4.2 Application to Cognition Analysis

Usually, instruction items mutually have the relation and the order. The relation between instruction items are the fuzzy relation, therefore we could analyze the instruction structure by the fuzzy cognition analysis method.

Here, we would explain the fuzzy cognition analysis by applying the fuzzy node fuzzy graph.

The instruction structure could be verified and/or modified by the fuzzy cognition analysis.

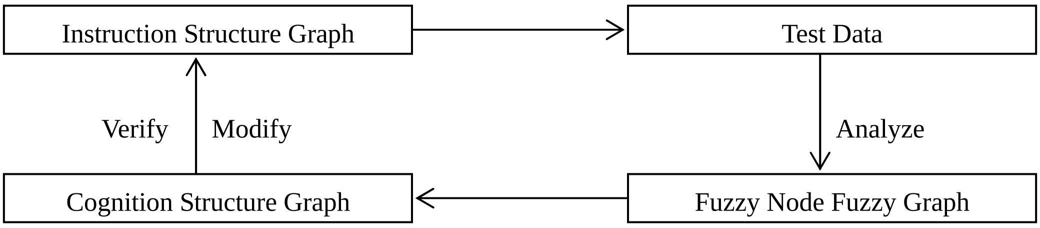


Figure 4.2.1 Instruction Structure Analysis Method

The fuzzy cognition structure could be analyzed by the following method.

If we execute the m problems test $\{P_i : 1 \le i \le m\}$ to n students $\{S_k : 1 \le k \le n\}$, we have a test score matrix $X = (x_{ki})$. Here, if S_k writes the correct answer to the problem P_i, then x_{ki}=1 and if S_k writes the wrong answer to the problem P_i, then x_{ki}=0.

According to the test score matrix X, we have obtained the contingency table in Figure 4.2.2.

P_i \ P_j	1	0	Sum
1	a	b	$a+b$
0	c	d	$c+d$
Sum	$a+c$	$b+d$	n

Figure 4.2.2 Contingency Table

where, $a = \sum_{k=1}^{n} x_{ki} x_{kj}$, $b = \sum_{k=1}^{n} x_{ki}(1 - x_{kj})$, $c = \sum_{k=1}^{n} (1 - x_{ki}) x_{kj}$, $d = \sum_{k=1}^{n} (1 - x_{ki})(1 - x_{kj})$.

If the problem P_i and P_j have similar quality, then the value of b and c are relatively small. Therefore, $(a+d)/n$ is nearly 1. If the problem P_i and P_j have different quality, then the value of b and c are relatively large. Therefore, $(a+d)/n$ is nearly 0.

Definition 4.2.1 Fuzzy Similarity Index s_{ij}

$$s_{ij} = \frac{a+d}{n}$$

From the fuzzy similarity index s_{ij}, we could obtain the fuzzy similarity structure graph (matrix) $S=(s_{ij})$.

If P_i is the premise item of P_j, then the value of c is small relatively. Therefore, $a/(a+c)$ and $d/(c+d)$ are nearly 1.

Definition 4.2.2 Fuzzy Relation Index t_{ij}

$$t_{ij} = \frac{a+d}{(a+c)+(c+d)}$$

where, if $a=c=d=0$, then $t_{ij}=1$.

From the fuzzy relation index t_{ij}, we could obtain the fuzzy relation structure graph (matrix) $T=(t_{ij})$.

By the cluster analysis of the fuzzy similarity structure graph $S=(s_{ij})$, we have obtained the fuzzy partition tree P. From the partition tree P, we could classify the instruction items.

From the fuzzy relation structure graph $T=(t_{ij})$, we have obtained the fuzzy node fuzzy graph $G=(P,T): P=\{P_i / u_i\}, T=(t_{ij})$. Here, u_i is the fuzziness of the problem P_i which could be calculated by the following:

$$u_i = \left[\frac{\phi(i)}{\underset{1 \le k \le m}{Max}\{\phi(k)\}} \right]^r, \ 0 < r \le 1$$

$$\phi(i) = \frac{\sum_{j=1}^{m} s'_{ij}}{\sum_{k=1}^{m}\sum_{l=1}^{m} s'_{kl}} \quad \text{(Shapley-value)}$$

$$S' = (s'_{ij}) = S - E$$

The fuzzy node fuzzy graph $G=(P. T)$ is usually complicated, we transform the fuzzy node fuzzy graph $G=(P, T)$ to the fuzzy graph sequence $\{G_\lambda\}$ by applying the quasi-logical product.

