Solving Problems from *Sangaku* with Technology

— For Good Mathematics in Education —

Hideyo MAKISHITA

1. Introduction

Wasan[1] started to be reevaluated and there are also some researchers who reported the use of *Wasan* in mathematics education. Furthermore, the number of reports of practical lesson trials or other research which attempt to make use of *Wasan* in mathematics education are increasing. I also use *Wasan* and *Sangaku*[2] as the history of mathematics in class. It is significant for students to learn the mathematical contents among with its history, and such materials encourage students to study mathematics more willingly and eagerly. Moreover, historical materials work well for making students construct the concept of mathematics. For example, *Sangaku* takes up a lot of problems about the figure, such as the properties of circles, which became intuitively obvious contents. But it is not easy for students to solve such problems. In such cases, we mathematics teachers advise students to draw the geometric constructions in problems, as students are poor at drawing geometric constructions. Therefore, I consider that using the technology, especially the use of a handheld graphic calculator, for drawing the figure is valuable.

The main aim of this paper is to report the possibility of using technology in order to draw the figure in *Sangaku*. In section 2, a brief history of *Wasan*, especially about "*Idai Keishou*" which contributed greatly to the development of *Wasan* will be introduced. In section 3, I will introduce the offering of *Sangaku*. Section 4 will be a description of *Sangaku* along with the existing *Sangaku* of Kon'nou Shrine. Section 5 will show how the graphic calculator is being used in *Sangaku* of Kon'nou Shrine.

2. *Wasan and Idai Keishou* (The Passing on of Difficult Problems)

Firstly, when introducing *Wasan,* it is necessary to introduce the *Wasan* treatise "*Jinkouki*" written and published by Mitsuyoshi Yoshida in 1627. "*Jinkouki*" is a mathematical treatise well-known to many people. Mitsuyoshi Yoshida, a member of the Kyoto Suminokura Clan of merchants, had studied the Chinese mathematical treatise "*Sanpou Tousou*" (1592, Cheng Dawei) under the tutelage of Ryoui Suminokura and Soan Suminokura. Soan Suminokura is well known for publishing the great reproduction of classical writings "*Sagabon*." Using "*Sanpou Tousou*" as a model, Yoshida wrote "*Jinkouki*" by creating mathematical problems which were intimately related to the realities of day-to-day life in Japan at the time. From its illustrations to its overall style, "*Jinkouki*" is an exceptional book that draws from the literary legacy of the great work "*Sagabon*." "*Jinkouki*" was also a lifestyle manual explaining, in both minute and considerate detail, methods for using the calculation tool called *Soroban* (abacus), which was necessary for people in their daily lives at that time. Among with the common use of *Soroban* in Japan, a slew of *Wasan* treatises imitating "*Jinkouki*" also began to appear. Since these imitations of "*Jinkouki*" continued to be published, Yoshida revised "*Jinkouki*" several times. In "Revised *Jinkouki*" published in 1641, Yoshida included 12 mathematical problems without answers. At the end of the book he wrote, "There are those out there who are teaching mathematics at the level of "*Jinkouki*." People studying mathematics probably have no way of knowing whether their teacher is competent or not, so I will teach you a method for judging the ability of your teacher. I shall write here twelve problems without answers. You may judge the ability of your teacher by whether or not he can solve these problems." These problems are called "*Idai*" (problems left behind) or "*Konomi*" (favorites).

The solutions to Yoshida's *Idai* were revealed 12 years later, in 1653, in "*Sanryouroku*" which was written by a young mathematician named Tomosumi Enami. Imitating Yoshida, Enami also wrote his own set of eight *Idai* problems at the end of his book. Enami's publication stimulated other mathematicians to publish their works with solutions to the *Idai* of "*Jinkouki*" along with their own sets of *Idai*. Examples include "*Enpou Shikanki*" written by Jyushun Hatsusaka in 1657, "*Sanpou Ketsugishou*" written by Yoshinori Isomura in 1661 and "*Sanpou Kongenki*" written by Masaoki Satoh in 1669.

