# Pairwise Comparison for Weight Restriction in DEA/ARI

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# Abstract

Strategic decision is important because it critically affects organization health and survival. The decision making can become more complex and often inherently uncertain, more so due to a large number of different alternatives with conflicting among criteria. Therefore, an efficiency of multi-criteria decision analysis for supporting strategic decision making plays a critical role in solving problem. This thesis presents an integrated methodology developed for dealing with the decision making problem that contains multiple alternative and multiple criteria with the target to get one best solution for an analysis.

Assurance Region of type I (ARI) technique which allows incorporating value judgements of decision maker into the assessment is integrated with Data Envelopment Analysis (DEA) method, which is a well-known tool for performance and efficiency analysis based on a mathematical programming approach, for purpose of imposing weight restriction on the traditional DEA with an intention to improve discrimination for the solution and to incorporate viewpoint of the decision maker into an assessment. The main challenge with the use of ARI is how to quantify the values of weight bounds which are in ratios of criteria weights determined by the decision maker and these bounds can provide feasible solution for linear programming models.

The thesis proposes a method for acquiring weight restriction constrains in the ARI by applying grade and pairwise comparison techniques to convert judgement of the decision maker on relative importance among decision criteria into values of the bound on ratios of criteria weights, which will be used as additional constraints in the DEA. The proposed method has an attempt to simplify setting bounds which the result could be more consistent with the direction of decision maker. The set of ARI constraints obtained from the proposed method also satisfy transitivity property which lead to the feasibility for the resulting linear programming model. From computational experiment of facility location problem containing many alternative and many criteria, the number of selected alternatives can be reduced from many to a few.

Analytic Hierarchy Process (AHP), which is another robust decision making tool based on mathematics and psychology, is also brought up in order to illustrate how the method is suitable for solving multi-criteria

decision making which contains few alternatives. This is shown by applying to a practical case study of route selection problem, which the AHP method proves to provide optimum solution.

In conclusion, this thesis targets to solve complex decision making problem that contains a large numbers of alternatives and criteria. The proposed method is useful in assisting the decision maker to determine the value of the bounds in the ARI technique. The work in this thesis shows that utilizing an integration of the DEA with the ARI technique along with the AHP serves as strategic decision support tool helping manager or decision maker effectively solve the decision making problem and they can be used as a tool for the enhancement of performance and organizational change.

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# 1 INTRODUCTION

Decision making has to be executed all the time, ranking from trivial issues like everyday choices such as what to have for dinner to complex policy decisions that have an impact on human life. Whether in daily lives or in professional contexts, there are typically many criteria which conflict to each other and they need to be evaluated in making decisions. Therefore the simplest sense of decision making, which is somehow a multi-criteria nature, is regarded as the cognitive process of the act of identifying and choosing between two or among several possible courses of action based on values and preferences of decision maker. The decision making can also be regarded as process of problem-solving involving finding a solution to a problem that deems to be satisfied. Real life decision problems involve a range of alternative options to be identified for the most preferred one. Usually more than one decision criteria are taken into account, and one alternative option rarely performs best with regard to all criteria.

The decision process which is more or less rational or irrational can be made through either an intuitive (or tacit knowledge), or reasoning (or explicit knowledge), or a combination of the two. Intuition is the ability to understand or know something, or an idea about what is true in a particular situation based on a feeling rather than considering facts. It is a combination of past experience and personal values which is worth to be taken into account while making decision. However, it is only one's perceptions on particular issues and is not always based on reality. Meanwhile, reasoning is a process of thinking carefully about something by using facts and figures in order to make a judgement. It can eliminate emotional aspects to the decision, however, issues from the past that may affect the decision are ignored.

The intuition is generally more appropriate and is acceptable means for making decision when the decision is simple or needs to be justified in a short time. The more complicated decisions certainly require more formal and structured approaches for evaluation. Table 1.1 summarizes a recommendation for the use of multi-criteria evaluation technique based on type of decision.

**Table 1.1** Recommendation of Multi-Criteria Evaluation Based On Type of Decision

| Type of Decision | Importance | Multi-Criteria Evaluation |
|---|---|---|
| Everyday decisions | Low | Not recommended |
| Important decisions | High | Recommended |
| Decisions which must be justified | High | Highly recommended |
| Strategic decisions | Extreme | Absolutely needed |

People make a lot of choices in daily lives. The multiple criteria are usually weighed implicitly and these low or unimportant decisions can be made based on only feelings or intuition of the decision maker because the consequences of decisions are generally insignificant and it will take much more time to proceed using an accurate multi-criteria supporting tool. By the way, it is worth to apply some multi-criteria methods to analyze important decisions and decisions which must be justified, although it requires some effort. This is because these two types of decision have significant impact on eventual outcome or future so the chances of making a good decision is preferable. For strategic decisions, it is absolutely important to properly structure the problem and explicitly evaluate multiple criteria because this type of decision is complex and difficult, containing high stakes, and its consequence could affect a large group of people. The strategic decisions are also infrequent decisions which made by the top leaders of an organization that critically have an effect on organizational health and survival (Eisenhardt and Zbaracki, 1992). Furthermore, the process of creating, evaluating, and implementing strategic decisions is typically characterized by the consideration of high levels of uncertainty, potential synergies between different alternatives, and long term consequences (Zopounidis, C. and Pardalos, 2010).

As most decision requires multi-criteria decision making (MCDM) methods for evaluation in order to gain better solutions, a wide variety of approaches and methods have been developed to overcome this problem of the optimization. Many of them have certain aspects in common (Chen and Hwang, 1991). The notions of alternatives and criteria of the decision problem are described by Triantaphyllou (2000) as follows:

*Alternatives*: alternatives represent different choices or options available to the decision maker to choose or use. The set of alternatives is usually assumed to be finite, which varying from a few to several. Each alternative has characteristic features with its own set of strengths, weaknesses, uncertainties and consequences, etc.

*Multiple criteria*: criteria represent different dimensions from which the alternatives can be viewed. Some multi-criteria decision making methods consider a hierarchical structure of the criteria when the number of criteria is large in a decision problem while most of the methods assume a single level of criteria.

*Conflict among criteria*: there can be many interrelated criteria to consider since the different criteria represent different dimensions of the alternatives. For example, cost has to be spent as less as possible while profit is preferable which is needed to be increased.

*Different units*: unit of measure may be different due to the different criteria. Having to consider these incommensurable units make multi-criteria decision making problem hard to solve.

*Decision weights*: weights of importance are required to be assigned to the criteria in most of the MCDM methods. These weights are usually normalized to sum up to one, however different methods have different techniques to attain the weights.

The MCDM approaches are considered a major part of decision theory and analysis. They are used as efficient tools for making critical decisions in many fields. The common purpose of these diverse methods is to be able to help the decision maker evaluates and chooses among alternatives based on multiple criteria by using systematic analysis that overcomes the observed limitations of unstructured decision making problem. The main role is to deal with the difficulties that human facing in handling large amounts of complex information in a consistent way. The decision making methods differ from one another in the way and their ability to handle problems. They also require different types of raw data and pursue different optimization algorithms. However, the methods are all based on the rational decision making process described below.

*Step 1: Define objective of the decision*

The most important step in solving any MCDM problem is first to correctly define the objective or problem of an analysis because attaining good decisions require clear and manifest objectives. The purpose of the decision being made has to be recognized, thoroughly analyzed and specific. A shared understanding of decision context also has to be established.

*Step 2: Identify alternatives for achieving the objectives*

The next step is to list the set of all possible and desirable alternatives to be considered. Including a number of different alternatives in an analysis may make the decision to be more complicated at the first place, however a wide range of alternatives stimulates the analysis to delve deeper into the issue and look at the problem from different aspects. The wider range of alternatives to be explored is also likely to give better outcome of the final solution.

*Step 3: Identify criteria to be used to compare the alternatives*

A set of decision criteria that reflect performance or efficiency of the alternatives in contributing to reach the objective has to be defined and developed. These criteria are used to distinguish or compare among the different alternatives to be evaluated. Each criterion must be possible and available to assess and measurable with at least in a qualitative manner.

*Step 4: Evaluate the alternatives*

In this step, judgement principles and decision criteria are used in evaluating each alternative. Each alternative need to be examined and compared in the sense that whether the objective identified in Step 1 would be solved or achieved through the use of a particular alternative. Certain alternatives which appear to have higher potential for reaching the objective seem to be more favourable for an analysis.

*Step 5: Select the best solution*

Once all the criteria are weighed, this step is to make a decision. The alternative which seems to be best suited to the objective or problem is usually selected.

The multi-criteria decision analysis tools are utilized whenever the decision maker is faced with difficulties in the decision due to the existence of more than one objective or criteria that have to be satisfied in order to arrive at a successful and final selection from the available alternatives (Belton, 1990). These decision making problem can contain a combination of varying number of criteria and alternatives. In this thesis, the decision making problem are defined into four categories as shown in Figure 1.1. Problems in the bottom left and bottom right corners of the figure that involve few criteria are likely easy to solve when compared with problems that involve many criteria displayed in top left and top right corners in which the strategic decisions usually fall into. Solving such problems with many criteria is in the focus of multi-criteria decision analysis. This area of decision making has still attracted the interest of many researchers and practitioners although it has been studied for long time. This leads to the continuous development of multi-criteria decision making methods for supporting strategic decision making.