By applying fuzzy decision, we could obtain the optimal fuzzy graph G_{λ_0} from the fuzzy graph sequence $\{G_\lambda\}$.

By summarizing the partition tree P and the optimal fuzzy graph G_{λ_0}, we have obtained the cognition structure graph ϕ^z. It shows the relational order among the instruction characteristics.

Here, we would present the case study of the instruction structure analysis about ‘The Number of the Permutation.’

The instruction structure of ‘the permutation’ is classified by next items.

(α) basic, (β) circular, (γ) necklace, (A) simple, (B) duplicated

According to these characteristics, the instruction structure graph I is constructed as Figure 4.2.3.

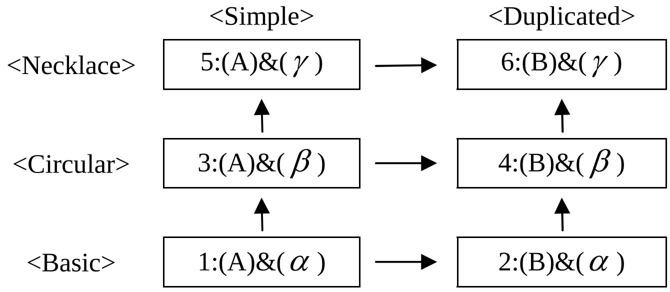


Figure 4.2.3　Instruction Structure Graph *I*

From the instruction structure graph *I*, we could guess the learning flows among instruction items.

For example, $[\alpha : 1 \to 2] \to [\beta : 3 \to 4] \to [\gamma : 5 \to 6]$, $[A : 1 \to 3 \to 5] \to [B : 2 \to 4 \to 6]$ and so on.

After teaching ‘the permutation’, we have executed the test in Figure 4.2.4 to 40 students, and have obtained the test score matrix *X* in Figure 4.2.5.

Write the number of the permutation about each letter sequences.

(1) SPLITE　(Original Permutation)
(2) COFFEE　(Original Permutation)
(3) SPRITE　(Circular Permutation)
(4) COFFEE　(Circular Permutation)
(5) SPLITE　(Necklace Permutation)
(6) COFFEE　(Necklace Permutation)

Figure 4.2.4　Test of ‘Permutation’

	$P1$	$P2$	$P3$	$P4$	$P5$	$P6$		$P1$	$P2$	$P3$	$P4$	$P5$	$P6$
$S01$	1	1	1	1	1	0	$S21$	1	1	1	1	1	0
$S02$	1	0	1	0	0	0	$S22$	1	0	1	0	0	0
$S03$	1	0	1	0	1	0	$S23$	1	1	1	0	1	0
$S04$	0	0	0	0	0	0	$S24$	1	1	1	0	1	0
$S05$	0	0	0	0	0	0	$S25$	1	1	1	1	1	0
$S06$	1	0	0	0	0	0	$S26$	1	0	1	0	1	0
$S07$	1	0	1	0	1	0	$S27$	1	0	1	0	1	0
$S08$	1	1	1	0	1	0	$S28$	1	1	1	0	1	0
$S09$	1	0	0	0	0	0	$S29$	1	1	1	1	1	0
$S10$	1	1	1	1	1	1	$S30$	1	1	1	1	1	0
$S11$	1	0	1	1	1	0	$S31$	0	0	0	0	0	0
$S12$	0	0	0	0	0	0	$S32$	1	1	1	1	0	0
$S13$	1	1	1	1	1	0	$S33$	1	1	1	1	1	1
$S14$	1	0	0	0	0	0	$S34$	1	0	0	0	0	1
$S15$	1	1	1	1	1	0	$S35$	1	1	1	1	1	1
$S16$	1	1	1	1	1	1	$S36$	1	1	1	1	1	0
$S17$	1	0	1	0	1	0	$S37$	0	0	0	0	0	0
$S18$	1	1	1	1	1	0	$S38$	1	1	1	0	1	0
$S19$	1	0	0	0	0	0	$S39$	0	0	0	0	0	0
$S20$	1	1	1	1	1	0	$S40$	1	1	1	1	1	1

