is possibly symmetric when the two countries are under similar infrastructure conditions. Thus, each firm’s cost function is indifferent to the location choice. Since the transport cost and fixed cost do not affect the qualitative analysis result in the third-market model, I assume them away for simplicity.

The properties of each country’s reaction function as well as its welfare function (i.e., the payoff) are shown to obtain the equilibrium.

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9Ishikawa and Komoriya (2009a,b), the two parallel papers examined the role of location-specific cost functions in affecting the firms’ location choices. The first one endogenized the location decisions, while the latter one focused on the domestic welfare analysis given plant locations.
3.4.1 Country 1’s Best Response

Let me first deal with country 1’s best response. Since country 1’s choice of subsidy rate affects firm 2’s decision on where to build the plant, country 1 must take account of firm 2’s reaction when choosing the best-response subsidy policy. The following strategies are undertaken to elucidate country 1’s best-response subsidy policy given $s_2$.

1st step Characterize country 1’s optimal subsidy given either (i) the policy of attracting firm 2 to the own country (hereafter the attracting policy) or (ii) the policy of refusing firm 2 (hereafter the non-attracting policy).

2nd step Choose the policy realizing the higher welfare between the attracting policy and the non-attracting policy.

Best Attracting Policy for Country 1

Consider country 1’s optimal decision on the subsidy rate when it succeeds in attracting firm 2 given $s_2$. Its associated welfare denoted as $V_1^a$ can be expressed as:

$$V_1^a(s_1) := W_1(s_1, s_1) - s_1x_2^B(s_1, s_1) = (\frac{(\beta_1 + s_1)^2}{9} - s_1(\frac{\beta_1 + \beta_2 + 2s_1}{3})$$

which is maximized at

$$s_1^a := \arg \max_{s_1} V_1^a(s_1) = -\frac{(\beta_1 + 3\beta_2)}{10} < 0.$$  

Since firm 2 never moves out of country 1, it is the best for country 1 to tax the duopoly rent of firm 2 through taxation, i.e., $s_1^a < 0$. Country 1’s best-response subsidy given its policy of attracting firm 2, denoted by $\Gamma_1^a(s_2)$ is $s_1^a$ when $s_2 < s_1^a$ and $s_2 + \epsilon$ otherwise. The best-response subsidy and the corresponding maximized welfare level expressed by $\bar{V}_1^a(s_2) := \sup_{s_1} \{V_1^a(s_1)|s_1 > s_2\}$ are shown in Table 3.2.

The associated equilibrium outputs of the firms are given by

$$x_1^B(s^a) = \frac{3\beta_2}{30} \left(\frac{3\beta_1}{\beta_2} - 1\right)$$

$$x_2^B(s^a) = \frac{\beta_2}{30} \left(7 - \frac{\beta_1}{\beta_2}\right),$$

in view of (2-25) and (3-2). The outputs of both firms are non-negative only when $0 \leq \beta_1 / \beta_2 \leq 3$. Likewise, it requires $\frac{\beta_2}{\beta_1} \in [\frac{1}{3}, 7]$ for subgame $CO$. Since the outputs of both firms are assumed to be non-negative, I replace Assumption 3.1 with the following stronger one throughout the rest of the chapter.

**Assumption 3.3.** $\frac{1}{3} \leq \beta_1 / \beta_2 \leq 3.$
Best Non-Attracting Policy for Country 1

Once country 1 bans any inward direct investment from abroad, its welfare is just the same as in the benchmark case of the BS model, i.e., \( W_1(s) \) and its best-response subsidy \( R^1(s_2) = \frac{\beta_1 - s_2}{4} \). However, as shown in Lemma 3.1, this best-response subsidy of country 1 exceeds country 2’s subsidy rate if \( s_2 < \hat{s}_1 \), so that country 1 is forced to accept firm 2. Given its non-attracting policy, country 1 cannot then employ \( R^1(s_2) \), but must match \( s_2 \) for its welfare maximization.

Therefore, country 1’s best-response subsidy against \( s_2 \) under the non-attracting policy, denoted by \( \Gamma^1_n(s_2) \) and the associated maximized welfare level denoted by \( \bar{V}^1_n(s_2) := \max_{s_1} \{ W_1(s) | s_1 \leq s_2 \} \) are summarized in Table 3.2.\(^{10}\)

<table>
<thead>
<tr>
<th>Range of ( s_2 )</th>
<th>Best response subsidy ( \Gamma^1_n(s_2) )</th>
<th>Maximum payoff ( \bar{V}^1_n(s_2) )</th>
<th>Range of ( s_2 )</th>
<th>Best response subsidy ( \Gamma^1_n(s_2) )</th>
<th>Maximum payoff ( \bar{V}^1_n(s_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_2 &lt; s_1^a )</td>
<td>( s_1^a )</td>
<td>( -\frac{5s_1^2 - (\beta_1 + 3\beta_2) s_1^2 + \beta_1^2}{9} )</td>
<td>( s_2 &lt; \hat{s}_1 )</td>
<td>( s_2 )</td>
<td>( \frac{\beta_1 - 2s_2}{(\beta_1 + s_2)} )</td>
</tr>
<tr>
<td>( s_2 \geq s_1^a )</td>
<td>( s_2 + \varepsilon )</td>
<td>( -\frac{5s_2^2 - (\beta_1 + 3\beta_2) s_2^2 + \beta_1^2}{9} )</td>
<td>( s_2 \geq \hat{s}_1 )</td>
<td>( s_2 - \varepsilon )</td>
<td>( \frac{\beta_1 - s_2}{4} )</td>
</tr>
</tbody>
</table>

Policy Switch for Country 1

Fig. 3.2 shows the associated maximized welfare for country 1 summarized in Table 3.2.\(^{11}\) The curve labeled \( A_1A_2B_3 \) illustrates the welfare under the attracting policy, while the curve labeled \( N_1B N_2 N_3 \) shows the welfare under the non-attracting policy.\(^{12}\)

Given country 2’s subsidy \( s_2 \), country 1 can choose whether to accept firm 2’s direct investment by strategically selecting its own subsidy rate. As shown in Figure 3.2, the two welfare curves for the two policies intersect at \( s_2 = 0 \), for country 1 cannot extract firm 2’s rent through zero subsidy rate. One can also prove that under Assumption 3.3 the curve \( N_1^2B \) is always below the curve \( A_1A_2B \), assuring a unique intersection of the two payoff curves at \( s_2 = 0 \).

Therefore, country 1’s best-response subsidy against \( s_2 \) when taking into account its choice between the attracting and non-attracting policies, denoted by \( \Gamma_1(s_2) \), is summarized in the following Lemma.

\(^{10}\)In the table, \( \varepsilon(>0) \) represents a sufficiently small positive number.

\(^{11}\)I set \( \beta_1 = 1 \) and \( \beta_2 = \frac{7}{6} \) when drawing the welfare curves in Figure 3.2. It is harmless to set the other values of \( \beta_1 \) and \( \beta_2 \) under the constrain in Assumption 3.3 and get the same result.

\(^{12}\)The curve \( N_1^2B N_2 N_3 \) associated with the function \( W_1 = \frac{\beta_1 - 2s_2}{(\beta_1 + s_2)} \) is tangent to the curve \( N_1N_2N_3 \) associated with \( W_1 = \frac{\beta_1 - s_2}{8} \) at \( s_2 = \hat{s}_1 = \beta_1/5 \). This is not a coincidence, for the best-response subsidy rates are the same both under the attracting and non-attracting policies.
Lemma 3.2. Country 1’s best response $\Gamma_1(s_2)$ should satisfy

$$\Gamma_1(s_2) = \begin{cases} 
\Gamma_1^a(s_2) & \text{for } s_2 < 0 \\
\Gamma_1^n(s_2) & \text{for } s_2 \geq 0 
\end{cases}$$

Or more precisely, it can be expressed as

$$\Gamma_1(s_2) = \begin{cases} 
s_1^a & \text{for } s_2 \in (-\infty, s_1^a) \\
s_2 + \varepsilon & \text{for } s_2 \in [s_1^a, 0) \\
0 & \text{for } s_2 = 0 \\
s_2 & \text{for } s_2 \in (0, \hat{s}_1] \\
R_1(s_2) & \text{for } s_2 \in (\hat{s}_1, +\infty) 
\end{cases}$$

where $\varepsilon(>0)$ is a sufficiently small positive number.

Country 1’s reaction curve is illustrated by the mixture of the thick real and broken curves, i.e., the curve labeled $A_1A_20A_3R_1$ in Fig. 3.3.

3.4.2 Country 2’s Best Response

Best Attracting Policy for Country 2

As to country 2’s best responses, first consider the case in which given $s_1$, country 2 succeeds in attracting (or more precisely keeping) firm 2 at home. The welfare is just the same as in the benchmark case of the BS model, i.e., $W_2(s)$. The best-response subsidy is also given by the reaction function (2.23), i.e., $R^{2B}(s_1)$. Likewise, as stated in Lemma 3.1, when $s_1$ is sufficiently high and greater than $\hat{s}_2$, country 2’s best-response subsidy $R^{2B}(s_1)$
becomes lower than country 1’s subsidy $s_1$. In this case, country 2 is forced to match its subsidy with country 1’s so as to keep firm 2 at home.

Given country 2’s attracting policy, its best-response subsidy rate denoted by $\Gamma_2^a(s_1)$ and the maximized welfare denoted by $\bar{V}_2^a(s_1)$ are summarized in Table 3.3.

Best Non-Attracting Policy for Country 2

Next consider the case in which country 2 has decided not to attract firm 2 (or more precisely, country 2 decided to keep firm 2 away from home). In this case, the subsidy rate chosen by country 2 does not affect the market outcomes at all. Thus, its maximized welfare denoted by $\bar{V}_2^n(s_1)$ depends only on country 1’s subsidy rate and exactly equals to firm 2’s profit, i.e., $\pi^2(s_1, s_1)$.

Since country 2 succeeds in keeping firm 2 away from home only with $s_2 < s_1$, its best-response subsidy against $s_1$ given the non-attracting policy, denoted by $\Gamma_2^n(s_1)$, is given by $(-\infty, s_1)$ as shown in Table 3.3.

| Tab. 3.3: Best-Response Subsidy and Welfare for Country 2 |
|---|---|---|---|---|---|
| Range of $s_1$ | Best response subsidy $\Gamma_2^a(s_1)$ | Maximum payoff $V_2^a(s_1)$ | Range of $s_1$ | Best response subsidy $\Gamma_2^n(s_1)$ | Maximum payoff $V_2^n(s_1)$ |
| $s_1 < \hat{s}_2$ | $\frac{3-s_1}{4}$ | $\frac{(3-s_1)^2}{8}$ | $s_1$ | $s_1$ | $\frac{(3-s_1)(3+s_1)}{9}$ |
| $s_1 \geq \hat{s}_2$ | $\frac{3-s_1}{4}$ | $\frac{(3-s_1)^2}{8}$ | all $s_1$ | $(-\infty, s_1)$ | $\frac{(3+s_1)^2}{9}$ |

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Policy Switch for Country 2

In Fig. 3.4, the curve named $A_1BCA_2'$ shows the maximized welfare of country 2 given the attracting policy and the curve named $N_1BN_2$ corresponds the non-attracting policy.

Country 2 chooses the attracting policy only when there holds $\bar{V}^a_2(s_1) > \bar{V}^n_2(s_1)$. \hspace{1cm} (3-3)

In view of the results in Table 3.3, two cases are discussed for solving the above inequality.

**Case 1:** When $s_1 \geq \hat{s}_2$, (3-3) can be rewritten as below.

$$\frac{(\beta_2 - 2s_1)(\beta_2 + s_1)}{9} > \frac{(\beta_2 + s_1)^2}{9}, \quad \text{or} \quad 0 > (\beta_2 + s_1)s_1.$$ 

The above inequality never holds for $s_1 > \hat{s}_2(> 0)$, so it is better for country 2 to employ the non-attracting policy, i.e., $(-\infty, s_1)$.

**Case 2:** When $s_1 < \hat{s}_2$, (3-3) now becomes

$$\frac{(\beta_2 - s_1)^2}{8} > \frac{(\beta_2 + s_1)^2}{9}, \quad \text{or} \quad s_1^2 - 34\beta_2 s_1 + \beta_2^2 > 0.$$ 

The inequality holds for $s_1 < (17 - 12\sqrt{2})\beta_2$ or $s_1 > (17 + 12\sqrt{2})\beta_2$. Since $0 < (17 - 12\sqrt{2})\beta_2 < \hat{s}_2 = \frac{\beta_2}{9} < (17 + 12\sqrt{2})\beta_2$, there holds $\bar{V}^a_2(s_1) > \bar{V}^n_2(s_1)$ for $s_1 < (17 - 12\sqrt{2})\beta_2$. In the following discussion, I define:

$$\bar{s}_1 := (17 - 12\sqrt{2})\beta_2 > 0 \hspace{1cm} (3-4)$$
The best-response subsidization policy of country 2 can be summarized as follows.

\[
\Gamma_2(s_1) = \begin{cases} \Gamma_a^2(s_1) = \Gamma_a^2(s_1) = R_2^2(s_1) & \text{for } s_1 < \bar{s}_1 \\ \{\Gamma_2^a(\bar{s}_1)\} \cup \{\Gamma_2^n(\bar{s}_1)\} = \{R_2^a(\bar{s}_1)\} \cup (-\infty, \bar{s}_1) & \text{for } s_1 = \bar{s}_1 \\ \{\Gamma_2^n(s_1) = (-\infty, s_1)\} & \text{otherwise} \end{cases}
\]

The associated reaction curve of country 2 is depicted as the segment \(R_2D\) and the shaded region excluding the dotted boundary in Figures 3.5 and 3.6. As Krishna (1989) addressing equivalence between quotas and tariffs in duopoly, it is discontinuous at \(s_1 = \bar{s}_1\).

\[\text{Fig. 3.5: Pure Strategy Equilibrium when } \beta_1/\beta_2 \leq \beta_{\text{mix}} \text{ in Subgame OC}\]

### 3.4.3 Equilibrium under Unilateral Capital Liberalization

The results in the previous sections imply several possible equilibria. But they are classified into the following two cases.

- **Case I:** Nash equilibrium in pure-strategy (See Figure 3.5)
- **Case II:** Nash equilibrium in mixed-strategy (See Figure 3.6)

\[\bar{s}_1/\beta_2 = 17 - 12\sqrt{2} \approx \frac{17}{12} - \sqrt{2} > 0.\]
Comparison of the two figures indicates that the pure-strategy equilibrium is possible if and only if $\tilde{s}_1 \geq s_1^{CC}$ holds, i.e., $\beta_1/\beta_2 \leq \beta_{mix} \left( := 64 - 45\sqrt{2} \in (0, 1) \right)$ holds. As shown in Krishna (1989), it is straightforward to prove the following proposition.

**Proposition 3.1.** Depending on the value of $\beta_1/\beta_2$, there emerge two types of equilibria for subgame OC as follows.

**P3.1.1** For $\beta_1/\beta_2 \leq \beta_{mix} \left( := 64 - 45\sqrt{2} \in (0, 1) \right)$, the same pure-strategy equilibrium as in subgame CC is realized.

**P3.1.2** Otherwise, a mixed-strategy equilibrium is realized where country 1 (having employed $O$) chooses $\tilde{s}_1$ with probability unity and country 2 (having employed $C$) randomizes over $R_{2B}(\tilde{s}_1)$ and $(-\infty, \tilde{s}_1)$.

The above Proposition implies that when the firm in the capital liberalizing country is much less productive than the rival firm, the capital liberalizing country behaves as in subgame CC, subsidizing the domestic firm to maximize its own welfare. The rival country, which is a much more productive country has strong incentive to subsidize the national firm itself. Otherwise, the capital liberalizing country sets a lower subsidy than the welfare-maximized subsidy in subgame CC, since the rival country is not so much productive and may set a lower subsidy or tax to let the firm move abroad to achieve higher welfare.

---

It is straightforward to derive $s_1^{CC} - \tilde{s}_1 = \frac{4\tilde{s}_1}{191} \left\{ \left( \frac{\beta_1}{\beta_2} \right) - (64 - 45\sqrt{2}) \right\}$. One should also note $\beta_{mix} > 1/3$, as shown by $\beta_{mix} - \frac{1}{3} = (64 - 45\sqrt{2}) - \frac{1}{3} \propto 191 - 135\sqrt{2} \times \frac{191}{135} - \sqrt{2} > 0$.
3.4.4 Characterization of Mixed-Strategy Equilibria for Subgame OC

Since the pure-strategy equilibrium for subgame OC is the same as in subgame CC, I focus on the mixed-strategy equilibrium as $\beta_1/\beta_2 > \beta_{mix}$ shown in Figure 3.6 and characterize its comparison with subgame CC.

**Mixed-Strategy Equilibria**

Start at the equilibrium subsidy pair given by point $E_{CC}$ with $s_{CC}^2 > s_{CC}^1$. Given $s_{CC}^1$, country 2 finds it better to let firm 2 go abroad and thus lowers its subsidy rate below $s_{CC}^1$. When country 2 chooses the subsidy rate slightly below $s_{CC}^1$ shown by point $F$, country 1 also has an incentive to cut its subsidy rate so as to spare the subsidy expenses to firm 2, which is then matched by the further subsidy decrease by country 2. This race of subsidy reduction continues along the $45^\circ$ line until reaching point $B$ where country 2 finds it indifferent to keeping firm 2 at home with $R_{2B}(\bar{s}_1)$ (point $D$) and letting it move abroad with $s_2 \in (-\infty, \bar{s}_1)$ (the segment $BB'$) when country 1 chooses $\bar{s}_1$.

Let me further characterize this mixed strategy of country 2. For this purpose, let $\rho$ represent the equilibrium probability of country 2 choosing $R_{2B}(\bar{s}_1)$ and $1 - \rho$ the probability of its choosing other subsidy rates smaller than $\bar{s}_1$. The equilibrium value of $\rho$ can be obtained by analyzing country 1’s optimization behavior. Denote $W_{1OC}^{OCm}(s_1)$ as the expected welfare of country 1 in the mixed-strategy, where the superscript $C_m$ represents that country 2 employs a mixed strategy on export subsidies. Given $\rho$, $W_{1OC}^{OCm}(s_1)$ is given by

$$W_{1OC}^{OCm}(s_1) = \rho W_1(s_1, R_{2B}(\bar{s}_1)) + (1 - \rho) V_1^a(s_1)$$

$$= \frac{\rho}{9} \left( \beta_1 + 2s_1 - R_{2B}(\bar{s}_1) \right) \left( \beta_1 - s_1 - R_{2B}(\bar{s}_1) \right) + (1 - \rho) \left( \frac{(\beta_1 + s_1)^2}{3} - s_1 \frac{(\beta_1 + \beta_2 + 2s_1)}{3} \right),$$

where use was made of (2-26) and (3-1). Differentiation with respect to $s_1$ yields

$$9 \frac{dW_{1OC}^{OCm}(s_1)}{ds_1} = \rho \left( \beta_1 - 4s_1 - R_{2B}(\bar{s}_1) \right) + (1 - \rho) \left( -\beta_1 - 3\beta_2 - 10s_1 \right).$$

Since it must hold that $\lim_{s_1 \to \bar{s}_1} \frac{dW_{1OC}^{OCm}(s_1)}{ds_1} = 0$, $\rho$ can be derived as

$$\rho = \frac{4(\beta_1 + 3\beta_2 + 10\bar{s}_1)}{8\beta_1 + 11\beta_2 + 25\bar{s}_1} = \frac{\beta_1/\beta_2 + 173 - 120\sqrt{2}}{2\beta_1/\beta_2 + 109 - 75\sqrt{2}},$$

(3-5)

\footnote{The following discussion assumes $s_2^{CC} > s_1^{CC}$. However, the same analysis applies even when $s_2^{CC} < s_1^{CC}$ with some modifications in Figure 3.6.}
by virtue of \( \bar{s}_1 = (17 - 12\sqrt{2}) \beta_2 \) in (3-4). Using \( \rho \) in the above equation, the expected welfare of each country in the mixed-strategy equilibrium is given by

\[
W_{1}^{OC_m} := \rho W_{1} (\bar{s}_1, R^2B(\bar{s}_1)) + (1 - \rho)V_{1}^{a}(\bar{s}_1), \quad (3-6)
\]

\[
W_{2}^{OC_m} := W_{2}(\bar{s}_1, R^2B(\bar{s}_1)) = (\beta_2 + \bar{s}_1)^2 / 9. \quad (3-7)
\]

### 3.4.5 Welfare Comparison between Subgames CC and OC

As implied by the discussion in the previous section, country 2 banning inward direct investment is sure to get better off in subgame OC than in subgame CC, i.e., \( W_{2}^{OC_m} > W_{2}^{CC} \). This is because its welfare at point \( D \) is strictly higher than at \( E \) along its reaction curve \( R^2R^2' \) in Figure 3.6.

However, as to country 1, it is not clear at the first glance at the figure whether it is better off in subgame OC than in subgame CC. This is because, compared with \( E \), there are (i) the losses both from the higher subsidy of country 2 and the failure to optimize its subsidy rate when country 2 sets \( R^2(\bar{s}_1) \), and (ii) the gain from lowering the subsidy and the loss from rent outflow due to subsidizing firm 2 when country 2 chooses \( s_2 \in (\bar{s}_1, -\infty) \). However, some tedious calculations in Appendix 3.A show that country 1 gets worse off in subgame OC than in subgame CC.

**Lemma 3.3.** *Given \( \beta_1/\beta_2 > \beta_{\text{mix}} \) (i.e., with a mixed-strategy equilibrium), \( W_{1}^{CC} > W_{1}^{OC_m} \) holds.*

Therefore, in the mixed-strategy equilibrium, unilateral capital liberalization makes the country liberalizing capital worse off and the country banning capital inflow better off.

Note that subgame CO can be solved in the same way as subgame OC. That is, subgame CO yields pure-strategy equilibrium for \( \beta_1/\beta_2 \geq 1/\beta_{\text{mix}} \) and mixed-strategy equilibrium for \( \beta_1/\beta_2 < 1/\beta_{\text{mix}} \). The other correspondent results also apply. Lastly, I turn to examine the equilibrium in subgame OO.
3.5 Subgame OO – Mutual Capital Liberalization

Subgame OO is an extension of the BS model explored by Janeba (1998) in which both exporting countries liberalize capital, i.e., the two exporting firms can freely choose their locations for production. The analysis makes sense only when both countries have already decided to accept inward direct investment from abroad.

Using the same approach as the previous section, I reexamine the result in Janeba (1998). When both exporting countries have liberalized capital, each firm’s strategic location choice depends on the subsidy rates chosen by the two countries. The country offering a higher subsidy can attract both firms, but suffers from the foreign rent outflow. Taxation can restrain this rent outflow, but induces both firms to go abroad, leading to a loss in tax revenue. To derive country 1’s best response subsidy, I first show the associated welfare as below.

\[
V_1(s_1, s_2) = \begin{cases} 
W_1(s_1, s_1) - s_1 x_2(s_1, s_1) & \text{when } s_1 > s_2 \\
W_1(s_1, s_2) & \text{when } s_1 = s_2 \\
W_1(s_2, s_2) + s_2 x_1(s_2, s_2) & \text{when } s_1 < s_2 
\end{cases}
\]

Based on the analyses in the previous section, country 1’s best-response subsidy in subgame OO is identical to its best attracting policy in subgame OC when \( s_1 > s_2 \), and its best non-attracting policy in subgame CO when \( s_1 < s_2 \). Thus, country 1’s best response subsidy against \( s_2 \) and the associated maximized welfare level are summarized in the following table.