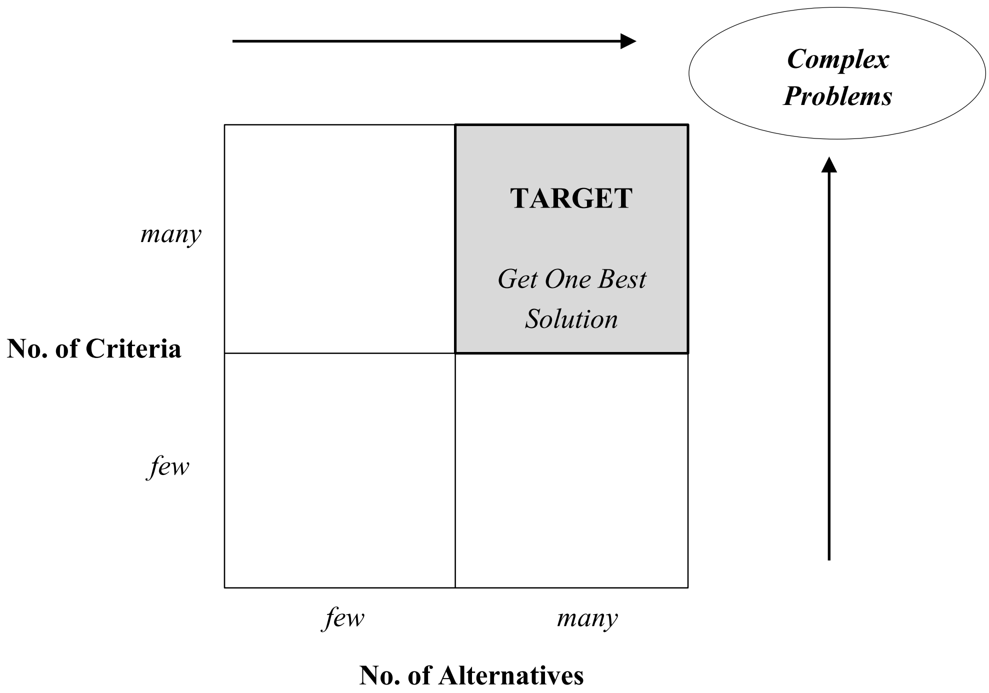



**Figure 1.1** Category of Decision Making Problem

A major part of the multi-criteria decision making involves the analysis of a finite set of alternatives which are described in terms of evaluative decision criteria. The task is either to make a ranking of alternatives in terms of how good or how attractive they are to the decision maker, or to select the best alternative when all the criteria are considered simultaneously. These problems are consequently more complex when multiple alternatives are accommodated in an analysis due to a large number of comparisons among criteria and alternatives that have to be taken into account during an evaluation.

According to Figure 1.1, the decision making problem consisting few alternative and many criteria shown in top left corner is named as FAMC (Few Alternatives Many Criteria), and problem with many alternative and many criteria shown in top right corner is named as MAMC (Many Alternatives Many Criteria). The objective of this thesis is to develop a framework based on utilization of multi-criteria decision analysis tools to support strategic decision making in resolution of MAMC with the overall purpose to improve the quality of decision making. Main part of the thesis is to propose a theoretical work on development of a method to determine values of Assurance Region of Type I (ARI) weight bound constraints. The ARI technique is incorporated in an employment of the conventional DEA in order to solve MAMC problem. The thesis also suggests a hybrid approach to get one best solution for decision making problem by an implementation of Analytic Hierarchy Process (AHP) to dealing with FAMC.

The thesis is organized into six chapters and is structured as follows:

- Chapter 2 outlines several multi-criteria decision making techniques used in this thesis. Firstly, the concept of Data Envelopment Analysis is described to show its capability in dealing with MAMC. Then technique of Assurance Region of Type I is introduced to be incorporated to DEA method for purpose of resolving some drawbacks associated with DEA. Lastly, Analytic Hierarchy Process is revealed in order to handle FAMC.
- Chapter 3 presents a proposed technique to determine weight bound values to be located in the ARI weight restriction constraints. Several important issues in setting bounds are raised, and the transitivity which is a key property to provide feasible solution in linear programming models is illustrated. The chapter explains the procedure of proposed method which is easy to follow and also shows how the inequality equations of ARI generated by the proposed method can reach the transitivity property.
- Chapter 4 offers two numerical applications to explain the utilization of decision making tools for solving real-world decision making problems. The first example demonstrates the use of proposed method for setting bounds on ARI constraints in order to deal with a large scale problem of facility location selection. The results shows that the proposed technique can effectively be applied to MAMC as it can improve solution of an analysis. The second example shows an application of AHP to select one best alternative from FAMC of route selection problem.
- Chapter 5 gives a discussion on the development and relevant issues of the proposed technique. The key contributions to the use of proposed method are highlighted. This chapter also presents an analysis of solution from numerical application using DEA with ARI and the proposed method.

- Chapter 6 concludes the research with an overall summary of the thesis and gives recommendation on supporting tools for strategic decision making. A reference is provided at the end of the thesis.

# 2 PREVIOUS STUDY

The target of this thesis is to solve MAMC by developing a new methodology based on the use of multi-criteria decision tools. Chapter 2 reviews the concept and application of decision making methods applied in this thesis. The drawbacks and difficulties associated with the use of the existing techniques are also discussed. This chapter is divided into the following sections: Section 2.1 represents Data Envelopment Analysis (DEA) method which is used as a main tool in dealing with MAMC, Section 2.2 analyzes Assurance Region of type I (ARI) technique for incorporating with the DEA, and Section 2.3 describes Analytic Hierarchy Process (AHP) which will be later applied to FAMC.

## 2.1 Data Envelopment Analysis (DEA)

This section explains one of the most useful tools which can deal with MAMC namely Data Envelopment Analysis or DEA. The method was initially proposed by Charnes, Cooper and Rhodes in early 1970s and has been one of the fastest growing areas of Operations Research and Management Science in the past decade (Cooper et al., 2007). DEA has also grown into a powerful analytical tool for measuring and evaluating the performance of many different types of entities engaged in a wide variety of activities in many contexts worldwide, including management analysis and economic problem situation in both public and private sectors (Seiford, 1994).

### 2.1.1 Concept of DEA and its model

**DEA concept**

The idea of efficiency measurement relies on production theory which an entity or unit is identified as a production system where inputs are the resources to be consumed in order to produce outputs. Then the measurement of relative efficiency of multiple inputs and multiple outputs was introduced by Farrell (1957) by assigning weights to the input and output variables. The overall relative efficiency score is therefore a ratio of the weighted sum of the outputs to the weighted sum of the inputs.

$$\text{Efficiency} = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}}$$

This efficiency measurement process considers multiple inputs and multiple outputs where equal importance is given to a particular input or output for all the selection alternatives. Considering $n$ units with $m$ inputs and $s$ outputs where $x_{ij}$ is quantity of input $i$ for unit $j$ and and $y_{rj}$ is quantity of output $r$ for unit $j$, the mathematical representation of the above expression of efficiency would be written as

$$\theta = \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \tag{2.1}$$

where $\theta$ is efficiency of unit $j$, $v_i$ is weight on input $i$, $u_r$ is weight on output $r$. Result of the above model is efficiency score of each unit in the range of zero to one.

The DEA method developed by Charnes, Cooper and Rhodes is an extension of Farrell's approach to measure efficiency $\theta$ by determining the best set of weights for each unit under consideration. This fractional programming model, known as CCR model, is a data-oriented approach for evaluating the performance or efficiency of a homogeneous set of peer entities in a data set of comparable units which are referred as Decision Making Units or DMUs (Cooper et al., 2011b). The definition of DMU in the DEA is general and allows flexibility in its use over wide range of possible applications. In general, a DMU is regarded as the entity responsible for converting multiple inputs into multiple outputs (Banker et al., 1984). It can be in a various forms of any businesses, operations, or entities under evaluation such as banks, hospitals, etc. (Charnes and Cooper, 1961). Figure 2.1 shows the DEA with multiple DMUs and multiple inputs and outputs.