So began the "relay" of mathematical questions and answers, in which one would publish a *Wasan* treatise with solutions to previous *Idai* while also including new *Idai* of one's own creation. This relay process is called "*Idai Keishou*" (the passing on of difficult problems). It goes without saying that these *Idai* became progressively more difficult, while at the same time spurring the development of new kinds of mathematical operations. It is obvious that "*Idai Keishou*" contributed greatly to the development of *Wasan*.

3. The Offering of *Sangaku*

The term *Sangaku* refers to *Ema* (votive tablets) on which mathematical problems were written and which were dedicated to shrines and temples. It is often said that the custom of offering *Sangaku* began in 1660, during the middle of the Edo Period. People at that time commonly dedicated *Sangaku* as expressions of gratitude to the gods for having been able to solve mathematical problems, and also as prayers to be able to apply themselves even more to their studies. With shrines and temples, as centers of social discourse for common people at the time, also being places for showing one's achievements, there also appeared *Sangaku* which were dedicated with only *Idai* without answers. Approximately 1,000

Sangaku exist up to this day. The custom of offering *Sangaku* is considered to be one of the unique aspects of Japanese culture.

By focusing on the background of *Sangaku*, it can be considered that the mathematical culture of the Edo Period was extremely advanced. The diverse contents ranging from mathematical games to fully-fledged mathematics also offers us a wealth of content and topics which are applicable to the modern mathematics taught in schools nowadays. Unfortunately, Japan's conversion to Western mathematics during the Meiji period, resulted as the decrease of the recognition of *Wasan*(3). However, *Wasan*, with its multitude of topics intimately related to daily life, is being reevaluated, as is the case with the application of *Wasan* in the mathematics taught in school. I would like, by all means, to apply *Sangaku* and *Wasan*, both invaluable cultural assets that were produced hundreds of years ago, to the mathematics classroom.

4. *Sangaku* of Kon'nou Shrine

Kon'nou Shrine is located in one of the more serene areas of Shibuya and is also filled with many historical artifacts. Here, I shall introduce the existing *Sangaku* of Kon'nou Shrine, along with translations of the texts of their problems into modern Japanese and show examples of methods for solving them for use in junior and senior high school classes. I shall also touch upon how the problems were solved using *Wasan*, as much as possible.

4.1. How to Look at *Sangaku*

In general, *Sangaku* has a figure drawn on it and the figure itself is the problem. It usually consists of:

(1) Problem text

(2) Figure

(3) Answer

(4) Explanation (*Jojutsu* formula)

(5) Date of dedication, name

The explanation/formula (4) lays out the method used to find the answer. These explanations often make use of *Jojutsu* (formulas) which are unique to *Wasan*, such as the Formula of the Pythagorean theorem, and contain parts of those formulas or their results.

4.2. The *Sangaku* of Tomitarou Noguchi (Minamoto-no-Sadanori): *Sangaku* 1

The following *Sangaku* was dedicated under the name Minamoto-no-Sadanori in 1864. Its fan-like shape is different from the rectangular shape most commonly seen in *Sangaku*, making it a distinctive and rare example. The problem presents three circles—one large, one medium and one small—and seeks to find the diameter of the large circle when those of the small and medium circle are given.

Figure 4.1. The Photograph of *Sangaku* 1 (1864)

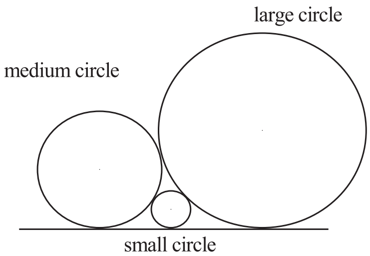


Figure 4.2. The Figure of *Sangaku* 1

如圖
　中圓径九寸
　小圓径四寸
大圓径幾何問
答 三十六寸
術曰置中圓径除小圓径
開平方内減一箇自之以
除中圓径得大圓径合問
關流 水野興七郎門人
　　野口富太郎
　　源 貞則
元治元 甲子年十一月吉日

Figure 4.3. The Text of *Sangaku* 1 in Japanese

4.2.1. Translation of Problem Text in *Sangaku* 1

As shown in the figure, if the diameter of the medium circle is 9 *sun* and the diameter of the small circle is 4 *sun*, what is the diameter of the large circle?

Answer: 36 *sun*.