<table>
<thead>
<tr>
<th>Range of ( s_1 )</th>
<th>Best Attracting Policy</th>
<th>Neutral Policy</th>
<th>Best Non-attracting Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_2 &lt; s_1^g )</td>
<td>( s_1^g )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s_1 \geq s_1^g )</td>
<td>( s_2 + \varepsilon )</td>
<td>( -5s_1^g - (\beta_1 + 3\beta_2)s_1^g + \beta_1^2 )</td>
<td>( \frac{(\beta_1 - 2s_2)(\beta_1 + s_2)}{g} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of ( s_1 )</th>
<th>Best Res.</th>
<th>Maximum Payoff</th>
<th>Range of ( s_1 )</th>
<th>Best Res.</th>
<th>Maximum Payoff</th>
<th>Range of ( s_1 )</th>
<th>Best Res.</th>
<th>Maximum Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_2 &lt; s_1^g )</td>
<td>( s_1^g )</td>
<td></td>
<td>( \infty, s_2 )</td>
<td>( s_1^g )</td>
<td></td>
<td>( \frac{(\beta_1 + s_2)^2}{g} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s_1 \geq s_1^g )</td>
<td>( s_2 + \varepsilon )</td>
<td>( -5s_1^g - (\beta_1 + 3\beta_2)s_1^g + \beta_1^2 )</td>
<td>( \infty, s_2 )</td>
<td>( s_1^g )</td>
<td></td>
<td>( \frac{(\beta_1 + s_2)^2}{g} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the above tabular, the best (non-)attracting policy is the policy (not) attracting the foreign firm, while the neutral policy is the policy setting the same subsidy rate as the rival country. In Figure 3.7, the curve named \( A_1A_2A_3 \) shows the maximized welfare of country 1 given the attracting policy, \( N_1N_2 \) the maximized welfare given the non-attracting policy and \( U_1U_2 \) the welfare given the equalized subsidy rates. From the figure, it is easy to show that the (non-)attracting policy always yields the highest welfare given \( s_2 < 0(> 0) \) and the three curves intersect at \( s_2 = 0 \).
Thus, country 1’s best response subsidization policy is given as below.

\[
\tilde{\Gamma}_1(s_2) = \begin{cases} 
  s_1^a & \text{for } s_2 \in (-\infty, s_1^a) \\
  s_2 + \varepsilon & \text{for } s_2 \in [s_1^a, 0] \\
  (-\infty, 0] & \text{for } s_2 = 0 \\
  (-\infty, s_2^\star) & \text{for } s_2 > 0 
\end{cases}
\]

Intuitively, country 1’s best response subsidization policy can be explained as below. When \( s_2 < 0 \), country 1 is better off to impose a lower tax to gain tax revenue. Given
the attracting policy, country 1’s welfare is maximized at \( s_1 = s^a_1 < 0 \). Thus, if \( s_2 < s^a_1 \), country 1 should set the subsidy rate at \( s^a_1 \), while if \( s_2 \geq s^a_1 \), country 1 reacts to set at \( s_2 + \varepsilon \) to attract both firms. However, when \( s_2 > 0 \), country 1 is better off to induce subsidy rent inflow to let both firms locate abroad. Thus, the best response is a bit lower subsidy \( s_2 - \varepsilon \). When the rival country is in free trade that \( s_2 = 0 \), country 1 is indifferent to the attracting and non-attracting policy.

Since both countries liberalize capital, country 2’s best response is symmetric to country 1. The unique equilibrium that both countries entail free trade can be shown in Figure 3.9.\(^\text{16}\)

\[ s_1 \]
\[ s_2 \]
\[ s^a_1 \]
\[ s^a_2 \]
\[ E \]
\[ R \]
\[ R' \]

**Fig. 3.9: Equilibrium in Subgame OO**

**Lemma 3.4** (Janeba (1998)). *When the two exporting countries open their domestic markets allowing foreign capital inflow, the equilibrium subsidy of each exporting country equals zero.*

There never exists an equilibrium with either strictly positive or negative subsidies. Janeba (1998)’s result elucidated how the mutual capital liberalization by both countries (or the relocatability of both firms) affects the governments’ subsidization incentives.

The associated equilibrium welfare of each exporting country is expressed by

\[
W_i^{OO} := \frac{\beta^2_i}{9} \quad (i = 1, 2). \tag{3-8}
\]

\(^{16}\)Figure 3.9 shows the case when \( c_1 < c_2 \). The unique equilibrium is irrespective of cost conditions.
In view of (2-22) and (3-8), the welfare difference between subgames \( CC \) and \( OO \) is given by \( W_{iCC} - W_{iOO} = \frac{7\beta_1^2 - 16\beta_1\beta_2 + 2\beta_2^2}{225} \). Given Assumption 3.3 that \( \beta_1/\beta_2 \in [\frac{1}{3}, 3] \), the following proposition holds.

**Proposition 3.2.** Mutual capital liberalization makes

**P3.2.1** both exporting countries strictly better off for \( \frac{\beta_1}{\beta_2} \in \left( \frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{7} \right) \),

**P3.2.2** country 1 better off but country 2 worse off for \( \frac{\beta_1}{\beta_2} \in \left[ \frac{1}{3}, \frac{8-5\sqrt{2}}{2} \right) \),

**P3.2.3** country 1 worse off but country 2 better off for \( \frac{\beta_1}{\beta_2} \in \left( \frac{8+5\sqrt{2}}{7}, 3 \right] \).

Janeba (1998) demonstrated that the exporting countries are better off when both allowing foreign capital inflow. However his result critically hinges on the assumption that both firms have the same cost conditions, i.e., \( \beta_1/\beta_2 = 1 \). When the cost conditions differ sufficiently to have \( \beta_1/\beta_2 \notin \left[ \frac{8-5\sqrt{2}}{2}, \frac{8+5\sqrt{2}}{7} \right] \), the more productive country gets worse off and the less productive country is better off. The intuition behind is as follows. de Meza (1986) showed that the more productive country has the greater incentive to subsidize its exports when both exporting countries ban inward direct investment. Large cost asymmetry causes large subsidy differential between the exporting countries, but this subsidy differential is eliminated when both countries allow capital inflow. Hence, the more productive country cannot secure the large duopoly rent through highly strong subsidization, and the less productive country no longer suffers from the cost disadvantage in the subsidy competition.
3.6 Full Equilibria for the Capital Liberalization Game

3.6.1 Second-Stage Subgame Equilibria

The subgame equilibria for the second stage are completed. In view of Proposition 3.1, concerning subgame OC and the range of $\beta_1/\beta_2$ in Assumption 3.3, the equilibria can be classified into 3 types as shown in Figure 3.10 where $E_{\sigma_1\sigma_2} = (W_1^{\sigma_1}, W_2^{\sigma_2}) (\sigma_i \in \{C, O\})$ denotes the equilibrium of subgame $\sigma_1\sigma_2$ in Table 3.1.

<table>
<thead>
<tr>
<th>Type M1</th>
<th>Type B</th>
<th>Type M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{OC} = (W_1^{OC}, W_2^{OC})$</td>
<td>$E_{OC} = (W_1^{OC}, W_2^{OC})$</td>
<td>$E_{OC} = (W_1^{OC}, W_2^{OC})$</td>
</tr>
<tr>
<td>$E_{CO} = (W_1^{CO}, W_2^{CO})$</td>
<td>$E_{CO} = (W_1^{CO}, W_2^{CO})$</td>
<td>$E_{CO} = (W_1^{CO}, W_2^{CO})$</td>
</tr>
</tbody>
</table>

- **Type B** for $\beta_1/\beta_2 \in (\beta_{mix}, 1/\beta_{mix})$: Unilateral capital liberalization yields mixed-strategy equilibria by the country banning inward direct investment.
- **Type $M_i$** ($i = 1, 2$) for $\beta_i/\beta_j \leq \beta_{mix}$ ($i, j = 1, 2; j \neq i$): Country $i$’s unilateral capital liberalization yields the same pure-strategy equilibrium as in subgame CC, while the other country $j(\neq i)$’s unilateral capital liberalization yields an equilibrium with a mixed strategy employed by country $i$.

Figure 3.10 also summarizes welfare comparisons among possible subgame equilibria by virtue of Proposition 3.2. One should note that for any possible values of $\beta_1/\beta_2$, at least one country will employ mixed strategy in either subgame OC or CO.

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17 See also footnote 14 to confirm $\beta_{mix} \in (1/3, 1)$ and thus $1/\beta_{mix} \in (1, 3)$

18 $\beta_{mix} < \frac{8-5\sqrt{2}}{2} < 1$ holds, which is given by:

$$\beta_{mix} - \frac{8-5\sqrt{2}}{2} = 64 - 45\sqrt{2} - \frac{8-5\sqrt{2}}{2} = 60 \left(1 - \frac{17\sqrt{2}}{24}\right) < 0.$$ 

The above inequation also implies $1/\beta_{mix} > \frac{2}{8-5\sqrt{2}} = \frac{8+5\sqrt{2}}{2}$. Thus $\beta_1/\beta_2 \in (\frac{2}{8-5\sqrt{2}}, \frac{8+5\sqrt{2}}{2})$ is always in the range of Type B as shown in Figure 3.10.
Proposition 3.3. For all the possible relevant values of $\beta_1/\beta_2 \in [1/3, 3]$, there always exist subgames of unilateral capital liberalization yielding mixed-strategy equilibria.

Lastly, let me deal with the full-game equilibria for the capital liberalization game with strategic subsidization. Due to qualitative symmetry, only Types $M_2$ and Type $B$ Equilibria are examined.

3.6.2 Type $M_2$ Equilibria

For $\beta_1/\beta_2 \in [1/\beta_{mix}, 3]$, the relevant payoff matrix at the first stage is shown by the following Table 3.5. I demonstrate first that $C$ strongly dominates $O$ for country 1. Lemma 3.3 indicates that $W_{1CC} > W_{1OC}$ when country 2 chooses $C$, while Figure 3.10 (or Proposition 3.2) shows that $W_{1CC} > W_{1OO}$ when country 2 chooses $O$. Thus $O$ is a dominated strategy for country 1 and can be deleted for consideration. Since country 2 is indifferent to $C$ and $O$ given $\sigma_1 = C$, there hold two equilibria (Close, Close) and (Close, Open), which yield the same payoffs ($W_{1CC}, W_{2CC}$).

<table>
<thead>
<tr>
<th>\sigma_2 = C</th>
<th>\sigma_2 = O</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{1CC}, W_{2CC}$</td>
<td>$W_{1CC}, W_{2CC}$</td>
</tr>
<tr>
<td>$W_{1OC}, W_{2OC}$</td>
<td>$W_{1OO}, W_{2OO}$</td>
</tr>
</tbody>
</table>

When the cost difference is large enough, the more productive country subsidizes the national firm itself and does not let the firm move aboard even though the rival country liberalizes capital. Therefore, the less productive country is indifferent to closing and opening the market without fear of attracting the foreign firm.

3.6.3 Type $B$ Equilibrium

For $\beta_1/\beta_2 \in (\beta_{mix}, 1/\beta_{mix})$, the relevant payoff matrix at the first stage is shown by Table 3.6. As for country 1, $W_{1CC} > W_{1OC}$ holds against country 2’s choice of $C$, while $W_{1OO} > W_{1OO}$ holds against country 2’s choice of $O$. Thus $C$ is the dominant strategy for country 1. Since the payoff structure is qualitatively symmetric, $C$ is also the dominant strategy for country 2. Thus (Close, Close) is the dominant strategy equilibrium. Note Type $B$ Equilibrium also includes the symmetric cost case when $\beta_1/\beta_2 = 1$.

\[W_{OC} = \frac{\beta_2^2}{\beta_1} \text{ in (3-8) and } W_{OC} = \frac{(\beta_2 + \bar{s}_1)^2}{\beta_1^2} \text{ in (3-7) yields } W_{OC} > W_{OO} \text{ since } \bar{s}_2 > 0 \text{ by virtue of (3-4). Symmetric formulas apply in subgame CO, so } W_{OC} > W_{OO} \text{ also holds.}\]
Tab. 3.6: 1st-Stage Payoff Matrix for Type B

\[
\begin{array}{c|cc|cc}
\text{Country 1} & \sigma_1 = C & \sigma_1 = O \\
\hline
\text{Country 2} & \sigma_2 = C & W^{CC}, W^{CC} & W^{CmO}, W^{CmO} \\
& W^{OCm}, W^{OCm} & W^{OO}, W^{OO} \\
\end{array}
\]

3.6.4 Welfare at Subgame Perfect Nash Equilibria

In view of the above results, the resulting equilibrium welfare of both countries are the same as the relevant equilibrium welfare in subgame CC.

The results of the full-game are summarized into the following Proposition.

**Proposition 3.4.** In the capital liberalization game in which both exporting countries can choose whether to liberalize capital,

- **P3.4.1** mutual capital liberalization (Open, Open) is never realized at a pure-strategy subgame-perfect Nash equilibrium, while mutual capital restriction (Close, Close) is always realized;

- **P3.4.2** in all pure-strategy subgame-perfect Nash equilibria, the equilibrium welfare of each country is the same as when both countries ban inward direct investment from abroad;

- **P3.4.3** unilateral capital liberalization is chosen by the less productive exporting country when the cost difference is large enough.

Although Janeba (1998) discussed mutual capital liberalization by exporting countries, such an outcome cannot be realized as a pure-strategy subgame-perfect Nash equilibrium when endogenizing each country’s decision on capital liberalization. Therefore, there need some additional coordination mechanisms for mutual capital liberalization. Furthermore, as implied by Proposition 3.2, mutual capital liberalization does not necessarily improve the welfare of both exporting countries if the cost difference is large enough.

3.6.5 World Welfare

As for the third country, which is a country importing the goods from Country 1 and 2, its welfare can be expressed as \( W^{CC}_3 = \frac{2(\beta_1+\beta_2)^2}{25} \) for subgame CC, and \( W^{OO}_3 = \frac{(\beta_1+\beta_2)^2}{18} \) for subgame OO.

Clearly, \( W^{CC}_3 > W^{OO}_3 \) holds, i.e., the third country is always worse off under mutual capital liberalization than mutual capital restriction. This is because in subgame CC, the two countries subsidize their exports and thereby expand the total sales to the third country, which means an improvement of the importing country’s terms of trade. Furthermore, comparison of world welfare between the two cases yields \( \sum_{i=1}^{3} W^{CC}_i > \sum_{i=1}^{3} W^{OO}_i \).
The result is in a sense obvious when the two exporting firms have the same cost conditions. That is, given the world social marginal cost of production, which is equal to the subsidy-exclusive marginal cost of each firm, the exporting countries’ subsidies expand the world output, causing less distortion in oligopoly pricing.

When the exporting firms exhibit cost heterogeneity, de Meza (1986) showed that the country with the more efficient firm has the greater incentive to subsidize its exports. Coupled with the gains from the total output expansion, the world also gains from the greater production efficiency, i.e., cost savings due to the output expansion by the more efficient firm and the output contraction by the less efficient one.

**Proposition 3.5.** The world welfare is higher under the exporting countries’ mutual capital restriction than under their mutual capital liberalization.
3.7 Discussion

The results in this chapter are constrained in Assumption 3.2. Relaxing such special conditions, I further discuss some other cases.

**Locating in the third country**  If the firms can choose to locate in a third country instead of the rival exporting country, it leads to the typical discussion in FDI literature: export or FDI? By locating in its home country, a firm can receive subsidization from the government, but will need to pay the transport cost to sell the product in the third-country market. Thus, transport cost plays a crucial role. Firms’ location choices are determined by the effective production cost difference between export and FDI. Assuming that the cost function is independent of location, each firm chooses to locate at home if the subsidy rate is higher than the transport cost. Therefore, in view of the values of both countries’ subsidy rates and transport costs, there are four patterns: (1) both firms export; (2) both firms accept FDI; and (3) and (4) one firm exports and another firm accepts FDI.

The exporting countries thereby use subsidy policies to implement their choices regarding whether to attract the domestic firm to locate in the home country or allow the domestic firm to invest in the third country. If the transport cost is relatively low, then exporting countries have incentives to subsidize the firms to maximize welfare. However, if the transport cost is relatively high, then letting the firm invest in another country to save the transport cost improves welfare. There exists a critical value of transport cost that optimizes welfare through its choice of attracting the domestic firm or not. Both pure and mixed strategy equilibria may be dependent on the value of the transport cost. Further, the exporting countries’ capital liberalization for outward direct investment can be examined.

**Multiple plants**  In the framework of a third-market model, no firm has an incentive to separate production in both exporting countries since the fixed cost doubles. Even if the fixed cost excluded, concentrating the production in a single country with higher subsidy rates is more profitable due to the assumption of location-independent cost function and identical transport cost.

It is not interesting to consider multiple plants in a third-market model in which the host countries have no domestic consumption. However, if the model is changed to a reciprocal trade model built by Brander (1981), each firm may have an incentive to set up a plant in both countries. Consider that both exporting countries have domestic consumption and the markets are equal in size and are segmented. The firms can choose to build one plant in the home country and serve the foreign market by exporting or building plants in both countries. In such a case, both transport cost and fixed cost are important. Without the fixed cost, firms build two plants if and only if subsidy-inclusive transport cost is positive. However, with fixed cost, firms build two plants if and only if subsidy-inclusive transport cost is higher than the fixed cost. Therefore, in view of the values of subsidy rates, trade cost, and fixed cost, there also exist four patterns: (1) both firms build a single plant; (2)
both firms build two plants; and (3) and (4) one firm builds a single plant and another firm builds two plants.

Governments’ subsidization incentives are dependent on the production cost asymmetry as well as the fixed cost and transport cost. If the fixed cost is relatively high, then the firms are willing to operate alone in their home countries. So, the country can subsidize the exports to maximize its welfare. If the transport cost is relatively high, then the low-cost country subsidizes its exports while the high-cost country is better off allowing the firm to invest outward. The key variable lies in the production cost asymmetry related with the fixed cost and transport costs. Both countries are likely to liberalize capital for inward direct investment since the subsidy is not needed for the foreign firm.

**Profit tax** Janeba (1998) discussed tax competition under capital liberalization. If each exporting country imposes profit tax on the firms located in its territory, then the firms choose to locate in the country with the lower profit tax. Janeba (1998) assumed that the costs are not tax deductible, which is different from the pure profit tax. However, when the costs are tax deductible, imposing profit tax does not affect firms’ output decisions. Each exporting country’s welfare is just its national firm’s profit plus tax revenue from the foreign firm. Since firms’ profits are independent of the profit tax, each country prefers a higher tax rate; however a higher tax leads to a loss of the foreign firm’s tax revenue. Thus, the race to the bottom tax rate constitutes the equilibrium state under capital liberalization. When restricting capital inflow, taxing or subsidizing the domestic firm does not change the social welfare. Thus, free trade is the equilibrium and both countries are indifferent to liberalizing and restricting capital. The result differs from that in the export subsidy case in which free trade is not realized shown in this chapter.
3.8 Concluding Remarks

This chapter examines the strategic subsidy policies under a four-stage capital liberalization game by endogenizing governments’ decisions on capital liberalization policy and considering asymmetric cost conditions between the exporting firms. Unlike Janeba (1998), who discussed the welfare effect of mutual capital liberalization between the exporting countries, subgame-perfection prevents both countries from liberalizing capital as far as focusing on pure-strategy equilibria. Furthermore, I show some pessimistic results concerning the welfare effects of capital liberalization in oligopoly when the cost difference is large enough between the exporting firms.

First, as stated in Proposition 3.2, mutual capital liberalization does not Pareto-dominate the initial state in which both countries ban inward direct investment. More specifically, when the cost conditions differ to a great extent, it benefits the less productive exporting country, but hurts the more productive one. In this case, even when some additional measures are available to coordinate capital policies between exporting countries, they do not choose to mutually liberalize capital.

Second, even when mutually capital liberalization benefits both exporting countries and there some measures made to improve both exporting countries’ welfare, capital mobility tends to prevent each exporting country’s incentive to subsidize more the lower cost firms as demonstrated in de Meza (1986) in the absence of capital mobility. The world gets worse off under mutual capital liberalization, for the exporting countries lose an incentive to subsidize their firms. In fact, when the exporting countries can coordinate their subsidy policies, they actually find it profitable to tax, rather than subsidize their firms so as to avoid excess competition and improve the terms of trade, which constitutes another factor to worsen world welfare.

The most important result underlying this chapter is the determinant to subsidize the exporting firms as discussed in de Meza (1986), i.e., the country with the more efficient firms has the greater incentive to subsidize its exports. Since this incentive works robustly in oligopoly, the analysis carries over to the more general model incorporating transportation costs specific to the shipping site of the exporting countries. The less productive exporting country with the lower subsidy rate does not care capital liberalization, but the more productive country does, which benefits from capital liberalization by both exporting countries. A problem is that any enforced capital liberalization, as is required when forming a common market, may hurt not only the exporting countries but also the region forming a common market.
Appendix

3.A Welfare Comparison $W_1^{CC}$ vs. $W_1^{OC_m}$

Given the probability of country 2 choosing $R_2^B(\bar{s}_1)$, the difference of country 1’s equilibrium welfare between subgames $CC$ and $OC$ is given by $W_1^{CC} - W_1^{OC_m} = 2\left(\frac{4\beta_1 - \beta_2}{15}\right)^2 - \rho\left(4\beta_1 - \beta_2 - 3\bar{s}_1\right)\left(4\beta_1 - \beta_2 + 9\bar{s}_1\right) - (1 - \rho)\frac{\beta_1^2 - \bar{s}_1(\beta_1 + 3\beta_2) - 5\bar{s}_1^2}{9}$, where use was made of (2-26) and (3-6).

Substituting (3-5) into the above equation yields

$$W_1^{CC} - W_1^{OC_m} = \frac{14b^3 + (531 - 375\sqrt{2})b^2 + (123,150\sqrt{2} - 174,165)b + (687,225\sqrt{2} - 971,882)}{225(109 - 75\sqrt{2} + 2b)},$$

where $b = \beta_1 / \beta_2$. Since $b > 0$ and $109 - 75\sqrt{2} > 0$, $W_1^{CC} > W_1^{OC_m}$ holds if and only if $f(b) > 0$, where $f(b) := 14b^3 + (531 - 375\sqrt{2})b^2 + (123,150\sqrt{2} - 174,165)b + (687,225\sqrt{2} - 971,882)$.

Therefore, there must have two positive and one negative solutions. Since the negative solution can be precluded for consideration, only two positive solutions are taken into account. Since $\beta_1 / \beta_2 = \beta_{mix} = 64 - 45\sqrt{2} \in (1/3, 1)$ is a critical value yielding both a pure-strategy and a mixed one in subgame $OC$ as stated in Proposition 3.1, it should be one of the solutions, which can be verified by rather tedious calculations. Thus $f(b)$ can be factorized as below.

$$f(b) = \left\{b - (64 - 45\sqrt{2})\right\} \left\{14b^2 + (1,427 - 1,005\sqrt{2})b + (7,613 - 5,385\sqrt{2})\right\}.$$  

Let $g(b) := 14b^2 + (1,427 - 1,005\sqrt{2})b + (7,613 - 5,385\sqrt{2})$. Then the discriminant for $g(b) = 0$ is given by $\Delta = 3,949,797 - 2,792,880\sqrt{2} > 0$, which implies that $g(b) = 0$ has two distinct real roots, $b_1, b_2(b_1 < b_2)$. Then the relation between the coefficients and the roots for quadratic equations implies $b_1 + b_2 = \frac{1}{14}(1,005\sqrt{2} - 1,427) < 0$ and $b_1b_2 = \frac{1}{14}(7,613 - 5,385\sqrt{2}) < 0$. These relations imply $b_1 < 0 < b_2$. Furthermore, since $g(1/3) = \frac{1}{9}(72,812 - 51,380\sqrt{2}) > 0$ holds, $b_2 < 1/3$ is beyond the scope of our discussion.

To sum up, $f(b)$ is factorized completely as below.

$$f(b) = \frac{1}{14}(b - b_1)(b - b_2)\left\{b - (64 - 45\sqrt{2})\right\},$$

where $b_1 < b_2 < 1/3 < \beta_{mix} = 64 - 45\sqrt{2} < 1$. The above demonstrations established that $f(b) > 0$ for $b \in (1/3, 3)$ if and only if $b > \beta_{mix}$. This completes the proof.
Chapter 4

International Cross Shareholding

4.1 Introduction

We have accelerated the globalization of economic activities over the last century. Rapid growth in commodity trade has not only given rise to new trade in services, but also the creation of a global production-marketing network that is supported by direct investments and the cross-licensing of technologies and know-how. This makes the shareholding structures of the firms more complicated than we have experienced before. With foreign investment liberalization, any firm financed by equities located in the home country is never totally owned by the domestic investors themselves. The equities are also partially owned by foreign investors of some other countries. In Japan, a number of major publicly traded firms are more than 30% owned by overseas investors. A report from Nihon Keizai Shimbun in 2004 said that the foreign shareholding of Japan’s major firms increased by 74% from the previous year and top firms such as Orix Corp. and Canon Corp. are controlled by more than 50% by the foreign investors. Mutual shareholding among firms is becoming widespread and foreign shareholding in firms is increasing.

Such internationalized shareholding of the firms should alter the standard welfare implications of trade and industrial policies. This is because traditional policy analysis is based on the explicit nationality of the firms involved. Domestic firms are assumed to be 100% “domestic” in the sense that their equities are all held by the domestic residents including the “domestic” firms. The well-known argument of the strategic trade policy toward international oligopoly shares the same assumption.

Thus, we are faced with the following question. If the shareholders of the firm are not purely domestic residents, does the internationalized shareholding of firms affect the decision of an individual country’s government seeking its own national interest or welfare? If it does, then how? In traditional literatures, Bhagwati and Brecher (1980), Brecher and Bhagwati (1981) and Brecher and Findlay (1983) studied the foreign ownership under perfect competition using the classical approaches. Lee (1990) examined the strategic trade policies and trade patterns by allowing the foreign ownership of firms. When both firms compete in the home market, an increase in foreign ownership makes both the optimal
export and import policies scale down at the same rate. The trade pattern, firm values, and world welfare are not affected by foreign ownership; however, the distribution of world welfare is changed. Other recent papers, such as Miyagiwa (1992), Dick (1993), and Welzel (1995) also discussed the weaker subsidization incentive in the presence of foreign ownership. Long and Soubeyran (2001) obtained the “neutrality result” in the presence of the cross ownership of a number of heterogeneous firms. Kang, Cheong, and Lee (2001) examined the trade patterns in a three-country model, but involving intra-industry trade.