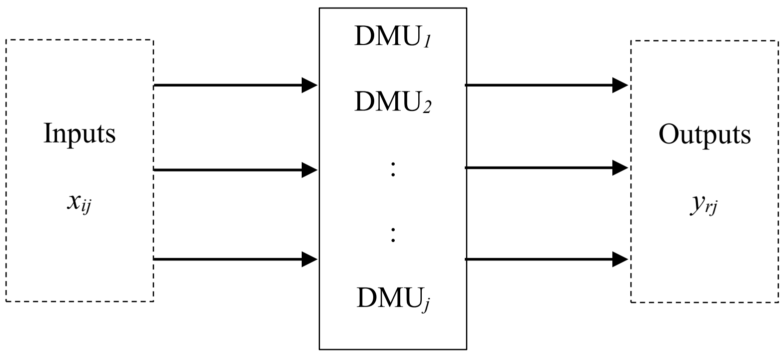



**Figure 2.1** DEA System with Multiple Inputs and Multiple Outputs

The DEA method is a nonparametric fractional linear programming technique that can be applied for the purpose of ranking or comparing the relative performance of DMUs which operate under comparable conditions. It is particularly effective in handling complex processes where DMUs use multiple input and output criteria. Unlike parametric methodologies such as regression model which assume that the same average equation applies to all samples or DMUs, the DEA can optimize particular DMU by arriving at an efficiency score for each of every DMU relative to the entire samples (Cooper et al., 2007). The different between DEA and regression methods in evaluation of the samples is shown in Figure 2.2.

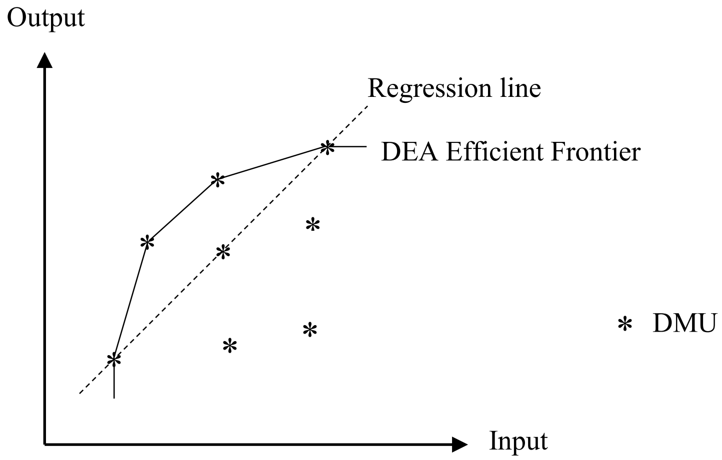



**Figure 2.2** DEA versus Regression Analysis

The method is designed to measure or assess the efficiencies in situations where the DMUs consume a variety of identical resources or inputs to produce a variety of identical products or outputs. The goal is to determine the productive efficiency of DMUs by comparing how well the DMU converts inputs into outputs (Charnes et al., 1978). The DEA produces a single comprehensive score for each DMU by calculating the ratio of the weighted sum of its outputs to the weighted sum of its inputs. The calculation is run for each DMU to determine the set of input and output weights which maximizes the efficiency of that assessed DMU subject to the condition that no DMU can have a relative efficiency score greater than unity for that set of weights. Thus, a unique set of weights is assigned for each DMU to maximize the score. The set of weights has the following characteristics: it maximizes the efficiency of the DMU for which it is calculated, and it is feasible for all DMUs. Each individual DMU, consequently, receives the highest score possible and the argument of using different weights is not valid when comparing final scores (Tandon et al., 2006). All DMUs also use the same set of non-negative weights. The final output of the DEA is a ranked efficiency score for each DMU.

The efficiency of each DMU is calculated in relation to all other DMUs and using actual observed input and output data, so the efficiency calculated in the DEA is called relative efficiency. Charnes, Cooper and Seiford (1994) define DEA as “DEA produces a piecewise empirical extremal production surface which in economic terms represents the revealed best-practice production frontier - the maximum output empirically obtainable from any DMU in the observed population, given its level of inputs.” In addition to calculating the efficiency scores, DEA also provides the level and amount of inefficiency for each of the inputs and outputs of each DMU. The amount of inefficiency is determined by comparison with a convex combination of two or more DMUs which lie on the efficient frontier that utilizing the same level of inputs and producing the same or higher level of outputs. Several models have been proposed in the DEA field where all of them utilize the concept of the DEA mentioned above. However, the CCR model which is the very first model of the DEA is still the most commonly referenced in the literature. It also will be used in the proposed method for incorporating with the weight restriction technique.

**DEA model**

The measure of efficiency of any DMUs is obtained as the maximum of a ratio of weighted output to weighted input subject to the condition that similar ratios for every DMU be less than or equal to unity. Assume that there are $n$ DMUs to be evaluated, where each DMU consumes varying amounts of $m$ different inputs to produce $s$ different outputs. $x_{ij}$ and $y_{rj}$ are respectively amounts of input $i$th and output $r$th of DMU$_j$, and they are in positive number $x_{ij} \geq 0$, $y_{rj} \geq 0$. Further assume that each DMU has at least one positive input and one positive output value. The ratio of outputs to inputs is used to measure the relative efficiency of the DMU$_j$ = DMU$_0$ to be evaluated relative to the ratios of all of the $j$ = 1, 2,…, $n$ DMU$_j$. And $h_0$ is the efficiency score of a particular DMU being evaluated. The mathematical formulation of input-oriented can thus be stated as

$$\max h_0(u,v) = \frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}} \tag{2.2}$$

s.t. :

$$\frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \leq 1 \qquad \forall j$$

$$u_r \ , \ v_i \ \geq \ 0 \qquad \forall r,i$$

where $v_i$ and $u_r$ are decision variables and are respectively called input and output multipliers or input and output weights. $x_{i0}$ and $y_{r0}$ are the observed input and observed output value of DMU$_0$. The objective function is to maximize the efficiency value, which is in a form of the ratio of weighted outputs to weighted inputs, of a particular DMU using the weights $v_i$ and $u_r$ for the inputs and the outputs respectively. The weights are determined by the model with an objective function to maximize the efficiency score of the DMU under consideration, and the same set of weights are applied to the other DMUs in the sample under constraint that their efficiency score cannot exceed one.

Since the model is a fractional program, it has to be converted into a linear program so that it can be solved easily. This is done by normalization, i.e. the denominator of the objective function is equated to one and the first constraint corresponding to efficiency ratios of all DMUs in the sample is also modified, which leads to the following equivalent linear programming problem.

$$\max h_0 = \sum_{r=1}^{s} \mu_r y_{ro} \tag{2.3}$$

s.t. :

$$\sum_{i=1}^{m} v_i x_{io} \ = \ 1$$

$$\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ \leq \ 0 \quad \forall j$$

$$u_r \ , \ v_i \ \geq \ 0 \qquad \forall r,i$$

The model is run $n$ times to identify the efficiency scores of all DMUs. Each DMU selects input and output weights that maximize its efficiency score. The DEA will identify the DMU(s) that produces the largest amounts of outputs by consuming the least amounts of inputs and then allocate efficiency score equal to one to the DMU(s). Thus, a DMU is considered to be efficient if it obtains a score of one ($h_0 = 1$) and other inefficient DMUs will be given efficiency scores relatively to the efficient DMU(s) which is less than one ($h_0 < 1$). Figure 2.3 shows a graphical of efficiency frontier of input-oriented model. It can be seen that the efficiency of the observed DMUs can be evaluated by forming a best practice frontier or efficiency frontier based on the performance of the best attaining DMU(s) and then comparing the rest DMUs to them.

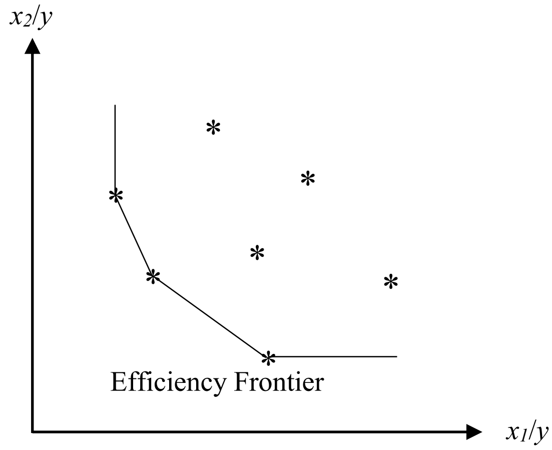



**Figure 2.3** Input-Oriented Model

The CCR model can also be written in output-oriented objective which contrasts to the above model as

$$\begin{aligned}
&\min\ h_0 = \sum_{i=1}^{m} v_i x_{io} \\
&\text{s.t. :} \\
&\sum_{r=1}^{s} \mu_r y_{r0} \ = \ 1 \\
&\sum_{i=1}^{m} v_i x_{ij} \ - \sum_{r=1}^{s} \mu_r y_{rj} \ \geq 0 \quad \forall j \\
&u_r\ ,\ v_i \ \geq\ 0 \qquad \forall r, i
\end{aligned} \tag{2.4}$$

Also Figure 2.4 shows a graphical of efficiency frontier of output-oriented model. Both input-oriented and output-oriented models yield the same optimum solution.

The topic of DEA model has increased in notoriety since 1978 (Seiford, 1996). There are also many issues related to its application which provide the basis for development of the technique in production efficiency and multi-criteria decision making research today.