Figure 4.2.5 Test Score Matrix *X*

From the test score matrix *X*, we have obtained the fuzzy similarity structure matrix *S* in Figure 4.2.6, and the partition tree *P* in Figure 4.2.7.

	1	2	3	4	5	6
1	1	0.68	0.88	0.58	0.80	0.30
2	0.68	1	0.80	0.85	0.83	0.58
3	0.88	0.80	1	0.70	0.93	0.38
4	0.58	0.85	0.70	1	0.73	0.68
5	0.80	0.83	0.93	0.73	1	0.45
6	0.30	0.58	0.38	0.68	0.45	1

Figure 4.2.6 Similarity Structure Matrix *S*

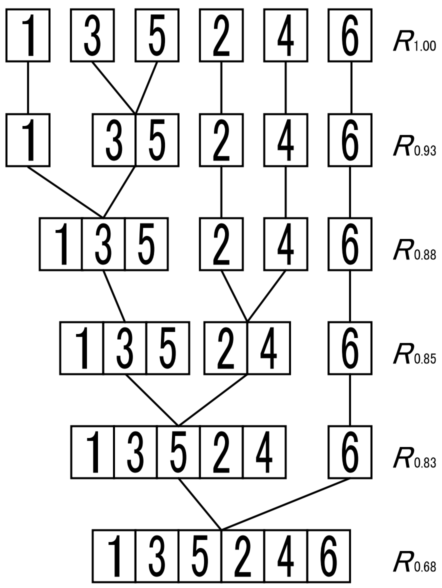


Figure4.2.7 Partition Tree *P*

By applying the fuzzy decision, we have obtained the optimal cut level c_0=0.88 in Figure 4.2.8.

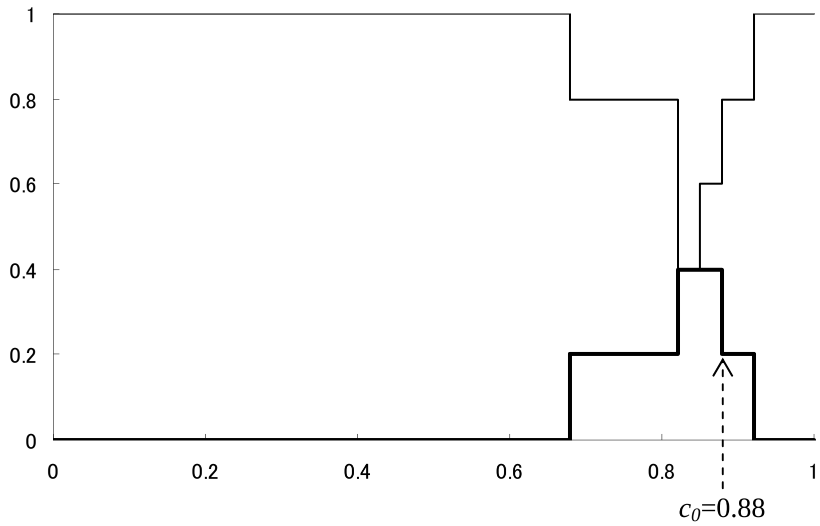


Figure 4.2.8 Fuzzy Decision

From the fuzzy relation matrix *T* and the similarity matrix *S*, we have obtained the fuzzy node fuzzy graph matrix *G* in Figure 4.2.9 and the fuzzy node fuzzy graph G in Figure 4.2.10. By applying the fuzzy node fuzzy graph analysis, we have obtained the fuzzy graph sequence $\{G_\lambda\}$ in Figure.4.2.11.1 to Figure.4.2.11.4.

