結び目理論の総合的研究

(課題番号：０８３０４００８)

平成８年度科学研究費補助金(基盤研究(B))

研究成果報告書

平成９年３月

研究代表者　　鈴　木　晋　一
(早稲田大学教育学部教授)

はしがき

本科学研究費は、下記の国際研究集会の開催のために使用された。

第５回日本数学会国際研究集会：結び目理論

英語名：The Fifth MSJ International Research Institute : Knot Theory

主催：日本数学会

共催：早稲田大学 (国際会議等開催助成費)

協賛：井上科学振興財団、鹿島学術振興財団、旭硝子財団、大和日英基金

会場：早稲田大学総合学術情報センター内・国際会議場

期間：International Conference 1996年7月22日～26日
International Workshop 1996年7月29日～31日

参加者数：１８３名 (内訳：国籍ではなく、所属機関の所在地による)
Australia 3, Canada 3, France 4, Germany 4, Italy 1, Japan 140, Korea 2, Russia 6, Spain 2, Switzland 1, U.K.5, U.S.A.12.
また、この中には、国内６１名、国外８名の大学院学生を含む。

前半の Conference では、plenary session において１９の４５分講演が、また３会場に分かれた parallel session においては飛び入りを含めて５７の２０分講演が行われた。典型結び目・空間グラフ・４次元結び目・３次元多様体等々多方面にわたり最新の話題が発表され、最近の結び目理論の発展ぶりを窺わせる集会となった。

後半の Workshop では、C.MaA.Gordon, Louis H.Kauffman, De Witt Sumners の３教授による３時間のサーベイレクチャーと、６名の日本人による１時間のサーベイレクチャーが行われた。

これらの全講演の内容（アブストラクト）は、この報告書に収録してある。

また、Conference における講演のうち４３は

Proceedings of Knots'96, World Scientific Publishing Co.

として、1997年4月に、Conference における講演２つと Workshop における８つの講義は

Lectures at Knots'96, World Scientific Publishing Co.

として、1997年5月に出版される。

会期を通して海外の研究者との活発な交流がなされ、国内の結び目理論研究のより一層の活性化と、次世代研究者の育成という目的も十分に達成されたものと考える。

なお本研究費は、(1) 国内からの講演者の旅費・滞在費、(2) 大学院学生の講演者の謝金、(3) 会場費、(4) 報告集作成のため等に使用された。

また本研究の組織と経費は次のようになっている。

研究組織

研究代表者：	鈴　木　晋　一	（早稲田大学教育学部教授）
研究分担者：	小　林　一　章	（東京女子大学文理学部教授）
研究分担者：	池　田　裕　司	（神戸大学理学部教授）
研究分担者：	川久保　勝　夫	（大阪大学理学部教授）
研究分担者：	松　本　幸　夫	（東京大学数理科学研究科教授）
研究分担者：	渋　谷　哲　夫	（大阪工業大学工学部教授）
研究分担者：	河　内　明　夫	（大阪市立大学理学部教授）
研究分担者：	丸　本　嘉　彦	（大阪産業大学教養部教授）
研究分担者：	中　西　康　剛	（神戸大学理学部教授）
研究分担者：	作　間　　　誠	（大阪大学理学部助教授）
研究分担者：	金　信　泰　造	（大阪市立大学理学部助教授）
研究分担者：	河　野　正　晴	（北見工業大学工学部教授）
研究分担者：	根　上　生　也	（横浜国立大学教育学部助教授）
研究分担者：	大　山　淑　之	（名古屋工業大学工学部助教授）
研究分担者：	横　田　佳　之	（九州大学数理学研究科講師）

研究経費　　平成８年度　　２、６００千円

目　次

結び目理論国際研究集会

List of Participants

Iain AITCHSON	University of Melbourne, AUSTRALIA (iain@mundoe.maths.mu.oz.au)
Hirotaka AKIYOSHI	Kyushu University, JAPAN (akiyoshi@math.kyushu-u.ac.jp)
Jürgen ALDINGER	Mathematisches Institut LMU, GERMANY (aldinger@rz.mathematik.uni-muenchen.de)
Daniel ALTSCHULER	Institute for Theoretical Physics, SWITZERLAND (altsch@itp.phys.ethz.ch)
Charilaos ANEZIRIS	DESY-Ifh, GERMANY (aneziris@hades.ifh.de)
Taro ASUKE	University of Tokyo, JAPAN (asuke@ms.u-tokyo.ac.jp)
Silvia BENVENUTI	Pisa University, ITALY (benvenut@dm.unipi.it)
Gerhard BURDE	University of Frankfurt am Main, GERMANY (burde@math.uni-frankfurt.de)
Sergei CHMUTOV	Program Systems Institute, RUSSIA (chmutov@math.botik.yaroslavl.su)
Tim COCHRAN	Rice University, U.S.A. (cochran@math16.rice.edu)
Oliver COLLIN	University of Oxford, U.K. (collin@maths.ox.ac.uk)
Isabel DAZEY	Florida State University, U.S.A. (dazey@math.fsu.edu)
Tetsuo DEGUCHI	Ochanomizu University, JAPAN (deguchi@phys.ocha.ac.jp)
Sergei DUZHIN	Program Systems Institute, RUSSIA (duzhin@botik.ru)
Toshiki ENDO	Waseda University, JAPAN (toshi2@mn.waseda.ac.jp)
Roger FENN	University of Sussex, U.K. (rogerf@sussex.ac.uk)
Lourent FREIDEL	ENSLAPP E.N.S-LYON, FRANCE (freidel@enslapp.ens-lyon.fr)
Stavros GAROUFALIDIS	Brown University, U.S.A. (stavros@math.mit.edu)
Patrick GILMER	Louisiana State University, U.S.A. (gilmer@marais.math.lsu.edu)
Hiroshi GODA	Science University of Tokyo, JAPAN (goda@ma.noda.sut.ac.jp)
Cameron GORDON	University of Texas at Austin, U.S.A. (gordon@fireant.ma.utexas.edu)

Victor V. GORYUNOV	University of Liverpool, U.K. (Victor.Goryunov@liverpool.ac.uk)
Masao HARA	Tokai University, JAPAN (masao@ss.u-tokai.ac.jp)
Toshio HARIKAE	Kwansei Gakuin University, JAPAN (harikae@kgupyr.kwansei.ac.jp)
Chuichiro HAYASHI	Gakushuin University, JAPAN (19952761@gakushuin.ac.jp)
Jonny W. HILL	University of Liverpool, U.K. (J.W.Hill@liverpool.ac.uk)
Mikami HIRASAWA	Osaka University, JAPAN (Hirasawa@math.sci.osaka-u.ac.jp)
Susumu HIROSE	Saga University, JAPAN (hirose@math.ms.saga-u.ac.jp)
Craig HODGSON	University of Melbourne, AUSTRALIA (cdh@mundoe.maths.mu.oz.au)
Fujitsugu HOSOKAWA	Kansai University, JAPAN (hosokawa@res.kutc.kansai-u.ac.jp)
Jim HOSTE	Pitzer College, U.S.A. (jhoste@pitzer.claremont.edu)
Toru IKEDA	NEC Co., JAPAN (ikeda@pat.cl.nec.co.jp)
Jyun IMAI	Tokyo Metroporitan University, JAPAN (ohara@math.metro-u.ac.jp)
Hisayoshi INABA	Tokyo Denki University, JAPAN (hisa@r.dendai.ac.jp)
Yuko ISHII	Tokyo Denki University, JAPAN (chan@r.dendai.ac.jp)
Makiko ISHIWATA	Tokyo Woman's Christian University, JAPAN (mako@zem.twcu.ac.jp)
Soichi IWAMOTO	Toyo University, JAPAN (iwamoto@krc.eng.toyo.ac.jp)
Gyo Taek JIN	Korea Institute of Technology, KOREA (trefoil@math.kaist.ac.kr)
Teruhisa KADOKAMI	Osaka City University, JAPAN (h1263@ocugw.cc.osaka-cu.ac.jp)
Osamu KAKIMIZU	Niigata University, JAPAN (kakimizu@ed.niigata-u.ac.jp)
Naoko KAMADA	Osaka City University, JAPAN
Seiichi KAMADA	Osaka City University, JAPAN (h1698@ocugw.cc.osaka-cu.ac.jp)
Tadahiro KANAUCHI	JAPAN

Taizo KANENOBU	Osaka City University, JAPAN (h1256@ocugw.cc.osaka-cu.ac.jp)
Takeshi KANETO	Tsukuba University, JAPAN (kaneto@math.tsukuba.ac.jp)
Jinko KANNO	Sirayuri Gakuen, JAPAN
Mitsuyoshi KATO	Kyushu University, JAPAN (kato@math.kyushu-u.ac.jp)
Louis H. KAUFFMAN	University of Illinois at Chicago, U.S.A. (Kauffman@uic.edu)
Eishin KAWAMOTO	Waseda University, JAPAN
Tomomi KAWAMURA	University of Tokyo, JAPAN (nn56014@komaba.ecc.u-tokyo.ac.jp)
Akio KAWAUCHI	Osaka City University, JAPAN (d54454@jpnkudpc.bitnet)
Takahiro KAYASHIMA	Kyushu University, JAPAN (kayap@math.kyushu-u.ac.jp)
Ken-ichiro KAZAMA	Kobe University, JAPAN
Yasushi KINOSHITA	University of Tokyo, JAPAN (GCH07325@niftyserve.or.jp)
Teruaki KITANO	Tokyo Institute of Technology, JAPAN (kitano@math.titech.ac.jp)
Ki Hyoung KO	Korea Institute of Technology, KOREA (knot@knot.kaist.ac.kr)
Kazuaki KOBAYASHI	Tokyo Woman's Christian University, JAPAN (kazuaki@twcu.ac.jp)
Kazushiro KOBAYASHI	Waseda University, JAPAN (693m5086@cfi.waseda.ac.jp)
Masako KOBAYASHI	Assumption Junior College, JAPAN (VFE16740@niftyserve.or.jp)
Kouji KODAMA	Kobe Kosen, JAPAN (kodama@kobe-kosen.ac.jp)
Toshitake KOHNO	University of Tokyo, JAPAN (kohno@ms.u-tokyo.ac.jp)
Masaharu KOUNO	Kitami Institute of Technology, JAPAN (kouno@ds1.comm.kitami-it.ac.jp)
Sofia LAMBROPOULOU	University of Goettingen, GERMANY (sofia@cfgauss.uni-math.gwdg.de)
Jerome LEVINE	Brandeis University, U.S.A. (levine@max.math.brandeis.edu)
Maria-Teresa LOZANO	Universityersidad de Zaragoza, SPAIN (tlozano@cc.unizar.es)

Natsuki MABUCHI	Tokyo Woman's Christian University, JAPAN (natsuki@zem.twcu.ac.jp)
Artem U. MACOVETSKY	Chelyabinsk State University, RUSSIA (sok@cgu.chel.su)
Yoshihiko MARUMOTO	Osaka Sangyo University, JAPAN (D52718@JPNKUDPC.BITNET)
Yves MATHIEU	University of Provence, FRANCE (mathy@gyptis.University-mrs.fr)
Saburo MATHUMOTO	University of Melbourne, AUSTRALIA (saburo@mundoe.maths.mu.oz.au)
Hiroshi MATSUDA	University of Tokyo, JAPAN (s50532@komaba.ecc.u-tokyo.ac.jp)
Yukio MATSUMOTO	University of Tokyo, JAPAN (yukiomat@ms.u-tokyo.ac.jp)
Takao MATUMOTO	Hiroshima University, JAPAN (r0013@math.sci.hiroshima-u.ac.jp)
Thomas MATTMAN	McGill University, CANADA (mattman@zaphod.math.mcgill.ca)
Sergei MATVEEV	Chelyabinsk State University, RUSSIA (matveev@cgu.chel.su)
Brigitte MEYER	Mathematisches Institut LMU, GERMANY (bmeyer@rz.mathematik.uni-muenchen.de)
David MILLER	William Paterson College, NETHERLANDS (damiller@pilot.njin.net)
Satoshi MIURA	Waseda University, JAPAN
Miki S. MIYAUCHI	NTT Co., JAPAN (miyauchi@ntt-20.ntt.jp)
Katsura MIYAZAKI	Tokyo Denki University, JAPAN (miyazaki@cck.dendai.ac.jp)
Haruko A. MIYAZAWA	Tsuda College, JAPAN (aida@tsuda.ac.jp)
Yasuyuki MIYAZAWA	Yamaguchi University, JAPAN (miyazawa@ccy.yamaguchi-u.ac.jp)
Shigeaki MIYOSHI	Waseda University, JAPAN (miyoshi@cfi.waseda.ac.jp)
Michael MONASTYRSKY	ITEP, RUSSIA (monastyrsky@vitep3.itep.ru)
Jose Maria MONTESINOS	University Complute de Madrid, SPAIN (montesin@eucmax.sim.ucm.es)
Takayuki MORIFUJI	University of Tokyo, JAPAN (morifuji@ms406ss5.ms.u-tokyo.ac.jp)
Kanji MORIMOTO	Takusyoku University, JAPAN (morimoto@hc.takushoku-u.ac.jp)

Shigeyuki MORITA University of Tokyo, JAPAN
(morita@ms.u-tokyo.ac.jp)
Hugh R. MORTON University of Liverpool, U.K.
(su14@liverpool.ac.uk)
Kimihiko MOTEGI Nihon University, JAPAN
(motegi@math.chs.nihon-u.ac.jp)
Tomoe MOTOHASHI Waseda University, JAPAN
(695m5081@cfi.waseda.ac.jp)
Hitoshi MURAKAMI Waseda University, JAPAN
(hitoshi@cfi.waseda.ac.jp)
Jun MURAKAMI Osaka University, JAPAN
(jun@math.sci.osaka-u.ac.jp)
Kunio MURASUGI University of Toronto, CANADA
(murasugi@math.toronto.edu)
Chbili NAFAA University of Paris 7, FRANCE
(chbili@mathp7.jussieu.fr)
Shigeru NAGAMATSU Osaka City University, JAPAN
(h1841@ocugw.cc.osaka-cu.ac.jp)
Hirofumi NAGASAKA Waseda University, JAPAN
(693m5163@cfi.waseda.ac.jp)
Maki NAGURA Yokohama National University, JAPAN
(maki@random.osu.seg.ynu.ac.jp)
Swatee NAIK University of Nevada, U.S.A.
(naik@math.unr.edu)
Hisanori NAKA Kwansei Gakuin University, JAPAN

Yoko NAKAGAWA Yamaguchi Prefectural University, JAPAN

Yasutaka NAKANISHI Kobe University, JAPAN
(nakanisi@math.s.kobe-u.ac.jp)
Masatoshi NAKANO Waseda University, JAPAN

Masahiro NAKAO Kwansei Gakuin University, JAPAN
(nakao@kgupyr.kwansei.ac.jp)
Tsutomu NAKATSURU Tokyo Institute of Technology, JAPAN
(tsutomu@is.titech.ac.jp)
Eiji NAKAYAMA Waseda University, JAPAN
(693m5150@cfi.waseda.ac.jp)
Seiya NEGAMI Yokohama National University, JAPAN
(negami@ms.ed.ynu.ac.jp)
Haruko NISHI Kyushu University, JAPAN
(nishi@math.kyushu-u.ac.jp)
Mitsuhiro NOMOTO Osaka University, JAPAN
(nomoto@nova.math.sci.osaka-u.ac.jp)

Eiji OGASA — University of Tokyo, JAPAN
(ogasa@tansei.cc.u-tokyo.ac.jp)

Tomotada OHTSUKI — Tokyo Institute of Technology, JAPAN
(tomotada@is.titech.ac.jp)

Yoshiyuki OHYAMA — Nagoya Institute of Technology, JAPAN
(h44685a@nucc.cc.nagoya-u.ac.jp)

Miyuki OKAMOTO — Waseda University, JAPAN
(miyuki@cfi.waseda.ac.jp)

Makoto OZAWA — Waseda University, JAPAN
(696m5050@cfi.waseda.ac.jp)

Muhammad Akmal PASHA — Natal University, SOUTH AFRICA
(ali@ph.und.ac.za)

Jozef H. PRZYTYCKI — George Washington University, U.S.A.
(przytyck@math.gwu.edu)

Tuyoshi SAKAI — Tsuda College, JAPAN

Makoto SAKUMA — Osaka University, JAPAN
(sakuma@math.wani.osaka-u.ac.jp)

Kazuhiro SASANO — Toyama Medical and Pharmaceutical University, JAPAN
(ksasano@toyama-mpu.ac.jp)

Akiko SHIMA — University of Tokyo, JAPAN
(nn57014@komaba.ecc.u-tokyo.ac.jp)

Koya SHIMOKAWA — University of Tokyo, JAPAN
(simokawa@poisson.c.u-tokyo.ac.jp)

Yaichi SHINOHARA — Kwansei Gakuin University, JAPAN

Jonathan SIMON — University of Iowa, U.S.A.
(jsimon@math.uiowa.edu)

Maxim V. SOKOLOV — Chelyabinsk State University, RUSSIA
(sok@cgu.chel.su)

Makoto SOMA — Osaka City University, JAPAN
(h1271@ocugw.cc.osaka-cu.ac.jp)

Teruhiko SOMA — Tokyo Denki University, JAPAN
(soma@r.dendai.ac.jp)

Yoichi SONODA — Waseda University, JAPAN
(695m0143@cfi.waseda.ac.jp)

Itsuki SUGIURA — Waseda University, JAPAN

De Witt SUMMERS — Florida State University, U.S.A.
(sumners@math.fsu.edu)

Noriyuki SUZUKI — Osaka University, JAPAN
(nsuzuki@nova.math.sci.osaka-u.ac.jp)

Shin'ichi SUZUKI — Waseda University, JAPAN
(sssuzuki@cfi.waseda.ac.jp)

Yasuhiro SUZUKI	Waseda University, JAPAN (695m0130@cfi.waseda.ac.jp)
Norihiro SYOUDA	Kyushu University, JAPAN (syouda@math.kyushu-u.ac.jp)
Satoshi TAGUCHI	Waseda University, JAPAN
Takashi TAKAMUKI	Kyushu University, JAPAN (takamuki@math.kyushu-u.ac.jp)
Toshie TAKATA	Kyushu University, JAPAN (takata@math.kyushu-u.ac.jp)
Kenichi TAMANO	Yokohama National University, JAPAN (tamano@math.sci.ynu.ac.jp)
Makoto TAMURA	Osaka University, JAPAN (tamura@math.sci.osaka-u.ac.jp)
Toshifumi TANAKA	Kyushu University, JAPAN (ttanaka@math.kyushu-u.ac.jp)
Kouki TANIYAMA	Tokyo Woman's ChristianUniversity, JAPAN (taniyama@twcu.ac.jp)
Masakazu TERAGAITO	Hiroshima University, JAPAN (teragai@top2.math.sci.hiroshima-u.ac.jp)
Hirotaro TOKUDA	JAPAN
Jinko TOMIKAWA	Tokyo Woman's Christian University, JAPAN (jinko@zem.twcu.ac.jp)
Ichirou TORISU	Osaka University, JAPAN (torisu@math.sci.osaka-u.ac.jp)
Yoshihiko TSUJII	Osaka City University, JAPAN (h1840@ocugw.cc.osaka-cu.ac.jp)
Emika TSUKAMOTO	Tokyo Woman's ChristianUniversity, JAPAN (emika@zem.twcu.ac.jp)
Tatsuya TSUKAMOTO	Waseda University, JAPAN (696m5091@cfi.waseda.ac.jp)
Yasuyoshi TSUTSUMI	Osaka City University, JAPAN (h1488@ ocugw.cc.osaka-cu.ac.jp)
Yasuyuki TUKUI	Senshu University, JAPAN (tsukui@isc.senshu-u.ac.jp)
Yoshiaki UCHIDA	Yamagata University, JAPAN (yuchida@kszaoh3.kj.yamagata-u.ac.jp)
Masaaki UE	Kyoto University, JAPAN (ue@math.h.kyoto-u.ac.jp)
Akira USHIJIMA	Osaka University, JAPAN (smv001ua@ex.ecip.osaka-u.ac.jp)
Michihisa WAKUI	Osaka University, JAPAN (wakui@math.sci.osaka-u.ac.jp)

Tatsuhiko YAGASAKI	Kyoto Institute of Technology, JAPAN (yagasaki@ipc.kit.ac.jp)
Tatsuya YAGI	Tokyo Institute of Technology, JAPAN (yagi@is.titech.ac.jp)
Shuji YAMADA	Kyoto Sangyo University, JAPAN (yamada@cc2000.kyoto-su.ac.jp)
Yuuich YAMADA	University of Tokyo, JAPAN (yyyamada@ms. u-tokyo.ac.jp)
Makoto YAMAMOTO	Chuo University, JAPAN (makotoy@math.chuo-u.ac.jp)
Masanaga YAMAMOTO	Osaka City University, JAPAN (h1118@ocugw.cc.osaka-cu.ac.jp)
Masakatsu YAMASHITA	Toyo University, JAPAN
Takeshi YAMAUCHI	Tokyo Institute of Technology, JAPAN (t-yama@is.titech.ac.jp)
Akira YASUHARA	Tokyo Gakugei University, JAPAN (yasuhara@u-gakugei.ac.jp)
Yoshiyuki YOKOTA	Kyushu University, JAPAN (jojo@math.kyushu-u.ac.jp)
Han YOSHIDA	Nara Woman University, JAPAN (han@vivaldi.nara-wu.ac.jp)
Kiyomi YOSHIDA	Tsuda College, JAPAN (kyoshida@tsuda.ac.jp)
Katsuyuki YOSHIKAWA	Kwansei Gakuin University, JAPAN (yoshikawa@kgupyr.kwansei.ac.jp)

Program

Conference

July 22(Mon)

9:00-10:00		REGISTRATION (COFFEE and DONUTS 9:30-10:00)
10:15-11:00		Kunio MURASUGI (with B. I. KURPITA)
		The Tutte polynomial and some of its applications to knot theory.
11:15-12:00		Tetuo DEGUCHI (with K. TSURUSAKI)
		Random knots and links and applications to polymer physics.
12:00-13:30		LUNCH
13:30-14:15		Jun MURAKAMI
14:40-15:00		*On the universal quantum invariant of 3-manifolds.*
	Room 1.	Shuji YAMADA
		Hyperbolic 3-manifolds and the four color theorem.
	Room 2.	Yoshihiko MARUMOTO
		Equivalence of ribbon presentations for knots.
	Room 3.	Susumu HIROSE
		Heegaard splittings of 3-manifolds and mapping class groups.
15:00-15:40		COFFEE BREAK
15:40-16:00		
	Room 1.	Hitoshi MURAKAMI
		A weight system derived from the Conway potential function.
	Room 2.	Kanji MORIMOTO
		Planar surfaces in a handlebody and a theorem of Gordon-Reid.
	Room 3.	Yasutaka NAKANISHI
		Alexander invariant and twisting operation.
16:10-16:30		
	Room 1.	Seiya NEGAMI
		Ramsey theorem for good spatial graphs.
	Room 2.	Tsuyoshi SAKAI
		A condition for a 3-manifold to be a knot exterior.
	Room 3.	Yuichi YAMADA
		Decomposition of S^4 as a twisted double of a certain manifold.

July 23(Tue)

9:15-10:00		Tim D. COCHRAN *TBA*
10:00-10:30		COFFEE and DONUTS
10:30-11:15		Patrick GILMER *Turaev-Viro modules of satellite knots.*
11:30-12:15		Daniel ALTSCHULER *Vassiliev knot invariants and Chern-Simons perturbation theory to all orders.*
12:15-13:30		LUNCH
13:30-14:15		Gerhard BURDE *Representation spaces of knnot groups.*
14:40-15:00		
	Room 1.	Swatee NAIK *Equivariant concordance of knots in dimension 3.*
	Room 2.	Tatsuya TSUKAMOTO *On inevitability of knots, links and spatial graphs.*
	Room 3.	Sergei DUZHIN *A quadratic lower bound for the number of primitive Vassiliev invariants.*
15:00-15:40		COFFEE BREAK
15:40-16:00		
	Room 1.	Yoshiyuki OHYAMA *Twisting of two strings and Vassiliev invariants.*
	Room 2.	Tomoe MOTOHASHI (with K. TANIYAMA) *Delta unknotting operation and vertex homotopy of graphs in R^3.*
	Room 3.	Jonny W. HILL *Vassiliev-type invariants in J^+-theory of planar fronts without dangerous self-tangencies.*
16:10-16:30		
	Room 1.	Taizo KANENOBU *Link polynomials as Vassiliev-type invariants.*
	Room 2.	Naoko KAMADA *Alternating links in the product space of a closed oriented surface and the real line.*
	Room 3.	Thomas MATTMAN *The fundamental polygons of twist knots and the (-2,3,7) pretzel knot.*

16:40-17:00

Room 1. Akira YASUHARA

Delta-unknotting operation and adaptability of certain graphs.

Room 2. Makoto OZAWA

Uniqueness of essential free tangle decompositions of knots and links.

Room 3. Charilaos ANEZIRIS

Developing computer programs for knot classification.

July 24(Wed)

9:15-10:00 Jonathan SIMON

Energy and thickness of knots.

10:00-10:30 COFFEE and DONUTS

10:30-11:15 Michael MONASTYRSKY

Statistics of knots and some relations with random walks on hyperbolic plane.

11:30-12:15 Louis H. KAUFFMAN

Invariants of links and three-manifolds from finite dimensional Hopf algebras.

12:15-13:30 LUNCH

13:30-14:15 Jerome LEVINE

Finite type invariants of 3-manifolds.

14:40-15:00

Room 1. Jun O'HARA

On the existence of the energy minimizing knots.

Room 2. Hiroshi GODA

Heegaard splitting for sutured manifolds and its application.

Room 3. Jürgen ALDINGER

Formulae for the calculation and estimation of writhe.

15:00-15:40 COFFEE BREAK

15:40-16:00

Room 1. Stavros GAROUFALIDIS

TQFT versus finite type invariants of 3-manifolds.

Room 2. Miyuki OKAMOTO & Haruko MIYAZAWA

Quantum SU(3)-invariants derived from the linear skein theory.

Room 3. Brigitte MEYER

Abelian Chern Simons theory and knots.

16:10-16:30

	Room 1.	Makoto SAKUMA
		Unknotting tunnels for knots and links.
	Room 2.	Eiji OGASA
		On the intersection of three spheres in a sphere.
	Room 3.	Isabel DAZEY (with DW SUMNERS)
		A strand passage metric for topoisomerase action.

16:40-17:00

	Room 1.	Oliver COLLIN
		Floer homology for orbifolds and gauge theory knot invariants.
	Room 2.	
	Room 3.	Katura MIYAZAKI & Kimihiko MOTEGI
		Seifert fibred manifolds and Dehn surgery on knots.

July 25(Thu)

9:15-10:00	Hugh R. MORTON
	Young diagrams, the Homfly skein of the annulus and unitary invariants.
10:10-10:30	COFFEE and DONUTS
10:30-11:15	Roger FENN
	Some new results in the theory of braids and generalised braids.
11:30-12:15	Iain AITCHISON
	Lifting surfaces to embeddings in covers.
12:15-13:30	LUNCH
13:30-14:15	Criag HODGSON
	Arithmetic invariants and hyperbolic Dehn filling.

14:40-15:00

	Room 1.	Sergei V. MATVEEV
		A finiteness theorem for surfaces in Haken 3-manifolds.
	Room 2.	Yasuyuki MIYAZAWA
		A Conway presentation and the coefficients of the Jones and Kauffman polynomials of a 2-bridge link.
	Room 3.	Saburo MATSUMOTO (with I. R. AITCHISON and J. H. RUBINSTEIN)
		Normal surfaces immersed in the figure-8 knot complement.

15:00-15:40	COFFEE BREAK

15:40-16:00

Room 1. Artem U. MACOVETSKY
Transformations on special spines of 3-manifolds and branched surfaces.

Room 2. Akiko SHIMA
On simply knotted tori in S^4 II.

Room 3. Makoto TAMURA
The average edge order of triangulations of 3-manifolds with boundary modification.

16:10-16:30

Room 1. Victor V. GORYUNOV (with S. CHMUTOV)
Kauffman bracket of plane curves.

Room 2. Sin'ichi SUZUKI (with F. HOSOKAWA)
Every 2-link with 2 components is link-homotopic to the trivial 2-link.

Room 3. Ichiro TORISU
The determination of the pairs of two-bridge knots or links with Gordian distance one.

16:40-17:00

Room 1. Teruhiko SOMA
Spatial-graph isotopy for trivalent graphs and minimally knotted embeddings.

Room 2.

Room 3. Ki Hyoung KO (with J. BIRMAN)
Band-generator presentation of the braid group and its advantage.

July 26(Fri)

9:15-10:00 Jóse Maria MONTESINOS
Arithmetic and geometry of some cone manifolds.

10:00-10:30 COFFEE and DONUTS

10:30-11:15 María -Teresa LOZANO (with H. HILDEN and J. M. MONTESINOS)
Geometric invariants of cone manifolds.

11:30-12:15 Józef H. PRZYTYCKI
Algebraic topology based on knots.

12:15-13:30 LUNCH

13:30-14:15 Yoshiyuki UCHIDA
Generalized unknotting number one two-bridge knot.

14:40-15:00

Room 1. Sergei CHMUTOV
A proof of Melvin-Morton conjecture.

Room 2. Yoshiyuki YOKOTA
On SU(n) invariants of knots and 3-manifolds.

Room 3. Jim E. HOSTE
Infinite framed link diagrams for open 3-manifolds.

15:00-15:40 COFFEE BREAK

15:40-16:00

Room 1. Maxim V. SOKOLOV
Which lens spaces can be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants.

Room 2. Han YOSHIDA
Invariant trace fields of hyperbolic 3-manifolds.

Room 3. Mikami HIRASAWA (with M.SAKUMA)
Minimal genus Seifert surfaces for alternating links.

16:10-16:30

Room 1. Yves MATHIEU (with M.BOILEAU & M.DOMERGUE)
On the complement of homotopically trivial knots.

Room 2. Teruhisa KADOKAMI
Seifert complex for links and 2-variable Alexander matrices.

Room 3. Koya SHIMOKAWA (with C. HAYASHI)
Symmetric knots satisfy the cabling conjecture.

18:00-20:00 **BANQUET** (at Okuma Garden House)

Workshop

July 29(Mon)

9:15-10:15 Cameron M. GORDON
Dehn Surgery, I.

10:15-11:00 COFFEE and DONUTS

11:00-12:00 Louis H. KAUFFMAN
Invariants of links, I.

12:00-13:00	LUNCH
13:00-14:00	De Witt SUMNERS *The topology of DNA, I.*
14:30-15:30	Toshitake KOHNO *Chern-Simons perturbative invariants.*
16:00-17:00	Akio KAWAUCHI *Topological imitations.*

July 30(Tue)

9:15-10:15	Cameron M. GORDON *Dehn Surgery, II*
10:15-11:00	COFFEE and DONUTS
11:00-12:00	Louis H. KAUFFMAN *Invariants of links, II.*
12:00-13:00	LUNCH
13:00-14:00	De Witt SUMNERS *The topology of DNA, II.*
14:30-15:30	Kouki TANIYAMA *On Spatial Graphs.*
16:00-17:00	Kanji MORIMOTO *Tunnel number and connected sum of knots.*

July 31(Wed)

9:15-10:15	Cameron M. GORDON *Dehn Surgery, III.*
10:15-11:00	COFFEE and DONUTS
11:00-12:00	Louis H. KAUFFMAN *Invariants of links, III.*
12:00-13:00	LUNCH
13:00-14:00	De Witt SUMNERS *The topology of DNA, III.*
14:30-15:30	Seiichi KAMADA *Surfaces in 4-Space.*
16:00-17:00	Kinihiko MOTEGI *Knot types of satellite knots and twisted knots.*

Abstract of Lectures

The Tutte Polynomial and some of its Applications to Knot Theory

by

B.I. Kurpita and K. Murasugi

We intend to discuss several properties of the Tutte polynomial for a graph. In particular, we shall show that the theorems below have applications in knot theory.

Let $X_G(x, y)$ be the Tutte polynomial of a graph G. It is known that if G is a plane graph, then $X_G(-t, -t^{-1})$ is equal to the Jones polynomial $V_K(t)$ (up to a factor $\pm t^m$) of an alternating knot K whose graph is G.

So, let us suppose that Γ is a finite graph that does not contain any multiple edges. We say that two graphs M_1 and M_2 of Γ are *independent* if they are disjoint, i.e., they do not have any vertices in common. A collection of mutually independent subgraphs of Γ, $\{M_1, M_2, \ldots, M_r\}$, is said to be of order k if

(1) each M_i is a complete graph with say n_i vertices, $(n_i \geq 1)$;

(2) $\displaystyle\sum_{i=1}^{r}(n_i - 1) = k.$

Further, if we consider a connected finite graph, G, with say n vertices, then we may imbed G in a complete graph K_m that has at least n, m, say, vertices. In a natural manner, we may define $H = K_m \backslash G$, the complement of G. H is also a finite graph. Let $\mu_k(H)$ be the number of collections (of mutually independent, complete subgraphs) of order k in H. Then the following equality holds:

Theorem 1

For any $m \geq n$.

$$X_G(2,0) = m!\mu_0(H) - (m-1)!\mu_1(H) + (m-2)!\mu_2(H) - \ldots$$
$$\ldots + (-1)^k(m-k)!\mu_k(H) + \ldots$$

where $\mu_0(H) = 1$

Let us, now, consider n disjoint sets $A_1, A_2, \ldots, A_n$ with $|A_i| = p_i > 0$. Suppose $G(p_1, p_2, \ldots, p_n)$ is the graph constructed by joining every vertex in A_i to every vertex of A_j, $1 \le i \ne j \le n$. (We do not allow a vertex to be joined to a vertex in the same set.) $G(p_1, p_2, \ldots p_n)$ will be called a *complete n-partite graph.* The following theorem holds for n-partite graphs:

Theorem 2

$$\sum_{k\ge 0}(-1)^k(p_1+p_2+\ldots,p_n-k)!\mu_k(G(p_1,p_2,\ldots,p_n)) = p_1!p_2!\ldots p_n!$$

We may further generalize Theorem 2. To this end, let us divide $E(G(p_1,p_2,\ldots,p_n))$, the set of edges of $G(p_1,p_2,\ldots p_n)$, into two subsets P and Q. Now, we may define an invariant, $\lambda(P)$, from the Tutte polynomial so that the following theorem holds:

Theorem 3

$$p_1!p_2!\ldots p_n!\lambda(P) = \sum_{k\ge 0}(-1)^k(p_1+p_2+\ldots+p_n-k)!\mu_k(Q).$$

It is straightforward to see that if $p_1 = p_2 = \ldots = p_n = 1$, then Theorem 3 reduces to Theorem 1; while if P is empty, then Theorem 3 reduces to Theorem 2.

For the simplest case, $n = 2$, Theorem 3 has the following form:

Theorem 4

In the case in Theorem 3 when $n = 2$,

$$\lambda(P) = \sum_{k\ge 0}\frac{\mu_k(P)}{k!}$$

and

$$\sum_{k\ge 0}\frac{\mu_k(P)}{k!} = \sum(-1)^k\frac{(p_1+p_2-k)!}{p_1!p_2!}\mu_k(Q).$$

More generally, for any integer $d \ge 0$, the following formula holds:

$$\sum_{k\ge 0}\frac{\mu_k(P)}{(k+d)!} = \sum(-1)^k\frac{(p_1+p_2+d-k)!}{(p_1+d)!(p_2+d)!}\mu_k(Q).$$

Mailing address: Dept. of Math., University of Toronto, Toronto, Ontario, M5S 3G3, CANADA.

Random Knots and Links and Applications to Polymer Physics

Tetsuo Deguchi and Kyoichi Tsurusaki[†]

Department of Physics, Ochanomizu University, Tokyo 112, Japan
[†]*Department of Physics, University of Tokyo, Tokyo 113, Japan*

Abstract

We discuss random knotting and linking probabilities by making an extensive use of topological invariants of knots and links. We show that the probability of an N-noded polygon having knot type K can be expressed by a simple scaling function of N. We define the linking probability $P_L(R; N)$ by the probability that a given pair of N-noded random polygons in a distance R gives a link L. We introduce a formula for the probability $P_L(R; N)$. We confirm it by numerical experiments. We also show some applications of random knotting and linking to statistical mechanics of polymers. From random knottiong and linking we show how the virial coefficients of a ring polymer solution depend on the degree of polymerization N.

Generating a large number of N-noded random polygons by computers and enumerating some knot invariants for each polygon, we can practically evaluate the statistical fraction of those polygons that have the same knot type K. The fraction is called knotting probability $P_K(N)$.

We make plots of the knotting probabilities $P_K(N)$ obtained numerically against the size N of the random polygons for various knot types. Then we find that the knotting probabilities are expressed by fitting curves given by the following formula

$$P_K(N) = C(K)N^{m(K)} \exp(-N/N(K))$$

where $m(K)$, $N(K)$ and $C(K)$ are fitting parameters. Our numerical experiments show that for a given model of random polygon the parameter $N(K)$ does not depend on the knot type K : $N(K) = N(0)$. Here 0 denotes the trivial knot.

We study the self-avoiding effect on the knotting probability by the bead-rod model, where the bead-radius is changed. The numerical results are consistent with our hypothesis that when a knot K is given the exponent $m(K)$ is universal: the same value of $m(K)$ gives a good fitting curve to any model of random polygon. [?]

Let us discuss statistical mechanics of a ring polymer solution, which has the topological constraint that the ring polymers are not linked each other in any thermal fluctuations. This constraint leads to an effective entropic force among the ring polymers.

The second virial coefficient of a ring polymer solution has been measured at the θ temperature of its linear polymer solution. The virial coefficient observed should be explained in terms of the topological effect. In fact, it is related to the linking probability $P_0(R; N)$ by the following

$$A_2 = \frac{N_A}{2M_w^2} \int 4\pi R^2 (1 - P_0(R; N)) dR, \qquad (1)$$

where N_A is Avogadro's number and M_w is the molecular weight of the ring polymer. As far as our numerical experiments are concerned, the experimental and numerical results are consistent. [?]

References

[1] T. Deguchi and K. Tsurusaki, Phys. Lett. A 174 (1993) 29

[2] T. Deguchi and K. Tsurusaki, J. Phys. Soc. Jpn. 62 (1993) 1411.

[3] T. Deguchi and K. Tsurusaki, J. Knot Theory and Its Ramifications **3** No. 3 (Special Issue, 1994) 321.

[4] K. Tsurusaki and T. Deguchi, J. Phys. Soc. Jpn. **64** (1995) 1506.

[5] K. Tsurusaki, Thesis University of Tokyo, 1995.

[6] T. Deguchi and K. Tsurusaki, On a universality of random knotting, preprint 1995.

[7] K.Tsurusaki and T. Deguchi , Numerical analysis on topological entanglements of random polygons, to appear.

On the universal quantum invariant of 3-manifolds

Jun Murakami
Osaka University, Japan

Hyperbolic 3-manifolds and the 4-color Theorem

SHUJI YAMADA

The 4-color theorem is one of the most well known theorem of graph theory. Appel and Haken proved the 4-color theorem after a long time computation by a computer. However, we don't have any mathematical proof of this theorem.

THE 4-COLOR THEOREM [AH]. *Any planar graphs are 4-colorable.*

The main result in this note is the following.

MAIN RESULTS. *The 4-color theorem is equivalent to the existence of some hyperbolic 3-dimensional manifolds.*

Through this note, we assume any graphs are simple graphs, i.e., graphs without multi-edges or loops.

We say a subset X of the vertex set of a connected graph G is an k-vertex cut set if $|X| = k$ and $G - X$ are disconnected. We say a connected graph G is k-connected if G has no $(k-1)$-vertex cut sets. We say a plane graph G is dual k-connected if the dual graph G^* is k-connected.

A plane graph G is maximal if there are no plane graphs which contain G as a subgraph. A k-coloring (of the vertices) of a graph G is an assignment of k colors to the vertices in such a way that adjacent vertices have distinct colors. If a graph G has a k-coloring then we say G is k-colorable.

THEOREM 1. *The 4-color theorem is equivalent to the following proposition*

PROPOSITION 1. *Any 5-connected maximal plane graphs are 4-colorable.*

PROOF: It is trivial that the 4-color theorem implies Proposition 1. We shall prove the reverse direction by an induction on the number of vertices of the graph. Assume G is a maximal plane graph which is not 5-connected. Let $X = \{v = 1, \ldots, v = k\}$ be a minimal k-vertex cut set with $k < 5$. We can assume that $\{v_1, \ldots, v_k\}$ constitutes the vertex set of a k-cycle C of G in this order. Let $G - X = A' \cup B'$ be the decomposition into connected components. Set $A = G - B'$ and $B = G - A'$ so that $A \cup B = G$ and $A \cap B = C$. Since A and B have vertices less than G, they have a 4-coloring by the hypothesis of the induction.

We shall consider two cases; $k \leq 3$ and $k = 4$. Assume $k \leq 3$. We can permute the colors of the coloring on A so that the colors on $C \subset A$ coincide with the colors on $C \subset B$. Then we have a 4-coloring on $G = A \cup B$. Assume $k = 4$. We classify A and B into following two types.

Type I It has a 4-coloring which assigns mutually distinct colors on the 4 vertices of C.

Type II Not type I.

Then, for A and B, the following two lemmas hold.

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

LEMMA 1. *If it is type I, then it has 4-coloring ϕ_1 such that $\phi_1(v_1) = \phi_1(v_3)$, $\phi_1(v_2) = \phi_1(v_4)$ or 4-coloring ϕ_2 such that $\phi_1(v_1) = \phi_1(v_3)$, $\phi_1(v_2) = \phi_1(v_4)$.*

LEMMA 2. *If it is type II, then it has 4-coloring ψ_1 such that $\psi_1(v_1) = \psi_1(v_3)$, $\psi_1(v_2) = \psi_1(v_4)$ and 4-coloring ψ_2 such that $\psi_1(v_1) = \psi_1(v_3)$, $\psi_1(v_2) = \psi_1(v_4)$.*

Now we shall complete the proof with four cases of type of A and B.

Case of A and B are type I. Take 4-colorings on A and B. After suitable permutation of colors we have a 4-coloring on G.

Case of A and B are type II. Take the 4-colorings on A and B which presented as ψ_1 in Lemma 2. After suitable permutation of colors we have a 4-coloring on G.

Case of A is type I and B is type II. Take a 4-coloring ϕ_1 or phi_2 on A which presented in Lemma 1. According to ϕ_1 or ϕ_2, take a 4-coloring ψ_1 or ψ_2 on B which presented in Lemma 2. After suitable permutation of colors we have a 4-coloring on G.

Case of A is type II and B is type I. It is same as the third case.

The following theorem is a different way of saying the well-known fact that the 4-Color Theorem is equivalent to Edge-3-Color Theorem.

THEOREM 2. *Let $G \subset S^2 \subset S^3$ be a trivalent plane graph on S^2 in S^3. The regions of $S^2 - G$ has 4-coloring if and only if there is a $Z_2 \times Z_2$ branched cover of S^3 whose branch set is G.*

The following theorem is derived from Andre'ev's theorem.

THEOREM 3. *Let G be a trivalent, dual 5-connected, plane graph. Then there is a hyperbolic polyhedron such that its 1-skelton is G and every face angle is right angle.*

THEOREM (ANDRE'EV [A1][A2]). *Let Q be an orbifold such that the base space X_Q is a 3-ball B^3 and the singular locus Σ_Q is $S^2 = \partial B^3$. Then Q is a hyperbolic orbifold if and only if Q has no incompressible sub-orbifold P whose Eular characteristic $\chi(P) \geq 0$.*

We can derive the following theorem from the above theorems.

THEOREM 4. *The 4-color theorem is equivalent to the following proposition.*

PROPOSITION. *Let P be a hyperbolic polyhedron whose dihedral angles are the right angle. Then P has a $Z_2 \times Z_2 \times Z_2$ fold manifold cover.*

REFERENCES

AH K. I. Appel and W. Haken, *Every planar map is four colorable*, Contemp. Math. 98, Amer. Math. Soc. (1989).

A1 E. M. Andreev, *Convex polyhedra in Lobacevskii space.*, Mat. Sb. **81** (1970), 445-478.

A2 E. M. Andreev, *Convex polyhedra of finite volume in Lobacevskii space.*, Mat. Sb. **83** (1970), 256-260.

Shuji Yamada
Department of Computer Science Kyoto Sangyo University,
Motoyama, Kamigamo, Kita-ku, Kyoto 603, JAPAN. (*E-mail address*: yamada@cc.kyoto-su.ac.jp)

EQUIVALENCE OF RIBBON PRESENTATIONS FOR KNOTS

YOSHIHIKO MARUMOTO

Department of Mathematics, Osaka Sangyo University

My purpose here is to state about a representing method of a knot, and discuss sevral topics around this.

The first question is "*Is a knot determined if a cross section of the knot is given?*" This is negative because the question itself is too ambiguous. For improving this point, we introduce a ribbon knot and its equator, and then we can restate the question above "*Is a ribbon knot determined by its equator?*" It is shown that there exist infinitely many counter examples to this question in 2-dim. knots, but it is not known in three or higher dimensions.

For attacking the last question, we introduce a notion of ribbon presentation, which describes a geometrical way to construct ribbon knots. We then need to define equivalence among ribbon presentations, and sevral equivalences are discussed. Using our definition, we modify the questions above to have "*Are any ribbon presenttations for a knot equivalent?*" We introduce results to this question. One of them is that there exist arbitraty finite many ribbon presentations for a knot in any dimension. The ribbon presenatations are, of course, different under an equivalence, but it is interesting that they are the same under another equvalence.

DAITO, OSAKA 574, JAPAN: E-MAIL: D52718@ JPNKUDPC.BITNET

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

HEEGAARD SPLITTINGS OF 3–MANIFOLDS AND MAPPING CLASS GROUPS

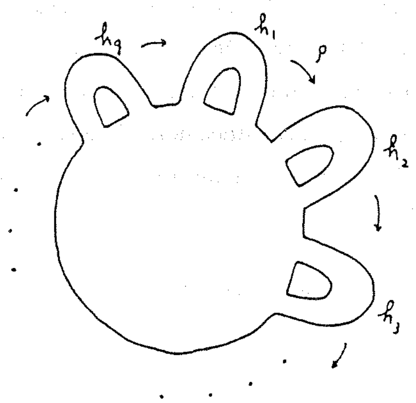
SUSUMU HIROSE

Let H_g, H_g^* be the 3-dimensional manifolds, each of which is constructed from a 3-ball with attaching g 1-handles, $S^3 = H_g \cup H_g^*$ be the Heegaard splitting of S^3. By Waldhausen [W], the genus g Heegaard splitting of S^3 is unique up to isotopy. We investigate the group $\mathcal{E}_g$, a group consisted of elements of mapping class group of ∂H_g which can be extended to orientation preserving diffeomorphisms of S^3. We give a system of generators of the group $\mathcal{E}_g$:

Theorem A. *$\mathcal{E}_g$ is generated by ρ, ω_1, ρ_{12}, θ_{12}.*

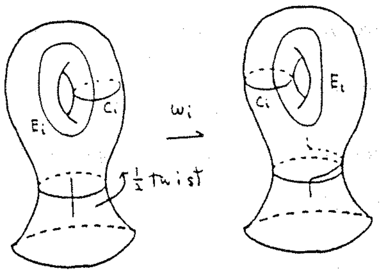
Remark 1. The maps ρ, ω_1, ρ_{12}, θ_{12} are defined as follows:

ρ : a cyclic translation of H_g

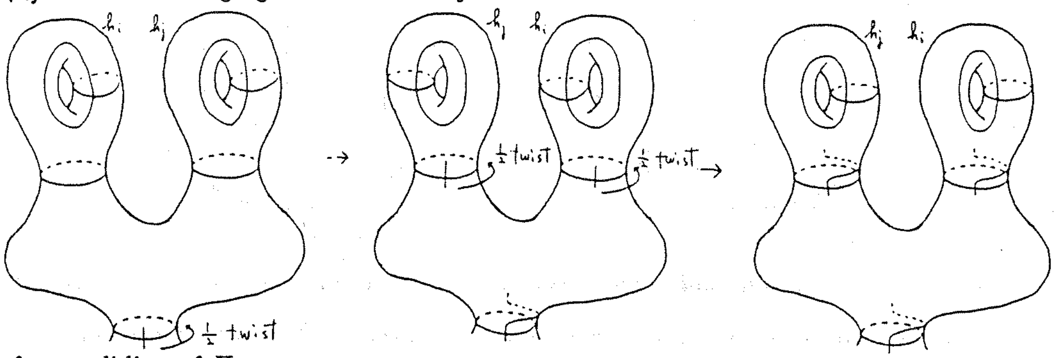


1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 57N05, 57N10.
Key words and phrases. Mapping class group, handlebody, Heegaard splitting.