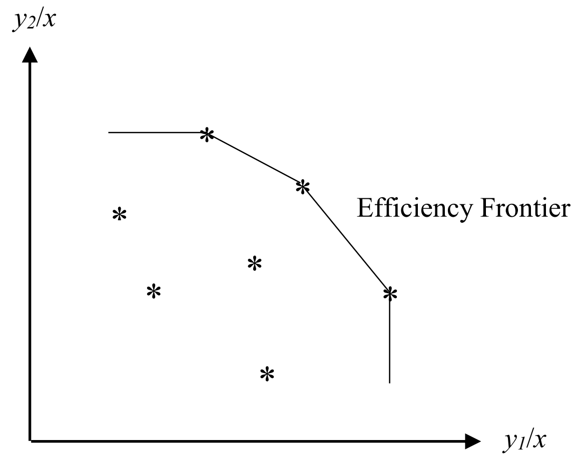



**Figure 2.4** Output-Oriented Model

**2.1.2 Characteristic of weights in DEA**

The DEA model calculates a unique set of weights for each DMU. The set of weights has two particular characteristics: it maximizes the efficiency of the DMU for which it is calculated, and it is feasible for all DMUs. The weights are assigned to individual input and output data to generate virtual input and output. These weights are not fixed, but are varied from each DMU in order to give the best combination of multiple weighted inputs and multiple weighted outputs for the purpose of maximizing the efficiency score of DMUs.

Assume that the input weight, $v^*$, and output weight, $u^*$, are variables obtained as an optimal solution for linear programming results which are in a set of optimal weights for the particular assessed DMU, i.e. $\mathrm{DMU}_0$. The ratio scale is evaluated by

$$h_0^* = \frac{\sum_r u_r^* y_{ro}}{\sum_i v_i^* x_{io}} \tag{2.5}$$

Considering that the denominator is equal to one, therefore $h_0^* = \sum_r u_r^* y_{ro}$. As mention that $v^*$ and $u^*$ are the most favorable weights for $\mathrm{DMU}_0$ in the sense of maximizing the ratio scale of the weighted sum of outputs to the weighted sum of inputs. $v_i^*$ is the optimal weight for the input item $i$ and its magnitude expresses how highly the item is evaluated. Similarly, $u_r^*$ is the optimal weight for the output item $r$.

The relative importance of each input criterion can be examined by reference to the value of each $v_i^* x_{io}$ in the virtual input $\sum_i v_i^* x_{io} = 1$. The same situation holds for $u_r^* y_{ro}$ where the $u_r^*$ provides a measure of the relative contribution of $y_{ro}$ to the overall value $h_o^*$. These weight values can show the extent of each criterion that contribute to the evaluation of DMU$_0$.

### 2.1.3 Advantage and problem of DEA

**Advantage**

The DEA has several strengths over other MCDM techniques. The main advantages are that the method only requires a set of actual observed input and output data belonging to DMUs or alternatives being evaluated. It can readily incorporate multiple inputs and outputs to calculate efficiency without any requirement of knowledge or a priori assumptions about the production function or a functional form relating inputs to outputs from decision maker before the analysis. The method also does not require decision maker to define weights to be attached to each input and output because it focuses on individual observations and optimizes the performance measure of each DMU (Coelli et al., 2005).

Another advantage is that the DEA can handle a large numbers of variables and constraints, therefore decision maker is able to choose several inputs and outputs without difficulties. It also relaxes conditions on the number of alternatives to be used in an evaluation which makes it easier for decision maker to deal with complex problems or other considerations that are likely to be confronted in many managerial and policy contexts.

Moreover, the method can accommodate multiple inputs and multiple outputs which are from different aspects and have different dimensions. These inputs and outputs can be non-discretionary or exogenous and can be in different units of measurement. The method can also handle multiple inputs and outputs simultaneously. Finally, the DEA has ability to identify the potential improvement for each inefficient DMU. Since the method compares the DMU enveloped by the frontier with a convex combination of the DMUs located on the frontier, this enables decision maker or analyst to be able to indicate the sources and the level of inefficiency for each of inputs and outputs of inefficient DMUs (Charnes et al., 1997).

**Drawback**

Although the DEA method offers attractive advantages, there are some drawbacks that have to be taken into consideration when applying the technique. Two problems that have long been recognized are poor discrimination in the assessments of the different DMUs and unrealistic weight variables assigned to criteria. These two problems are inter-related in that they often occur simultaneously (Li and Reeves, 1999).

The DEA method could not provide optimum solution especially for selection or ranking purpose due to the problem of lack of discrimination which many of the DMUs are classified as efficient or are rated near the maximum efficiency score. This situation can occur when the number of DMUs under evaluation is not large enough as compared to that of the total number of input and output criteria.

Since the DEA model places no constraints other than positive values to the weights $u$ and $v$, it has complete flexibility to assign any weight value to each item of input and output data for each DMU. These values of the input and output weights are determined directly from the data and are varied from one DMU to another in order to give the best combination of multiple weighted inputs and multiple weighted outputs for the purpose of maximizing the efficiency score of the assessed DMU. This ability of total weight flexibility has been considered to be one of major advantage in application of the DEA in that there is no requirement for a priori knowledge of the input and output weights (Cooper et al., 2011b). However, it often leads to unreasonable results due to several weighting issues as follows:

- Large differences in weights of the same criteria are assigned to different DMUs.

This means that the weights chosen by the DEA in assessing efficiency of one DMU may be completely different from the weights selected for another DMU (Thanassoulis et al., 2004). This issue might become unacceptable from the viewpoint of decision maker in the sense that most of the DMUs employ similar inputs to produce the same kind of outputs under the same overall objectives (Pedraja et al., 1997).

- Very low or very high weight values assigned to some of unfavorable inputs and outputs.

Since the efficiency measured in DEA is derived relative to the performance of other DMUs, a DMU that is superior to all other units in only a single or few output and/or input ratio will receive an efficiency score equal to one by placing very high weights on that particular output and/or input ratio. As a result, criteria of secondary importance may dominate a DMU's efficiency assessment (Thanassoulis, 2001).

- Some variables are ignored by assigning value of zero to the weights.

In extreme case, some input and output criteria which may be considered very important by decision maker as well as by analyst are completely ignored from assessment. This may be unacceptable given the fact that all input and output criteria are meticulously selected but some of them being completely neglected by DMUs. Moreover, the efficiency of a DMU may not really reflect its real performance with respect to the considered inputs and outputs taken as a whole (Dyson and Thanassoulis, 1988).

Lastly, weights assigned by the DEA may be inconsistent with prior knowledge or accepted viewpoint of decision makers on the relative values of the inputs and outputs (Allen et al., 1997). In addition, the DEA model which is unbounded weight restriction do not allow decision maker to incorporate any a priori information, viewpoint, or judgement that might be available regarding the importance of inputs and outputs into the analysis.

An inappropriate estimation of efficiency scores due to complete flexibility of the weight in original DEA model is found to be nonsensical or unacceptable from managerial point of view when using the model in some certain applications. This leads to the development of various approaches to control the variations in weights. A whole new series of models called weight restriction DEA models in which constraints imposing bounds on the input or output weights are added to the original model. This imposition of restrictions on the weights implies the formulation of value judgements about the relative importance of the different inputs or different outputs. The weight restrictions can also reduce the region of search for the weights thus possibly reducing the efficiency of the DMUs.

**Explanatory example**

The following explanatory example illustrates the use of DEA model and its problem due to weight flexibility. Table 2.1 shows six DMUs, i.e. *A*, *B*, *C*, *D*, *E*, and *F* with two inputs and one output, where the output value is equal to one for each DMU.

**Table 2.1** Data

| | DMU | *A* | *B* | *C* | *D* | *E* | *F* |
|---|---|---|---|---|---|---|---|
| Input | $x_1$ | 4 | 7 | 8 | 4 | 2 | 10 |
| | $x_2$ | 3 | 3 | 1 | 2 | 4 | 1 |
| Output | $y$ | 1 | 1 | 1 | 1 | 1 | 1 |

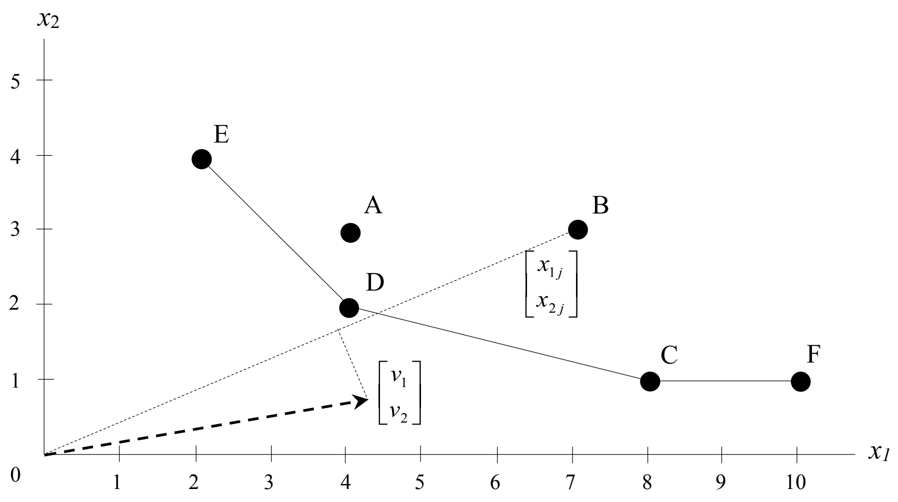


**Figure 2.5** Graphical

This two input-single output problem is easy to analyze graphically as shown in Figure 2.5. It can be seen that DMU *C*, *D*, *E*, and *F* are positioned on efficient frontier and are considered as efficient DMUs by receiving efficient score equal to 1 (one). In an evaluation of the DEA, the weight vector is allowed to move freely in order to find the best combination of multiple weighted inputs and multiple weighted outputs for the purpose of maximizing the efficiency score of the assessed DMU. If there are many more alternatives in an evaluation, it tends to have more efficient DMU along the efficient frontier.

