

Appendix. Fuzzy Node Fuzzy Graph Drawing System Applying Genetic Algorithm

In general, as to the fuzzy node fuzzy graph drawing, it is not easy to find an optimal solution. However, by applying the fuzzy Shapley value and the genetic algorithm, we could effectively draw the fuzzy node fuzzy graph on a latticed plane.

1. Genetic Algorithm

There are three basic operations in the genetic algorithm as follows:

- (1) Selection: Select pairs of genes according to the fitness value in the group
- (2) Crossover: Create the new gene from the pair of genes
- (3) Mutation: Change the part of the gene compulsorily

A basic flow of the genetic algorithm is shown in figure.1.

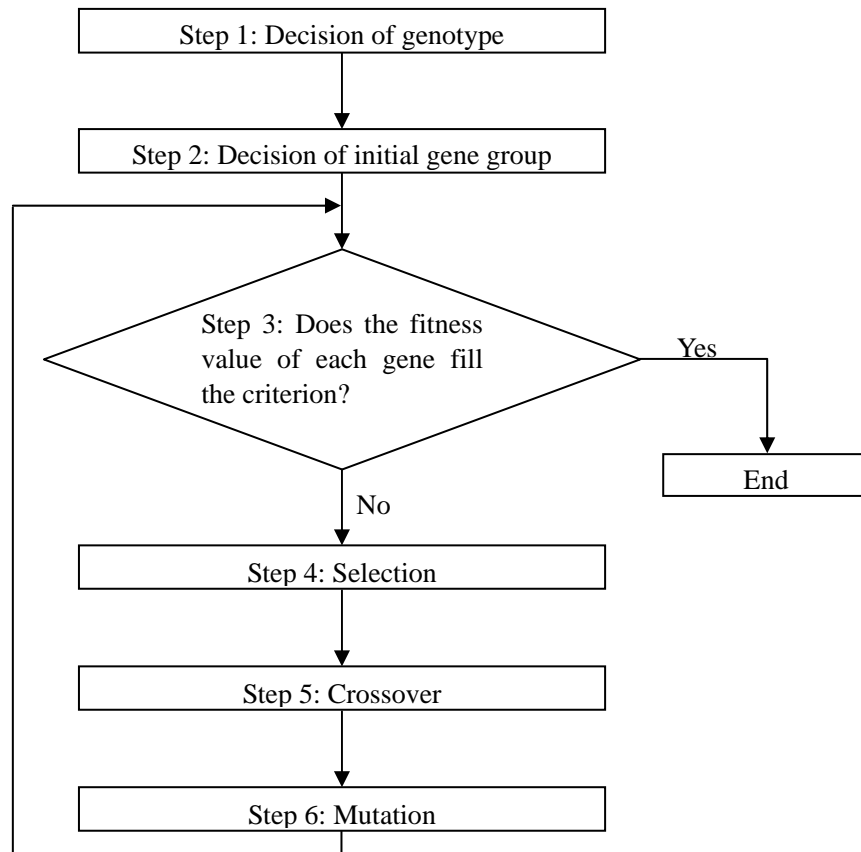


Figure.1 Basic Flow of Genetic Algorithm

In the genetic algorithm, the information is being composed by PTYPE and GTYPE.

GTYPE (genotype) corresponds to the chromosome in the cell, then, we usually treat it as one dimensional sequence of numbers. PTYPE (phenotype) shows the appearance of the structure which GTYPE has.

Basically, we apply the genetic algorithm operation(Crossover, Mutation and so on) as to GTYPE, and calculate the fitness value as to PTYPE.

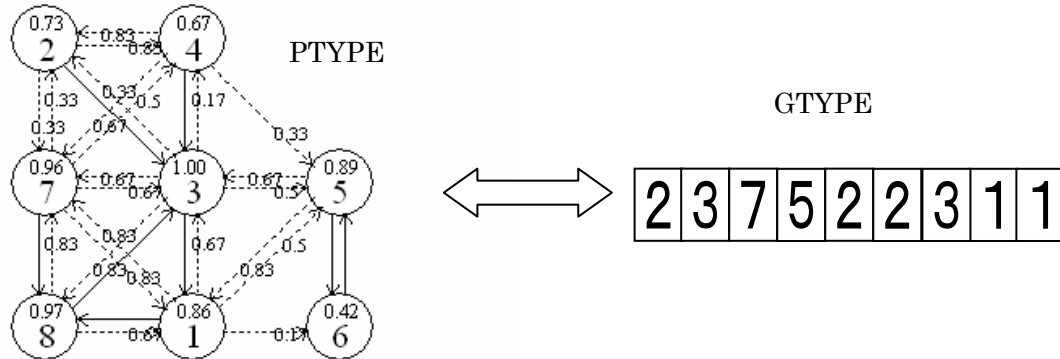


Figure.2 Example of GTYPE and PTYPE

2. Fuzzy Node Fuzzy Graph Drawing

There are various rules in the graph drawing. Here, we proposed the following rules.