	1	2	3	4	5	6	
1	1	1.00	1.00	1.00	1.00	1.00	0.86
2	0.51	1	0.67	0.94	0.73	1.00	1.00
3	0.78	1.00	1	1.00	1.00	1.00	0.99
4	0.40	0.77	0.54	1	0.59	1.00	0.94
5	0.67	0.94	0.86	0.94	1	1.00	1.00
6	0.16	0.43	0.25	0.54	0.31	1	0.66

Figure 4.2.9 Fuzzy Node Fuzzy Graph Matrix *G*

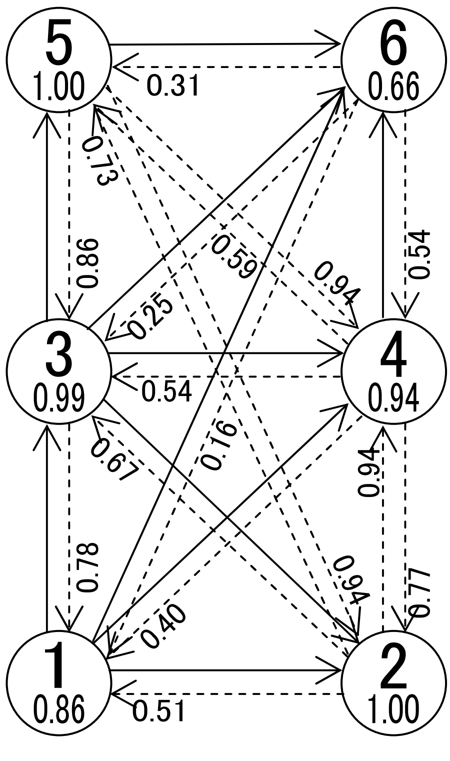
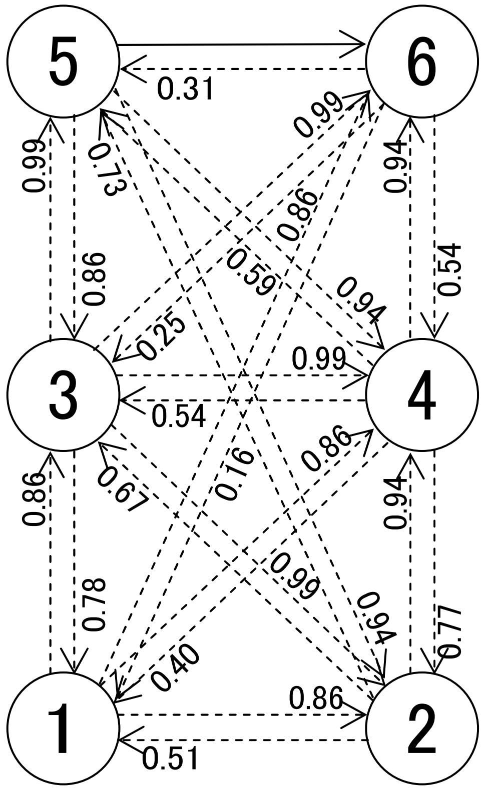