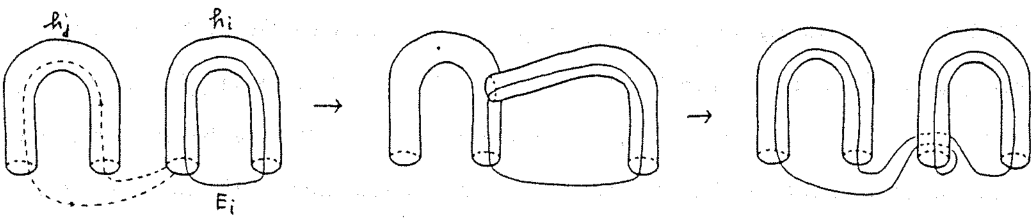
ω_i : a twisting a knob of H_g



ρ_{ij} : an interchanging two knobs of H_g



θ_{ij} : a sliding of H_g



Remark 2. By Suzuki [S], a system of generators of the mapping class group of H_g is given. When we embed H_g standardly in S^3, as is easily seen, two of these elements can not be extended to S^3. We prove that the other elements generate $\mathcal{E}_g$. In the paper by Powell [P], it is shown that $\mathcal{E}_g$ is generated by ρ, ω_1, ρ_{12}, θ_{12} and "ν".

Each element of $\mathcal{E}_g$ acts on $\pi_1(H_g, *)$, a free group of rank g, as an isomorphism, hence, there is a natural homomorphism $\alpha : \mathcal{E}_g \to \mathrm{Out}(\pi_1(H_g, *))$. For this homomorphism, $\{\alpha(\rho),\ \alpha(\omega_1),\ \alpha(\rho_{12}),\ \alpha(\theta_{12})\}$ is a system of generator of $\mathrm{Out}(\pi_1(H_g,*))$ given by Nielsen (see [MKS]), therefore, α is surjective. Let $\mathcal{EK}_g$ be the kernel of α. We give the following theorem for this group $\mathcal{EK}_g$:

Theorem B. *(1)$\mathcal{EK}_g$ is generated by ball-twistings on H_g.*
(2)$\mathcal{EK}_g$ is not finitely generated.

Remark 3.

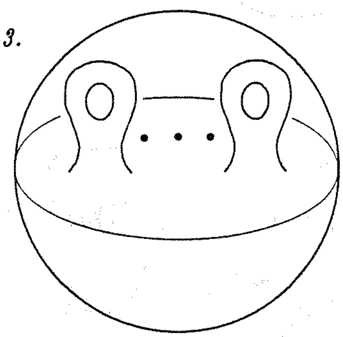


Figure 1

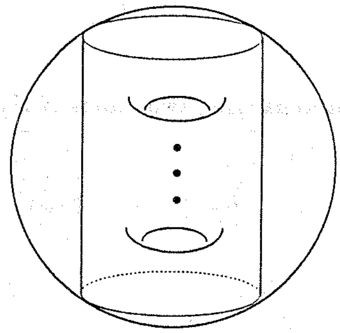


Figure 2

A *ball-twisting of first kind* (resp. a *ball-twisting of second kind*) on H_g (resp. H_g^*) is the map defined as follows: Let $\mathcal{B}$ be a 3-ball embedded in S^3 whose intersection with H_g (resp. H_g^*)is as in Figure 1 (resp. Figure 2). Give a polar coordinate (r,θ,φ) on this 3-ball such that, in Figure 1, $\partial\mathcal{B}\cap H_g$(resp. H_g^*) is identified with $\{(1,\theta,\varphi)|\pi/2\le\theta\le\pi, 0\le\varphi\le 2\pi\}$, and, in Figure 2, $\partial\mathcal{B}\cap H_g$(resp. H_g^*) is identified with $\{(1,\theta,\varphi)|0\le\theta\le\pi/2-\epsilon \text{ or } \pi/2+\epsilon\le\theta\le\pi,\quad 0\le\varphi\le 2\pi\}$ where $0<\epsilon<\pi/2$. Let ϵ' be the sufficiently small number, a *ball-twisting of first kind* (resp. *ball-twisting of second kind*) on H_g (resp. H_g^*) along $\mathcal{B}$ is given as a map

$$(r,\theta,\varphi)\mapsto\begin{cases}(r,\theta,\varphi+2\pi(1-r)/\epsilon') & 1-\epsilon'\le r\le 1\\ (r,\theta,\varphi) & 0\le r\le 1-\epsilon'\end{cases}$$

Remark 4. By Luft [L] and McCullough [McC], the same kind of results for mapping class groups of handlebodies are shown. We show Theorem B(1) (resp. (2)), by applying the same kind of technique of Luft (resp. McCullough).

We have a homomorphism from $\mathcal{E}_g$ to $\mathrm{Aut}(H_1(H_g;\mathbb{Z}))$, induced by the action of the elements of $\mathcal{E}_g$ on $H_1(H_g;\mathbb{Z})$. Let $\mathcal{EI}_g$ be the kernel of this homomorphism. We give the following theorem about $\mathcal{EI}_g$:

Theorem C. *(1) $\mathcal{EI}_g$ is generated by ball-twistings on H_g and ball-twistings on H_g^*.*
(2) $\mathcal{EI}_g = \mathcal{E}_g \cap$ the Torelli group of genus g Riemann surface.

References

[L] E. Luft, *Actions of the homeotopy group of an orientable 3-dimensional handlebody*, Math. Ann. **234** (1978), 279–292.

[MKS] W. Magnus, A. Karras, D. Solitar, *Combinatorial group theory*, Interscience, New York, 1966.

[McC] D. McCullough, *Twist groups of compact 3-manifolds*, Topology **24** (1985), 461–474.

[P] J. Powell, *Homeomorphisms of S^3 leaving a Heegaard surface invariant*, Trans. Amer. Math. Soc. **257** (1980), 193–216.

[S] S. Suzuki, *On homeomorphisms of a 3-dimensional handlebody*, Canad. J. Math. **29** (1977), 111–124.

[W] F. Waldhausen, *Heegaard-Zerlegungen der 3-Sphäre*, Topology **7** (1968), 195–203.

Department of Mathematics Faculty of Science and Engineering Saga University Saga, 840 Japan
E-mail: hirose@ms.saga-u.ac.jp

A weight system derived from the Conway potential function

Hitoshi Murakami

Department of Mathematics
School of Science and Engineering
Waseda University

A weight system is defined from the Conway potential function. We also show that it can be calculated recursively by using five axioms.

Planar surfaces in a handlebody and a theorem of Gordon-Reid

by

Kanji Morimoto

Department of Mathematics, Takushoku University
Tatemachi, Hachioji, Tokyo 193, Japan
e-mail : morimoto@hc.takushoku-u.ac.jp

In this talk, we show some results on incompressible planar surfaces in a handlebody, and show a sufficient condition for an orientable closed 3-manifold to have a lens space summand. As an application, we reprove a theorem of Gordon-Reid ([GR]).

Proposition 1.1. *Let V be a genus $g > 1$ handlebody, and P an incompressible planar surface with $\ell > 1$ boundary components properly embedded in V. If ∂P consists of mutually parallel separating loops in ∂V, then $\ell = 2$ and P is a ∂-parallel annulus.*

Proposition 1.2. *Let V be a genus two handlebody, and P an incompressible planar surface with $\ell > 1$ boundary components properly embedded in V. If ∂P consists of mutually parallel non-separating loops in ∂V, then $\ell = 2$ and P is an annulus.*

Let P be a compact 2-manifold properly embedded in a 3-manifold. Then we say that P is essential if P is incompressible and has a component which is not ∂-parallel.

Proposition 1.3. *Let M be an orientable closed 3-manifold with a genus $g > 1$ Heegaard splitting (V_1, V_2). If M contains a 2-sphere S such that $S \cap V_1$ has an annulus component A which is separating and essential in V_1 and cuts off a solid torus W from V_1 with $W \cap S = A$, then M has a lens space summand.*

Remark 1.4 In the hypothesis of Proposition 1.3, if $g = 2$ then we can get rid of the condition that "which cuts off a solid torus from V_1". Because a separating essential annulus properly embedded in a genus two handlebody cuts off a solid torus ([Ko, Lemma 3.2]).

Let B be a 3-ball and t a family of n arcs properly embedded in B and finitely many simple closed curves in B. Then we say that the pair (B, t) is an n-string tangle, and that (B, t) is essential if $cl(\partial B - N(t))$ is incompressible and boundary incompressible in $cl(B - N(t))$. Let L be a knot or a link in the 3-sphere S^3. Then according to [GR], we say that L is n-string composite if (S^3, L) can be decomposed into two essential n-string tangles, and that L is n-string prime if (S^3, L) is not n-string composite. Then Gordon and Reid have proved in [GR] :

Theorem 1.5 ([GR, Corollary 1.2]). *Every tunnel number one knot in S^3 is n-string prime for all $n > 0$.*

Remark 1.6 If $n = 1$ this was shown by Norwood ([No]), and if $n = 2$ this was shown by Scharlemann ([Sc]).

The key proposition to prove the above theorem is :

Proposition 1.7 ([GR, Proposition 2.1]). *Let M be an orientable closed 3-manifold with a genus two Heegaard splitting (V_1, V_2). If M contains a 2-sphere S such that each component of $S \cap V_1$ is a non-separating disk in V_1 and $S \cap V_2$ is an essential planar surface in V_2, then M has a lens space or $S^2 \times S^1$ summand.*

Remark 1.8 In [GR], by some technical reason the proposition has been proved under the hypothesis that $S \cap V_2$ is a "homotopically essential" planar surface instead of "essential". But there is no essential difference.

In [GR], Gordon and Reid have proved the proposition by using Scharlemann cycle argument ([CGLS, Sc]) and all types argument ([GL, Pa]). As an application of Propositions 1.1, 1.2 and 1.3, we reprove it not using those arguments, although the proof does not become more simple or easier.

References

[CGLS] M. Culler, C. McA. Gordon, J. Lueke and P. B. Shalen, *Dehn surgery on knots*, Ann. of Math. **115**, (1987) 237-300

[GL] C. McA. Gordon and J. Lueke, *Knots are determined by their complements*, J. A.M.S. **2**, (1989) 371-415

[GR] C. McA. Gordon and A. Reid, *Tangle decompositions of tunnel number one knots and links*, J. Knot Rami. 4 (1995) 389-409

[Ko] T. Kobayashi, *Structures of the Haken manifolds with Heegaard splittings of genus two*, Osaka J. Math. **21**, (1984) 437-455

[Mo] K. Morimoto, *Planar surfaces in a handlebody and a theorem of Gordon-Reid*, in preparation

[No] F. H. Norwood, *Every two generator knot is prime*, Proc. A. M. S. **86**, (1982) 143-147

[Pa] W. Parry, *All types implies torsion*, Proc. A.M.S. **110**, (1990) 871-875

[Sc] M. Scharlemann, *Tunnel number one knots satisfy the Poenaru conjecture*, Topology Appl. **18**, (1984) 235-258

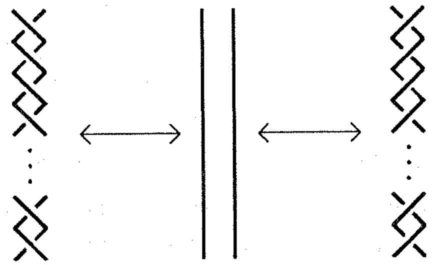
Alexander Invariant and Twisting Operation

Yasutaka Nakanishi

Department of Mathematics, Kobe University

Abstract

We consider an operation τ^n, which is a local move canceling n full-twists on a link diagram as in the figure below. We study a necessary condition for two links to be deformable to each other by a finite sequence of operations τ^n's in term of Alexander invariant. As a corollary, the Borromean rings and a 3-component trivial link are never deformed to each other by a finite sequence of operations τ^2's and link-homotopies.



Outline. For a μ-component link $L = K_1 \cup \cdots \cup K_\mu$, let $E = E(L) = S^3 - L$ and take the universal abelian covering $p : \widetilde{E_a} \to E$, associated with the epimorphism $\pi_1(E) \to \langle t_1, \ldots, t_\mu \rangle$ sending each meridian of K_i to t_i $(i = 1, \ldots, \mu)$, where $\langle t_1, \ldots, t_\mu \rangle$ is the free abelian group with a basis $t_1, \ldots, t_\mu$. The first integral homology group $H_1(\widetilde{E_a}; \mathbf{Z})$ is a finitely generated $\mathbf{Z}\langle t_1, \ldots, t_\mu \rangle$-module, which is called the Alexander invariants, has a presentation matrices as a $\mathbf{Z}\langle t_1, \ldots, t_\mu \rangle$-module, written $P_L(t_1, \ldots, t_\mu)$.

Theorem 1. *Let two links L_1 and L_2 be deformable to each other by a finite sequence of operations τ^n's. Then, for properly chosen $P_{L_1}(t_1, \ldots, t_\mu)$ and $P_{L_2}(t_1, \ldots, t_\mu)$, we have $P_{L_1}(t_1, \ldots, t_\mu) \equiv P_{L_2}(t_1, \ldots, t_\mu) \bmod \{\sigma_n(t_p t_q), \sigma_n(t_p t_q^{-1}),\ (1 \leq p, q \leq \mu)\}$.*

In the above, $\sigma_n(t)$ means $(1 - t^n)/(1 - t) = 1 + t + t^2 + \cdots + t^{n-1}$, and $(f_{ij}) \equiv (g_{ij}) \text{mod}\{h_1, \ldots, h_\nu\}$ means that each pair of corresponding entries f_{ij} and g_{ij} are in the same class of the quotient $\mathbf{Z}\langle t_1, \ldots, t_\mu \rangle / (h_1, \ldots, h_\nu)$, where $(h_1, \ldots, h_\nu)$ is the ideal generated by $h_1, \ldots h_\nu$ in $\mathbf{Z}\langle t_1, \ldots, t_\mu \rangle$. Theorem 1 is reduced to the following form by rewriting t_i's to the same t.

Claim 1. *Let two links L_1 and L_2 be deformable to each other by a finite sequence of oerations τ^2's. Then, for properly chosen $P_{L_1}(t_1, \ldots, t_\mu)$, and $P_{L_2}(t_1, \ldots, t_\mu)$, we have $P_{L_1}(t, \ldots, t) \equiv P_{L_2}(t, \ldots, t) \bmod \{2, 1 + t^2\}$.*

On the other hand, by a surgical view of Alexander invariant, we have the following:

Theorem 2. *If a 3-component link L is link homotopic to a trivial 3-component link, then, for properly chosen $P_L(t_1, t_2, t_3)$, P_L is characterized by the following type of matrices:*

$$\begin{pmatrix} 1-t_1 & 1-t_2 & 1-t_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & g_{11} & \cdots & g_{1\nu} \\ 0 & 0 & 0 & g_{21} & \cdots & g_{2\nu} \\ f_{11} & f_{21} & f_{31} & m_{11} & \cdots & m_{1\nu} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{1\nu} & f_{2\nu} & f_{3\nu} & m_{\nu 1} & \cdots & m_{\nu\nu} \end{pmatrix}$$

where each entry is a Laurent polynomial with variables t_1, t_2, t_3 such that the following conditions are satisfied:

(1) $m_{ij}(t_1,t_2,t_3) = m_{ji}(t_1^{-1}, t_2^{-1}, t_3^{-1})$,

(2) $|m_{ij}(1,1,1)| = \delta_{ij} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$,

(3) $g_{1k} = f_{3k}(t_1^{-1}, t_2^{-1}, t_3^{-1})(1-t_1^{-1})(1-t_2^{-1}) - f_{2k}(t_1^{-1}, t_2^{-1}, t_3^{-1})(1-t_3^{-1})(1-t_1^{-1})$, *and* $g_{2k} = f_{1k}(t_1^{-1}, t_2^{-1}, t_3^{-1})(1-t_2^{-1})(1-t_3^{-1}) - f_{3k}(t_1^{-1}, t_2^{-1}, t_3^{-1})(1-t_1^{-1})(1-t_2^{-1})$.

We consider the reduced version on $P_L(t,t,t)$ as follows.

Claim 2. *If a* 3*-component link L is link homotopic to a trivial* 3*-component link, then, for properly chosen $P_L(t_1,t_2,t_3)$, $P_L(t,t,t)$ is equivalent* mod $\{2, 1+t^2\}$ *to* $\begin{pmatrix} 1-t & 0 & 0 \end{pmatrix}$.

Let two links L_1 and L_2 be deformable to each other by a finite sequence of operations τ^2's and link-homotopies. As an operation τ^2 and a link homotopy can be interchanged, there exists a link L^* such that L_1 and L^* are deformed to each other by a finite sequence of operations τ^2's and L^* and L_2 are link-homotopic.

Here, let L_1 be the Borromean rings, and L_2 a trivial 3-component link. $P_{L_1}(t,t,t)$ is equivalent mod $\{2, 1+t^2\}$ to $\begin{pmatrix} 1-t & 0 & 0 \\ 0 & 1-t & 0 \\ 0 & 0 & 1-t \end{pmatrix}$. And so is $P_{L^*}(t,t,t)$, by Claim 1. On the other hand, L^* is link-homotopic to a trivial 3-component link L_2, and $P_{L^*}(t,t,t)$ is equivalent mod $\{2, 1+t^2\}$ to $\begin{pmatrix} 1-t & 0 & 0 \end{pmatrix}$ by Claim 2. Since $1-t \not\equiv 0$ mod $\{2, 1+t^2\}$, both matrices are not equivalent mod $\{2, 1+t^2\}$, a contradiction. Hence, the Borromean rings and a trivial 3-component link are never deformable to each other by a finite sequence of operations τ^2's and link-homotopies.

Remarks. In 1986, the author made a preprint [1], which consists of two long parts and two short parts. There were gaps in the two short parts. So, the preprint went to pieces. The two long parts were published as [2] and [3]. The gaps in the first short part is filled up and this abstract is on such a topic. However, the gaps in the second short part is not removed until now.

References

[1] Y. Naknaishi, *Fox's congruence classes and Conway's potential functions of knots and links*, preprint, 1986.

[2] Y. Naknaishi, *Three-variable Conway potential function of links*, Tokyo J. Math. **13** (1990), 163–177.

[3] Y. Naknaishi, *On Fox's congruence classes of knots, II*, Osaka. J. Math. **27** (1991), 207–215.

Ramsey Theorem for Good Spatial Graphs

Seiya Negami
Department of Mathematics, Faculty of Education, Yokohama National University
156 Tokiwadai, Hodogaya-Ku, Yokohama 240, JAPAN.
E-mail address: negami@ms.ed.ynu.ac.jp

A *spatial embedding* of a graph G is an embedding $f : G \to \mathbf{R}^3$ of G into the 3-space $\mathbf{R}^3$ and its image $f(G)$ is called a *spatial graph*. We denote by K_n the *complete graph* over n vertices, that is, one each pair of whose vertices are joined by an edge. The following theorem, proved by Conway and Gordon in [1], is the starting point of our subject in this talk.

THEOREM 1. (Conway-Gordon [1])

(i) *Every spatial embedding of K_6 contains a nonsplittable 2-component link.*

(ii) *Every spatial embedding of K_7 contains a nontrivial knot.*

This theorem presents an unavoidable phenomenon in sufficiently large complete graphs K_n. Such a theorem is called a "Ramsey theorem" in combinatorics or discrete mathematics. (See [2] for general observations on Ramsey theory.)

A natual question arises: Does K_n with sufficient large n contain a prescribed knot type or link type? The answer to this question is negative in general. For example, if one makes a local knot on each edge of a spatial embedding of K_n, then any cycle in K_n will involve a number of such local knots, which restricts the knot types and link types.

To give a positive answer, Negami [4] has proved the following theorem, restricting spatial embeddings of K_n to be *rectilinear*, that is, to be spatial graphs each of whose edge is a straight line segment in $\mathbf{R}^3$.

THEOREM 2. (Negami [4]) *Given a spatial graph H, there exists a positive integer n such that any rectilinear spatial embedding of K_n contains a subgraph which is ambient isotopic to H.*

Moreover, Miyauchi [3] has shown that the same theorem holds for complete bipartite graphs $K_{n,m}$ and pointed out that her theorem implies the above as its corollary since K_{n+m} contains $K_{n,m}$. In this talk, we shall give a generalized form of their theorems, relaxing the rectilinearity of embeddings.

The projection of a spatial graph $f(G)$ is its image $p(f(G))$ via the canonical projection $p : (x, y, z) \to (x, y, 0)$ and is said to be *rectilinear* if each edge is a straight line segment on the xy-plane. The rectilinearity of projections also excludes local knots on edges. Recently, Negami [5] has given a very simple proof of a theorem for spatial embeddings of $K_{n,m}$ with rectilinear projections, in the same style as above.

We define another property on the projections to exclude local knots, as follows. A *good drawing* of a graph G on the plane is a drawing of G such that:

(i) The points presenting vertices are all distinct.

(ii) Each edge is a simple arc.

(iii) Any adjacent pair of edges intersect only in their ends.

(iv) Any nonadjacent pair of edges cross each other in at most one point.

This is a familiar notion in topological graph theory, related to the "crossing numbers" of graphs on the plane. For example, a rectilinear projection satisfies these conditions. A spatial graph G is said to be *good* if its projection is a good drawing after carrying out an ambient isotopic deformation of G.

THEOREM 3. *Given a spatial graph H, there exists a pair of positive integers (n, m) such that any good spatial embedding of $K_{n,m}$ contains a subgraph which is ambient isotopic to H.*

A proof of this theorem has been already shown in [6] with arguments on a Ramsey theorem for good drawings in the plane. In this talk, we shall show some good spatial graphs which admit no rectilinear projections. Thus, the last theorem generalizes the others strictly.

References

[1] J.H. Conway and C.McA. Gordon, Knots and links in spatial graphs, *J. Graph Theory* **7** (1983), 445-453.

[2] R.L. Graham, B.L. Rothschild and J.H. Spencer, *"Ramsey Theory"*, Wiley, 1980.

[3] M.S. Miyauchi, Topological Ramsey theorem for complete bipartite graphs, *J. Combin. Theory, Ser. B* **62** (1994), 164–179.

[4] S. Negami, Ramsey theorems for knots, links and spatial graphs, *Trans. Amer. Math. Soc.* **324** (1991), 527–541.

[5] S. Negami, Ramsey-type theorems for spatial graphs, preprint.

[6] S. Negami, Ramsey-type theorems for spatial graphs and good drawings, preprint.

A CONDITION FOR A 3-MANIFOLD TO BE A KNOT EXTERIOR

Tsuyoshi Sakai

Institute of mathematics and computer science, Tsuda College, Tokyo 187, Japan

We consider the following problem:

Problem. Research conditions for a given 3-manifold to be a knot exterior.

To state our result, we need some preliminaries.

Definition. A pseudo knot diagram $\mathcal{L}$ is an oriented knot projection with indication of positive or negative crossing for some of the double points(see Fig.1).

Note that a usual knot diagram is a pseudo knot diagram whose double points are all crossing points(i.e. double points with indication as above).

For a finite set Ω, $W(\Omega)$ denotes the word semi-group with $\Omega \cup \Omega^{-1}$ as alphabets.

Definition. We associate a presentation $SP(\mathcal{L})$ with a given pseudo knot diagram as follows. We draw a sufficiently small squares at each crossing point of $\mathcal{L}$, and on the intersections of $\mathcal{L}$ and the squares we put distinct labels and also put arrows according to the orientation of $\mathcal{L}$ as in Fig.2. Let Ω denote the set of all labels. We associate two elements of $W(\Omega)$ to each square as in Fig.3. We also associate an element of $W(\Omega)$ to each subarc of $\mathcal{L}$ connecting two squares as in Fig.4. Let R denote the set of all words associated as above. Then $SP(\mathcal{L})$ is defined to be the pair $< \Omega : R >$.

For two presentations P and P^*, we denote $P \equiv P^*$ if P^* is obtained from P by renaming the generators.

Let $\mathcal{H}$ be a normal handle decomposition of a 3-manifold M consisting of 0-handle E, 1-handles $\{X_1, \cdots, X_n\}$ and 2-handles $\{H_1, \cdots, H_m\}$. Let $x_i (i = 1, \cdots, n)$ denote the oriented belt circles of X_i respectively and let Ω denote the set $\{x_1, \cdots, x_n\}$. Let $h_i (i = 1, \cdots, m)$ denote the oriented attaching circle of H_i with a marked point. We now proceed on h_i, according to its orientation, starting from the marked point. Then, by reading the intersections of h_i and

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

$x(\in \Omega)$ weighted as in Fig.5, we obtain an element in $W(\Omega)$. We denote it by $[h_i]$. Then

$$SP(M, \mathcal{H}) = < \Omega | [h_1], \cdots, [h_m] >$$

is a presentation of $\pi_1(M)$.

Theorem. *Let M be a 3-manifold with $H_1(M) \cong \mathbf{Z}$ and $\partial M \cong T^2 \cup S^2$ and let $\mathcal{H}$ be a handle decomposition of M as above. Suppose that there exists a pseudo knot diagram $\mathcal{L}$ such that $SP(\mathcal{L}) \equiv SP(M, \mathcal{H})$. Then M is homeomorphic to a punctured knot exterior. That is, there exists a knot k in S^3 such that M is homeomorphic to $S^3 - \{intN(k) \cup (\text{an open 3-cell})\}$.*

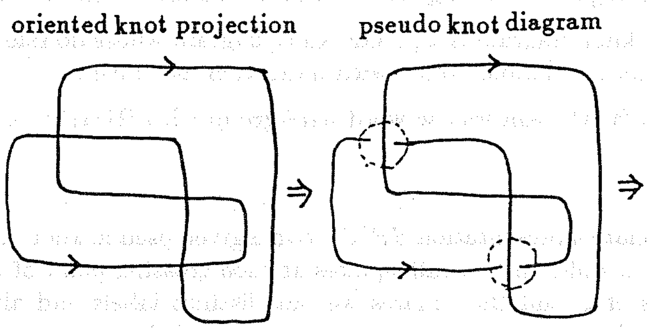


Fig.1

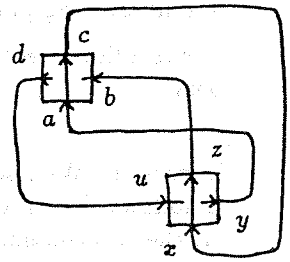


Fig.2

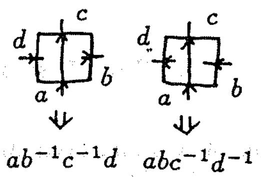


Fig.3

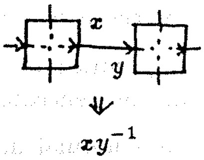


Fig.4

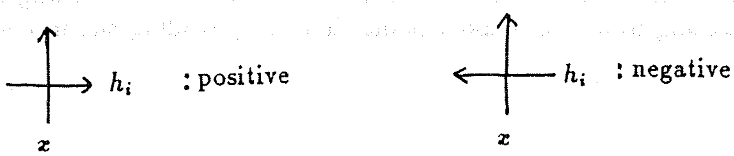


Fig. 5

DECOMPOSITION OF S^4 AS A TWISTED DOUBLE OF A CERTAIN MANIFOLD

YUICHI YAMADA

We will work in the PL category.

Definition. (see [L]) Let N be a compact oriented 4-manifold with a boundary. We say that S^4 decomposes as a twisted double of N if $S^4 = N \cup_\partial -N$.

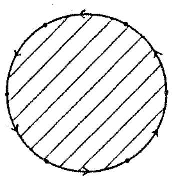
We use the word "twisted double" because we allow that the gluing map between the boundaries is not $id|_{\partial N}$. This conception is a kind of an extension of Heegaard splitting of S^3.

This story is an example of an extension of the spacial graph theory in a sence, for we treat a 2-complex in S^4.

Motivation and History. (see[L] and its references) Let N_2 be a tubular neighborhood of a (+)-standard $\mathbf{R}P^2$ in S^4. It is well known that the closure of $S^4 \backslash N_2$ is also homeomorphic to N_2 by an orientation reversing homeomorphism, i.e., (1) $S^4 = N_2 \cup_\partial -N_2$. Thus S^4 decomposes as a twisted double of N_2. N_2 can be characterized as a total space of a 2-disk bundle over $\mathbf{R}P^2$ whose normal Euler number is 2. (2) The boundary of N_2, which we call Q_2, is a Seifert rational homology 3 sphere. (3) It is known that the 2 covering of S^4 branched along a $(-)$-standard $\mathbf{R}P^2$ is $\mathbf{C}P^2$.

Here we extend these facts to the case of a 2-complex X_n $(n \geq 2)$ defined below instead of $\mathbf{R}P^2$. We note that $X_2 = \mathbf{R}P^2$.

$$X_n = D^2 \ / \ e^{2\pi\sqrt{-1}\theta} \sim e^{2\pi\sqrt{-1}(\theta + \frac{1}{n})},$$
where $D^2 = \{ \, |z| \leq 1 \mid z \in \mathbf{C} \}$.



$(n = 6)$

We define a standard realization of the complex X_n in S^4 and let N_n be a regular neighborhood of the realization. N_n is a compact oriented 4-manifold with a boundary.

Theorem. *For any n, S^4 decomposes as a twisted double of N_n.*

The boundary of our manifold N_n, which we call Q_n, is a Seifert rational homology sphere. The author think Q_n as a typical example among prime 3-manifolds which can be embedded in S^4.

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

Proposition. *Q_n admits a Seifert structure ([O]) whose invariants are* $\{-1;\ (o_1,0)\ ;\ (n,1)\ (n,1)\ (n,n-1)\}$, *and* $\pi_1(Q_n)\ \cong <\alpha,\beta \mid \alpha^n=\beta^n=(\alpha\beta)^n>$.

When $n=2$, this is the quaternion group, and when $n>2$, this is an infinite group.

We also study a (kind of) covering of S^4 branched along $-X_n$ using framed links. A known complex manifold (a neighborhood of a singular fiber of a Fermat-type Surface) appears.

REFERENCES

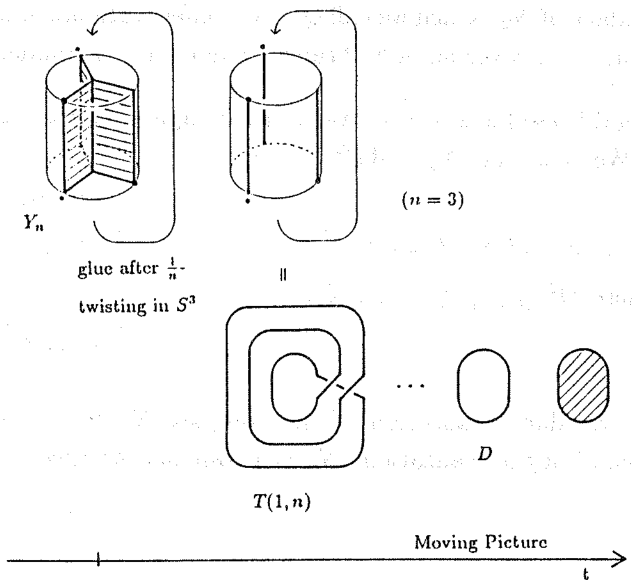
[L] T.Lawson, *Splitting S(4) on* **R***P(2) via the branched cover of* **C***P(2) over S(4)*, Proc. Amer. Math. Soc. **86** (1982), no.2, 328–330.

[O] P.Orlick, "Seifert Manifolds," Lecture Notes in Math. 291, Springer, 1972.

[Y1] Y.Yamada, *DECOMPOSITION OF S^4 AS A TWISTED DOUBLE OF A CERTAIN MANIFOLD*, to apper.

[Y2] ———, *Some Seifert 3-manifolds which decompose S^4 as a twisted double*, preprint.

A stsndard realization of X_n in S^4



Graduate School of Mathematical Sciences
University of Tokyo
3-8-1 Komaba Meguro-ku
Tokyo, 153, Japan.

TBA

Tim D. Cochran
Rice University, U.S.A.

Turaev-Viro Modules of Satellite knots

Patrick Gilmer

Let (V, Z) be a Topological Quantum Field Theory over a field f defined on a cobordism category whose morphisms are oriented 3-manifolds perhaps with extra structure. Let (M, χ) be a closed oriented 3-manifold M with this extra structure together with $\chi \in H^1(M)$. Let M_∞ denote the infinite cyclic cover of M given by χ. Consider a fundamental domain E for the action of the integers on M_∞ bounded by lifts of a surface Σ dual to χ, and in general position. E can be viewed as a cobordism from Σ to itself. $Z(E)$ can be viewed as an endomorphism of $V(\Sigma)$. Let V_0 be the generalized 0-eigenspace for the action of $Z(E)$ on $V(\Sigma)$. $Z(E)$ induces an automorphism $Z_*(E)$ of $V(\Sigma)/V_0$. The Turaev-Viro module (M, χ) associated to (V, Z) is simply is $V(\Sigma)/V_O$ viewed as a $f[t, t^{-1}]$-module where t acts by $Z_*(E)$. Turaev-Viro showed that this module does not depend on the choice of E. See my e-print "Invariants for 1-dimensional cohomology classes arising from TQFT" available as q-alg/9501004.

Given an oriented knot K in S^3, let $M(K)$ denote zero framed surgery to S^3 along K. Let χ denote the cohomology class which evaluates to one on a positive meridian of K. We are interested in the Tureav-Viro modules of $(M(K), \chi)$. We will take (V, Z) to be the Witten-Reshetikhin-Turaev TQFT associated to $SU(2)$ and level r. Actually we work within the framework developed by Blanchet, Habegger, Masbaum and Vogel with $p = 2r$ (Topology Vol 34,1995, 883-927.) Let $M(K, c)$ denote $M(K)$ with a meridian colored by an integer c, $0 \le c \le r-2$. Let $TV_r(K, c)$ denote the Turaev-Viro module of $(M(K, c), \chi)$. In q-alg/9501004, we showed how the Witten-Reshetikhin-Turaev invariants of the branched cyclic covers of K can be computed from $TV_r(K, c)$.

Suppose now that S is a satellite knot with companion C , orbit K and axis A. Here we use the terminology of Litherland, "Cobordism of Satellite

Knots", Contemp. Math. 35,1984,327-362. There is a well known decomposition of the infinite cyclic cover of the complement of S into pieces coming from the infinite cyclic cover of C and the infinite cyclic cover of K. See Livingston and Melvin's "Abelian invariants of satellite knots", Springer Lecture Notes 1167,217-227, where the decomposition is attributed "in essence" to Seifert.

We use this decomposition to calculate $TV_r(S, c)$ for some satellite knots S from $TV_r(C, i)$ for $0 \leq i \leq r-2$ and similar information coming from the pair (A, K). This will generalize a formula given for Turaev-Viro modules of a connected sum of two knots given in q-alg/9501004.

Vassiliev knot invariants and Chern-Simons perturbation theory to all orders

Daniel Altschuler
Institut für Theoretische Physik
ETH-Hönggerberg
CH-8093 Zürich, Switzerland

In this talk I report on a joint work with L. Freidel [1]. Chern-Simons theory is the most popular example of topological field theory in 3 dimensions. Given a compact Lie group G, a compact, oriented 3-manifold M, a link $L \subset M$, and for each component of L a representation of G, this theory associates topological invariants to these data. There are several ways to define the invariants, which are all closely related. First of all there are the non-perturbative definitions: Witten [2] used fundamental properties of quantum field theory, in particular the path integral formulation, and Reshetikhin and Turaev [3] used quantum groups. These two definitions are equivalent.

Then there are the perturbative definitions, the first of which were given by Guadagnini et al. [4] in the case $M = S^3$, $L \neq \emptyset$, using propagators and Feynman diagrams. This approach was then elaborated by Bar-Natan [5]. The case $M \neq S^3$, $L = \emptyset$ was treated by Axelrod and Singer [6]. A common feature of all these works is the Feynman diagram expansion familiar in perturbative quantum field theory. Invariants are defined at every order in the expansion, each is a sum of several terms corresponding to the diagrams of the given order. The contribution of any diagram is the product of two factors, the first depends only on the group G and the representations associated to the components of L, and the second is independent of G and its representations, it is an integral over the configuration space of the vertices of the diagram, some of which are constrained to lie on L, while the others can lie anywhere in the complement of L. When L is a knot in S^3, several properties of the invariant arising from the contributions of order two were already discussed in [4], although the invariance itself was shown in [5]. Bar-Natan also studied the properties of the group-dependent contributions, and among them he found relations between the contributions of different diagrams which are the same for all groups G. This led him [8] to define abstract objects, which we call BN diagrams, by these relations, and abstract invariants which take their values in the space of BN diagrams. To every choice of group G and representations corresponds a linear functional on the space of BN diagrams. Applying this functional to the abstract invariants gives back the ordinary group-dependent invariants.

In order to show that the contributions of a given order sum up to an invariant, one must compute the variation of these integrals under a small change of the embedding of L, and this proved to be quite difficult and lengthy. However, Bott and Taubes [7] greatly improved this situation. They showed that the variation can be split in two terms, the "diagrammatic" and the "anomalous" variations. As its name indicates, the diagrammatic variation can be read at once from the Feynman diagram. It corresponds to the differential of Kontsevich's graph complex, obtained by collapsing the edges. The anomalous variation is more difficult to compute, but it is proportional to the variation of the first order contribution, the "self-linking number". The constant of proportionality, is still unknown in general, but independent of the embedding. These results of Bott and Taubes are powerful enough, as we will show, to prove invariance at all orders.

During the same period, the subject of Vassiliev knot invariants, also known as finite type invariants, was developing rapidly. The starting point of Vassiliev [9] was the space of all immersions of S^1 in S^3. In this space, a knot type is a cell whose faces are singular knots with a finite number of transverse double points. Any knot invariant can be extended to such singular knots. It is said to be a finite type invariant of order $\leq N$, if it vanishes on all singular knots with more than N double points. Let V^N be the space of invariants of order $\leq N$. Unexpectedly at first, Bar-Natan found that V^N/V^{N-1} embeds in the dual of the space of BN diagrams of degree N. Kontsevich [10] showed that the two spaces are in fact isomorphic. His proof [8] involved the construction of a universal Vassiliev invariant, a formal power series in the space of BN diagrams whose coefficients are finite type invariants, based on the Knizhnik-Zamolodchikov equations of the WZW model of conformal field theory.

In this talk we start from the results of Bott and Taubes [7] to construct a universal Vassiliev knot invariant, given by the perturbative expansion of the expectation value of a Wilson loop in Chern-Simons theory on $\mathbb{R}^3$. The basic ingredient in the integrals obtained from the Feynman rules is the propagator of the gauge field, which is given in the Lorentz gauge by the Gauss two-form, the pullback of the volume form on S^2.

In more details, the contents of the talk are as follows: we first define the graphs appearing in the perturbative expansion, which are equipped with an additional structure called vertex orientation, and state some simple combinatorial lemmas. Then we give the Feynman rules, in which the vertex orientation plays an important role. They allow us to define unambiguously the signs of the contributions of graphs appearing in the perturbative expansion. Next we define the expectation value of a Wilson loop Z, which is a sum over trivalent graphs, and prove that it is invariant under the changes of embedding corresponding to the collapse of a single edge. After that we consider the other variations of the embedding, called "anomalous". We improve some results of [7], which allow us to conclude that a suitably corrected version of Z becomes a framed knot invariant $\hat{Z}$. In the last part we prove that $\hat{Z}$ is a universal invariant. In particular, the N-th order contribution to $\hat{Z}$ is a finite type invariant of order $\leq N$. Although it is stated explicitly in the literature, we have never seen a proof of this going beyond the second order. The question whether the KZ and the Chern-Simons universal invariants are equal is still open. The answer would be positive if one could show that the Chern-Simons invariant extends functorially to the category of tangles, as in the case of the KZ invariant, but at least to us it is not obvious that it has this extension property.

References

[1] D. Altschuler and L. Freidel, *Vassiliev knot invariants and Chern-Simons perturbation theory to all orders*, preprint ETH-TH/95-35.

[2] E. Witten, Commun. Math. Phys. **121** (1989) 351.

[3] N. Reshetikhin and V.G. Turaev, Inv. Math. **103** (1991) 547.

[4] E. Guadagnini, M. Martellini and M. Mintchev, Nucl. Phys. **B330** (1990) 575.

[5] D. Bar-Natan, *Perturbative Aspects of the Chern-Simons Topological Quantum Field Theory*, Ph. D. thesis, Princeton University, June 1991.
D. Bar-Natan, *Perturbative Chern-Simons theory*, Princeton University preprint, 1990, to appear in Journal of Knot Theory and its Ramifications.

[6] S. Axelrod and I. Singer, *Chern-Simons perturbation theory*, int the proceedings of the XXth International conference on differential geometric methods in theoretical physics, June 3-7, 1991, New York City (World Scientific, Singapore, 1992).
S. Axelrod and I. Singer, *Chern-Simons perturbation theory II*, Jour. Diff. Geom. **39** (1994) 173.

[7] R. Bott and C. Taubes, *On the self-linking of knots*, Jour. Math. Phys. **35** (1994).

[8] D. Bar-Natan, *On the Vassiliev knot invariants*, Topology **34** (1995) 423.

[9] V. A. Vassiliev, *Cohomology of knot spaces*, in "Theory of singularities and its applications" (ed. V. I. Arnold), Advances in soviet mathematics, AMS, 1990.

[10] M. Kontsevich, *Vassiliev's knot invariants*, Adv. in Sov. Math., **16(2)** (1993), 137.

Representation spaces of knot groups

GERHARD BURDE

Abstract

The character varieties of representations of 2-bridge knots (and links) or rather a real section of these are considered from a geometrical point of view. Information on the geometry of the representing real algebraic curves can be derived by interpreting their points as isometries of the hyperbolic plane of certain types resp. rotations in $\mathbb{R}^3$. Further information is drawn from a first step to desingularize the ideal critical points. This is used to prove an inequality which gives an upper bound for the genus of the representation curve.

Equivariant Concordance of Knots in Dimension 3

Swatee Naik
Department of Mathematics
University of Nevada, Reno, NV 89557
U. S. A.

Extended abstract of a talk to be presented at
The Fifth MSJ International Research Institute on Knot Theory
Tokyo
July 22–26, 1996

Abstract

A knot K in S^3 is said to be periodic with a period $q > 1$ if there exists an an orientation preserving diffeomorphism $f : S^3 \to S^3$ such that $f(K) = K$, order$(f) = q$, and the fixed point set of f is a circle disjoint from K. Any such f is called a period q transformation for K. Two knots are said to be concordant if they cobound a smooth 1-manifold $C \cong S^1 \times I$ properly embedded in $S^3 \times I$. We say that two period q, concordant knots K_1 and K_2 are equivariant concordant if there are period q transformations f_i for K_i, $1 \leq i \leq 2$, and an order q diffeomorphism of $S^3 \times I$ which leaves the concordance C invariant and restricts to f_i on the boundary component containing K_i.

A natural question is whether all period q, concordant knots are equivariant concordant. The answer is no. We observe that if K_1 and K_2 are equivariant concordant, then the links $K_i \cup \mathrm{Fix}(f_i)$, $1 \leq i \leq 2$, are concordant. It follows that the linking numbers $\mathrm{lk}(K_1, \mathrm{Fix}(f_1))$ and $\mathrm{lk}(K_2, \mathrm{Fix}(f_2))$ are equal. These linking numbers are determined by q and the Alexander polynomials of the knots K_i. We use this condition and give examples of concordant knots which are periodic with the same period but not be equivariant concordant.

An equivariant slice knot is equivariant concordant to the trivial knot. We discuss classical obstructions to sliceness in the equivariant setting. In particular, we show that if K is equivariant slice, then the map induced by a periodic transformation on the homology of an equivariant Seifert surface preserves a metabolizer of the Seifert form.

Finally, a knot K is called q-equivariant ribbon if it is periodic with period q and it bounds an equivariant ribbon disk in S^3. We give examples of equivariant ribbon knots as well as of knots which are equivariant slice but it is unknown whether or not they are equivariant ribbon.

On inevitability of knots, links and spatial graphs

Tatsuya TSUKAMOTO (Waseda University)

Let G be a finite graph. We consider it as a topological space i.e. a 1-dimensional CW complex. A *spatial embedding* of G is an embedding $g : G \to \mathbf{R}^3$of G into the 3-dimensional Euclidian space $\mathbf{R}^3$, and its image $g(G)$ is called a *spatial graph*. If it consists of a single cycle or a disjoint union of cycles, then it is called a *knot* or a *link*. A graph is called *planar* if there is an embedding of it into $\mathbf{R}^2$. A spatial graph $g(G)$ is called *unknotted* if it is ambient isotopic to a graph in $\mathbf{R}^2 \subset \mathbf{R}^3$. Thus $g(G)$ is unknotted only if G is planar. Of course, if it is a knot or a link, then it is usually called *trivial*.

A *regular projection* of G is an image of a continuous map from G to $\mathbf{R}^2$ whose multiple points are only finitely many transversal double points of edges. Then, we obtain 2^c diagrams from a regular projection of G by giving information about the over crossings at all double points of it, where c is a number of double points of the projection. It is well known that if G is a single cycle (or a disjoint union of cycles), then there is a trivial diagram i.e. a diagram of a trivial knot (or link) in diagrams obtained from a regular projection of it. However, this is not always true for planar graphs. In fact, Taniyama showed that there is a regular projection of a planar graph such that every diagram obtained from it, is not a unknotted diagram ([1], see the figure); yet, we obtain the following proposition since every diagram obtained from the projection in Figure 1 contains a Hopf link as like Figure 2.

Proposition There is a regular projection of a planar graph such that every diagram obtained from it contains a subdiagram which is ambient isotopic to Hopf link.

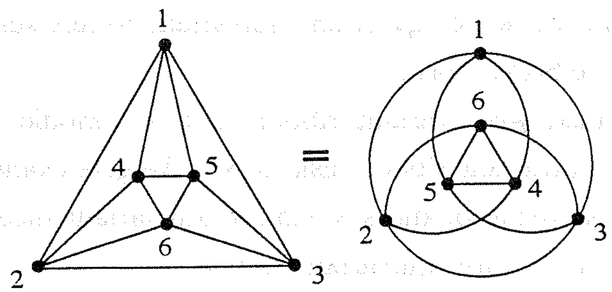


Figure 1

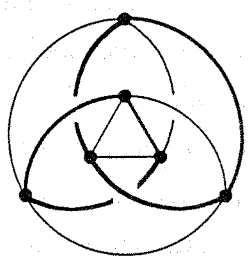


Figure 2

Starting from this proposition, Taniyama and the author, in [2], introduced the concept of inevitability of spatial graphs and obtained the following theorem.

Definition Let H be a spatial graph. A regular projection of a graph is called *H-inevitable* if every diagram obtained from it contains a subdiagram that is ambient isotopic to a diagram of H.

Theorem 1 (Taniyama-Tsukamoto [2]) Let T_n be a $(2,n)$-torus knot or link according as n is odd or even. Then, for each integer n, there is a T_n-inevitable projection of a planar graph.

Therefore a question naturally arises: For a prescribed knot K, is there a K-inevitable projection of a planar graph? For this question, we obtain the following results.

Theorem 2 Given a planar graph embedded in $\mathbf{R}^3$, denoted by H, there is an H-inevitable projection of a planar graph.

Corollary 1 Given a knot K, there is a K-inevitable projection of a planar graph.