Explanation (formula): first divide the segment of the medium circle by that of the small circle and take the square root of that number. Then, subtract 1 from that number and square the result. By dividing that number by the segment of the medium circle, we can find the segment of the large circle, and simplified,

$$\text{Diameter of large circle} = \frac{\text{medium}}{\left(\sqrt{\frac{\text{medium}}{\text{small}}}-1\right)^2}$$

The resulting number is equal to the diameter of the large circle.

Moreover, the *Sangaku* expresses the contact relationship between the large, medium and small spheres. That is, all three spheres are centered on a level plane.

4.3. The *Sangaku* of Takataka Youzaburou Yamamoto: *Sangaku* 2

The following *Sangaku* was dedicated by Takataka Youzaburou Yamamoto in 1859. It is thought to be dedicated to Kon'nou Shrine because of the proximity of the shrine to the Saijou Domain's official residence in Edo. Also, as can also be seen from the photograph, this *Sangaku* consists of three problems (Problem 1, 2, and 3 from right).

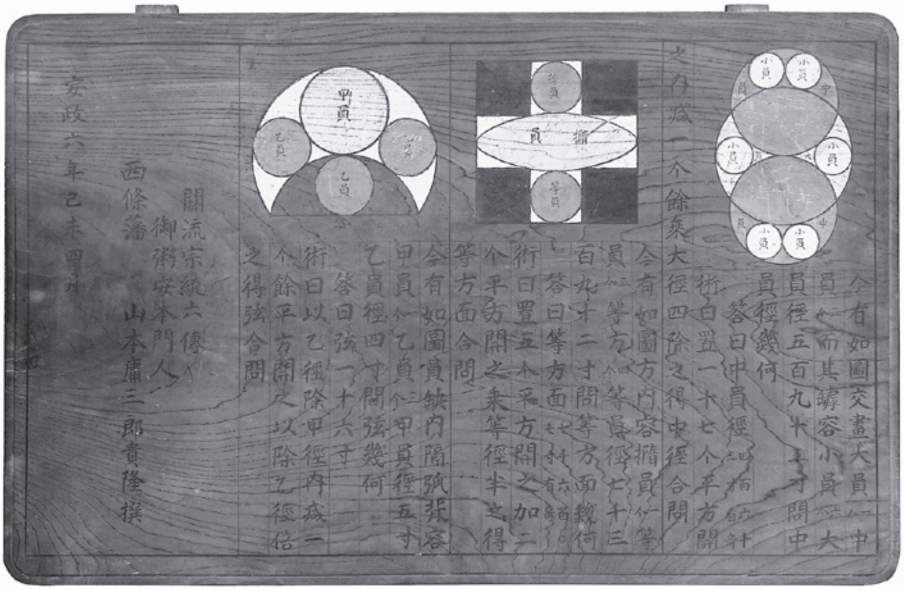


Figure 4.4. The Photograph of *Sangaku* 2 (1859)

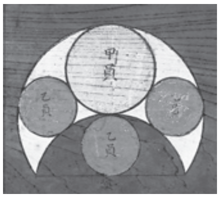


Figure 4.5. Problem 3

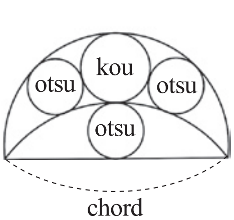


Figure 4.6. The Figure of Problem 3

（第三問）
今有如圖員鋏内隔弧背容
甲員一個乙員三個甲員径五寸
乙員径四寸問弦幾何
答曰弦一十六寸
術曰以乙径除甲径内減一個
餘平方開之以除乙径倍之得弦合問
關流宗統六傳
御粥安本門人
西條藩 山本庸三郎貴隆撰
安政六年己未四月

Figure 4.7. The Text of Problem 3 in Japanese

4.3.1. Translation of Problem Text in *Sangaku* 2 (Problem 3)

As shown in the figure, there is a chord and four circles—one labeled *kou* and three labeled *otsu*, within a larger circle. If the diameter of *kou* is 5 *sun* and the diameter of *otsu* is 4 *sun*, then what is the length of the chord?

Answer: The length of the chord is 16 *sun*.