Meanwhile, internationalized shareholding also takes the form of mutual shareholding among the firms themselves as a long-running practice. It has been typically seen that the firms hold each other’s shares in order to stabilize their management and maintain business ties especially in Japan where is is known as keiretsu, i.e., see Lichtenberg and Pushner (1994) and Weinstein and Yafeh (1995). Reynolds and Snapp (1986) and Malueg (1992) studied firms’ collusive behavior under mutual cross-firm shareholding. Macho-Stadler and Verdier (1991) examined the managerial incentive where owners and managers had separate identities and the owners showed collusive behavior among themselves. Inoue (1998) and Kuroki (2001) examined the mutual shareholding among financial banks. However, studies on the strategic trade policies under mutual shareholding among firms are very limited and it is of great interest to see whether the mutual shareholding among firms yields any different implications for strategic trade policies as compared to the one among the residents of the countries. To distinguish between these two types of mutual shareholding, the former only among the firms is referred to as cross-firm shareholding, and the latter only among the residents is referred to as cross-country shareholding throughout the chapter.

Furthermore, in this chapter I introduce a mixed international cross shareholding structure, which allows a part of a firm’s shares to be owned by both foreign residents and foreign firms. This chapter investigates how the cross shareholding structure affects the firms’ market performances and government’s strategic subsidization incentives. It is shown that increasing the ratio of foreign firm’s shareholding is more likely to strengthen the country’s subsidization incentive and facilitate collusion between the firms as in Krishna (1989).\(^1\) Although the dual types of distortions imply that cross-firm shareholding should be regulated or banned, when exporting governments engage in subsidy competition, firms’ collusion does not occur and world welfare benefits from export subsidization. Therefore, the cross-firm shareholding structure should not necessarily be banned when governments subsidize the exports.

In the classical financial and industrial organization theory, the diverse shareholders of a firm may cause a unanimity problem. This chapter excludes this complexity and assumes that shareholder unanimity is satisfied, that is, all of a firm’s shareholders who make the production decision unanimously desire to maximize the value of the firm’s shares. Previous studies showed that under the assumption of no trade and complete information, shareholder unanimity is supported by the so-called spanning condition: any production

\(^{1}\)However, in the repeated-game setting, Malueg (1992) showed that increasing the degree of cross-firm shareholding in the market may reduce the likelihood of collusion under certain demand conditions.
plan of the firm can be written as a linear combination of the production plans of the other firms (See Ekern and Wilson (1974), Leland (1974), and Radner (1974)). When there is trading in the shares of firms, Grossman and Stiglitz (1980) indicated that spanning does not imply unanimity due to new information. However, if firms behave as perfect competitors, the spanning condition still leads to unanimity. Hart (1979) generalized the result to an economy with incomplete markets and showed that the competitive condition ensures shareholder unanimity independent of the spanning condition. DeAngelo (1981) demonstrated intuitive conditions for unanimity and formulated a general definition of market competition. This chapter does not consider the trading of the stocks and incomplete information, so shareholder unanimity is assumed in order to simplify the analysis.

The analysis in this chapter is based on Kiyono and Wei (2004), Wei and Kiyono (2005). The remainder of the chapter is organized as follows. Section 2 introduces the cross shareholding model. Section 3 studies the optimal subsidy policies under international cross-country shareholding and employs some specific cases to work out the results in great details. Section 4 synthesizes both cross-country and cross-firm shareholding analysis into a mixed shareholding structure and analyzes how increasing foreign firms’ shareholding affects the optimal outputs and strategic subsidization decisions. The effects of subsidy competition on national welfare and world welfare under cross-country and cross-firm shareholding structures are discussed in Section 5. Finally, the conclusion is summed up in Section 6.
4.2 The Model

The model is a two-stage game involving governments and firms as in the BS model. Let me first explain the difference between the two international cross shareholding structures: cross-country and cross-firm shareholding. When the equities of a firm are internationally owned by the residents of both countries, it is called the cross-country shareholding structure. When the equities are internationally owned by the shareholders of both firms, it is called the cross-firm shareholding structure.

Given the subsidy profile $s$, the profit earned by firm $i$ through its own export is expressed by

$$
\pi_i(x, s_i) = \{ P(x_1 + x_2) - c_i + s_i \} x_i \quad (i, j = 1, 2; j \neq i),
$$

where $x = (x_1, x_2)$ denotes the output profile. Let $\sigma_i \in [0, 1]$ denote the percentage share of firm $i$’s equities owned by domestic residents where $i = 1, 2$. Values of $(\sigma_1, \sigma_2)$ are assumed to be exogenously given. Then, the national welfare of country $i$ is expressed by

$$
W_i(x, s, \sigma) = \sigma_i \pi_i(x, s_i) + (1 - \sigma_j) \pi_j(x, s_j) - s_i x_i,
$$

where $s = (s_1, s_2)$ denotes the subsidy profile and $\sigma = (\sigma_1, \sigma_2)$ the bilateral shareholding structure of the firms.

Without loss of generality, I assume

**Assumption 4.1.** $\sigma_i \in (\frac{1}{2}, 1)$ for $i = 1, 2$.

Without this assumption, there is no essential meaning to refer the home firm as the "home" firm.\(^2\)

Although each government cares about its own welfare as defined in (4-1) under both shareholding structures, the objectives of the firms are critically different. When there is cross-country shareholding between the exporting countries, each firm maximizes its own profit defined in (2-1). However, when there is cross-firm shareholding, each firm now maximizes not its own profit but the value of the firm which is defined as the sum of the dividends from both firms’ exporting activities. The value function of firm $i$ is given by

$$
V_i(x, s, \sigma) = \sigma_i \pi_i(x, s_i) + (1 - \sigma_j) \pi_j(x, s_j).
$$

Such value maximization is incomplete, for each firm can choose the amount of its own export only and has no explicit measures for coordinating its exports and production.

Keep in mind the difference in the objective and the constrained set of actions imposed on the two structures. Let me first explore the properties of cross-country shareholding equilibrium.

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\(^2\)Normally, when a firm holds more than a half of the shares of the other firm, it becomes the parent firm and thus can control it as its subsidiary firm. Corporate laws in most countries ban cross-firm shareholding between the parent company and its subsidiary, and prescribe the maximum mutual shareholding ratio among the firms. For example, in France is 10%; in German 25%; in Korea 40%; and in Japan 50%. However, in Hong Kong, cross-firm shareholding is strictly prohibited.
4.3 Cross-Country Shareholding Equilibrium

4.3.1 Second-Stage Equilibrium

As mentioned earlier, the equilibrium is solved by backward induction from the second-stage. Firm $i$’s reaction function denoted by $r^C(x_j, s_i)$ is equivalent to the one in the BS model. The superscript $C$ shows the variables associated with the equilibrium values under the cross-country shareholding structure:

$$r^C(x_j, s_i) = \frac{1}{2} (a - c_i + s_i - x_j) = r^B(x_j, s_i) \quad (i, j = 1, 2; j \neq i). \quad (2-19)$$

Denote $x^*_i(s_i, s_j)$ ($i, j = 1, 2; j \neq i$) as firm $i$’s equilibrium output under cross-country shareholding that is expressed by

$$x^*_i(s) = \frac{1}{3} (\beta_i + 2s_i - s_j) = x^*_B(s) \quad (i, j = 1, 2; j \neq i). \quad (2-20)$$

The equilibrium profit of firm $i$ can be rewritten as

$$\pi^*_i(s) = \pi^i(x^*_i(s), x^*_j(s), s_i) = \pi^*_B(s).$$

Under the cross-country shareholding structure, the familiar comparative statics results are equivalent to the BS model. This is because no changes in the bilateral mutual shareholding structure affect either firm’s output decision.

4.3.2 Government’s Subsidy Incentive

In the first stage, the governments can predict the resulting second-stage equilibrium, given their own choices regarding the subsidies. Country $i$’s welfare is now given by

$$W^*_i(s, \sigma) \overset{\text{def}}{=} \sigma_i \pi^*_i(s) + (1 - \sigma_i) \pi^*_j(s) - s_i x^*_i(s).$$

Each government maximizes the national welfare by choosing the optimal export subsidy, taking into account the responses of both firms. The Nash solution for the FOC should satisfy

$$0 = \frac{\partial W^*_i(s, \sigma)}{\partial s_i} = \sigma_i \frac{\partial \pi^*_i}{\partial s_i} + (1 - \sigma_i) \frac{\partial \pi^*_j}{\partial s_i} - x_i - s_i \frac{\partial x^*_i}{\partial s_i}$$

$$= \sigma_i \left( x_i P'(X) \frac{\partial x^*_i}{\partial s_i} + x_i \right) - x_i - s_i \frac{\partial x^*_i}{\partial s_i} + (1 - \sigma_j) x_j P'(X) \frac{\partial x^*_i}{\partial s_i}, \quad (4-3)$$

The SOC for each country’s welfare maximization can be examined by using (4-5):

$$\frac{\partial^2 W^*_i(s, \sigma)}{\partial s_i^2} = \frac{1}{9} [8 \sigma_i - 10 - 2 \sigma_j] < 0,$$

which shows that the welfare function of each country is strictly concave with respect to its own export subsidy.
where use was made of (2-10) and (2-11). The terms on the right-hand side of (4-3) show the decomposition of strategic export subsidization under the cross-country shareholding structure. The subsidy incentive is easy to understand when it is compared to the standard form in the BS model \(\sigma_i = 1\) for \(i = 1, 2\). For this purpose, (4-3) is rewritten as below.

\[
\frac{\partial W^*_i}{\partial s_i}(s, \sigma) = x_i P'(X) \frac{\partial x^*_j}{\partial s_i} - s_i \frac{\partial x^*_i}{\partial s_i} - (1 - \sigma_i) x_i P'(X) \frac{\partial x^*_j}{\partial s_i} - (1 - \sigma_i) x_i + (1 - \sigma_j) x_j P'(X) \frac{\partial x^*_i}{\partial s_i}.
\]

The terms on the first line show the subsidy incentives found in the BS model in (2-12), while those on the second line show the new sources of subsidy incentives specific to cross-country shareholding. For later reference, the terms on the second line are called the additional subsidization incentives under cross-country shareholding and are denoted as \(I^C_i(s, \sigma)\) for country \(i\), that is,

\[
I^C_i(s, \sigma) = -(1 - \sigma_i) x_i P'(X) \frac{\partial x^*_j}{\partial s_i} - (1 - \sigma_i) x_i + (1 - \sigma_j) x_j P'(X) \frac{\partial x^*_i}{\partial s_i}.
\]

The above three terms on the right-hand side are all strictly negative in view of (2-8) and (2-9). The first term \(-(1 - \sigma_i) x_i P'(X) \frac{\partial x^*_j}{\partial s_i}\) represents the cross rent-shifting effect. Export subsidy to the domestic firm, through the standard rent-shifting effect, increases its profit, but it leads to an increase in the dividend given to the rival firm. This effect becomes smaller in absolute terms as the domestic firm’s own shares increases. In other words, the higher the domestic firm’s own share, the less the government cares about the outflow of the domestic firm’s rent due to the cross rent-shifting effect.

The second term \(-(1 - \sigma_i) x_i\) shows the subsidy outflow effect, for it represents the portion of the subsidy expense going abroad as increased dividend to the foreign residents. Further, its absolute value decreases as the domestic firm’s own share increases.

The last term \((1 - \sigma_j) x_j P'(X) \frac{\partial x^*_i}{\partial s_i}\) shows the dividend suppression effect, for it represents the portion of the decrease in the dividend received from the shared rival firm.

The above three effects weaken the subsidy incentives in the presence of cross-country shareholding.

From (2-21) in the linear demand function, (4-3) can be written as

\[
0 = \frac{\partial W^*_i}{\partial s_i}(s, \sigma) = \frac{1}{3} [(4\sigma_i - 3)x_i - 2s_i - 2(1 - \sigma_j)x_j].
\]

Let \(R^C_i(s_j, \sigma)\) represent country \(i\)’s reaction function. Since no changes in the cross-country shareholding structure affect the comparative statics results for the market outcomes, it is straightforward to derive the effect of a change in the cross-country shareholding

\[\text{Welzel (1995) and Dick (1993) also identified three similar effects on the optimal export subsidization. Here, these three effects are used to conduct a comparison with the cross-firm shareholding case for later analysis.}\]
structure on the optimal response subsidy. In view of the SOC for national welfare maximization, an application of the implicit function theorem using the result in (4-5) yields

\[
\frac{\partial R^C(s_j, \sigma)}{\partial \sigma_i} = -\frac{\partial^2 W^*_i C}{\partial s_i^2} \frac{\partial^2 W^*_i C}{\partial \sigma_i^2} = \frac{4}{3} x_i > 0.
\]

An increase in the domestic residents’ share over the domestic firm makes the government care more about the profit of the domestic firm, and thereby, strengthens the government’s strategic export subsidy incentive.

Similarly, an increase in the foreign ownership of foreign firm leads to

\[
\frac{\partial R^C(s_j, \sigma)}{\partial \sigma_j} = -\frac{\partial^2 W^*_i C}{\partial s_i^2} \frac{\partial^2 W^*_i C}{\partial \sigma_j^2} = \frac{2}{3} x_j > 0
\]

which implies that when there is a decrease in the domestic residents’ claim over the foreign firm’s profits, the domestic country’s government then cares less about the decrease in the foreign firm’s profits, which gives rise to a stronger export subsidy incentive a la Brander-Spencer.

I summarize the above results into the following Lemma.

**Lemma 4.1.** An increase in the residents’ ownership share in the domestic firm induces both governments to increase the strategic export subsidy rates.

The full-game Nash equilibrium subsidy is a solution to the following reaction function.

\[
R^C(s_j, \sigma) = \frac{4\sigma_j - 4\sigma_i - 1}{10 - 8\sigma_i + 2\sigma_j} s_j + \frac{(4\sigma_i - 3)\beta_i - 2(1 - \sigma_j)\beta_j}{10 - 8\sigma_i + 2\sigma_j},
\]

where \( \beta_i = a - 2c_i + c_j (i, j = 1, 2; j \neq i) \) is defined in Section 2.5.

The equilibrium subsidy, denoted as \( s^C_i(\sigma, \sigma_j) \) depends on the cross-country shareholding structure.

\[
s^C_i(\sigma) = \frac{4(10\sigma_i + 4\sigma_j - 6\sigma_i\sigma_j - 7)\beta_i + (8\sigma_i + 20\sigma_j - 12\sigma_i\sigma_j - 17)\beta_j}{3(33 - 20\sigma_1 - 20\sigma_2 + 12\sigma_1\sigma_2)}. \tag{4-6}
\]

**Proposition 4.1.** The equilibrium export subsidy rate under the cross-country shareholding structure is strictly lower than that in its absence, that is, \( s^C_i(\sigma) < s^C_i(1) = s^B_i \).

**Proof:** See Appendix 4.A.

To clarify how the cross-country shareholding structure governs the equilibrium export subsidy profile, let me focus on the marginal subsidy rate when \( s_i = 0 \) and investigate what factors induce the government to impose export subsidies. In view of (4-5), it follows that

\[
\frac{\partial W^*_i C(s, \sigma)}{\partial s_i} \bigg|_{s_i=0} = 4\sigma_i x_i + 2\sigma_j x_j - 3x_i - 2x_j. \tag{4-7}
\]
Denote the market share of firm $i$ as $\theta_i = x_i/X$ ($i = 1, 2$) where $\theta_1 + \theta_2 = 1$. Thus the government has an incentive to subsidize the domestic firm if and only if $4\theta_i\sigma_i + 2\theta_j\sigma_j - (3 - \theta_j) > 0$. (4-7) also yields the following lemma specific to the cross-country shareholding structure.

**Lemma 4.2.** Under the cross-country shareholding structure, the government’s subsidy incentive is dependent on market share $\theta_i$.

Figure 4.1 that allows the pairs of the shares letting both countries to find a zero subsidy optimal with respect to (4-7), where curve $S_1S_1'$ stands for country 1 and curve $S_2S_2'$ for country 2 in the case of symmetric cost conditions. The unit square $[\frac{1}{2}, 1] \times [\frac{1}{2}, 1]$ is divided into four areas, each representing both governments’ incentives for subsidization or taxation. Figure 4.1 implies that both governments have an incentive to provide a subsidy only when the domestic residents’ share of the domestic firm is sufficiently high as represented by the shaded area. The BS model corresponds to the top corner $(1, 1)$. In the presence of cross-country shareholding, each government may rather tax the domestic firm even though the residents’ equity share constitutes over half of the total share. In fact, when the residents’ share is half for both firms, the governments would definitely tax the exports. This is exactly the case when there is a single exporting country holding two firms. When the two domestic firms compete in the export market, the competition becomes excessive from the viewpoint of joint profit maximization. The government thus has an incentive to suppress, rather than promote the competition so as to maximize national welfare.
4.4 Cross-Firm and Mixed Shareholding Equilibrium

Next, I consider the cross-firm shareholding case in which the equities of a firm are mutually owned by the shareholders of both firms. In the same notations, the heterogeneous cross shareholding rates of the firms are denoted by $\sigma = (\sigma_1, \sigma_2)$.

Firm $i$’s value function under cross-firm shareholding is defined as the sum of fractional profits of both firms accruing to the domestic shareholders and is denoted as:

$$V^{iF}(x, s, \sigma) = \sigma_i \pi^i(x, s_i) + (1 - \sigma_j) \pi^j(x, s_j) \quad (i, j = 1, 2; \ j \neq i),$$

where superscript $F$ shows the variables associated with the values under the international cross-firm shareholding.

Strictly speaking, cross-firm shareholding is a special cross-country shareholding structure since the shareholders of both firms are the residents of both countries.\(^5\) In order to distinguish between both shareholding structures, the cross-country shareholding discussed in the previous section is defined strictly as the mutual shareholding structure among residents who are irrelative to the shareholding of the rival firms. However, due to the capital liberalization in the stock market, a fraction of the shares of firms can be owned not only by the foreign rival firms’ shareholders, but also by the irrelative foreign residents. Separating the cross-country and cross-firm shareholding structures is not realistic. In this chapter, I further develop a mixed cross shareholding structure in which the foreign investors are constituted by foreign individual residents and foreign firm’s shareholders. I use an exogenous variable $\alpha_i \in [0, 1]$ to represent the weight of the foreign shareholding portion owned by firm $i$’s shareholders. Therefore, under the mixed shareholding structure, when a fraction of firm $j$’s shares are not only owned by firm $i$’s shareholders, but also by the irrelative country $i$’s residents. Firm $i$’s value function is given by

$$V^{i\alpha}(x, s, \sigma, \alpha_i) = \sigma_i \pi^i(x, s_i) + \alpha_i(1 - \sigma_j) \pi^j(x, s_j) \quad (i, j = 1, 2; \ j \neq i).$$

The superscript $\alpha$ represents the variables associated with the values under the mixed shareholding structure. When $\alpha_i = 1$, firm $j$’s foreign shareholding is entirely owned by firm $i$’s shareholders, which is the cross-firm shareholding. When $\alpha_i = 0$, firm $j$’s foreign shareholding portion is entirely constituted by country $i$’s residents, which is the cross-country shareholding. The greater the value of $\alpha_i$, the higher the weight of foreign firms’ shareholding. Using the form of mixed shareholding defined in (4-8) enables me to analyze both the cross-country and cross-firm shareholding structures and compare the equilibrium values with respect to $\alpha_i$.

\(^5\)Considering shareholders from a third country is out of the scope of my analysis.
4.4.1 Second-Stage Equilibrium

Denote each firm’s reaction function under mixed shareholding as \( r^{ia}(x_j, s_i, \sigma, \alpha_i) \). Given that the SOC is satisfied,\(^6\) the FOC for maximizing (4-2) with respect to its own output yields

\[
0 = \frac{\partial V^{ia}(r^{ia}(x_j, \sigma, s_i), x_j, s, \sigma, \alpha_i)}{\partial x_i} \\
= \sigma_i \left( P(X) - c_i + s_i + r^{ia}(.)P'(X) \right) + \alpha_i(1 - \sigma_j)x_jP'(X) \\
= \sigma_i \left( a - 2r^{ia}(.) - x_j - c_i + s_i \right) - \alpha_i(1 - \sigma_j)x_j. 
\]

(4-9)

Solving for each firm’s reaction function yields

\[
r^{ia}(x_j, \sigma, s_i, \alpha_i) = \frac{1}{2} \left[ a - c_i + s_i - (1 + \alpha_i\mu_i)x_j \right] \quad (i, j = 1, 2; \ j \neq i), 
\]

(4-10)

where \( \mu_i(\sigma) \stackrel{\text{def}}{=} \frac{1 - \sigma_i}{\sigma_i} \) denotes the relative shareholding ratio of firm \( i \). Note that Assumption 4.1 assures \( \mu_i \in (0, 1) \). Under cross-country shareholding, when \( \alpha_i = 0 \), each firm’s reaction function is independent of the cross shareholding structure \((\sigma_1, \sigma_2)\) as shown in (2-19). With the presence of foreign firm’s shareholding, each firm’s best response actually depends on the cross shareholding structure \((\sigma_1, \sigma_2)\) and the weight of the foreign firm’s shareholding \( \alpha_i \). For the convenience of later analysis, one can show the following properties of each firm’s reaction function by using the implicit function theorem in (4-10).

\[
r^{ia}_{x}(x_j, \sigma, s_i, \alpha_i) \stackrel{\text{def}}{=} \frac{\partial r^{ia}}{\partial x_j} = -\frac{1 + \alpha_i\mu_i}{2} < 0, 
\]

(4-11)

\[
r^{ia}_{s}(x_j, \sigma, s_i, \alpha_i) \stackrel{\text{def}}{=} \frac{\partial r^{ia}}{\partial s_i} = \frac{1}{2} > 0, 
\]

(4-12)

\[
r^{ia}_{\sigma_i}(x_j, \sigma, s_i, \alpha_i) \stackrel{\text{def}}{=} \frac{\partial r^{ia}}{\partial \sigma_i} = \frac{\alpha_i(1 - \sigma_j)x_j}{2(\sigma_i)^2} > 0, 
\]

(4-13)

\[
r^{ia}_{\sigma_j}(x_j, \sigma, s_i, \alpha_i) \stackrel{\text{def}}{=} \frac{\partial r^{ia}}{\partial \sigma_j} = \frac{\alpha_ix_j}{2\sigma_i} > 0, 
\]

(4-14)

\[
r^{ia}_{\alpha_i}(x_j, \sigma, s_i, \alpha_i) \stackrel{\text{def}}{=} \frac{\partial r^{ia}}{\partial \alpha_i} = -\frac{\mu_ix_j}{2} < 0. 
\]

(4-15)

**Lemma 4.3.** Given \( \alpha_i \in [0, 1] \), each firm’s reaction function under cross shareholding structure satisfies as following.

(i) Each firm’s optimal output is a strategic substitute to the other’s.

---

\(^6\)The SOC for the value function to be strictly concave in its own output can be verified as follows.

\[
\frac{\partial^2 V^{ia}(x, s, \sigma, \alpha_i)}{\partial x_i^2} = -2\sigma_i < 0.
\]
(ii) An increase in the subsidy rate increases each firm’s best-response output.

(iii) An increase in any firm’s own share $\sigma_1$ or $\sigma_2$ increases each firm’s best-response output.

(iv) An increase in the weight of the foreign firm’s shareholding $\alpha_i$ reduces each firm’s best-response output.

Results in Lemma 4.3 (i) and (ii) are satisfied under both the cross-country and cross-firm shareholding structures. The intuition behind (iii) that is attributed to cross-firm shareholding can be explained as follows. An increase in $\sigma_1$ or $\sigma_2$ raises the weight of the domestic firm’s marginal profit or reduces the weight of the rival firm’s marginal profit in the FOC defined in (4-9). For the FOC to satisfy, the marginal profit of the rival firm should decrease and the domestic firm’s output should increase. (iv) shows that cross-firm shareholding is likely to facilitate collusion with the shared firms. Since foreign firms’ profits also constitute a part of the market value of the firm under cross-firm shareholding, each firm takes into account the effect of the domestic firm’s output to suppress the shared foreign firms’ profits.

The equilibrium under cross shareholding is globally stable in the standard Cournot output adjustment process. After imposing (4-11),

$$\Delta^a(\sigma, \alpha) \overset{\text{def}}{=} 1 - r_i^{\alpha}r_j^{\alpha} = \frac{1}{4}(3 - \alpha_i\mu_i - \alpha_j\mu_j - \alpha_i\alpha_j\mu_i\mu_j) > 0 \quad (4-16)$$

is satisfied in view of Assumption 4.1. Hence, for all the relevant equilibria, the condition for the stability process is satisfied under the mixed cross shareholding structure when $\alpha_i \in [0, 1]$.