Corollary 2 Given a link L, there is an L-inevitable projection of a planar graph.

References

[1] K. Taniyama, Knotted projections of planar graphs, *Proc. Amer. math. Soc.* , **123** (1995) 3575-3579.

[2] K. Taniyama and T. Tsukamoto, Knot-inevitable projections of planar graphs, *J. Knot Theory and its Ramifications*, to appear.

[3] T. Tsukamoto, On inevitability of knots, links and spatial graphs, preprint.

A quadratic lower bound for the number of primitive Vassiliev invariants

S. Duzhin*

Extended abstract

I will speak about a result obtained in the course of my joint work with S. Chmutov. Adding one more trick to the construction described below we know how to prove a quartic lower bound ($\sim cn^4$) for the same number — and we hope that after overcoming some technical difficulties we will be able to establish a bound $c_m n^m$ for any natural number m.

Dimensions of the filtered space of Vassiliev knot invariants $\mathcal{V}_n$ are presently known only up to order 9 [BN1]. The exact asymptotics is not known either, but there are some estimates from above and from below ([CD], [Ng], [CDL], [MM]). Note that the associated graded space $A = \oplus \mathcal{V}_n/\mathcal{V}_{n-1}$ (called the algebra of *weight systems*) is a commutative Hopf algebra generated by its graded subspace of *primitive elements* $\mathcal{P}$ as a free polynomial algebra ([BN2]). Thus it is sufficient to study the sequence of dimensions of $\mathcal{P}_n = \mathcal{P} \cap \mathcal{A}_n$. The best known lower bound for $\dim \mathcal{P}_n$ is linear in n ([MM]). The paper [K] mentions a better estimate, but there is a circular reference between [K] and [BN3] about this, and the proof is not given anywhere.

In this talk, I will sketch the proof of a lower bound for the dimension of $\mathcal{P}_n$ which is quadratic in n. The argument is based on the explicite construction of an ample linearly independent family of elements in $\mathcal{P}_n$.

We will use the characterization of the primitive space $\mathcal{P}_n$ given by D. Bar-Natan [BN2]: *P_n is the space spanned by all connected Chinese character diagrams.* To prove linear independence of the families of diagrams, we will use the weight function with values in the universal enveloping algebra of the Lie algebra gl_N due to M. Kontsevich [K]:

$$\kappa : A \to U(gl_N)$$

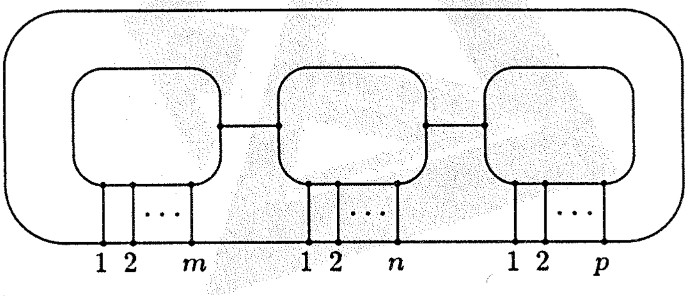
for a sufficiently large N. It is known that the image of κ belongs to the center of $U(gl_N)$ which, by Harish-Chandra theorem, coincides with the set of all polynomials in the variables x_1, ..., x_N, where x_i is an element of the center

*University of Aizu, Aizu-Wakamatsu, 965, JAPAN, on leave from Program Systems Institute, Pereslavl-Zalessky, 152140, RUSSIA, duzhin@botik.ru.

having degree i. Thus, for a given Chinese character diagram D, the value $\kappa(D)$ can be rewritten as a polynomial in x_1, ..., x_N. We have found explicite formulas for the highest homogeneous part of these polynomials, when D is a *polycycle diagram* (to understand what it means see the picture below that shows a tricycle diagram).

Now we will state the main result of this note.

Theorem. *Let $W_{m,n,p}$ be the tricycle diagram with 3 wheels of m, n and p spokes:*



Its order (half the number of triple points) is $m+n+p+2$. We claim that, for a given order d, the family of all $W_{m,n,p}$ where m and p are odd, n even, $m<p<n/2$ and $m+n+p=d-2$, is linearly independent in $\mathcal{P}_d$. Hence the dimension of the subspace of 3 wheel primitive Chinese character diagrams in $\mathcal{P}_d$ is asymptotically greater than or equal to $d^2/96$.

Outline of the proof. The assertion follows from the fact that, for a sufficiently large N, the gl_N-polynomials of these diagrams are linearly independent.

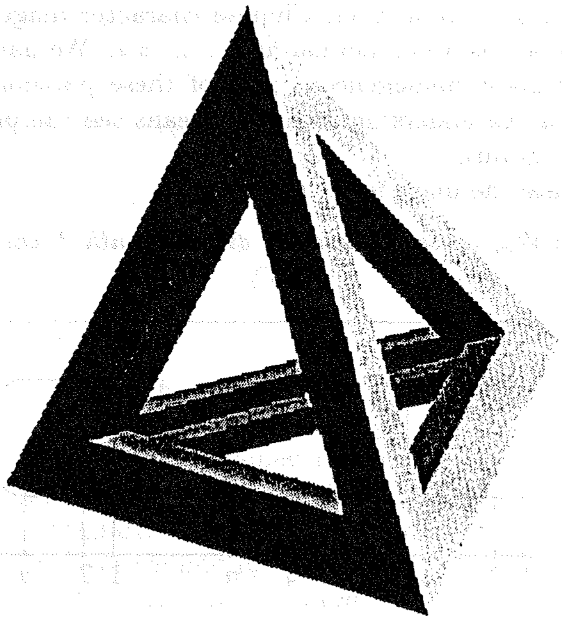
A direct computation yields the following explicite formula for the highest homogeneous part of these polynomials:

$$\tilde{\kappa}(W_{m,n,p}) = \sum(-1)^{i+j+k}\binom{m}{i}\binom{n}{j}\binom{p}{k}(x_i x_j x_k x_{i'+j'+k'} + x_i x_{i'+j'} x_{j+k} x_{k'}).$$

where $i' = m-i$, $j' = n-j$, $k' = p-k$ and the summation is over all i from 0 to m, all j from 0 to n and all k from 0 to p.

To understand why the family of such polynomials for the triples (m,n,p) described above is linearly independent, let us represent every nonzero term $x_\alpha x_\beta x_\gamma x_\delta$ as the point $(\alpha,\beta,\gamma,\delta) \in \mathbb{Z}^3 = \{\alpha+\beta+\gamma+\delta = const\} \in \mathbb{Z}^4$. The set of all such points forms the *support* of the polynomial (a notion different from the Newton polyhedron because we use the numbers of the variables, not their exponents, as coordinates). Since the variables x_i are commutative, we have to take all the permutations of the four indices and then consider the resulting set modulo the natural action of the symmetric group S_4 on $\mathbb{Z}^4$.

The following picture shows that the supports of $\kappa(W_{m,n,p})$ are independent, if m and p are small, while n is big:



References

[BN1] D. Bar-Natan, *Some computations related to Vassiliev knot invariants*, Preprint, September 1995.

[BN2] D. Bar-Natan, *On the Vassiliev knot invariants*, Topology **34**, (1995), 423–472.

[BN3] D. Bar-Natan, *On the Vassiliev knot invariants*, Pre-preprint, August 21, 1992.

[CD] S. Chmutov, S. Duzhin. *An upper bound for the number of Vassiliev knot invariants*, Jour. of Knot Theory and its Ramifications, **3(2)** (1994), 141–151.

[CD] S. Chmutov, S. Duzhin, S. Lando. Vassiliev knot invariants III. Forest algebra and weighted graphs. — Adv. in Soviet Math., v. 21 (1994).

[K] M. Kontsevich, *Vassiliev's knot invariants*, Adv. in Soviet Math., vol. 16, Part 2, pp. 137–150, 1993.

[MM] P. M. Melvin, H. R. Morton, *The coloured Jones function*, Comm. Math. Physics. **169** (1995) 501–520.

[Ng] K. Y. Ng, *Groups of ribbon knots*, q-alg/9502017 and Columbia University preprint, February 1995.

Twisting of two strings and Vassiliev invariants

Yoshiyuki OHYAMA

Department of Mathematics, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya, 466, Japan

In 1990, V. A. Vassiliev defined a sequence of knot invariants that is at least as powerful as all of the quantum group invariants of knots. The set of all Vassiliev invariants of order less than or equal to n forms a vector space, which is denoted by V_n. Only the case that the order is small, the dimensions and the bases are known.

Some classical invariants are not Vassiliev invariants. To show this fact and to study uniform limits of Vassiliev invariants, R. Trapp defined a twist sequence $\{K_i\}$, which is a sequence of knots that differ only by a full twist of two strings and showed that the space of all Vassiliev invariants on $\{K_i\}$ of order at most n, $V_n \mid_{\{K_i\}}$, has dimension at most $n+1$, that is also obtained by J. Dean independently.

Moreover Trapp obtained that when a twist sequence $\{K_i\}$ is a sequence of $(2, 2i+1)-$torus knots, the dimension of $V_n \mid_{\{K_i\}}$ is exactly n and determined the topological information that Vassiliev invariants give on $\{K_i\}$. There are two cases for twist sequences according to an orientation of two strings.

In this talk, we show the following theorems by using an $n-$similarity of knots.

Theorem A *For a twist sequence* $\{K_i\}$ *where two strings have parallel orientation, either*

(i) $dim(V_n\,|_{\{K_i\}}) = n$ *(for all* $n \geq 2$*)*

or

(ii) *there exists an integer* $k \geq 3$, *such that*

$$dim(V_n\,|_{\{K_i\}}) = \begin{cases} n & (for\ n \leq k) \\ n+1 & (for\ n > k) \end{cases}.$$

In the case (i) in Theorem A, the topological information that Vassiliev invariants give on $\{K_i\}$ is obtained from the invariants of order 2 and order 3. A sequence of $(2, 2i+1)-$torus knots is an example of the case (i) and we have an example for any integer $k \geq 3$ that the case (ii) occurs.

Theorem B *For a twist sequence* $\{K_i\}$ *whose two strings are oriented against each other, there exists an integer* $k \geq 1$ *such that*

$$dim(V_n\,|_{\{K_i\}}) \begin{cases} = 1 & (for\ n \leq k) \\ \leq n+1-k & (for\ n > k) \end{cases}.$$

In the case that $k = 1$ and equality holds in Theorem B, the k^{th} derivatives($k \leq n$) of the Jones polynomial of $\{K_i\}$ evaluated at 1 can be taken to be the bases. And we also indicate an example for any $k \geq 1$ that the equality holds in Theorem B.

Delta unknotting operation and vertex homotopy of graphs in R^3

Tomoe Motohashi and Kouki Taniyama

Throughout of this report we work in the piecewise linear category. Let G be a finite simple graph and R^3 three-dimensional Euclidean space.

Two embeddings $f, g : G \to R^3$ are *vertex homotopic* if f and g are transformed into each other by crossing changes between adjacent edges, and ambient isotopy.

Two embeddings $f, g : G \to R^3$ are *homologous* if there is a two-dimensional complex Λ and an embedding $\Phi : \Lambda \to R^3 \times [0,1]$ which satisfies the following three conditions.

(1) Λ is a connected sum of $G \times [0,1]$ and a finite number of closed connected orientable surfaces. More precisely, each surface is connected summed to some open disk int $e \times (0,1)$ where e is an edge of G.

(2) There is a real number $\varepsilon > 0$ such that if we consider $G \times ([0,\varepsilon] \cup [1-\varepsilon, 1])$ as a natural subspace of Λ, then $\Phi(x,t) = (f(x),t)$ for any $x \in G$, $0 \le t \le \varepsilon$ and $\Phi(x,t) = (g(x),t)$ for any $x \in G$, $1-\varepsilon \le t \le 1$.

(3) Φ is *locally flat.* That is, each point of the image of Φ has a neighbourhood N such that the pair $(N, N \cap \Phi(\Lambda))$ is homeomorphic to either the standard disk pair (D^4, D^2) or $(D^3 \times [0,1], X_n \times [0,1])$ of some non-negative integer n.

The second author proved:

Proposition 1[2] *If two embeddings $f, g : G \to R^3$ are vertex homotopic then they are homologous.*

Theorem 2[5] *Two embeddings $f, g : G \to R^3$ are homologous if and only if they have the same Wu invariant.*

Therefore the next end is the vertex homotopy classification of embeddings of G into R^3. In this report we give a vertex homotopy classification for *almost pseudo adjacent graphs.*

Two edges e_1, e_2 of G are *adjacent* if they have a common vertex. We define recursively that

(1) adjacent edges are *pseudo adjacent,*

(2) if $e_1, e_2, \cdots, e_n$ is the edges incident to a vertex of G and an edge e are pseudo adjacent to each of $e_1, e_2, \cdots, e_{n-2}$ and e_{n-1} then e and e_n are *pseudo adjacent.*

Namely pseudo adjacency is the maximal relation on the set of edges of G generated by (1) and (2).

A graph G is *pseudo adjacent* if any two edges of G are pseudo adjacent.

A graph G is *almost pseudo adjacent* if any three edges of G contain two pseudo adjacent ones.

Main Theorem *Let G be an almost pseudo adjacent graph. Then two embeddings of G into R^3 are vertex homotopic if and only if they are homologous.*

We give a proof of Main Theorem using Theorem 3.

Theorem 3 *Two embeddings $f, g : G \to R^3$ are homologous if and only if they are transformed into each other by delta unknotting operation and ambient isotopy.*

Let $C(G)$ be the set of all cycles of G. We say that $C(G)$ is *spatially independent* if for any set of embeddings $\{f_c : c \to R^3 | c \in C(G)\}$, there is an embedding $f : G \to R^3$ such that the restriction map $f|_c$ is ambient isotopic to f_c for each $c \in C(G)$. Then $C(\theta_n)$ and $C(K_4)$ are spatially independent by [1] and [7] respectively where θ_n is the graph on two vertices and n edges joining them. See also [8].

As an application of Main Theorem and an invariant defined in [4], we obtain next Theorem.

Theorem 4 *Let G be a nonplanar graph. Then $C(G)$ is spatially dependent.*

In the end we give a complete characterization of almost pseudo adjacent graphs.

Theorem 5 *G is almost pseudo adjacent if and only if G does not contain any subdivision of the 17 graphs in Fig.1.*

Fig.1

References

[1] S. Kinoshita: On θ_n-curves in R^3 and their constituent knots, in Topology and Computer Science edited by S. Suzuki, Kinokuniya, 211-216, 1987.

[2] K. Taniyama: Cobordism, homotopy and homology of graphs in R^3, Topology, 33, 509-523, 1994.

[3] K. Taniyama: On embeddings of a graph in R^3, Comtemp. Math., 164, 239-246, 1994.

[4] K. Taniyama: Link homotopy invariants of graphs in R^3, Rev. Mat. Univ. Complut. Madrid, 7, 129-144, 1994.

[5] K. Taniyama: Homology classification of spatial embeddings of a graph, Topology Appl., 65, 205-228, 1995.

[6] W. T. Wu: On the isotopy of complexes in a Euclidean space I, Science Sinic, 9, 21-46, 1960.

[7] M. Yamamoto: Knots in spatial embeddings of the complete graph on four vertices, Topology Appl., 36, 291-298, 1990.

[8] A. Yasuhara: Delta unknotting operation and adaptability of spatial graphs, preprint.

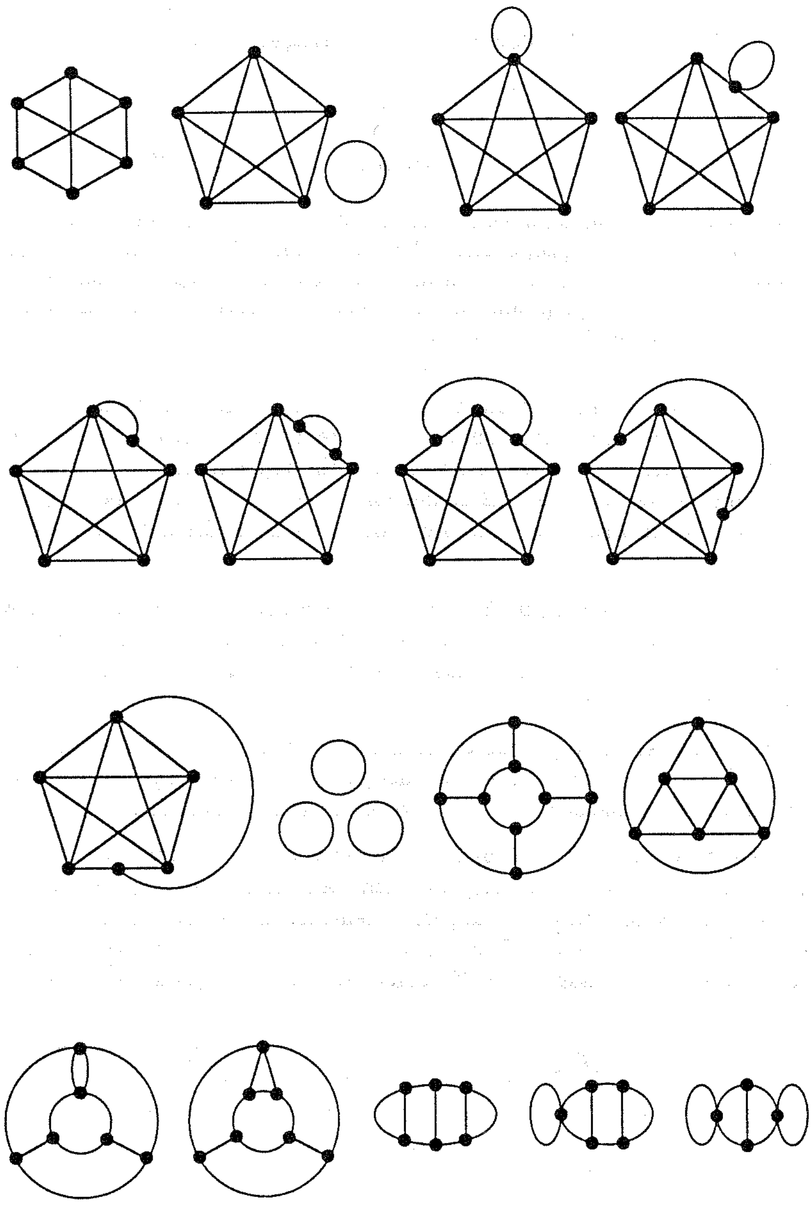


Fig.1

Vassiliev-type invariants in J^+ -theory of planar fronts without dangerous self-tangencies

J.W.Hill

Abstract

We obtain a combinatorial description of all complex-valued finite type invariants in Arnold's J^+-theory of planar fronts. The isomorphism of the order n graded part of the space of $\mathbb{C}$-valued invariants with the space spanned by specially marked n-chord diagrams modulo an appropriate four-term relation is provided by the introduction of the universal Vassiliev-Kontsevich invariant for fronts.

The manifold M^3 of cooriented contact elements of the plane is diffeomorphic to the solid torus $\mathbb{R}^2 \times S^1$. M^3 has a natural contact structure given by zeros of the form $\cos(\varphi)dx + \sin(\varphi)dy$ where x, y are coordinates on $\mathbb{R}^2$ and φ is the angle made by the coorienting normal vector with some fixed direction. An immersion $f : S^1 \longrightarrow M^3$ is called *Legendrian* if it is tangent to the contact structure. The projection of a Legendrian curve to $\mathbb{R}^2$ is called its *front*.

The front of a generic Legendrian curve has only semicubical cusps and transversal double points as singularities. A Legendrian curve in M^3 is uniquely determined by its cooriented front in $\mathbb{R}^2$. Thus the study of invariants of cooriented fronts is equivalent to the study of invariants of Legendrian curves.

There are two standard integer characteristics of a front: the *winding index* (or *winding number*) and the *Maslov index*. These two integers enumerate connected components of the space Ω of all C^∞-immersions $S^1 \longrightarrow M^3$. We identify Ω with the space of cooriented fronts.

Consider the hypersurface $\Sigma \subset \Omega$ formed by the fronts with finitely many points of *dangerous self-tangency* (i.e. points of self-tangency with parallel cooriénting normal vectors). An *invariant* of fronts which have no dangerous self-tangencies (of which Arnold's J^+ is an example [1]) is a locally constant function on $\Omega \setminus \Sigma$. We extend these invariants to fronts which have finitely many dangerous self-tangencies via the relations of figure 1 (cf. [8]).

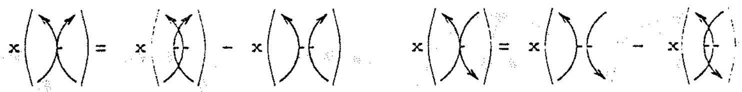


Figure 1:

Proposition 1 *Any extended invariant of fronts satisfies the two-term relations of figures 2(a) and (b) and the four-term relation of figure 3.*

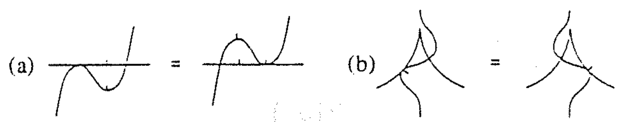


Figure 2:

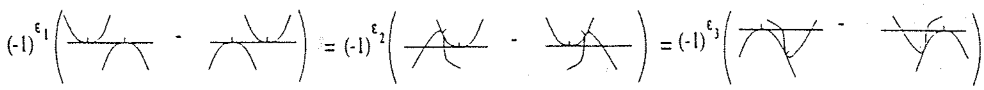


Figure 3:

We denote by Σ_n the set of all fronts which have exactly n points of dangerous self-tangency, and by Σ_n^1 and Σ_n^2 the sets of fronts having $n-1$ self-tangencies together with (respectively) one cubical dangerous self-tangency and one point of coincidence of a dangerous self-tangency with a cusp point. We assume that the fronts in the above sets have otherwise only 'moderate' singularities. These moderate singularities can be any of transversal double points, semicubical cusps, triple points (with pairwise transversal branches), non-dangerous self-tangencies, cusp crossings, cusp births (with a local normal form being a parabola of degree 4/3), third branches passing through points of dangerous self-tangency (transversally to the two tangent branches).

We say that two fronts in Σ_n are *related* if they can be connected by a smooth homotopy mostly in Σ_n and otherwise passing through $\Sigma_n^1 \cup \Sigma_n^2 \cup \Sigma_{n+1}$ in a generic way.

The *μ-marked chord diagram* of a front with dangerous self-tangencies (figure 4) is an anticlockwise oriented circle with a finite number of chords which join the pairs of points on the preimage circle that are mapped to the points of dangerous self-tangency of the front. Each chord is marked with the winding index of the 'subfront' which is parametrised by the arc on the preimage circle that faces the chord. The entire circle is marked with the winding index and Maslov index of the whole front. In a μ-marked chord diagram the sum of the markings either side of any one chord equals the first marking on the whole circle. Chord diagrams are considered modulo orientation preserving diffeomorphisms of the circle.

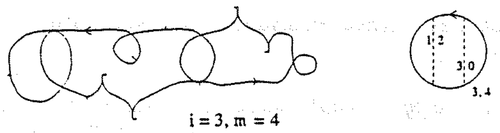


Figure 4:

Theorem 1 *Two fronts are related if and only if their μ-marked chord diagrams coincide*

We say that an extended invariant of fronts is of *order* $\leq n$ if it vanishes on all fronts which have $> n$ points of dangerous self-tangency. We denote the space of order $\leq n$ invariants by X_n. An invariant of order n is an element of the set $X_n \setminus X_{n-1}$. The *symbol* of an invariant of order n is its restriction to fronts which have exactly n dangerous self-tangencies. Theorem 1 and Proposition 1 imply

Proposition 2 *The value of the symbol of an invariant of order n on a front with exactly n points of dangerous self-tangency depends only on the μ-marked chord diagram of the front.*

Proposition 3 *The values of symbols of invariants on μ-marked chord diagrams of fronts without dangerous self-tangencies obey the μ-marked four-term relation of* figure 6.

Let $\mathcal{M}_n^\mu$ be the space of all $\mathbb{C}$-linear combinations of finitely many elements of the set of all μ-marked n-chord diagrams modulo the μ-marked four-term relation. Let the space of all $\mathbb{C}$-linear functions on $\mathcal{M}_n^\mu$ be $\mathcal{M}_n^{\mu*}$. We denote by $X_{n,\mathbb{C}}$ the space of order $\leq n$ complex-valued invariants of fronts

Theorem 2 $$X_{n,\mathbb{C}}/X_{n-1,\mathbb{C}} = \mathcal{M}_n^{\mu*}$$

$X_{n,\mathbb{C}}/X_{n-1,\mathbb{C}}$ is a subspace of $\mathcal{M}_n^{\mu*}$ by Proposition 3. The equality is provided by the introduction of the universal Vassilev-Kontsevich invariant for fronts, which is obtained from that used in [5] (cf. [7, 4, 6]) by addition of the Maslov index as a marking. Evaluation of such a modified invariant on a front whose μ-marked chord diagram $\mathcal{D}$ is a series in $\prod_{n\geq 0}\mathcal{M}_n^{\mu}$ whose lowest term is a non-zero multiple of $\mathcal{D}$.

References

[1] Arnold, V.I, *Plane curves, their invariants, perestroikas and classifications*, Adv.Sov.Math. **21** (1994), AMS, Providence, RI, 33-91.

[2] Arnold, V.I, *Topological invariants of plane curves and caustics*, University Lecture Series **5** (1994) AMS, Providence, RI.

[3] Arnold, V.I, *Invarianty i perestroiki ploskikh frontov*, Trudy Math.Inst. Steklova (1994)

[4] Bar-Natan, D, *On the Vassiliev knot invariants*, Topology **34** (1995), 423-472.

[5] Goryunov, V.V, *Vassiliev type invariants in Arnold's J^+-theory of plane curves without direct self-tangencies*, The University of Liverpool, preprint (1995)

[6] Goryunov, V.V, *Vassiliev type invariants of knots in a solid torus*, The University of Liverpool, preprint (1995)

[7] Kontsevich, M, *Vassiliev's Knot invariants*, Adv.Sov.Math. **16** (1993), part 2, AMS, Providence, RI, 137-150.

[8] Vassiliev, V.A, *Cohomology of Knot spaces*, In: *Theory of Singularities and its applications* (V.I.Arnold ed.), Adv.Sov.Math. **1** (1990), AMS, Providence, RI, 23-69.

LINK POLYNOMIALS AS VASSILIEV-TYPE INVARIANTS

TAIZO KANENOBU

Let V_n denote the vector space consisting of all Vassiliev knot invariants of order less than or equal to n. There is a filtration

$$V_0 \subset V_1 \subset V_2 \subset \cdots \subset V_n \subset \cdots$$

in the entire space of Vassiliev knot invariants. Each V_n is finite-dimensional. Vassiliev studied for the special cases when n is small: $V_0 = V_1$, which consists of a constant map, and V_2/V_1 is a one-dimensional vector space, whose basis is the second coefficient of the Conway polynomial. The dimensions for small n are found by using the computer by Bar-Natan and Stanford: For $n = 1, 2, 3, 4, 5, 6, 7$, $\dim V_n/V_{n-1} = 0, 1, 1, 3, 4, 9, 14$, respectively.

On the other hand, Bar-Natan showed that the n-th coefficient of the Conway polynomial is of order less than or equal to n. Birman and Lin proved that the Jones, HOMFLY, and Kauffman polynomials of a knot can be interpreted as an infinite sequence of Vassiliev knot invariants. Stanford generalized this for a link. From this, we have: Let $V_L(t)$ be the Jones polynomial of a link L. Then the n-th derivative of $V_L(t)$ evaluated at 1, $V_L^{(n)}(1)$, is a Vassiliev invariant of order n (cf. [KM, Theorem 1]). Let $P_k^{(\ell)}(L; 1)$ be the ℓ-th derivative of the k-th coefficient polynomial of the HOMFLY polynomial of a link L evaluated at 1. In particular, $P_k(L; 1) = a_k(L)$, the k-th coefficient of the Conway polynomial. Then $P_k^{(\ell)}(L; 1)$ is a Vassiliev link invariant of order less than or equal to $\max\{k + \ell, 0\}$ [KM, Lemma 1]. Furthermore, we have:

Theorem 1 [KM, Main Theorem]. *Let $s = \min\{\ n,\ [(n + r - 1)/2]\ \}$. Then the dimension of the subspace of the Vassiliev invariants for an r-component link of order n spanned by the following Vassiliev invariants is s:*

$$P_{2i-r+1}^{(n+r-2i-1)}, \qquad i = 0, 1, \ldots, s.$$

Here $[\,]$ denotes the greatest integer function.

Combining an upper bound of $\dim V_n/V_{n-1}$ given by Ka Yi Ng, we have

Corollary. *If $n > 5$, then $[n/2] \leq \dim V_n/V_{n-1} \leq (n-2)!/2$.*

Let $F_k^{(\ell)}(L; i)$ be the ℓ-th derivative of the k-th coefficient polynomial of the Kauffman polynomial of a link L evaluated at $i = \sqrt{-1}$. If $k + \ell > 0$ and $\ell > 0$, then $i^{k+\ell}F_k^{(\ell)}(L;i)$ is a Vassiliev-type invariant of order less than or equal to $k+\ell$, otherwise it is of order 0.

We give a basis for the space V_5 in terms of the invariants derived from the HOMFLY and Kauffman polynomials. Let v be a Vassiliev invariant of order ≤ 5 and K be a knot. Let U be a trivial knot and 3_1, 4_1,... the knots in the table of [R]. We denote by $K!$ the mirror image of K. Then we have

$$v(K) = \begin{bmatrix} v(U) \\ v(3_1!) \\ v(3_1) \\ v(4_1) \\ v(5_1!) \\ v(5_2!) \\ v(5_2) \\ v(6_1!) \\ v(6_2!) \\ v(6_3) \end{bmatrix}^T X \begin{bmatrix} 1 \\ a_2(K) \\ P_0^{(3)}(K;1)/24 \\ a_2(K)^2 \\ a_4(K) \\ P_0^{(4)}(K;1) \\ a_2(K)P_0^{(3)}(K;1)/24 \\ P_0^{(5)}(K;1)/120 \\ P_4^{(1)}(K;1) \\ F_4^{(1)}(K;i)/i \end{bmatrix},$$

where

$$X = \begin{bmatrix} 1 & -\frac{3}{4} & \frac{7}{48} & -\frac{3}{4} & \frac{3}{2} & \frac{17}{48} & -1 & \frac{13}{48} & -\frac{5}{6} & -\frac{4}{3} \\ 0 & \frac{31}{32} & \frac{1}{24} & \frac{3}{8} & -\frac{5}{4} & \frac{5}{96} & \frac{1}{4} & -\frac{1}{48} & \frac{7}{12} & \frac{1}{3} \\ 0 & \frac{13}{48} & \frac{73}{288} & \frac{5}{24} & -\frac{5}{4} & -\frac{55}{288} & \frac{5}{12} & -\frac{41}{288} & \frac{25}{36} & \frac{5}{18} \\ 0 & -\frac{19}{96} & -\frac{137}{288} & \frac{1}{12} & -\frac{1}{2} & -\frac{11}{36} & \frac{2}{3} & -\frac{53}{288} & \frac{11}{18} & \frac{17}{18} \\ 0 & -\frac{1}{32} & -\frac{1}{96} & 0 & 0 & -\frac{1}{48} & 0 & -\frac{1}{96} & \frac{1}{6} & \frac{1}{6} \\ 0 & -\frac{5}{48} & -\frac{23}{288} & -\frac{1}{24} & \frac{1}{4} & \frac{5}{288} & -\frac{1}{12} & \frac{7}{288} & -\frac{17}{36} & -\frac{7}{18} \\ 0 & -\frac{7}{96} & -\frac{5}{72} & -\frac{1}{24} & \frac{1}{4} & \frac{11}{288} & -\frac{1}{12} & \frac{5}{144} & -\frac{5}{36} & -\frac{1}{18} \\ 0 & \frac{1}{96} & \frac{65}{288} & \frac{1}{6} & 0 & \frac{17}{144} & -\frac{1}{6} & \frac{17}{288} & -\frac{1}{9} & -\frac{4}{9} \\ 0 & -\frac{1}{16} & -\frac{1}{48} & 0 & 0 & -\frac{1}{24} & 0 & -\frac{1}{48} & -\frac{1}{6} & \frac{1}{3} \\ 0 & -\frac{1}{32} & -\frac{1}{96} & 0 & 1 & -\frac{1}{48} & 0 & -\frac{1}{96} & -\frac{1}{3} & \frac{1}{6} \end{bmatrix}.$$

Using this, we may obtain various relations among polynomial invariants.

Fact 1. A Vassiliev knot invariant of order ≤ 3 is determined by the Jones polynomial, but one of order 4 is not. Let K_1 and K_2 be the two 2-bridge knots with 10 crossings $S(49,-15)$ and $S(49,27)$, respectively, which are 10_{22} and 10_{35} in the table of [R]. They share the same Jones polynomial, but they have distinct Vassiliev invariants of order 4; $P_n^{(4-n)}(K_1,1) \neq P_n^{(4-n)}(K_2,1)$ for $n = 0,2,4$.

Fact 2. A Vassiliev knot invariant of order ≤ 4 is determined by the HOMFLY polynomial, but one of order 4 is not. Let K_3 and K_4 be the two 2-bridge knots with 14 crossings $S(297,215)$ and $S(297,233)$, respectively. They share the same HOMFLY polynomial [KS], but they have distinct Vassiliev invariants of order 5; $F_n^{(5-n)}(K_3,i) \neq F_n^{(5-n)}(K_4,i)$ for $n = 1,2,3,4$.

Fact 3. A Vassiliev knot invariant of order ≤ 5 is determined by the Kauffman polynomial, but one of order 6 is not. Let K_5 and K_6 be the two 2-bridge knots with 13 crossings $S(245,-103)$ and $S(245,137)$, respectively. Then they share the same Kauffman polynomial [KS], but they have distinct Vassiliev invariants of order 6; $a_6(K_5) = 4$ and $a_6(K_6) = -1$.

References

[KM] T. Kanenobu and Y. Miyazawa, *HOMFLY polynomials as Vassiliev link invariants*, preprint.

[KS] T. Kanenobu and T. Sumi, *Polynomial invariants of 2-bridge knots through 22 crossings*, Math. Computation **60** (1993), 771-778, S17-S28.

[R] D. Rolfsen, *Knots and Links*, Lecture Series no. 7, Publish or Perish, Berkeley, 1976.

Department of Mathematics, Osaka City University, Sugimoto, Sumiyoshi-ku, Osaka 558, Japan

E-mail address: h1256@ocugw.cc.osaka-cu.ac.jp

Alternating links in the product space of a closed oriented surface and the real line

Naoko KAMADA

Osaka City University

L.H. Kauffman and K. Murasugi proved the following theorem.

Theorem 1.1 (Kauffman;Murasugi). *Any proper alternating connected link diagrams of a given link in S^3 have the same number of double points.*

Let F be a closed oriented connected surface. A result similar to Theorem 1.1 holds.

Theorem 1.2. *Any proper alternating connected link diagrams of a given link in $F \times R$ whose complements in F consist of open disks have the same number of double points.*

This is proved by using the Kauffman bracket polynomials (or the Jones polynomials) of link diagrams on F.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

The fundamental polygons of twists knots and the $(-2,3,7)$ pretzel knot

Thomas Mattman, McGill University

We briefly review the machinery developed in [CGLS] and [BZ] for analyzing cyclic/finite Dehn surgeries on a hyperbolic knot and explain how it can be applied to determine the fundamental polygons of twist knots and the $(-2,3,7)$ pretzel.

Let $M = S^3 \setminus K$ denote the exterior of a hyperbolic knot. By [CGLS, Chapter 1], there exists a norm $\|\cdot\|$ on $V = H_1(\partial M; \mathbf{R})$ satisfying the following.

1. $\|\cdot\|$ is positive integer valued for each non-trivial element $\delta \in L$ where $L = H_1(\partial M; \mathbf{Z})$.

2. Let $s = min\{\|\delta\|; \delta \in L, \delta \neq 0\}$ and let B be the disc of radius s in V. Then B is a compact, convex, finite-sided polygon whose vertices are rational multiples of strict boundary classes in L.

3. If $\alpha \in L$ is a primitive element which is not a strict boundary class and if $M(\alpha)$ has cyclic fundamental group, then $\|\alpha\| = s$.

We will call B the fundamental polygon of the norm $\|\cdot\|$.

In [BZ, Theorem 2.3], these results are extended to finite surgeries, that is, surgeries α such that $M(\alpha)$ has finite fundamental group. Property 3 above states that cyclic surgeries realize the minimal norm s. Although this is not true in general for the non-cyclic finite surgeries, it almost is. For example, the $(-2,3,7)$ pretzel admits an Icosahedral-type surgery along slope 17. For such "I-type" surgeries the norm is bounded by $s+8$.

In addition to these bounds on the norms of cyclic/finite slopes, certain other aspects of the geometry of the fundamental polygon B are known. For example, B is symmetric to the origin $(0,0)$ of V (i.e. $-B = B$) and the Euclidean area of B is no larger than 4 ([CGLS,page 244]). In [BZ], Boyer and Zhang give the possible shapes of B for $s < 10$ and show that slopes in B have longitude coordinate bounded by 2. They also argue that $\|\cdot\|$ is even integer valued.

This information, along with a list of boundary slopes, is enough to exactly determine the fundamental polygon in some cases. In unpublished work, Boyer has calculated B for the twist knots. His analysis depends on an explicit calculation of the norm for the small Seifert slopes $-1, -2, -3$. Since the twist knots are 2-bridge knots, their boundary slopes may be found in [HT], and together these fix B. In [BZ], the authors examine the (-2,3,7) pretzel. They show that it admits exactly 4 finite surgeries and $s \geq 8$. We will complete the analysis by showing the exact shape of the fundamental polygon of this knot.

References

[BZ] S. Boyer and X. Zhang, 'Finite Dehn surgery on knots', (to appear in *Journal of the A.M.S*).

[CGLS] M. Culler, C.M. Gordon, J. Luecke and P.B. Shalen, 'Dehn surgery on knots', *Ann. of Math* **125** (1987) 237-300.

[HT] A.E. Hatcher and W. Thurston, 'Incompressible surfaces in 2-bridge knot complements', *Inv. Math.* **79** (1985) 225-246.

Delta-unknotting operation and adaptability of certain graphs

Akira Yasuhara

Department of Mathematics, Tokyo Gakugei University, Nukuikita 4-1-1, Koganei, Tokyo 184, Japan
e-mail: yasuhara@u-gakugei.ac.jp

Let G be a graph and $\mathcal{C} = \{c_1, c_2, ..., c_n\}$ the set of cycles of G. The graph G is *adaptable* if for any knots $k_1, k_2, ..., k_n$, there is an embedding $f : G \longrightarrow \mathbf{R}^3$ such that $f(c_i) \cong k_i$ $(i = 1, 2, ..., n)$. In [1] Kinoshita showed that the θ_p-curves $(p \geq 3)$ are adaptable, and in [5] Yamamoto showed that the complete graph K_4 on four vertices is adaptable. Recently, Motohashi and Taniyama showed that any nonplanar graph is not adaptable [3].

To state our result we define the following sets of cycles of a graph. Let G be a graph and $\mathcal{C}$ the set of cycles of G. For a positive integer r, let $\mathcal{C}_r \subset \mathcal{C} \setminus \bigcup_{t=1}^{r-1} \mathcal{C}_t$ (resp. $\mathcal{C}_1 \subset \mathcal{C}$) be the set of cycles with the following property; for each $c \in \mathcal{C}_r$, there are three edges e_1, e_2, e_3 (possibly $e_p = e_q$ for some e_p, e_q with $p \neq q$) such that c is the unique cycle in $\mathcal{C} \setminus \bigcup_{t=1}^{r-1} \mathcal{C}_t$ (resp. $\mathcal{C}$) that contains $e_1 \cup e_2 \cup e_3$.

Theorem. *Let G be a graph and l a positive integer. Let $\mathcal{C}$ and $\mathcal{C}_r$ $(r = 1, 2, ..., l)$ as above. Set $\bigcup_{t=1}^{l} \mathcal{C}_t = \{c_1, c_2, ..., c_m\}$. Then for any embedding $f : G \longrightarrow \mathbf{R}^3$ and any knots $k_1, k_2, ..., k_m$, there is an embedding $f' : G \longrightarrow \mathbf{R}^3$ such that*

(i) *$f'(c_i) \cong k_i$ for any i $(i = 1, 2, ..., m)$ and*

(ii) *$f'(c) \cong f(c)$ for any $c \in \mathcal{C} \setminus \bigcup_{t=1}^{l} \mathcal{C}_t$.*

In particular, if $\mathcal{C} = \bigcup_{t=1}^{l} \mathcal{C}_t$, then G is adaptable.

If G is θ_p-curve or K_4, then $\mathcal{C} = \mathcal{C}_1$. Hence by Theorem, θ_p-curves and K_4 are adaptable. By applying Theorem, we have the following examples. In fact, for the all cases below, we have that $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$.

Examples. (1) The wheels W_p $(p \geq 3)$ are adaptable.
(2) Let e be an edge of the complete graph K_5 on five vertices. The graph $K_5 - e$ obtained from K_5 by removing e is adaptable. This implies that any proper subgraph of K_5 is adaptable.
(3) The trianglar prism is adaptable. □

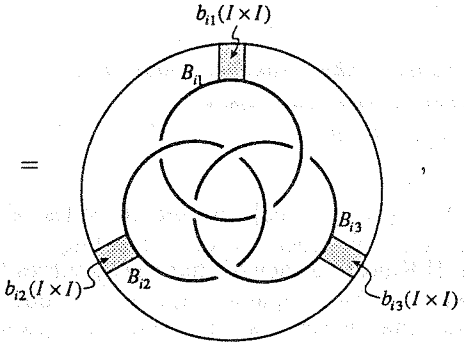
In order to prove Theorem, we need the following lemma.

Lemma.(cf. [4][5].) *Let k and k' be knots in $\mathbf{R}^3$ such that k is obtained from k' by n times Δ-unknotting operations. Let $L_i = B_{i1} \cup B_{i2} \cup B_{i3}$ $(i = 1, 2, ..., n)$ be the Borromean rings and L a link that is the split sum of $L_1, L_2, ..., L_n$ and k. Then there are mutually disjoint three arcs α_1, α_2 and α_3 in k and mutually disjoint embeddings $b_{ij} : I \times I \longrightarrow \mathbf{R}^3$ $(i = 1, 2, ..., n,\ j = 1, 2, 3)$ such that*

(i) $b_{ij}(I \times I) \cap k = b_{ij}(I \times \{0\}) \subset \alpha_j$, $b_{ij}(I \times I) \cap (\bigcup_{l=1}^{n} L_l) = b_{ij}(I \times \{1\}) \subset B_{ij}$ *for any* i, j $(i = 1, 2, ..., n,\ j = 1, 2, 3)$,

(ii) *there are mutually disjoint 3-disks $D_1, D_2, ..., D_n$ such that for each i $(i = 1, 2, ..., n)$,*

$$(D_i, D_i \cap (L \cup \textstyle\bigcup_{i',j} b_{i'j}(I \times I))$$
$$= (D_i, D_i \cap (L_i \cup b_{i1}(I \times I) \cup b_{i2}(I \times I) \cup b_{i3}(I \times I))$$

$=$  ,

(iii) $(L - \bigcup_{i,j} b_{ij}(I \times \partial I)) \cup (\bigcup_{i,j} b_{ij}(\partial I \times I)) \cong k'$.

In the lemma above, Δ-unknotting operation is an unknotting operation defined by Murakami and Nakanishi in [2].

References

[1] S. Kinoshita, On θ_n-curves in R^3 and their constituent knots, Topology and Computer Science, (ed. S. Suzuki), Kinokuniya, Tokyo Japan, 1987, pp. 211-216.

[2] H. Murakami and Y.Nakanishi, On a certain move generating link-homology, Math. Ann. **284**(1989), 75-89.

[3] T. Motohashi and K. Taniyama, Delta unknotting operation and vertex homotopy of spatial graphs, preprint.

[4] S. Suzuki, Local knots of 2-spheres in 4-manifolds, Proc. Japan Acad. **45**(1969), 34-38.

[5] M. Yamamoto, Knots in spatial embeddings of the complete graph on four vertices, Topology Appl. **36**(1990), 291-298.

Uniqueness of essential free tangle decompositions of knots and links

Makoto OZAWA (Waseda University)

An *n-string tangle* is a pair (B, t), where B is a 3-ball, and t is a union of mutually disjoint n arcs properly embedded in B. We say that (B, t) is *trivial* if (B, t) is homeomorphic to $(D \times I, \{x_1, \ldots, x_n\} \times I)$ as pairs, where D is a 2-disk and x_i is a point in $intD(i = 1, \ldots, n)$. We say that (B, t) is *essential* if $cl(\partial B - N(t))$ is incompressible and boundary-incompressible in $cl(B - N(t))$. We say that (B, t) is *free* if $\pi_1(B - t)$ is a free group. We note that (B, t) is free if and only if $cl(B - N(t))$ is a genus n handlebody.

Let L be a knot or link in S^3, and let (B, t) and (B', t') be n-string tangles. Then we say that $(B, t) \cup (B', t')$ is an *n-string tangle decomposition* of L if $B \cup B' = S^3$, $B \cap B' = \partial B = \partial B'$, $\partial t = \partial t'$ and $t \cup t' = L$. An n-string tangle decomposition $(B, t) \cup (B', t')$ of L is said to be *essential* (*free* resp.) if both (B, t) and (B', t') are essential (free resp.). Let $(B, t) \cup (B', t')$ and $(C, s) \cup (C', s')$ be n-string tangle decompositions of L. Then we say that these tangle decompositions are *mutually isotopic* if there is an ambient isotopy $\{f_t\} : S^3 \to S^3 (t \in [0, 1])$ such that $f_0 = id$, $f_1(\partial B) = \partial C$ and $f_t(L) = L$ for any $t \in [0, 1]$.

Theorem *Let L be a knot or link in S^3 which admits an essential free 2-string tangle decomposition. Then L admits non-isotopic essential 2-string tangle decompositions if and only if L is a 2-component Montesinos link $M(e; (\alpha_1, \beta_1), (2, 1), (\alpha_2, \beta_2), (2, 1))$, where e is an integer, α_i and β_i are coprime integers and $|\alpha_i|$ is an odd integer greater than 1 $(i = 1, 2)$.*

Moreover, if L is the Montesinos link, then L admits exactly two essential free 2-string tangle decompositions up to isotopy, and any essential 2-string tangle decomposition of L is isotopic to one of those two.

Corollary *If a knot K admits an essential free 2-string tangle decomposition, then essential 2-string tangle decompositions of K are unique up to isotopy.*

DEVELOPING COMPUTER PROGRAMS FOR KNOT CLASSIFICATION

Charilaos Aneziris
Institut für Hochenergienphysik Zeuthen
DESY Deutsches Elektronen-Synchrotron
Platanenallee 6
15738 Zeuthen

Abstract

In this paper we summarise the work discussed in Ref. [1] and [2], in which we introduced a method helpful in solving the problem of knot classification. We also present results obtained since then.

1. INTRODUCTION

Knot Theory has attracted significant attention during recent years, both among mathematicians, and among areas of applied science such as Physics, Chemistry and Biology. In fact, a number of problems that were previously considered unrelated to each other, have been connected through applications of Kont Theory. While enormous progress has been achieved in the study of knots and of their applications, the problem of a complete classification remains still open, in spite of recent successes (Ref. [3]). In this paper we describe and discuss an algorithmic approach that could be useful in solving the problem. With the help of an algorithm which is presented in this paper, a computer program was developed, resulting to the classification of all knots with crossing number up to 11.