- (1) We put the node with the highest Shapley value on the center.
- (2) We put the node with high Shapley value on the vicinity of the center.
- (3) We put the node in a coordinates of a $k \times k$ square.
- (4) We only draw the arc between the adjacent nodes.

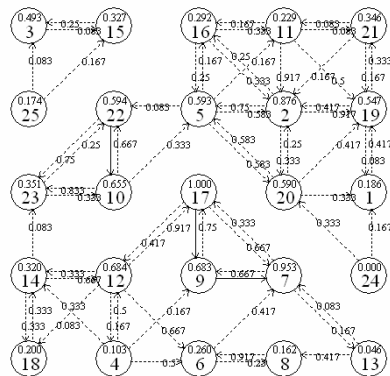


Figure.3 Example of Fuzzy Node Fuzzy Graph Drawing

According to these rules, we decide placement of a node. Firstly, we decide the placement of the first stratum by applying genetic algorithm. Secondly, we decide the placement of the second stratum.

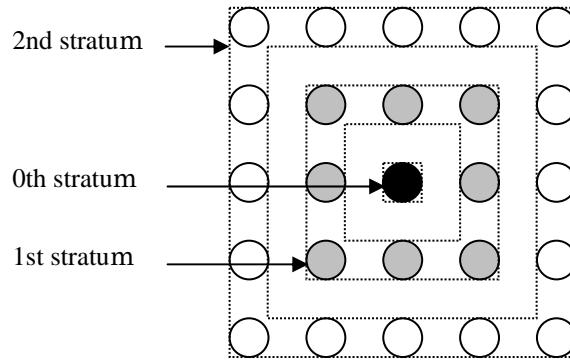


Figure.4 Decision Order of Placement

We would obtain the optimal solution through the next steps.

Step 1: We create 100 GTYPE by random numbers.

Step 2: We create PTYPE corresponding to GTYPE by the ordinal representation.

Step 3: We calculate the fitness value corresponding to PTYPE.

Step 4: We select 50 pairs of GTYPE by the roulette method.

Step 5: We apply the crossover to the pair selected in Step 4.

Step 6: We apply the mutation to GTYPE at a constant probability.

Step 7: It returns Step 3.

Then, we would define the fitness value as follows:

$$fitness(GTYPE) = \sum_{a=0}^{2m} \sum_{b=0}^{2m} \sum_{y=-1}^1 \sum_{x=-1}^1 \mu(X(k-m+a, k-m+b), X(k-m+a+x, k-m+b+y)) \cdot (|x+y| - xy)$$

where, $k = -\left\lceil \frac{3 - \sqrt{N}}{2} \right\rceil + 1$, $\#(S) = N$, $\mu(x_i, x_j) = f_{ij}$ and $X(s, t)$ expresses a node placed on

the coordinate (s, t) .

Here, we would introduce the practical application concerning the following fuzzy matrix.

	1	2	3	4	5	6	7	8	S.V.
1	1	0.00	0.67	0.33	0.50	0.17	0.83	1.00	0.86
2	0.00	1	1.00	0.83	0.67	0.00	0.33	0.50	0.73
3	1.00	0.33	1	0.17	0.50	0.00	0.67	0.83	1.00
4	0.00	0.83	1.00	1	0.33	0.17	0.67	0.50	0.67
5	0.83	0.50	0.67	0.00	1	1.00	0.33	0.17	0.89
6	0.00	0.00	0.00	0.00	1.00	1	0.00	0.00	0.42
7	0.83	0.33	0.67	0.50	0.17	0.00	1	1.00	0.96
8	0.67	0.17	1.00	0.50	0.33	0.00	0.83	1	0.97

Figure.3 Fuzzy Node Fuzzy Graph Matrix

If we would apply the genetic algorithm to this matrix by the above mentioned method, then we have obtained the optimal solution in figure.5. We could present GTYPE simulation of the steps in Figure.7 and Figure.8 which shows the initial and the final situation.