Explanation (formula): first divide the diameter of *kou* circle by that of *otsu* circle and subtract 1 from that number. Then take the square root of that number. Second divide the double of the diameter of *otsu* circle by that number, and simplified,

$$\text{chord} = \frac{2otsu}{\sqrt{\frac{kou}{otsu} - 1}}$$

5. Explore *Sangaku* with a Graphic Calculator

Sangaku includes the contact relations between a circle and a line, between a circle and a circle. Of course, we can draw geometric construction to the figure in *Sangaku* with a compass and a ruler, but it is difficult to draw the figure in such way. I used Cabri Geometry for drawing the figures in this paper. Some figures are drawn approximately, though it connot be recommended from the perspective of mathematics education as many teachers would agree. Therefore, many students are stuck at the entrance of the problem, and it is not easy to consider the problem going ahead.

I have developed mathematical teaching materials using technology in the field of algebra and analysis, and I have wanted a good technology in the field of geometry as well. In order to satisfy such demands, I would like to introduce Casio fx-9860GII graphic calculator.

The advantage in using this calculator is its easiness to draw geometric constructions. For example, in order to draw a fixed radius of a circle and its' tangent:

(1) First, draw a circle and a line loosely.

(2) Set the radius of the circle.

(3) Set to touch the circles and the line.

Drawing a circle and its' tangent requires only this operation.

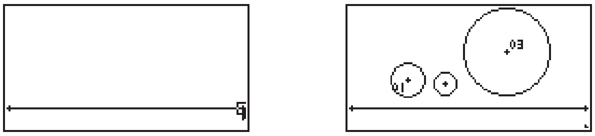
The figures below are presented in *Sangaku* 1 problem and *Sangaku* 2 problem, which are drawn by fx-9860GII Manager PLUS.

5.1. Apply for Problem of *Sangaku* 1

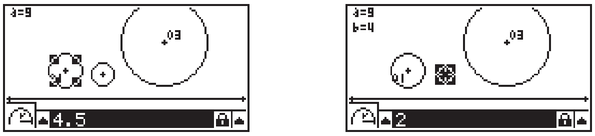
The procedure to draw the figure in *Sangaku* 1 problem with using fx-9860G II, is as follows:

<Process>

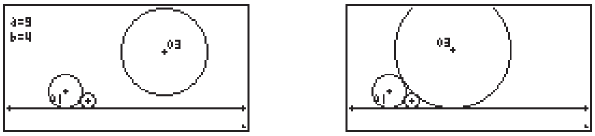
(1) Draw a line (tangent). Draw three circles on a tangent, medium one and small one and large one.



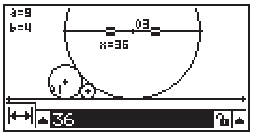
(2) Set the length, diameter of medium circle a = 9. Set the length, diameter of small circle b = 4.



(3) Set three circles contact onto tangent, and set three circles to contact each other.



(4) fx-9860GⅡ shows the length, diameter of large circle x = 36.

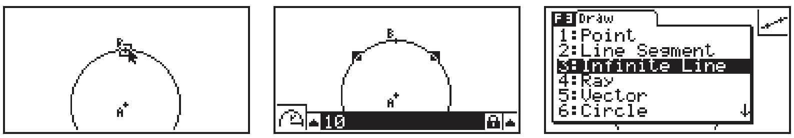


5.2. Apply for Problem of *Sangaku* 2

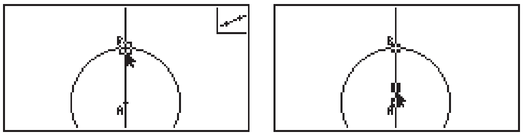
The procedure to draw the figure in *Sangaku* 2 problem 3 with using fx-9860G II, is as follows:

<Process>

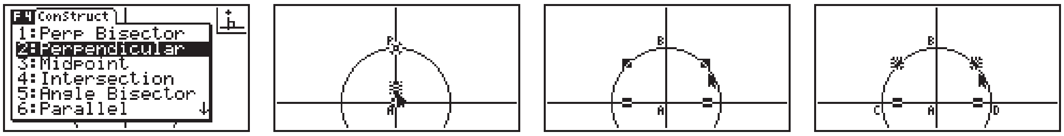
(1) Draw circle A whose center is point A and radius is about 10.