In Section 2 we present the main points of the algorithm, while in Section 3 we introduce a suitable notation and show how through this notation it is possible to classify knot projections. In Section 4 we show how Reidemeister moves can be used to identify projections of equivalent knots, so that ambient isotopic knots may not appear more than once at the output. In Section 5 we generate a series of "color tests" in order to demonstrate knot inequivalence; such a procedure is necessary since equivalent since equivalent knots may fail to be identified through the procedure of Section 4. Finally in Section 6 we show the results obtained through this computer program. Ideally, any two knot projections should either be shown to belong to knots shown equivalent through Reidemeister moves, or to knots shown inequivalent due to different responses in one or more "color tests". This would be the case if the computer program could run for ever; in practice the results depend on the two input parameters, one indicating the maximum crossing number considered, the other indicating the ultimate "color test" to be used. Currently this has been achieved for all knots whose crossing number does not exceed 11.

2. THE ALGORITHMIC PROCESS

In this Section we present the main steps of the algorithm. First, a suitable method to denote knot projections is introduced. Second, once the set of possible such notations has been obtained, one needs a method to distinguish notations that correspond to actual knot projections, from notations that do not. Third, notations that correspond to identical knot projections must be identified. Once these steps are completed, knot projections are fully classified. This however is not identical to classifying knots, since distinct knot projections may correspond to ambient isotopic knots, and such knots are considered equivalent.

The next step therefore is to identify such projections. It is well known that projections of ambient isotopic knots are related through Reidemeister moves (Ref. [4]). It is thus necessary to know how a notation is affected by a Reidemeister move. Once this is known, one may use such moves to identify ambient isotopic knots. In order for the program to be finite, one may establish an upper limit to the number of Reidemeister moves to be applied; it A turns out however to be simpler to set an upper limit not to the actual number of such moves, but to the number of crossings of the knot projections involved. This upper limit is one of the two input parameters used in the program.

Since however no upper limit to either this number, or the number of necessary moves is known, there is no guarantee that projections not found connected through Reidemeister moves, actually belong to inequivalent knots. Therefore once as many equivalent knots as possible have been identified, one proceeds by selecting one knot from each equivalence class, conventionally the knot appearing first, and by calculating knot characteristics, in order to establish inequivalences between selected knots. As such characteristics we shall use the so called "color tests", which are a generalisation of the "tricolorisation" through which the trefoil's non-triviality may easily be shown. Each color test consists of an $n \times n$ matrix whose elements take values in $\{1, 2, ..., n\}$; the strands of each knot projections are mapped to elements of $\{1, 2, ..., n\}$ (the n "colors"). Acceptable mappings are the ones where the three strands meeting at each crossing, are mapped to numbers satisfying relations determined through the $n \times n$ matrix. Once certain constraints among the matrix elements are satisfied, the number of acceptable mappings is invariant under Reidemeister moves. Therefore if two projections yiled different results for one or more such color tests, they definitely belong to inequivalent knots.

Not all knots on which such "color tests" are applied, are going to yield distinct results and thus shown inequivalent. This is due to two reasons. First, some of these knots are actually equivalent, but due to the limitations in the Reidemeister moves considered, the program failed to identify them. Second, even if two knots are actually inequivalent, they may not yield distinct results due to the finite number of color tests applied. The second input parameter indicates in fact the color tests that are applied.

Having presented the main steps of the program, we now proceed with a detailed discussion.

3. THE CLASSIFICATION OF KNOT PROJECTIONS

Knot projections are denoted as sets of n pairs of natural numbers $\{(a_1, a_2), (a_3, a_4), ..., (a_{2n-1}, a_{2n})\}$, where n is the crossing number, such that $a_i \in \{1, 2, ..., 2n\}$ and $i \neq j \Leftrightarrow a_i \neq a_j$. This set is obtained as follows. First one chooses a starting point and an orientation. Then, as one travels around the projection, one assigns successive natural numbers to the crossing points, starting from 1 and ending to $2n$. Each crossing is eventually assigned two numbers, a_{over} for the overcrossing and a_{under} for the undercrossing. The set of the pairs (a_{over}, a_{under}) denotes the projection.

Not all possible notations yield actual knot projections, the simplest counterexample being $\{(1,3),(2,4)\}$. One necessary condition is that odd numbers are always paired to even numbers. This condition is not sufficient, as the counterexample $\{(1,4),(3,6),(5,8),(7,10),(9,2)\}$ demonstrates. The necessary and sufficient condition is that any two loops obtained from an actual projection, must either share one or more line segments, or intersect at an even number of points, vertices not being counted. This condition is due to the *Jordan Curve Theorem* (Ref. [5]) which states that any loop on R^2 or S^2 which does not intersect itself, divides R^2 or S^2 into two disjoint pieces. In these two counterexamples, the loops 1-2-3 and 3-4-1, in the first case, and 1-2-3-4 and 5-6-7-8 in the second case, violate this rule by not sharing any common segment and intersecting at exactly one point. The maximum number of loops obtained from an n crossing knot projection is 3^n, since each crossing may be a vertex of the loop or may not, and if it is, there are two possible direction changes. Therefore, checking whether a notation yields an actual knot projection, is a finite process.

Once "drawable" notations have been separated from "undrawable" ones, one needs to identify notations leading to the same knot projection. For an n crossing projection, there are at most $4n$ such projections, corresponding to $2n$ possible starting segments and to 2 possible orientations. By altering the starting place and/or the orientation, each pair (a_i, a_j) becomes $(k + \epsilon a_i, k + \epsilon a_j)$, where k indicates the change of the starting point and $\epsilon = \pm 1$ indicates a possible change of orientation; $\epsilon = 1$ indicates that the orientation remains the same, while $\epsilon = -1$ indicates that the orientation has been reversed. One may thus identify all such notations and keep just one, conventionally the one appearing first. This too is a finite process, and since it is less time consuming than checking the notation's "drawability", the program becomes more efficient if this step precedes the previous one.

At this point the procedure of classifying two-dimensional knot projections has been completed.

4. IDENTIFYING AMBIENT ISOTOPIC KNOTS

As mentioned earlier, two projections correspond to ambient isotopic knots if and only if they can be connected through Reidemeister moves. There are three kinds of Reidemeister moves, and their pictorial forms can be found in a number of relevant books (see for example Ref. [6]). Here we present their "numerical" form, by showing how each Reidemeister move affects a notation.

A first Reidemeister move, which increases the crossing number by 1, adds a pair $(i, i+1)$ or a pair $(i+1, i)$ to the notation, while replacing any other number j which is larger or equal to 2, with $j+2$. A second Reidemeister move, which increases the crossing number by 2, adds two pairs (i, j) and $(i+1, j+1)$, or $(i+1, j)$ and $(i, j+1)$, to the notation. Numbers larger or equal to i and smaller than j, increase by 2; numbers larger or equal to j increase by 4. A first or second Reidemeister move which decrease the crossing number, will have the converse effect. Finally a third Reidemeister move, which keeps the crossing number constant, replaces pairs (i, j), (i', k) and (j', k') with the pairs (i, k'), (i', j') and (j, k), where $|i' - i| = |j' - j| = |k' - k| = 1$, while all other $n - 3$ pairs remain the same.

The process of identifying equivalent knot goes as follows. First, one obtains through the procedure of Section 3, all distinct knot projections whose crossing number does not exceed some maximum value N. Then, on each projection one applies Reidemeister moves that do not increase the crossing number. Projections that cannot be connected to ones appeared before, are stored in the computer memory and are assigned two natural numbers, a "temporary" and a "permanent" one. Initially these numbers are equal. The permanent numbers assigned to such projections, are successive natural numbers. Projections connected to ones appeared before, are not stored in the memory, but help obtain equivalences among projections already stored. If for example some projection P is found equivalent to projections P_1, P_2, ..., P_k which have been assigned the permanent numbers p_1, p_2, ..., p_k and the temporary numbers t_1, t_2, ..., t_k, the permanent numbers do not change, while the temporary numbers are replaced by $\min(t_1, t_2, ..., t_k)$.

When all projections have been checked, only the ones stored in the memory with equal temporary and permanent numbers are going to appear at the output, since only these have not been found equivalent to preceding projections. As stated earlier, such projections may or may not be equivalent, and one thus procedes by developing "color tests" in order to distinguish inequivalent knots.

5. ESTABLISHING "COLOR TESTS"

A simple method to show the existence of non-trivial knots is through "tricolorisation". One maps the strands s_i of a knot projection to a number $n_i \in \{1, 2, 3\}$, so that at each crossing, the strands involved, s_i, s_{i+1} and s_j satisfy the relation $n_i + n_{i+1} + n_j = 0 \text{mod} 3$. If the projection is altered by a Reidemeister move, to each mapping of the old projection corresponds exactly one mapping of the new. Therefore the number of mappings is a knot invariant; if a projection P_1 admits m_1 mappings, while a projection P_2 admits m_2, and $m_1 \neq m_2$, then P_1 and P_2 definitely belong to inequivalent knots. For the trefoil three such mappings are possible, each mapping the only strand to one of the elements of $\{1, 2, 3\}$. In contrast, for the trefoil nine such mappings are possible; three map all strands to the same number, while the other six map the strands to three different numbers. Therefore the non-triviality of the trefoil is established (Ref. [7]).

Starting from this "three color test", one may generalise to obtain more such color tests in order to distinguish inequivalent knots whose responses to tricolorisation are identical. Each such color test is defined through an $n \times n$ matrix M_{ij}, so that if at some crossing the strands involved, s_i, s_{i+1} and s_j are mapped to n_i, n_{i+1} and n_j, then either $n_{i+1} = M_{n_i n_j}$, or $n_i = M_{n_{i+1} n_j}$, depending on whether the crossing is positive or negative. Only mappings $s_k \longrightarrow n_k$, where this property is satisfied at every crossing, are considerd acceptable and are counted for the corresponding knot invariant. For the "three color test" mentioned before, one may notice that $n = 3$, $M_{ii} = i$, while for $i \neq j$, $M_{ij} = k$, where $k \neq i$ and $k \neq j$.

Not all possible matrices however are suitable. A matrix may only be used to define a "color test" if for any two knot projections P and P' differing by Reidemeister moves, to each acceptable mapping for P corresponds exactly one mapping for P'. To ensure this property, one considers the constraints that each Reidemeister move imposes. One may easily observe that these constraints are the following.

1^{st} move: $M_{ii} = i \quad \forall \quad i \in \{1, 2, ..., n\}$

2^{nd} move: $M_{ij} = M_{i'j} \Leftrightarrow i = i'$

3^{rd} move: $M_{ij} = k \quad \wedge \quad M_{li} = m \quad \wedge \quad M_{lj} = n \quad \Rightarrow M_{nk} = M_{mj}$

In addition, an n-color test is not considered if there is a subset S of $\{1, 2, ..., n\}$ other than the empty set and $\{1, 2, ..., n\}$ itself, such that $i \in S \Rightarrow M_{ij} \in S \quad \forall \quad j \in \{1, 2, ..., n\}$, since such a test may be reduced to simpler ones. Finally, two tests are considered identical if one may be obtained from the other by permutation, or if they are defined through matrices M and M' such that $M_{ij} = k \Rightarrow M'_{kj} = i$, since in such a case they are related through mirror symmetry.

Subsequently, a computer program was developed that recorded the matrices that yield distinct valid color tests. The running time grew exponentially with the number of colors; to obtain all color tests for up to 11 colors the time needed was a few days, while for 12 colors it would exceed one month. The number of color tests per number of colors came out to be as follows.

Number of Colors	Number of Tests
1	1
2	0
3	1
4	1
5	2
6	2
7	3
8	2
9	6
10	1
11	5

As shown in Section 6, these tests are not sufficient for distinguishing knots of high crossing numbers, and the method of establishing tests by explicitly checking every possible matrix is not efficient enough. Instead, one obtains an infinite number of tests by generalising from the tests already established. One such class of tests is defined through matrices $M(i, j) = (k + 1)j - ki \mod n$, where the Greatest Common Divisor GCD(k, n)=GCD($k + 1, n$)=1. The existence of non-trivial mappings depends on the determinant of the linear homogeneous system that is defined through the equations satisfied at each crossing. This determinant is the *Alexander-Conway* polynomial (Ref.[8]). One may thus calculate and compare the Alexander-Conway polynomials of various knots, and apply additional color tests only for knots whose Alexander-Conway polynomials are identical.

A second class of tests associates the "colors" to group elements g_i, and is defined through the matrix $M(g_i, g_j) = g_j g_i g_j^{-1}$ (Ref. [9]). In particular, one may use as groups the permutation groups S_n; each conjugacy class, defined through a partition $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_k$ of n, $(\lambda_1 + \lambda_2 + ... + \lambda_k = n)$, defines a valid color test.

6. COMPUTER RESULTS

The maximum value of the crossing number of the projections studied, was set equal to $N = 14$. In order for the program to run, the CPU time needed was 8 days, and the memory required was about 10 MBytes. The number of knots that were not connected through Reidemeister moves, came out as follows.

Number of Crossings	Number of Knots
0	1
1	0
2	0
3	1
4	1
5	2
6	3
7	7
8	21
9	49
10	165
11	552
12	2191
13	29781

Due to memory constraints, 14 crossing knots were not recorded. As pointed out earlier, these numbers are mere upper limits, since it is certain that many of them although equivalent, may only be connected through Reidemeister moves involving more than 14 crossings. One thus proceeds by applying the color tests in order to obtain topologically inequivalent knots.

When the color tests listed in the table of Section 5 were applied, which are all the color tests involving at most 11 colors, all knots with crossing number up to 7 were shown inequivalent. This was not the case however with knots whose crossing number is 8, and therefore this method is good enough for only the first 15 knots.

When the Alexander-Conway polynomials were calculated, the results were slightly better; all 36 knots whose crossing number does not exceed 8, possess distinct Alexander-Conway polynomials. When knots with crossing number 9 are also considered, one faces the first cases of inequivalent knots with identical Alexander-Conway polynomials.

We later applied color tests derived from permutation groups, as discussed at the end of Section 5. Permutation groups up to S_5 were sufficient to demonstrate the inequivalence of all knots that possess identical Alexander polynomials and whose crossing number does not exceed 10. For crossing number 11, one has to go up to S_7, until all 802 knots with crossing number not exceeding 11 were shown inequivalent. For a complete list of all these knots and the characteristics through which these knots were distinguished, the reader is referred to Ref. 10.

For crossing numbers 12 and 13 it is almost certain that equivalent knots do exist, which would require the study of projections with crossing number higher than 14. The basis of this assumption is the fact that 11 crossing knots may only be distinguished once 14 crossing projections are studied; if the maximum value is set equal to 13 crossings, then 3 pairs of 11 crossing ambient isotopic knots cannot be identified.

At this point we have derived a full list of all knots whose crossing number does not exceed 11. In principle the method discussed could lead to extending this list to an arbitrary high crossing number; the CPU time and computer memory however rise very rapidly with the crossing number.

ACKNOWLEGMENTS

I would like to thank the Brookhaven National Laboratory, the Weizmann Institute of Science and the University of Göttingen for their kind hospitality during parts of the work. I would also like to thank Wei Chen, K. Anagnostopoulos, J. Przytycki, S. Lambropoulou, D. Westbury, C. Shubert for their help and/or interesting discussions concerning the problem discussed in this paper.

REFERENCES

1) C. Aneziris, DESY-Ifh preprint DESY-94-230, November 1994.

2) C. Aneziris, q-alg/9505003, May 1995.

3) M.B. Thistlethwaite, (1985) L.M.S. Lecture Notes no 93 pp. 1-76, Cambridge University Press.

4) K. Reidemeister, *Abh. Math. Sem. Univ. Hamburg* **5** (1927) 7-23.

5) C. Jordan, (1893) *Course d' Analyse*, Paris.

6) D. Rolfsen, (1976) *Knots and Links*, Berkeley, CA: Publish or Perish, Inc., L.H. Kauffman, (1991) *Knots and Physics*, World Scientific Publishing Co. Pte. Ltd. and references therein.

7) R.H. Fox, *Canadian J. Math.*, XXII(2) 1970, 193-201.

8) J.W. Alexander, *Trans. Amer. Math. Soc.* **30** (1926) 275-306.

9) R. Fenn and C. Rourke, *Journal of Knots and its Ramifications*, 1(4) 1992, 343-406.

10) C. Aneziris, http://sgi.ifh.de/~aneziris/contents/

ENERGY AND THICKNESS OF KNOTS

Jonathan Simon. Department of Mathematics, University of Iowa, Iowa City, IA 52240, (319)335-0768, jsimon@math.uiowa.edu.

Overview Knots are idealized 1-dimensional loops that tangle themselves in 3-space. They have been studied, for more than 100 years, primarily as abstract mathematical objects even though the original interest in the subject seems to be based in physics. There is now interest in re-investing the mathematical abstractions with physical-like properties such as *thickness* or self-repelling *energy*. The motivation is partly chemistry/biology and partly the lure of the mathematics itself. By modeling knots with physical properties, new invariants of knots can be defined and there is hope for better understanding of how knotted and tangled filaments (simple loops, links of several loops, or tangled spatial graphs) behave in real systems such as DNA gel electrophoresis.

This paper considers several notions of energy and other measures of geometric complexity for knots. Theorems show that various energies are related to each other, e.g. by inequalities saying that one energy is less than some function of another, and that they also are related to intuitive geometric measures of knot complexity such as *compaction* (a long knot contained in a small ball) or *average crossing number.* Another idea, *thickness* (or, rather, its reciprocal, the *rope-length* of a knot) may be viewed either as an energy or as a naive geometric measure of complexity; in any case, it also is related to the others by various inequalities. The general pattern of the theorems is that knots which seem complicated according to one measure, also must be complicated according to others. This leads us to believe that while the various energies etc. are defined differently (and are different), they all are capturing, at least approximately, the same intuit! ive idea of one knot being more complicated than another. One theorem common to the various energy functions is that there are only finitely many knot types that can be realized by knots below a given energy level, and that all knots below some level are unknotted..

There also are interesting questions about existence of minimum-energy conformations. The situation is clearer for polygonal knots (minima exist for each knot type, for each number of segments) than for smooth knots. M. Freedman *et al* showed that minima exist for prime knots under O'Hara's energy, and there is a widely believed conjecture that minimum energy conformations do not exist for smooth composite knots; the problem appears to be that the limiting energy is the sum of the energies of the factors, but that as a sequence of knots tries to realize that limit, one factor of the knot gets pulled tight to become singular in the limit.

The fact that one type of knot seems more tight or more complicated than another manifests itself in the laboratory. When DNA loops (of the same length) are tied into different types of knots, the loops move with different velocities in gel-electrophoresis experiments. Initially, it was observed that the crossing numbers of the knot types largely determine relative velocity; that is, six crossing knots move faster in the gel than five crossing knots, etc. One reason this observation is surprising is that the crossing number of a knot type is a property of a special "ideal" conformation of the knot, whereas the loops in the gel are moving and bending in many ways, assuming many conformations. Nevertheless, the tendency for velocity and minimum crossing number to be related is well-documented. However, subsequent studies have shown that while the qualitative correlation between crossing number and gel velocity is excellent for knot-types with few crossings, when one gets t! o seven and eight crossings, the re are eight crossing knot types that move more slowly than most seven crossing types. Knot energies provide an extra level of discrimination that seems to be a better predictor of gel velocity than crossing number; in particular, the energies successfully predict which of the two five-crossing knots is faster in the gel, and which eight-crossing knots should be the slow ones.

Background Many people have speculated informally about what would happen if a knotted string were somehow given an electric charge and allowed to repel itself. Seminal papers by S. Fukuhara and by J. O'Hara have helped lead to a large body of work involving different energy functions for smooth or polygonal knots. When we try to make the thought-experiment ("charge the string and let go") mathematically precise, the most naive definition of a potential energy function for a charged knot has mathematically unpleasant properties. So the functions defined by various investigators have departed from what seems like "true" physics, in order to obtain functions that are finite and prevent curves from changing knot type.

Approximate definitions The energies that have been studied for smooth knots generally are of the following kind: For each pair of points x, y on the curve K, one computes a number that depends on [some power, usually 2, of] the reciprocal distance between the points, and then integrates over $K \times K$. To prevent a near-neighbor effect that would make the integral infinite, one needs to regularize, either by subtracting something equally divergent (e.g. the same quantity computed for points on a standard circle used to parametrize

K) or by multiplying by a factor that has a zero of the appropriate order when points approach each other along the curve. The O'Hara energy used the first approach, and other energies explored in this paper use the second.

A simple energy for a polygonal knot K is defined as follows: For each pair X, Y of nonconsecutive segments of K, compute the minimum distance between the segments, $MD(X, Y)$; then take the sum over all non-adjacent pairs X,Y of the numbers

$$\frac{length(X) \times length(Y)}{[MD(X, Y)]^2}.$$

If one wants to consider knots with varying numbers of segments, then it is helpful to normalize the number of segments, e.g. by subtracting the energy associated to a standard regular n-gon.

Another kind of "energy" comes from visualizing a knot as actually made of some rope, with a positive thickness. Given a smooth curve K in 3-space, we can associate a number, R(K) that bounds the thickness of a uniform tube that can be placed around K without self-intersection. (Formally, this is defined in terms of the normal bundle $N(K; R^3)$). To correct for varying lengths, we either normalize K to have its length equal 1 or define R(K) to be a ratio of radius/length. To get an energy, e.g. something that would become large as knots get more complicated, we use the reciprocal of thickness, which we may call the *rope-length* of K. Our main theorem on rope-length is that the thickness of a knot K equals the minimum of two numbers: the min radius of curvature, and the min critical self-distance on K.

When a knot in 3-space is projected into a plane, for almost all choices of direction, the projected curve is immersed and one can count the number of self-crossings. This can be averaged over all directions to produce the average crossing number. M. Freedman *et al* showed that $acn(K)$ can be computed as double integral over $K \times K$, which facilitates comparison with energies.

Acknowledgments This paper is an exposition of joint work by various combinations of G. Buck, O. Durumeric, R. Litherland, E. Rawdon, and the author. The author is partly supported by NSF No. DMS9407132, "Energy Functions for Knots".

Statistics of knots and some relations with random walks on hyperbolic plane

Michael Monastyrsky, ITEP, Moscow, Russia

Abstract

In this talk we consider some relations between knot theory, conformal field theory and random walks on the plane with punctures. More precisely we study the simpliest nontrivial case where knots generated by Bruid group B_3 and the plane is considered with 3 punctures. Very scrappy our approach based on the following observation. From the one hand side we generate the knots invariants using the well-known relation between knots and braids and from the other hand side we consider the representation of braid groups as the monodromy matrices on the Riemann surfaces with punctures. For all such surfaces we construct conformal field models and calculate correlation n-point functions.

For special case of surfaces with three punctures we consider four point correlation functions.It is known the remarkable but not completely understandable relation between the indices of sub- factors II_1 generated Jones polynomials and the discrete triangle subgroups of $SL(2, R)$. Using this relation we construct conformal models related to these groups and calculate their critical dimensions.We discuss lso the relation between knots invariants and spectral properties of corresponding Riemann surfaces.

The main results of this talk based on joint paper with Sergei Nechaev (Landau Institute,Moscow). I would like to thank S.Nechaev for very fruthful collaboration.

M. Monastyrsky,Institute of Theoretical and Experimental Physics, 117259 , Moscow , Russia.E:mail:Monastyrsky@vxitep.itep.Ru

Invariants of links and three-manifolds from finite dimensional Hopf algebras

Louis. H. Kauffman

Abstract: A convenient category, $Cat(A)$, using immersed diagrams in the plane, is constructed for a (finite dimensional) Hopf algebra A. By using this category one can construct invariants of knots, links and three manifolds, characterize a large class of elements in the center of a quasi-triangular Hopf algebra, see algebraic identities diagrammatically. Invariants distinct from those of Reshetikhin-Turaev are constructed via integrals on the Hopf algebra. Numerous questions about the nature of these invariants can be formulated. The relationship with Kuperberg's formalism and invariants will be discussed.

ABSTRACT: FINITE TYPE INVARIANTS OF 3-MANIFOLDS

JEROME LEVINE

The notion of finite-type invariants, as applied to knots or 3- manifolds emerges from the following approach to computation. One first looks for formulae which measure how the invariant changes when certain types of alterations are performed on a manifold (or knot). These formulae should ultimately depend on the scheme of the alterations but not on the manifold (or knot). Using these formulae would then allow one to reduce the computation to knowing the value of the invariant on a smaller class of manifolds (or knots), equipped with such a scheme, to which any such object can be changed by the allowed alterations. These values would be presented in an *actuality table* and we would then say the invariant is of *finite type* if this table were of finite size.

An explicit definition of finite type for integral homology spheres was first given by Ohtsuki using alterations in the form of surgeries on *unit-framed, algebraically split* links. Given an invariant v of oriented integral homology spheres, one then extends it to pairs (M, L), where L is such a link in M, by the formula:

$$v(M, L) = \sum_{L'} (-1)^{|L'|} v(M_{L'})$$

where L' ranges over all sublinks of $L, |L'|$ is the number of components in L' and $M_{L'}$ is the result of doing surgery on L'. One says that v is of type n if $v(M, L) = 0$ whenever $|L| > n$. To satisfy the philosophical requirements of finite-type described above, one would want the space of invariants of type n to be finite-dimensional and to have a nice explicit set of generators. This has now been verified and culminates in a universal finite-type invariant defined by Le-Murakami-Ohtsuki using a generalization of Kontsevich's universal finite-type knot invariant.

A variation of Ohtsuki's notion of finite type was proposed by Garoufalidis using the smaller class of boundary links instead of all algebraically split links. This turns out to be more efficient and *seems* to be equivalent to Ohtsuki's notion. Garoufalidis-Levine have shown that an Ohtsuki type $3n$ invariant is of Garoufalidis type n and that a Garoufalidis type n invariant, *which is assumed to be of some finite*

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Ohtsuki type is of Ohtsuki type $3n$. The question remaining is whether finite Ohtsuki type implies finite Garoufalidis type.

Recently Garoufalidis-Levine have considered a new notion of finite type using alterations defined by cutting and pasting along separating surfaces (Heegard decomposition). If $\mathcal{G}$ is some subgroup of the mapping class group whose members, after a cut and paste, don't change the homology of the manifold, then any decreasing filtration $\mathcal{G}_n$ of $\mathcal{G}$ defines a notion of finite type. If v is an invariant of closed oriented 3-manifolds then it is of type n if $v(M_h) = v(M)$ whenever $h \in \mathcal{G}_{n+1}$, where M_h denotes here the result of cutting and repasting via h along a separating surface in M. Our first example might be $\mathcal{G}$ =Torelli group and $\mathcal{G}_n$ =lower central series of $\mathcal{G}$. Question: What is the relation between these notions of finite type and those using surgery on links? It turns out they coincide if we replace the lower central series filtration by a closely related filtration of the *group algebra of* $\mathcal{G}$ by powers of the fundamental ideal. Then the Ohtsuki and Garoufalidis notions coincide with that defined using two appropriate choices for $\mathcal{G}$ (neither of which is the Torelli group) and the Torelli group notion of finite type coincides with that defined by surgery on a third class of links (called *blinks*). Finally we know that type $3n$ in the Ohtsuki (algebraically split links) sense coincides with type $2n$ in the Torelli group (or blink) sense. Thus all these notions of finite type seem to be equivalent (except for the remaining question mentioned earlier).

ON THE EXISTENCE OF THE ENERGY MINIMIZING KNOTS

JUN O'HARA

Department of Mathematics, Tokyo Metropolitan University

Abstract of abstract. *Energy of knots* is a kind of real valued function on the space of knots, originally inspired by the static electric energy of charged knots. The goal is to define good-looking conformations for any given knot type as the embeddings that attain the minimum value of the energy within their knot type. Several results are known when the total space in which the knots are embedded is the euclidian 3-space. We give conjectures when the total space is the 3-sphere or the hyperbolic 3-space.

1. INTRODUCTION

In this note we focus our attention on the problem to define *the canonical conformation* for any given knot type, i.e. (hopefully unique) good-looking representative for its knot type. Our approach is to find a suitable real valued functional on the space of knots to define the canonical conformation as the embedding that attains the (local) minimum value of that functional within its knot type. One might naively lead to the notion of the static electric energy of charged knots, which turns out to blow up for any knot. Thus we are obliged to make kinds of regularizaion to obtain well-defined functionals.

Our functionals should satisfy that if a knot is approaching to have self-intersections then its value of the functional is blowing up. Such a functional is called *energy functional of knots* or *knot energy* for short. This property is required to assure that the knot type is kept unchanged during the process where the knot evolves itself along the gradient flow of that functional, though it is not sufficient because of the *pull-tight* phenomena may happen.

This subject has inclined people to do numerical experiments from the beginning. Several kinds of energy functionals for the *polygonal knots* were introduced in the very first paper of this subject by Fukuhara [Fu] accompanied by his computer program, followed by the papers and programs by Ahara, Buck and Orloff, Gunn, Kim and Kusner, Kusner and Sullivan, Ligocki and Sethian, Scharein, Simon, and Wu.

The first example of the energy functionals for the smooth knots was given in [O1] and studied in [FHW]. Other kinds of energy functionals were studied by Buck and Orloff, Chui and Moffatt, and [O2]. Some generalization to higher dimension was given in Auckly and Sadun, and Kusner and Sullivan.

1991 *Mathematics Subject Classification.* 57M25, 49Q10, 53A04.

Key words and phrases. knot, energy, minimizer.

† The author's ?th birthday!

2. Known results.

Let $f : S^1 \to \mathbb{R}^3$ be an embedding with $|f'(t)| \equiv 1$ $(\forall t)$. Let

$$E_j{}^p(f) = \int\int_{S^1\times S^1} \left\{ \frac{1}{|f(x)-f(y)|^j} - \frac{1}{\delta(f(x),f(y))^j} \right\}^p dxdy,$$

where $\delta(f(x), f(y))$ is the shorter arclength between $f(x)$ and $f(y)$. Then $E_j{}^p$ is an energy functional of knots if and only if

$$p \geq \frac{2}{j} \quad (0 < j \leq 2) \qquad \text{or} \qquad \frac{1}{j-2} > p \geq \frac{2}{j} \quad (2 < j < 4).$$

Let $E(f) = E_2{}^1(f)$.

Theorem [FHW]. *There is an E-minimizing knot in any prime knot type.*

Remark. Since E is Möbius invariant [FHW], there are infinitely many E-minimizing knots in any prime non-trivial knot type.

Remark. In the case of composite knot type, Kusner and Sullivan's experiments indicate that there are no E-minimizing knots in any composite knot type because all the component tangles are going to pull tight to points if we evolve the knot so as to decrease its energy E.

If $jp > 2$ the value of $E_j{}^p$ blows up if pull-tight phenomena happen. Thus there hold;

Theorem [O2]. *(1) If $jp > 2$, then there is an $E_j{}^p$- minimizing knot in any knot type.*

(2) For any knot type, the number of the "rough shapes" of the $E_j{}^p$-minimizing knots is finite.

Remark. When one tries numerical experiments for polygonal knots, it sometimes seems better to use higher power than 2 to get good-looking conformations.

3. Conjectures on the spherical or hyperbolic cases.

There are two ways to generalize the energy E to the space of knots in a 3 dimensional manifold M. If we assume that "*the electric density* ρ"$\equiv 1$, we get $E_{M,\rho}$, and if we assume that "*the total charge* t"$\equiv 1$, we get $E_{M,t}$. Let $f \to M$ be an embedding. Then

$$E_{M,\rho}(f) = \int\int_{S^1\times S^1} \left(\frac{1}{d_M(f(x),f(y))^2} - \frac{1}{\delta(f(x),f(y))^2} \right) |f'(x)||f'(y)|dxdy,$$

$$E_{M,t} = \frac{1}{L_f{}^2} E_{M,\rho}(f),$$

where $d_M(f(x), f(y))$ is the distance in M between $f(x)$ and $f(y)$, i.e. the length of the shortest path in M joining $f(x)$ and $f(y)$, $\delta(f(x), f(y))$ is the shorter arclength on the knot between $f(x)$ and $f(y)$, and L_f is the length of the knot $f(S^1)$.

In this section, we use $E_{\mathbb{R}^3}$ instead of E to specify the total space in which the knots are embedded.

(1) Spherical case. Let $M = S^3 \subset \mathbb{R}^4$.

Example 1. Consider the family of circles c_r in S^3 of radius r $(0 < r \leq 1)$. Then the great circle c_1 gives the absolute minimum value 0 of both $E_{S^3,\rho}$ and $E_{S^3,t}$. Both $E_{S^3,\rho}(c_r)$ and $E_{S^3,t}(c_r)$ increase as r goes down to 0 to 4 which is the value of $E_{\mathbb{R}^3}$ of the circle and ∞ respectively.

Example 2. Consider a trefoil t_{pt} in S^3 which is almost the great circle except a very small tangle which makes it trefoil. Then $E_{S^3,\rho}(t_{pt})$ is approximately same as the value of $E_{\mathbb{R}^3}$ of the 'open' trefoil with the same tangle. Since numerical experiments shows

$$\inf_{f \text{ is a trefoil}} E_{\mathbb{R}^3}(f) \fallingdotseq 74,$$

Möbius invariance of $E_{\mathbb{R}^3}$ shows that $E_{S^3,\rho}(t_{pt}) \geq 70 = 74 - 4$.

Consider next the family of trefoils on the Clifford tori. Numerical computation by Ligocki shows that $E_{S^3,\rho}$ attains the minimum value approximately 54.3 which is much smaller than 70. This indicates that in the spherical case the energy clearly decreases as a tangle is relaxed, which is different from the euclidian case.

If we use $E_{S^3,t}$ instead of $E_{S^3,\rho}$, this tendency seems to become more definite.

Thus we are lead to the following conjecture:

Conjecture. *There are (hopefully, finitely many) $E_{S^3,\rho}$-(or $E_{S^3,t}$-)minimizing knots in any knot type.*

(2) Hyperbolic case. Let $M = H^3 \subset \mathbb{R}^4$.

Example. Consider the family of circles c_r in H^3 of radius $r > 0$. Then $E_{H^3,\rho}(c_r)$ decreases as r goes down to 0 to 4, and $E_{H^3,t}(c_r)$ decreases as r goes up to ∞ to 0.

Thus we are lead to the following conjecture:

Conjecture. *There are no $E_{H^3,\rho}$-(nor $E_{H^3,t}$-)minimizing knots in any knot type.*

REFERENCES

[[FHW]]M. H. Freedman, Z.-H. He, and Z. Wang, *Möbius energy of knots and unknots*, Ann. of Math. **139** (1994), 1–50.

[[Fu]] Fukuhara, S., *Energy of a knot*, A Fête of Topology, Y. Matsumoto, T. Mizutani, and S. Morita, eds., Academic Press, 1987, pp. 443–452.

[[O1]] J. O'Hara, *Energy of a knot*, Topology **30** (1991), 241–247.

[[O2]] ———, *Energy functionals of knots II*, Topology Appl. **56** (1994), 45–61.

2-1 MINAMI-OHSAWA, HACHIOUJI-SHI, TOKYO 192-03, JAPAN.
E-mail address: ohara@math.metro-u.ac.jp

HEEGAARD SPLITTING FOR SUTURED MANIFOLDS AND ITS APPLICATION

HIROSHI GODA

1. Heegaard splitting for sutured manifolds

A manifold pair (N, δ) is a *product sutured manifold* if (N, δ) is homeomorphic to $(S \times [0,1], \partial S \times [0,1])$ with $R_+(\delta) = S \times \{1\}, R_-(\delta) = S \times \{0\}, \delta = \partial S \times [0,1]$, where S is a compact surface with boundary. Let L be a non-split oriented link in S^3, $E(L)$ the exterior of L and R denotes a Seifert surface of L. Since $R \cap E(L)$ is homeomorphic to R, we abbreviate $R \cap E(L)$ to R. $(N, \delta) = (N(R; E(L)), N(\partial R; \partial E(L)))$ has a product sutured manifold structure $(R \times [0,1], \partial R \times [0,1])$. Here, $N(X;Y)$ means a regular neighborhood of X in Y. $(M, \gamma) = (\text{cl}(E(L) - N), \text{cl}(\partial E(L) - \delta))$ with $R_\pm(\gamma) = R_\mp(\delta)$ is called *the sutured manifold for* R, where $\text{cl}(X)$ denotes the closure of X.

A *compression body* W is a cobordism rel ∂ between surfaces $\partial_+ W$ and $\partial_- W$ such that W is homeomorphic to $\partial_+ W \times [0,1] \cup$ 2-handles $\cup$ 3-handles and $\partial_- W$ has no 2-sphere components. We can see that if $\partial_- W \neq \phi$ and W is connected, W is obtained from $\partial_- W \times [0,1]$ by attaching a number of 1-handles along disks on $\partial_- W \times \{1\}$ where $\partial_- W$ corresponds to $\partial_- W \times \{0\}$. We denote by $h(W)$ the number of these attaching 1-handles.

A *Heegaard splitting for a sutured manifold* (M, γ) is a pair of compression bodies (W, W') such that $M = W \cup W', W \cap W' = \partial_+ W = \partial_- W', \partial_- W = R_+(\gamma)$ and $\partial_- W' = R_-(\gamma)$. Any sutured manifold has this splitting. *The handle number* $h(R)$ is defined by $h(R) = \min\{h(W) | (W, W') \text{ is a Heegaard splitting for } (M, \gamma)\}$.

Suppose that L is a fibered link in S^3. Then $E(L)$ is a surface bundle over S^1 such that a Seifert surface R represents a fiber. This Seifert surface is called *a fiber surface.* We can easily see that:

Proposition 1.1. $h(R) = 0 \iff R$ *is a fiber surface.*

A $2n$*-Murasugi sum* is a Murasugi sum attaching along $2n$-gon. In particular, a 2-Murasugi sum is called a connected sum and 4-Murasugi sum is called a plumbing. Let R be a Seifert surface obtained from Seifert surfaces R_1 and R_2 by a $2n$-Murasugi sum. Then we have:

Theorem 1.2 ([4]). $h(R_1) + h(R_2) - n + 1 \leq h(R) \leq h(R_1) + h(R_2)$.

Theorem 1.3 ([4]). *If* $h(R_1) = 0$, *then* $h(R) = h(R_2)$.

Corollary 1.4 ([2], [10]). *If R_1 and R_2 are fiber surfaces, then R is a fiber surface.*

The inequality of Theorem 1.2 is the best possible for any n. Further, we have a sufficient condition to realize the upper equality $h(R) = h(R_1) + h(R_2)$ of Theorem 1.2 in the case of $n = 2$.

Theorem 1.5 ([5]). *Let R be a Seifert surface obtained by a 4-Murasugi sum of Seifert surfaces R_1 and R_2 and $(M_i, \gamma_i, A_i)(i = 1, 2)$ the marked sutured manifold for R_i associated with the 4-Murasugi sum. If there exists a product disk in M_1 with A_1 as an edge, then $h(R) = h(R_1) + h(R_2)$.*

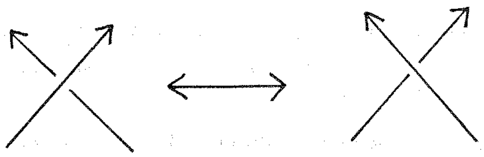
For the definition of a marked sutured manifold, see [5].

By using these theorems, we can completely determine the handle numbers of incompressible Seifert surfaces for prime knots of ≤ 10 crossings. This result involves some results of [3] and [7]. In addition, we see that there is a knot which admits two minimal genus Seifert surfaces whose handle number are mutually different.

2. APPLICATION

As a characterization of the handle number 1 case, we have:

Proposition 2.1 ([6],[8]). *Suppose that L has a Seifert surface R such that $h(R) = 1$. Then L can be changed into a fibered knot L' by a crossing change. Moreover, (the genus of L')=(the genus of R)+2.*



a crossing change

Example 2.2. Except fibered knots, every prime knot of ≤ 10 crossings has a minimal genus Seifert surface R such that $h(R) = 1$. Thus all prime knots of ≤ 10 crossings can be changed into fibered knots by a crossing change.

Moreover, we have:

Proposition 2.3 ([6]). *Every tunnel number one genus one non-fibered knot in S^3 has a genus one Seifert surface R such that $h(R) = 1$.*

Thus we have:

Theorem 2.4 ([6]). *Let K be a tunnel number one genus one knot in S^3. Then K can be changed into a genus three fibered knot by a crossing change.*

REFERENCES

1. A.J. CASSON, C.MCA. GORDON: *Reducing Heegaard splitting*, Topology and its Applications, **27** (1987), 275-283.
2. D. GABAI: *The Murasugi sum is a natural geometric operation*, Contemp. Math., **20** (1983), 131-143.
3. D. GABAI: *Detecting fibred links in S^3*, Comm. Math. Helv., **61** (1986), 519-555.
4. H. GODA: *Heegaard splitting for sutured manifolds and Murasugi sum*, Osaka J. Math., **29** (1992), 21-40.
5. H. GODA: *On handle number of Seifert surfaces in S^3*, Osaka J. Math., **30** (1993), 63-80.
6. H. GODA: *Genus one knots with unknotting tunnels and unknotting operations*, preprint.
7. T .KANENOBU: *The Augmentation Subgroup of a Pretzel Link*, Math. Seminar Notes, Kobe Univ., **7** (1979), 363-384.
8. T. KOBAYASHI: *Fibered links and unknotting operations*, Osaka J. Math., **26** (1989), 699-742.
9. D. ROLFSEN: Knots and Links, Mathematical Lecture Series **7**, Publish or Perish Inc. Berkeley, 1976.
10. J. STALLINGS: *Constructions of fibered knots and links*, Proc. Symp. Pure. Math., AMS **27** (1975), 315-319.

DEPARTMENT OF MATHEMATICS, FACULTY OF SCIENCE AND TECHNOLOGY, SCIENCE UNIVERSITY OF TOKYO, NODA,CHIBA, 278 JAPAN

E-mail address: goda@ma.noda.sut.ac.jp

FORMULAE FOR THE CALCULATION AND ESTIMATION OF WRITHE

JÜRGEN ALDINGER

Mathematisches Institut LMU
Theresienstr. 39
80333 München

Abstract

In applications to the biological and physical sciences one of the most useful results in knot theory is the formula

$$Lk = Tw + Wr \tag{1}$$

which gives the well known relationship between the linkage (Lk), twist (Tw), and writhe (Wr) of a closed ribbon. The ideas behind this formula were developed by Calugareanu [5, 6] and Pohl [18, 19] and proved in full generality by White [24]. Work by Fuller [8, 9] also presented this result with particular emphasis on its applications to DNA structure. We concentrate here on the writhe. Unlike Lk and Tw, the writhe Wr is independent of the choice of ribbon and is a global geometric characteristic of the base curve itself. This fact is of physical significance in certain circumstances.

One of the best known consequences of (1) arises in the context of the knotting and unknotting of circular double-stranded DNA. Without strand passage Lk is a topological invariant which implies by (1) that the only geometric freedom is between Tw and Wr. The twist in a closed ring can be converted to writhe; this conversion can lead to various crossings (i.e. contact points) between the strands which in turn can be passed through each other by the action of the emzyme topoisomerase II. This twist to writhe conversion followed by strand passage is the basic mechanism by which circular DNA is knotted and unknotted (see, for example, Wasserman and Cozzarelli [23]). The onset of the "writhing instability," i.e. the occurence of the critical

twist needed to trigger a conversion to writhe for simple closed filaments, has been considered by Zajac [26] and Benham [1, 2]. The role of (1) in characterizing fluid mechanical and magnetic helicity has been discussed by Berger and Field [3] and Moffatt and Ricca [17].

In terms of actual computation the challenge typically lies in evaluating the geometric quantities. White and Bauer [25], for example, have discussed the properties and explicit computation of Tw and Wr for a variety of ribbon geometries. Here our main interest lies in the general properties and computation of Wr. Given Wr, as Lk is a topological invariant, one can obtain Tw (which is often of energetic consequence) using (1). Fuller [8, 9] gives a number of explicit expressions by which Wr can be calculated or approximated. We will state these formulae together with several new ones. The first is the geometrically appealing construction

$$1 + Wr = A/2\pi \mod 2$$

where A is the area enclosed by the curve on the unit sphere traced out by the tangents along a given closed, non-intersecting space curve $\mathbf{X}$. The main computational formula relates the writhe between two closed curves $\mathbf{X}_0$ and $\mathbf{X}_1$ that can be deformed into each other:

$$Wr(\mathbf{X}_1) - Wr(\mathbf{X}_0) = \frac{1}{2\pi} \oint_0^1 \frac{\mathbf{T}_0(t) \times \mathbf{T}_1(t)}{1 + \mathbf{T}_0(t) \cdot \mathbf{T}_1(t)} \cdot \frac{d}{dt}\Big(\mathbf{T}_0(t) + \mathbf{T}_1(t)\Big) dt.$$

These formulae are frequently cited and used in the literature but, to the best of our knowledge, rigorous proofs have not been provided. Fuller provides hints for a few of the proofs, some of which we have used here. Here our aim is to bring together a detailed analysis of these formulae along with other known results concerning directional writhing number and estimates of writhe for nearly planar curves. In addition to providing some new insights into these results our analysis also leads to an apparently new result concerning the rate of change of writhe with deformation parameter λ, namely

$$\frac{d}{d\lambda} Wr(\mathbf{X}_\lambda) = -\frac{1}{2\pi} \oint_0^1 \left(\frac{\partial}{\partial\lambda}\mathbf{T}(t,\lambda) \times \mathbf{T}(t,\lambda)\right) \cdot \frac{\partial}{\partial t}\mathbf{T}(t,\lambda) dt.$$

As will be discussed in the conclusion this result has a number of important applications; these include the description of the dynamics of elastic filaments subject to external twisting forces and the conservation of writhe in closed curves evolving under certain integrable evolution equations.

In addition, it seems that Wr can be considered as a measure of the energy of a knot and we will expand on some ideas in this direction.

Important references concerning the topics mentioned include the following:

References

[1] C.J. Benham, *An elastic model of the large-scale structure of duplex-DNA*, Biopolymers **18** (1979), 609-623.

[2] C.J. Benham, *Geometry and mechanics of DNA superhelicity*, Biopolymers **22** (1983), 2477-2495.

[3] M.A. Berger and G.B. Field, *The topological properties of magnetic helicity*, J. Fluid Mech. **147** (1984), 133-148.

[4] M. Berger and B. Gostiaux, *Differential geometry: manifolds, curves and surfaces*, GTM **115**, Springer, New York (1988).

[5] G. Călugăreanu, *L'integral de Gauss et l'analyse des nœuds tridimensionnels*, Rev. Math. Pures Appl. **4** (1959), 5-20.

[6] G. Călugăreanu, *Sur les classes d'isotopie des nœuds tridimensionnels et leurs invariants*, Czechoslovak Math. J. **11** (1961), 588-625.

[7] B.A, Dubrovin, A.T. Fomenko, and S.P. Novikov, *Modern geometry - methods and applications I and II*, GTM **93** and **104**, Springer, New York (1985).

[8] F.B. Fuller, *The writhing number of a space curve*, Proc. Natl. Acad. Sci. USA **68** (1971), 815-819.

[9] F.B. Fuller, *Decomposition of the linking of a closed ribbon: a problem from molecular biology*, Proc. Natl. Acad. Sci. USA **75** (1978), 3557-3561.

[10] A. Goetz, *Introduction to differential geometry* Addison-Wesley, Reading, Massachusetts, (1970).

[11] V. Guillemin and A. Pollack, *Differential topology*, Prentice-Hall, Englewood Cliffs, New Jersey (1974).

[12] H. Hasimoto, *A soliton on a vortex filament*, J. Fluid Mech. **51** (1972), 477-485.

[13] M.W. Hirsch, *Differential topology*, GTM **33**, Springer, New York (1976).

[14] L.H. Kauffman, *Knots and Physics*, World Scientific Publishing Co., Singapore (1991).

[15] I. Klapper and M. Tabor, *A new twist in the kinematics and elastic dynamics of thin filaments and ribbons*, preprint (1993).

[16] J. Langer and R. Perline, *Poisson geometry of the filament equation*, J. Nonlinear Sci. **1** (1991), 71-93.

[17] H.K. Moffatt and L.R. Ricca, *Helicity and the Călugăreanu invariant*, Proc. Roy. Soc. A **439** (1992), 411-429.

[18] W.F. Pohl, *Some integral formulas for space curves and their generalization*, Amer. J. Math. **90** (1968), 1321-1345.