### 4.4.2 Firms’ Best-Response Outputs

From the comparative statics analysis in (4-15), the best-response output of each firm declines with an increase in the weight of foreign firm’s shareholding $\alpha_i$. For the case under the cross-firm shareholding, each firm’s output increase suppresses the shared foreign firm’s profit and its best-response output is no larger than that under cross-country shareholding. The equilibrium output is equal under both shareholding structures only when (i) the shared rival firm takes no production as $x_j = 0$, or (ii) each firm fully holds the domestic firm’s shares as $\sigma_i = 1(i = 1, 2)$. Thus, the cross-firm shareholding structure facilitates collusion with the shared firms.

Summarize the above results into the following lemma:

**Lemma 4.4.** Each firm’s best-response output under cross-firm shareholding is no larger than that under cross-country shareholding. More specifically, given $(\sigma_i, \sigma_j)$ and $x_j > 0$

$$r_i^{\alpha}(x_j, \sigma_i, s_i) < r_i^{\alpha}(x_j, \sigma, s_i, \alpha_i) < r_i^{\alpha}(x_j, s_i),$$

where $r_i^{\alpha}(x_j, \sigma, s_i) \overset{\text{def}}{=} r_i^{\alpha}(x_j, \sigma, s_i, 1)$ and $r_i^{\alpha}(x_j, s_i) \overset{\text{def}}{=} r_i^{\alpha}(x_j, \sigma, s_i, 0)$. 

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Figure 4.2 illustrates the related results in Lemma 4.4. \( r^{iC}x_i^m \) and \( r^{iF}x_i^m \) denote the downward sloping reaction curves under cross-country and cross-firm shareholding, respectively. \( r^{i\alpha}x_i^m \) represents the reaction curve under mixed cross shareholding and lies between \( r^{iC}x_i^m \) and \( r^{iF}x_i^m \). All the reaction curves start from the same output when the firm is a monopolistic producer in the market. Given the rival firm’s output decision, the best-response output of each firm is always larger under cross-country shareholding than that under cross-firm shareholding. Points \( N^C(x_C^1, x_C^2), N^\alpha(x_\alpha^1, x_\alpha^2), \) and \( N^F(x_F^1, x_F^2) \) in the figure denote the corresponding equilibrium outputs, respectively.

\[ x_i^\alpha(s, \sigma, \alpha) = \frac{2(a - c_i + s_i) - (1 + \alpha_i\mu_i)(a - c_j + s_j)}{4\Delta^\alpha(\sigma, \alpha)}. \] (4-17)
Differentiating (4-17) with respect to $s_i$ and $\alpha_i$ yields the following results.

$$\frac{\partial x^*_{i}(s, \sigma, \alpha)}{\partial s_i} = \frac{1}{2\Delta^\alpha} > 0,$$

$$\frac{\partial x^*_{j}(s, \sigma, \alpha)}{\partial s_i} = -\frac{1 + \alpha_j \mu_j}{4\Delta^\alpha} < 0,$$

$$\frac{\partial x^*_{i}(s, \sigma, \alpha)}{\partial \alpha_i} = -\frac{\mu_i x_j}{2\Delta^\alpha} < 0,$$

$$\frac{\partial x^*_{j}(s, \sigma, \alpha)}{\partial \alpha_i} = \frac{(1 + \alpha_j \mu_j) \mu_i x_j}{4\Delta^\alpha} < 0,$$

where use was made of (4-16).

Given the cross shareholding structure $(\sigma, \alpha)$, (4-18) and (4-19) show that the standard effect of strategic export subsidization is unchanged. In (4-20) and (4-21), an increase in the weight of foreign firm’s shareholding $\alpha_i$ decreases the domestic firm’s optimal output, but increases the foreign firm’s optimal output. However, the total effects of cross-firm shareholding structure on the individual firm’s equilibrium output is ambiguous.

Moreover, the equilibrium total output under mixed cross shareholding can be derived as follows.

$$X^*(s, \sigma, \alpha) = \frac{(1 - \alpha_2 \mu_2)(a - c_1 + s_1) + (1 - \alpha_1 \mu_1)(a - c_2 + s_2)}{4\Delta^\alpha(\sigma, \alpha)}$$

$$= \sum_{k=1,2} \frac{(1 - \alpha_k \mu_k)(a - c_k + s_k)}{4\Delta^\alpha(\sigma, \alpha)}.$$

Note that the equilibrium total output depends on the sum of the subsidies for the countries under cross-country shareholding when $\alpha_1 = \alpha_2 = 0$.

Differentiation with $\alpha_i$ yields the following result.

$$\frac{\partial X^*(s, \sigma, \alpha)}{\partial \alpha_i} = \frac{\partial x^*_{i}(s, \sigma, \alpha)}{\partial \alpha_i} + \frac{\partial x^*_{j}(s, \sigma, \alpha)}{\partial \alpha_i} = -\frac{(1 - \alpha_j \mu_j) \mu_i x_j}{4\Delta^\alpha(\sigma, \alpha)} < 0.$$  

(4-22)

Thus, increasing the weight of the foreign firm’s shareholding always reduces the equilibrium output, that is,

$$X^C(s) > X^*(s, \sigma, \alpha) > X^F(s, \sigma),$$

where $X^C(s) \overset{\text{def}}{=} X^*(s, \sigma, 0)$ and $X^F(s, \sigma) \overset{\text{def}}{=} X^*(s, \sigma, 1)$. Such results can be summarized into the following Proposition.

**Proposition 4.2.**

P4.2.1 The equilibrium total output and market price depend on the sum of the subsidy-inclusive unit costs over the industry under cross-country shareholding when $\alpha_i = 0(i = 1, 2)$.

P4.2.2 Given the same subsidy rate, the equilibrium total output under cross-country shareholding is always larger than that under cross-firm shareholding.
4.4.4 Firms’ Equilibrium Profits

The equilibrium profit of each firm can be expressed as below.

\[ \pi^*_i(s, \sigma, \alpha) = [P(X^*\alpha(s, \sigma, \alpha)) - c_i + s_i]x^*_i(s, \sigma, \alpha). \]

Differentiating the above equation with respect to \( s_i \) yields

\[
\frac{\partial \pi^*_i(s, \sigma, \alpha)}{\partial s_i} = (P(X) - c_i + s_i + x_iP'(X)) \frac{\partial x^*_i}{\partial s_i} + x_i \frac{\partial x^*_j}{\partial s_i} + x_i \\
= \alpha_i \mu_j x_j \frac{\partial x^*_i}{\partial s_i} - x_i \frac{\partial x^*_j}{\partial s_i} + x_i > 0, \tag{4-23}
\]

where use was made of (4-9), (4-18), and (4-19). The above results yield the following Proposition.

**Proposition 4.3.** Under international mixed cross shareholding structure, an increase in the subsidy rate (i) increases the domestic firm’s equilibrium output and equilibrium profit, but (ii) reduces the other firm’s equilibrium output and profit.

4.4.5 Government’s Subsidy Incentive

Next solve the resulting equilibrium subsidy. In the first stage, each government makes its own choice over subsidy by predicting the resulting second-stage equilibrium. Under international cross shareholding, the national welfare is measured by the value accrued to the domestic shareholders minus the total subsidy payment made by the government. Country \( i \)'s welfare function is given by

\[ W^*_i(s, \sigma, \alpha) \overset{\text{def}}{=} \sigma_i \pi^*_i(s, \sigma, \alpha) + (1 - \sigma_j) \pi^*_j(s, \sigma, \alpha) - s_i x^*_i(s, \sigma, \alpha). \]

Note that although the welfare function has the same expression as the one under cross-country shareholding, the equilibrium output and profit are actually dependent on the weight of the foreign firm's shareholding (\( \alpha_i, \alpha_j \)), and so each country’s welfare function is also dependent on (\( \alpha_i, \alpha_j \)).

After a little manipulation by using (4-9), the FOC for welfare maximization yields

\[
0 = \frac{\partial W^*_i(s, \sigma, \alpha)}{\partial s_i} \\
= \sigma_i \frac{\partial \pi^*_i}{\partial s_i} - x_i - s_i \frac{\partial x^*_i}{\partial s_i} + (1 - \sigma_j) \frac{\partial \pi^*_j}{\partial s_i} \\
= -(1 - \sigma_i)(1 - \sigma_j)x_j \frac{\partial x^*_i}{\partial s_i} - (1 - \sigma_i)x_i - \sigma_i(1 - \mu_i \mu_j \alpha_j)x_i \frac{\partial x^*_i}{\partial s_i} - s_i \frac{\partial x^*_i}{\partial s_i}, \tag{4-25}
\]

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where use was made of (4-23) and (4-24).

Similar to the case of cross-country shareholding, the additional incentive terms under the mixed cross shareholding can be extracted as below.

\[ I_i^a(s, \sigma, \alpha) = (1 - \sigma_i)x_i \frac{\partial x_j^{s\sigma}}{\partial s_i} - (1 - \sigma_j)x_j \frac{\partial x_i^{s\sigma}}{\partial s_i} + \alpha_j \mu_j(1 - \sigma_j)x_i \frac{\partial x_j^{s\sigma}}{\partial s_i} + \alpha_i(1 - \sigma_j)x_j \frac{\partial x_i^{s\sigma}}{\partial s_i}. \]  

(4-26)

The first three terms on the first line represent the **cross rent-shifting effect**, **subsidy-outflow effect**, and **dividend-suppression effect** shown in the cross-country shareholding case in the previous section, respectively. All of them are not dependent on the foreign firm’s shareholding ratio \((\alpha_1, \alpha_2)\).

The last two terms show the additional strategic subsidization effects in the presence of mixed cross shareholding. The fourth term \(\mu_j \alpha_j(1 - \sigma_j)x_i \frac{\partial x_j^{s\sigma}}{\partial s_i}\) represents the magnified **cross rent-shifting effect**, and the fifth term \(\alpha_i(1 - \sigma_j)x_j \frac{\partial x_i^{s\sigma}}{\partial s_i}\), the minified **dividend suppression effect**. Due to the collusive effect under cross-firm shareholding, a further decrease in the foreign firm’s outputs increases the domestic firm’s profit through the rent-shifting effect; however, it also increases the dividend outflow to the foreign firm. Meanwhile, a decrease in the domestic firm’s output mitigates the foreign firm’s profit reduction and thereby increases the dividend inflow to the domestic firm.

Under cross-firm shareholding when \(\alpha_1 = \alpha_2 = 1\), the additional incentive effect in (4-26) can be simplified as below.

\[ I_i^F(s, \sigma) = \mu_j x_i \frac{\partial x_j^{s\sigmaF}}{\partial s_i} - (1 - \sigma_i)x_i. \]  

(4-27)

A comparison with (4-4) under cross-country shareholding yields some insights as given below. First, the **dividend suppression effect** vanishes, for the effect is already taken into account by the domestic firm’s output decision in value maximization; the foreign firm’s receipt of subsidy increases the domestic firm’s value through an increase in dividend. Second, the **subsidy outflow effect** remains intact, though its value may differ due to a change in the equilibrium output. Last, the **cross rent-shifting effect** is magnified by the factor \(1/\sigma_{jj}\). This multiplier effect is peculiar to cross-firm shareholding.

When the marginal subsidy rate is set as \(s_i = 0\), the FOC for national welfare maximization under cross-firm shareholding yields the following result:

\[
\left. \frac{\partial W_i^{sF}(s, \sigma)}{\partial s_i} \right|_{s_i=0} = \frac{\sigma_i + \sigma_j - 1}{\sigma_j} x_i P'(X) \frac{\partial x_j^{s\sigmaF}}{\partial s_i} - (1 - \sigma_i)x_i
\]

\[
= \left( \frac{\sigma_i(\sigma_j - \sigma_i + 1)}{\sigma_j(\sigma_i + \sigma_j + 1)} - (1 - \sigma_i) \right) x_i
\]

\[
= \frac{\psi_i^F(\sigma)}{\sigma_j(\sigma_i + \sigma_j + 1)} x_i,
\]

\[7\] The second-order condition for welfare maximization is satisfied. See Appendix 4.B.
which follows from (4-16) and (4-19). $\psi_i^F(\sigma)$ is defined as

$$\psi_i^F(\sigma) := \sigma_i\sigma_j(\sigma_i + \sigma_j - 1) + (\sigma_i - \sigma_j)(\sigma_j - \sigma_i + 1),$$

which shows that the government’s subsidy incentive depends only on mutual shareholding structure $(\sigma_1, \sigma_2)$.

Under the symmetric cross-firm shareholding structure, as $\sigma_1 = \sigma_2 = \sigma$,

$$\psi_i^F(\sigma, \sigma) = (\sigma)^2(2\sigma - 1) > 0,$

where $\sigma > \frac{1}{2}$ in Assumption 4.1. Thus, it is straightforward to establish the following proposition.

**Proposition 4.4.** Under cross-firm shareholding,

P4.4.1 Each government’s incentive for export subsidies is independent of the cost conditions of the firms.

P4.4.2 When the percentage share of each firm’s holding of the other firm’s equity is equal as $\sigma_1 = \sigma_2 = \sigma$, each country has an incentive to subsidize its domestic firm given $\sigma > \frac{1}{2}$.

Then, what about the subsidization incentive under both equity structures? Intuitively, under cross-firm shareholding, each firm has to take into account the shared firm’s profit and has less incentive to increase its own output. The oligopolistic competition becomes milder, and thereby, the government has stronger incentive to subsidize its exports to maximize national welfare.

From (4-7) and (4-28), Figure 4.3 depicts both shareholding structures under the symmetric cost structure as $c_1 = c_2 = c$. The shaded areas show the positive subsidy incentives. From Figure 4.3, it is obvious that under cross-firm shareholding, each country’s government has stronger incentive to subsidize its own firm than under cross-country shareholding.

Thus, the following Proposition can be established.

**Proposition 4.5.** Under cross-firm shareholding, each government has stronger incentive to subsidize its own exports than under cross-country shareholding structure.

**Proof:** See Appendix 4.C. □
4.4.6 Optimal Subsidy under Symmetric Cost and Shareholding Structure

Let $R^i_\alpha$ be a solution to (4-25), which represents country $i$’s reaction function under the mixed cross shareholding structure. Then, the full-game Nash Equilibrium subsidy profile denoted as $s^i_\alpha(\sigma, \alpha)$ is thus a solution to

$$s^i_\alpha(\sigma, \alpha) = R^i_\alpha(s^j_\alpha(\sigma, \alpha), \sigma, \alpha).$$

Under the symmetric cost and shareholding structure, when $c_1 = c_2 = c$, $\sigma_1 = \sigma_2 = \sigma$ and $\alpha_1 = \alpha_2 = \alpha$,

$$R^i_\alpha(s_j, \sigma, \alpha) = -\frac{(1 + \mu)(1 - \alpha\mu)s_j + (1 + \alpha\mu^2 + 3\alpha\mu - 5\mu)(a - c)}{2(2 + 5\mu + \mu^2)},$$

where use was made of (4-17), (4-18), and (4-19).

The properties of the above reaction function are shown as follows.

$$\frac{\partial R^i_\alpha(s_j, \sigma, \alpha)}{\partial s_j} = -\frac{(1 + \mu)(1 - \alpha\mu)}{2(2 + 5\mu + \mu^2)} < 0,$$

$$\frac{\partial R^i_\alpha(s_j, \sigma, \alpha)}{\partial \alpha} = \frac{2\mu[2 + 6\mu + \mu^2(1 - \alpha\mu)] + 4\mu(5\mu + 3)(\mu + 1)(a - c)}{4(2 + 5\mu + \mu^2)^2} > 0.$$

Each country’s reaction curve is downward sloping. An increase in the weight of the foreign firm’s shareholding increases each country’s best-response subsidy. Given $s_j$, country $i$’s
best-response subsidy under cross-firm shareholding is always larger than under cross-
country shareholding, that is, \( R^\alpha(s_j, \sigma, 0) = R^{iC}(s_j, \sigma) < R^{iF}(s_j, \sigma) = R^\alpha(s_j, \sigma, 1) \). This
is because under cross-firm shareholding, the firms behave more collusively, the equilibrium
outputs are suppressed, and the governments are likely to provide a higher subsidy rate.

Solving for the optimal subsidies yields

\[
s_i^\alpha(\sigma, \alpha) = \frac{1 - 5\mu + 3\alpha\mu + \alpha\mu^2}{5 + 11\mu - \alpha\mu + \alpha\mu^2}(a - c).
\]

An increase in the weight of the foreign firm’s shareholding always increases each coun-
try’s subsidy, which is given by

\[
\frac{\partial s_i^\alpha(\sigma, \alpha)}{\partial \alpha} = \frac{16(1 + \mu)^2}{(5 + 11\mu - \alpha\mu + \alpha\mu^2)^2} > 0.
\]

It is straightforward to show that under cross-firm shareholding, each country’s optimal
subsidy is always higher than that under cross-country shareholding given the symmetric
structures. In more detail,

\[
s_i^C(\sigma) = s_i^\alpha(\sigma, 0) = \frac{6\sigma - 5}{11 - 6\sigma} \(a - c\),
\]

\[
s_i^\alpha(\sigma, \alpha) = \frac{6\sigma^2 - 5\sigma + \alpha(1 - \sigma(1 + 2\sigma))}{-6\sigma^2 + 11\sigma + \alpha(1 - \sigma)(1 - 2\sigma)} \(a - c\),
\]

\[
s_i^F(\sigma) = s_i^\alpha(\sigma, 1) = \frac{(2\sigma - 1)^2}{-4\sigma^2 + 8\sigma + 1} \(a - c\).
\]

In Figure 4.4, the horizontal axis represents each firm’s share owned by the domestic
residents. The vertical axis represents the subsidy rate. Under cross-country sharehold-
ing, the optimal subsidy denoted as \( s_i^C(\sigma) \) is positive only when the domestic residents’
shareholding is very large. Under cross-firm shareholding, the optimal subsidy denoted as
\( s_i^F(\sigma) \) is always nonnegative and larger than that under cross-country shareholding. The
optimal subsidy under mixed cross shareholding denoted as \( s_i^\alpha(\sigma, \alpha) \) lies between these
two curves. The three curves intersect at \( s_i^B \) when \( \sigma_1 = \sigma_2 = 1 \). Therefore, the higher
the foreign shareholding owned by the rival firm’s shareholders, the stronger the country’s
subsidization incentive.
4.5 Welfare Implication under Cross-Country vs. Cross-Firm Shareholding

This section examines the national welfare and world welfare under both mutual shareholding structures and discusses the subsidy competition effects. Here $W_1 (= W_2)$ denotes the exporting country’s welfare, $W_3$ the importing country’s welfare and $W_T$ the world welfare. The cost structure and mutual shareholding structure is symmetric as $c_1 = c_2 = c$, $\sigma_{11} = \sigma_{22} = \sigma$. The results are summarized into the following table and figures.

<table>
<thead>
<tr>
<th></th>
<th>Cross-Country</th>
<th>Cross-Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. Output</td>
<td>$x^{C^*}(\sigma, 0) = \frac{1}{3} (a - c)$</td>
<td>&gt; $x^{F^*}(\sigma, 0) = \frac{\sigma}{2\sigma + 1} (a - c)$</td>
</tr>
<tr>
<td></td>
<td>$x^{C^*}(\sigma, s^C(\sigma)) = \frac{1}{11 - 6\sigma} (a - c)$</td>
<td>&lt; $x^{F^*}(\sigma, s^F(\sigma)) = \frac{2\sigma}{11\sigma - 8\sigma + 1} (a - c)$</td>
</tr>
<tr>
<td>Ex. Country $W_1^{C^*}(\sigma, 0) = \frac{1}{9} (a - c)^2$</td>
<td>&lt; $W_1^{F^*}(\sigma, 0) = \frac{(2\sigma + 1)^2}{9} (a - c)^2$</td>
<td></td>
</tr>
<tr>
<td>Im. Country $W_3^{C^*}(\sigma, s^C(\sigma)) = \frac{2(7 - 6\sigma)}{(11 - 6\sigma)^2} (a - c)^2$</td>
<td>&gt; $W_3^{F^*}(\sigma, s^F(\sigma)) = \frac{2\sigma(-4\sigma^2 + 4\sigma + 1)}{(11\sigma + 8\sigma + 1)^2} (a - c)^2$</td>
<td></td>
</tr>
<tr>
<td>World Welfare $W_T^{C^*}(\sigma, 0) = \frac{4(a - c)^2}{9}$</td>
<td>&gt; $W_T^{F^*}(\sigma, 0) = \frac{2\sigma(\sigma + 1)}{(2\sigma + 1)^2} (a - c)^2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$W_T^{C^*}(\sigma, s^C(\sigma)) = \frac{12(3 - 2\sigma)}{(11 - 6\sigma)^2} (a - c)^2$</td>
<td>&lt; $W_T^{F^*}(\sigma, s^F(\sigma)) = \frac{4\sigma(-4\sigma^2 + 6\sigma + 1)}{(11\sigma + 8\sigma + 1)^2} (a - c)^2$</td>
</tr>
</tbody>
</table>
4.5.1 Exporting Country

Analyses for the exporting country welfare in Figure 4.5 yield the following results:

1. In the case of cross-country shareholding without subsidy provision, each exporting country’s welfare is constant.

2. In the other cases, an increase in the foreign shareholding ratio $1 - \sigma$ improves each exporting country’s welfare.

3. $W_{1^*}^F(\sigma, 0) > W_{1^*}^C(\sigma, 0)$. Without subsidy competition, cross-firm shareholding improves each exporting country’s welfare. Since the exporting firms behave more collusively, cross-firm shareholding benefits the exporters’ national welfare.

4. $W_{1^*}^F(\sigma, s^F(\sigma)) < W_{1^*}^C(\sigma, s^C(\sigma))$. With subsidy competition, cross-firm shareholding worsens each exporting country’s welfare due to the higher subsidy payments.

4.5.2 Importing Country

Analyses for the importing country welfare in Figure 4.6 yield the following results:

1. In the case of cross-country shareholding without subsidy provision, the importing country’s welfare is constant.

2. In the other cases, an increase in the foreign shareholding ratio $1 - \sigma$ deteriorates the importing country’s welfare.

3. $W_{3^*}^F(\sigma, 0) < W_{3^*}^C(\sigma, 0)$. Without subsidy competition, cross-firm shareholding worsens the importing country due to the exporting firm’s collusive behavior.

4. $W_{3^*}^F(\sigma, s^F(\sigma)) > W_{3^*}^C(\sigma, s^C(\sigma))$. With subsidy competition, cross-firm shareholding improves the importing country’s welfare from the higher subsidy benefits.
4.5.3 World Welfare

Analyses for world welfare in Figure 4.7 yield the following results:

1. In the case of cross-country shareholding without subsidy provision, world welfare is constant.

2. In the other cases, an increase in the foreign shareholding ratio \((1 - \sigma)\) deteriorates world welfare.

3. Without subsidy competition, cross-firm shareholding makes world welfare worse off. Due to the collusively lower total output, cross-firm shareholding structure should be banned or regulated as shown in the traditional industrial organization theory.

4. With governments’ subsidy competition, the equilibrium output under cross-firm shareholding is larger than that under cross-country shareholding. Firms’ collusion does not occur and world welfare is improved. Therefore, cross-firm shareholding structure should not always be banned or regulated. With governments’ subsidy provision, cross-shareholding structure should be encouraged between exporting firms.

Proposition 4.6. Without subsidy competition, cross-firm shareholding structure makes world welfare worse off due to the collusive behavior of the exporting firms. However, with subsidy competition, cross-firm shareholding structure leads to higher subsidy rates and makes world welfare better off.
4.6 Concluding Remarks

This chapter examines how the market performances and strategic export subsidy incentives are affected by the international mutual shareholding structures, especially under cross-country and cross-firm shareholding structure. The new findings can be summarized as follows.

First, the market outcome is independent of the cross-country shareholding structure. However, under the cross-firm shareholding, since the foreign firm’s profits also constitute a fraction of the firm’s market value, each firm should take into account its output decision on the foreign firm’s profit. This suggests that cross-firm shareholding actually plays a role in determining the output decisions of the firms and makes the firms behave more collusively.

Second, this chapter clarifies that new sources of strategic subsidy incentives are different from that in the standard export subsidy model without mutual shareholding. Additional subsidy incentive effects dampen the traditional rent extraction of export subsidization and mitigate the subsidy competition between the two exporting countries.

Third, compared with cross-country shareholding, the cross-firm shareholding is more likely to strengthen each government’s subsidy incentive, and it also improves world welfare. Hence, cross-firm shareholding should not always be regulated or banned with government subsidization.