Figure 4.2.10 Fuzzy Node Fuzzy Graph *G*



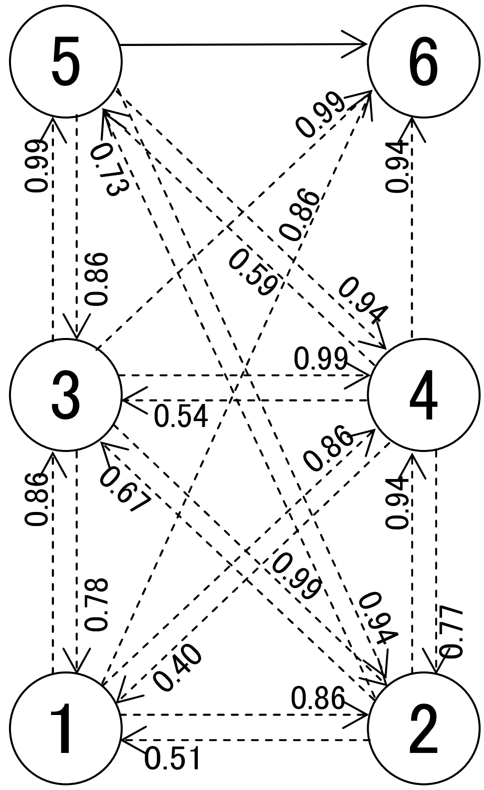
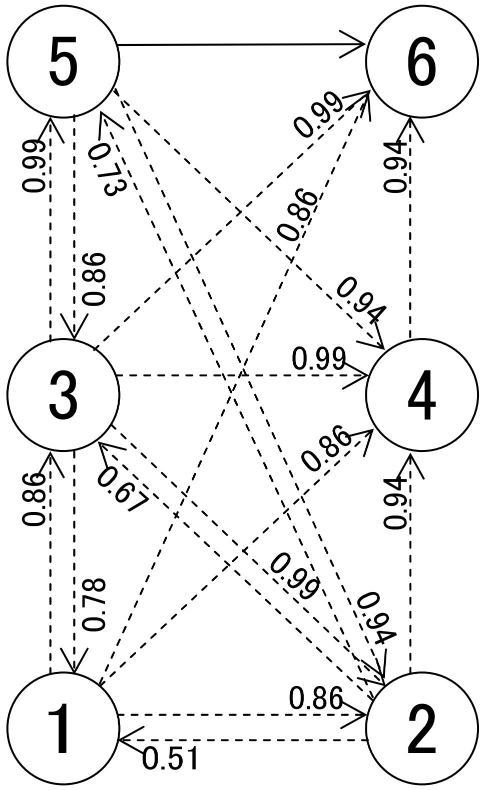


Figure 4.2.10.1 Fuzzy Graph $G_\lambda, \lambda \in [0.34, 1.00]$

Figure 4.2.10.2 Fuzzy Graph $G_\lambda, \lambda \in [0.06, 0.34)$



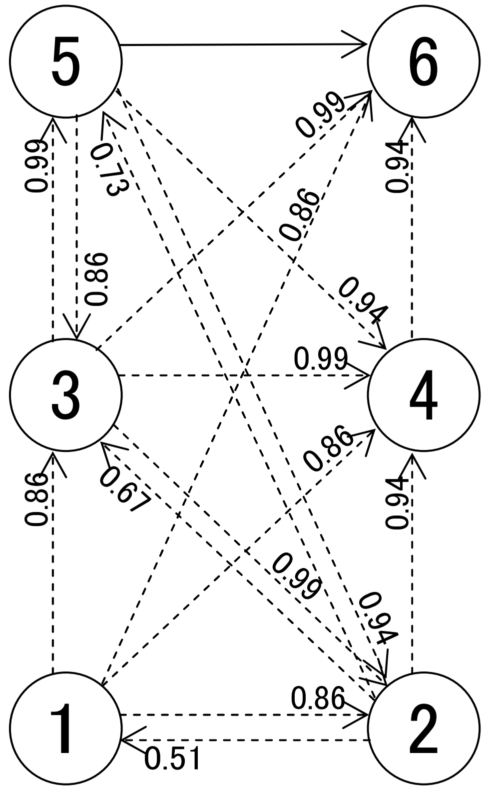


Figure 4.2.10.3 Fuzzy Graph $G_\lambda, \lambda \in [0.01, 0.06)$

Figure 4.2.10.4 Fuzzy Graph $G_\lambda, \lambda \in [0.00, 0.01)$

By applying the fuzzy decision, we have obtained the optimal value $\lambda_0 = 0.01$ in Figure 4.2.11.