[19] W.F. Pohl, *The self-linking number of a closed space curve*, J. Math. and Mech. **17** (1968), 975-985.

[20] W.F. Pohl, *DNA and differential geometry*, Math. Intelligencer **3** (1980), 20-27.

[21] D. Rolfsen, *Knots and links*, Publish or Perish, Berkeley California (1976).

[22] M. Spivak, *Comprehensive Introduction to Differential Geometry, Vol. 3*, Publish or Perish, Berkeley California (1975).

[23] S.A. Wasserman and N.R. Cozzarelli, *Biochemical topology: applications to DNA recombination and replication*, Science **232** (1986), 951-960.

[24] J.H. White, *Self-linking and the Gauss-integral in higher dimensions*, Amer. J. Math. **91** (1969), 693-728.

[25] J.H. White and W.R. Bauer, *Calculation of the twist and the writhe for representative models of DNA*, J. Mol. Biol. **189** (1989), 329-341.

[26] E.E. Zajac, *Stability of two planar loop elasticas*, J. Appl. Mech. **29** (1962), 136-142.

TQFT versus finite type invariants of 3-manifolds
Abstract of a talk of Stavros Garoufalidis

In the past 15 years there is a plethora of numerical knot and 3-manifold invariants. At present, there are two sources of such invariants: *topological quantum field theory* ($TQFT$ for short), and *finite type*.

$TQFTs$, as axiomatized by M. Atiyah [At], produce representations of mapping class groups of compact surfaces of arbitrary genus, with an arbitrary number of boundary components. Traces of such representations give (complex valued) invariants of (framed, colored) knots and (framed) 3-manifolds. Examples of such $TQFTs$ were introduced by E. Witten [Wi] in his seminal 1989 article. Using a Chern-Simons Lagrangian (on an appropriate space of connections), he constructed, for every compact simple Lie group G, and integer k, a $TQFT$ in 3-dimensions. Witten's idea, even though it invloves a not-yet-defined integration over an infinite dimensional affine space of connections, unified the various skein theory approaches to the various Jones-like polynomials, and shortly afterwards was made rigorous by a number of authors: T. Kohno [Kh], N. Reshetikhin, V. Turaev [RT1], [RT2] and O. Viro to mention a few.

The specific $TQFT$ that was introduced by E. Witten has the specific feature that it depends on an integer k, or alternatively, on a complex root of unity $q = exp(\frac{2\pi i}{k})$. As $k \to \infty$ the stationary phase approximation of the theory produces knot and 3-manifold invariants (depending on a Lie group G and a G-representation of an appropriate fundamental group). Due to the presence of a Chern-Simons Lagrangian with quadratic and cubic terms only, the Feynmann diagrams of these perturbative knot and 3-manifold invariants are graphs with univalent and trivalent vertices only, additionally equipped with a vertex orientation, and considered modulo two relations: an antisymmetry relation, and another one resembling the Jacobi identity.

A few years after Witten's fundamental contribution, A. Vassiliev [Va] (originally motivated by singularity theory) axiometized the above mentioned knot invariants, and M. Kontsevich [Ko] showed that a universal such (finite type) invariant exists.

Last year, T. Ohtsuki [Oh] in pioneering work, introduced the notion of finite type invariant of integral homology 3-spheres. Subsequently, joint work of T.Q.T. Le, J. Murakami and T. Ohtsuki [LMO], [L] showed that the universal such invariant exixsts. The first such nontrivial type 3 invariant is the Casson invariant, considered in earlier work of H. Murakami [Mu1], [Mu2] and S. Morita [Mo].

In the talk we will discuss reformulation of the notions of finite type invariants of knots and 3-manifolds. We will also compare the two notions of invariants, namely $TQFT$ and finite type. We will comment on conjectures relating the two notions as well as implications of such conjectures on arithmetic, combinatorics, geometry, physics and topology.

REFERENCES

[At] M. F. Atiyah, *Topological Quantum Field Theories*, I.H.E.S. Publ. Math. **68** (1988) 175-186.

[Ga1] S. Garoufalidis, *On finite type 3-manifold invariants I*, M.I.T. preprint 1995, to appear in J. Knot Theory and its Ramifications

[Ga2] ______, *Comparing finite type invariants of knots and integral homology 3-spheres*, preprint February 1996.

[GL1] S. Garoufalidis, J. Levine, *On finite type 3-manifold invariants II*, Brandeis Univ. and M.I.T. preprint June 1995, to appear in Math. Annalen.

[GL2] ______, *On finite type 3-manifold invariants IV: comparison of definitions*, Brandeis Univ. and M.I.T. preprint September 1995, to appear in Proc. Camb. Phil. Soc.

[GL3] ______, *Finite type 3-manifold invariants, the mapping classs group and blinks*, Brandeis Univ. and M.I.T. preprint March 1996, submitted to Invantiones.

[GO1] S. Garoufalidis, T. Ohtsuki, *On finite type 3-manifold invariants III: manifold weight systems*, Tokyo Institute of Technology and M.I.T. preprint August 1995.

[GO2] ______, *On finite type 3-manifold invariants V: rational homology 3-spheres* , Tokyo Institute of Technology and M.I.T. preprint August 1995, to appear in Proceedings of the Aarhus conference.

[Kh] T. Kohno, *Topological invariants of 3-manifolds using representations of mapping class groups I*, Topology, **31** (1992) 203-230.

[Ko] M. Kontsevich, *Vassiliev's knot invariants*, Adv. in Sov. Math., **16(2)** (1993), 137–150.

[LMO] T.Q.T. Le, J. Murakami, T. Ohtsuki, *A universal quantum invariant of 3-manifolds*, preprint, November 1995.

[L] T.Q.T. Le, *An invariant of integral homology 3-spheres which is universal for all finite type invariants*, preprint January 1996.

[Mo] S. Morita, *Casson's invariant for homology 3- spheres and characteristic classes of vector bundles I*, Topology, **28** (1989) 305-323.

[Mu1] H. Murakami, *Quantum $SU(2)$ invariants dominate Casson's $SU(2)$ invariant*, Math. Proc. Cambridge Phil. Soc. **117** (1995) 237-250.

[Mu2] ______ *Quantum $SO(3)$ invariants dominate $SU(2)$ invariants of Casson and Walker*, Math. Proc. Cambridge Phil. Soc. **117** (1995) 237-250.

[Oh] T. Ohtsuki, *Finite type invariants of integral homology 3-spheres*, preprint 1994, to appear in J. Knot Theory and its Ramifications

[RT1] N. Reshetikhin, V. Turaev, *Ribbon graphs and their invariants derived from quantum groups*, Commun. Math. Phys. **127** (1990) 1-26.

[RT2] ______, *Invariants of 3-manifolds via link polynomials and quantum groups*, Invent. Math. **103** (1991) 547-597.

[Va] V. A. Vassiliev, *Complements of discriminants of smooth maps*, Trans. of Math. Mono. **98** Amer. Math. Society., Providence, 1992.

[Wi] E. Witten, *Quantum field theory and the Jones polynomial*, Commun. Math. Phys. **121** (1989) 360-376.

Quantum $SU(3)$-invariants derived from the linear skein theory

Haruko A.Miyazawa and Miyuki Okamoto

Witten [5] introduced topological invariants of a compact oriented 3-manifold using quantum field theory. These invariants, which are associated with a compact Lie group G, are called quantum G-invariants.

In the case of $G = SU(2)$, Lickorish [1] gave an elementary construction of invariants using the linear skein theory. The basic idea of the consruction is the following: To begin with, consider the skein module $\mathcal{S}(S^1 \times I)$ of the annulus and define the element ω of $\mathcal{S}(S^1 \times I)$ satisfying "some" condition.

It is well known that any closed oriented 3-manifold M can be obtained by a surgery along a framed link in S^3. Such a surgrey is represented by its diagram D in S^2, where the framing of each component of the link is represented by the writhe of its image in the diagram. Given D with μ-components, there is a multilinear map

$$<\ ,\ ,\ldots,\ >_D: \mathcal{S}(S^1 \times I)^{\otimes \mu} \longrightarrow \mathcal{S}(S^2) \cong \mathbf{Z}[A, A^{-1}]$$

given by replacing neighbourhoods of its components by elements of $\mathcal{S}(S^1 \times I)$.

Evaluating this map at primitive $4r$-th roots of unity, we have its value as a complex value. Then the quantity

$$< \omega, \omega, \ldots, \omega >_D < \omega >_{U_+}^{-\sigma_+} < \omega >_{U_-}^{-\sigma_-}$$

is a topological invariant of M, where $U_\pm$ denote planar diagrams representing the unknots with framings ± 1 respectively, and $\sigma_\pm$ denote the numbers of positive and negative eigenvalues of the linking matrix of the link respectively.

Moreover he [2] showed that the evaluations at primitive $2r$-th roots of unity, r odd, turn out to be quantum $SU(2)$-invariants, too.

From the point of view above, a construction of quantum $SU(3)$-invariants using the linear skein theory was given by Ohtsuki and Yamada [3]. (In the case of $G = SU(N)$, Yokota [4] established invariants in the same way.) They obtained invariants for evaluations at primitive $6r$-th roots of unity.

Inspecting the construction of invariants by Ohtsuki and Yamada, we find possibility of another choice of roots of unity. Our purpose is to establish that evaluations at primitive $3r$-th roots of unity, r odd, also give them.

Theorem. Let M be a closed oriented 3-manifold obtained by Dehn surgery on S^3 along a framed link which is represented by a planar diagram D. Suppose that r is an odd integer, $r \geq 5$, and that A is a primitive $3r$-th root of unity. Then there exists $\omega \in \mathcal{S}(S^1 \times I) \cong \mathbf{Z}[A, A^{-1}][x, y]$ which is the same as ω in [3], and the quantity

$$< \omega, \omega, \ldots, \omega >_D < \omega >_{U_+}^{-\sigma_+} < \omega >_{U_-}^{-\sigma_-}$$

is a topological invariant of M. Here $U_\pm$ are planar diagrams representing the unknots with framings ± 1 respectively, and $\sigma_\pm$ are the numbers of positive and negative eigenvalues of the linking matrix of the link respectively.

References

1. Lickorish, W.B.R., *Three-manifolds and the Temperley-Lieb algebra*, Math. Ann. **290** (1991), 657–670.

2. Lickorish, W.B.R., *The skein method for three-manifold invariants*, J. Knot Theory and its Rami. **2** (1993), 171–194.

3. Ohtsuki, T., Yamada, S., *Quantum* $SU(3)$ *invariants via linear skein theory*, J. Knot Theory and its Rami. (to appear).

4. Yokota, Y., *Skeins and quantum* $SU(N)$ *invariants of 3-manifolds*, preprint.

5. Witten, E., *Quantum field theory and the Jones polynomial*, Commun. Math. Phys. **121** (1989), 351–399.

Abelian Chern Simons theory and knots

Brigitte MEYER
Mathematisches Institut LMU
Theresienstr. 39
80333 München

Unknotting tunnels for knots and links

MAKOTO SAKUMA

In this talk, I will report on my joint work with Kanji Morimoto and Yoshiyuki Yokota [MSY], that with Elena Klimenko [KS], and the master thesis of Yoshiyuki Nakagawa [N] written under my supervision on unknotting tunnels and related topics.

Theorem 1. *A Montesinos knot or link* $K = M(b; (\alpha_1, \beta_1), \cdots, (\alpha_r, \beta_r))$ *has tunnel number one, if and only if one of the following conditions holds up to cyclic permutaion of the indices:*

(1) $r = 2$.

(2) $r = 3$, $\alpha_1 = 2$, *and* α_2 *or* $\alpha_3 \equiv 1 \pmod 2$.

(3) $r = 3$, $\beta_2/\alpha_2 \equiv \beta_3/\alpha_3 \in \mathbf{Q}/\mathbf{Z}$, *and* $e(K) = \pm 1/(\alpha_1\alpha_2)$.

This theorem was obtained in [MSY] and [KS] for knot case and was obtained in [N] for two component link case. In [N], the unknotting tunnels of some classes of two component Montesinos links are determined by using a method of [AR]. To prove this theorem, we use the idea of [BM] and the following result which is proved in [KS].

Theorem 2. *The extended triangle group* $[p, q, r] =< x, y, z | x^2 = y^2 = z^2 = (xy)^p = (yz)^q = (zx)^r = 1 >$ *is generated by two elements if and only if one of the following conditions are satisfied up to permutation of the indices:*

(1) $p = 2$ *and* $q \not\equiv 0 \pmod 2$.

(2) $p = q = 3$ *and* $r \not\equiv 0 \pmod 3$.

In fact, [KS] gives an answer to the following problem by using the method of Matelski [M]: Let f and g be elements of the isometry group $Isom(\mathbf{H}^2)$ of the

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

hyperbolic plane $\mathbf{H}^2$, and assume that one of them is orientation-reversing. Let $G =< f, g >$ be the group they generate. Then when is G discrete?

I also hope to discuss on our experimental observation concerning relation between the unknotting tunnels and the canonical decompositions of hyperbolic knot complements.

References

[**AR**] C. Adams and A. Reid, *Unknotting tunnels in two-bridge knot and link complement*, preprint.

[**BM**] S. A. Bleiler and Y. Moriah, *Heegaard splittings and branched coverings of* B^3, Math. Ann. 281 **281** (1988), 531–543.

[**KS**] E. Klimenko and M. Sakuma, *Two-generator discrete subgroups of* $Isom(\mathbf{H}^2)$ *containing orientation-reversing elements*, preprint.

[**M**] J. P. Matelski, *The classification of discrete 2-generator subgroups of* $PSL(2, \mathbb{R})$, Israel J. Math. **42** (1982), 309–317.

[**MSY**] K. Morimoto, M. Sakuma, and Y. Yokota, *Identifying tunnel number one knots*, to appear in J. Math. Soc. Japan.

[**N**] Y. Nakagawa, *Unknotting tunnels for Montesinos links*, Master thesis, Osaka Univ. 1996.

Department of Mathematics, Faculty of Science, Osaka University,
Machikaneyama-cho 1-16, Toyonaka, Osaka, 560, Japan
e-mail: sakuma@math.wani.osaka-u.ac.jp

ON THE INTERSECTION OF THREE SPHERES IN A SPHERE

EIJI OGASA*

Department of Mathematical Sciences, University of Tokyo
Komaba, Tokyo 153, Japan

Let S_1^3 and S_2^3 be 3-spheres embedded in the 5-sphere S^5 and intersect transversely. Then the intersection C is a disjoint collection of circles. Thus we obtain a pair of 1-links C in S_i^3, and a pair of 3-knots S_i^3 in S^5.

Conversely let (L_1, L_2) be a pair of 1-links and (X_1, X_2) be a pair of 3-knots. It is natural to ask whether (L_1, L_2) is obtained as the intersection of X_1 and X_2.

In this paper we give a complete answer to this question.

Furthermore we discuss some modified problems of this.

An *(oriented) (ordered) m-component n(-dimensional) link* is a smooth, oriented submanifold $L = \{K_1, ..., K_m\}$ of S^{n+2}, which is the ordered disjoint union of m manifolds, each PL homeomorphic to the standard n-sphere (if $m = 1$, then L is called a *knot.*)

§1

Definition.(L_1, L_2, X_1, X_2) is called a *4-tuple of links* if the following conditions (1), (2) and (3) hold.
(1) $L_i = (K_{i1}, ..., K_{im_i})$ is an oriented ordered m_i-component 1-dimensional link $(i = 1, 2)$. (2) $m_1 = m_2$. (3) X_i is an oriented 3-knot.

Definition.A 4-tuple of links (L_1, L_2, X_1, X_2) is said to be *realizable* if there exists a smooth transverse immersion $f : S_1^3 \coprod S_2^3 \looparrowright S^5$ satisfying the following conditions.
(1) $f|S_i^3$ is a smooth embedding and defines the 3-knot $X_i (i = 1, 2)$ in S^5.
(2) For $C = f(S_1^3) \cap f(S_2^3)$, the inverse image $f^{-1}(C)$ in S_i^3 defines the 1-link $L_i (i = 1, 2)$. Here, the orientation of C is induced naturally from the preferred orientations of S_1^3, S_2^3, and S^5, and an arbitrary order is given to the components of C.

The following theorem characterizes the realizable 4-tuples of links.

Theorem 1. *A 4-tuple of links* (L_1, L_2, X_1, X_2) *is realizable if and only if* (L_1, L_2, X_1, X_2) *satisfies one of the following conditions i) and ii).*

*This research was partially suppported by Reseach Fellowships of the Promotion of Science for Young Scientists.

i) Both L_1 and L_2 are proper links, and

$$\mathrm{Arf}(L_1) = \mathrm{Arf}(L_2).$$

ii) Neither L_1 nor L_2 is proper, and

$$lk(K_{1j}, L_1 - K_{1j}) \equiv lk(K_{2j}, L_2 - K_{2j}) \mod 2 \quad \text{for all } j.$$

Let $f : S^3 \looparrowright S^5$ be a smooth transverse immersion with a connected self-intersection C in S^5. Then the inverse image $f^{-1}(C)$ in S^3 is a knot or a 2-component link. For a similar realization problem, we have:

Theorem 2.
(1) All 2-component links are realizable as above.
(2) All knots are realizable as above.

Remark. By Theorem 1 a 4-tuple of links (L_1, L_2, X_1, X_2) with K_1 being the trivial knot and K_2 being the trefoil knot is not realizable. But by Theorem 2, the two component split link of the trivial knot and the trefoil knot is realizable as the self-intersection of an immersed 3-sphere.

§**2** We next discuss high dimensional case.

Definition. (K_1, K_2) is called a *pair of n-knots* if K_1 and K_2 are n-knots. (K_1, K_2, X_1, X_2) is called a *4-tuple of n-knots and $(n+2)$-knots* or a *4-tuple of $(n, n+2)$-knots* if K_1 and K_2 compose a pair of n-knots (K_1, K_2) and X_1 and X_2 are diffeomorphic to the standard $(n+2)$-sphere.

Definition. A 4-tuple of $(n, n+2)$-knots (K_1, K_2, X_1, X_2) is said to be *realizable* if there exists a smooth transverse immersion $f : S_1^{n+2} \coprod S_2^{n+2} \looparrowright S^{n+4}$ satisfying the following conditions.

(1) $f|S_i^{n+2}$ defines X_i (i=1,2).
(2) The intersection $\Sigma = f(S_1^{n+2}) \cap f(S_2^{n+2})$ is PL homeomorphic to the standard sphere.
(3) $f^{-1}(\Sigma)$ in S_i^{n+2} defines an n-knot K_i $(i = 1, 2)$.

A pair of n-knots (K_1, K_2) is said to be *realizable* if there is a 4-tuple of $(n, n+2)$-knots (K_1, K_2, X_1, X_2) which is realizable.

The following theorem characterizes the realizable pair of n-knots.

Theorem 3. *A pair of n-knots (K_1, K_2) is realizable if and only if (K_1, K_2) satisfies the condition that*

$$\begin{cases} (K_1, K_2) \text{ is arbitrary} & \text{if } n \text{ is even,} \\ \mathrm{Arf}(K_1) = \mathrm{Arf}(K_2) & \text{if } n = 4m+1, \quad (m \geqq 0, m \in \mathbb{Z}). \\ \sigma(K_1) = \sigma(K_2) & \text{if } n = 4m+3, \end{cases}$$

There exists a mod 4 periodicity in dimension similar to the periodicity which appears in the knot cobordism theory and the surgery theory. ([CS1,2] and [L1,2].)

We have the following results on the realization of 4-tuple of $(n, n+2)$-knots.

Theorem 4. *A 4-tuple of $(n, n+2)$-knots $T = (K_1, K_2, X_1, X_2)$ is realizable if K_1 and K_2 are slice.*

Kervaire proved that all even dimensional knots are slice ([Ke]). Hence we have:

Corollary 5. *If n is even, an arbitrary 4-tuple of $(n, n+2)$-knots $T = (K_1, K_2, X_1, X_2)$ is realizable.*

§**3** We next discuss the case when three spheres intersect in a sphere.

Let F_i be closed surfaces $(i = 1, 2, ..\mu)$. A *surface-$(F_1, F_2, ..., F_\mu)$-link* is a smooth submanifold $L = (K_1, K_2, ..., K_\mu)$ of S^4, where K_i is diffeomorphic to F_i. If F_i is orientable we assume that F_i is oriented and K_i is an oriented submanifold which is orientation preserving diffeomorphic to F_i. If $\mu = 1$, we call L *surface-F_1-knot.*

A (F_1, F_2)-link $L = (K_1, K_2)$ is called a *semi-boundary link* if $[K_i] = 0 \in H_2(S^4 - K_j; \mathbb{Z})$ $(i \neq j)$ ([S]).

A (F_1, F_2)-link $L = (K_1, K_2)$ is called a *boundary link* if there exist Seifert hypersurfaces V_i for K_i $(i = 1, 2)$ such that $V_1 \cap V_2 = \phi$.

A (F_1, F_2)-link (K_1, K_2) is called a *split link* if there exist B_1^4 and B_2^4 in S^4 such that $B_1^4 \cap B_2^4 = \phi$ and $K_i \subset B_i^4$.

Definition. Let $L_1 = (K_{12}, K_{13})$, $L_2 = (K_{23}, K_{21})$, and $L_3 = (K_{31}, K_{32})$ be surface-links. (L_1, L_2, L_3) is called a *triple of surface-links* if K_{ij} is diffeomorphic to K_{ji}. ((i,j)=(1,2),(2,3),(3,1).) (Note that the knot type of K_{ij} is different from that of K_{ji}.)

Definition. Let $L_1 = (K_{12}, K_{13})$, $L_2 = (K_{23}, K_{21})$, and $L_3 = (K_{31}, K_{32})$ be surface-links. A triple of surface-links (L_1, L_2, L_3) is said to be *realizable* if there exists a transverese immersion $f : S_1^4 \amalg S_2^4 \amalg S_3^4 \looparrowright S^6$ such that (1)$f|S_i^4$ is an embedding(i=1,2,3), and (2) $(f^{-1}(f(S_i^4) \cap f(S_j^4))$, $f^{-1}(f(S_i^4) \cap f(S_k^4))$) in S_i^4 is L_i. ((i,j,k)=(1,2,3), (2,3,1), (3,1,2).)

Note. If (L_1, L_2, L_3) is realizable, then K_{ij} are orientable and are given an orientation naturally. From now on we assume that, when we say a triple of surface-links, the triple of surface-links consists of oriented surface-links.

We state the main theorem.

Theorem 6. *Let L_i $(i = 1, 2, 3)$ be semi-boundary surface-links. Suppose the triple of surface-links (L_1, L_2, L_3) is realizable. Then we have the equality*

$$\beta(L_1) + \beta(L_2) + \beta(L_3) = 0,$$

where $\beta(L_i)$ is the Sato-Levine invariant of L_i.

Refer the Sato-Levine invariant for [S]. Since there exists a triple of surface-links (L_1, L_2, L_3) such that $\beta(L_1)$=0, $\beta(L_2)$=0 and $\beta(L_3)$=1 ([R] and [S]), we have:

Corollary 7. *Not all triple of oriented surface-links are realizable.*

We have sufficient conditions for the realization.

Theorem 8. *Let L_i $(i = 1, 2, 3)$ be split surface-links. Then the triple of surface-links (L_1, L_2, L_3) is realizable.*

Theorem 9. *Suppose L_i are (S^2, S^2)-links. If L_i are slice links$(i = 1, 2, 3)$, then the triple of surface-links (L_1, L_2, L_3) is realizable.*

It is well known that there exists a slice-link which is neither a boundary link nor a ribbon link. Hence we have:

Collorary 10. *There exists a realizable triple of surface-links (L_1, L_2, L_3) such that neither L_i are boundary links and all L_i are semi-boundary links.*

Besides the above results, we prove the following triple are realizable.

Theorem 11. *There exists a realizable triple of surface-links (L_1, L_2, L_3) such that neither L_i are semi-boundary links.*

Here we state:

Probrem 12(1). *Suppose $\beta(L_1)+\beta(L_2)+\beta(L_3)=0$. Then is the triple of surface-links (L_1, L_2, L_3) realizable?*

Using a result of [O], we can make another problem from Probrem 12(1).

Probrem 12(2). *Is every triple of (S^2, S^2)-links realizable?*

Note. By Theorem 9, if the answer to Problem 12(2) is negative, then the answer to an outstanding problem: "Is every (S^2, S^2)-link slice?" is "no." (Refer [CO] to the slice ploblem.)

References

[CS1] Cappell and Shaneson, *The codimension two placement problem and homology equivalent manifolds*, Ann of Math **99** (1974).

[CS2] Cappell, S and Shaneson, J. L., *Singular spaces, characteristic classes, and intersection homology*, Ann. Math. (1991).

[CO] Cochran,T.D and Orr,K. E, *Not all links are concordant to boundary links*, Ann. of Math. **138** (1993), 519–554.

[Ke] Kervaire,M, *Les noeudes de dimensions supéreures*, Bull.Soc.Math.France **93** (1965), 225-271.

[L1] Levine,J, *Polynomial invariants of knots of codimension two*, Ann. Math. **84** (1966), 537-554.

[L2] Levine,J, *Knot cobordism in codimension two*, Comment. Math. Helvetici. **44** (1969), 229-244.

[Og1] Ogasa,E, *On the intersection of spheres in a sphere I*, University of Tokyo preprint (1995).

[Og2] Ogasa,E, *On the intersection of spheres in a sphere II*, University of Tokyo preprint (1995).

[Og3] Ogasa,E, *The intersection of more than three spheres in a sphere*, In preparation.

[O] Orr. K.E, *New link invariants and applications*, Commentarii **62** (1987), 542-560.

[R] Ruberman, D, *Concrdance of links in S^4*, Contmp.Math. **35** (1984), 481-483.

[S] Sato, N, *Cobordisms of semi-boundary links*, Topology and its applications **18** (1984), 225-234.

A STRAND PASSAGE METRIC FOR TOPOISOMERASE ACTION

Isabel K. Dazey[1,2], De Witt Sumners[1,2].

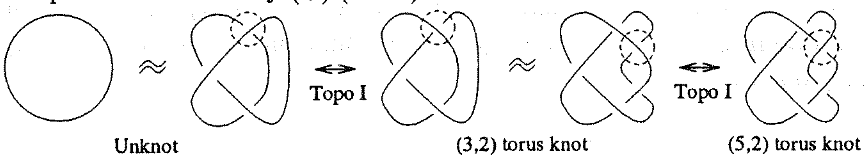
[1] *Department of Mathematics, Florida State University, Tallahassee FL USA.*
[2] *Program in Mathematics and Molecular Biology, University of California, Berkeley CA USA.*

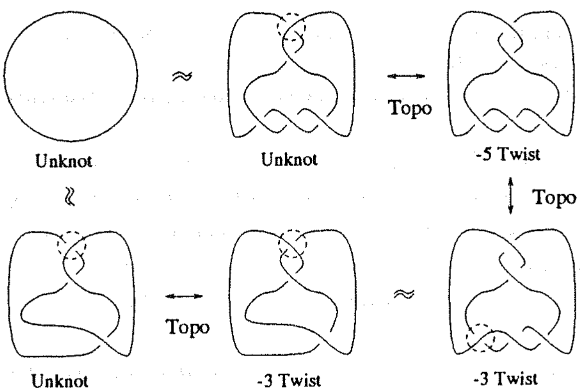
Strand passage of DNA mediated by topoisomerases may change the knot or catenane type of covalently closed circular DNA substrate. The unknotting number of a knot and other similar knot invariants can be generalized to study these topoisomerase reactions. For example, the unknotting number of a knot is defined to be the minimum number of crossing changes needed to convert the unknot (trivial catenane) into the knot (catenane); in biological terms, it is the minimum number of times the topoisomerase must perform strand passage in order to convert unknotted DNA substrate into an observed DNA knot (catenane). In a topoisomerase experiment performed on circular DNA, however, the enzyme may interconvert nontrivial knots. This enzyme action can be studied using a generalization of the unknotting number: a strand passage metric on knots [Murakami, Math. An., 270, 35 - 45] in which one computes the minimum number of strand passages necessary to interconvert a pair of knots (catenanes). Study of this metric yields:

1.) The minimum number of times topoisomerase must perform strand passage to interconvert DNA knots and catenanes.

2.) All possible minimal reaction pathways in a topoisomerase experiment, if all intermediates are known.

Values of the strand passage metric are known for all (2n+1) torus knots and some twist knots. We analyze the T4 topoisomerase data of Wasserman and Cozzarelli (JBC 266, 567-573) in which exclusively (-) twist knots are produced by T4 topoisomerase acting on negatively supercoiled unknotted substrate, and unpulished data of J. Wang in which topoisomerase I acting on single stranded DNA produces exclusively (+) (2n+1) torus knots.





Floer Homology for Orbifolds and Gauge Theory Knot Invariants

OLIVIER COLLIN

In recent years, Gauge Theory has provided the field of Low-dimensional Topology with many new invariants which, in some cases, have given a new breath to the field. Some examples are Casson's invariant and Floer Homology for 3-manifolds, and also Donaldson invariants for four-manifolds and their embedded surfaces. A Floer Homology for knot complements seems hard to achieve for various reasons, but using 3-orbifolds (or cone manifolds), (Y^3, K, n), which are singular along a knot with $\mathbb{Z}_{\kappa}$ isotropy, a Floer Homology can be constructed and associated to the singular knot. The case of primary interest is that of knots in S^3, but the construction is in general valid for knots in homology 3-spheres. This Floer Homology for orbifolds generalizes Floer's original construction for manifolds which are homology spheres, and once the hard analysis underpinning Floer's construction is understood, there are few complications that arise in the case of orbifolds. Moreover, this construction ties in well with the four-dimensional work of Kronheimer and Mrowka on Donaldson invariants of embedded surfaces in four-manifolds.

In this talk, our aim is to introduce this new gauge theoretic knot invariant. We will start by setting up rapidly the relevant Gauge Theory for orbifolds, and then proceed to give the definition of the Floer Homology for 3-orbifolds singular along a knot, denoted $HF_*^{(k)}(S^3, K, n)$. The knot invariant $HF_*^{(k)}(S^3, K, n)$ consists of a collection of four groups for each integer $k \leq n$. We will also be interested in the relations between this invariant and other recent invariants for knots. In particular Lin's invariant and it's generalisation by D. Austin and by C. Herald will be seen as the Euler characteristic of a relevant Floer Homology for 3-orbifolds, and we will try to explain the relation to the recently developped symplectic Floer Homology for knots of W. Li. If time

Date: 27th of May 1996.

allows, we shall consider the problem of computing the invariant and describe some possible applications.

MATHEMATICAL INSTITUTE, UNIVERSITY OF OXFORD, 24-29 ST.GILES', OXFORD OX1 3LB, U.K.
E-mail address: collin@maths.ox.ac.uk

SEIFERT FIBRED MANIFOLDS AND DEHN SURGERY ON KNOTS

KATURA MIYAZAKI AND KIMIHIKO MOTEGI

Let K be a knot in the 3-sphere S^3. We denote by $(K;r)$ the manifold obtained from S^3 by r-Dehn surgery on a knot K, where $r \in \mathbb{Q} \cup \{\infty\}$. In general the manifold obtained from a 3-manifold M by Dehn surgery on a knot K in M with slope γ is denoted by $M(K;\gamma)$. A slope of K is called *integral* if a representative of it intersects a meridian of K exactly once; for knots in S^3 integral slopes correspond to integers using preferred meridian-longitude pair.

For a (p,q)-torus knot K $(q > |p| > 1)$, $(K;r)$ is a Seifert fibred manifold unless $r = pq$. If K is a (p,q)-cable of a torus knot $(q \geq 2)$, then $(K;(pqn \pm 1)/n)$ is a Seifert fibred manifold for any integer n. It is conjectured that if K is nontrivial and none of these knots, then $(K;r)$ $(r \neq \infty)$ is a Seifert fibred manifold only for $r \in \mathbb{Z}$.

It is well known that 1-surgery on the figure eight knot K is a Seifert fibred manifold over S^2 with three exceptional fibres. In this example we can see an interesting phenomenon : a trivial knot $c \subset S^3$ disjoint from K become an exceptional fibre in $(K;1)$.

Question. Suppose $M = (K;r)$ is a Seifert fibred manifold. Is there a trivial knot c in S^3 disjoint from K such that when regarding $c \subset M$, c is a regular or exceptional fibre of some Seifert fibration of M?

Surprisingly all the known exaples of Seifert fibring surgery, the question has the affirmative answer. With these in mind we prove:

Corollary 1.1. *Let K be a knot in S^3. Suppose that $(K;r)$ $(r \neq \infty)$ is a Seifert fibred manifold such that Question has the affirmative answer for $(K;r)$. Then one of the following holds.*

(1) *K is a trivial knot, a torus knot or a cable of a torus knot.*
(2) *r is an integer.*

Corollary 1.1 is a consequence of Theorem 1.2 on Seifert fibring surgery on knots in a solid torus.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

A 0-*bridge braid* in a solid torus is a simple closed curve isotopic to a curve in the boundary of the solid torus.

Theorem 1.2. *Let K be a knot in a solid torus V such that K is not contained in a 3-ball in V. Suppose that $V(K;\gamma)$ is a Seifert fibred manifold where the slope γ is not meridional. Then one of the following holds.*

(1) *K is 0-bridge braid in V or a cable of a 0-bridge braid in V.*
(2) *γ is an integral slope.*

The next result is an application of Theorem 1.2 for Dehn surgery on periodic knots.

Theorem 1.3. *Let K be a nontrivial periodic knot in S^3 with period p which is neither a torus knot nor a cable of a torus knot in S^3. If $p > 2$, then $(K;r)$ cannot be a Seifert fibred manifold with infinite fundamental group for any r.*

Remark. The condition "$p > 2$" cannot be deleted. In fact the figure eight knot K has period 2 and admits Dehn surgery producing a Seifert fibred manifold with infinite fundamental group.

Young diagrams, the Homfly skein of the annulus and unitary invariants.

H.R.Morton

Department of Mathematical Sciences, University of Liverpool, PO Box 147, Liverpool, L69 3BX, UK.

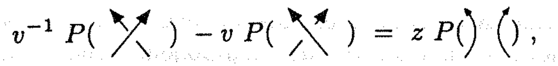
INTRODUCTION. The aim of this talk is to describe how the algebra of Young diagrams underlies both the Homfly and the unitary invariants of links, and how it features in the construction of manifold invariants from either route.

The Young diagram algebra Y is well-known classically in the context of representations of the unitary groups $SU(N)$. The algebra Y consists of linear combinations of Young diagrams, where the product of two diagrams λ and μ is given as an integer linear combination of diagrams by the Littlewood-Richardson rules for multiplication. The algebra Y itself is isomorphic to the polynomial algebra $\mathbf{C}[c_1, c_2, \ldots, c_j, \ldots]$, where c_j is the Young diagram consisting of a single column with j cells. There is an explicit determinantal formula for any Young diagram λ as a polynomial in $\{c_j\}$.

The representation ring $\mathcal{R}_N$ of $SU(N)$ can be described in terms of this algebra by means of a surjective homomorphism $Y \to \mathcal{R}_N$ in which $c_N \mapsto 1$ and $c_j \mapsto 0$ for $j > N$, giving $\mathcal{R}_N \cong \mathbf{C}[c_1, c_2, \ldots, c_{N-1}]$. In this setting the fundamental N-dimensional representation corresponds to the Young diagram c_1 with a single cell, and its jth exterior power to the Young diagram c_j. Each irreducible representation is the image of a single Young diagram with at most $N-1$ rows, and the tensor product in $\mathcal{R}_N$ of irreducibles comes from the product of Young diagrams in Y.

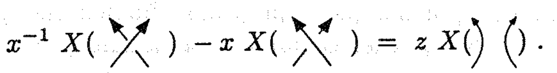
THE FRAMED HOMFLY SKEIN OF THE ANNULUS.

The Homfly polynomial $P_L(v, z)$ of an oriented link is determined by the skein relation



with the normalisation $P = 1$ for the empty knot.

There is a framed version $X_L(x, v, z) = (xv^{-1})^{w(D)} P_L(v, z)$, where $w(D)$ is the writhe of any diagram having the required framing. Altering the framing by insertion or deletion of curls changes X by multiples of xv^{-1}. The skein relation for X is



The framed Homfly skein of the annulus $\mathcal{C}$ consists of linear combinations of framed diagrams in the annulus, up to the skein relations for X, with coefficient ring $\Lambda = \mathbf{C}[x^{\pm 1}, v^{\pm 1}, z^{\pm 1}]$. When a framed knot K is decorated by a pattern Q in the annulus the polynomial $X_{K * Q}$ of the resulting satellite gives an invariant of K which depends on Q only as an element of $\mathcal{C}$. Write $X(L; Q_1, \ldots, Q_k)$ for the invariant of a k-component link L with patterns $Q_1, \ldots, Q_k$ applied to its components.

The skein $\mathcal{C}$ forms an algebra under the product induced by placing patterns side by side in parallel annuli. Write $\mathcal{C}_{\pm j}$ for the subspaces spanned by the closures of braids on j strings, oriented either in the same or the opposite sense to the core of the annulus. The algebra $\mathcal{C}$ has been studied by Turaev [6] who showed it to be the free polynomial algebra on generators $\{\alpha_j\}, j \in \mathbf{Z}$ for an explicit choice of elements $\alpha_j \in \mathcal{C}_j$. I shall restrict attention to the subalgebra $\mathcal{C}_+$ generated by $\{\alpha_j\}, j \geq 0$, which contains $\mathcal{C}_j$ for each $j \geq 0$.

Linear combinations of braids on j strings, modulo the skein relation for X, form an algebra under braid composition isomorphic to the Hecke algebra H_j, [2], and braid closure induces a surjective linear map from this to the subspace $\mathcal{C}_j \subset \mathcal{C}$. The elementary braids $\sigma_i \in H_j$ satisfy a quadratic equation $x^{-1}\sigma - x\sigma^{-1} = z$ with roots $xs, -xs^{-1}$, where $z = s - s^{-1}$. Two suitably weighted sums of the positive permutation braids, one for each choice of root, determine idempotent elements of H_j which become the anti-symmetriser and the symmetriser respectively in the symmetric group algebra on setting $x = s = 1$, [1]. Denote the closures of these two elements in $\mathcal{C}_j$ by C_j and D_j. It is easy to use Turaev's result to show that $\mathcal{C}_+$ is also freely generated by $\{C_j\}, j \geq 0$, or equally by $\{D_j\}, j \geq 0$; at this stage the coefficient ring Λ must be extended to consist of rational functions in x, v and s with denominators of the form $s^k - s^{-k}$.

We may then formally define an isomorphism from Y to $\mathcal{C}_+$ by $c_i \mapsto C_i$. It is very satisfactory to find that this isomorphism carries each single-row Young diagram d_j to D_j. This can be proved using skein theory to establish the relation $C(X)D(X) = 1$, where $C(X) = \sum(-1)^i C_i X^i$ and $D(X) = \sum D_j X^j$ as formal power series with coefficients in $\mathcal{C}$. Indeed the isomorphism maps a general Young diagram λ to an element $Q_\lambda \in \mathcal{C}_{|\lambda|}$ which can be constructed very appealingly from a template in the shape of λ using the two sorts of idempotent.

Quantum invariants. Reshetikhin and Turaev showed [4] how to use a finite-dimensional module V over a suitable quantum group to construct an invariant $J(K; V)$ of a framed knot K which is a power series in the quantum group parameter h. It can usually be expressed easily in terms of $q = e^h$ or $s = e^{h/2}$.

The construction extends to determine an invariant of framed oriented links when 'coloured' by a choice of module for each component. The invariants are multilinear under direct sums of modules, while a knot K coloured with a tensor product $V \otimes W$ of two modules has the same invariant as the link $K^{(2)}$ made up of two parallel copies of K when coloured by V and W respectively on the two components. It is thus usual to regard the invariants for a framed link L with k components as elements $J(L; w_1, \ldots, w_k)$ of the power series ring $\mathbf{C}[[h]]$ parametrised by a choice of $w_1, \ldots, w_k$ in the representation ring of the quantum group. Further results allow the quantum invariants of a satellite $K * Q$ when coloured by a module V to be calculated in terms of the quantum invariants of K itself, coloured by a suitable linear combination of summands of tensor product $V^{\otimes j}$ where the pattern Q is the closure of an oriented j-braid, or (j, j) tangle T. This combination can be interpreted as an element $\varphi_V(T)$ of the representation ring of the quantum group, giving $J(K * Q; V) = J(K; \varphi_V(T))$. The element $\varphi_V(T)$ depends only on Q, rather than the choice of T, although this is not immediately clear unless T is a braid.

Reshetikhin and Turaev [3,5,4] established a direct connection between the invariants determined by the quantum unitary groups $SU(N)_q$ and the Homfly polynomial invariants. They showed that the invariant $J(L; V_\Box, \ldots, V_\Box)$ for a link coloured by the fundamental N-dimensional $SU(N)_q$-module $V_\Box$ equals $e_N(X_L)$, where

$e_N : \Lambda \to \mathbf{C}[[h]]$, is the substitution in which $v \mapsto s^{-N} = e^{-Nh/2}$, $x \mapsto e^{-h/2N}$ and $z \mapsto s - s^{-1} = e^{h/2} - e^{-h/2}$.

When $V = V_{\square}$ the element $J(K * Q; V)$ depends only on Q as an element of $\mathcal{C}$, once the substitution e_N is made, as indeed does $\varphi_V(T)$ as an element of the representation ring of $SU(N)_q$, which is isomorphic to $\mathcal{R}_N$ for generic q. We thus have a map $\varphi_N : \mathcal{C}_+ \to \mathcal{R}_N$ with the property that

$$e_N(X(L; Q_1, \ldots, Q_k)) = J(L; \varphi_N(Q_1), \ldots, \varphi_N(Q_k)).$$

This map $\varphi_N : \mathcal{C}_+ \to \mathcal{R}_N$, with coefficients altered by e_N, is an algebra homomorphism and carries the element Q_λ to the irreducible $SU(N)_q$-module V_λ.

SUBSTITUTIONS. The element $Q_\lambda \in \mathcal{C}$, when written as a polynomial in Turaev's generators $\{\alpha_j\}$, has coefficients which are rational functions of x, v and s, with $z = s - s^{-1}$. The denominators have factors only of the form $s^k - s^{-k}$ with $1 \leq k < \rho(\lambda) + c(\lambda)$, where λ has $\rho(\lambda)$ rows and $c(\lambda)$ columns. Skein calculations show immediately that $X(K; Q_\lambda)$ is also a rational function whose denominator is restricted in the same way for every choice of K.

Write $\delta(\lambda)$ for the rational function $\delta(\lambda) = X(U; Q_\lambda)$, where U is the unknot with zero framing. The induced map $\delta : \mathcal{C} \to \Lambda$ is a ring homomorphism, determined by the values $\delta(C_i)$. Direct skein calculation shows that

$$\delta(C_i) = (-1)^i \frac{v - v^{-1}}{s - s^{-1}} \frac{vs - v^{-1}s^{-1}}{s^2 - s^{-2}} \cdots \frac{vs^{i-1} - v^{-1}s^{-i+1}}{s^i - s^{-i}}.$$

This gives $\delta(C(X)) = \sum (-1)^i \delta(C_i) X^i = \prod_{k=0}^{\infty} \dfrac{1 - vs^{2k+1}X}{1 - v^{-1}s^{2k+1}X}$ and hence

$$\delta(D_j) = (-1)^j \frac{v - v^{-1}}{s - s^{-1}} \frac{vs^{-1} - v^{-1}s}{s^2 - s^{-2}} \cdots \frac{vs^{-j+1} - v^{-1}s^{j-1}}{s^j - s^{-j}}.$$

It can be shown that if $e : \Lambda \to \Lambda'$ is a ring homomorphism, thought of as substituting the values of $e(x), e(v)$ and $e(s)$ for x, v and s, and if $e(\delta(\lambda)) = 0$ then $e(X(L; \ldots, Q_\lambda, \ldots)) = 0$ for any link L with one component decorated by Q_λ, provided that none of the denominators in $X(L; \ldots)$ evaluate to zero. Thus when $e(\delta(\lambda)) = 0$ the evaluation $e(X(L; Q))$ depends only on Q modulo the ideal in $\mathcal{C}_+$ generated by Q_λ.

If $v = s^{-N}$ after the substitution e, as in the case of e_N above, then $e(\delta(C_j)) = 0$ for $j > N$, and the ideal generated by $C_j, j > N$ has no effect on the value of $e(X(L; \ldots))$. Further skein calculation shows that if $x^{-N} = s$ after e then the ideal generated by $C_N - 1$ can also be factored out of the decorating algebra. Write $\mathcal{I}_N \subset \mathcal{C}_+$ for the ideal generated by $\{C_j\}, j > N$ and $C_N - 1$; then $\mathcal{I}_N$ is indeed the kernel of the map $\varphi_N : \mathcal{C}_+ \to \mathcal{R}_N$ discussed earlier in the context of $SU(N)_q$ and the substitution e_N.

ROOTS OF UNITY.

As a further example of the use of substitutions in X in relation to the Young diagram algebra we can combine the substitution e_N with the choice of $s = e^{\frac{m\pi i}{r}}$ where $(m, r) = 1$. Then $s^r = s^{-r}$, but $s^k \neq s^{-k}$ for $0 < k < r$. We can see that $\delta(d_j) = 0$ for $l + 1 \leq j \leq r - 1$ after this substitution, where $l = r - N$ is called the level for the substitution. In this range the denominators remain non-zero, and we may factor out the ideal $\mathcal{I}_{N,l}$ of $\mathcal{C}_+$ generated by $\{D_j\}, l+1 \le j \le r-1$, in addition to $\mathcal{I}_N$, without changing the value of the invariant after the substitution. The natural parameter space for $SU(N)_q$ invariants when they are to be evaluated at $s = e^{\frac{m\pi i}{r}}, r = N+l$ is thus the quotient algebra $V_{N,l} = \mathcal{C}_+/\mathcal{I}_{N,l}$. The Verlinde algebra $V_{N,l}$ is a finite-dimensional algebra of dimension $\binom{r-1}{N-1}$, seen by first factoring out $\mathcal{I}_N$ to get $\mathbf{C}[C_1, \ldots, C_{N-1}]$ and then the $N-1$ polynomials $D_{l+1}, \ldots, D_{r-1}$.

The algebra $V_{N,l}$ has a basis which can be represented by the Young diagrams λ with $\rho(\lambda) \le N-1$ and $c(\lambda) \le l$. In terms of this basis the structure constants $b^{\nu}_{\lambda\mu}$, where $\lambda\mu = \sum b^{\nu}_{\lambda\mu}\, \nu$, are integers with important symmetry properties which can be established quickly within the context of the Young diagram algebra, Y.

Construction of an invariant of a 3-manifold M presented by surgery on a framed link L uses the symmetry property above. Start with the element $\Omega = \sum \delta(\lambda) Q_\lambda$, where λ runs over the Young diagram basis for $V_{N,l}$. Then decorate all components of L with Ω. The element $X(L; \Omega, \ldots, \Omega)$, when evaluated with the substitution above gives a complex number which, after a simple normalisation, can be shown by the symmetry property to depend only on M.

In the description here it is primarily the properties of the Young diagram algebra which govern the behaviour of the manifold invariant; it is clear that it can be calculated using either a choice of Homfly invariants or of generic $SU(N)_q$ invariants and then making a suitable choice of substitution.

REFERENCES.

1. Morton, H.R. 'Invariants of links and 3-manifolds from skein theory and from quantum groups'. In 'Topics in knot theory', the Proceedings of the NATO Summer Institute in Erzurum 1992, NATO ASI Series C 399, ed. M.Bozhüyük. Kluwer 1993, 107-156.
2. Morton, H.R. and Traczyk, P. 'Knots and algebras'. In 'Contribuciones Matematicas en homenaje al profesor D. Antonio Plans Sanz de Bremond', ed. E. Martin-Peinador and A. Rodez Usan, University of Zaragoza, (1990), 201-220.
3. Reshetikhin, N. Y. 'Quantized universal enveloping algebras, the Yang-Baxter equation and invariants of links I and II ', preprint, LOMI E-4-87, 1987.
4. Reshetikhin, N. Y. and Turaev, V. G. 'Ribbon graphs and their invariants derived from quantum groups', *Commun. Math. Phys.* 127 (1990), 1-26.
5. Turaev, V.G. 'The Yang-Baxter equation and invariants of links', *Invent. Math.* 92 (1988), 527-553.
6. Turaev, V. G. 'The Conway and Kauffman modules of a solid torus', *Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov.* (LOMI), 167 (1988), Issled. Topol., 6, 79-89,

Version 1.2 April 1996

Some New Results in the Theory of Braids and Generalised Braids.