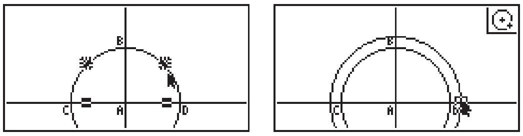
(2) Draw a line through from point A to point B.



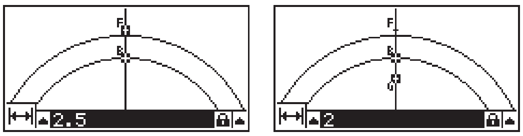
(3) Draw a perpendicular line to AB through point A, and intersect at circle A for label C and D.



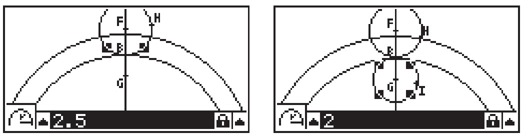
(4) Draw a concentric circle at point A, and draw two circles whose centers are on the line AB as the below figure shows.



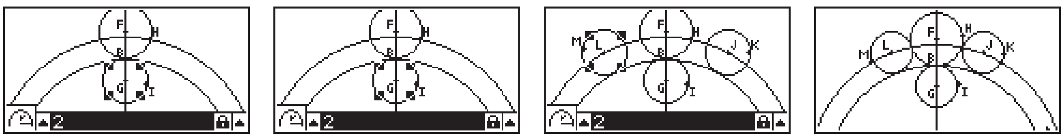
(5) Set two points F and G on the line AB, and set lengths as BF = 2.5, BG = 2.



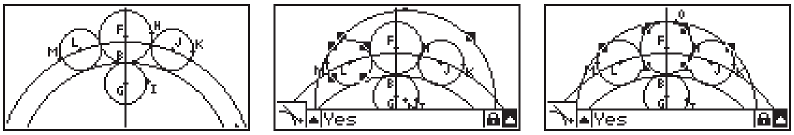
(6) Draw two circles whose centers are point F and G on the line AB, and set circle F as radius 2.5, and circle G as radius 2.



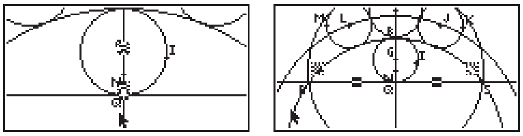
(7) Draw two circles whose centers are on the concentric circle as radius 2. And set circle L and circle J contact circle F.



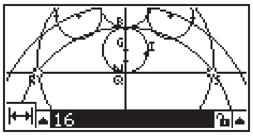
(8) Draw circle N whose center is on the line AB, and set three circles F, J and L contact to circle N.



(9) Draw a perpendicular line to the line AB, and set circle G contact to it. Make intersection points circle N and it as R and S.



(10) fx-9860GⅡ shows the distance from R to S for 16. Of course, the value fits.



5.3. The Knowledge with Using fx-9860GⅡ

The positive effects of using fx-9860GⅡ to draw *Sangaku* figures in mathematics education is considered to be as follows:

(1) In the solution of geometric problems, it will be very important to try to draw a figure from the beginning. By thinking about the method of the figure, students can approach the solution by the drawing requirements.

(2) Therefore, by using fx-9860GⅡ which supports many students in their mathematical activities, it is expected to help in the following in mathematics education.

(3) It is able to visualize the geometric construction.

(4) It is able to show the geometric construction steps.

(5) Using this graphic calculator, it is possible to avoid the difficulty of problems.

(6) Students will be able to understand clearly what is required in the problem in order for them to operate this graphic calculator. And this experience will extend their mathematical thinking.

(7) It will be possible for students to read the geometric construction in the guidance.

(8) Some conditions are necessary for drawing the geometric construction, and it will be linked to the solving of algebraic equations.

6. Conclusion and Future Implications

Ushering in a new era of mathematics education, I have considered the content of *Wasan* as a breakthrough, and have conducted demonstrative research in this field, including the usage of *Wasan* geometry teaching materials, the making of *Sangaku,* and so on. I have also published my results through various organs, including this school's Research for "*Super Science High Schools* (SSHs)", and the Japan Society for Mathematical Education.