The efficiency of DMU $A$ is evaluated by solving linear programing formulation below:

$$\max \ h_A = u \tag{2.6}$$

subject to

$$4v_1 + 3v_2 = 1$$
$$u \leq 4v_1 + 3v_2$$
$$u \leq 7v_1 + 3v_2$$
$$u \leq 8v_1 + v_2$$
$$u \leq 4v_1 + 2v_2$$
$$u \leq 2v_1 + 4v_2$$
$$u \leq 10v_1 + v_2$$

$v_1, \ v_2, \ v_3, \ u \ \geq \ 0$ ; where all variables are constrained to be nonnegative.

After solving the linear programming problem above, the optimal solution is $v_1^*$ = 0.1429, $v_2^*$ = 0.1429, $u^*$ = 0.8571, $h_A^*$ = 0.8571 and the efficiency of $A$ is 0.8571. The efficiency of DMU $B$, $C$, $D$, $E$, and $F$ can similarly evaluated from the data in Table 2.1. The optimal solution of each DMU is shown in Table 2.2. Each DMU is assigned a best set of weights with values that vary from one DMU to another DMU.

From Table 2.2, considering the difference between the optimal weights of DMU $B$ where $v_1^*$ = 0.0526 and $v_2^*$ = 0.2105. The ratio $v_2^*/v_1^*$ = 0.2105/0.0526 = 4 represents that it is advantageous for the DMU $B$ to weight input $x_2$ four times more than input $x_1$ in order to maximize the ratio scale measured by output to input.

**Table 2.2** Result

| DMU | $x_1$ | $x_2$ | $y$ | Efficiency | $v_1$ | $v_2$ | $u$ |
|---|---|---|---|---|---|---|---|
| A | 4 | 3 | 1 | 0.8571 | .1429 | .1429 | .8571 |
| B | 7 | 3 | 1 | 0.6316 | .0526 | .2105 | .6316 |
| C | 8 | 1 | 1 | 1 | .0833 | .3333 | 1 |
| D | 4 | 2 | 1 | 1 | .1667 | .1667 | 1 |
| E | 2 | 4 | 1 | 1 | .2143 | .1429 | 1 |
| F | 10 | 1 | 1 | 1 | 0 | 1 | 1 |

The result has poor discrimination as four out of six DMUs get efficiency score equal to one. The complete flexibility to assign any weight value to each item of input and output criteria for each DMU may lead to unrealistic or unreasonable results as can be seen in DMU *F*. The DEA defines DMU *F* as efficient by assigning weight $v_1^* = 0$ to input $x_1$ and give maximum weight of one, $v_2^* = 1$, to input $x_2$, while the DMU consumes abundant amount of input $x_1$, $x_1 = 10$, and only consumes one unit of input $x_2$. Hence, input $x_1$ which may be considered very important by decision maker is completely ignored from the assessment and the efficiency of the DMU may not reflect its real performance with respect to the considered inputs and outputs.

## 2.2 The Use of DEA Weight Restriction for MAMC (Many Alternative Many Criteria)

As stated in previous section, one of the recognized advantages of the DEA method is that a priori specification of the weights is not required, and each DMU can be evaluated in the best possible light which is to maximize its efficiency as high as possible. However, this full flexibility in identifying weights or values to be assigned to each input and output in the way that maximize efficiency of assessed DMU as high as possible can be seen as disadvantages in the identification of efficiency and can lead to undesirable consequences. For this reason, a number of approaches have been developed to control or limit the complete freedom of the original DEA. One such development is the use of weight restrictions and value judgements. The weight restrictions allows for the integration of managerial preferences in terms of relative importance levels of various inputs and outputs. The intention of this incorporating value judgements of decision maker is to include prior views, opinion, or information regarding the assessment of efficiency of DMUs into an analysis. Methods for incorporating weight restrictions which have been suggested by several researchers will be summarized in the latter section.

### 2.2.1 Incorporate judgement or a priori knowledge

In an analysis, management often has strong preferences about the relative importance of different criteria and what determines best practice. Also there occur many cases where additional information is available and decision maker is willing to make assumptions or incorporate them into the model. And when a number of DMUs under evaluation are very small, the DEA might fails to discriminate DMUs by giving them all as efficiency. These situations have proven beneficial to impose some control on the weight in an analysis (Cooper et al., 2011b).

Allen et al. (1997) state that value judgements are considered as logical constructs incorporated within an efficiency assessment study, reflecting preferences of decision maker in the process of assessing efficiency. Most methodological extensions of the DEA and evolution of value judgements in the assessment of efficiency have arisen as a result of application of the DEA method on real life problems. The intention of incorporating value judgements is to involve prior views or information regarding the assessment of efficiency of DMUs. This prior information can be incorporated in several different ways having different implications on the assessed relative efficiency of DMUs. A number of reasons motivating the use of value judgements in the DEA discussed in Thanassoulis and Allen (2004) are listed as follows:

- To incorporate prior views on the value of individual inputs and outputs

In assessing the performance of rates departments of Dyson and Thanassoulis (1988), some local authorities are evaluated as efficient because the excessively high weights are assigned to some outputs such as the numbers of rebates of taxes and court summonses of tax payers while other outputs such as tax accounts administered are effectively ignored. Thus top management perspectives on the relative importance of the inputs and outputs are incorporated. Chilingerian and Sherman (1997) evaluate practice patterns of primary care physicians in a large Health Maintenance Organization. The weight restrictions are used to enclose the factor weights in a cone which is constructed by incorporating a clinical manager's directives.

- To relate the values of certain inputs and/or outputs

Thanassoulis et al. (1995) assesse the efficiency of perinatal care units in the UK. A weight restriction model is developed to include the ratio of the number of survivals which is output criterion to babies at risk which is input criterion in the assessment. The adopted approach also allows the importance of the survival rate ratio to be varied. Beasley (1990) establishes several relationships between the weights of inputs and outputs in the assessment of university efficiency. For example, the belief that the value of a postgraduate is higher for the university than the value of an undergraduate student is incorporated into the analysis, so the model can prevent universities from weighting undergraduates more than postgraduates.

- To incorporate prior views on efficient and inefficient DMUs

Charnes et al. (1990) realize that management often have prior perceptions about the efficiency or performers of DMUs under assessment. Therefore, the cone-ratio weight restriction model is developed to include managerial view in evaluating the performance of banks in the USA. The efficiency of banks is assessed on the basis of the input and output values of three preselected banks which are determined as very good performers.

- To enable discrimination between efficient units

Efficiency results of the DEA do not always reflect the desired degree of discrimination between DMUs. Thompson et al. (1986) use the DEA to determine the best location for a nuclear physics facilities in Texas. Five out of six alternative facilities are found relatively efficient by the free weights of the DEA model. The assurance intervals is developed as additional constraints by defining ranges of acceptable weights for each site in order to select one efficient site.

- To ensure incorporation of all inputs and outputs in the assessment

Having weight restrictions by imposing upper and lower bounds on the weights is the way to ensure that all criteria are considered in the analysis.

- To ensure that widely differing weights are not assigned to the same criterion

The complete weight flexibility allows the DEA model to assign extremely large or extremely small weights to certain input or output criterion while evaluating different DMUs. This may not be acceptable when management are interested to know the performance of all DMUs when using similar sets of weights. Imposing some constraints on weights can be applied to ensure that all the DMUs are evaluated with similar sets of weights. Roll et al. (1991) propose Common Set of Weights (CSW) procedure which assumes that all DMUs face the same circumstances.