Roger Fenn

In this talk we look at recent advances in the theory of braids and generalised braids.

An old question asked by Artin is "when do two braids commute?" This has now been answered when one of the braids involved is a standard generator. The answer has a satisfactory geometric flavour in that the other braid must have a ribbon or band lying between two consecutive strings and disjoint from the rest of the braid.

It is a curious fact that mathematics often answers questions by generalising the theory concerned. One only has to think of the proof of the prime number theorem and the above result was in part inspired by a desire to solve a conjecture concerning singular braids.

The idea of looking at knots with singular points or self intersections, due to Vassiliev, leads to invariants which include the so called quantum invariants. Recent work by Bar Natan and others show that Vassiliev invariants distinguish braids.

We note that singular knots and links have a cubical cell structure and that Vassiliev invariants depend on the resulting homology. We also look at how three classical results in the theory of braids, namely: embedding positive braids in the braid group, Alexander's theorem and Markov's theorem, can be reformulated for generalised braids.

Lifting surfaces to embeddings in covers

Iain Aitchison
University of Melbourne

We will describe briefly various conjectures and definitions concerning the existence of immersed π_1-injective surfaces in 3-manifolds. In particular, we address whether or not certain non-Haken manifolds are virtually Haken.

The most familiar classes of non-Haken manifolds are surgeries on 2-bridge knot complements, and surgeries on certain once-punctured torus bundles. Although it is expected that many surgeries on knot and link complements in S^3 will yield non-Haken manifolds, a difficulty lies in proving that there are no embedded π_1-injective surfaces. Hass and Menasco analysed the complement of the link 8^3_4 in Rolfsen's tables, and showed that many surgeries yielded non-Haken manifolds. This example was investigated since the link complement admits a polyhedral decomposition into cubes, and contains an immersed π_1-injective surface totally geodesic with respect to the polyhedral metric of non-positive curvature. This surface continues to be an immersed π_1-injective surface in essentially any manifold M^3 obtained by surgery on the link, and since the surface satisfies the 4-plane, 1-line condition of Hass and Scott, the manifold M^3 is topologically rigid: it is determined among irreducible, infinite π_1 3-manifolds by its fundamental group.

We will show that a number of manifolds in this class are virtually Haken. The technique used generalizes to manifolds obtained by certain surgeries on the large class of prime, non-splittable alternating links with all complementary regions of even degree, and with exactly one bigon region at each crossing. The essential idea is that for cubed manifolds of non-positive curvature, with all edges of even degree, all canonical immersed π_1-surfaces can be shown to lift to embeddings in a specific 6-fold cover.

Examples will also be described of new constructions of manifolds containing immersed 'virtual fibres': π_1-injective surfaces which lift to fibres of a fibration over S^1. Again this exploits cubed manifolds and the implications of edge parity on the 'holonomy' defined on surfaces continued from one cube to the next.

Some of this work is joint with Hyam Rubinstein; some is joint with Saburo Matsumoto and Hyam Rubinstein.

Arithmetic invariants and hyperbolic Dehn filling

Craig Hodgson

Abstract: This talk will describe some arithmetic invariants of finite volume hyperbolic 3-manifolds including: the invariant trace field, the invariant quaternion algebra, and the Bloch invariant. These invariants are very useful for studying commensurability and "cut and paste" equivalence of hyperbolic 3-manifolds. We will discuss some basic properties of these invariants, and describe methods for their computation. We will also give results on the behaviour of some arithmetic invariants during hyperbolic Dehn filling.

A finiteness theorem for surfaces in Haken 3-manifolds

Sergei V. Matveev (Chelyabinsk)

Let F be a torus or a proper annulus in a 3-manifold M. By a *twist* of M along F we mean a homeomorphism of M which is the identity outside a neighborhood of F.

THEOREM. *Let M be an irreducible, boundary irreducible 3-manifold and χ_0 be an integer. Then, up to isotopies and twists along tori and proper annuli, there exists only a finite number of incompressible, boundary incompressible connected surfaces in M with Euler characteristic $\geq \chi_0$.*

The Theorem is well-known [1,2]. We give an alternative proof.

Step 1. Choose a handle decomposition ξ for M. It was proved by W. Haken that there is a (unique) finite set of *fundamental* (with respect to ξ) surfaces in M. Any incompressible and boundary incompressible surface in M is isotopic to a surface $F = k_1F_1 + k_2F_2 + \ldots + k_nF_n$ where each F_i is a fundamental surface and $k_i > 0$ for $1 \leq i \leq n$. The surfaces $F_1, F_2, \ldots, F_n$ have no triple intersection points, and all double intersection curves are contained in the union of 1- and 2-handles. The coefficients k_i and plus signs in the above expression mean that one should take k_i parallel copies of each F_i and make regular switches along all double curves.

Step 2. It follows from Step 1 that F is contained in a regular neigborhood U of $P = \cup_{i=1}^n F_i$. The set $S(P)$ of singular points of P is a union of simple curves. Connected components of $P \setminus S(P)$ are called *2-components of P*. Suppose that there is a 2-component α homeomorphic to a 2-cell. Let the curve $\partial\alpha = \mathrm{Cl}(\alpha) \setminus \alpha$ lie in $F_i \cap F_j$ such that $\alpha \subset F_i$. Then we cut F_j along $\partial\alpha$ and glue two parallel copies of α to it. We get a new polyhedron P' having a smaller number of singular curves. Since F is incompressible and boundary incompressible, it can be isotoped into a regular neighborhood of P'.

Step 3. According to Step 2, we may assume that P has no 2-components with positive Euler characteristics. The surface F can be isotoped within U

so that afterwards, outside a regular neighborhood of $S(P)$, F is a union of *sheets* presenting cross-sections of normal bundles of 2-components. Since $\chi(F) \geq \chi_0$, the number of sheets corresponding to 2-components with strictly negative Euler characteristics has an upper bound depending on ξ and χ_0. It means that, up to a finite number of possibilities, we know the exact behavior of F near the union of 2-components with negative Euler characterictics.

Step 4. Note that 2-components with zero Euler characterictics are tori, annuli or Möbius bands. A regular neigborhood V of the union of such 2-components is a disjoint union of Seifert manifolds and I-bundles over surfaces. Recall that, up to a finite number of possibilities, we know the boundary of the surface $F' = F \cap V$. We conclude the proof of the Theorem by the following easy observation: since each connected component of V is either a Seifert manifold or an I-bundle over a surface, the number of possibilities for F' is finite up to isotopies and twists along tori and proper annuli.

References

1. G. HEMION, Equivalence relation for incompressible surfaces, *Topology*, **15** (1976), 41 - 50.

2. K. JOHANNSON, Topologie und Geometrie von 3-Mannigfaltigkeiten, *Jahresber. Deutsch. Math.-Ver.*, *86*, (1984), 37-68.

A Conway presentation and the coefficients of the Jones and Kauffman polynomials of a 2-bridge link

Yasuyuki Miyazawa

Let L be a knot and link and D be a diagram of L. The Jones and Kauffman polynomials of L are calculated from D by the recursive formulas. So choosing a suitable diagram, we may find the properties of these polynomials. In this talk, we restrict L to a 2-bridge knot or link and investigate a relation between a diagram and the coefficients of the Jones and Kauffman polynomials of L. There are two kinds of diagram for 2-bridge knot or link: the Schubelt and the Conway presentations. In this talk, we consider the Conway presentation as a diagram D of L and denote it by $C(b_1, b_2, \cdots, b_n)$ ([1]). The Jones and Kauffman polynomials are defined by the recursive formulas as in [2]. Let c be the minimal crossing number of L and w be the writhe of L. Then the Jones and Kauffman polynomials of L can be written as $V_L(t) = t^d \sum_{i=0}^{c} e_i t^i$ and $F_L(a, z) = a^{-w} \sum_{i=1-\nu}^{c} f_i(a) z^i$, where $\nu = 1$ or 2, $e_i, 2d \in Z$ and $f_i(a) \in Z[a^{\pm 1}]$. Then we obtain the following theorems.

Theorem 1. Let L be a 2-bridge knot or link with a diagram $C(b_1, b_2, \cdots, b_n)$ satisfying $b_i > 0,\ 1 \leq i \leq n$, and $b_1, b_n \neq 1$. Then
(1) If $c \geq 3$, then $| e_1 | = [\frac{n}{2}]$, $| e_{c-1} | = [\frac{n+1}{2}]$.
(2) (i) If $c \geq 4$ and c is odd, then

$$| e_2 | = \frac{1}{8}(n^2 + 4n + 3) - \sum_{j=2}^{n-1} \mu_j(1),$$

$$| e_{c-2} | = \frac{1}{8}(n^2 + 8n - 1) - \mu_1(2) - \mu_n(2) - \sum_{j=2}^{n-1} \mu_j(1).$$

(ii) If $c \geq 4$ and c is even, then

$$| e_2 | = \frac{1}{8}(n^2 + 6n) - \mu_n(2) - \sum_{j=2}^{n-1} \mu_j(1),$$

$$| e_{c-2} | = \frac{1}{8}(n^2 + 6n) - \mu_1(2) - \sum_{j=2}^{n-1} \mu_j(1).$$

Here $\mu_j(l) = \begin{pmatrix} 1 & if\ b_j \leq l, \\ 0 & if\ b_j > l. \end{pmatrix}$

Theorem 2 (cf. [3],[4]) Let L be a 2-bridge knot or link with a diagram $C(b_1, \cdots, b_n)$ satisfying $b_i > 0,\ 1 \leq i \leq n$, and $b_1, b_n \neq 1$. Then
(1) $f_{c-1}(a)/(a^{-1}+a) = 1$.
(2) If $c \geq 3$, then $f_{c-2}(a)/(a^{-1}+a) = [\frac{n+1}{2}]a^{-1} + [\frac{n}{2}]a$.
(3) (i) If $c \geq 4$ and n is odd, then

$$\begin{aligned}\frac{f_{c-3}(a)}{a^{-1}+a} &= (\frac{n^2+4n+3}{8} - \sum_{1<i=odd<n} \mu_i(1) - \mu_1(2) - \mu_n(2))a^{-2} \\ &+ (-c + \frac{n^2+4n-1}{4} - \sum_{i=2}^{n-1} \mu_i(1)) \\ &+ (\frac{n^2-1}{8} - \sum_{i=even} \mu_i(1))a^2.\end{aligned}$$

(ii) If $c \geq 4$ and n is even, then

$$\begin{aligned}\frac{f_{c-3}(a)}{a^{-1}+a} &= (\frac{n^2+2n}{8} - \sum_{1<i=odd} \mu_i(1) - \mu_1(2))a^{-2} \\ &+ (-c + \frac{n^2+4n}{4} - \sum_{i=2}^{n-1} \mu_i(1)) \\ &+ (\frac{n^2+2n}{8} - \sum_{i=even<n} \mu_i(1) - \mu_n(2))a^2.\end{aligned}$$

References

[1] T. Kanenobu and Y. Miyazawa: *2-bridge link projections*, Kobe J. Math. **9** (1992), 171-182.

[2] L. H. Kauffman: *An invariant of regular isotopy*, Trans. Amer. Math. Soc. **318** (1990), 417-471.

[3] W. B. R. Lickorish: *Linear skein theory and link polynomials*, Topology Appl. **27** (1987), 265-274.

[4] M. B. Thistlethwaite: *Kauffman's polynomial and alternating links*, Topology **27** (1988), 311-318.

Normal Surfaces Immersed in the Figure-8 Knot Complement

I.R. Aitchison, S. Matsumoto, J.H. Rubinstein

ABSTRACT

Non-Haken hyperbolic 3-manifolds are conjectured to be virtually Haken, that is, to contain immersed π_1-injective surfaces which lift to embeddings in some finite-sheeted cover. A result of D. Long implies that if a closed hyperbolic 3-manifold contains *totally geodesic immersed surfaces*, then this conjecture is true. This gives rise to further studies of immersed surfaces, incompressibility, and related topics. We consider these issues in the context of the figure-8 knot complement, its 5-fold cover, and more generally, cubed 3-manifolds.

One of the well-known constructions due to W. Thurston is the realization of various alternating knot- and link-complements as the union of two polyhedra glued together. The figure-8 knot complement is a prime example of this, consisting of two ideal tetrahedra where the faces are identified by isometries of the hyperbolic space. We refer to this manifold, along with this polyhedral structure, as M_8. Thurston analyzed this specific space and concluded that there are no closed incompressible surfaces *embedded* in it except for the boundary torus. This led to the question of the existence of *immersed* incompressible surfaces.

The role of incompressible surfaces in the theory of 3-manifolds is very crucial, particularly in light of the many well-known results by Haken, Waldhausen, and others. One way to study such surfaces is to examine *normal* surfaces, consisting of normal triangles and normal quadrilaterals in the tetrahedra. Every essential surface in M_8 can be realized in this way, i.e., by the combinatorics of triangles and quadrilaterals. Conversely, given a certain fixed number of these normal triangles and quadrilaterals, one can ask whether there is any gluing of them which gives a *regular* surface, or a surface without branch points. We will briefly refer to and describe a computer program that answers this question, due to R. Rannard of Sydney University. Through this program, we were able to find many regular normal surfaces in M_8. We will also discuss some combinatorial conditions which are necessary for regular surfaces.

Having discovered many immersed surfaces, we want to determine whether these surfaces are incompressible or not. We first give a criterion for compressibility, which we will apply to a series of examples called "homogeneous surfaces."

From the work of A. Reid, we know that M_8 actually contains infinitely many families of immersed *totally geodesic* and hence incompressible surfaces. One of the homogeneous surfaces we describe (first discovered by W. Thurston but was not published) turns out to be totally geodesic. Another totally geodesic surface was found by A. Skinner (a former student of Rubinstein) using a totally different method (described below). Trying to understand totally geodesic surfaces in M_8 from a combinatorial standpoint seems to be a very interesting question as well.

We will also point out that A. Skinner constructed a five-fold covering space for M_8 which admits a cubing structure of non-positive curvature. In fact, this is how he found the totally geodesic surface mentioned earlier. The concept of cubing was studied extensively by Aitchison and Rubinstein, and many significant facts are known about it. Hence, our analysis of normal surfaces in M_8 sheds some light on immersed surfaces in cubed 3-manifolds as well. We conclude the paper by giving a criterion for incompressibility of immersed surfaces in these cubed manifolds under certain conditions and by showing some applications of it.

Transformations on special spines of 3-manifolds and branched surfaces

Artem U. Macovetsky *

There are different methods for presenting 3-manifolds. These are, for examples, triangulations of manifolds and Heegaard decompositions of manifolds.

It is known that for two triangulations of the same manifold there is a common star subdivision. Also, there is stable equivalence of any two Heegaard decomposition of same manifold.

Besides two above-mentioned methods for presenting 3-manifolds there is a well-known method for presenting 3-manifolds by special spines. The relation between 3-manifolds and spines was studed in the [C]. In [M] it was investigated $T_0^{\pm 1}, T_2^{\pm 1}$ moves for spines. We put next question. Is there result for special spines like two above-mentioned results for triangulations and Heegaard decompositions? The following theorem is answer.

DEFINITION. *A special spine Q of manifold M^3 dominates a spine S of M^3 if one can pass from S to Q by moves T_0 and T_2 .*

THEOREM. *Let P and S be special spines of a manifold M^3. Then there is a special spine Q of M^3 such that Q dominates P and S.*

In [MR] it was investigated such additional structures for the special spines as orientation and branched surface structure.

THEOREM. *Let P be a special spine with a branched surface structure and let a spine Q be obtained from P by moves T_0, T_2. Then Q has a branched surface structure too.*

THEOREM. *Let P be a special spine with an orientation and a spine Q is obtained from P by moves T_0, T_2. Then spine Q is orientable too.*

---

*Research is supported in part by INTAS, grant No. 94201.

We discuss connections between these two structures on spines.

It is known ([MR]) that if a spine P is acyclic then P admits an orientation. We prove a theorem that describes some connection between different orientations on an acylic spine.

References

[C] B.G.Casler, An imbedding for connected 3-manifolds with boundary, *Proc. Amer. Math. Soc.* 16(1965), 559–566.

[M] S.V. Matveev, Transformations of special spines and the Zeeman conjecture, *Math. USSR Izvestia*, Vol. 31, 2, 1988, 423–434.

[MR] S.Matveev, D.Rolfsen. Zeeman's collapsing conjecture. *Two-dimensional Homotopy and Combinatorial Group Theory.* London Mathematical Society Lecture Note Series 197, Cambridge University Press, Cambridge, 1993. [Chapter XI]

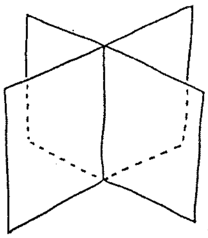
ON SIMPLY KNOTTED TORI IN S^4 II

Akiko Shima

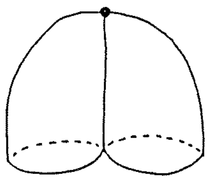
We will work in the PL category. We may assume that all manifolds are locally flat. Let S^n be n-dimensional sphere, $p : S^4 \setminus \{\infty\} \longrightarrow S^3 \setminus \{\infty\}$ be the projection, and T be an embedded torus in $S^4 \setminus \{\infty\}$. Put $\Gamma(T^*) = cl\{x \in p(T);\ |p^{-1}(x) \cap T| \geq 2\}$ and $\Gamma(T) = p^{-1}(\Gamma(T^*)) \cap T$.

Moreover we may assume that each point x of $\Gamma(T^*)$ there exists a regular neighborhood N of x in S^3 such that $N \cap p(T)$ satisfies either (i) or (ii).

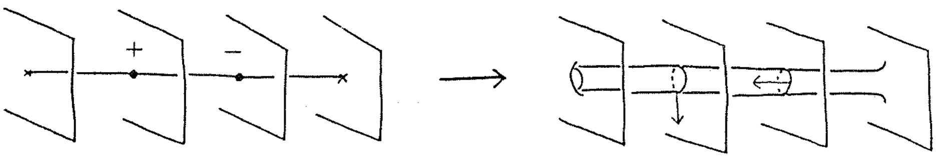
(i)



(ii)



Let S be an embedded surface in S^4, and α be an arc in S^3 such that $\partial\alpha \subset p(S)$, $\alpha \cap \Gamma(T^*) = \phi$, $int\alpha$ is transverse to $p(S)$, and each point of $int\alpha \cap p(S)$ has a signature $+$ or $-$. Then we call (S, α) *a surface with an arc*. We construct an embedded surface $F(S, \alpha)$ in S^4 from (S, α) as follows (See Figure 1).



(S, α) Figure 1 $F(S, \alpha)$

Put $S_i^2 = \{(x,y,z)|x^2+(y-4i)^2+z^2=1\}$ and $B_i^3 = \{(x,y,z)|x^2+(y-4i)^2+z^2 \leq 1\}$. Let $\mathfrak{S} = (\cup_{i=1}^n S_i^2, \cup_{i=1}^n \alpha_i)$ be a surface with arcs, and a_i^1, a_i^2 be endpoints of α_i. We call $F(\mathfrak{S})$ *an ribbon torus* if $\mathfrak{S}$ satisfies that

(1) all integers i there exist subarcs ε_i, δ_i in α_i with $(\varepsilon_i \cup \delta_i) \cap (\cup_{i=1}^n B_i^3) = \partial\alpha_i$,

(2) there exists an integer m $(1 \leq m \leq n)$ such that

if $1 \leq i \leq m-1$, then $a_i^1 \subset S_i^2$ and $a_i^2 \subset S_{i+1}^2$,

if $i = m$, then $a_m^1 \subset S_m^2$ and $a_m^2 \subset S_1^2$,

if $m+1 \leq i \leq n$, then $a_i^1 \subset S_i^2$ and $a_i^2 \subset S_j^2$ $(1 \leq j \leq m)$.

Let $T^a(K_b)$ be a symmetry-spun torus in S^4 (For the definition $T^a(K_b)$, see [T]). Let $\mathfrak{S} = (T^a(K_b) \cup (\cup_{i=1}^m S_i^2), \cup_{i=1}^m \alpha_i)$ be a surface with arcs, and $a_i^1; a_i^2$ be endpoints of α_i. We call $F(\mathfrak{S})$ *a torus obtained by m-fusions of a symmetry-spun torus* if $\mathfrak{S}$ satisfies that

(1) $p(T^a(K_b)) \subset B_0^3$,

(2) each an integer i there exists a subarc ε_i in α_i with $\varepsilon_i \cap (\cup_{i=1}^m B_i^3) = a_i^1$,

(3) $a_i^1 \subset S_i^2$ and $a_i^2 \subset p(T^a(K_b))$ for all i.

Main Theorem ([S4]). *If $\Gamma(T^*)$ dose not contain 3rd singular points, then T can be moved to either a ribbon torus or a torus obtained by m-fusions of a symmetry-spun torus.*

Corollary. *Suppose that $\Gamma(T^*)$ consists of only 2nd singular points, and all components of $\Gamma(T)$ are not contractible in T.*

If $\pi_1(S^4 \setminus T) \cong \mathbb{Z}$, then T is unknotted (i.e. T is a boundary of a solid torus in S^4).

Remarks. (1) There is a classification of symmetry-spun tori in [T].

(2) For a mean of arrows in Figure 1, see [Y].

REFERENCES

[C-S] S. Carter and M. Saito, *Canceling branch points on projections of surfaces in 4-space*, Proc. of the AMS **116** (1992), 229–237.

[H-N] T. Homma and T. Nagase, *On elementary deformation of maps of surface into 3-manifolds I*, Yokohama Math. J. **33** (1985), 103–119.

[R] D. Rolfsen, *Knots and Links*, Publish or Perish, Berkeley, Calif., 1976.

[S1] A. Shima, *An unknotting theorem for tori in S^4*, preprint.

[S2] A. Shima, *An unknotting theorem for tori in S^4 II*, preprint.

[S3] A. Shima, *On simply knotted tori in S^4*, preprint.

[S4] A. Shima, *On simply knotted tori in S^4 II*, preprint.

[T] M. Teragaito, *Symmetry-spun tori in the four-sphere*, Knots 90, 163–171.

[Y] T. Yajima, *On simply knotted spheres in $\mathbb{R}^4$*, J. of Math. Osaka **1** (1964), 133–152.

THE AVERAGE EDGE ORDER OF TRIANGULATIONS OF 3-MANIFOLDS WITH BOUNDARY MODIFICATION

MAKOTO TAMURA

Let M be a compact, connected 3-manifold and K a triangulation of M. Note that we distinguish a triangulation from a cell complex, that is, such a cell complex is a triangulation when the intersection of any two simplices is actually a face of each of them. Suppose M is closed. Then the *average edge order* $\mu_0(K)$ of K is defined to be $3F_0(K)/E_0(K)$, where $E_0(K)$ and $F_0(K)$ are the numbers of edges and faces in K, respectively. This is equal to the average of the orders of edges of K, where the order of an edge is the number of triangles incident to that edge. Feng Luo and Richard Stong showed in [1] that for a closed 3-manifold M, the average edge order being small implies that the topology of M is fairly simple and restricts the triangulation K of M. In fact, they proved the following theorem.

Theorem 1 (LS). *Let K be any triangulation of a closed connected 3-manifold M. Then*

(*a*) $3 \leq \mu_0(K) < 6$, *equality holds if and only if K is the triangulation of the boundary of a 4-simplex.*

(*b*) *For any $\varepsilon > 0$, there are triangulations K_1 and K_2 of M such that $\mu_0(K_1) < 4.5 + \varepsilon$ and $\mu_0(K_2) > 6 - \varepsilon$.*

(*c*) *If $\mu_0(K) < 4.5$, then K is a triangulation of S^3. There are an infinite number of distinct such triangulations, but for any constant $c < 4.5$ there are only finitely many triangulations K with $\mu_0(K) \leq c$.*

(*d*) *If $\mu_0(K) = 4.5$, then K is a triangulation of $S^3, S^2 \times S^1$, or $S^2 \tilde{\times} S^1$. Furthermore, in the last two cases, the triangulations can be described.*

For compact 3-manifolds with boundary, we modify the definition of the average edge order as follows. Put $E_0(K) = E_i(K) + E_\partial(K)/2$ and $F_0(K) = F_i(K) + F_\partial(K)/2$, where $E_i(K)$ (resp. $F_i(K)$) is the number of edges (resp. faces) in $intK = K \backslash \partial K$, and $E_\partial(K)$ (resp. $F_\partial(K)$) is the number of edges (resp. faces) on ∂K. Then we define the *average edge order* $\mu_0(K)$ of a triangulation K of a 3-manifold with boundary to be $3F_0(K)/E_0(K)$. This is the average of the orders of edges of K, where for each edge e in ∂K

(1) the order of e is defined to be the number of triangles incident to e, where we count triangles on the boundary with weight $1/2$, and
(2) the order of e is counted with weight $1/2$ when we take the avarage.

By using this average edge order, we have the following theorem.

Theorem 2. *Let K be any triangulation of a compact connected 3-manifold M with non-empty boundary. Then*

(a) $2 \leq \mu_0(K) < 6$, *equality holds if and only if K is the triangulation of one 3-simplex.*

(b) *For any rational number r with $4 < r < 6$, there is a triangulation K' of M such that $\mu_0(K') = r$.*

(c) *If $\mu_0(K) < 4$, then K is a triangulation of B^3. There are an infinite number of distinct such triangulations, but for any constant $c < 4$ there are only finitely many triangulations K with $\mu_0(K) \leq c$.*

(d) *If $\mu_0(K) = 4$, then K is a triangulation of $B^3, D^2 \times S^1$, or $D^2 \tilde{\times} S^1$. Furthermore, in the last two cases, the triangulations can be described.*

References

1. F. Luo and R. Stong, *Combinatorics of triangulations of 3-manifolds*, Trans. Amer. Math. Soc. **337** (1993), 891-906.
2. C. P. Rourke and B. J. Sanderson: Introduction to Piecewise-Linear Topology, Springer, Berlin, 1972
3. M. Tamura, *The average edge order of triangulations of 3-manifolds*, preprint
4. M. Tamura Th *average edge order of triangulations of 3-manifolds with boundary modification,*
5. D. [illegible] *ower bound problem for 3 and 4 manifolds*, Acta math. **125** (1970), 75-107.

Department [illegible] Mathematics, Osaka University, Toyonaka, Osaka 560, Japan
E-mail address: tamura@math.sci.osaka-u.ac.jp

Kauffman bracket of plane curves

S. Chmutov,* V. Goryunov†

There exists a straightforward way to get an invariant of an immersed cooriented hypersurface C in a smooth manifold N. We lift C to the manifold M of cooriented contact elements of N, that is to the spherisation ST^*N of the cotangent bundle of N. This gives us an embedded submanifold L_C in M. Now we take the value of a known invariant of embeddings on $L_C \hookrightarrow M$ as the invariant of our initial immersion $C \looparrowright N$.

This general approach was used in [5, 6] to define an invariant of an immersed plane curve $C \hookrightarrow \mathbf{R}^2$. There a Kontsevich type integral [4] was taken as a known invariant of knots L_C in the solid torus $M = ST^*\mathbf{R}^2$.

In fact, the described procedure allows to induce invariants not only on immersed $C \looparrowright N$ but also on submanifolds with certain "admissible" singularities. Namely, the manifold $M = ST^*N$ has a natural contact structure. Our lifting L_C is a Legendrian submanifold with respect to this structure. The hypersurface C is called *the front* of L_C. So it is natural to permit C to have singularities which may appear as singularities of fronts of smooth Legendrian submanifolds generically embedded into M.

In the simplest case of $N = \mathbf{R}^2$, the lifted submanifolds L_C are Legendrian links in the solid torus $M = ST^*\mathbf{R}^2$ and the "admissible" singularities of the underlying plane fronts are cusps. Thus we can induce an invariant on collections of closed cooriented plane curves which may have only transverse

*Program Systems Institute, Pereslavl-Zalessky, 152140, Russia. E-mail: chmutov@math.botik.yaroslavl.su. Partialy supported by the Fellowship "Emma e Giovanni Sansone", the International Science Foundation and the Russian Foundation for Fundamental Research.

†Department of Applied Mathematics, Moscow Aviation Institute, Volokolamskoe sh., 4, 125871 Moscow, Russia and Department of Mathematical Sciences, The University of Liverpool, Liverpool, L69 3BX, UK. E-mail: goryunov@liv.ac.uk. Supported by an RDF grant of The University of Liverpool.

double points and cusps as singularities. We call such collections *normal fronts*. Their invariant may come from any invariant of links in the solid torus.

In the talk we will take the Kauffman bracket of links in a solid torus (see [7]) as a known invariant to be induced on plane fronts.

Generic one-parameter families of plane fronts contain various types of bifurcations: triple-point and self-tangency perestroikas, passing of a cusp through a branch, birth/death of a pair of cusps. Self-tangencies can be subdivided into two types: *dangerous* (when the coorientations of the branches coincide) and safe (when they are opposite). The corresponding Legendrian link in the solid torus changes its topology only under dangerous self-tangencies.

So any invariant of plane fronts induced from a link invariant should not change under any perestroikas except for dangerous self-tangencies. The first invariant with this property was introduced by Arnold [1] and called J^+. Following this we call any invariant of plane fronts having similar restrictions on its changes a *J^+-type invariant.*

Legendrian lowering of the skein relations of the Kauffman bracket produces a set of skein relations in terms of plane fronts. It turns out that they are sufficient to inambiguously calculate the invariant (we call it the *Kauffman bracket* as well) of any normal front completely staying on the plane, without any mentioning of the links in the solid torus.

Theorem 1. *There exists a unique J^+-type invariant $< C > \in \mathbf{Z}[A^{\pm 1}, h]$ of a normal front C satisfying the following properties:*

1) 

2) $\langle \infty \rangle = -A^3$;

3) $\langle \bigcirc \rangle = -A^3 h$;

4) $< C_1 \cdot C_2 >= -(A^2 + A^{-2}) < C_1 > \cdot < C_2 >$, *for* $C_1 \neq \emptyset, C_2 \neq \emptyset$.

Here $C_1 \cdot C_2$ is a collection of two fronts C_1 and C_2 which lie in different half-planes with respect to a certain line in $\mathbf{R}^2$.

Similar to the definition of a finite order invariant of knots in [9, 3], one can introduce a notion of a J^+-type invariant of finite order. Consideration of the coefficients of the Kauffman bracket from this point of view provides

Theorem 2. *Set $A = e^t$ in the Kauffman bracket of a plane front C and expand the result in a power series in t. Then the coefficient at t^n in the series $< C > |_{A=e^t}$ is a J^+-type invariant of order at most n in Vassiliev sense.*

Their exists a way to calculte the weight systems of the above coëfficients. It turns out that the first coefficient is basicly the quantum deformation of the Bennequin invariant [2] introduced recently by M.Polyak [8]. The involved space generated by corresponding chord diagrams is more complicated then that of the original Vassiliev theory. For example, each graded part of this space is infinite-dimensional.

References

[1] Arnold, V.I.: Topological invariants of plane curves and caustics. University Lecture Series **5**. Providence, RI: AMS 1994

[2] Bennequin, D.: Enterlacements et équation de Pfaff. Astérisque **107-108**, 87–162 (1983)

[3] Birman, J., Lin, X.-S.: Knot polynomials and Vassiliev invariants. Invent.Math. **111**, 225–270 (1993)

[4] Goryunov, V.: Vassiliev invariants of knots in $\mathbf{R}^3$ and in a solid torus. Preprint 1-95, The University of Liverpool (1995)

[5] Goryunov, V.: Vassiliev type invariants in Arnold's J^+–theory of plane curves without direct self-tangencies. Preprint 2-95, The University of Liverpool (1995)

[6] Goryunov, V.: Finite order invariants of framed knots in a solid torus and in Arnold's J^+-theory of plane curves. To appear in: *Proceedings on Geometry and Physics. Aarhus 1995*, Marcel Dekker

[7] Hoste, J., Przytycki, J.H.: An invariant of dichromatic links. Proc. AMS **105**, 1003–1007 (1989)

[8] Polyak, M.: On the Bennequin invariant of Legendrian curves and its quantization. Preprint, Bonn: Max-Planck-Institut für Mathematik (1995)

[9] Vassiliev, V.A.: Cohomology of knot spaces. In: *Theory of Singularities and its Applications* (V.I.Arnold ed.), Adv.Sov.Math. 1, 23–69. Providence, RI: AMS 1990

Every 2-link with 2 components is link-homotopic to the trivial 2-link

Fujitsugu HOSOKAWA and Shin'ichi SUZUKI

Abstract

In the previous paper [1], we asserted that every link of two 2-spheres $K_1 \cup K_2$ in the 4-sphere S^4 is link-homotopic to the trivial 2-link, but as J. P. Levine pointed out in [5], there is a gap in the proof. In fact, the link-homotopy did not define in $R^3[3, \infty)$. In this talk, we will construct a required link-homotopy, and complete the proof.

Definition 1. An *n-link with c components* is an embedding $L : S^n \coprod \cdots \coprod S^n \to S^{n+2}$ of the disjoint union of c n-spheres into the $(n+2)$-sphere S^{n+2}. In particular, an n-link with one component will be called an n-knot.

An n-link $L : S^n \coprod \cdots \coprod S^n \to S^{n+2}$ is called *trivial*, iff there exists an embedding $\widetilde{L} : D^{n+1} \coprod \cdots \coprod D^{n+1} \to S^{n+2}$ of the disjoint union of $(n+1)$-disks with $\widetilde{L}|\partial D^{n+1} \coprod \cdots \coprod \partial D^{n+1} = L$.

As an n-link $L : S^n \coprod \cdots \coprod S^n \to S^{n+2}$ with c components, we refer to the image $L(S^n \coprod \cdots \coprod S^n)$ by $K_1 \cup \cdots \cup K_c$. We shall use the motion picture method to describe the configuration of subspaces of R^4, $R^4 \cup \{\infty\} = S^4$, and we use the same notation and definitions as [4] and [6]. Then we have the following:

Proposition. *Any locally flat* 2*-link* $(K_1 \cup \cdots \cup K_c \subset R^4)$ *can be deformed into a* 2*-link* $(\widetilde{K_1} \cup \cdots \cup \widetilde{K_c} \subset R^4)$ *in the normal form by an ambient isotopy of* R^4*; that is,* $(\widetilde{K_1} \cup \cdots \cup \widetilde{K_c} \subset R^4)$ *is in the following position:*
(1) *all maximum-disks of* $\widetilde{K_1} \cup \cdots \cup \widetilde{K_c}$ *are in the hyperplane* $R^3[3]$,
(2) *all minimum-disks of* $\widetilde{K_1} \cup \cdots \cup \widetilde{K_c}$ *are in the hyperplane* $R^3[-3]$,
(3) *all saddle-bands of* $\widetilde{K_1} \cup \cdots \cup \widetilde{K_c}$ *are in the hyperplanes* $R^3[-1]$ *and* $R^3[1]$, *and so,*
(4) *the equatorial cross-sectional* 1*-link* $((\widetilde{K_1} \cup \cdots \cup \widetilde{K_c}) \cap R^3[0] \subset R^3)$ *is a* 1*-link with c components.*

We use the following criterion of 2-knots due to A. Kawauchi [3].

Theorem 1 [3]. *Let* $(K \subset R^4)$ *be a locally flat* 2*-knot in the normal form. If the equatorial cross-sectional* 1*-knot* $(K \cap R^3[0] \subset R^3)$ *is a trivial* 1*-knot, then the second* *homotopy group $\pi_2(S^4 - K)$ is trivial.*

Definition 2. Let $P_1, \cdots, P_\mu$ be polyhedra, and let $\mathcal{P} = P_1 \coprod \cdots \coprod P_\mu$ be their disjoint union, and let X be a manifold.

A continuous map $f : \mathcal{P} \to X$ is said to be a *link-map*, iff $f(P_i) \cap f(P_j) = \emptyset$ for $i \neq j$.

Two link-maps $f_0, f_1 : \mathcal{P} \to X$ will be called *link-homotopic*, iff there exists a homotopy $\{\eta_t\}_{t\in I} : \mathcal{P} \to X$ such that $\eta_0 = f_0, \eta_1 = f_1$ and η_t is also link-map for each $t \in I$.

We can now formulate our main theorem.

Theorem 2. *Every 2-link $L : S^2 \coprod S^2 \to S^4$ is link-homotopic to a trivial 2-link.*

A key lemma is the following:

Lemma [2]. *Let $\mathcal{O}_i = O_{i1} \cup \cdots \cup O_{in(i)}$ be a trivial 1-link with $n(i)$ components in $S^3 = \partial D^4$ for $i = 1, \cdots, \mu$, such that $\mathcal{O}_1 \cup \cdots \cup \mathcal{O}_\mu$ is also a trivial 1-link with $n = n(1) + \cdots + n(\mu)$ components. Let $\mathcal{P}_i = D_{i1} \coprod \cdots \coprod D_{in(i)}$ be the disjoint union of $n(i)$ 2-disks for $i = 1, \cdots, \mu$, and let $\mathcal{P} = \mathcal{P}_1 \coprod \cdots \coprod \mathcal{P}_\mu$. Let f and k be non-degenerate link-maps of $\mathcal{P}$ into S^3 such that $f(\partial D_{ij}) = O_{ij} = k(\partial D_{ij})$ for $i = 1, \cdots, \mu$ and $j = 1, \cdots, n(i)$. Then, f and k are link-homotopic in D^4 keeping $\mathcal{O}_1 \cup \cdots \cup \mathcal{O}_\mu$ fixed.*

References

[1] F. Hosokawa and S. Suzuki: Linking 2-spheres in the 4-sphere, *Kobe J. Math.*, 4(1987), 193-208.

[2] F. Hosokawa and S. Suzuki: On singular cut-and-pastes in the 3-space with applications to link theory, *Revista Mat. Univ. Complut. Madrid*, 8(1995), 155-168.

[3] A. Kawauchi: A partial Poincare duality theorem for infinite cyclic coverings, *Quart. J. Math. Oxford*(2), **26**(1975), 437-458.

[4] A. Kawauchi, T. Shibuya and S. Suzuki: Descriptions on surfaces in four-space I, *Math. Sem. Notes Kobe Univ.*, **10**(1982), 75-125.

[5] J. P. Levine: *Math. Review* **89j:57016**.

[6] S. Suzuki: Knotting problems of 2-spheres in 4-sphere, *Math. Sem. Notes Kobe Univ.*, 4(1976), 241-371.

THE DETERMINATION OF THE PAIRS OF TWO-BRIDGE KNOTS OR LINKS WITH GORDIAN DISTANCE ONE

BY ICHIRO TORISU

Department of Mathematics, Osaka University
Toyonaka, Osaka
560, Japan

1. INTRODUCTION

For any two knots or links K, K' in S^3, we can define the Gordian distance from K to K', denoted by $d_G(K, K')$, to be the minimal number of crossing changes needed to deform a diagram of K into that of K', where the minimum is taken over all diagrams of K from which one can obtain a diagram of K'. Then d_G defines a metric on the space of the equivalence classes of knots or links. If O is a trivial knot or link, then $d_G(K, O)$ is the unknotting or unlinking number of K, denoted by $u(K)$ (see [4]).

In this paper we determine the pairs of two-bridge knots or links with Gordian distance one. This result can be thought as a generalization of those of Kanenobu-Murakami [2] and Kohn [3].

2. MAIN THEOREM

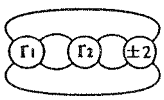
Let $S(p, q)$ be the two-bridge knot or link whose two-fold branched cover is the lens space $L(p, q)$, where p and q are relatively prime and $p \geq 0$. When p is even, $S(p, q)$ is a two component link, for p odd, $S(p, q)$ is a knot.

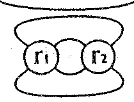
Our main theorem is then the following.

Theorem 1. *Let $S(p, q)$ and $S(r, s)$ be two-bridge knots or links with $ps < rq$. Then the following conditions are equivalent:*
(i) $d_G(S(p, q), S(r, s)) = 1$.
(ii) There exist pairs of relatively prime integers (m, n) and (a, b) such that $rb - sa = 1$ and

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} a & r \\ b & s \end{pmatrix} \begin{pmatrix} 2n^2 \\ 2mn \pm 1 \end{pmatrix}.$$

(iii) There exist rational numbers r_1 and r_2 such that

$S(p, q) =$ 

$S(r, s) =$ 

where 

is a rational tangle of slope r_i (for the definition of a rational tangle, see [1, Chapter 12]).

Remark 2. (i) $S(1,0)$ is a trivial knot. So the condition (ii) of Theorem 1 says that $d_G(S(p,q),S(1,0))=1$ if and only if

$$\begin{aligned} S(p,q) &= S(2an^2+2mn\pm 1, 2n^2) \\ &= S(2m'n\pm 1, 2n^2) \end{aligned}$$

where $m' = m + an$. Therefore Theorem 1 is a generalization of Kanenobu-Murakami's theorem [2].

(ii) $S(0,-1)$ is a trivial link. So the condition (ii) of Theorem 1 also says that $d_G(S(p,q),S(0,-1))=1$ if and only if

$$\begin{aligned} S(p,q) &= S(2n^2, 2bn^2-(2mn\pm 1)) \\ &= S(2n^2, 2m'n\pm 1) \end{aligned}$$

where $m' = -m+bn$. Therefore Theorem 1 is also a generalization of Kohn's theorem [3].

Acknowledement

The author would like to thank K. Kawakubo for his kind encouragement. He is also grateful for several valuable conversations concerning the material of this paper offered by M. Sakuma.

References

1. Burde, G. and Zieschang, H., *Knots*, de Gruyter Studies in Mathematics, no. 5, Walter de Gruyter, Berlin, 1985.
2. Kanenobu, T. and Murakami, H., *Two-bridge knots with unknotting number one*, Proc. Amer. Math. Soc. **98** (1986), 499-502.
3. Kohn, P., *Two-bridge links with unlinking number one*, Proc. Amer. Math. Soc. **113** (1991), 1135-1147.
4. Murakami, H., *Some Metrics on Classical Knots*, Math. Ann. **270** (1985), 35-45.
5. Torisu, I., *A note on Montesinos links with unlinking number one (conjectures and partial solutions)*, preprint.
6. Torisu, I., *The determination of the pairs of two-bridge knots or links with Gordian distance one*, preprint.

E-mail address: torisu@math.sci.osaka-u.ac.jp

Spatial-Graph Isotopy for Trivalent Graphs and Minimally Knotted Embeddings

TERUHIKO SOMA

In this talk, we will discuss spatial-graph isotopy for trivalent graphs, and give a connection between this equivalence relation and minimally knotted embeddings of graphs into 3-space.

A *graph* G is a finite, 1-dimensional CW-complex. Here, we also assume that graphs have no isolated vertices. Let $\mathcal{S}(G)$ be the set of all piecewise linear embeddings $\Gamma : G \longrightarrow \mathbf{R}^3$. For a planar graph G, an embedding $\Gamma \in \mathcal{S}(G)$ is said to be *minimally knotted* if Γ itself is knotted but, for any proper subgraph H of G, the restriction $\Gamma|_H$ is unknotted. Examples of minimally knotted embeddings were presented by Kinoshita [2], Suzuki [4] among others.

In [5], Taniyama introduced several kinds of equivalence relations in $\mathcal{S}(G)$: ambient isotopy, isotopy, cobordism, I-equivalence, homotopy, weak-homotopy, homologous and $\mathbf{Z}_2$-homologous. Rather weak equivalence relations among them such as homotopy, weak-homotopy or homology were studied by some authors. Here, we will study stronger equivalence relations (especially spatial-graph isotopy).

Consider a pair $\Gamma, \Gamma' : G \longrightarrow \mathbf{R}^3 \in \mathcal{S}(G)$ admitting a piecewise linear embedding $\Phi : G \times I \longrightarrow \mathbf{R}^3 \times I$ such that $\Phi(x,t) = (\Gamma(x), t)$ for any $(x,t) \in G \times [0,\varepsilon]$, $\Phi(x,t) = (\Gamma'(x), t)$ for any $(x,t) \in G \times [1-\varepsilon, 1]$, where ε is a sufficiently small positive number and I is the closed interval $[0,1]$. We say that Γ is *ambient isotopic* to Γ' if Φ is locally flat and level-preserving, Γ is *cobordant* to Γ' if Φ is locally flat, and Γ is *isotopic* to Γ' if Φ is level-preserving. Note that an isotopy is quite different from an ambient isotopy. For example, all knots in $\mathbf{R}^3$ are isotopic to the trivial knot in our case. For our convenience, we denote by $[\Gamma]_{\text{cobor}}$ (resp. by $[\Gamma]_{\text{isotopy}}$) the subset of $\mathcal{S}(G)$ consisting of all elements cobordant to (resp. isotopic to) $\Gamma \in \mathcal{S}(G)$, and call it the *cobordism class* (resp. the *isotopy class*) of Γ.

Kawauchi [1] and Wu [6] proved independently that any planar graph G without free edges admits a minimally knotted embedding. The following theorem implies that such an embedding can be constructed in the cobordism class of a planar embedding.

Theorem 1. *Suppose that G is any graph without free edges and admitting a planar embedding $\Gamma_0 : G \longrightarrow \mathbf{R}^2 \subset \mathbf{R}^3$. Then, the cobordism class $[\Gamma_0]_{\text{cobor}}$ contains a minimally knotted embedding.*

Our proof is based on Wu's. However, we use a rather simple tangle to construct minimally knotted embeddings.

Any isotopy between two elements $\Gamma, \Gamma' \in \mathcal{S}(G)$ is realized by a sequence of blowing-downs $\searrow$ and ups $\nearrow$, for example:

$$\Gamma = \Lambda_0 \nearrow \Lambda_1 \nearrow \Lambda_2 \searrow \Lambda_3 \nearrow \Lambda_4 \searrow \Lambda_5 \searrow \Lambda_6 \nearrow \Lambda_7 = \Gamma'.$$

It is useful for the study of spatial-graph isotopy to rearrange the order of blowing-ups and downs. In the case of G trivalent, we have the following rearrangement theorem which is a basic result for further investigation.

Theorem 2. *Let G be a trivalent graph, and let $\Gamma_1, \Gamma_2 : G \longrightarrow \mathbf{R}^3$ be embeddings isotopic to each other. Then, there exists an embedding $\Gamma_3 : G \longrightarrow \mathbf{R}^3$ and a sequence of blowing-downs followed by blowing-ups such that $\Gamma_1 \searrow \cdots \searrow \Gamma_3 \nearrow \cdots \nearrow \Gamma_2$.*

For a trivalent graph G, an element $\Gamma^{\text{red}} \in \mathcal{S}(G)$ is said to be *isotopically reduced* if the ambient-isotopy type of Γ^{red} can not be changed by any blowing-down of Γ^{red}. By using Haken's Finiteness Theorem, it is shown that the isotopy class $[\Gamma]_{\text{isotopy}}$ of any $\Gamma \in \mathcal{S}(G)$ contains an isotopically reduced element. Corollary 1 gives an interaction between the two equivalence relations, isotopy and ambient isotopy, in $\mathcal{S}(G)$.

Corollary 1. *Let G be a trivalent graph, and let $\Gamma_1, \Gamma_2 : G \longrightarrow \mathbf{R}^3$ be embeddings. Suppose that Γ_i^{red} is any isotopically reduced element in $[\Gamma_i]_{\text{isotopy}}$ for $i = 1, 2$. Then, Γ_1 is isotopic to Γ_2 if and only if Γ_1^{red} is ambient isotopic to Γ_2^{red}.*

Corollary 1 is restated as follows.