Observing genuine *Sangaku* alongside my students while doing fieldwork, I sensed the potential of *Wasan* and with it, a strong sense of interest in the fact that such mathematical research was carried out during the Edo Period. Furthermore, the students' experience of making *Sangaku*, that is, having my own students make *Sangaku*, added a new layer of depth to the content of my daily classes.

Lastly, I introduced a graphic calculator for geometric constructions. It should be fruitful to consider further methods of geometric constructions as future implications. It is also necessary for high school students to consider how to describe the figure. I showed another example of how to solve *Sangaku* 1 in appendixes by using parabolas (see appendix 2).

Acknowledgements I would like to thank my colleagues for their valuable comments and input. In addition, I have received significant advice from Dr. Shin'ichi Suzuki and Dr. Kimio Watanabe who are professors at Waseda University.

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Note (1) Books and other documents from the Edo Period which remain to this day show us that a diverse group of people at the time were seriously involved with mathematical undertakings. By perusing their contents, we can see that a wide variety of mathematics was being studied by the people of the Edo Period: from mathematical games, such as age-guessing quizzes and puzzles; to mathematics necessary for daily life, such as mathematics used for work dealing with money changing, land surveying or measuring rice harvests; to the work of mathematicians such as Shigekiyo Matsumura who calculated pi in his treatise on "*Sanso*" in 1663. By the end of the Edo Period, the content of Japanese mathematics had developed as far as the level of what is covered by modern calculus. In order to distinguish between the mathematics of the Edo Period and Western mathematics, the mathematics of the Edo Period is referred to as *Wasan.*

(2) *Sangaku* (mathematics tablets) discussed in this article are a part of Japan's unique cultural heritage. They were written onto *Ema* (wooden votive tablets) and presented as offerings at shrines and temples in thanks to the gods. This was done out of the religious devotion that is peculiar to Japan and for which I too offer thanks to the gods for allowing me to conduct such research and create such mathematical problems.

(3) Western mathematics was introduced in the Japanese school system at the beginning of the Meiji Period.

With the introduction of Western mathematics in Japanese schools the number of people learning *Wasan* gradually declined. *Wasan* had been studied mainly in research. At present, with *Wasan* researchers are at the forefront, and research on *Wasan* is being actively conducted through writings and events, *Wasan* culture is being spread to the world.

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Appendix

(1) A Modern Mathematical Solving the Problem from *Sangaku* 1

(Formula)

Let a be the radius of medium circle O_1, b be the radius of small circle O_2 and r be the value of the radius of large circle O_3.

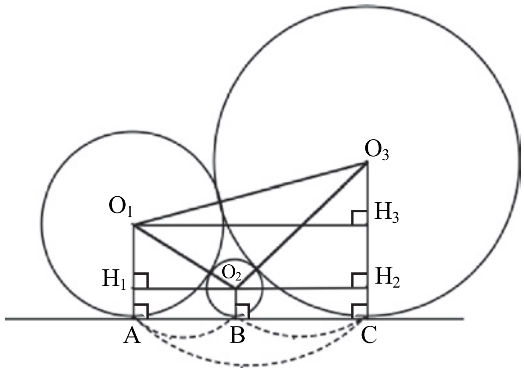

Figure 1 The Figure of the problem from *Sangaku* 1

(Proof)

Let A, B and C be the points of contact between the shared plane and the medium circle, small circle and large circle, respectively. Then: AB + BC = AC

So, by the Pythagorean theorem:

$$\mathrm{AB}=\mathrm{H_1O_2}=\sqrt{(a+b)^2-(a-b)^2}=2\sqrt{ab}$$

$$\mathrm{BC}=\mathrm{O_2H_2}=\sqrt{(r+b)^2-(r-b)^2}=2\sqrt{br}$$

$$\mathrm{AC}=\mathrm{O_1H_3}=\sqrt{(r+a)^2-(r-a)^2}=2\sqrt{ar}$$

Thus:

$$2\sqrt{ab}+2\sqrt{br}=2\sqrt{ar}$$

Thus, we get:

$$\left(\sqrt{a}-\sqrt{b}\right)\sqrt{r}=\sqrt{ab}$$

$$\sqrt{r}=\frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}$$

$$r=\left(\frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}\right)^2 \qquad (*)$$

Moreover, the equation used (∗) is:

$$r=\left(\frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}\right)^2=\frac{a}{\left(\sqrt{\frac{a}{b}}-1\right)^2}$$

This equation is exactly the same as what is described in the explanation written on the *Sangaku* 1.