Finally, it would be interesting to examine how strategic subsidy incentives are affected when cross shareholding structures \((\sigma_i, \sigma_j)\) are endogenously determined and take into account the timing of the game. Further, the model can be extended from an exporting firm to a multinational firm with a subsidiary in the foreign country. When the subsidiary is internationally owned, one can examine the export competition in the third country or foreign country. These are the topics for future research.
Appendix

4.A Optimal Subsidy under Cross-Country Shareholding

When $\sigma_1 = \sigma_2 = 1$, it is the BS model. The optimal subsidy rate is shown by

$$s_i^C(1) = \frac{4\beta_i - \beta_j}{15} > 0 \quad \text{if} \quad x_i^C > 0. \quad (2-13)$$

Comparing with $s_i^C(\sigma_i, \sigma_j)$ in (4-6) leads to

$$\delta(\sigma) = s_i^C(\sigma_i, \sigma_j) - s_i^C(1, 1)$$

$$\propto 20(10\sigma_i + 4\sigma_j - 6\sigma_i\sigma_j - 7)\beta_i + 5(8\sigma_i + 20\sigma_j - 12\sigma_i\sigma_j - 17)\beta_j$$

$$- (33 - 20\sigma_i - 20\sigma_j + 12\sigma_i\sigma_j)(4\beta_i - \beta_j)$$

$$= 8(35\sigma_i + 20\sigma_j - 21\sigma_i\sigma_j - 34)\beta_i + 4(5\sigma_i + 20\sigma_j - 12\sigma_i\sigma_j - 13)\beta_j.$$  

Due to $\frac{\partial \delta(\sigma)}{\partial \sigma_i} = 8(35 - 21\sigma_j)\beta_i + 4(5 - 12\sigma_j)\beta_j > 8(35 - 21)\beta_i + 4(5 - 12)\beta_j = 28(4\beta_i - \beta_j) > 0$, it follows

$$\delta(\sigma_i, \sigma_j) < \delta(1, \sigma_j) = 8(1 - \sigma_j)\beta_i + 4(-8 + 8\sigma_j)\beta_j = 8(1 - \sigma_j)(\beta_i - 4\beta_j) < 0,$$

in view of (2-13). Therefore, $s_i^C(\sigma) < s_i^C(1)$ is proved.

Using (4-25), the SOC for the welfare maximization can be examined as below.

\[
\frac{\partial^2 W^*_i(s, \sigma, \alpha)}{\partial s_i^2} = -\sigma_i[(1 - \alpha_i)\mu_i + (1 - \mu_i \mu_j \alpha_j)] \frac{\partial x^*_i}{\partial s_i} \frac{\partial x^*_j}{\partial s_i} - (2 - \sigma_{ii}) \frac{\partial x^*_i}{\partial s_i} \\
= \left\{ -\sigma_i[(1 - \alpha_i)\mu_i + (1 - \mu_i \mu_j \alpha_j)] \right\} \frac{\partial x^*_i}{\partial s_i}
\]

Since (4-18) yields \(\frac{\partial x^*_i}{\partial s_i} > 0\), the sign is determined by the value in the square bracket defined as below.

\[
A \overset{\text{def}}{=} -\sigma_i[(1 - \alpha_i)\mu_i + (1 - \mu_i \mu_j \alpha_j)] \frac{\partial x^*_j}{\partial s_i} - (2 - \sigma_{ii})
\]

\[
= 2(1 - \mu_i \mu_j)(1 - \alpha_j \mu_j) + \mu_j(1 - \mu_i)(4 - 2\alpha_i \mu_i - 2\alpha_i \alpha_j \mu_i \mu_j) - \mu_i(1 - \mu_j)(1 - \alpha_j \mu_j^2)
\]

\[
= \mu_j(1 - \mu_i)(4 - 2\alpha_i \mu_i - 2\alpha_i \alpha_j \mu_i \mu_j) + (1 - \alpha_j \mu_j)[(1 - \mu_j)(2 - \mu_i - \mu_i \mu_j \alpha_j) + 2\mu_j(1 - \mu_i)]
\]

\[
< 0
\]

where use was made of (4-19) and \(\sigma_i = \frac{1-\mu_i}{1-\mu_i \mu_j}\).

Note $\sigma_i$ can be expressed as $\sigma_i = \frac{1 - \mu_j}{\Delta^\mu}$ and $\Delta = 1 - \mu_i\mu_j$. Then, the FOCs for welfare maximization in both regimes yield

$$\Delta^\mu \cdot \left. \frac{\partial W^*_C}{\partial s_i} \right|_{s_i=0} = x_i \left( (1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} - \mu_j(1 - \mu_i) \right) + \mu_i(1 - \mu_j)x_j P'(X) \frac{\partial x^*_i}{\partial s_i},$$

$$\Delta^\mu \cdot \left. \frac{\partial W^*_F}{\partial s_i} \right|_{s_i=0} = x_i \left( \Delta^\mu(1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} - \mu_j(1 - \mu_i) \right).$$

In view of $\Delta^\mu > 0$, to assure $\left. \frac{\partial W^*_C}{\partial s_i} \right|_{s_i=0} \geq 0$,

$$x_i \left( (1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} - \mu_j(1 - \mu_i) \right) \geq -\mu_i(1 - \mu_j)x_j P'(X) \frac{\partial x^*_i}{\partial s_i} > 0$$

should be satisfied. Thus, it follows

$$(1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} > \mu_j(1 - \mu_i).$$

Substituting the above result into $\left. \frac{\partial W^*_F}{\partial s_i} \right|_{s_i=0}$ yields the following results.

$$\Delta^\mu \cdot \left. \frac{\partial W^*_F}{\partial s_i} \right|_{s_i=0} = x_i \left( \Delta^\mu(1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} - \mu_j(1 - \mu_i) \right)$$

$$> x_i \left( \Delta^\mu(1 - \mu_j) P'(X) \frac{\partial x^*_j}{\partial s_i} - (1 - \mu_j) P'(X) \frac{\partial x^*_i}{\partial s_i} \right)$$

$$= x_i (1 - \mu_j) P'(X) \left( \Delta^\mu \frac{\partial x^*_j}{\partial s_i} - \frac{\partial x^*_i}{\partial s_i} \right)$$

$$= x_i (1 - \mu_j) \left( \frac{(1 - \mu_i\mu_j)(1 + \mu_j)}{3 - \mu_i - \mu_j - \mu_i\mu_j} - \frac{1}{3} \right)$$

$$= x_i (1 - \mu_j) \frac{F_i(\mu_i, \mu_j)}{3(3 - \mu_i - \mu_j - \mu_i\mu_j)}$$

where $F_i(\mu_i, \mu_j) := \mu_i + 4\mu_j - 2\mu_i\mu_j - 3\mu_i\mu_j^2$.

To prove $F_i(\mu_i, \mu_j) \geq 0$, note that $\mu_i := \frac{1 - \sigma_i}{\sigma_i} \in (0, 1)$ when $\sigma_i (i = 1, 2)$ runs over $(\frac{1}{2}, 1)$. Given $\mu_i \in (0, 1)$, $F_i(\mu_i, \mu_j)$ is strictly concave in $\mu_j$ given by

$$\frac{\partial F_i(\mu_i, \mu_j)}{\partial \mu_j} = -2\mu_i - 6\mu_i\mu_j + 4,$$

$$\frac{\partial^2 F_i(\mu_i, \mu_j)}{\partial \mu_j^2} = -6\mu_i < 0.$$
Thus, $F_i (\mu_i, \mu_j) = \min\{F_i (\mu_i, 0), F_i (\mu_i, 1)\} = \min\{\mu_i, 4 (1 - \mu_i)\} \geq 0$ must hold. This establishes the desired result as below.

\[
\frac{\partial W^*_{iC}}{\partial s_i} \bigg|_{s_i=0} \geq 0 \quad \Rightarrow \quad \frac{\partial W^*_{iF}}{\partial s_i} \bigg|_{s_i=0} > 0,
\]

which conveys the message that under cross-firm shareholding, the government has stronger incentive to subsidize the own exports than under cross-country shareholding.
Chapter 5
Separation of Ownership and Management

5.1 Introduction

Over 70 years ago, Berle and Means (1932) first argued that large corporations are characterized by the separation of ownership and management. They criticized that firms’ own profit-maximization behavior is oversimplified in the traditional economic and industrial organization theories. Based on Berle and Means (1932)’s argument, Baumol (1958) suggested that firm managers may have certain objectives other than pure profit maximization and assumed a sales maximization hypothesis. His work emphasized the behavioral theory of the firm, and a number of economists examined different managerial objectives to analyze firms’ optimal behavior (See Simon (1964), Williamson (1964), etc.).

However, the above studies focused on the internal organization of the firm and regarded the firm as a simple monopolizer. When a greater number of firms compete in the market, each firm’s managerial objectives are determined by taking into consideration the rival firms’ behavior. A strategic managerial decision analysis in the oligopolistic market was first conducted by Vickers (1985) and stylized by Fershtman and Judd (1987) and Sklivas (1987) (hereafter the FJS model). They considered a two-stage model where, in the first stage, profit-maximizing owners offer compensation schemes to their managers and in the next stage, managers compete in quantities or prices under precommitted compensation schemes. The FJS model clarified managers’ nonprofit-maximizing behavior from the game-theoretical point of view, indicating that delegating a manager with distorted objective functions affects the strategic performance of the firm and induces it to act as a Stackelberg-leader (or follower) in the quantity (or price) competition.

Managerial delegation attains the equivalent effect as the strategic subsidization shown in the BS model. The rent-shifting effect of the strategic subsidization can also be explained by the firms’ distorted objective functions as a similar principal-agent model. Government subsidization induces the firms to maximize the subsidy-inclusive profits and wins a Stackelberg-leader position in the quantity competition, thus improving their own wel-
fare. In that sense, government’s export subsidy policy can be replaced by the owner’s managerial delegation in the oligopolistic competition.

Although Fershtman and Judd (1987) have pointed out the similarity between the BS and FJS models, few studies have considered this view seriously. Recently, a number of papers analyzed strategic managerial delegation involving international trade in a duopoly market. Das (1997) applied an FJS-style delegation in both quantity and price settings to the standard strategic trade policy models and showed that the magnitude of the optimal export subsidy or tax is smaller in the presence of managerial delegation in both the quantity and price competition. Miller and Pazgal (2005), which is distinguished from the analyses in Brander and Spencer (1985) and Eaton and Grossman (1986), introduced the so-called – ”Relative Performance” contract – a linear combination of own profit and competitor’s profit. Collie (1997) examined the domestic government’s incentive to delegate the trade policy to a policy-maker when two firms compete in the domestic market and revealed that the domestic government should choose delegation so as to improve both countries’ welfares. However, the above research did not discuss the nature of the equivalent strategic behavior between government’s trade policy and owner’s managerial delegation under oligopolistic competition. In addition, they considered the two policies as independent instruments and did not explore their total effects on the behavior of the firms.

This chapter combines the BS and FJS models and reexamines Das (1997)’s study by focusing on the owner’s subsidy effect hidden in the managerial delegation process and clarified the result of oversubsidization of the firm with government intervention. Although Das (1997) has already investigated such a strategic export subsidy model coupled with managerial delegation, the study in this chapter is explicitly different from Das (1997). First, my study focuses on the owner’s subsidization incentives by designing a managerial incentive contract. Das (1997) has indicated that the owner’s delegation itself is a profit-shifting mechanism, he did not clearly explain this mechanism. This chapter further shows the equivalence result that the owner’s delegation behavior has the same effect as government subsidization on the own firm in the duopoly market. Second, my study discusses how government intervention affects the owner’s profit-shifting performance. Das (1997) simply compared the magnitude of government subsidy in equilibrium with the BS model and disregarded the role of the owner’s rent-shifting performance in a strategic export subsidy competition. This chapter clarifies that each owner’s strategic subsidization incentive is strengthened with government intervention if their own subsidy-inclusive marginal cost is lower than the rival firm’s marginal cost. Third, my study examines the total subsidy effect summing up both government subsidization and owner’s delegation behavior. Under symmetric cost conditions, each exporting firm is over-subsidized in equilibrium and the Cournot competition between the firms becomes more fierce. Each exporting country’s welfare worsens and world welfare improves.

This chapter elucidates how the traditional subsidization incentives studied in the BS model are affected in the presence of separation of ownership and management. Although managerial delegation can replace export subsidy policy to yield the same profit shifting effect, the export competing governments still have incentives to subsidize the own firms.
The study emphasizes the result in de Meza (1986), who showed that the more effective country has stronger incentive to subsidize the firm. Government’s positive subsidization lies in that it makes the own firm more competitive and thereby strengthens the owner’s subsidization incentives to grab more rent from the foreign firm. Thus, in the presence of separation of ownership and management in the duopoly market, export subsidy policy weakens its role as a rent-shifting instrument, but intensifies its cost-reduction effect to gain cost advantage. The study is a challenge to clarify the interdependent relationship connecting government’s policy decision with the organization of the firm. It shows new implications on the traditional strategic trade policy related with modern corporate structure.

Furthermore, this chapter investigates the unilateral delegation case and endogenizes the owners’ delegation decisions at the very first stage. In the FJS model framework, when letting the owners decide whether or not to hire a manager, Basu (1995) showed that a Stackelberg equilibrium may be realized if the cost difference between the firms is large enough. White (2001) examined this issue in a mixed oligopoly and concluded that only private firms hire managers. Constantine, Evangelos, and Emmanuel (2006) endogenized the owner’s choice between the two types of managerial incentive contracts: Profit-Revenues contract (introduced in the FJS model) and Relative-Performance contracts (introduced in Miller and Pazgal (2001, 2002)). The above research showed that prisoner’s dilemma result in the FJS model may not occur if the firm is able to arrive at the managerial delegation decision. This chapter clarifies that when governments are involved, both owners have no incentive to delegate a manager, and a Pareto-efficient result is realized in the symmetric cost conditions.

The analysis in this chapter is based on Wei (2008). The remaining sections proceeds as follows. Section 2 describes a three-stage government-owner-manager game and discusses the role of owner’s subsidy equivalent and total subsidy to the firms. Section 3 solves the bilateral delegation model in owner’s subsidy equivalent approach and reveals some new results not discussed in Das (1997). Section 4 examines the unilateral delegation case and Section 5 extends the model to add one more stage to endogenize the owners’ delegation decisions. Concluding remarks are summed up in section 6.
5.2 Model Setup

Following the framework of the BS model, firm $i$’s profit function is given by

$$\pi^i(x, s_i) = (P(X) - c_i + s_i) x_i. \quad (2-1)$$

Each exporting firm has one owner and one manager. Each owner designs an incentive contract to compensate its manager, which is expressed as a linear combination of the firm’s profit and revenue as in the FJS model.

$$M^i(x, \alpha_i, s_i) = \alpha_i \pi^i(x, s_i) + (1 - \alpha_i) P(X) x_i$$

$$= [P(X) - \alpha_i(c_i - s_i)] x_i, \quad (5-1)$$

where $\alpha_i$ denotes the contract term of firm $i$ and is the weight on the firm’s profit in the contract. If $\alpha_i = 1$, (5-1) is simply firm $i$’s pure profit function. The firm faces a managerial subsidy-inclusive cost of $\alpha_i(c_i - s_i)$ in (5-1).

Note that $M_i$ does not represent a manager’s rewards in general. In fact, the manager is paid $A_i + B_i M_i$ for some constants $A_i$ and $B_i$ with $B_i > 0$. The owner must offer his manager a contract under which the participation constraint is satisfied, i.e., $A_i + B_i M_i = \tilde{K}$ such that $\tilde{K}$ equals the manager’s reservation income or opportunity cost and is a constant.\(^1\)

Without loss of generality, $\tilde{K}$ is normalized to 0, i.e., $A_i + B_i M_i = 0$.

In view of the weighed-average combination of profit and sales in (5-1), the manager is able to determine a more (or less) aggressive output, since unlike in the pure profit maximization case, the manager faces the marginal cost of $\alpha_i(c_i - s_i)$ in the incentive contract. $\alpha_i(c_i - s_i)$ is termed as firm $i$’s managerial marginal cost. Each firm acts as though it were subsidized (or taxed) by an amount equivalent to the cost difference between the actual marginal cost $c_i$ and the managerial marginal cost $\alpha_i(c_i - s_i)$. I define this cost difference as total subsidy (or tax) of firm $i$, $S_i$:\(^2\)

$$S_i := c_i - \alpha_i(c_i - s_i).$$

Total subsidy (or tax) can be divided into two parts. One is government subsidy $s_i$ set at the first stage, which is the cost difference between government intervention and non-intervention behavior. The other is nonpecuniary subsidy caused by the owner’s manipulated incentive contracts designed in the second stage, which is the cost difference between the owner’s delegation and non-delegation behavior given a precommitted government subsidization.\(^3\) The nonpecuniary subsidy is defined as owner’s subsidy (or tax) equivalent of firm $i$, $d_i$:\(^4\)

$$d_i := S_i - s_i = (1 - \alpha_i)(c_i - s_i).$$

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\(^1\)See Chapter 14 in Basu (1993).

\(^2\)If $S_i < 0$, the firm is taxed in total.

\(^3\)With government subsidy commitment at the first stage, the marginal cost is $c_i - s_i$ without managerial delegation and $(1 - \alpha_i)(c_i - s_i)$ with managerial delegation. I define the cost difference due to managerial delegation as owner’s subsidy (tax) equivalent.

\(^4\)I regard $d_i$ as the owner’s subsidy equivalent if $d_i > 0$ or the owner’s tax equivalent if $d_i < 0$. 

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Owner’s subsidy (or tax) equivalent appears to be a debatable concept since the owner cannot subsidize (or tax) the firm itself. However, by manipulating an incentive contract, the owner can divert the manager’s objective from strict profit maximization to attain the subsidization (or taxation) objective. Owing to the separation of ownership and management, the firm faces a marginal cost that is reduced by \( d_i \) (or increased by \(-d_i\)) comparing to the pure profit-maximization behavior. Hence, the owner’s behavior of delegating a manager with contract term \( \alpha_i \) is equivalent to subsidizing the firm with a unit production subsidy \( d_i \) (or taxing the firm with unit production tax \(-d_i\)).

This chapter explores a three-stage government-owner-manager game. In the first stage, each exporting country’s government simultaneously determines the country-specific subsidy rate to the own firm. In the second stage, given both the countries’ subsidy rates, each owner delegates a manager and decides his/her owner subsidy (or tax) equivalent \( d_i \). In the third stage, each manager – being aware of his incentive scheme and that of the rival – decides the production quantity to export to the third country competing à la Cournot. Unlike Das (1997), my study lets each owner decide \( d_i \) instead of contract term \( \alpha_i \) in the second stage. Given that \( s_i \) is determined in the first stage, \( d_i \) is a monotonic function of \( \alpha_i \) since \( c_i - s_i > 0 \).^5 Although the model results in the same equilibrium values as those in Das (1997), the owner’s subsidy equivalent approach clarifies the total effects on the firms’ outputs and social welfare in the proceeding analysis.

(5-1) can be rewritten as follows:

\[
\tilde{M}^i(x, S_i) = [P(X) - c_i + S_i] x_i,
\]

where \( S_i = d_i + s_i \). The game is solved by backward induction from the third stage.

---

^5I do not consider the case \( c_i - s_i \leq 0 \) when \( c_i \) is very small.
5.3 Model Solution

5.3.1 Output Stage Equilibrium

After observing each country’s government subsidy rate and each firm’s incentive contract, the managers decide their optimal outputs under the precommitted contract in (5-2). Given that the SOC is satisfied,\(^6\) the FOC for maximizing (5-2) with respect to its own output yields

\[
0 = \frac{\partial \tilde{M}^i(x, S_i)}{\partial x_i} = MR_i - (c_i - S_i),
\]

(5-3)

where \(MR_i = P(X) + x_i P'(X)\) denotes the marginal revenue of firm \(i\). Given its rival’s output, each firm’s manager ascertains the best response obtained by equating the marginal revenue with the marginal cost net of total subsidy, i.e., \(MR_i = c_i - S_i\).

Define \(r^{iD}(x_j, S_i)\) as manager \(i\)’s reaction function and the superscript \(D\) denotes the equilibrium values under managerial delegation.

\[
r^{iD}(x_j, S_i) = \arg \max_{x_i} \tilde{M}^i(x, S_i) = \frac{1}{2}(a - x_j - c_i + S_i).
\]

(5-4)

Thus, \(r^{iD}_x \overset{\text{def}}{=} \frac{\partial r^i(x_j, S_i)}{\partial x_j} = -\frac{1}{2} < 0\) shows that each firm’s optimal output is a strategic substitute to the other’s and \(r^{iD}_S \overset{\text{def}}{=} \frac{\partial r^i(x_j, S_i)}{\partial S_i} = \frac{1}{2} > 0\).

Solving for each firm’s optimal output at the third-stage equilibrium yields

\[
x^*_D(S) = \frac{1}{3}[\beta_i + 2S_i - S_j],
\]

(5-5)

where \(S = (S_i, S_j)\) represents the total subsidy profile and \(\beta_i = a - 2c_i + c_j(i, j = 1, 2; j \neq i)\). Note that each firm’s equilibrium output depends on the total subsidies of both firms. Differentiating (5-5) with \(S_i\) yields:

\[
\frac{\partial x^*_D(S)}{\partial d_i} = \frac{\partial x^*_D(S)}{\partial S_i} = \frac{2}{3} > 0, \quad \frac{\partial x^*_D(S)}{\partial d_i} = \frac{\partial x^*_D(S)}{\partial S_i} = r^{iD}_S \frac{\partial x^*_D(S)}{\partial S_i} = -\frac{1}{3} < 0.
\]

(5-6)

An increase in the domestic owner’s subsidy reduces the domestic marginal cost and induces the domestic manager to act more aggressively under Cournot competition. Hence, the domestic firm’s output increases and foreign firm’s output decreases as a strategic substitute.

\(^{6}\)It is easily verified that:

\[
\frac{\partial^2 \tilde{M}^i(x, S_i)}{\partial x_i^2} = -2 < 0.
\]

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5.3.2 Contract Stage Equilibrium

BS Subsidy Equivalence Result

Without government intervention, the nonintervention two-staged owner-manager model is the FJS model. The optimal owner’s subsidy equivalent in the FJS model is identical to à la Brander-Spencer government subsidy, i.e.,

\[ d_{i}^{FJ} = \beta_{i}' = s_{i}^{B}, \]

where the superscripts \( FJ \) denotes the equilibrium values in the FJS model and \( \beta_{i}' := a - 3c_{i} + 2c_{j}(i, j = 1, 2; j \neq i) > 0 \) due to the positive equilibrium output in (5-7) below.

The resulting equilibrium output and national welfare also yield the equivalence results in view of (2-25) and (2-26).

\[
\hat{x}_{i}^{FJ} = \frac{2\beta_{i}'}{5} = \hat{x}_{i}^{B}, \quad (5-7)
\]

\[
\hat{W}_{i}^{FJ} = \frac{2\beta_{i}'}{25} = \hat{W}_{i}^{B}, \quad (5-8)
\]

**Proposition 5.1.** In the absence of government intervention, strategic managerial delegation induces each firm to act as though it were subsidized with an optimal government subsidy in the BS model, i.e., \( d_{i}^{FJ} = s_{i}^{B} \) and \( \hat{x}_{i}^{FJ} = \hat{x}_{i}^{B} (i = 1, 2) \).

The above result also holds true under a general demand function when each firm’s product is a strategic substitute to that of the other. The BS and FJS models can be regarded as similar principal-agent models, in which agents play Nash against all others, and principals play Stackelberg against agents and Nash against all other principals. In the BS model, the governments’ precommitments to pay an export subsidy distort firms’ incentives to advance the own national welfare. Similarly, in the FJS model, owners’ strategic managerial delegation also distorts managers’ incentives to achieve higher profits. Note that the objective functions in both the models are the same, i.e., since principals maximize the own firm’s subsidy-exclusive profit functions and agents maximize the own firm’s subsidy-inclusive profit functions. Thus, under the same duopolistic market performance, owner’s optimal nonintervention subsidy equivalent in the FJS model is equivalent to the government’s optimal subsidy in the BS model.