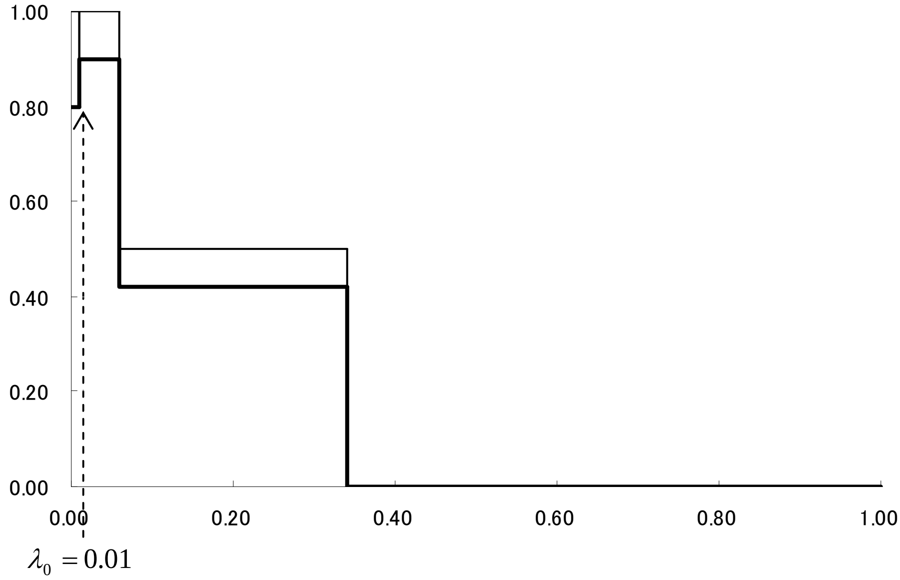


Figure 4.2.11. Fuzzy Decision

We would illustrate the fuzzy cognition structure graph $\phi^z, z = 0.88$ in Figure 4.2.12 which is based on the fuzzy graph $G_{\lambda_0}, \lambda_0 = 0.01$ in Figure 4.2.10.3.

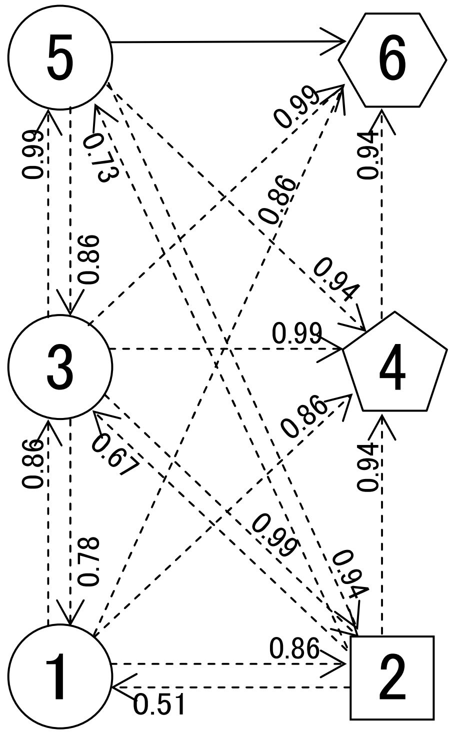


Figure 4.2.12 Fuzzy Cognition Structure Graph $\phi^z, z = 0.88$

From the cognition structure graph ϕ^z of ‘the permutation’, we could consider the followings:

(1) The graph ϕ^z has the cluster {1, 3, 5}, so, the properties of (A)|(B) is stronger than (α)|(β)|(γ).

(2) The graph ϕ^z has the arc $1 \to 2, 3 \to 4$ and $5 \to 6$, so, the order of instruction items is (A)$\to$(B).

(3) The graph ϕ^z has the path $1 \to 3 \to 5$ and $2 \to 4 \to 6$, so, the flow of instruction items is $(\alpha) \to (\beta) \to (\gamma)$.

(4) The graph ϕ^z has a spanning path $1 \to 3 \to 5 \to 2 \to 4 \to 6$ whose fuzzy level is 0.94, so, the learning flow is
(A : $1 \to 3 \to 5$)$\to$(B : $2 \to 4 \to 6$).

In this way, we could effectively analyze the learning structure concerning the subject matter by this analysis method.