### 2.2.2 Approaches for incorporating value judgements in DEA

Since the original DEA model allows decision maker to introduce the restrictions on the input and output weights which can then affect solutions that will be obtained from the corresponding models, many methods have been proposed for incorporating value judgements to the DEA model (Cooper et al, 2011b). For example, the problem of unrealistic weights has been tackled mainly by the techniques of weight restrictions including imposing upper and lower bounds on individual multipliers introduced by Dyson and Thanassoulis (1988). Thompson et al. (1990) suggest an AR (assurance region) constraints by imposing bounds directly on ratios of multipliers. Wong and Beasley (1990) propose to set bounds on multiplier inequalities which are proportions of individual inputs (or outputs) to total input (or output). Cone-ratio model developed by Charnes et al. (1990) attempts to impose a set of linear restrictions that define a convex cone of efficient DMU. Golany (1988) tries to incorporate ordinal relationships of among the weights without adding additional constraints. And Pedraja-C et al. (1997) introduce contingent weight restrictions by using AR to restrict virtual inputs/outputs rather than weights. These large diversity of methods that can be applied to incorporate value judgments in the original DEA and to reduce the flexibility of DMUs in choosing their weight values can be classified into the following three broad categories:

1. Direct restrictions on the weights
2. Adjusting the observed input-output levels to capture value judgements
3. Restricting the virtual inputs and outputs

*Direct restrictions on the weights* is applied by adding additional constraints that involve weights to the original basic DEA model. Three approaches in which direct restrictions have been applied in the literature are as follows:

- Absolute weights restrictions: this type of model uses constraints which impose upper and lower limits on the individual input and output weights. These constraints are mainly applied to prevent the inputs or outputs from being over or under emphasized, or being ignored in the analysis. The value of the restriction is dependent as it may represent either the maximum or minimum value of the associated criterion.
- Assurance regions of type I (ARI): this type of restriction is introduced to incorporate the relative ordering of the inputs and outputs into the analysis. The value of upper and lower bounds are imposed on the ratios of input weights and the ratios of output weights.
- Assurance regions of type II (ARII): the input and output weights are imposed in term of relationship for this type of restriction. So the bounds are imposed on the ratios of output weights to input weights.

*Adjusting the observed input-output levels to capture value judgements* modifies the existing input-output data to stimulate weights restrictions.

- Cone Ratio: the approach involves generating an artificial data set. The optimal virtual multipliers of efficient DMUs are used as a restricted cone span which satisfy some certain conditions specified by the decision maker.
- Ordinal Relations or Golany method: the method incorporates ordinal relationships among the input weights or output weights without allowing the weights to take a zero value.

*Restricting the virtual inputs and outputs* imposes limitations on weights by restricting the weighted inputs and outputs.

- Contingent weight restrictions: weight restrictions are imposed by taking into account the levels of inputs and outputs chosen by the DMU. This type of constraints require the proportion of total costs or benefits ascribed to an input or output of a DMU not exceed another input or output by more than a certain multiple.
- Restriction on relative importance of factors to a DMU: rather than restricting the actual DEA weights, the method involves putting restrictions on the "importance" attached to a certain output or input measure

by a DMU, i.e. the importance attached to a particular output by a DMU is the proportion of the total output devoted to that output.

These all mentioned approaches require a priori information that involves human value judgement. However, the first two approaches, i.e. direct restrictions on the weights and adjusting the observed input-output levels to capture value judgements are applies more in the applications of DEA than the last one. Some of the models can be applied to solve more than one of the problems. For example, using weight restrictions to incorporate value judgements regarding the relative importance of different variables may help improve discrimination and/or reduce weights dispersion (Cooper et al., 2011a). Nowadays, the issues of weights restrictions and value judgements are still one of an important parts of the research on DEA and it applications without showing any signs of saturation.

### 2.2.3 Assurance Region of Type I (ARI) and its problem

Amongst the numerous types of weight restriction DEA models, Assurance Regions of Type I or ARI is found a popular technique of weight restrictions which has been vastly discussed and applied in real-life applications for the performance measurement (Dyson et al., 2001). The technique of ARI itself is easy to be integrated with the DEA method. The ARI was developed by Thompson et al. (1986) to help in choosing a best site for the location of a high-energy physics laboratory when other approaches proved to be deficient in evaluating output criteria like contributions to fundamental knowledge. The approach of ARI is to restrict the regions of weights by imposing constraints on the relative magnitude of the weights to some special area. Additional inequality constraints of the following form are introduced to incorporate into the analysis.

$$a_{ii'} \leq \frac{v_i}{v_{i'}} \leq b_{ii'} \qquad \text{for input criteria} \tag{2.7}$$

and

$$c_{rr'} \leq \frac{u_r}{u_{r'}} \leq d_{rr'} \qquad \text{for output criteria} \tag{2.8}$$

for $i$, $r$ = 1,…, $m$-1, $i'$, $r'$ = $i$+1,…, $m$ where $a_{ii'}$ and $b_{ii'}$ are lower and upper bounds on the ratios between each pair of input weights, and $c_{rr'}$ and $d_{rr'}$ are lower and upper bounds on the ratios between each pair of output weights. They are user-specified constants to reflect value judgements or opinion that the decision maker wishes to incorporate into the assessment. The name assurance region comes from the constraint which limits the region of weights to some specific area. Generally, the DEA efficiency score in the corresponding envelopment model is worsened by the additions of these ARI constraints and a DMU previously characterized as efficient may subsequently be found as inefficient after such constraints have been imposed (Cooper et al., 2007). Charnes et al. (1990) and Thompson et al. (1990) note that when imposing the ARI, there will always exist at least one efficient DMU. Moreover, whether the input or output-oriented model is used, a DEA model incorporating ARI produces the same relative efficiency scores (Liu, 2008). It also notes that the ratio of the weights is likely to coincide with the upper or lower bound in an optimal solution, consequently it requires some concern when choosing these bounds.

The assurance region method is formulated for the DEA model by adding additional constraints for pairs of criteria if needed. Rearranging the terms (2.7) and (2.8), the original DEA model is augmented by the following linear inequality which are most commonly used form of ARI constraints.

$$\begin{aligned} &v_i - b_{ii'} v_{i'} \leq 0 \\ &- v_i + a_{ii'} v_{i'} \leq 0 \qquad \text{for input criteria} \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} &u_r - d_{rr'} u_{r'} \leq 0 \\ &- u_r + c_{rr'} u_{r'} \leq 0 \qquad \text{for output criteria} \end{aligned} \tag{2.10}$$

Figure 2.6 shows an example of a graphical of the ARI constraint for weight restrictions. The technique of the ARI is applied to restrict the regions of ratios of input weights and ratios of output weights to some specific values in order to reduce the region of search for the weights. The weight vector can only move within specific area which resulting in possibly reducing the efficiency of the DMUs. From the figure, the number of efficient DMUs is reduced from four (C, D, E, F) to one (D).

The weight restrictions can also avoid the assessed DMUs from ignoring some criteria or relying too much on any criteria in an evaluation.

The generality of the ARI constraints provides flexibility for utilization. Different types of measures can also be accommodated and can mixtures of such concepts. Moreover, in case decision maker cannot state the values for their preferences in a priori manner, the ARI technique allow the decision maker to first try to examine provisional solutions and then tighten or loosen the bounds until one or more solutions that appears to be reasonably satisfactory are attained. Further, the approach greatly relaxes the conditions and also widens the scope for the use of a priori conditions (Cooper et al., 2011b).

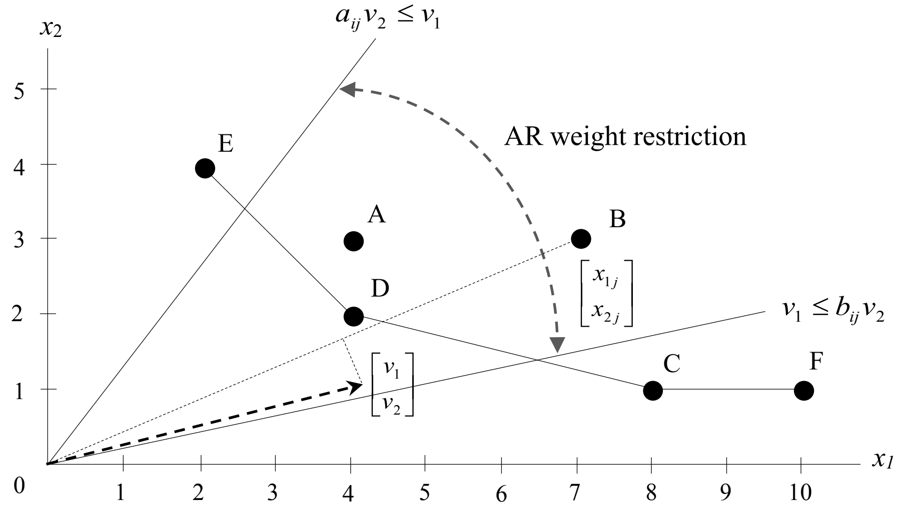



**Figure 2.6** Graphical of ARI Weight Restriction

The technique of ARI is assumed that decision maker is capable of expressing his/her opinion of the relative importance of each pair of criteria in terms of ratio of criteria weights. Determining bounds requires discretion of the expert or decision maker in conjunction with available information such as economic data about cost and price ranges of the input and output criteria (Thompson et al., 1992). When the information is insufficient, unavailable, or cannot be used for determining values of relative importance of the criteria, setting bounds is solely based upon managerial preferences or a priori information such as previous experience, expert opinion, and common sense (Cooper et al., 2011a). These weight bound values are decided from a basis of perceived regarding the relative importance levels or worth of various inputs and output criteria. For example, if input $x_1$ is at least twice as important as input $x_2$, then the linear constraint $v_1 \geq 2v_2$ can be incorporated into the DEA model. Moreover, the bound values for the ARI are dependent on the scaling of the inputs and outputs, therefore they are sensitive to the units of measure of the related criteria (Allen et al., 1997). Considering these characteristics of the bounds, the way to set the bounds to be associated with the ARI constraints becomes one of the key issues in this technique. The method is somehow difficult for decision maker to put the judgements into quantitative bounds because the lower and upper bounds in ARI have to represent the relative values between each pair of criteria weights while evaluation process usually comprises complicated inputs and outputs where many criteria cannot be measured in ratios. Moreover, usually there is uncertainty due to subjective opinion of decision maker which leads to inconsistency when compare each pair of criteria. This eventually results in infeasibility of solution which is a potential problem of the method.