(2) Another method for drawing geometric constructions in *Sangaku* 1

(Explanation)

Many problems concerning the construction of a figure can be solved using loci of points. When we want to find the position of a point, we draw two loci which satisfy two given conditions respectively, and find where they intersect. Such a method is called the intersection of loci.

In *Sangaku*1, we introduce how to describe the circle O_3 by the intersection of loci with software, Cabri Geometry Ⅱ Plus. The follow shows its process, and the figure illustrates also. Where, let the line ℓ be the common tangent line.

(1) Firstly, describe the circle O_1, and describe the parabola C_1 at O_1 as a focal point.

(2) Secondly, describe the circle O_2, and describe the parabola C_2 at O_2 as a focal point.

(3) The parabola C_1 meets the parabola C_2 at O_3. Finally, we can describe the circle O_3 at an intersection point O_3.

(Proof)

Let the common tangent line ℓ be $y = 0$, let the circle O_1 be $x^2 + (y - a)^2 = a^2$ in a plane.

Let the focus be $(0, a)$, the equation of the directrix be $y = -a$.

Then, the parabola C_1 is expressed $y = \frac{1}{4a}x^2$.

In the same way, let the circle O_2 be $\left(x - 2\sqrt{ab}\right)^2 + (y - b)^2 = b^2$.

Let the focus be $(2\sqrt{ab}, b)$, the equation of the directrix be $y = -b$.

Then, the parabola C_2 is $y = \frac{1}{4b}\left(x - 2\sqrt{ab}\right)^2$.

Let find the intersection point between C_1 and C_2 by the following equation.

$$\frac{1}{4b}\left(x - 2\sqrt{ab}\right)^2 = \frac{1}{4a}x^2$$

$$x = \frac{2a\sqrt{b}\left(\sqrt{a} \pm \sqrt{b}\right)}{a-b} = \frac{2a\sqrt{b}}{\sqrt{a}-\sqrt{b}}, \frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$$

In this case, $x = \frac{2a\sqrt{b}}{\sqrt{a}-\sqrt{b}}$, the point O_3 is $\left(\frac{2a\sqrt{b}}{\sqrt{a}-\sqrt{b}}, \frac{ab}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)$.

So, in the above figure 1, $AC = O_1H_3 = 2\sqrt{ar}$.

$$2\sqrt{ar} = \frac{2a\sqrt{b}}{\sqrt{a}-\sqrt{b}}$$

$$\sqrt{r} = \frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}$$

$$r = \left(\frac{\sqrt{ab}}{\sqrt{a}-\sqrt{b}}\right)^2 = \frac{a}{\left(\sqrt{\frac{a}{b}}-1\right)^2} \qquad (*)$$

This expression above (∗) is the same one in (1) A Modern Mathematical Solving the Problem from *Sangaku* 1.

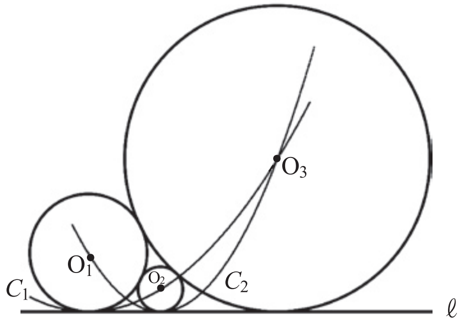

Figure 2 How to describe the circle O_3 by the intersection of loci with software

Another case, $x = \frac{2a\sqrt{b}}{\sqrt{a}+\sqrt{b}}$: The intersection point exists inside the area where the circle O_1, O_2 and the common tangent line ℓ.

(3) A Modern Mathematical Solving the Problem from *Sangaku* 2

Let A be the center point of circle ***kou*** and B be the center point of the circle ***otsu*** (on the right).

Also, as shown in the illustration, let C be the center point of the third circle, D be the center point of the outer circle, H be a segment of a perpendicular line drawn from B to the diameter of the outer circle, G be the point of contact between the central ***otsu*** circle and E and F be the start and end points of the chord whose length we are seeking.