Corollary 2. *For any embedding $\Gamma : G \longrightarrow \mathbf{R}^3$ of a trivalent graph G, the class $[\Gamma]_{\text{isotopy}}$ contains a unique isotopically reduced element up to ambient isotopy.*

It is easy to show that any embedding $\Gamma : \Theta \longrightarrow \mathbf{R}^3$ of a theta-curve Θ is isotopic to a planar embedding $\Gamma_0 : \Theta \longrightarrow \mathbf{R}^2 \subset \mathbf{R}^3$, i.e. $[\Gamma_0]_{\text{isotopy}} = \mathcal{S}(\Theta)$. In particular, $[\Gamma_0]_{\text{isotopy}}$ contains a minimally knotted embedding. The following theorem implies the converse.

Theorem 3. *Let G be a trivalent graph admitting a planar embedding $\Gamma_0 : G \longrightarrow \mathbf{R}^2 \subset \mathbf{R}^3$. If the isotopy class $[\Gamma_0]_{\text{isotopy}}$ contains a minimally knotted embedding, then G is a theta-curve.*

Corollary 3 follows immediately from Theorems 1 and 3.

Corollary 3. *Suppose that G is a trivalent graph admitting a planar embedding* $\Gamma_0 : G \longrightarrow \mathbf{R}^2 \subset \mathbf{R}^3$. *If G is not a theta-curve, then* $[\Gamma_0]_{\text{isotopy}}$ *does not contain* $[\Gamma_0]_{\text{cobor}}$.

Final Remark. The speaker knows some sporadic results extending Theorems 2-3 and Corollaries 1-3 to certain non-trivalent graphs, and feels that it would be possible to prove such theorems and corollaries for any graphs.

References

1. A. Kawauchi, Almost identical imitations of (3,1)-dimensional manifold pairs, Osaka J. Math. 26 (1989), 743-758.
2. S. Kinoshita, On elementary ideals of polyhedra in the 3-sphere, Pacific J. Math. 42 (1972) 89-98.
3. T. Soma, Spatial-graph isotopy for trivalent graphs and minimally knotted embeddings, Topology Appl. (to appear).
4. S. Suzuki, Almost unknotted θ_n-curves in the 3-sphere, Kobe J. Math. 1 (1984) 19-32.
5. K. Taniyama, Cobordism, homotopy and homology of graphs in $\mathbf{R}^3$, Topology 33 (1994) 509-523.
6. Y.Q. Wu, Minimally knotted embeddings of planar graphs, Math. Z. 214 (1993) 653-658.

Band-generator presentation of the braid group and its advantage

Ki Hyoung Ko

The elements of the n-string braid group B_n defined by:

$$a_{ts} = (\sigma_{t-1}\sigma_{t-2}\cdots\sigma_{s+1})\sigma_s(\sigma_{t-1}\sigma_{t-2}\cdots\sigma_{s+1})^{-1}.$$

for $1 \leq s > t \leq n$ together with 4-term defining relations:

$$a_{ts}a_{rq} = a_{rq}a_{ts} \qquad \text{if} \quad (t-r)(t-q)(s-r)(s-q) > 0$$

$$a_{ts}a_{sr} = a_{tr}a_{ts} = a_{sr}a_{tr} \qquad \text{for all } r, s, t \quad \text{with} \quad 1 \leq r < s < t \leq n$$

give a presentation of B_n.

The word problem for this presentation is solved by decomposing a word into a canonical form. The complexity of this decomposition is quadratic in both the braid index and the word length. The conjugacy problem is also solved by generating a "super summit set" uniquely determined by a conjugacy class. A linear bound with respect to the braid index for the number of conjugations required to get an element in the super summit set is found. For B_4, the shortest word problem up to conjugacy can be solved in this presentation.

Arithmetic and geometry of some cone manifolds

J.M.Montesinos(U.C.Madrid)

In this talk joint work with Hugh Hilden and María Teresa Lozano will be discussed.

Let K be a hyperbolic knot in S^3. The character variety of representations of $\pi_1(S^3 - K)$ into $PSL(2, \mathbb{C})$ is an algebraic variety, denoted by $\mathcal{C}(K)$, whose points can be interpreted as holonomies of geometric structures in $S^3 - K$. Among these geometric structures are the cone-manifold structures (K, α) in S^3, with singular set the knot K and angle α around K. In particular for angle $\alpha = (2\pi)/n$, the cone-manifold $(K, (2\pi)/n)$ is the orbifold structure in S^3, with singular set the knot K and cyclic isotropy group of order n.

Let α_e be an angle such that the cone-manifold (K, α_e) is Euclidean. Then, we have prove that $2\cos\alpha_e$ is an algebraic number. Its minimal polynomial (called the h-polynomial) is a knot invariant and can be computed from $\mathcal{C}(K)$.

We use also $\mathcal{C}(K)$ to detect the arithmeticity of some orbifolds structures.

Geometric invariants of cone manifolds

María Teresa Lozano

This is joint work with Hugh Hilden and José María Montesinos.

After the Mostow Rigidity Theorem, each geometric invariant of a hyperbolic 3-manifold is a topological invariant. We are interested in computation of the volume and the Chern-Simons invariant of hyperbolic 3-manifolds.

Some hyperbolic 3-manifolds are obtained by covering of S^3 branched over a hyperbolic knot K. Given a hyperbolic knot K, there exists an algebraic variety, $\mathcal{C}(K)$, parametrizing geometric structures in $S^3 - K$. We use $\mathcal{C}(K)$ to obtain the volume and the Chern-Simons invariant of the orbifold structure (K, n) in S^3 with singular set the knot K and cyclic isotropy group of order n. Then, we can obtain the volume and the Chern-Simons invariant of some hyperbolic 3-manifolds, those obtained by certain coverings of S^3 branched over the knot K.

The method to compute volumes of the orbifold structure (K, n), is to apply Schlaffli Formula for the volume to the family of cone-manifold structures (K, α) in S^3, with singular set the knot K and angle α around K.

To follow the same program to compute the Chern-Simons invariant, we have prove a "Schlaffli Formula" for a generalized Chern-Simons function on the family of cone-manifold structures (K, α).

(observed by D.Bullock).

(iv) We introduce the concept of a skein algebra of an abstract group and show that for finitely generated group, modulo nil-radical, it is equal to the coordinate ring of the $SL(2, C)$ character variety of the group. We show several examples for which the nil-radical is trivial (e.g. surface groups, abelian groups, finite groups).

(v) We discuss torsion in Kauffman bracket skein modules. We show that a manifold with an incompressible 2-sphere or torus has often a torsion in its skein module. We consider, in more details, the case of the double of the figure eight knot complement. Here to detect a torsion, we use the hyperbolic structure on the figure eight knot complement.

Algebraic topology based on knots

by Józef H. Przytycki

Abstract.

Our goal is to build an algebraic topology based on knots. We call the main object used in the theory *a skein module* and we associate it to any 3-dimensional manifold. In short skein modules are quotients of free modules over ambient isotopy classes of links in 3-manifolds by properly chosen local (skein) relations.

These new objects are not sufficiently understood yet, however their properties seem to be topologically very significant. In particular, one should look for their features similar to Seifert-Van Kampen or Mayer-Vietoris theorems. Another interesting question concerns the relation between the skein modules of the base and the skein modules of the covering space, for coverings and branched coverings. At present we can say something about the above question only in a very special situation and then the result concerns symmetries of links. As in the case of homologies, one should try to understand the free and torsion part of the module. In particular, the torsion of the module seems to reflect the geometry of the manifold (i.e. their incompressible surfaces).

We concentrate, in this talk, on the skein module related to the Jones polynomial (via the Kauffman bracket), $\mathcal{S}_{2,\infty}(M)$, describing, in particular, the recent work with A.Sikora.

(i) We describe several examples of manifolds for which the Kauffman bracket skein module is fully computed (e.g. lens spaces, $(2,n)$ torus knot complements).

(ii) We describe the algebra structure of the Kauffman bracket skein module for a surface cross interval. We show that the algebra has no zero-divisors.

(iii) We discuss the relation of the Kauffman bracket skein module with the $SL(2,C)$ character variety of the fundamental group of a 3-manifold

Generalized unknotting number one two-bridge knot

YOSHIAKI UCHIDA

Let k be a knot in S^3. An ordinary unknotting operation is an operation which changes the overcrossing and undercrossing at a double point of a diagram of k. The unknotting number of a knot k in S^3 is the minimun number of this unknotting operations needed to deform k into a trivial knot. The unknotting number is not easy to calculate. For two-bridge knots, Kanenobu-Murakami [KM] determined unknotting number one.

Theorem[KM]. *Let k be a two-bridge knot with unknotting number one. Then k is equivalent to*

(1) *$S(p, 2n)$ where p is an odd integer (> 1) and m and n are coprime positive integers with $2mn = p \pm 1$, or*
(2) $C(a, a_1, a_2, \cdots, a_k, \pm 2, -a_k, \cdots, -a_2, -a_1)$.

Here $S(p, q)$ is Schubert's notation for a two-bridge knot, and $C(c_1, c_2, \cdots, c_r)$ is Conway's notation. If the continued fraction

$$c_1 + \frac{1}{c_2 +} \cdots \frac{1}{+c_r}$$

is equal to p/q, then $C(c_1, c_2, \cdots, c_r)$ is equivalent to $S(p, q)$.

Moreover Kohn determened two-bridge links with unlinking number one.

Theorem [K]. *Let k be a two-bridge link with unlinking number one. Then k is equivalent to*

(1) *$S(2n, 2m \pm 1)$, where m and n are coprime, or*
(2) $C(a_1, a_2, \cdots, a_k, \pm 2, -a_k, \cdots, -a_2, -a_1)$.

Now we can consider the unknotting operation is an operation that exchanges a trivial tangle with a rational tangle $1/2$. (See Figure 1.)

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

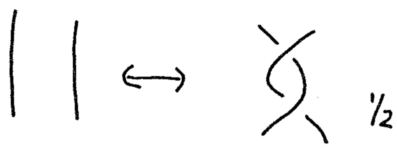


Figure 1.

So, we can consider an operation which exchanges a trivial tangle with a rational tangle b/a, and we call this operation b/a-type operation. Note that it is not known that b/a-type operation is unknotting operation or not, except 1/2-type. But Y. Nakanishi proved that for two-bridge knots 1/3-type, and 1/4-type operations are unknotting operations. We can determine b/a-unknotting number one two-bridge knot.

Theorem. *Let k be a b/a-unknotting number one two-bridge knot. Then $b = 1$ and k is equivalent to* $C(a_0, a_1, a_2, \cdots, a_k, \pm a, -a_k, \cdots, -a_2, -a_1)$

References

[K] P. Kohn, *Two-bridge links with unlinking number one*, Proc. Amer. Math. Soc. **113** (1991), 1135-1147.

[KM] T. Kanenobu and H. Murakami, *Two-bridge knots with unknotting number one*, Proc. Amer. Math. Soc. **98** (1986), 499-502.

Department of Mathematical Sciences
Faculty of Science
Yamagata University
Yamagata 990
e-mail yuchida@kszaoh3.kj.yamagata-u.ac.jp

A proof of Melvin-Morton conjecture

S. Chmutov*

Abstract

The coloured Jones function $J_k(K)$ is an invariant of a framed knot K defined by the irreducible k-dimensional representation of the quantum group $U_q(sl_2)$. The unframed coloured Jones function $J_k^u(K)$ is obtained from $J_k(K)$ by multipication by $q^{-C\cdot writhe(K)}$, where C is the quadratic Casimir number of the representation. The ordinary Jones polynomial corresponds to the case $k = 2$ and the standard 2-dimensional representation of sl_2.

Set $q = e^h$. Write $J_k^u(K)$ as a power series in h:

$$J_k^u(K) = \sum_{d=0}^{\infty} J_{d,k}^u(K)h^d.$$

The coefficient $J_{d,k}^u(K)$ is a Vassiliev invariant of order d. $J_k^u(K)$ tends to k as h tends to zero. So $J_{0,k}^u(K) = k$

First Melvin-Morton conjecture ([MM],[BNG]). *$J_{d,k}^u(K)/k$ is a polynomial in k of degree at most d:*

$$J_{d,k}^u(K)/k = \sum_{0\le j\le d} b_{d,j}(K)k^j; \qquad b_{0,0}(K) = 1.$$

The conjecture has been proved in [BNG]. Also it easily follows from results of [ChV].

Define the *Melvin-Morton function* $MM(K)$ as the highest degree part of the coloured Jones function:

$$MM(K) = \sum_{d=0}^{\infty} b_{d,d}(K)h^d.$$

*Program Systems Institute, Pereslavl-Zalessky, 152140, RUSSIA (**chmutov@botik.ru**).

Second Melvin-Morton conjecture ([MM], [BNG]).

$$MM(K) \cdot A_K(e^h) = \frac{e^{h/2} - e^{-h/2}}{h},$$

where $A_K(q)$ is the Alexander-Conway polynomial.

The conjecture has been proved in [BNG]. In fact, D.Bar-Natan and S.Garoufalidis reduced this second conjecture to the following statement about weight systems. Let $A_K(e^h) = \sum_{d=0}^{\infty} a_d(K)h^d,\ a_0(K) = 1$ be the power series expansion of the Alexander-Conway polynomial. Consider the symbols $S_{MM,d}$ and $S_{A,d}$ of the coefficients $b_{d,d}(\cdot)$ and $a_d(\cdot)$ respectively. These are some weight systems, i.e. functions on chord diagrams satisfying the four-term relations. The whole space $\mathcal{W}$ of weight systems carries a graded Hopf algebra structure

$$\mathcal{W} = \mathcal{W}_0 \oplus \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \dots$$

which is dual to the Hopf algebra of chord diagrams

$$\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1 \oplus \mathcal{A}_2 \oplus \dots .$$

The symbol of a Vassiliev invariant of order d belongs to $\mathcal{W}_d$. So $S_{MM,d} \in \mathcal{W}_d$ and $S_{A,d} \in \mathcal{W}_d$. Let S_{MM} and S_A be the formal power series

$$S_{MM} = \sum_{d=0}^{\infty} S_{MM,d}; \qquad S_A = \sum_{d=0}^{\infty} S_{A,d}.$$

They belong to the graded completion $\overline{\mathcal{W}}$ of $\mathcal{W}$.

The second Melvin-Morton conjecture is equivalent to the equality:

$$S_{MM} \cdot S_A = 1. \tag{1}$$

The most difficult part of the paper [BNG] concentrated on the proof of this equality.

I give a new proof of the equality (1) based on the combinarorial description ([ChV]) of the primitive space of Hopf algebra $\mathcal{A}$. The equality (1) follows from

Proposition. *Let $\mathcal{P}_d \subset \mathcal{A}_d$ be the primitive space. $d > 0$. Then*

i) $(S_{MM,d} + S_{A,d})|_{\mathcal{P}_d} = 0;$

ii) for even d $S_{MM,d}|_{\mathcal{P}_d}$ *takes two values* : 0 *and* 2,
for odd d $S_{MM,d}|_{\mathcal{P}_d} = 0.$

The function $S_{MM,d}|_{\mathcal{P}_d}$ was essentialy described in [ChV]. So one needs to describe only the function $S_{A,d}|_{\mathcal{P}_d}$. This can be done in a pure combinatorial way.

References

[ChV] S. V. Chmutov, A. N. Varchenko, *Remarks on the Vassiliev knot invariants coming from sl_2*, preprint of the University of Pisa (February, 1995), submitted to Topology.

[BNG] D. Bar-Natan, S. Garoufalidis, *On the Melvin-Morton-Rozansky conjecture*, Preprint (July, 1994).

[MM] P. M. Melvin, H. R. Morton, *The coloured Jones function*, Comm. Math. Physics. **169** (1995) 501-520.

ON $SU(N)$ INVARIANTS OF KNOTS AND 3-MANIFOLDS

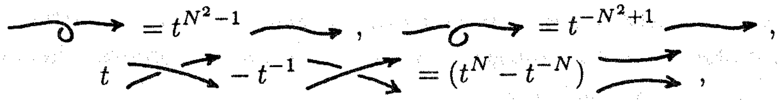
YOSHIYUKI YOKOTA

Graduate School of Mathematics
Kyushu University
Fukuoka, 812, Japan

The purpose of this talk is to give an elementary approach to the quantum $SU(N)$ invariant of knots and 3-manifolds, and to give a brief account of the quantum $PSU(N)$ invariant of 3-manifolds, together with its level-rank duality. Furthermore, an observation on the invariants of mutant knots and 3-manifolds is also given.

We only suppose the existence of the Homfly polynomial for oriented links in a 3-sphere S^3, and first introduce the linear skein $\mathcal{S}(F)$ of a planar surface F which is a complex vector space made up with diagrams in F quotiented by

$$D \cup \bigcirc = \frac{t^N - t^{-N}}{t - t^{-1}} D,$$



where D is arbitrary diagram in F and $\bigcirc$ stands for the boundary of a disk in F disjoint from D. In this talk, F is choosen to be an annulus A, a disk B or a 2-sphere S^2, and t is supposed to be a root of unity.

Let L be a framed link in S^3 represented by a diagram D in S^2. By decorating each component of D with an element of $\mathcal{S}(A)$, we have an element of $\mathcal{S}(S^2)$. By identifying this element with a polynomial in t via the skein relations above, we obtain a multilinear form

$$< \underbrace{\ ,\dots,\ }_{\sharp D} >_D : \underbrace{\mathcal{S}(A) \times \cdots \times \mathcal{S}(A)}_{\sharp D} \to \mathbb{C},$$

where $\sharp D$ denotes the number of components of D. Of course, this multilinear form is invariant under regular isotopy, and so invariant of L.

Now, associated with each Young diagram, say λ, we define an element $\hat{e}_\lambda$ of $\mathcal{S}(A)$ so that, for any $\sharp D$-tupple of Young diagrams, $\lambda_1, \dots, \lambda_{\sharp D}$ say, the quantity

$$< \hat{e}_{\lambda_1}, \dots, \hat{e}_{\lambda_{\sharp D}} >_D$$

behaves nicely under Reidemeister move I. We shall call such quantities the $SU(N)$ invariants of L. The invariants of a 3-manifold M will be expressed as a linear sum of such $SU(N)$ invariants of a framed link representing M.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Abstract of talk to be given at Knots96
Tokyo, Japan, July 1996

Infinite framed link diagrams for open 3-manifolds

Jim Hoste
Pitzer College, Claremont CA 91711

We prove that every open, orientable 3-manifold, M, may be obtained from S^3 by first removing a tame 0-dimensional set X homeomorphic to the space of ends of M, and then performing surgery on a locally finite link in $S^3 - X$ with possibly infinitely many components. If M is non-orientable a similar, but slightly more complicated, situation exists. In either case, the Kirby calculus must be enlarged. We will discuss the general theory of such infinite framed link diagrams.

Which lens spaces can be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants.

M. Sokolov*

We give a criterion that answers a question whether or not two lens spaces can be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants τ_r ([RT]).

We mean that two lens spaces L_{p_1,q_1} and L_{p_2,q_2} can be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants iff there is a level $r \geq 3$ such that $|\tau_r(L_{p_1,q_1})| \neq |\tau_r(L_{p_2,q_2})|$.

S. Yamada [Y] has derived the following explicit formula for the absolute values of the Witten-Reshetikhin-Turaev invariants for any lens spaces for any level $r \geq 3$:

$$|\tau_r(L_{p,q})|^2 = \begin{cases} \frac{1}{r}(1-\cos\frac{2\pi p^*}{r}), & \text{if } d=1; \\ \frac{2}{r}(1-\cos\frac{\pi p'^*}{r}), & \text{if } d=2 \text{ and } c \text{ is even}; \\ \frac{d}{2r}, & \text{if } d>2 \text{ and } c \text{ is even and } q \equiv \pm 1 \pmod{d} \\ & \text{or } d>2 \text{ and } c \text{ is odd and } q \equiv \frac{d}{2} \pm 1 \pmod{d}; \\ 0, & \text{otherwise,} \end{cases}$$

where $d = \gcd(p, 2r)$, $c = \frac{2pr}{d^2}$, $p' = \frac{p}{2}$, and p^*, p'^* are integers such that

$$pp^* \equiv 1 \pmod{r}, \quad p'p'^* \equiv 1 \pmod{r}, \quad \text{and } \frac{p'p'^*-1}{r}p'^* \text{ is even.}$$

Our criterion is derived from this formula directly.

THEOREM. *Lens spaces L_{p_1,q_1} and L_{p_2,q_2} can be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants iff one of the following holds:*

1) $p_1 \neq p_2$;

2) $p_1 = p_2 = p$, q_1 is equal to 1 or $p-1$, and $q_2 \neq 1, p-1$;

3) $p_1 = p_2 = p$, q_1, $q_2 \neq 1, p-1$, and there are $k, v \in \mathbf{N}$ such that $v > 2$, v devides p, and one of the following conditions holds:

a) v is equal to $\frac{q_1+1}{k}$ or $\frac{q_1-1}{k}$, $q_2 \not\equiv \pm 1 \pmod{v}$;

b) v is equal to $\frac{q_2+1}{k}$ or $\frac{q_2-1}{k}$, $q_1 \not\equiv \pm 1 \pmod{v}$.

*Research is supported in part by INTAS, grant No. 94201.

CORROLARY 1. *If p is a prime integer, q_1, $q_2 \neq 1, p-1$, then lens spaces L_{p,q_1} and L_{p,q_2} can not be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants.*

REMARK. It was proved in [J] that if p is prime integer and lens spaces L_{p,q_1} and L_{p,q_2} are not homeomorphic then they can be distinguished by the Witten-Reshetikhin-Turaev invariants.

CORROLARY 2. *The pair $L_{11,2}$ and $L_{11,4}$ are the lens spaces with the minimal first parameter that are homotopic but not homeomorphic and can not be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants. The pair $L_{13,3}$ and $L_{13,5}$ are the lens spaces with the minimal first parameter that are not homotopic and can not be distinguished by the absolute values of the Witten-Reshetikhin-Turaev invariants.*

References

[J] L. C. Jeffrey, *Chern-Simons-Witten invariants of lens spaces and torus bundles, and the semiclassical approximation,* Commun. Math. Phys. 1992. N 147, 563–604.

[RT] N. Reshetikhin and V. Turaev, *Invariants of three-manifolds via link polynomials and quantum groups,* Invent. Math. Vol. 103 (1991), 547–597.

[Y] S. Yamada, *The absolute value of the Chern-Simons-Witten invariants of lens spaces,* J. of Knot Theory and Ramif. 1995. Vol. 4, N 2. 319–327.

INVARIANT TRACE FIELDS OF HYPERBOLIC 3-MANIFOLDS

HAN YOSHIDA

Department of Mathematics
Nara Women's University

We say that two hyperbolic manifolds are commensurable if they have finite sheeted covers which are diffeomorphic to each other. It is known that if two cusped hyperbolic manifolds are commensurable, the invariant trace fields of them coincide. The reverse is not true in general. For example, the 5_2 knot complement and the $(-2,3,7)$ pretzel knot complement have the same invariant trace field $\mathbb{Q}(\theta)$ where $\theta^3-\theta^2+2\theta+1=0$, but they are noncommensurable [3].

In [6], Thurston has shown the following theorem.

Theorem. *There exist infinitely many pairs of noncommensurable hyperbolic 3-manifolds which admit decompositions into the same set of ideal polyhedra.*

Neumann and Reid give a geometric description of the invariant trace field of a cusped hyperbolic 3-manifold M. Namely it is the field generated by the tetrahedral parameters of an ideal triangulation of M. Hence by subdividing ideal polyhedra into ideal tetrahedra, the above theorem implies the following corollary.

Corollary. *There exist infinitely many pairs of noncommensurable hyperbolic 3-manifolds which have the same invariant trace field.*

In this paper we show that there exists a set of arbitrary number of mutually noncommensurable hyperbolic 3-manifolds which admit decompositions into the same set of ideal polyhedra. Therefore we can obtain the following theorem.

Main Theorem. *There exists a set of arbitrary number of mutually noncommensurable hyperbolic 3-manifolds with the same invariant trace field.*

REFERENCES

1. C.Adams:, *Thrice punctured spheres in hyperbolic 3-manifolds*, Trans.Amer.Math.Soc. **287** (1985), 645–656.
2. D.B.A. Epstein, R.C. Penner:, *Euclidean decomposition of non-compact hyperbolic manifolds*, J. Diff. Geom. **27** (1988), 67–80.
3. W.Neumann, A.Reid:, *Arithmetic of hyperbolic 3-manifolds*, O.S.U.Math.Research Inst. Topology '90 Proceedings of the Research Semester in Low Dimensional Topology at Ohio State Univ. (de Gruyter Berlin) (1991).
4. W.Neumann, A.Reid:, *Amalgamation and the invariant trace field of Kleinian group*, Math. Proc.Camb.Phil.Soc. **109** (1991), 509–515.
5. A.Reid:, *A note on trace-fields of Kleinian groups*, Bull.London Math.Soc. **22** (1990), 349–352.
6. W.P. Thurston:, *The geometry and topology of 3-manifolds*, Mimeographed lecture notes Princeton Univ. (1977).

Minimal Genus Seifert Surfaces for Alternating Links

Makoto SAKUMA (Osaka University)
Mikami HIRASAWA (Osaka University)

Let L be an oriented, prime alternating link in S^3 with a reduced alternating diagram D. We denote by $S(D)$ a Seifert surface for L obtained by applying Seifert's algorithm to D. Generally the algorithm doesn't uniquely present a surface (this corresponds to the duality with respect to Murasugi sum). However when D is a special alternating diagram, i.e., no Seifert circles are nested, $S(D)$ is uniquely decided and we call L a special alternating link.

Murasugi [4] and Crowell [1] showed that if D is alternating then $S(D)$is of minimal genus for L. And Gabai [2] presented a straightforward proof. The question is on the converse; whether all minimal genus Seifert surfaces for an alternating link are obtained from alternating diagrams by Seifert's algorithm. We show an affirmative answer to this question for special alternating links applying the technique of [2]. Moreover we show that we can find all minimal surfaces by consequently deforming a diagram by so-called flypes. Namely we show;

Theorem 1. Let D be a reduced, special alternating diagram for the prime special alternating link L. Then for every minimal Seifert surface F for L, there exists a finite sequence of flypes from D to another reduced, special alternating diagram D' for L such that F is isotopic to $S(D')$.

We remark that the same result is announced by Schrijver [5] without full proof. Then we also determine the structure of the complex $MS(L)$ for special alternating links defined by Kakimizu [3]. The complex reflects the behaviors of the surfaces in the link exterior of S^3.

Definition. The complex $MS(L)$ for a link L is a simplicial complex constructed from the set of minimal surfaces for L, defined as follows;

(1) the vertex set $MS(L)^{(0)}$ consists of the equivalence classes of the minimal surfaces for L, and

(2) a set of $n+1$ vertices $\{v_0, \ldots, v_n\}$ spans an n-simplex if and only if there exist representatives $\{F_0, \ldots, F_n\}$ such that $F_i \cap F_j = \emptyset$ for every $i \neq j$

Theorem 2. The complex $MS(L)$ for every prime special alternating link L is homeomorphic to a ball.

Note that this result supports Kakimizu's conjecture [3] stating that $MS(L)$ for any link is contractible.

On the other hand, we show the answer to the above question is generally negative for non-special alternating links.

Theorem 3 . There are infinitely many prime alternating links with a minimal surface which does not arise from alternating diagrams.

To be specific, we have the following;

(1) There are infinitely many prime alternating links with arbitrarily many minimal surfaces which do not arise from alternating diagrams, and that such surfaces are disjoint (except for the boundaries) from one another and the minimal surfaces which arise from alternating diagrams.

(2) There are infinitely many prime alternating links with a minimal surface which does not arise from alternating diagrams, and that it intersects, in arbitrarily large complexity, with every minimal surface arising from alternating diagrams, i.e., the distance in terms of $MS(L)$ from surfaces of alternating diagrams is arbitrarily large.

References

[1] R. Crowell, *Genus of alternating link types*, Ann. of Math., 69, 258-275.

[2] D. Gabai, *Genera of alternating links*, Duke Math. J., 53 (1986), 677-681.

[3] O. Kakimizu, Talk at the meeting *"Knot theory and related topics"*, held in Osaka, 1989

[4] K. Murasugi, *On the genus of the alternating knot, I, II*, J. Math. Soc. of Japan, 10 (1958), 94-105, 235-248.

[5] A. Schrijver, *Tait's flyping conjecture for well-connected links*, J. Combinatorial Theory, 58 (1993), 65-146.

Makoto SAKUMA (Osaka university)

sakuma@math.wani.osaka-u.ac.jp

Mikami HIRASAWA (Osaka university)

hirasawa@math.sci.osaka-u.ac.jp

ON THE COMPLEMENT OF HOMOTOPICALLY TRIVIAL KNOTS

M.Boileau-M.Domergue-Y.Mathieu

In 1988, Gordon and Luecke [G-L] showed knots in the 3-sphere are determined by their complement. David Gabai [Ga] had proved the same results for knots in $S^1 \times S^2$ in 1985, but Domergue and Mathieu [D-M] gave tw= o knots k and k' in the solid torus V with homeomorphic complements and = no homeomorphismof the pair (V, k) to the pair (V, k'). For closed 3-manifolds, Mathieu [M] constucts in Seifert fiber spaces examples of pairs of knots with an orientation reversing homeomorphism of the complement which are not equivalent .The question :

are knots determined by their complement ?

is still open for knots in arbitrary 3-manifolds.

We give now a "surgical" point of view of the complement problem. Let X an oriented compact 3-manifold with boudary a torus, $T = 3D\partial X$; a *slope* on T is the isotopy class of a simple closed curve of T. Dehn filling on X along a slope α is obtained by gluing to X a solid torus $V = 3DS^1 \times D^2$ by its boundary ∂V identified with= T, so that α bounds a disk in $X(\alpha) = 3DX \cup_\alpha V$. In the case= of X is the exterior of a regular neighborhood $N(k)$ of a knot k in the 3-sphere, $X = 3DS^3 - int(N(k))$, Gordon-Luecke's theorem is:

Theorem [G-L] : *Let X be a 3-manifold with incompressi= ble boundary a torus and a slope μ so that $X(\mu) = 3DS^3$. If $X(\alpha) == 3DS^3$, then $\alpha = 3D\mu$*

Corollary : *Any knot in S^3 is determined by its complement.*

Let now k be a knot in an oriented compact 3-manifold M and $X =$

$3DM - int(N(k))$ with $X(\mu) = 3DM$ for μ meridian of k. Let be $M' = 3D = X(\mu')$ for any slope μ' on ∂X.

Property (H) *A knot k has property (H) if μ' different from μ implies $\pi_1(M')$ not isomorphic to $\pi_1(M)$.*

In the case of $M = 3DS^3$, property (H) is known as property (P).

Remark : if a knot k a property (H), then k is determin= ed by its complement.

1) Manifolds which are not rational homology spheres.

The first part concerns homotopically trivial (nul-homotop) knots in oriented 3-manifolds with non trivial rational homology.

We use Gabai's results [Ga] to prove the following :

Lemma 1 : *Let M be a 3-manifold with hopfian fundament= al group, k a homotopically trivial knot in M, and k' the core of the surgery in $M' = 3DX(\mu')$, then $\pi_1(M')$ is isomorphic to $\pi_1(M)$ if a= nd only if k' is homotopically trivial in M'.*

Theorem 1 : *Let k be a homotopically trivial knot in M, with $X = 3DM - int(N(k))$ irreducible, boundary incompressible.*

If M is not a rational homology sphere, then k has property (H).

2) Large knots.

In this part, the hypothesis "M is not a rational homology sphere" is replaced by a geometrical condition on the knot complement :

Definition 1 : *A knot k in M is sufficiently large i= f the complement of $k, M - k$, contains an incompressible surface F.*

Definition 2 : *A sufficiently large knot k is strictly nul-homotop if k is homotopically trivial in $M - F$.*

We prove the fundamental proposition :

Proposition 1 : *let k be a sufficiently large and homotopically trivial knot in M and $M' = 3DX(\mu')$.*

If $\pi_1(M')$ is isomorph to $\pi_1(M)$, then F separates M (and M'), and F compresses both in M and M'.

By using Proposition 1 and Scharleman's result [S] we can prove :

Theorem 2 : *Let k be a homotopically trivial knot in $M =$ with $M - int(N(k))$ irreducible boundary incompressible. Let F an incompressible surface in $M - k$, no boundary parallel.*

(i) if k is strictly nul-homotop, then k as property (H).

(ii) if F is an incompresible torus and $\pi_1(M)$ infinite hopfian, then k as property (H) except maybe if k is a (1,2)-cable of an atoroidal knot K. In this case k is not determined by is complement if and only if K also.

3) Detecting strictly nul-homotop knots.

As we see above, a knot k which has not property (H), cannot be in a manifold which is not a rational homology sphere, and cannot be strictly nul-homotop.

We give a sufficient condition to detect strictly nul-homotop knots :

Proposition 2 : *Let k be a homotopically trivial knot in M with $M - int(N(k))$ irreducible boundary incompressible.*

The knot k is strictly nul homotop if there is a singular disk ∂ with $\partial\partial = 3Dk$, so that for a regular neighborhood $N(\partial)$ of ∂ :

(i) either $i_*(\pi_1(N(\partial))$ is different from $\pi_1(M)$, where i_* is induced by inclusion

$i : N(\partial) \subset M$

(ii) or $g(\partial N(\partial)) < h(M)$, where $g(\partial N(\partial))$ is the genus of the surface $\partial N(\partial)$, and $h(M)$ is the Heegaard's g= enus of M

Definition 3 : *A knot k is totally nul-homotop in $M =$ if there is a submanifold N so that k is nul-homotop in N and the inclusion $i : N \subset M$ induces a trivial homomorphism on fundamental groups $i_*(\pi_1(N) = 3D\{1\}$.*

It is clear that totally nul-homotop knots are strictly nul-homotop knots, but the class of totally nul-homotop knots is easier to recognize, in particular for manifolds given by a Heegaard splitting. We have :

Corollary 1 : *Let M be a rational homology sphere containing a nul-homotop knot with irreducible boundary-incompressible exterior. Suppose M is not a homotopy sphere.*

If k is totally nul-homotop, then k is strictly nul-homotop. In particular k has property (H).

References

[D-M] M.DOMERGUE, Y.MATHIEU : "Noeuds qui ne sont pas d=E9termin=E9s par leur compl=E9ment...", Bull. Soc. Math. France, 119= (1991) p. 327-341.

[Ga] D.GABAI : "Foliations and the topology of 3-manifolds II" J. of Diff. Geom. 26 (1987), p. 461-478.

[G-L] C. Mc A GORDON, J.LUECKE : "Knots are determined by their complements" J. of A.M.S. vol.2-2 (1989), p. 371-415.

[M] Y.MATHIEU : "Closed 3-manifolds unchanged by Dhen surgery", J. of knot Theory and its ramifications, vol.1 n°3 (1992),= p. 279-296.

[S] M.SCHARLEMAN : "Producing reducible 3-manifolds by surgery on a knot", Topology 29 (1990), p. 481-500.

SEIFERT COMPLEX FOR LINKS AND 2-VARIABLE ALEXANDER MATRICES

TERUHISA KADOKAMI

Dept.of Mathematics,Osaka City Univ.,D2

ABSTRACT

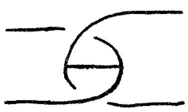
We introduce a general theory to extend a method for calculating an one-variable Alexander matrix from a Seifert surface to a multi-variable one by increasing components of surfaces, and especially characterize the matrix in the 2-variable case. A multi-component Seifert surface has singularities in general, so we define it here a *Seifert complex.* This idea is mentioned by J.H.Conway and applicated firstly by his student,D.Cooper,to a theory of knot cobordism invariants [6,7]. This is our original to use it for a characterization theory of Alexander matrices.

For a statement of a main theorem,we introduce some terms.

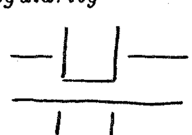
Definition1. *(Types of a singularity)*

Types of intersections of surfaces embedded in S^3 in general positions are shown below.

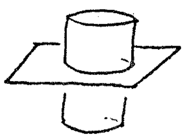
(1)*clasp singularity*



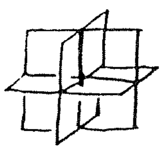
(2)*ribbon singularity*



(3)*circle singularity*



(4)*triple point singularity*



Definition2. *($S-complex, C-complex, R-complex, RC-complex$)*

If there are n Seifert surfaces in general positions,then we define the union of them as *an n-component S-complex(Seifert complex).* And if its singularities are all clasps,we define it *a C-complex*,if they are all ribbons,we define it *an R-complex*,and if they are clasps and ribbons, we define it *an RC-complex.*

Lemma. *An $S-$complex can be transformed to a $C-$complex by isotopies of each component with their boundaries fixed.*

Main Theorem. *Let $L = L_1 \cup L_2$ be an $n-$component link such that $L_i (i = 1, 2)$ is an n_i-component link with $n = n_1 + n_2$,and is a Seifert surface of L_i,then a $2-$variable Alexander matrix of L associated to $S, A_L(t_1, t_2)$,is calculated in the following form up to basis changes.*

$$A_L(t_1,t_2) = \begin{pmatrix} \overbrace{\begin{matrix} C_1 & & \\ & \ddots & \\ & & C_1 \end{matrix}}^{2g_1} & \overbrace{}^{2g_2} & \overbrace{0}^{m} \\ & \begin{matrix} C_2 & & \\ & \ddots & \\ & & C_2 \end{matrix} & \\ 0 & & B \end{pmatrix} + \begin{pmatrix} (1-t_1)M_1 \\ (1-t_2)M_2 \\ (1-t_1)(1-t_2)M_3 \end{pmatrix}$$

Here $C_i = \begin{pmatrix} 0 & 1 \\ -t_i & 0 \end{pmatrix} (i = 1, 2)$, $B = t_1t_2B_{--} - t_1B_{-+} - t_2B_{+-} + B_{++}$,an $m \times m-$type matrix (This needs a precise explanation,but some preparation is needed,so we omit it here.It will be explained in my talk.Sorry!),and $M = {}^tM = \begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix}$,a symmetric matrix over $\mathbb{Z}$ such that M_1 is a $2g_1 \times (2g_1 + 2g_2 + m)-$type, M_2 is a $2g_2 \times (2g_1 + 2g_2 + m)-$type,and M_3 is an $m \times (2g_1 + 2g_2 + m)-$type.

Conversely,as M can be taken arbitrary,this is a characterization of $2-$variable Alexander matrices.

REFERENCES

1. Torres,G., *On the Alexander polynomials*, Ann. of Math. **57** (1953), 57–89.
2. Hillman,J.A., *The Torres conditions are insufficient*, Math. Proc. Cambridge Phil. Soc. **89** (1981), 19–22.
3. ———, *Alexander ideals of links*, Lect. Notes. Math. 895.Springer-Verlag, 1981.
4. Traldi,L., *Milnor's invariants and the completions of link modules*, Trans. Amer. Math. Soc. **284** (1984), 401–429.
5. Bailey,J.L., *Alexander invariants of links*, Ph.D.Thesis,Univ.British Columbia. Vancouver (1977).
6. Cooper,D., *The universal abelian cover of a link*, In:Low-Dim. Top.(Bangor,1979) London Math. Soc. Lect. Note Ser. **48** (1982), Springer-Verlag, 51–56.
7. ———, *Signatures of surfaces in 3-manifolds and applications to knot and link cobordism*, Ph.D.Thesis,Univ. Warwick (1982).
8. Nakanishi,Y., *Alexander invariants of links*, Master Thesis,Univ. Kobe (1980).

Symmetric knots satisfy the cabling conjecture

CHUICHIRO HAYASHI[1] and KOYA SHIMOKAWA[2]

Abstract

The cabling conjecture states that a knot K in S^3 is a cable knot or a torus knot if some Dehn surgery on K yeilds a reducible manifold. We prove that symmetric knots satisfy this conjecture. (Gordon and Luecke also prove this independently ([G-L4])).

1 Introduction

Let X be an orientable 3-manifold, $K \subset X$ a knot, $N(K)$ a regular neighbourhood of K in X, r a slope on the torus $\partial N(K)$. We let $X(K;r)$ denote the 3-manifold obtained from X by r-surgery on K, i.e., the result of attaching a solid torus W to $X-\text{int}N(K)$ by identifying ∂W with $\partial N(K)$ so that r bounds a disc in W. We use K^* to denote the core of W in $X(K;r)$. Surgery slopes along a knot in S^3 (in solid torus) are in one to one correspondence with rational numbers with respect to the standard (a prefered) meridian-longitude coordinates. A knot $K \subset X$ is said to be *cabled* in X if there is another knot K' in X such that $K \subset \partial N(K')$ and $[K] = \omega[K']$ in $H_1(N(K');Z)$ with $\omega \geq 2$.

In this paper we consider the case where a Dehn surgery on K produce an essential sphere in $S^3(K;r)$, that is, $S^3(K;r)$ is a reducible manifold.

For restrictions on reducing slopes (i.e. slopes on which one can get reducible manifolds by Dehn surgeries) C.McA. Gordon and J. Luecke proved that they are integral [G-L] and if there are two such slopes then their distance is at most one [G-L3]. For restrictions on types of knots F. González-Acuña and H. Short proposed the following cabling conjecture.

The Cabling Conjecture *If $S^3(K;r)$ is reducible, then K is cabled.*

The cabling conjecture is known to be ture for some knots including satellite knots [Sch], strongly invertible knots [E], alternating knots [M-T] and arborescent knots [Wu].

A symmetry for a knot K in X is a finite group G acting nontrivially on X such that $g(K) = K$ for each element g of G. In [L-Z], E. Luft and X. Zhang showed that "symmetric knots satisfy the cabling conjecture" except for periodic knots of order $2, 3$ or 5. It is confirmed in [H-M] that periodic knots of order 3 or 5 also satisfy the cabling conjecture. We consider periodic knots of order 2 and get the following result.

Theorem 1 ([G-L4], [H-S]) *Symmetric knots satisfy the cabling conjecture.*

[1]This research was partially supported by Fellowships of Japan Society for the Promotion of Science for Japanese Junior Scientists.

[2]This research was partially supported by Fellowships of Japan Society for the Promotion of Science for Japanese Junior Scientists.

Since symmetric knots other than periodic knots satisfy the cabling conjecture [L-Z], we consentrate our attention to periodic knots.

The basic idea of the proof of Theorem 1 owes to [L-Z]. Theorem 1 follows from Theorem 2 below. Let $\tilde{K}$ be a non-cabled periodic knot with a periodic automorphism f of S^3 of order n. Then f is a rotation of S^3 and $Fix(f)$ is a trivial knot disjoint from $\tilde{K}$. Hence $\tilde{K}$ is contained in the unknotted solid torus $\tilde{W} = S^3-$ int $N(Fix(f))$. Passing through the branched covering $p : S^3 \to S^3/\{f\}(\cong S^3)$, we obtain the factor knot $K = p(\tilde{K})$. Luft and Zhang show that if $\tilde{K}$ produces a reducible manifold by m-surgery, then $W(K; m/n)$ contains a seperating essential annulus whose every boundary component intersects ∂D (D is a meridian disc of W) once. They proved if K is not cabled, then $\Delta(m/n, 1/0) = |n| \leq 6$. C. Hayashi and K. Motegi [H-M] showed that $\Delta(m/n, 1/0) \leq 2$ and here we have $\Delta(m/n, 1/0) = 1$.

Theorem 2 ([H-S]) *Let V be a solid torus, D a meridian disc of V, and $K \subset V$ a non-cable knot. Assume that ∂V is incompressible in $V-$int$N(K)$. If the manifold $V(K; r)$ contains a separating essential annulus A such that each component of ∂A intersects ∂D transversely in one point, then $\Delta(\mu, r) = 1$, where μ is the meridian slope of K.*

From now on we assume $\Delta(\mu, r) = 2$ by means of [H-M]. This assumption will lead us to a contradiction.

2 Preliminaries

The method we used to prove Theorem 2 is analysis of intersections of a meridian disc and a seperating essential annulus. We breifly describe it.

We take the meridian disc D of V so that $n_D = |D \cap K|$ is minimal over all meridian discs. We have an incompressible and ∂-incompressible planar surface $P_D = D \cap (V-\text{int}N(K))$.

We are given a boundary slope ∂A on the torus ∂V. We take the annulus A in $V(K; r)$ so that $n_A = |A \cap K^*|$ is minimal over all essential annuli in $V(K; r)$ with such a boundary slope as above. Then the surface $P_A = A \cap (V-\text{int}N(K))$ is incompressible and ∂-incompressible.

Hence we can further take D and A so that $|\partial P_D \cap \partial P_A|$ is minimal and $P_D \cap P_A$ consists of loops and arcs which are essential on both P_D and P_A.

As in [G-L2] we will form graphs G_D and G_A on D and A. In the following we assume that $\{i, j\} = \{D, A\}$.

We orient arbitrarily the knots K and K^*. Number the components of $\partial P_i \cap \partial N(K)$, $\{1, 2, ..., n_i\}$ in the order in which they appear on $\partial N(K)$. Thus K and K^* are divided into n_D and n_A *intervals* $[1, 2], [2, 3], ..., [n_i, 1]$.

We label the end points of arcs of $P_D \cap P_A$ in P_i with the corresponding boundary components of P_j. Thus around each component of $\partial P_i \cap \partial N(K)$ we see the *labels* $\{1, 2, ..., n_j\}$ appearing sequentially Δ times.

We regard the discs $D \cap N(K)$ and $A \cap N(K^*)$ as forming the "fat vertices" of graphs G_D and G_A. The edges of G_i are corresponding to the arcs of $P_D \cap P_A$ in P_i except for the arcs whose both end points are in ∂i. We call the closure of a component of ∂(fat vertex)$-$(end points of edges) a *corner*. If an edge e connects a vertex to a vertex, then we say e is an ***interior edge***, otherwise a ***boundary edge***. If an interior edge e has both two end points in the same fat vertex, then we say e is a ***loop***. The graph G_i contains no ***trivial loops***, i.e., a 1-sided face (no arc of $P_A \cap P_D$ is boundary parallel in P_i). We assign arbitrary orientations to P_i. Then every fat vertex v is assigned a sign $+$ or $-$ according to the induced orientation of ∂v as it lies on $\partial N(K)$. If an interior edge e connects vertices of the same sign, then we say e is a ***homo-edge***, We thus obtain two labeled graphs in D and A, whose edges are in one to one correspondence. We call components of $i - G_i$ ***faces*** of G_i. A face P is called a ***disc face*** if P is an open disc. For every face P, let ∂P be the subgraph which consists of vertices and edges intersecting $\bar{P} - P$.

Let x be a label of G_i. An ***x-edge*** in G_i is an interior edge with label x at an end point. A subgraph σ is an ***x-edge cycle*** if all its edges are homo x-edges and if there is a disc face P of the subgraph σ such that $\sigma = \partial P$. A ***Scharlemann cycle*** is an x-edge cycle for some label x which bounds a disc face of G_i. The ***length*** of a Scharlemann cycle is the number of edges contained in the cycle.

We call a Scharlemann cycle ***a Scharlemann cycle for the interval*** $[x, x+1]$ if its corners are in the interval $[x, x+1]$. We call the subgraph of G_j consisting of the vertices x and $x+1$ and the edges corresponding to those of σ a ***Scharlemann co-cycle*** of σ. A Scharlemann co-cycle in G_j is ***inessential*** if it is contained in a disc imbedded in the surface j.

Lemma 2.1 ([C-G-L-S, Lemma 2.5.2.(a)]) *The graph G_A does not contain a Scharlemann cycle. The graph G_D does not contain a Scharlemann cycle whose Scharlemann co-cycle is inessential.*

3 Great x-webs in G_D

An ***x-web*** is a connected subgraph Σ of G_D such that all the edges of Σ are x-edges, all the vertices of Σ have the same sign, and such that there exists a vertex v_0 of Σ with the property that

1. for any vertex v of Σ other than v_0, there is an edge of Σ incident to v at each occurrence of the label x at v, and

2. there is an edge of Σ incident to v_0 at some occurrence of the label x at v_0.