**Owner’s Subsidy Equivalent in the Second-Stage Equilibrium**

In the second stage, each firm’s owner decides \( d_{i} \) in the incentive contract to maximize its own profit. Since the cost of delegating a manager is assumed to zero, i.e., \( A_{i} + B_{i}M_{i} = 0 \), the owner acts as a pure profit maximizer. Evaluating the equilibrium output in (5-5) yields the following expression for each firm’s profit function:

\[
\pi_{i}^{*,D}(d, s) = \pi^{*,D}(x_{i}^{*,D}(d + s), x_{j}^{*,D}(d + s), s_{i}) \]

\[
= \frac{1}{9} [a - (2c_{i} + S_{i}) + (c_{j} - S_{j}) + 3s_{i}] [a - 2(c_{i} - S_{i}) + (c_{j} - S_{j})].
\]

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Given the SOC is satisfied, the FOC for maximizing the profit function is given by

\[
0 = \frac{\partial \pi^*_D(d, s)}{\partial d_i} = \frac{\partial \pi^*_i}{\partial x_i} \frac{\partial x^*_i}{\partial d_i} + \frac{\partial \pi^*_i}{\partial x_j} \frac{\partial x^*_j}{\partial d_i} = (MR_i - c_i + s_i) \frac{\partial x^*_i}{\partial d_i} + x_i P'(X) \frac{\partial x^*_j}{\partial d_i}. \tag{5-9}
\]

The first term in (5-9) represents the marginal profit-loss through the excess competition effect. An increase in the domestic firm’s production results in a further decrease in the marginal revenue as compared to the subsidy-inclusive marginal cost. Hence, the own output expansion leads to a domestic profit loss. The second term in (5-9) represents the marginal profit gain through the rent-shifting effect, which shows that a decrease in the foreign firm’s output improves the terms of trade and thus shifts the rent from the foreign firm to the domestic firm.

Denote \(\gamma^i_D(d, s)\) as owner \(i\)’s reaction function to maximize its own profit:

\[
\gamma^i_D(d, s) := \arg \max \pi^*_i(d, s) = \frac{1}{4}(\beta_i + 2s_i - s_j - d_j).
\]

Although the properties of the above reaction function can be easily derived in the linear demand function, I provide an intuitive explanation in view of (5-9).

Owner \(i\)’s reaction curve is depicted as \(\gamma^i\gamma^i(i = 1, 2)\) in Figure 5.2. Each firm’s reaction curve is downward sloping, which is given by

\[
\frac{\partial \gamma^i_D(d, s)}{\partial d_j} \propto \frac{\partial^2 \pi^*_i(d, s)}{\partial d_j \partial d_i} = \frac{\partial MR_i}{\partial d_j} \frac{\partial x^*_i}{\partial d_i} + \frac{\partial (x_i P'(X))}{\partial d_j} \frac{\partial x^*_j}{\partial d_i} = 0 - \frac{\partial x^*_j}{\partial d_j} \frac{\partial x^*_j}{\partial d_i} < 0.
\]

In view of (5-9), an increase in the rival firm’s owner’s subsidy equivalent does not affect the excess competition effect since the manager always equates its marginal revenue to the marginal cost exclusive of the total subsidy. However, its terms of trade deteriorates due to an increase in the rival firm’s output, and the rent-shifting effect becomes weaker. Hence, each firm’s owner’s subsidy equivalent is a strategic substitute to that of the rival. The above result also clarifies that an increase in the rival country’s government subsidy shifts the reaction curve inward as below:

\[
\frac{\partial \gamma^i_D(d, s)}{\partial s_j} = \frac{\partial \gamma^i_D(d, s)}{\partial d_j} < 0.
\]

\(^7\)The SOC can be derived as follows:

\[
\frac{\partial^2 \pi^*_i(d, s)}{\partial d_i^2} = \frac{4}{9} < 0.
\]

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Meanwhile, an increase in the own government’s subsidy shifts the reaction curve outward:

$$\frac{\partial \gamma_i^{D}(d_j,s)}{\partial s_i} \propto \frac{\partial^2 x_i^{*D}(d,s)}{\partial s_i \partial d_i} \cdot \frac{\partial x_i^{*D}}{\partial S_i} \cdot \frac{\partial x_j^{*D}}{\partial S_i} = 0 - \frac{\partial x_i^{*D}}{\partial S_i} \frac{\partial x_j^{*D}}{\partial S_i} < 0.$$ 

An increase in the own government subsidy does not affect the excess competition effect. However, it strengthens the rent-shifting effect; this is because the rival firm’s output contracts further and improves the terms of trade, thus shifting the reaction curve outward.

The intersection of the two reaction curves labeled $B$ in Figure 5.2 represents the optimal owner’s subsidy equivalent of firm $i$ in the second-stage equilibrium, $d_i^{eD}(s)$ which is given by

$$d_i^{eD}(s) = x_i^{*D} = \frac{\beta_i' + 3s_i - 2s_j}{5}, \quad (5-10)$$

where the superscript $e$ represents the delegation stage equilibrium values. Without government intervention, Point B shows the equilibrium subsidies in the BS model, or the equilibrium owner’s subsidy equivalent in the FJS model, i.e., $d_i^{FJ} = d_i^{eD}(0) = s_i^B$.

The comparative static results yield:

$$\frac{\partial d_i^{eD}(s)}{\partial s_i} = \frac{3}{5} > 0, \quad \frac{\partial d_j^{eD}(s)}{\partial s_i} = -\frac{2}{5} < 0.$$ 

An increase in the domestic government subsidy makes the domestic firm more efficient than the rival firm due to the reduction in marginal cost. Thus, the domestic owner has a stronger subsidization incentive as indicated by de Meza (1986). Meanwhile, the rival firm becomes less efficient and its owner’s subsidization incentive weakens.

**Equilibrium Output Change**

The resulting second-stage equilibrium output is given by

$$x_i^{eD}(s) : = x_i^{*D}(d_i^{eD}(s) + s_i, d_j^{eD}(s) + s_j)$$

$$= \frac{2}{5} [\beta_i' + 3s_i - 2s_j] \quad (5-11)$$

Differentiating firm $i$’s equilibrium output $x_i^{eD}(s)$ with respect to $s_i$ yields

$$0 < \frac{\partial x_i^{eD}}{\partial s_i} = \frac{\partial x_i^{*D}}{\partial S_i} + \frac{\partial x_i^{*D}}{\partial d_i} \frac{\partial d_i^{eD}}{\partial S_i} + \frac{\partial x_j^{*D}}{\partial S_j} \frac{\partial d_j^{eD}}{\partial S_i}.$$ 

An increase in the domestic government subsidy affects the domestic equilibrium output in three ways: (1) it reduces the domestic marginal cost; (2) strengthens the domestic owner’s subsidization incentive; and (3) weakens the foreign owner’s subsidization incentive. Since
the three effects work in the same direction, the overall effect is reinforced, and the domestic firm acts more aggressively than it does without government intervention.

Likewise, the foreign firm’s equilibrium output is affected in the same three ways.

\[
0 > \frac{\partial x_j^{eD}}{\partial s_i} = \frac{\partial x_j^{eD}}{\partial s_i} + \frac{\partial x_j^{eD}}{\partial s_i} \frac{\partial d_j}{\partial s_i} + \frac{\partial x_j^{eD}}{\partial s_i} \frac{\partial d_j}{\partial s_i} = r_j^{eD} \frac{\partial x_j^{eD}(s)}{\partial s_i} + \Delta D \frac{\partial x_j^{eD}(s)}{\partial s_i}.
\]

Using (5-6), (5-11) and \(\Delta D = 1 - r_i^{eD}r_j^{eD} > 0\), foreign output change can be rewritten into two parts as shown in (5-12). The first part represents the foreign firm’s output decrease as a strategic substitute to the domestic output, and the second part represents the foreign firm’s excess output decrease due to strategic managerial delegation competition between the owners. Note that the second part does not hold true when the foreign owner does not compete to delegate a manager.

**Equilibrium Profit Change**

Note that \(x_i^{eD}(s) = r_i^{eD}(x_j^{eD}(s), S_i^{eD}(s))\) where \(S_i^{eD}(s) = s_i + d_i^{eD}(s)\), then equilibrium output change can be rewritten as follows.

\[
\frac{\partial x_i^{eD}(s)}{\partial s_i} = r_i^{eD} \frac{\partial x_j^{eD}(s)}{\partial s_i} + r_i^{eD} \frac{\partial S_i^{eD}(s)}{\partial s_i}
\]

\[
\frac{\partial x_j^{eD}(s)}{\partial s_i} = r_j^{eD} \frac{\partial x_i^{eD}(s)}{\partial s_i} + r_j^{eD} \frac{\partial S_j^{eD}(s)}{\partial s_i}
\]

Each firm’s profit function can be rewritten as \(\pi_i^{eD}(s) = \pi_i(x_i^{eD}(s), x_j^{eD}(s), s_i)\). Differentiating \(\pi_i^{eD}(s)\) with \(s_i\) yields

\[
\frac{\partial \pi_i^{eD}(s)}{\partial s_i} = \frac{\partial \pi_i}{\partial x_i} \frac{\partial x_i^{eD}}{\partial s_i} + \frac{\partial \pi_j}{\partial x_j} \frac{\partial x_j^{eD}}{\partial s_i} + \frac{\partial \pi_i}{\partial s_i} = -d_i^{eD} \frac{\partial x_i^{eD}}{\partial s_i} + x_i P'(X) \frac{\partial x_j^{eD}}{\partial s_i} + x_i > 0,
\]

\[
\frac{\partial \pi_j^{eD}(s)}{\partial s_i} = \frac{\partial \pi_j}{\partial x_j} \frac{\partial x_j^{eD}}{\partial s_i} + \frac{\partial \pi_j}{\partial x_i} \frac{\partial x_i^{eD}}{\partial s_i} = -d_j^{eD} \frac{\partial x_j^{eD}}{\partial s_i} + x_j P'(X) \frac{\partial x_i^{eD}}{\partial s_i} = x_j P'(X) r_j^{eD} \frac{\partial S_i^{eD}}{\partial s_i} < 0.
\]
where use was made of (5-3), (5-13) and (5-14). With managerial delegation, government subsidization increases domestic firm’s profit and reduces the rival firm’s profit. The rent-shifting effect of strategic subsidization is not dampened in the presence of separation of ownership and management.

5.3.3 Subsidy Stage Equilibrium

**Government Subsidy in Equilibrium**

Each country’s welfare function is expressed by the product surplus less the subsidy payment.

\[
W_{ei}^{eD}(s) = \pi_{ei}^{eD}(s) - s_i x_{ei}^{eD}(s)
\]

\[
= \frac{2}{25} (\beta'_i - 2s_i - 2s_j)(\beta'_i + 3s_i - 2s_j)
\]

Given that the SOC is satisfied,\(^8\) the FOC for welfare maximization is solved as follows.

\[
0 = \frac{\partial W_{ei}^{eD}(s)}{\partial s_i} = \frac{\partial \pi_{ei}^{eD}}{\partial s_i} - x_i - s_i \frac{\partial x_{ei}^{eD}}{\partial s_i}
\]

\[
= x_i P'(X) \left( r_{iD} \frac{\partial S_{ej}^{eD}}{\partial s_i} \right) - s_i \frac{\partial x_{ei}^{eD}}{\partial s_i}, \quad (5-19)
\]

where (5-16) was used. The parenthetical term in (5-19) represents the foreign firm’s excess output decreases due to the strategic managerial delegation competition shown by the second part in (5-14). This term times \((-P')\) represents the price rise and times domestic output \(x_i\) represents the domestic marginal revenue increase caused by the improved terms of trade. Given any \(s_j\), a small subsidy benefits the own country as shown by

\[
\left. \frac{\partial W_{ei}^{eD}(s)}{\partial s_i} \right|_{s_i=0} = x_i P'(X) r_{iD} \frac{\partial S_{ej}^{eD}}{\partial s_i} > 0.
\]

**Lemma 5.1.** When both exporting firms strategically delegate a manager, both governments have positive incentives to subsidize the own firms.

Each country’s reaction function is derived as below.

\[
R_{iD}(s_j) = \frac{1}{12} (\beta'_i - 2s_j)
\]

\(^8\)Again, the SOC is easily verified:

\[
\frac{\partial^2 W_{ei}^{eD}(s)}{(\partial s_i)^2} = -\frac{24}{25} < 0.
\]
Denote $s_i^D$ as the equilibrium government’s subsidy of country $i$. Calculation under linear demand function yields

$$s_i^D = a - 4c_i + 3c_j = \frac{\beta''_i}{14} > 0. \tag{5-20}$$

where $\beta''_i = a - 4c_i + 3c_j (i, j = 1, 2; j \neq i) > 0$. The positive sign is assured by the duopolistic output in the equilibrium, i.e.,

$$\hat{x}_i^D = x_i^{cD}(S) = \frac{3\beta''_i}{7}. \tag{5-21}$$

Since the firms are subsidized by the owners in the second-stage equilibrium, there may be a doubt as to why the governments do not tax the firms to reduce welfare distortion in the first-stage equilibrium. The paradox is resolved by noting that only the rent-shifting effect induces a shift in each owner’s reaction curve shown in the previous subsection. Taxation increases the marginal cost, owing to which domestic owner has less incentive to subsidize the firm. The profit of the domestic firm decreases and the rent shifts to the foreign firms, thus deteriorating the domestic country’s welfare. Although each firm’s owner subsidizes the firm through manipulating the separation of ownership and management, each country’s government still has a positive incentive to subsidize the own firm to prevent rent outflow.

In view of (5-20), it is shown that the optimal government subsidy is definitely lower than the subsidy à la Brander-Spencer under the asymmetric cost conditions, i.e.,

$$s_i^D - s_i^B = \frac{\beta''_i}{14} - \frac{\beta'_i}{5} = -\frac{2\beta'_i + 4\beta''_i + 3(a - c_j)}{70} < 0,$$

where (5-7) and (5-21) were used.

**Lemma 5.2.** Strategic managerial delegation competition suppresses both governments’ subsidization incentives, i.e., $s_i^D < s_i^B (i = 1, 2)$.

The intuition behind can be explained as below. In the absence of government intervention, each owner manipulates the incentive scheme to grant the firm a subsidy à la Brander-Spencer. However, when the governments are involved, each country’s government subsidization strengthens the domestic owner’s subsidization incentive and weakens that of the foreign owner. The quantity competition between the exporting firms becomes more fiercer, which deteriorates the terms of trade and worsens the welfare of the exporting countries. Therefore, each country’s government has a weaker incentive to subsidize the own firm.

Comparing the magnitude of government subsidy in equilibrium is not enough in our analysis. In view of (5-5), the firms’ outputs, as well as social welfare\(^9\) are dependent

\(^9\)Das (1997) does not show this result explicitly.

\(^{10}\)The welfare function can be rewritten as:

$$W_i^{*,D}(S) = (P(X^{*,D}(S)) - c_j)x_i^{*,D}(S).$$
on the total subsidies of both firms. Therefore, I proceed to examine the owner’s subsidy equivalent and total subsidy in equilibrium.

**Owner’s Subsidy Equivalent in Equilibrium**

The owner’s subsidy equivalent in equilibrium can be rewritten as:

\[ \hat{d}_i^D = d_i^D(s^D) = \frac{3\beta''}{14}. \]  

(5-22)

Comparing the owner’s subsidy equivalent \( \hat{d}_i^D \) to the subsidy à la Brander-Spencer \( s_i^B \) yields

\[ \hat{d}_i^D - s_i^B = \frac{a - 18c_i + 17c_j}{70}. \]

Evidently, \( \hat{d}_i^D > s_i^B \) under the symmetric cost function. However, under the asymmetric cost function, I find that

\[ \hat{d}_i^D \gtrless s_i^B \iff s_i^D \gtrless c_i - c_j. \]

Note that if the foreign firm is not as efficient as the domestic firm, i.e., \( c_i \leq c_j \), \( \hat{d}_i^D \) is always larger than \( s_i^B \) due to the positive value of \( s_i^D \) shown in (5-20). Then, consider the case wherein the foreign firm is more efficient than the domestic firm, i.e., \( c_i > c_j \). The above condition can be rewritten as follows:

\[ \hat{d}_i^D \gtrless s_i^B \iff c_i - s_i^D \gtrless c_j. \]

It is shown that if the domestic firm’s subsidy-inclusive marginal cost is lower than the foreign firm’s marginal cost, the domestic owner’s subsidy equivalent in equilibrium is higher than the subsidy à la Brander-Spencer and vice versa. The intuition can be shown by the result in de Meza (1986). When the strategic government subsidization makes the domestic firm more efficient than the foreign firm, the domestic owner has a stronger subsidization incentive than it does without government intervention.

**Proposition 5.2.** Each firm’s equilibrium owner’s subsidy equivalent is higher than the subsidy à la Brander-Spencer if and only if its government-subsidy-inclusive marginal cost is lower than the rival firm’s marginal cost.

**Total Subsidy In Equilibrium**

Using (5-15), (5-18) can be rewritten as below.

\[
0 = \frac{\partial W_i^D(s)}{\partial s_i} = -d_i \frac{\partial x_i^D}{\partial s_i} + x_i P'(X) \frac{\partial x_i^D}{\partial s_i} - s_i \frac{\partial x_i^D}{\partial s_i} \\
= -s_i \frac{\partial x_i^D}{\partial s_i} + x_i P'(X) \frac{\partial x_i^D}{\partial s_i}
\]
Solving for total subsidy in the above equation yields

\[ \hat{S}_D^i = x_i P'(X) \frac{\partial x_j^D}{\partial s_i} \frac{i \partial x_i^D}{\partial s_i} = \frac{2\beta_i''}{7} > 0. \]

Comparing total subsidy with the subsidy à la Brander-Spencer yields\(^\text{11}\)

\[ \hat{S}_D^i \geq s_i^B \iff s_i^D \geq \frac{1}{6} (c_i - c_j). \]

Note that only if the domestic firm is not considerably less efficient than the foreign firm does \( s_i^D > s_i^B \) hold. However, if the analysis is confined under the symmetric cost conditions, each firm owner’s subsidy and total subsidy in equilibrium is higher than the subsidy à la Brander-Spencer. In other words, strategic subsidy competition between the exporting countries strengthens both firms’ owner’s subsidization incentives and leads to oversubsidization to the firms.

### Welfare in Equilibrium

Country \( i \)’s welfare in equilibrium is given by:

\[ \hat{W}_i^D = W_i^D(s^D) = \frac{3}{49} \beta_i''^2, \]

which is lower than the welfare in the BS model shown in (5-8) when the cost conditions are symmetric, i.e., \( \hat{W}_i^D < \hat{W}_i^B \). However, the third country is at an advantage due to an improvement in the importing country’s terms of trade. Further, world welfare improves as well, i.e., \( \sum_{i=1}^{3} \hat{W}_i^D > \sum_{i=1}^{3} \hat{W}_i^B \).

**Proposition 5.3.** Under strategic managerial delegation and export subsidy competition, each exporting country’s welfare worsens in comparison to the BS model due to excess subsidization in the symmetric cost conditions. However, the third country benefits from an improvement in the terms of trade and world welfare improves.

---

\(^{11}\)It is given by

\[ \hat{S}_D^i - s_i^B = \frac{3a - 19c_i + 16c_j}{35} = \frac{6}{5} \left[ s_i^D - \frac{1}{6} (c_i - c_j) \right]. \]
5.4 Unilateral Manager Delegation

Next, consider the unilateral manager delegation case in which only firm 1’s owner delegates a manager. In the third stage, firm 1’s manager decides the output under the precommitted contract, while firm 2’s owner decides the output as a pure profit maximizer. The equilibrium output in (5-5) is modified as $S_1 = d_1 + s_1$ and $S_2 = s_2$.

In the second stage, only firm 1’s owner delegates a manager. The equilibrium owner’s subsidy equivalent is

$$d_{1}^{eU1}(s) = \frac{\beta_1 + 2s_1 - s_2}{4},$$

where the superscript $U1$ denotes the equilibrium when only firm 1 delegates a manager.

As for the equilibrium output change, the second part in (5-12) does not hold true.

$$\frac{\partial x^{eU1}_2}{\partial s_1} = \frac{r_2^D}{\partial x^{eU1}_1 \partial s_1}.$$ 

By substituting the above equations into (5-18), the FOC for country 1 can be derived as below.

$$0 = \frac{\partial W^{eU1}_1(s)}{\partial s_1} = s_1 \frac{\partial x^{eU1}_1}{\partial s_1}.$$ 

It is shown that country 1’s equilibrium subsidy $s_{1}^{U1} = 0$. Country 1, which has a firm following separation of ownership and management, has no incentive to subsidize its exports. The reason can be shown in Figure 1. Firm 1’s reaction curve is sloping downward as $\gamma_1^\gamma 1^\gamma$. Since $d_2 = 0$, firm 1’s owner decides the optimal subsidy at the intersection of the reaction curve and horizontal axis. Government subsidy (or tax) will shift firm 1’s reaction curve outward (or inward) and lead to lower subsidy-exclusive profit (or national welfare). Hence, country 1’s government has no incentive to intervene.

As for country 2, since $\frac{\partial x_2}{\partial x_2} = 0$, the FOC of (5-18) satisfies

$$0 = \frac{\partial W^{eU1}_2(s)}{\partial s_2} = x_2 P'(X) \frac{\partial x^{eU1}_1}{\partial s_2} - s_2 \frac{\partial x^{eU1}_2}{\partial s_2}.$$ 

Country 2’s equilibrium subsidy can be derived as

$$s_{2}^{U1} = x_2 P'(X) \frac{\partial x^{eU1}_1}{\partial s_2} / \frac{\partial x^{eU1}_2}{\partial s_2} = \frac{\beta_2'}{3} = \hat{S}_{2}^{U1} > \hat{S}_{2}^{D},$$

which has the same form as the total subsidy $S_{2}^{D}$, but results in a larger value in equilibrium.\textsuperscript{12} Country 2, which has no firm following separation of ownership and management, has stronger incentive to subsidize its exports.

\textsuperscript{12}It is shown as

$$\hat{S}_{2}^{U1} - \hat{S}_{2}^{D} = \frac{\beta_2'}{3} - \frac{2\beta_2''}{7} = \frac{\beta_2''}{21} > 0.$$
Country 2’s government subsidy weakens firm 1’s owner subsidization incentive. Solving for firm 1’s owner’s subsidy equivalent in equilibrium yields

\[
\hat{d}_{U1} = d_{E1}(0, s_{U2}) = \frac{\beta'^{''}}{6} = \hat{S}_{U1}.
\]

In the absence of government intervention, the Stackelberg equilibrium in managerial delegation yields

\[
d_{Li} \overset{\text{def}}{=} \arg \max_{d_i} \pi^*_i(d_i, \gamma^j(d_i), 0),
\]

\[
d_{Fj} \overset{\text{def}}{=} \gamma^j(d_{Li}), \quad (i, j = 1, 2; j \neq i).
\]

It is shown that \(s_{U1} = d_{L2}^1\) and \(\hat{d}_{U1} = d_{F1}^L\). When only firm 1 delegates a manager, country 1’s government has no incentive to subsidize, while country 2’s government strengthens its subsidization incentive, playing as a Stackelberg leader to firm 1’s owner in the subsidy competition. Hence, firm 1’s unilateral delegation puts itself at a disadvantage as a Stackelberg follower. Since total subsidy of each firm is just the Stackelberg leader-follower subsidy, the equilibrium outputs also yield Stackelberg solution.

**Proposition 5.4.** Unilateral manager delegation entails free trade equilibrium in the country with the managerial firm. Meanwhile, the rival country’s government strengthens the subsidization incentive, playing as a Stackelberg leader to the managerial firm in the subsidy competition.

Equilibrium total subsidies under bilateral and unilateral delegation can be summarized into Figure 5.2. Point \(B\), the intersection of \(\gamma^1\gamma^1'\) and \(\gamma^2\gamma^2'\), represents the equilibrium.
when no firm delegates a manager, the BS model. Point $U_i (i = 1, 2)$ represents the equilibrium when only firm $i$ delegates a manager. It is characterized by the tangency of firm $j (j \neq i)$’s isoprofit curve to firm $i$’s reaction curve, that is, the Stackelberg equilibrium point when firm $j$ behaves as a leader. Point $D$ represents the bilateral delegation equilibrium in that both countries’ governments play as Stackelberg leaders to the rival firm’s owner.

![Diagram](image)

Fig. 5.2: Equilibrium Total Subsidies in Bilateral and Unilateral Managerial Delegation

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13The equilibrium government subsidy in the BS model is equivalent to the equilibrium owner’s subsidy equivalent in the FJS model.
5.5 Managerial Delegation Game

The FJS model ends up in a prisoner’s dilemma in the symmetric cost condition. When endogenizing owners’ delegation decisions, Basu (1995) discussed that unilateral delegation equilibrium maybe dependent on the firms’ cost difference and delegation costs. However, in our model wherein governments play Stackelberg against owners, the owners’ managerial delegation effects on the firms’ behavior are dampened by government intervention. Unilateral managerial delegation pushes the rival country to achieve Stackelberg advantage and damage its own profit. To examine the owners’ delegation incentives, I add one more stage to let both owners decide whether or not to delegate a manager at the zero stage. The game can be divided into four subgames, and the firms’ payoffs in each subgame are shown as below in the symmetric cost case.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Delegation</th>
<th>Non-delegation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delegation</td>
<td>( \frac{a}{18}(a-c)^2 ), ( \frac{a}{18}(a-c)^2 )</td>
<td>( \frac{1}{18}(a-c)^2 ), ( \frac{1}{18}(a-c)^2 )</td>
</tr>
<tr>
<td>Non-delegation</td>
<td>( \frac{1}{18}(a-c)^2 ), ( \frac{1}{18}(a-c)^2 )</td>
<td>( \frac{1}{18}(a-c)^2 ), ( \frac{1}{18}(a-c)^2 )</td>
</tr>
</tbody>
</table>

It is shown that \( \hat{\pi}_{ij}^{Uj} > \hat{\pi}_i^B > \hat{\pi}_i^D > \hat{\pi}_i^{Uj}(i, j = 1, 2; j \neq i) \). In the equilibrium, both firms do not delegate a manager, and the Pareto-efficient equilibrium is realized. Therefore, when both governments commit to intervene, both owners do not choose to delegate a manager followed by government subsidization, since unilateral delegation leads to Stackelberg-follower payoff.