The use of ARI technique can be found in many applications of performance measurement and decision making and most of researches have applied it to solve their specific problems which could somehow guide decision maker on the practice to determine values of the bounds. The followings are examples of researches that applied the ARI technique to incorporate with the DEA method.

Beasley (1990) compares performance of university departments in UK by using direct judgement on relative importance of inputs and outputs to create AR constraints. Chilingerian and Sherman (1997) define AR bounds from the optimal weights obtained by running the unbounded DEA model in evaluating practice patterns of primary care physicians. The bound values are developed based on the ratios of marginal rates of input factors. Schaffnit et al. (1997) measure productivity of branch personnel of large Canadian banks. The upper and lower bounds for all output activities are estimated by management based on information of ranges of the standard transaction and maintenance times. Zhu (1996) evaluates industrial performance of textile factories in China by applying the basis of pairwise comparison judgement in Analytic Hierarchy Process to develop matrices of input and output criteria. The results from matrices are used to establish bounds on the weights. Takamura and Tone (2003) also apply AHP-like method to weight the importance of the criteria in evaluation of site for relocating Japanese government organizations out of Tokyo. The AR lower and upper bounds are derived from the minimum and maximum ratios of weights on criteria estimated by Council members. Thompson et al. (1992) deal with major oil and gas companies in USA by considering total production of crude oil, natural gas, total production cost, proven reserves of crude oil and natural gas, and number of wells drilled. The AR principle is used to place bounds on the modeled prices. Ray et al. (1998) assess iron and steel Chinese firms using total salary and worker benefits divided by the total number of workers by firm to set the range for the price of labour force, and use dual price system created by the Chinese economic reforms to determine bounds for all the variables. Olesen and Petersen (2002) estimate the cost efficiency of 70 Danish hospitals. The analysis relates to a cost function based on 483 outputs in combination with a set of probabilistic assurance regions defined by the cost distributions for each output.

Among a number of application of ARI technique, few of them emphasize on how the values of the bounds are determined and there is little attention to illustrate the process to derive the bounds. Also issue related to possibility of infeasibility is even less mentioned. Setting bounds on the weights is necessary to account for potential inconsistency. The linear program can become infeasibility due to the new restrictions that are imposed on the weights. Thus, these limits between weights that can vary would have to be relaxed until a feasible result is obtained. Roll et al. (1991) mention that the setting bounds and its effects on feasibility is an interesting field to develop and it has not been studied. Considering the case when an analysis consists of a large number of criteria, the decision will become more complicated and difficult for decision maker to evaluate the relative importance of each pair of criteria.

This is a challenge to determine the values of bounds on weights with reflecting the judgement or opinion of decision maker and the bounds also have to overcome a potential problem of infeasibility. So far there has been no literature for procedure to obtain feasible solution from setting possible bounds in ARI, therefore it is necessary to develop a practical method to set bounds that can hold transitivity in ARI constraints so that it can guarantee the feasibility for the resulting linear programming model. This leads to the development of proposed method which will be described in next chapter.

### 2.3 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) is a powerful tool for systematic and easily understood assessment that has been used in almost all the applications related with decision making. It is designed to cope with both the rational and the intuitive to select the best from a number of alternatives evaluated with respect to several criteria. The method is one of the most widely used and has been successfully applied to many practical decision making problems because it enables decision maker to resolve complex problems by simplifying and expediting the natural decision making processes.

#### 2.3.1 Concept of AHP

The AHP method provides relative ease but theoretically strong multi-criteria methodology for evaluating alternatives in an analysis. The technique enables decision maker to use a simple hierarchy structure to analyze a complicated problem and to evaluate both quantitative and qualitative data in a systematic way under conflicting multi-criteria (Lee et al., 2001). It is basically designed to examine complex issues by breaking down the complicated, unstructured problem into four stages: constructing a hierarchy, pairwise comparisons, priority vector generation, and synthesis (Saaty, 1980). During the evaluation process, the decision maker carries out simple pairwise comparison judgements which are then used to develop overall priorities for ranking the alternatives. Saaty (1990) explains a basic procedure to carry out the AHP as following steps:

*1. Structuring a decision problem and selection of criteria*

The first step is to decompose a decision problem into its constituent parts. The simplest form used to structure a decision problem is a hierarchy consisting of three main levels. An objective or goal of a decision which is an only one element reflecting the overall objective of the system is at the topmost level, followed by a criteria, and subcriteria if applied, at the intermediate levels by which the alternatives will be evaluated. The lowest level contains all alternatives. An example of a three level hierarchy is shown in Figure 2.7.

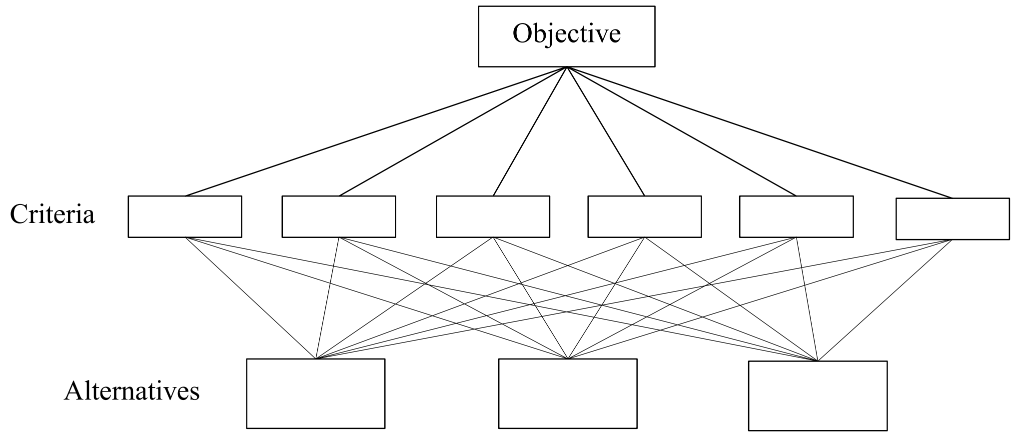



**Figure 2.7** A Three Level Hierarchy

Hierarchical decomposition of all complex components in a system provides an overall view of the complex relationships and helps the decision maker to assess whether the elements in each level are of the same magnitude so that they can be compared accurately. A purpose of the hierarchical structure is to make it possible for the decision maker to give judgement on the importance of the elements in a given level with respect to some or all of the elements in the adjacent level above. The AHP is simple to apply when the structuring is completed (Saaty and Vargas, 2012).

*2. Pairwise comparison of the criteria (weighing)*

The next step is to allocate priority weights to the criteria within each level of the hierarchy. The weights have to be determined successively by pairwise comparison of the relevant criteria. Usually matrix is applied for the pairwise comparison. For each pair of criteria, the decision maker is required to give judgement on degree of importance between the two criteria. He/she has to response a question such as "How important is criterion $x_1$ relative to criterion $x_2$?" Each of the judgements is assigned a number on scale which can be exemplified as in Table 2.3. The weighing are then normalized into the sum of one and averaged in order to obtain an average weight for each criterion.

**Table 2.3** Rating Scale

| Intensity of Importance | Definition |
|---|---|
| 1 | Equal importance |
| 3 | Somewhat more importance |
| 5 | Much more importance |
| 7 | Very much more importance |
| 9 | Absolutely more importance |

In the AHP, multiple pairwise comparisons are normally based on a standardized comparison scale of nine levels. Let C = {$C_j$ | $j$ = 1,…, $n$} be the set of criteria. The result of an evaluation matrix in which every element $a_{ij}$, where $i, j$ = 1,…, $n$, is the quotient of weights of the criteria as

$$A = (a_{ij})_{nxn} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \tag{2.11}$$

where $a_{11} = 1$, $a_{ji} = 1/a_{ij}$ for $j \neq i$, $a_{ij} \neq 0$.

*3. Pairwise comparison of alternatives on each criterion (scoring)*

The decision maker again has to give judgement on each pair of alternatives corresponding to each criterion. The scale between one (equally good) to nine (absolutely better) similar to Table 2.1 can also be applied for rating the alternatives. For this step, the decision maker is required to answer a question such as "How well alternative A meets criteria $x_1$ when compared to alternative B?" The ratings are also normalized and averaged afterwards.

In a process of pairwise comparison, the decision maker has to deal with the structure of an $m \times n$ matrix, where $m$ is the number of alternatives and $n$ is the number of criteria. The matrix is constructed by using the relative importance of the alternatives in terms of each criterion.