Let U be the component of $D - \Sigma$ which contains ∂D. A ***great x-web*** satisfies the additional condition that all the vertices of G_D in $D - U$ are those of the x-webs itself.

The loops ∂A divides the torus $\partial V(K;r)$ into two annuli. We attach one of them, say A' to A and obtain a torus T. We push T slightly into $\text{int}V(K;r)$. This torus T may be compressible in $V(K;r)$. As in [Go], we construct new graphs Γ_T and Γ_D on this torus T and the meridian disc D, such that they contain no boundary edges. Let P_T be the punctured torus $T\cap(V-\text{int}N(K))$. The curves of intersection of P_T and P_D in $V-\text{int}N(K)$ is derived from those of P_A and P_D.

The two boundary edges of G_D and G_A are connected by a subarc of ∂D in Γ_D and by an essential arc in A' in Γ_T, and are amalgamated into an edge e_0.

The next Lemma follows from results of [Go] and [P].

Lemma 3.1 *Assume $\Delta(\mu, r) \geq 2$. Then Γ_D contains a great x-web.*

4 In case $n_A \geq 4$

Theorem 2 under the assumption that $n_A \geq 4$ follows from three lemmas.

Lemma 4.1 ([G-L3, Theorem 2.3]) *Assume that $n_A \geq 4$. If Γ_D contains a great x-web, then Γ_D contains two Scharlemann cycles for distinct intervals.*

Lemma 4.2 *Assume that $n_A \geq 4$. Then we can choose the annulus A' so that Γ_D does not contain a Scharlemann cycle which contains the edge e_0 derived from the two boundary edges of G_D. (We regard a trivial loop as a Scharlemann cycle of length 1.)*

Lemma 4.3 ([H-M2, Lemma 3.1]) *Suppose that G_D contains two Scharlemann cycles for distinct intervals. Then $n_A = 2$.*

5 In case $n_A = 2$

Let Σ be a great x-web in Γ_D, and U be the face of Σ which contains ∂D. A vertex v of Σ is called a *cut-vertex* if D contains a simple closed curve C such that $C\cap\Sigma = C\cap v$ and Σ has edges in both sides of C. We say that Σ is *composite* if D contains a simple closed curve C such that $C\cap\Sigma = C\cap\{\text{edges of }\Sigma\}$, $|C\cap\Sigma| = 2$ and Σ has vertices on both sides of C. Otherwise Σ is *prime*. It is clear that if Γ_D contains a great x-web, then we can find prime one without cut-vertices.

One can prove the following Lemma using the Euler's formula.

Lemma 5.1 *Let Σ be a prime great x-web of Γ_D without cut-vertices. Then Σ consists of two kinds of Scharlemann cycles of length a and b placed in the checkerboard-like way and $(a, b) = (2, b)$, $(3, 3)$, $(3, 4)$ or $(3, 5)$.*

Using the above lemma, we can show the next lemma, whose conclusion contradicts Lemma 2.1.

Lemma 5.2 *Each great x-web of Γ_D contains a Scharlemann cycle whose Scharlemann co-cycle is inessential.*

References

[C-G-L-S] Culler, M., Gordon, C. McA., Luecke, J. and Shalen, P. B. : Dehn surgery on knots. Ann. of Math. **125**, 237–300 (1987)

[E] Eudave-Muñoz, M. : Band sums of links which yield composite links. The cabling conjecture for strongly invertible knots. Trans. Amer. Math. Soc. **330**, 463–501 (1992)

[Go] Goodman-Strauss, C. : On composite twisted unknots. to appear in Trans. Amer. Math. Soc.

[G-S] González-Acuña, F. and Short, H. : Knot surgery and primeness Math. Proc. Camb. Phil. Soc. **99**, 89–102 (1986)

[G-L] Gordon, C. McA. and Luecke, J. : Only integral Dehn surgeries can yield reducible manifolds. Math. Proc. Camb. Phil. Soc. **102**, 97–101 (1987)

[G-L2] ——— : Knots are determined by their complements. J. Amer. Math. Soc. **2**, 371–415 (1989)

[G-L3] ——— : Reducible manifolds and Dehn surgery. Topology **35**, 385–409 (1996)

[G-L4] ——— : Dehn surgery on knots. Oral announcement in Workshop on 3-dimensional manifolds in Montreal, June 12–16, 1995

[H-M] Hayashi, C. and Motegi, K. : Dehn surgery on knots in solid tori creating essential annuli. to appear in Trans. Amer. Math. Soc.

[H-S] Hayashi, C. and Shimokawa, K. : Symmetric knots satisfy the cabling conjecture. preprint (1995)

[L-Z] Luft, E. and Zhang, X. : Symmetric knots and the cabling conjecture. Math. Ann. **298**, 489–496 (1994)

[M-T] Menasco, W. and Thistlethwaite, M. : Surfaces with boundary in alternating knot exteriors. J. Reine Angew. Math. **426**, 47–65 (1992)

[P] Parry, W. : All types implies torsion. Proc. Amer. Math. Soc. **110**, 871–875 (1990)

[Sch] Scharlemann, M. : Producing reducible 3-manifolds by surgery on a knot. Topology **29**, 481–500 (1990)

[Wu] Wu, Y.-Q. : Dehn surgery on arborescent knots. J. Diff. Geom. **43**, 171–197 (1996)

Dehn surgery

Cameron Gordon

We will give a survey of some known results and remaining problems concerning Dehn surgery on knots, and. more generally, Dehn filling on 3-manifolds along a torus boundary component. We will then focus on the technique of combinatorial analysis of intersections of punctured surfaces, and outline the proofs of some results obtained by these methods.

Invariants of links

Louis H. Kauffman

1 Title of First Talk: Integral Heuristics and Vassiliev Invariants

Abstract: This talk discusses how simple heuristics involving Witten's functional integral formulation of link invariants lead to the form of the weight systems for Vassiliev invariants. We shall also discuss numerical experiments (with Sostenes Lins) related to the large k asymptotics of Witten's integral.

2 Title of Second Talk: Invariants of Links and Quantum Groups

Abstract: This talk discusses the relationship of Hopf algebras (aka quantum groups) with the structure of link invariants and with link diagrams. Specific quantum groups are implicated by the simplest invariants and the categorical structure of link diagrams corresponds to the axiomatics of quasi-triangular Hopf algebras. This gives rise to a natural functor from the category of tangles to a category naturally associated with any finite dimensional quasi-triangular Hopf algebra.

3 Title of Third Talk: Finite Dimensional Hopf Algebras and Invariants of Three Manifolds

Abstract: This talk uses techniques built in the previous talk to discuss invariants of three manifolds obtained via integrals on finite dimensional Hopf algebras. The talk will discuss the existence and applications of these integrals (joint work with D. Radford). We also discuss applications of this point of view to finding elements in the center of a Hopf algebra that are represented by knots (jouint work with D. Radford and S. Sawin).

The Topology of DNA

De Witt Sumners
Department of Mathematics
Florida State University
sumners@math.fsu.edu

The DNA of all organisms has a complex and fascinating topology. It can be viewed as two very long, closed curves that are intertwined millions of times, linked to other closed curves, tied into knots, and subjected to four or five successive orders of coiling to convert it into a compact form for information storage. For information retrieval and cell viability, some geometric and topological features must be introduced, and others quickly removed. Some enzymes maintain the proper geometry and topology by passing one strand of DNA through another via an enzyme-bridged transient break in the DNA; this enzyme action plays a crucial role in cell metabolism, including segregation of daughter chromosomes at the termination of replication and in maintaining proper in vivo (in the cell) DNA topology. Other enzymes break the DNA apart and recombine the ends by exchanging them. These enzymes regulate the expression of specific genes, mediate viral integration into and excision from the host genome, mediate transposition and repair of DNA, and generate antibody and genetic diversity. These enzymes perform incredible feats of topology at the molecular level; the description and quantization of such enzyme action absolutely requires the language and computational machinery of topology. In the topological approach to enzymology, circular DNA is incubated with an enzyme, producing an enzyme signature in the form of DNA knots and links. By observing the changes in

DNA geometry (supercoiling) and topology (knotting and linking) due to enzyme action, the enzyme mechanism and substrate conformation in solution can often be characterized. These lectures will discuss analytic topological models and Monte Carlo simulation models for the structure of DNA and the active enzyme-DNA complex.

The long-range goal of the topological approach to enzymology is to develop a complete set of experimentally observable topological parameters with which to describe and compute enzyme mechanism and the structure of the active enzyme-DNA synaptic intermediate. One of the important unsolved problems in biology is the three-dimensional structure of proteins, DNA and active protein-DNA complexes in solution (in the cell), and the relationship between structure and function. It is the 3-dimensional shape in solution which is biologically important, but difficult to determine. The topological approach to enzymology is an indirect method in which the descriptive and analytical powers of topology and geometry are employed in an effort to infer the structure of active enzyme-DNA complexes in vitro (in a test tube) and in vivo.

In the topological approach to enzymology, the topological invariance of knotted and linked (catenated) DNA during experimental workup and the computational power of topology are exploited to capture information on enzyme action. In in vitro (in a test tube) experiments, an enzyme extracted from living cells is reacted with circular DNA substrate produced by cloning techniques. Similar experiments have been done in vivo in which various experimental manipulations (mutation, drugs, heat shock) can be used to turn off genes which produce enzymes whose action would confound the analysis. The enzyme reaction produces a topological signature in the form of an enzyme-specific family of supercoiled DNA knots and links (catenanes). By observing changes in DNA geometry (supercoiling) and topology (knotting and linking) by gel electrophoresis and electron microscopy of recA-coated products the enzyme mechanism can be described and quantized. Because of the enormous variety of knot and catenane structure, fine details of DNA structure and enzyme action can be selectively assayed.

The topological approach to enzymology poses a number of challenges and problems for mathematics:

1. How can one best describe and compute enzyme mechanism? There are (mathematically) infinitely many ways to change one DNA knot or link into another. Of all these ways, only a few make biological "sense". This requires the construction of mathematical models for enzyme action.

2. How much experimental information is necessary in order to uniquely characterize enzyme mechanism? Mathematical proof of biological structure can significantly reduce the amount of lab work required.

3. How can incomplete and sometimes conflicting experimental information be accommodated in a mathematical model for enzyme action? The model can often enumerate all possible enzyme mechanisms and enzyme-DNA structures which could give rise to the observed products. At this stage, collaboration with biologists is crucial in order to incorporate supercoiling energetics and other biological reasoning methods to produce hybrid mathematics/biology arguments for selecting one answer instead of another.

4. What is the best mix of gel and EM data? Gel electrophoresis yields node (minimum crossing) number of reaction products, gel velocity of a product can often be used to determine its exact topology; electrophoresis can detect vanishingly small amounts of product. Electron microscopy yields exact topological information (when the EM can be unambiguously scored); EM is technically difficult, and requires a large amount of product. What is

the best mix of gel and EM data in order to uniquely characterize enzyme structure/mechanism?

The tangle model has been developed to aid in the mathematical analysis of these DNA experiments. A tangle is a model for two (or more) DNA strands bound to a globular protein. The (2-string) tangle model can be used to analyze experimental results for any enzyme which operates by binding to DNA at two places, such as topoisomerase and site-specific recombinase. The action of topoisomerase is to non-specifically bind to the DNA substrate at two places, form an enzyme-bridged transient break (of one or two DNA backbone strands), and pass the other binding site through the enzyme-bridged transient break, releasing the DNA product at the termination of the process. In site-specific recombination, recombinase recognizes and binds to duplex DNA substrate at two specific places (sites), performs double-stranded enzyme-bridged breaks at each site, and exchanges the ends (one or more times) in an enzyme-specific manner, releasing the product when the process terminates.

Tangles can be added together to form other tangles, and can be closed up to yield either a knot or a link by the numerator construction, in which one forgets the defining 3-ball for the tangle. The numerator construction is the mathematical analog of deproteinization, when the enzyme releases the bound DNA. The numerator and addition operations on tangles can be used to write tangle equations which describe enzyme action. The relevant observation is that most DNA conformations are produced by the plectonemic interwinding of pairs of DNA strands; such interwinding templates are exactly how rational tangles and 4-plats are constructed. Rational tangles and 4-plats admit classification schemes by vectors which represent minimal alternating diagrams; these vector classifications can be used to construct algorithms for computing all rational tangle solutions to tangle equations.

The tangle model provides mathematical proof of the topological structure of the enzyme-DNA synaptic complex, both before and after enzyme action, the algorithmic calculation of that structure, and the precise prediction of the results of further experiments. The tangle model uses experimental information to write down tangle equations which quantize changes in DNA topology. The solution of these tangle equations uses some recently-developed knot theory, such as the cyclic surgery theorem, property R, and the knot complement theorem.

References

[1] C. Ernst and D.W. Sumners, *A calculus for rational tangles: applications to DNA recombination*, Math. Proc. Camb. Phil. Soc., 108(1990), 489-515.

[2] D.W. Sumners, *Lifting the Curtain: Using topology to probe the hidden action of enzymes*, Notices of the AMS 42 (1995), 528-537.

[3] E.J.Janse van Rensburg, E. Orlandini, D.W. Sumners, M.C. Tesi and S.G. Whittington, *Lattice ribbons: A model of double-stranded polymers*, Phys. Rev. E. 50 (1994), R4279-R4282.

[4] D.W. Sumners, C. Ernst, S.J. Spengler and N.R. Cozzarelli, *Analysis of the mechanism of DNA recombination using tangles*, Quarterly Reviews of Biophysics 28 (1995), 253-313.

Chern-Simons perturbative invariants

Toshitake KOHNO
Department of Mathematical Sciences, University of Tokyo

This is an expository lecture on recent developments in Chern-Simons perturbative theory for knots and 3-manifolds. Let M be a 3-manifold and $\mathcal{A}$ the space of G connections on M for a simply connected Lie group G. The critical points of the Chern-Simons functional $CS : \mathcal{A} \to \mathbf{R}$ are the flat connections. The original idea due to Witten is to compute the asymptotic expansion of the partition function of the Chern-Simons functional with respect to the level k, around a flat connection. Each term of such expansion gives, in principle, a toplogical invariant of a framed 3-manifold described by Feynman diagrams. A direct construction of such invariants was established by Axelrod-Singer, Kontsevich, Taubes and other authors. Here the invariants are expressed as an integral of a wedge product of Green forms, which are 2-forms on $M \times M$ having singularities along the diagonal set, over the configuration space of points on M. A different approach based on Morse homotopy was persued by Fukaya.

1. Integral representations of knot invariants

Applying Chern-Simons perturbative theory to knots in $\mathbf{R}^3$, we obtain interesting integral representations of classical knot invariants, which might be considered to be a generalization of the Gauss formula for the linking number. The integral representation of the 2nd coefficient of the Alexander-Conway polynomial was obtained by Guadagnini-Martellini-Mintchev, and has been developed by Bar-Natan, Bott-Taubes, Lin-Wang and others. We discuss such integral representations in terms of the graph complex. We also describe combinatorial formulae due to Polyak-Viro and the relationship with Arnold theory of invariants of plane curves.

2. Finite type invariants of 3-manifolds

On the other hand, a number theoretic expansion of the Witten invariants has been investigated by H. Murakami and T. Ohtsuki. This is the expansion of the Witten invariants with respect to $q - 1$ where q is a root of unity. It was shown by H. Murakami that for a rational homology 3-sphere, the leading coefficient is expressed by the Casson-Walker invariant. We review developments on such finite type invariants appearing in the expansion, especially the work of Rozansky on the relation with asymptotic expansion with respect to the level k and the work of Lawrence on the adic convergence of the above number theoretic expansion. It should be noted that, more recently, Le Tu Quoc Thang, H. Murakami, J. Murakami and T. Ohtsuki established a universal way to construct finite type invariants with values in the algebra of chord diagrams.

3. Chord diagrams on surfaces

We present an approach to understand the Chern-Simons perturbation theory of a 3-manifold with boundary as a topological quantum field theory. Our method is based on the structure of a Poisson algebra on the space of chord diagrams on a surface discovered by Reshetikhin and others. We discuss Vassiliev invariants for links in the product of a surface and the unit interval, with values in the quantization of the space of functions on the moduli space of flat G connections on the surface. This approach leads us to a generalization of Kontsevich's integral to the case of higher genus. We discuss a relation between the asymptotic behaviour of the action of the mapping class group on the space of conformal blocks for large k limit and topological invariants of links in the product of a surface and the unit interval defined associated with chord diagrams on surfaces. In particular, we describe an integrable system on the configuration space of the torus whose holonomy defines Vassiliev invariants of links in the product of the torus and the unit interval.

Bibliographies

[1] S. Axelrod and I. M. Singer, Chern-Simons perturbation theory, Proc. XXth DGM Conference, 1992, 3–45.

[2] D. Bar-Natan, On Vassiliev knot invariants, Topology 34-2, 1995, 423–472.

[3] R. Bott and C. Taubes, On the self-linking of knots, preprint.

[4] T. Kohno, Vassiliev invariants and de Rham complex on the space of knots, Contemp. Math. 179, 1994, 123–138.

[5] T. Kohno, Elliptic KZ system , braid group of the torus and Vassiliev invariants, to appear in Topology and its Applications.

[6] M. Kontsevich, Vassiliev's knot invariants, Advances in Soviet Math. 16, 1993, 137–150.

[7] T. Q. T. Le, J. Murakami and T. Ohtsuki, On a universal quantum invariant of 3-manifold, preprint.

[8] X. S. Lin and Z. Wang, Integral geometry of plane curves and knot invariants, preprint.

[9] M. Polyak and O. Viro, Gauss diagram formulae for Vassiliev invariants, Int. Math. Res. Notes 11, 1994, 445–454.

[10] L. Rozansky, The trivial connection contribution to Witten's invariant and finite type invariants of rational homology 3-spheres, preprint.

[11] E. Witten, Quantum field theory and the Jones polynomial, Commun. Math. Phys. 121, 1989, 360–379.

TOPOLOGICAL IMITATIONS

Akio Kawauchi

Department of Mathematics, Osaka City University, Osaka 558, Japan

Abstract

By a $(3,1)$-*manifold pair*, we mean a pair (M, L) such that M is a smooth connected oriented 3-manifold and L is a proper (possibly disconnected) oriented smooth 1-submanifold.

A topological imitation of a $(3,1)$-manifold pair (M, L) is a $(3,1)$-manifold pair (M^*, L^*) together with a smooth map $q : (M^*, L^*) \to (M, L)$ with some properties close to a diffeomorphism (cf. [**1,2,3,4,5,6,7**]). A useful concept of the topological imitation is an almost identical imitation, which we call here an *AID imitation.* It is roughly a topological imitation $q : (M^*, L^*) \to (M, L)$ with the following properties (1) and (2):

(1) There exist tubular neighborhoods $N(L^*)$ and $N(L)$ of L^* and L in M^* and M, respectively, such that the restrictions $q|_{(N(L^*),L^*)} : (N(L^*), L^*) \to (N(L), L)$ is a diffeomorphism and $q(M^* - \mathrm{int}N(L^*)) = M - \mathrm{int}N(L)$.

(2) The restriction $q|_{M^*-\mathrm{int}N(L^*-a^*)} : M^* - \mathrm{int}N(L^* - a^*) \to M - \mathrm{int}N(L - a)$ is (boundary-relatively) homotopic to a diffeomorphism for every pair a^*, a of components of L^*, L with $q(a^*) = a$, where $N(L^* - a^*)$ and $N(L - a)$ are the tubular neighborhoods of $L^* - a^*$ and $L - a$ obtained from $N(L^*)$ and $N(L)$ in (1) by removing the components which contain a^* and a, respectively.

When (M, L) is a trivial link, the AID imitation $q : (M^*, L^*) \to (M, L)$ is closely related to the concept of an *almost trivial link* (or, in other words, a *Brunnian link*) in knot theory which is defined as follows:

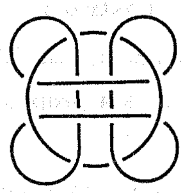
DEFINITION. A link L in S^3 with $r(\geq 2)$ components $K_i (i = 1, 2, \ldots, r)$ is *almost trivial* if the sublink $L - K_i$ is trivial in S^3 for all i. (In the case $r = 2$, we further impose that *the linking number of the link* L *is* 0.)

Trying to strengthen the concept of an almost trivial link, we also have the following concept of an *almost trivial link in the strong sense*:

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

DEFINITION. A link L in S^3 with $r(\geq 2)$ components $K_i(i = 1, 2, \ldots, r)$ is *almost trivial in the strong sense* if it is almost trivial and every cyclic covering link of L (i.e., the lifting of the sublink $L - K_i$ to every finite cyclic covering space over S^3 branched over K_i for every i) is an almost trivial link.

For example, the Whitehead link is almost trivial, but not almost trivial in the strong sense. On the other hand, the Milnor link, shown below, is almost trivial in the strong sense.



Although the Milnor link is not any imitation of a trivial link (because the link module is distinct from that of a trivial link), this concept leads us to the following concept(cf. [**7**,**9**]):

DEFINITION. A *strongly AID imitation* is an AID imitation $q : (M^*, L^*) \to (M, L)$ of a $(3,1)$-manifold pair (M, L) such that the lifting of this imitation to every finite regular covering $p : (\tilde{M}, \tilde{L}) \to (M, L)$ branched over a proper subfamily of the components of L such that $\tilde{M}$ is connected is still an AID imitation after missing the branch set.

In this lecture, we shall explain these concepts and recent applications to some 3-manifold invariants (cf. [**7**,**8**]) and the skein link polynomial.

REFERENCES

[1] A. Kawauchi, *Imitations of (3,1)-dimensional manifold pairs*, Sugaku Expositions **2** (1989), Amer. Math. Soc., 141-156.

[2] A. Kawauchi, *An imitation theory of manifolds*, Osaka J. Math. **26** (1989), 447-464.

[3] A. Kawauchi, *Almost identical imitations of (3,1)-dimensional manifold pairs*, Osaka J. Math. **26** (1989).

[4] A. Kawauchi , *Almost identical imitations of (3,1)-dimensional manifold pairs and the branched coverings*, Osaka J. Math. **29** (1992), 299-327.

[5] A. Kawauchi, *Almost identical imitations of (3,1)-dimensional manifold pairs and the manifold mutation*, J. Austral. Math. Soc. (Seri. A) **55** (1993), 100-115.

[6] A. Kawauchi, *Topics in Knot Theory*, Kluwer Academic Publishers, 1993, pp. 69-83.

[7] A. Kawauchi, *The 3rd Korea-Japan School of Knots and Links*, Proc. Applied Math. Workshop 4, Korea Advanced Institute of Science and Technology, 1994, pp. 43-52.

[8] A. Kawauchi, *Mutative hyperbolic homology 3-spheres with the same Floer homology*, Geom. Dedicata (to appear).

[9] A. Kawauchi, *A stronger concept of almost identical imitation of (3,1)-dimensional manifold pair*, preprint.

On spatial graphs

Kouki Taniyama

Department of Mathematics, College of Arts and Sciences

Tokyo Woman's Christian University

Zempukuji 2-6-1, Suginamiku, Tokyo, 167, Japan.

e-mail: taniyama@twcu.ac.jp

First we review some recent results of the author on spatial graphs.

Let G be a finite graph without loops and multiple edges. Let I be the unit interval. We say that two spatial embeddings $f, g : G \to R^3$ are

(1) *ambient isotopic* if there is a level preserving locally flat embedding of $G \times I$ into $R^3 \times I$ between f and g.

(2) *cobordant* if there is a locally flat embedding of $G \times I$ into $R^3 \times I$ between f and g.

(3) *isotopic* if there is a level preserving embedding of $G \times I$ into $R^3 \times I$ between f and g.

(4) *I-equivalent* if there is an embedding of $G \times I$ into $R^3 \times I$ between f and g.

(5) *edge homotopic* if f and g are transformed into each other by self-crossing changes and ambient isotopy. Here a self-crossing change is a crossing change on an edge of G.

(6) *vertex homotopic* if f and g are transformed into each other by crossing changes of adjacent edges and ambient isotopy. Here a crossing change of adjacent edges is a crossing change between two edges that have a common vertex.

(7) *homologous* if there is a locally flat embedding of $(G \times I)\# \bigcup_{i=1}^{n} S_i$ into $R^3 \times I$ between f and g where n is a natural number, S_i is a closed orientable surface and $\#$ means the connected sum. More precisely, there is an edge e of G for each i such that S_i is attached to the open disk $\mathrm{int}(e \times I)$ by the usual connected sum of surfaces.

Then we have the following fundamental relationship.

Theorem 1 [4].

$$\begin{array}{ccccc} & & (2) & & \\ & \nearrow & & \searrow & \\ (1) & & & & (4) \to (5) \to (6) \to (7). \\ & \searrow & & \nearrow & \\ & & (3) & & \end{array}$$

Thus we are able to approach the ambient isotopy classification problem step by step. Roughly speaking (7) homology is an 'abelianization'of (1) ambient isotopy and free from the difficulty of low dimensional topology. In fact it is classified by an algebraic invariant as follows.

For a topological space X let $C_2(X)$ be the configuration space of ordered pair of (distinct) points in X. Let σ be the involution on $C_2(X)$ that exchanges the order of the two points, i.e. $\sigma(x, y) = (y, x)$. Let $f : G \to R^3$ be an embedding. Let $f^2 : C_2(G) \to C_2(R^3)$ be the map defined by $f^2(x, y) = (f(x), f(y))$. Then f^2 induces a homomorphism

$$(f^2)^\# : H^2(C_2(R^3), \sigma) \to H^2(C_2(G), \sigma)$$

where $H^2(C_2(X), \sigma)$ denotes the skew-symmetric second cohomology of the pair $(C_2(X), \sigma)$. Namely $H^2(C_2(X), \sigma)$ is the second cohomology of the subcomplex $A_*(C_2(X), \sigma)$ of the singular chain complex $A_*(C_2(X))$ defined by $A_*(C_2(X), \sigma) = \{a \in A_*(C_2(X)) \mid \sigma(a) = -a\}$. It is known that $H^2(C_2(R^3), \sigma)$ is an infinite cyclic group, see [6] or [5]. Let τ be a fixed generator of $H^2(C_2(R^3), \sigma)$. Then Wu defined an invariant of f by $(f^2)^\#(\tau)$. We will denote this element of $H^2(C_2(G), \sigma)$ by $\mathcal{L}(f)$.

Theorem 2 [5].

Two embeddings f and g of a finite graph G into R^3 are homologous if and only if $\mathcal{L}(f) = \mathcal{L}(g)$.

Next we study the dependence and independence of the knot types of the subgraphs in a spatial graph. This is motivated by the following two pioneering works. In [1] Conway showed that every spatial embedding of the complete graph on six vertices contains a pair of nontrivially linked cycles, and Gordon showed that every spatial embedding of the complete graph on seven vertices contains a nontrivially knotted cycle. On the contrary, Kinoshita showed in [2] that any $n(n-1)/2$ knot types are realized by a spatial embedding of θ_n at once where θ_n is the graph on two vertices and n edges joining them. This result is followed by [7] and [8]. Thus we are interested in the dependence and the independence of all knot types in a spatial graph.

We show that under certain conditions, the Vassiliev type invariants of knots in a spatial graph are dependent. We will show in particular that the knots in a spatial embedding of a nonplanar graph are dependent. See [3].

Next we generalize the problem and give a complete answer to the realization problem of knotted subgraphs in a spatial graph under (7) homology.

References

[1] J. H. Conway and C. McA. Gordon: Knots and links in spatial graphs, *J. Graph Thory*, 7, 445-453, 1983.

[2] S. Kinoshita: On θ_n-curves in R^3 and their constituent knots, in Topology and Computer Science edited by S. Suzuki, Kinokuniya, 211-216, 1987.

[3] T. Motohashi and K. Taniyama: Delta unknotting operation and vetex homotopy of graphs in R^3, preprint.

[4] K. Taniyama: Cobordidim, homotopy and homology of graphs in R^3, *Topology*, 33, 509-523, 1994.

[5] K. Taniyama: Homology classification of spatial embeddings of a graph, *Topology Appl.*, 65, 205-228, 1995.

[6] W. T. Wu: On the isotopy of complexes in a eucledian space I, *Science Sinica*, 9, (1960), 21-46.

[7] M. Yamamoto: Knots in spatial embeddings of the complete graph on four vertices, *Topology Appl.*, 36, 291-298, 1990.

[8] A. Yasuhara: Delta-unknotting operation and adaptability of certain graphs, preprint.

Tunnel number and connected sum of knots

by

Kanji Morimoto

Department of Mathematics, Takushoku University
Tatemachi, Hachioji, Tokyo 193, Japan
e-mail : morimoto@hc.takushoku-u.ac.jp

Let K be a knot in the 3-sphere S^3 and $t(K)$ the tunnel number of K, where $t(K)$ is the minimal number of arcs in S^3 whose end points are in K such that the exterior of K and those arcs in S^3 is a handlebody. We call the family of those arcs an unknotting tunnel system for K. Let K_1 and K_2 be two knots in S^3, then we denote the connected sum of K_1 and K_2 by $K_1 \# K_2$.

In this talk, we report on study of the behavior of tunnel numbers of knots under connected sum.

By the definition of tunnel number and connected sum of knots, and by taking an arc contained in the decomposing 2-sphere for the connected sum, the following follows immediately.

Fact 1.1. $t(K_1 \# K_2) \leq t(K_1) + t(K_2) + 1$ *for any knots* K_1 *and* K_2.

By the above inequality, the following two conjectures had been made.

Conjecture A. *Tunnel numbers of knots cannot go down under connected sum, i.e., the inequality* $t(K_1) + t(K_2) \leq t(K_1 \# K_2)$ *holds for any knots* K_1 *and* K_2.

Conjecture B. *Tunnel numbers of knots can go up under connected sum, i.e., there are knots* K_1 *and* K_2 *such that* $t(K_1 \# K_2) = t(K_1) + t(K_2) + 1$.

Concerning the above conjectures, the first result is :

Theorem 1.2 ([No, Sc]). *Tunnel number one knots are prime, i.e., if* $t(K_1 \# K_2) = 1$ *then one of* K_1 *and* K_2 *is a trivial knot (tunnel number zero).*

The above result was obtained in the first half in 1980's. And this shows that Conjecture A is true if $t(K_1 \# K_2) = 1$.

In 1991, we studied the case when $t(K_1 \# K_2) = 2$ and got the following.

Theorem 1.3 ([Mo1, Theorem]). *Let K_1 and K_2 be non-trivial knots in S^3. Suppose $t(K_1 \# K_2) = 2$. Then :*

(1) *if neither K_1 nor K_2 is a 2-bridge knot, then $t(K_1) = t(K_2) = 1$.*

(2) *if one of K_1 and K_2, say K_1, is a 2-bridge knot, then $t(K_2) \leq 2$ and K_2 is prime.*

And in 1992, we showed that the estimate of the above theorem is the best possible by constructing knots K having the property that $t(K) = 2$ and $t(K \# K') = 2$ for any 2-bridge knot K'. In fact, we got the following.

Theorem 1.4 ([Mo2, Theorem 3]). *Let n be a positive integer, and K_n the knot illustrated in Figure 1.1. Then we have :*

(1) $t(K_n) = 2$.

(2) *$t(K_n \# K) = 2$ for any 2-bridge knot K.*

(3) *K_n and $K_{n'}$ are different types if $n \neq n'$.*

The examples in Theorem 1.4 show that Conjecture A is false. On the other hand, in the same year, Moriah and Rubinstein got the following.

Theorem 1.5 ([MR, Theorem 0.6]). *For any positive integers t_1 and t_2, there are infinitely many pairs of knots K_1 and K_2 such that $t(K_1) = t_1$, $t(K_2) = t_2$ and $t(K_1 \# K_2) = t_1 + t_2 + 1$.*

Theorem 1.5 shows that Conjecture B is true. And the theorem was proved by using argument from hyperbolic geometry, and those examples are corresponding to sufficiently complicated Dehn surgeries along some pretzel knots in S^3.

Concerning Conjecture B, the author, Sakuma and Yokota proved independently of [MR] by using another method that there are infinitely many pairs of knots K_1 and K_2 such that $t(K_1) = 1$, $t(K_2) = 1$ and $t(K_1 \# K_2) = 3$. This result is indeed contained in that of Moriah-Rubinstein's. But the examples we got would be more conclete than those of Moriah-Rubinstein's. In fact, we got the following.

Theorem 1.6 ([MSY, Theorem 2.1]). *Let m be an integer and K_m the knot illustrated in Figure 1.2. Then $t(K_m) = 1$, $t(K_{m'}) = 1$ and $t(K_m \# K_{m'}) = 3$ for any integers m and m'.*

References

[Mo1] K. Morimoto, *On the additivity of tunnel number of knots*, Topology Appl. **53**, (1993) 37-66

[Mo2] ________, *There are knots whose tunnel numbers go down under connected sum*, Proc. A. M. S. **123**, (1995) 3527-3532

[MSY] K. Morimoto, M. Sakuma and Y. Yokota, *Examples of tunnel number one knots which have the property " 1 + 1 = 3 "*, Math. Proc. Camb. Phil. Soc. **119**, (1996) 113-118

[MR] Y. Moriah and H. Rubinstein, *Heegaard structures of negatively curved 3-manifolds*, Preprint

[No] F. H. Norwood, *Every two generator knot is prime*, Proc. A. M. S. **86**, (1982) 143-147

[Sc] M. Scharlemann, *Tunnel number one knots satisfy the Poenaru conjecture*, Topology Appl. **18**, (1984) 235-258

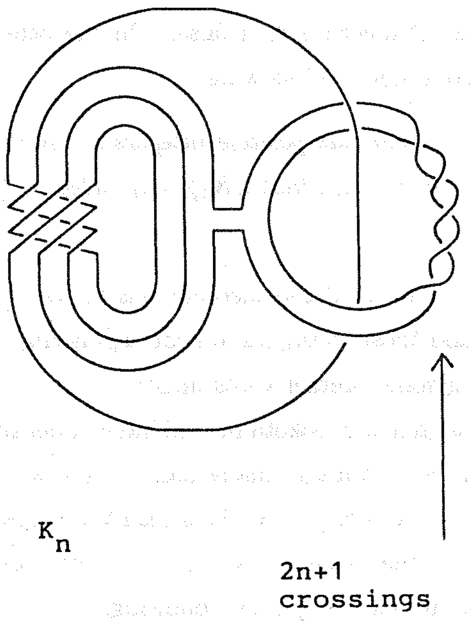


Figure 1.1

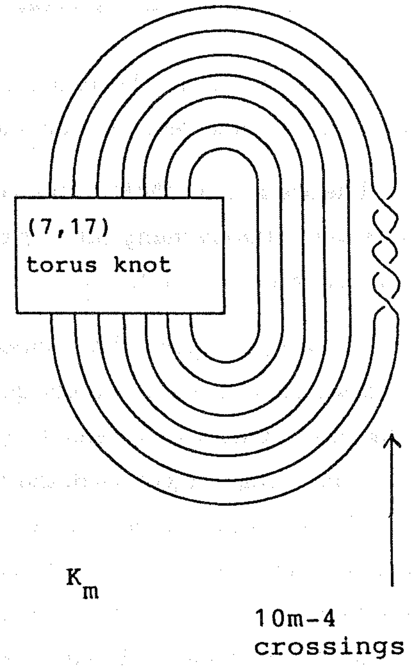


Figure 1.2

SURFACES IN 4-SPACE

SEIICHI KAMADA

Department of Mathematics, Osaka City University

Classical knot theory deals with simple closed curves in 3-space R^3 (or in the 3-sphere S^3) and asks when they are ambient isotopic or not. It is generalized into higher dimensional ones. We treat surfaces in 4-space R^4.

By a *surface in* R^4 we mean a closed oriented (or non-orientable) surface embedded in Euclidean 4-space R^4. Two surfaces in R^4 are *equivalent* if they are ambient isotopic. For given two surfaces in R^4 that are homeomorphic, it is very difficult to determine whether they are equivalent or not. It is the main problem in 2-knot theory.

The purpose of the talk is to explain some basic notions and fundamental results which might help one to work in 2-knot theory. First we treat of a method to describe surfaces in R^4 by considering their intersections with parallel hyperplanes R^3_t $(-\infty < t < \infty)$, which is called *hyperplane cross sections* in R. H. Fox's article "A quick trip through knot theory". Any surface F in R^4 is deformed, up to equivalence, such that the projection of the surface to the t-axis is a Morse function. Critical points of index 0, 1 and 2 are called *minimal, saddle* and *maximal* points, respectively.

Projecting each hyperplane cross section into 2-space R^2_{xy} we have a series of like knot (link) diagrams with some nodes. It is called a *movie* of F. A movie corresponds to a projection of F into R^3 (in precise R^3_{xyt}). Then Reidemeister

Typeset by $\mathcal{A}_\mathcal{M}\mathcal{S}$-TEX

type moves are introduced by D. Roseman and their completeness is proved by J. S. Carter and M. Saito.

Every example in Fox's article above is a 2-sphere F in R^4 such that $F \cap R^3_0$ is a knot and as t goes through a saddle point with increasing absolute value, the number of components increases. He asked in the article (p. 134) whether any 2-sphere in R^4 is obtainable in this way. This is solved affirmatively. Actually, for any surface in R^4 a natural analogy holds; namely it is defomed into a certain kind of configuration called *normal form*. (A. Kawauchi, T. Shibuya and S. Suzuki gave a concrete proof to it.) The first half of the talk concerns with normal forms.

As every classical knot (or link) is equivalent to a closed braid, every oriented surface in R^4 is equivalent to *a closed 2-dimensional braid*. This fact is announced by O. Ya. Viro in the lecture given at Osaka City University in 1990. (It is a motivation for my reserch on this field.) L. Rudolph also considered a similar notion called *braided surface* and gave some applications to knot theory. I would like to devote the latter half to explaining some results on such braidings of oriented surfaces in R^4.

KNOT TYPES OF SATELLITE KNOTS AND TWISTED KNOTS

KIMIHIKO MOTEGI

Department of Mathematics, Nihon University

Let V be a standardly embedded solid torus in S^3 and K a knot in S^3 contained in V. The pair (V, K) is called a pattern and the minimal geometric intersection number (resp. algebraic intersection number) of K and a meridian disk of V is called the *wrapping number* (resp. *winding number*) of K in V and denoted by $\mathrm{wrap}_V(K)$ (resp. $\mathrm{wind}_V(K)$). Let f be an orientation preserving embedding from V into S^3. Then using f we obtain a new knot $f(K)$ in S^3. If $f(V)$ is a knotted solid torus in S^3, then $f(K)$ is a *satellite knot* with a companion knot $f(C_V)$, where C_V denotes a core of V. On the other hand, if $f(V)$ is also unknotted, then the above operation is essentially same as *twisting* several times along a meridian disk of V.

First we consider the case where $f(V)$ is a knotted soloid torus. We assume that knots are oriented and we write $K_1 \cong K_2$ if K_1 and K_2 are equivalent as oriented knots. The knot obtained from K by inverting its orientation is denoted by $-K$. If $\mathrm{wrap}_V(K) = 1$, then the operation corresponds with a "product" of knots. In such a situation, Schubert's unique factorization theorem implies that if $f(K) \cong g(K)$ for two embeddings f and g, then their companions $f(C_V)$ and $g(C_V)$ are equivalent.

It is this result we generalize to any pattern. In the following, $\mathrm{twist}(f)$ is defined to be $\ell k(f(C_V), f(\ell_V))$, where ℓ is a longitude of $V (\subset S^3)$. When $\mathrm{wrap}_V(K) = 1$, $\mathrm{twist}(f)$ is irrelevant.

Theorem 1 ([Kouno-Motegi]). *Let (V, K) be a pattern with $\mathrm{wrap}_V(K) \geq 2$ and $f : V \hookrightarrow S^3$ an orientation preserving embedding such that $f(V)$ is knotted. Let $g : V \hookrightarrow S^3$ be an orientation preserving embedding satisfying $g(K) \cong f(K)$. Then $g(C_V) \cong f(C_V)$, or $f(C_V) \cong K_0 \sharp K_1$ and $g(C_V) \cong (-K_0) \sharp K_1$, where K_0 and K_1 are the knots uniquely determined by f and the pattern (V, K). Furthermore, in any case $\mathrm{twist}(f) = \mathrm{twist}(g)$.*

As a corollary, we have:

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

Corollary 2. *Let (V, K) and f be as in Theorem 1. Then up to isotopy there is at most one orientation preserving embedding $g : V \hookrightarrow S^3$ which is not isotopic to f and satisfies $g(K) \cong f(K)$.*

There is an example in which $f(K) \cong g(K)$ but f and g are not isotopic ; in fact $f(C_V)$ and $g(C_V)$ cannot be equivalent even in the weakest sense.

In what follows we restrict our attention to the case where the original knot K is unknotted in S^3.

For a given pattern (V, K) (K is unknotted in S^3), Theorem 1 can be simply stated as follows.

Corollary 3. *Let (V, K) and f be as in Theorem 1 and assume that K is unknotted in S^3. If $f(K) \cong g(K)$, then $f(C_V) \cong \pm g(C_V)$ and twist(f) =twist(g).*

Let us fix an embedding f and exchange the pattern (V, K) to (V, K').

Can $f(K')$ be ambient isotopic to $f(K)$?

Theorem 4. *Let (V, K) and (V, K') be patterns such that K is unknotted and K' is knotted in S^3. If wind$_V(K') \neq 0$, then $f(K) \ncong f(K')$ for any embedding f.*

There is an example showing the necessity of the condition "wind$_V(K') \neq 0$".

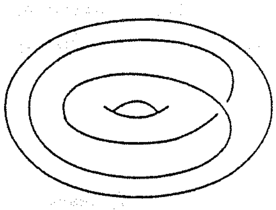
We now turn to the case where both K and K' are unknotted in S^3.

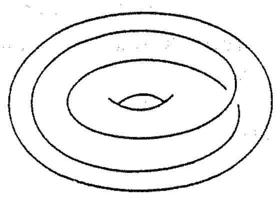
Theorem 5. *Let (V, K) and (V, K') be patterns such that K and K' are unknotted in S^3.*
(1) If wind$_V(K) \neq$ wind$_V(K')$, then $f(K) \ncong f(K')$ for any emdedding f.
(2) If wind$_V(K)$ = wind$_V(K') \neq 0$ and wrap$_V(K) \neq$wrap$_V(K')$, then $f(K) \ncong f(K')$ for any embedding f.

If wind$_V(K)$ = wind$_V(K') = 0$, then there is a counterexample for this theorem.

Now let us assume that $f(V)$ is an unknotted solid torus in S^3. In this situation we may assume that $f(V) = V$ and f is a twisted homeomorphism f_n ($f_n(\ell) = \ell + nm$). In the following we assume that wrap$_V(K) \geq 2$ and write $K_n = f_n(K)$.

Theorem 6 ([Mathieu], [Kouno-M-Shibuya]). *Suppose that K is a trivial knot in S^3. If K_n is again a trivial knot in S^3 for some $n \neq 0$, then (V, K) is the following.*





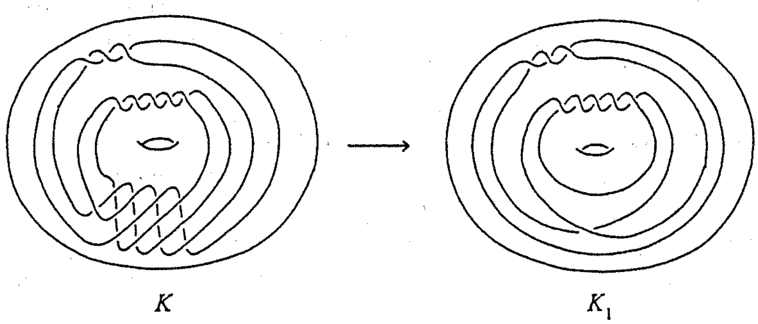
The following question was raised by Y.Mathieu.

Can we obtain a composite knot from a trivial knot by twisting?

If $\text{wrap}_V(K) = 2$, then Scharlemann and Thompson, Gordon, and Zhang had proved that K_n is prime.

However in general we have:

Example ([Motegi-Shibuya]).



Ohyama, Yasuhara and Teragaito also found other examples. In any example $n = \pm 1$.

Theorem 7 ([Goodman-Strauss], [Hayashi-Motegi]). *Suppose that K is a trivial knot in S^3. If K_n is a composite knot, then $n = \pm 1$.*

Goodman-Strauss shows further that K_{-1} and K_1 cannot both be composite.

In the case where a twisted knot K_n is a torus knot, we have:

Theorem 8 ([Miyazaki-Motegi]). *Suppose that K is a trivial knot in S^3. If K_n is a torus knot, then except for trivial examples $n = \pm 1$.*

A Weight System for 3-d Manifold

Laurent FREIDEL

The Kontsevich integral gives rise to an invariant of framed links with value into the space of chord diagrams satisfying 4-term relation. T.Le, H.Murakami, J.Murakami, T.Ohtsuki showed that it is possible to constract from this universal invariant a 3-dim. manifold invariant under the condition that chord diagrams satisfied some new relations: essentially orientation independence and handle sliding independence relations. Using the recoupling theory of $SU(2)$ we constract a weight system, i.e. a linear functional on chord diagrams with value in Z/pZ, p odd prime which satisfies these relations.

Knot Tabulation Progress Report
July, 1996

Jim Hoste, Morwen Thistlethwaite, Jeff Weeks

We have presently tabulated all prime knots through 15 crossings and all prime 16-crossing alternating knots. A provisional table of prime 16-crossing non-alternating knots awaits final confirmation and will be ready soon. The number of prime knots with a given crossing number are given in Table 1.

Numbers of Prime Knots by Crossing Number

crossings	alternating	non-alternating	total
3	1	0	1
4	1	0	1
5	2	0	2
6	3	0	3
7	7	0	7
8	18	3	21
9	41	8	49
10	123	42	165
11	367	185	552
12	1288	888	2176
13	4878	5110	9988
14	19536	27436	46972
15	85263	168030	253293
16	379799		

Table 1

The tables have been created by two completely independent efforts, each employing significantly different methods. Both Jim Hoste and Morwen Thistlethwaite have written computer programs to generate complete lists of knots with a given crossing number and containing relatively few repetitions. In order to remove dulpicates from these lists two approaches are taken. Hoste has teamed with Jeff Weeks and applied methods from hyperbolic geometry. Using his computer program *SnapPea*, Weeks computes the canonical triangulation of each hyperbolic knot on Hoste's list. Since this is a complete knot invariant, duplications are easily recognized and removed. The few knots that are not hyperbolic are handled separately. Thistlethwaite relies on a battery of knot invariants (including for example, skein polynomials and representations of knot groups into permutation groups) to distinguish different knots and isolate suspected duplications. Ultimately, duplicates are shown to be the same by applying a more intensive computer search for "moves" that will relate the diagrams. Remarkably, only a handful of discrepancies have arisen between the two tabulations, and in each case have been successfully resolved.

A computer program called *KnotScape* is being written to allow easy access to the tables. An alpha version may be downloaded from http://www.math.utk.edu/~morwen/knotscape.html. This version only includes knots through 15 crossings, does not yet contain a graphical knot editor which is still in a preliminary state, and presumably contains bugs! It runs in a UNIX environment and requires Tcl 7.4 / Tk 4.0.

Markov theorem with one move and Markov theorem in 3-manifolds

Sofia Lambropoulou (with C. Rourke)

We show that braid equivalence reflecting knot/link isotopy in S^3 can be generated by one geometric move, the L-move (instead of the well-known conjugation and stabilizing moves in the braid groups). We further explain how to obtain braid equivalence for isotopic links in arbitrary c.c.o. 3-manifolds using braid equivalence in knot/link complements.

On the invariants of lens knots

Nafaa Chbili

A knot in S^3 is called a (p, s)-lens knot if it's invariant by the (p, s)-lens action on S^3. The main purpose of this paper is to study the invariants of these knots. We prove that the Vassiliev-Gusarov invariants of lens knots are related to those of torus knots, in the same way as in Przytycki's criteria for periodic knots. We use this result to prove that in the case $p = 3$, the HOMFLY polynomial satisfies a powerful criteria.