**Proposition 5.5.** In the government-owner-manager game, each firm has no incentive to delegate a manager, and the Pareto-efficient equilibrium can be realized.

The above Proposition is based on the analyses in the previous sections. Given government intervention, the managerial delegation competition leads to the oversubsidization and lowers the firms’ profits; however, unilateral delegation makes the domestic country lose subsidization incentive and strengthens the rival country’s subsidization incentive. Thus, when the owners determine whether they should delegate a manager or not before government interventions, no firm has incentive to delegate.
5.6 Concluding Remarks

This chapter reexamines the strategic export subsidy competition with the separation of ownership and management in a third market model. I explore the owners’ subsidization incentives in designing a managerial incentive contract and discuss the total subsidy effect on the firms’ performance in the market. Although the model is constructed in the same way as that of Das (1997), this chapter conveys some new implications that are not clarified in Das (1997). The essence in Das (1997) is that both the firms are subsidized with a smaller government subsidy as compared to the case without delegation. However, this result cannot explain the change in the firm’s output and social welfare since lower government subsidy merely lowers the total output and increases the monopoly rent in the exporting country, while the model ends up in a contradictory result. The rather paradoxical result can be explained by the total subsidy defined in my model. I show that firms are subsidized in a larger total subsidy, and both firms overproduce in equilibrium. The nature of strategic managerial delegation in the export subsidy competition lies in the fact that it intensifies the competition between the exporting firms and reduces the distortions in oligopoly pricing, thus improving world welfare. This is the main point that my model has emphasized differently from Das (1997).

This chapter also recognizes owner’s subsidization incentive through managerial delegation. Indicating the equivalence result between the FJS model and the BS model, I regard owners’ managerial delegation as subsidization behavior. It elucidates the result in Das (1997) as to why the governments weaken the subsidization incentives in the presence of managerial delegation. It also clarifies the Stackelberg solution in the unilateral delegation case, which resulted in the government playing Stackelberg against the owner in the subsidy competition.

The extension of delegation game shows that no firm has incentive to delegate under governments’ commitments to intervene. However, the results are largely dependent on the order of the moves. If firm owners move first and the governments subsequently, the total subsidy in equilibrium is a subsidy à la Brander-Spencer. This is because the governments always determine the optimal subsidy rates to maximize the total subsidy exclusive profit of the national firm. Irrelevant of firm owners’ subsidy rates, the governments always decide the total subsidy rate to à la Brander-Spencer subsidy. Bearing in mind this subsidization behavior, the firm owners actually choose to delegate and greatly tax the firms to induce higher government subsidy. The analysis that the owners move as leaders against governments is somewhat difficult and is left for future research.
Appendix

5.A Price Competition

Wei (2009a) examined the strategic trade policy and managerial delegation under Bertrand competition. The model is constructed in the framework of Eaton and Grossman (1986), a price competition version of the BS model.

Each firm produces a differentiated good. The demand function of good $i$ is given by

$$x_i(p) = a - p_i + bp_j \quad (i, j = 1, 2; j \neq i),$$

where $a > 0$ and $0 < b < 1$.

To simplify the analysis, the cost conditions are symmetric, i.e., $c_i = c_j = c$. Each firm’s profit function is given by

$$\pi_i(p, t_i) = (p_i - c + t_i)x_i(p).$$

Both exporting countries’ governments tax their exports at a special tax of $t_i$. Under price competition, firms’ managerial delegations yield owner’s tax equivalent denoted as $\tau_i$.

$$\tau_i := (\beta_i - 1)(c_i + t_i).$$

Total tax $T_i$ is defined as a sum of government tax and owner’s tax equivalent.

$$T_i := t_i + \tau_i = \beta_i(c_i + t_i) - c_i$$

Price Stage Equilibrium The equilibrium price of good $i$ in the third-stage is given by

$$p_i^*(T) = \frac{a(2 + b) + 2(c_i + T_i) + b(c_j + T_j)}{4 - b^2}.$$ 

The equilibrium output can be derived as below.

$$x_i^*(T) = \frac{(2 + b)a - (2 - b^2)(c_i + T_i) + b(c_j + T_j)}{4 - b^2}.$$ 

Taxation lowers domestic production and expands foreign production.
**Contract Stage Equilibrium** Without government intervention, the model is the FJS model. The equilibrium owner’s tax equivalent is identical to the optimal government tax in Eaton and Grossman (1986).

\[
\tau^F_{iJ} = \frac{b^2}{2}(p_i - c_i) = t^EG_i \quad (i = 1, 2),
\]

where superscript \( EG \) represents the equilibrium values in Eaton and Grossman (1986).

With government intervention, firm \( i \)'s reaction function is given by

\[
\gamma^i(\tau_j, t) = \frac{b^2[(2 + b)a - (2 - b^2)(c_i + t_i) + b(c_j + \tau_j + t_j)]}{4(2 - b^2)},
\]

which shows that \( \frac{\partial \gamma^i(\tau)}{\partial \tau_j} > 0 \) and \( \frac{\partial \gamma^i(t)}{\partial t_i} < 0 \). Each firm’s owner subsidy equivalent is a strategic complementary to the rival’s. Government’s taxation weakens domestic firm’s owner taxation incentive and strengthens foreign firm’s owner taxation incentive.

Denote

\[
Z_i(t) := (4 + 2b - b^2)a - (4 - 3b^2)(c_i + t_i) + b(2 - b^2)(c_j + t_j),
\]

\[
Y := (4 + 2b - b^2)(4 - 2b - b^2) = 16 - 12b^2 + b^4 > 0.
\]

The optimal owner’s tax equivalent of firm \( i \) is

\[
\tau^e_i(t) = \frac{b^2Z_i(t)}{Y} > 0 \quad (i = 1, 2).
\]

The positive taxation incentive is assured by the duopoly equilibrium given by

\[
x^e_i(t) = x^*_{i}(\tau^e_i(t) + t_i, \tau^e_j(t) + t_j) = \frac{(2 - b^2)Z_i(t)}{Y}.
\]

It is evident to show that \( \frac{\partial x^e_i(t)}{\partial t_i} < 0 \) and \( \frac{\partial x^e_j(t)}{\partial t_i} > 0 \). Government taxation reduces domestic firm’s owner tax equivalent and increases foreign firm’s tax equivalent in the equilibrium. In the quantity competition discussed in Section 5.3, since government subsidization raises domestic firm’s subsidy equivalent, government intervention policy seem as a complement to managerial delegation. However, in the price competition, government intervention policy acts as a substitution to managerial delegation.

**Tax Stage Equilibrium** Each country’s welfare function is given by

\[
W^c_i(t) = \pi_i(p^c_i(t), p^c_j(t), t_i) + t_i x^e_i(t).
\]

Optimal government tax yields

\[
\hat{t}_i = \frac{b^4C}{(2 - b^2)D} > 0,
\]
where $C := a - (1 - b)c > 0$ and $D := 8 - 4b - 4b^2 + b^3 > 0$. Although each firm’s owner taxes its exports, each government further has incentives to tax the exports, pushing the domestic firm to yield Stackelberg leader’s profit.

Comparing $\hat{t}_i$ with $t_i^{EG}$ yields

$$\hat{t}_i - t_i^{EG} = -\frac{b^3(1 - b)(2 + b)(8 - 6b^2 + b^3)C}{(2 - b^2)(4 - 2b - b^2)D} < 0.$$ 

That is, each country’s government taxes in a lower rate in the presence of managerial delegation.

Equilibrium owner’s tax equivalent can be derived as below.

$$\hat{\tau}_i = \tau_i^e(\hat{t}) = \frac{b^2(4 - 3b^2)C}{(2 - b^2)D} > 0.$$ 

Simple calculation yields $\hat{\tau}_i < t_i^{EG} = \tau_i^{FJ}$. Equilibrium owner’s subsidy equivalent also results in a lower value with government intervention.

However, total tax is larger than $t_i^{EG}$ shown as below.

$$\hat{T}_i - t_i^{EG} = \hat{\tau}_i + \hat{\tau}_i - t_i^{EG} = \frac{b^4(2 - b)C}{(4 - 2b - b^2)D} > 0.$$ 

Each country’s equilibrium welfare is given by

$$\hat{W}_i = W_i^e(\hat{t}) = \frac{(4 - b^2)(4 - 3b^2)C^2}{D^2}.$$ 

Comparing with the equilibrium welfare without managerial delegation in Eaton and Grossman (1986) yields

$$\hat{W}_i - W_i^{EG} = \frac{b^5(16 - 16b - 4b^2 + 5b^3)C^2}{(4 - 2b - b^2)^2D^2} > 0.$$ 

In the presence of separation of ownership and management, each good’s price rises up due to a larger total tax. The two firms behave close to a monopolistic firm. The exporting countries’ welfare improves and the third country is in a welfare loss. Thus, with government intervention, managerial delegation in the price competition increases distortions in the oligopoly competition and worsens world welfare.
Chapter 6

International Separation of Ownership and Management

6.1 Introduction

Chapters 4 and 5 concern the ownership and management structures of the firms. To the best of my knowledge, no paper has examined the traditional strategic export policies in the presence of both international cross shareholding and separation of ownership and management. This chapter combines the analyses in the previous two chapters and discusses how the strategic subsidization incentives are affected by managerial delegation when the shares of the firms are internationally owned by the residents of both countries, i.e., in the presence of international separation of ownership and management. This chapter attempts to study how the complexity of managerial decision process and cross shareholding structure alter the standard welfare implication of strategic export promotion policies.

The works related to this chapter is summarized in the following table.

<table>
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<th>Tab. 6.1: Literature Summary</th>
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<tr>
<td>Subsidy</td>
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<td>Brander and Spencer (1985)</td>
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The analysis in this chapter is based on Wei (2009b). The rest of this chapter is organized as follows. Section 2 investigates the equilibrium subsidy rate in the presence of both cross shareholding and separation of ownership and management. Sections 3 and 4 discuss the effects of cross shareholding and managerial delegation on the strategic subsidy
decisions, respectively. Section 5 compares the equilibrium results and their values in Chapters 4 and 5. Section 6 shows two special cases of symmetric and partial ownership structures. The concluding remarks are summed in section 7.
6.2 Subsidy Stage Equilibrium

The timing of the game is the same as that of the three-stage game of Das (1997). In this chapter, I only consider the international cross-country shareholding, i.e., the shares of both firms are internationally owned by the residents of both countries. Although shareholders are from different nations, shareholder unanimity on managerial delegation to maximize its holding firm’s profit is assumed to be satisfied. Hence, the output and contract stage equilibria are the same as in the previous chapter. I solve the game in the first stage in which the governments decide the optimal subsidy rates. As in Chapter 4, \(\sigma_i\) denotes the percentage share of firm’s \(i(i = 1, 2)\) equities owned by domestic residents and is exogenously given.

Evaluating at the equilibrium in (5-11) and (5-16), each country’s welfare function is expressed by:

\[
W^E_i(s; \sigma_i) = \sigma_i \pi_i^{eD}(s) + (1 - \sigma_j) \pi_j^{eD}(s) - s_i x_i^{eD}(s).
\]

Each government maximizes its national welfare by choosing the optimal export subsidy, taking into account the response of both firms. The FOC for welfare maximization should satisfy:

\[
\begin{align*}
\frac{\partial W^E_i(s; \sigma_i)}{\partial s_i} &= \sigma_i \frac{\partial \pi_i^{eD}(s)}{\partial s_i} + (1 - \sigma_j) \frac{\partial \pi_j^{eD}(s)}{\partial s_i} - x_i - s_i \frac{\partial x_i^{eD}}{\partial s_i} \\
&= \sigma_i \left( x_i P'(X) r^{SD}_j \frac{\partial S^eD_j}{\partial s_i} + x_i \right) + (1 - \sigma_j) \left( x_j P'(X) r^{SD}_i \frac{\partial S^eD_i}{\partial s_i} \right) - x_i - s_i \frac{\partial x_i^{eD}}{\partial s_i} \\
&= x_i P'(X) r^{SD}_j \frac{\partial S^eD_j}{\partial s_i} - s_i \frac{\partial x_i^{eD}}{\partial s_i} - (1 - \sigma_i) x_i + (1 - \sigma_j) x_j P'(X) r^{SD}_i \frac{\partial S^eD_i}{\partial s_i}. \tag{6-1}
\end{align*}
\]

Comparing the subsidy incentive in the above equation with the one without cross shareholding in (5-19), I can show that the terms in (6-1) represent the subsidy incentives under managerial delegation without cross shareholding as in Das (1997) and the terms in (6-2) the subsidy incentives specific to cross-country shareholding. Further, (6-2) can be decomposed into three parts.

The first part \(-(1 - \sigma_i) x_i P'(X) r^{SD}_j \frac{\partial S^eD_j}{\partial s_i}\) shows the **cross rent-shifting effect through managerial delegation**. Export subsidy to the home firm, through the standard rent-shifting effect, increases its profit, but it leads to an increase in the dividend given to the foreign firm. Note that \(r^{SD}_j \frac{\partial S^eD_j}{\partial s_i} < \frac{\partial x_j^{eD}}{\partial s_i}\), and hence, export subsidy increases the

\[
\frac{\partial^2 W^E_i(s; \sigma_i)}{\partial s_i^2} = -\frac{4}{25} (11 - 9 \sigma_i + 4 \sigma_j) < 0.
\]

\(\text{1}\) The SOC for welfare maximization is satisfied.
domestic owner’s subsidy and leads to fiercer competition in the output market. The domestic firm’s profit gain shrinks and the dividend given to the foreign firm decreases.

The second part \(-(1 - \sigma_i)x_i\) shows the **subsidy outflow effect**, which has the same expression as in the case without managerial delegation.

The third part \((1 - \sigma_j)x_jP'(X)r^D_S\frac{\partial s_i^D}{\partial s_i}\) shows the **dividend suppression effect through managerial delegation**. Further, note that \(r^D_S\frac{\partial s_i^D}{\partial s_i} < \frac{\partial \pi^D_e}{\partial s_i}\), and hence, managerial delegation scales down the decrease in the dividend from the shared foreign firm.

The above three negative parts weaken subsidy incentives in the presence of cross-country shareholding.

Denote \(R^{IE}(s; \sigma)\) as country \(i\)'s reaction function, where superscript \(E\) represents the values under international separation of ownership and management.

An increase in the domestic residents’ ownership share over the domestic firm (or the foreign residents’ ownership share over the foreign firm) strengthens both countries’ subsidy incentives, leading to an increase in the optimal export subsidy. The results follow from

\[
R^i_{\sigma_i}(s; \sigma) = \frac{\partial R^i_{E}(s; \sigma)}{\partial \sigma_i} = -\frac{\partial^2 W_i(s; \sigma)}{\partial \sigma_i \partial s_i} = -\frac{\partial \pi^D_e(s; \sigma)}{\partial s_i} > 0,
\]
\[
R^j_{\sigma_j}(s; \sigma) = \frac{\partial R^j_{E}(s; \sigma)}{\partial \sigma_j} = -\frac{\partial^2 W_j(s; \sigma)}{\partial \sigma_j \partial s_i} = -\frac{\partial \pi^D_j(s; \sigma)}{\partial s_i} > 0,
\]

where use was made of (5-16) and (5-17).
6.3 Effects of Cross-Country Shareholding

6.3.1 Equilibrium Government Subsidy

Denote \( s_i^E(\sigma) \) as the equilibrium government’s subsidy of country \( i \). The full-game Nash equilibrium subsidy profile is thus defined as a solution to

\[
s_i^E(\sigma) = R_i^E(s_j^E(\sigma); \sigma) \quad (i, j = 1, 2; j \neq i).
\]

Under the symmetric cost conditions, that \( c_i = c_j = c \),

\[
s_i^E(\sigma) = \frac{(16\sigma_i + 13\sigma_j - 10\sigma_i\sigma_j - 18)(a - c)}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]}.
\]  \hfill (6-3)

The optimal subsidy rate is dependent on the cross shareholding structure \( (\sigma_i, \sigma_j) \). From Assumption 4.1, the denominator in (6-3) is positive, i.e., \( 24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j > 0 \), and hence \( s_i^E(\sigma) \) is positive if and only if \( \sigma_j > \frac{18 - 16\sigma_i}{13 - 10\sigma_i} \). Figure 6.1 illustrates \( (\sigma_1, \sigma_2) \) for which country 1’s government finds zero subsidy optimal with origin \((0.5, 0.5)\). In view of Figure 6.1, when the domestic shares of both firms are large enough, each country’s government subsidizes the firm; otherwise, export tax is the optimal policy.

Without cross-country shareholding, Das (1997) showed that \( s_i^E(1, 1) = s_i^D = \frac{1}{14}(a - c) > 0 \) in (5-20), and each government always has a positive incentive to subsidize its exports. In the presence of cross-country shareholding, (6-3) shows that \( \frac{\partial s_i^E(\sigma, \sigma_j)}{\partial \sigma_k} > 0 (k = i, j) \), which yields

\[
s_i^E(\sigma_i, \sigma_j) < s_i^E(1, 1).
\]

**Proposition 6.1.** Given managerial delegation, the presence of cross-country shareholding weakens both countries’ subsidization incentives, i.e., \( s_i^E(\sigma_i, \sigma_j) < s_i^E(1, 1) \).

6.3.2 Equilibrium Owner’s Subsidy Equivalent

In view of (5-10), the owner’s subsidy equivalent is always positive in the duopolistic market. Solving for the owner’s subsidy equivalent in the equilibrium yields

\[
d_i^E(\sigma) = d_i^D(s_i^E(\sigma), s_j^E(\sigma)) = \frac{3(2 - \sigma_j)(a - c)}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]} > 0.
\]  \hfill (6-4)

It is easy to show that \( \frac{\partial d_i^E(\sigma)}{\partial \sigma_i} > 0 \). Increasing domestic ownership strengthens the domestic owner’s subsidy incentives.

Differentiating (6-4) with \( \sigma_j \) yields

\[
\frac{\partial d_i^E(\sigma)}{\partial \sigma_j} = \frac{-3(2 - \sigma_i)}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]^2}(a - c) < 0.
\]
The above equation is equivalent to \( \frac{\partial d_i^E(\sigma)}{\partial(1-\sigma)} > 0 \). Increasing the shares of foreign equities owned by domestic residents also increases the domestic owner’s subsidy equivalent.

Comparing \( d_i^E(\sigma_i, \sigma_j) \) with \( d_i^E(1, 1) \) yields

\[
d_i^E(\sigma_i, \sigma_j) - d_i^E(1, 1) = \frac{3(2 - \sigma_j)(a - c)}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]} - \frac{3(a - c)}{14}.
\]

Thus, \( d_i^E(\sigma_i, \sigma_j) < d_i^E(1, 1) \) if only if \( \sigma_i < \frac{10 - 4\sigma_j}{11 - 5\sigma_j} \), or equivalently, \( \Delta \sigma_i = \sigma_i - \sigma_j < \frac{5(1-\sigma_j)(2-\sigma_i)}{11 - 5\sigma_j} \). The owner’s subsidy equivalent may be larger in the presence of cross-country shareholding if the two firms’ domestic share difference is large enough.

### 6.3.3 Equilibrium Output

Under the linear demand function, the equilibrium output yields

\[
x_i^E(\sigma) = 2d_i^E(\sigma) = \frac{3(2 - \sigma_j)(a - c)}{24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j}.
\]

The same result holds for the equilibrium output, i.e., \( x_i^E(\sigma_i, \sigma_j) < x_i^E(1, 1) \) if and only if \( \sigma_i < \frac{10 - 4\sigma_j}{11 - 5\sigma_j} \).

**Proposition 6.2.** Given managerial delegation, the presence of cross-country shareholding may increase firm i’s equilibrium output if firm i’s domestic share \( \sigma_i \) is large enough.

\[
d_i^E(\sigma_i, \sigma_j) \gtrless d_i^E(1, 1) \iff x_i^E(\sigma_i, \sigma_j) \gtrless x_i^E(1, 1) \iff \sigma_i \gtrless \frac{10 - 4\sigma_j}{11 - 5\sigma_j}.
\]

Note that if \( \sigma_i < \frac{16}{19} \), cross-country shareholding always lowers firm i’s equilibrium output, i.e., \( x_i^E(\sigma_i, \sigma_j) < x_i^E(1, 1) \) irrespective of values of \( \sigma_j \).

### 6.3.4 Equilibrium Total Subsidy

Solving for total subsidy, it yields

\[
S_i^E(\sigma) = s_i^E(\sigma) + d_i^E(\sigma) = \frac{(8\sigma_i + 5\sigma_j - 5\sigma_i\sigma_j - 6)(a - c)}{24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j}.
\]

\( S_i^E(\sigma) \) is positive if and only if \( \sigma_j > \frac{2(3-4\sigma_i)}{5(1-\sigma_i)} \). \( (\sigma_1, \sigma_2) \) for \( S_i^E(\sigma) = 0 \) are depicted in Figure 6.1. Although the owner’s subsidy equivalent is always positive, total subsidy may become negative when foreign residents’ ownership in the home firm’s shares is large enough. That is, the government’s optimal tariff outweighs the owner’s optimal subsidy.

Since \( \frac{\partial S_i^E(\sigma_j)}{\partial \sigma_k} > 0 (k = i, j) \), \( S_i^E(\sigma_i, \sigma_j) < S_i^E(1, 1) \) always holds.

\( ^2d_i^E(1, 1) \) is equivalent to \( d_i^D \) in (5-22).
6.4 Effects of Managerial Delegation

Next, I examine the effects of managerial delegation under cross-country shareholding. Without managerial delegation, the optimal government subsidy (see Dick (1993), Welzel (1995)) yields

\[
s^C_i(\sigma) = \frac{(16\sigma_i + 12\sigma_j - 12\sigma_i\sigma_j - 15)(a - c)}{33 - 20(\sigma_i + \sigma_j) + 12\sigma_i\sigma_j}.
\]

\(s^C_i(\sigma) > 0\) is positive if and only if \(\sigma_j > \frac{15 - 16\sigma_i}{12(1 - \sigma_1)}\). \((\sigma_1, \sigma_2)\) for \(s^C_i(\sigma) = 0\) are shown by the dashed line in Figure 6.1, which lies between the lines for \(s^E_1(\sigma) = 0\) and \(S^E_1(\sigma) = 0\). Figure 6.1 shows that without managerial delegation, the government is more likely to subsidize its own firm.

\[
x^C_i(\sigma) = \frac{2(3 - 2\sigma_j)(a - c)}{33 - 20(\sigma_i + \sigma_j) + 12\sigma_i\sigma_j}.
\]

Fig. 6.1: \(s^E_1\) and \(S^E_1\)

Without managerial delegation, the equilibrium output yields
6.4.1 Optimal Subsidy

Comparing $s_i^E(\sigma)$ with $s_i^C(\sigma)$ yields

$$s_i^E(\sigma) - s_i^C(\sigma) = \frac{(16\sigma_i + 13\sigma_j - 10\sigma_i\sigma_j - 18)(a - c) - (16\sigma_i + 12\sigma_j - 12\sigma_i\sigma_j - 15)(a - c)}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]} - \frac{33 - 20(\sigma_i + \sigma_j) + 12\sigma_i\sigma_j}{2[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j]}(a - c).$$

(6-5)

Since the denominator in (6-5) is positive, the sign of $s_i^E(\sigma) - s_i^C(\sigma)$ is determined by the numerator. Define

$$f(\sigma) = 4(1 - 7\sigma_i)\sigma_j^2 - (117 - 216\sigma_i + 32\sigma_i^2)\sigma_j + 126 - 210\sigma_i + 32\sigma_i^2$$

(6-6)

which is a quadric equation of $\sigma_j$. The discriminant for $f(\sigma)$ is given by $\Delta(\sigma_i) \defeq (117 - 216\sigma_i + 32\sigma_i^2)^2 - 16(1 - 7\sigma_i)(126 - 210\sigma_i + 32\sigma_i^2)$. There are four solutions for $\Delta(\sigma_i) = 0$, but because of Assumption 4.1, I only consider the range around $\sigma_i \in (0.5, 1)$. Since $\Delta(0.5) > 0$ and $\Delta(1) < 0$, there exist at least one solution satisfying $\Delta(\sigma_i) = 0$ for all $\sigma_i \in (0.5, 1)$ in view of the intermediate-value theorem.