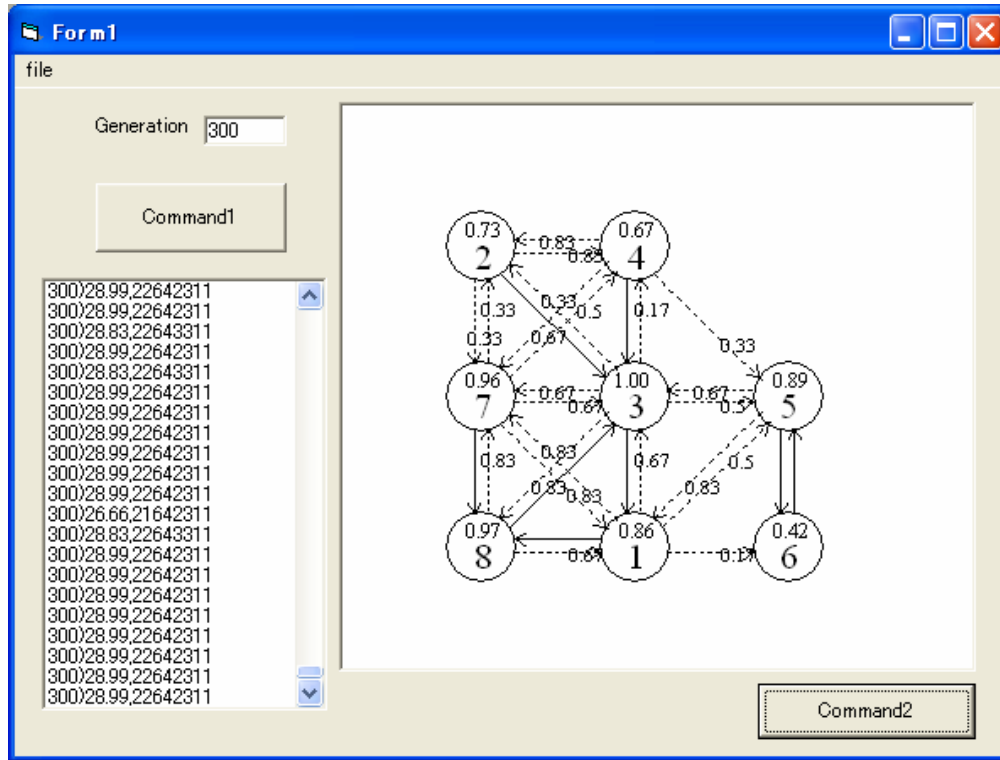


Figure.5 Optimal Solution

Here, from the fuzzy matrix S of the fuzzy node fuzzy graph matrix, we have obtained the partition tree in Figure.6.

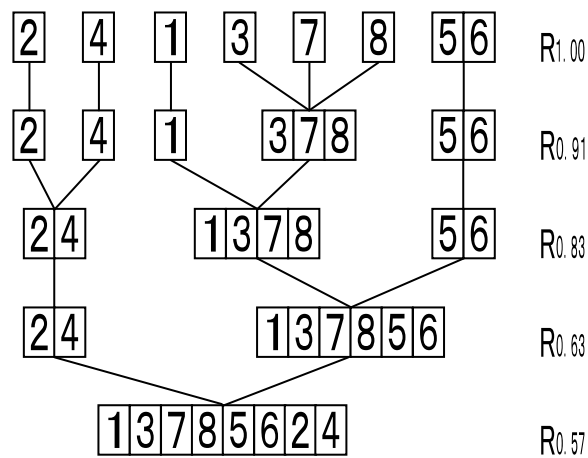


Figure.6 Partition Tree

Applying the fuzzy decision, we have the optimal clustering level $z=0.83$ of the partition tree. In this level, there are three clusters $\{1, 3, 7, 8\}$, $\{5, 6\}$ and $\{2, 4\}$.

Comparing with this result, the fuzzy node fuzzy graph in Figure.5 should be the optimal solution, which could show the practical effectiveness of this drawing system.

Table with 4 columns: ID, Genetic Code, Value, and Position. The table contains 100 rows of data, representing the GTYPE Situation from the 298th to the 300th generation. Each row lists a unique identifier, a sequence of genetic codes (e.g., 23864715, 24363, 778699), a numerical value, and a position number.

Figure.8 GTYPE Situation from 298th Generation to 300th Generation