The diameters of the ***kou*** circle, the ***otsu*** circle, the third circle and the outer circle are expressed as $2R$, $2r$, $2a$ and $2b$, respectively. Also, the length of the chord is expressed as x and the length of AH is expressed as y.

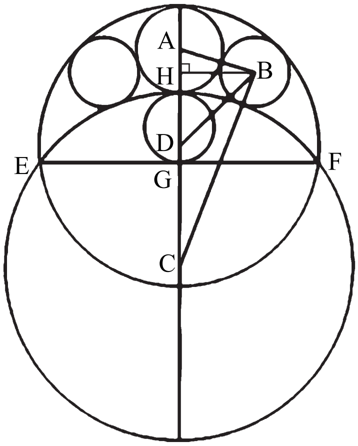

Figure 3 The Figure of the problem from *Sangaku* 2

By the similarity of triangle:

$$x^2 = 4 \cdot 2r\,(2a - 2r) \tag{1}$$

Because the power of point of G is equal for both the outer circle and the third circle:

$$(2R + 2r)(2b - 2R - 2r) = 2r\,(2a - 2r) \tag{2}$$

In short,

$$(2R + 2r) \cdot 2b - \left\{ (2R)^2 + 2 \cdot 2R \cdot 2r + 2r \cdot 2a \right\} = 0 \tag{3}$$

By applying the *Wasan's* Formula of like cosine law to triangle △ABD, we get:

$$(2b - 2r)^2 = (2R + 2r)^2 + (2b - 2R)^2 - 4(2b - 2R)y$$
$$(2R)^2 + 2R \cdot 2r - 2b \cdot 2R + 2b \cdot 2r - 2\,(2b - 2R)y = 0 \tag{4}$$

By applying the *Wasan's* Formula of like cosine law to triangle △ABC, we get:

$$(2a + 2r)^2 = (2R + 2r)^2 + (2a + 2R)^2 - 4(2a + 2R)y$$
$$(2R)^2 + 2R \cdot 2r + 2a \cdot 2R - 2a \cdot 2r - 2(2a + 2R)y = 0 \tag{5}$$

If we eliminate y from (4) and (5), we get:

$$\left\{ (2R)^2 + 2R \cdot 2r - 2b \cdot 2R + 2b \cdot 2r \right\} (2a + 2R) = \left\{ (2R)^2 + 2R \cdot 2r + 2a \cdot 2R - 2a \cdot 2r \right\} (2b - 2r)$$

Putting these into order gives us:

$$\left\{ (2R)^2 + 2a \cdot 2R - 2a \cdot 2r \right\} \cdot 2b - \left\{ (2R)^3 + 2r \cdot (2R)^2 + 2a \cdot (2R)^2 \right\} = 0 \tag{6}$$

If we eliminate the outer parts of the problem by applying the ratio from (3) and (6), we get:

$$(2R + 2r) \left\{ (2R)^3 + 2r \cdot (2R)^2 + 2a \cdot (2R)^2 \right\} = \left\{ (2R)^2 + 2 \cdot 2R \cdot 2r + 2r \cdot 2a \right\} \left\{ (2R)^2 + 2a \cdot 2R - 2a \cdot 2r \right\}$$
$$(2R - 2r) \cdot (2a)^2 + \left\{ (2R)^2 - 2 \cdot 2R \cdot 2r \right\} \cdot 2a - (2R)^2 \cdot 2r = 0$$

Factoring gives us:

$$\left\{ 2(2R - 2r)a - 2R \cdot 2r \right\} (2a + 2R) = 0 \tag{7}$$

Thus,

$$2a=\frac{2R\cdot 2r}{2R-2r}$$

Substituting this into (1) gives us:

$$x^2=4\cdot 2r\left(\frac{2R\cdot 2r}{2R-2r}-2r\right)=\frac{4(2r)^3}{2R-2r}$$

Thus,

$$x=\frac{2\cdot 2r}{\sqrt{\frac{2R}{2r}-1}}$$

Input the value: $2R=5$ *sun*, $2r=4$ *sun*, so:

$$x=\frac{2\times 4}{\sqrt{\frac{5}{4}-1}}=16 \quad (sun)$$