**Lemma 6.1.** For all $\sigma_i \in (0.5, 1)$, there exists a unique $\sigma_i = \hat{\sigma}$ satisfying $\Delta(\hat{\sigma}) = 0$.

**Proof.** (Reduction to absurdity) If there are two solutions $\sigma_a, \sigma_b (0.5 < \sigma_a < \sigma_b < 1)$ satisfying $\Delta(\sigma_a) = \Delta(\sigma_b) = 0$, by the mean-value theorem, there must exist $\sigma_c \in (\sigma_a, \sigma_b)$ satisfying $\Delta'(\sigma_c) = 0$. Since $\Delta''(\sigma_i) = 64(941 - 960\sigma_i + 192\sigma_i^2) > 0$ for all $\sigma_i \in (0.5, 1)$, $\Delta'(\sigma_i) > 0$ for all $\sigma_i > \sigma_c$. This leads to $\Delta(\sigma_i) > 0$ for all $\sigma_i > \sigma_b$, which contradicts the result that $\Delta(1) < 0$.

From above and Lemma 6.1, it yields

$$\Delta(\sigma_i) \begin{cases} > 0 & \text{when } \sigma_i > \hat{\sigma} \\ \leq 0 & \text{when } \sigma_i \leq \hat{\sigma} \end{cases}$$

(6-7)

Since $1 - 7\sigma_i < 0$ in (6-6), from the results in (6-7), $s_i^E(\sigma)$ vs. $s_i^C(\sigma)$ can be shown for the following three cases.

(I) When $\sigma_i > \hat{\sigma}$, $\Delta(\sigma_i) < 0$ holds.

$$f(\sigma) < 0 \iff s_i^E(\sigma) < s_i^C(\sigma)$$

(II) When $\sigma_i < \hat{\sigma}$, $\Delta(\sigma_i) > 0$ holds. There exist two real roots $\sigma_j(\sigma_i)$ and $\sigma_{\overline{j}}(\sigma_i)$ satisfying $f(\sigma_i, \sigma_j) = f(\sigma_i, \sigma_{\overline{j}}) = 0$.

$$\begin{cases} f(\sigma) > 0 \iff s_i^E(\sigma) > s_i^C(\sigma) & \text{when } \sigma_j(\sigma_i) < \sigma_j < \sigma_{\overline{j}}(\sigma_i) \\ f(\sigma) = 0 \iff s_i^E(\sigma) = s_i^C(\sigma) & \text{when } \sigma_j = \sigma_j(\sigma_i) \text{ or } \sigma_{\overline{j}}(\sigma_i) \\ f(\sigma) < 0 \iff s_i^E(\sigma) < s_i^C(\sigma) & \text{when } \sigma_j(\sigma_i) < \sigma_j \text{ or } \sigma_j > \sigma_{\overline{j}}(\sigma_i), \end{cases}$$

where $\sigma_j(\sigma_i) = \frac{117 - 216\sigma_i + 32\sigma_i^2 + \sqrt{\Delta(\sigma_i)}}{8(1 - \sigma_i)}$ and $\sigma_{\overline{j}}(\sigma_i) = \frac{117 - 216\sigma_i + 32\sigma_i^2 - \sqrt{\Delta(\sigma_i)}}{8(1 - \sigma_i)}$. 

"\text{105}"
When \( \sigma_i = \hat{\sigma}, \Delta(\sigma_i) = 0 \) holds and \( \overline{\sigma}_j(\hat{\sigma}) = \underline{\sigma}_j(\hat{\sigma}) \cdot \)

\[
\left\{ \begin{array}{ll}
 f(\sigma) < 0 & \iff \quad s_i^E(\sigma) < s_i^C(\sigma) \quad \text{when} \quad \sigma_j \neq \overline{\sigma}_j(\hat{\sigma}) \\
 f(\sigma) = 0 & \iff \quad s_i^E(\sigma) = s_i^C(\sigma) \quad \text{when} \quad \sigma_j = \overline{\sigma}_j(\hat{\sigma}) 
\end{array} \right.
\]

The values of the two roots, \( \overline{\sigma}_j(\sigma_i) \) and \( \underline{\sigma}_j(\sigma_i) \), for \( \Delta(\sigma_i) = 0 \) yield the following lemma.

**Lemma 6.2.** For all \( \sigma_i \in (0.5, \overline{\sigma}) \), \( \overline{\sigma}_j(\sigma_i) \) and \( \underline{\sigma}_j(\sigma_i) \) satisfy:

(i) \( \overline{\sigma}_j'(\sigma_i) < 0 \), \( \underline{\sigma}_j'(\sigma_i) > 0 \).

(ii) \( \overline{\sigma}_j(\sigma_i) \gtrless 1 \iff \sigma_i \gtrless \sigma_m \).

(iii) \( \underline{\sigma}_j(\sigma_i) \lessgtr 0.5 \iff \sigma_i \lessgtr \sigma_n \).

Here \( 0.5 < \sigma_m < \sigma_n < \hat{\sigma} < 1 \).

**Proof.** See the Appendix.

The curves for \( \overline{\sigma}_2(\sigma_1) \) and \( \underline{\sigma}_2(\sigma_1) \) obtained from Lemma 6.2, are depicted in Figure 6.2. The two curves intersected at \( \overline{\sigma}_1 = \hat{\sigma} \), where \( \overline{\sigma}_2(\hat{\sigma}) = \underline{\sigma}_2(\hat{\sigma}) \). Figure 6.2 is largely divided into two areas by the curves of \( \overline{\sigma}_2(\sigma_1) \) and \( \underline{\sigma}_2(\sigma_1) \). The left area shows \( s_i^E(\sigma) > s_i^C(\sigma) \) and the right area shows the opposite one; the curves represent \( s_i^E(\sigma) = s_i^C(\sigma) \). Note that regardless the value of \( \sigma_2 \), if \( \sigma_1 > \hat{\sigma} \), \( s_i^E(\sigma) < s_i^C(\sigma) \) always holds; and if \( \sigma_1 < \sigma_m \), \( s_i^E(\sigma) > s_i^C(\sigma) \) always holds.

**Proposition 6.3.** Given cross-country shareholding, managerial delegation may raise or lower the governments’ optimal subsidy rates depending on the cross shareholding structure \( (\sigma_i, \sigma_j) \) when \( \sigma_m \leq \sigma_i \leq \overline{\sigma} \). Furthermore, if the domestic shareholding ratio is small enough that \( \sigma_i < \sigma_m \), managerial delegation always strengthens the government’s subsidization incentive; if the domestic shareholding ratio is large enough that \( \sigma_i > \hat{\sigma} \), the government’s subsidization incentive is always weakened under managerial delegation.

Without cross shareholding, as shown in the previous chapters, managerial delegation always weakens the government’s subsidization incentive. When the domestic shareholding ratio is large, a small fraction of foreign shareholding does not change this result. However, when the domestic shareholding is small enough, a nearly half, the large portion of foreign shareholding induces the government to tax the exports. Under managerial delegation, the negative cross-rent shifting and dividend suppression effects are dampened, and as such, the government’s tax incentive is also weakened.
6.4.2 Owner’s Subsidy Equivalent and Total Subsidy

d^E_i(\sigma) > s^C_i(\sigma) and S^E_i(\sigma) > s^C_i(\sigma) hold for any given (\sigma_i, \sigma_j). With managerial delegation, both the owner’s subsidy equivalent and total subsidy always result in higher subsidy rates regardless of the cross shareholding structure.

6.4.3 Output Decision

Comparing \( x^E_i(\sigma) \) with \( x^C_i(\sigma) \) yields

\[
x^E_i(\sigma) - x^C_i(\sigma) = \frac{2(1 - \sigma_i)(27 - 29\sigma_j + 8\sigma_j^2) + \sigma_j}{[24 - 11(\sigma_i + \sigma_j) + 5\sigma_i\sigma_j][33 - 20(\sigma_i + \sigma_j) + 12\sigma_i\sigma_j]} > 0.
\]

Given \( \sigma_k(k = i, j) > 0.5 \), \( x^E_i(\sigma) > x^C_i(\sigma) \) holds. Managerial delegation increases the equilibrium output irrespective of the cross shareholding structure.

**Proposition 6.4.** Given cross-country shareholding, managerial delegation always increases each firm’s equilibrium output, i.e., \( x^E_i(\sigma) > x^C_i(\sigma) \).
6.5 Equilibrium Results under Four Cases

In this section, I summarize the equilibrium results analyzing the implication of shareholding structure and managerial delegation on the governments’ subsidy incentives under the following four cases.

- Case B: without managerial delegation and cross shareholding,
- Case C: with only cross-country shareholding,
- Case D: with only managerial delegation,
- Case E: with both managerial delegation and cross-country shareholding.

The above four cases are discussed in the previous chapters and this chapter. The related papers are shown in the following table.

### Tab. 6.2: Related Papers

<table>
<thead>
<tr>
<th>Delegation</th>
<th>Cross-Country Shareholding</th>
<th>No Shareholding</th>
</tr>
</thead>
</table>

6.5.1 Optimal Subsidy

First, the government’s optimal subsidy rate under the four cases is summarized as below.

### Tab. 6.3: Government’s Optimal Subsidy Rate

<table>
<thead>
<tr>
<th>Delegation</th>
<th>Cross-Country Shareholding</th>
<th>No Shareholding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s_i^E(\sigma) = \frac{(16\sigma_i + 12\sigma_j - 18)}{2(24 - 11(\sigma_i + \sigma_j) + 5\sigma_i \sigma_j)} (a - c) )</td>
<td>( s_i^D = \frac{1}{19}(a - c) )</td>
</tr>
<tr>
<td></td>
<td>( s_i^C(\sigma) = \frac{(16\sigma_i + 12\sigma_j - 18)}{33 - 20(\sigma_i + \sigma_j) + 12\sigma_i \sigma_j} (a - c) )</td>
<td>( s_i^B = \frac{1}{5}(a - c) )</td>
</tr>
</tbody>
</table>

As shown in Propositions 4.1 and 6.1, cross-country shareholding always lowers the government’s optimal subsidy rate irrespective of the managerial decision, i.e., \( s_i^C(\sigma) < s_i^B \) and \( s_i^E(\sigma) < s_i^D \).

As shown in Lemma 5.2, managerial delegation always lowers government’s optimal subsidy rate in the absence of cross shareholding, i.e., \( s_i^D < s_i^B \). However, with cross-country shareholding, the effects of managerial delegation on the optimal subsidy are dependent on the cross shareholding structure \( (\sigma_i, \sigma_j) \) shown in Figures 6.2 and 6.3.

Furthermore,

\[
 s_i^C(\sigma) - s_i^D = \frac{(244\sigma_i + 188\sigma_j - 180\sigma_i \sigma_j - 243)}{14(33 - 20(\sigma_i + \sigma_j) + 12\sigma_i \sigma_j)} (a - c),
\]
which shows that \( s_i^C(\sigma) > s_i^D \) if and only if \( \sigma_j > \frac{243 - 244\sigma_i}{188 - 180\sigma_i} \). \((\sigma_1, \sigma_2)\) satisfying \( s_i^C(\sigma) = s_i^D \) are depicted in Figure 6.3.

Summarizing the above results, the ranking of equilibrium subsidy rate under the four cases as shown in Figure 6.3 is as below.

- \((\sigma_1, \sigma_j) \in \text{Region (I)}: s_i^C(\sigma) < s_i^E(\sigma) < s_i^D < s_i^B\),
- \((\sigma_1, \sigma_j) \in \text{Region (II)}: s_i^E(\sigma) < s_i^C(\sigma) < s_i^D < s_i^B\),
- \((\sigma_1, \sigma_j) \in \text{Region (III)}: s_i^E(\sigma) < s_i^D < s_i^C(\sigma) < s_i^B\).

Note that the government’s equilibrium subsidy rate is the highest under the BS model, the case without managerial delegation and cross shareholding. The presence of both managerial delegation and cross shareholding weakens the government’s subsidization incentive. The lower the domestic shareholding ratio, the stronger the effect of cross shareholding and the weaker the effect of managerial delegation.

### 6.5.2 Output Decision

The following table summarizes the individual firm’s equilibrium output under the four cases.
Tab. 6.4: Individual Firm’s Equilibrium Output

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Cross-Country Shareholding</th>
<th>No Shareholding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delegation</td>
<td>$x_i^D(\sigma) = \frac{3^{2-\sigma_i}}{24-11(\sigma_i+\sigma_j)+5\sigma_i\sigma_j} (a-c)$</td>
<td>$x_i^D = \frac{3}{7} (a-c)$</td>
</tr>
<tr>
<td>No Delegation</td>
<td>$x_i^E(\sigma) = \frac{2^{3-2\sigma_i}}{33-20(\sigma_i+\sigma_j)+12\sigma_i\sigma_j} (a-c)$</td>
<td>$x_i^B = \frac{2}{5} (a-c)$</td>
</tr>
</tbody>
</table>

From Proposition 6.4 and the results of the table, managerial delegation always increases the firms’ equilibrium outputs irrespective of the cross shareholding stricture, i.e., $x_i^D > x_i^B$ and $x_i^E(\sigma) > x_i^C(\sigma)$.

Proposition 6.2 shows that cross-country shareholding may increase the firms’ equilibrium outputs if the domestic share is large enough and there is separation of ownership and management. That is, $x_i^E(\sigma) > x_i^D$ if and only if $\sigma_i > \frac{10-4\sigma_j}{11-5\sigma_j}$.

The ranking of the equilibrium output under the four cases is shown in Figure 6.4.

Note that the equilibrium output is the highest under the case with only managerial delegation. The presence of cross shareholding increases (reduces) the equilibrium output when the domestic shareholding ratio is high (low). The lower the domestic shareholding ratio, the stronger the negative effect of cross shareholding on reducing the equilibrium output.
6.6 Special Shareholding Structures

In this section, I examine the equilibrium subsidy rates under two special shareholding structures.

6.6.1 Symmetric Shareholding Structure when $\sigma_i = \sigma_j = \sigma$

In view of Figure 6.1, $s^E_i(\sigma, \sigma) > 0$ if $\sigma > 0.9$ and $s^E_i(\sigma, \sigma) > 0$ if $\sigma > 0.6$.

Comparing $s^E_i(\sigma, \sigma)$ with $s^C_i(\sigma, \sigma)$ yields

$$s^E_i(\sigma, \sigma) - s^C_i(\sigma, \sigma) = \frac{3(7 - 10\sigma)(a - c)}{264 - 254\sigma + 60\sigma^2}.$$

Thus, $s^E_i(\sigma, \sigma) > s^C_i(\sigma, \sigma)$ if $\sigma > 0.7$. The ranking of equilibrium subsidy rates are shown as below.

- $\sigma \in [0, 0.7]: s^C_i(\sigma, \sigma) \leq s^E_i(\sigma, \sigma) < s^D_i < s^B_i$
- $\sigma \in (0.7, 0.9]: s^E_i(\sigma, \sigma) < s^C_i(\sigma, \sigma) \leq s^D_i < s^B_i$
- $\sigma \in (0.9, 1]: s^E_i(\sigma, \sigma) < s^D_i < s^C_i(\sigma, \sigma) \leq s^B_i$

which are shown in Figure 6.3.

6.6.2 Partial Shareholding Structure when $\sigma_i = 1$

When firm $i$’s shares are totally owned by its own country’s residents as $\sigma_i = 1$, $s^E_i(1, \sigma_j) > 0$ if $\sigma_j > \frac{2}{3}$ and $s^E_i(1, \sigma_j) > 0$ if $\sigma_j > \frac{5}{6}$ shown in Figure 6.1. Total subsidies of both firms ($s^E_i, s^E_j$) are always positive.

In view of Figure 6.3, when $\sigma_i = 1$, $s^E_i(1, \sigma_j) < s^D_i < s^C_i(1, \sigma_j) < s^B_i$ holds regardless of the value of $\sigma_j$.

However, comparing $s^E_j(1, \sigma_j)$ with $s^C_j(1, \sigma_j)$ yields

$$s^E_j(1, \sigma_j) - s^C_j(1, \sigma_j) = \frac{13 - 22\sigma_j}{338 - 364\sigma_j + 96\sigma_j^2}(a - c).$$

Since the denominator is always positive, $s^E_j(1, \sigma_j) < s^C_j(1, \sigma_j)$ if and only if $\sigma_j > \sigma_m$.

- $\sigma_j \in [0.5, \sigma_m]: s^C_j(1, \sigma_j) \leq s^E_j(1, \sigma_j) < s^D_j < s^B_j$
- $\sigma_j \in (\sigma_m, \frac{55}{64}]: s^E_j(1, \sigma_j) < s^C_j(1, \sigma_j) \leq s^D_j < s^B_j$
- $\sigma_j \in (\frac{55}{64}, 1]: s^E_j(1, \sigma_j) < s^D_j < s^C_j(1, \sigma_j) \leq s^B_j$

which are shown in Figure 6.3.
6.7 Thesis Conclusion

This thesis examines the traditional strategic trade policies in the presence of capital liberalization, international cross shareholding and separation of ownership and management in an international duopolistic market. Although the analyses are limited in the framework of the BS model, the essential results are not changed when considering domestic consumer surplus. In Chapter 3, unilateral capital liberalization may lower the domestic country’s subsidy rate, but it raises the rival country’s subsidy rate. However, bilateral capital liberalization dampens both countries’ subsidization incentives and leads to free trade. Strategic subsidization strongly reacts to capital liberalization policy in a negative way, since firm relocation ability directly affects the subsidy expenses. In Chapter 4, under international cross shareholding when a part of equities of both firms are owned by the foreign country’s residents or the rival firm’s shareholders, government subsidization is weakened, but not disappears. Both governments still have positive incentives to subsidize their exports, since cross shareholding only causes a part of domestic profit in change of foreign profit and the governments need not subsidize the foreign products. Strategic subsidization reacts negatively, but not so strongly to cross shareholding. In Chapter 5, under managerial delegation, the owners have positive incentives to subsidize their products. When governments involve in, government subsidization is weakened, but it actually raises the owner’s subsidy incentives and leads to higher total subsidy. Thus, strategic subsidization reacts positively to managerial delegation.

Chapter 6 combines the analyses in Chapters 4 and 5 and examines the implication of the cross shareholding structure and managerial decision of firms on the strategic export subsidy incentives. Although the presence of either cross shareholding or managerial delegation weakens the exporting countries’ subsidization incentives, their combined presence does not. It is found that the cross shareholding structure always weakens both countries’ subsidization incentives irrespective of the managerial delegation. However, managerial delegation may raise or lower the optimal subsidy (or tax) rate depending on the cross shareholding structure.

Chapters 4 to 6 focus on the separation of the firm’s structure: separation of ownership and management and separation of stocks among different nationals. The separation of ownership and management makes the firms behave more aggressively, and the separation of stocks among the rival firm’s shareholders makes them collude with each other. As for the government’s subsidy policy decision, both separation effects weaken the government’s subsidization incentives. The strength of the two separation effects is dependent on the cross shareholding structure. The lower the domestic shareholding ratio, the larger the effect of cross shareholding.

Besides separation, merger is also often found in modern enterprises. In contrast to the analyses in Chapters 4 to 6, it is interesting to examine integration in the firm’s structure: integration of management and integration of stocks (merger). The study to examine the integration effect on the firms’ behavior and government’s policy decision complements the research in this thesis and gives some new implications on the analyses of trade policy dealing with the firm’s structure.
This thesis provides new insights into the traditional strategic export subsidy studies. The flow of foreign direct investment and the diversified ownership and management structure of the firm make the subsidy policy analysis more complicated and they give the standard rent shifting effect a whole new meaning. The studies in this thesis can be applied into R&D subsidy competition, environment regulation policies and public firm analysis. Furthermore, the thesis examines the welfare effects of export subsidy policy and discusses what kind of coordination policies is necessary from the viewpoint of world welfare maximization. The study is expected as a cornerstone research toward institution-building for international harmonization.
Appendix

6.A Proof in Lemma 6.2

(i) Differentiating $\sigma_j(\sigma)$ and $\overline{\sigma}_j(\sigma)$ with $\sigma_i$ yields

$$\sigma_j'(\sigma_i) = \frac{A(\sigma_i) + B(\sigma_i)}{8\sqrt{\Delta(\sigma_i)(1 - 7\sigma_i)^2}}, \quad \overline{\sigma}_j'(\sigma_i) = \frac{A(\sigma_i) - B(\sigma_i)}{8\sqrt{\Delta(\sigma_i)(1 - 7\sigma_i)^2}}$$

where $A(\sigma_i) := (603 + 64\sigma_i - 224\sigma_i^2)\sqrt{\Delta(\sigma_i)}$ and $B(\sigma_i) := \frac{1}{2}(1 - 7\sigma_i)\Delta'(\sigma_i) + 7\Delta(\sigma_i).$

Given Assumption 4.1, $A(\sigma_i) > 0$ and $B(\sigma_i) > 0$ since $\Delta(\sigma_i) > 0$ and $\Delta'(\sigma_i) < 0$ for all $\sigma_i \in (0.5, \hat{\sigma})$. Thus, it is evident that $\sigma_j'(\sigma_i) > 0$.

The sign for $\sigma_j'(\sigma_i)$ is dependent on the difference between $A(\sigma_i)$ and $B(\sigma_i)$. Since $A(\sigma_i) + B(\sigma_i) > 0$, the difference of their squares is shown as below.

$$\begin{align*}
A(\sigma_i)^2 - B(\sigma_i)^2 &= -32(1 - 7\sigma_i)^2(105399 + 261024\sigma_i - 221248\sigma_i^2 - 115712\sigma_i^3 + 78848\sigma_i^4) \\
&= -32(1 - 7\sigma_i)^2[105399(1 - \sigma_i^3) + 221248\sigma_i^2(1 - \sigma_i) + 10313\sigma_i(1 - \sigma_i) + 29464\sigma_i^2 + 78848\sigma_i^4] \\
&< 0
\end{align*}$$

Thus, $A(\sigma_i) < B(\sigma_i)$, which yields $\overline{\sigma}_j'(\sigma_i) < 0$.

(ii) Comparing $\overline{\sigma}_j(\sigma_i)$ with 1 yields

$$\overline{\sigma}_j(\sigma_i) - 1 = \frac{109 - 160\sigma_i + 32\sigma_i^2 - \sqrt{\Delta(\sigma_i)}}{8(1 - 7\sigma_i)},$$

where the sign is determined by the numerator. Define $\{\sigma_m : 109 - 160\sigma_m + 32\sigma_m^2 - \sqrt{\Delta(\sigma_m)} = 0\}$. Under constraint in Assumption 4.1, one solution is obtained: $\sigma_m = \frac{13}{22}$. Simple computation shows $\Delta(\sigma_m) > 0$, then $\sigma_m < \hat{\sigma}$ holds. In view of $\overline{\sigma}_j'(\sigma_i) < 0$, $\overline{\sigma}_j(\sigma_i) \leq 1$ if and only if $\sigma_i \geq \frac{13}{22}$ for all $\sigma_i \in (0.5, \hat{\sigma})$.

$$\overline{\sigma}_j(\sigma_i) \leq 1 \iff \sigma_i \geq \sigma_m = \frac{13}{22}, \quad \forall \sigma_i \in (0.5, \hat{\sigma})$$
(iii) Comparing $\sigma_j(\sigma_i)$ with 0.5 yields

$$
\frac{\sigma_j(\sigma_i) - 0.5}{8(1 - 7\sigma_i)} = \frac{113 - 188\sigma_i + 32\sigma_i^2 + \sqrt{\Delta(\sigma_i)}}{8(1 - 7\sigma_i)}
$$

where the sign is determined by the numerator. Define $\{\sigma_n : 113 - 188\sigma_n + 32\sigma_n^2 - \sqrt{\Delta(\sigma_n)} = 0\}$. One solution is $\sigma_n = \frac{109 - 21\sqrt{17}}{32}$ under the constraint in Assumption 4.1. Simple computation shows $\Delta(\sigma_n) > 0$, then $\sigma_n < \sigma_m < \hat{\sigma}$ holds. In view of $\sigma_j'(\sigma_i) > 0$, $\sigma_j(\sigma_i) \gtrless 0.5$ if and only if $\sigma_i \gtrless \frac{109 - 21\sqrt{17}}{32}$ for all $\sigma_i \in (0.5, \hat{\sigma})$.

$$
\sigma_j(\sigma_i) \gtrless 0.5 \iff \sigma_i \gtrless \sigma_n = \frac{109 - 21\sqrt{17}}{32}, \quad \forall \sigma_i \in (0.5, \hat{\sigma})
$$
REFERENCE


——— (2009b): “Strategic Trade Policy and Managerial Incentives with Cross Ownership,” a paper presented at Asia Pacific Trade Seminars 5th Annual Meeting held at University of HongKong.