*4. Obtaining an overall relative score for each option*

The final step is to combine the alternative scores with the criterion weights in order to produce an overall score for each alternative which is done by simple weighted summation. The extent to which the alternatives satisfy the criteria is weighed according to the relative importance of the criteria.

Note that the less important elements can be taken out from the consideration after judgements have been made on the impact of all the elements in a hierarchy and the priorities of the alternatives have been computed. This is because they have relatively small impact on the overall objective and it has to be done with care.

The AHP methodology has been applied to support decision process in various areas including logistics and supply chain management. For example, Gaudenzi and Borghesi (2006) assess risks in the supply chain in order to improve customer value. Yurimoto and Masui (1995) select plant location in European countries for Japanese company. Levary (2008) ranks potential foreign suppliers located in China for a Midwest manufacturer concerning reliability and risk. Wei et al. (2005) select a suitable ERP system for an electronics company in Taiwan. Banai (2006) evaluates light rail transit corridor and route alternatives for public transportation decision in Memphis. Kengpol et al. (2012) design a decision support system for selecting multi transportation route within GMS countries by incorporating the AHP to translate users' viewpoint of decision criteria into weight. Banomyong and Beresford (2001) explore various alternative routes for garment exporters in Lao PDR.

### 2.3.2 Advantage and problem of AHP

**Advantage**

Unlike the DEA method that each alternative is evaluated individually so more than one alternatives can get efficient score equal to one, the AHP can always give optimal solution to the selection problem because the technique tries to integrate different measures into a single overall score of one so that all the alternatives under evaluation can be put into ranking. The alternative which has highest score is then considered the best and is normally selected.

Moreover, the AHP is quite easy for most decision makers to understand because the methodology of the AHP is similar to that used in common sense decision making. The technique decompose a decision problem into its constituent parts and uses hierarchical structuring of criteria, consequently it can simplify the complex problem and the importance of each criterion becomes clear for evaluation (Macharis et al., 2004). It also helps capturing both subjective and objective evaluation measures, and it can mix quantitative and qualitative criteria into a decision. In addition, the method provides a useful mechanism for checking the consistency of the evaluation criteria and alternatives (Ramanathan, 2001).

The AHP reduces bias in decision making and is able to support group decision making through consensus by calculating the geometric mean of the individual pairwise comparisons (Zahir 1999). Furthermore, the method is flexible to be integrated with other different techniques such as Linear Programming, Quality Function Deployment, Fuzzy Logic, etc. Therefore user is able to extract benefits from all the combination of the methods in order to achieve the desired solution in a better way (Vaidya and Kumar, 2006).

**Drawback**

Despite the benefit of the AHP, some certain issues are expressed for consideration. Firstly, the method is somewhat difficult to apply because it requires decision maker to make comparisons between both alternatives and criteria. The decision problem is decomposed into a number of subsystems and the decision maker has to complete a number of pairwise comparisons which can make an analysis quite inconvenient. With MAMC, a number of judgement on pairwise comparisons to be made may become very large, and performance of all alternatives has to be compared under each criterion. Thus, the paired comparisons take considerable time and the decision making turns to be a lengthy and inconvenient task.

Another concern is that the AHP heavily bases on the experience, knowledge and judgment of decision maker. It requires management to be involved in every process. Sometimes, he/she might find difficult to distinguish among the elements especially when there are many selection alternatives and decision criteria in an analysis. The assumption of comparability may be invalid due to lack of necessary information or unwillingness to make comparison (De Boer et al., 1998). In addition, the human judgment is always subjective so it will usually be bias towards the intuitive of the decision maker thought the processes (Rebstock and Kaula, 1996).

# 3 PROPOSED TECHNIQUE FOR MAMC

### 3.1 Domain of the Thesis

This thesis classifies the decision making problem into four categories regarding a number of alternatives and a number of criteria involved in an analysis as introduced in Chapter 1. The main focus of the study is on solving MAMC which is the most complex problem and is considered very difficult problem to handle due to a set of large number of alternatives which are described in terms of various conflicting evaluative criteria.

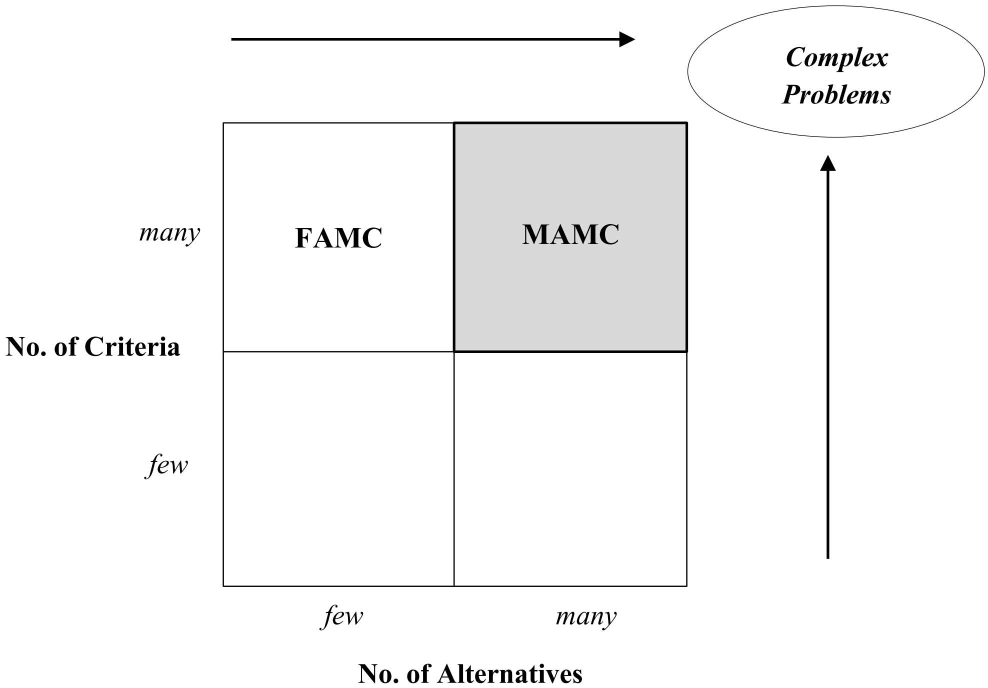



**Figure 3.1** Category of Decision Making Problem

To tackle the problem, the DEA which is one of the powerful methods for dealing with multi-criteria decision making is chosen as a primary tool because it can provide a means of calculating apparent efficiency levels of a group of alternatives and can readily incorporate multiple inputs and outputs into an assessment. However, the main shortcomings include the following:

- The method has poor discrimination in an assessment of alternatives thus it could not provide optimum solution for a selection problem.

- Due to property of weight flexibility of the conventional DEA, the method assigns unrealistic weight variables to criteria which are often contradiction and unreasonable from viewpoint of the decision maker. Usually the decision maker has valuable opinion or a priori information and willing to incorporate them into an analysis.

The ARI which is a technique for weight restrictions by incorporating judgement or a priori knowledge of decision maker is integrated with the original DEA for purpose of fixing the DEA problems. The region of search for the DEA weights is reduced by placing constraints on these weights which might lead to non-zero weights, reduce the variation in weights, and improve the discrimination among the efficient DMUs. Nevertheless, an obvious difficulty of the AR technique is the way to determine the values of the weight bounds. Many applications of the ARI constraints can be found in the literature which might help the user or decision maker in deriving appropriate bounds but these bounds are mostly presented in illustrative manner. The research so far has paid relative little attention on the way to derive values of the bounds on the ARI constraints. The difficulties and problems in determining bounds will be described in the following section.

### 3.1.1 Issue of setting bounds

The ARI approach involves imposing bounds on the ratio between various input weights and the ratio between various output weights by using available information and/or expert opinion. It also widens the scope for use of prior conditions. The technique helps solving unreasonable results due to the flexibility in assigning weight of the classical DEA model. However, the DEA model that incorporates weight restrictions in turn creates problems of infeasibility for the corresponding linear programming. The difference of units of measurement and orders of magnitude may complicates the setting of meaningful bounds without causing the calculation to be infeasible (Sarrico and Dyson, 2004). Moreover, when the number of criteria to be processed in the analysis is large, the number of possible comparisons between each pair of input weights and output weights will increase likewise, and it can be too large. Suppose that there are $n$ criteria to be analyzed, then a complete set of pairwise comparisons is of size $n(n - 1)/2$. This makes an estimation of relative importance of the criteria more complicated and difficult. It easily happens that the decision maker will lose consistency in making sensible decisions due to a number of various pairs of weights. The relative importance value of one pair of weight may be given conflict to other pairs which results in infeasibility of solution when calculating by the DEA with additional ARI constraints. Therefore such pairwise comparisons should be consistent and thus need to satisfy transitivity in order to have feasible solution.

For the ARI constraints, the values of lower and upper bounds have to represent the relationship between each pair of criteria weights where the ratio scale contains most information and the magnitude of measured criteria. In addition, these values of the bounds have to reflect the information obtained from opinion of expert or decision maker. Nonetheless, due to the nature of an evaluation process that it usually comprises complicated inputs and outputs where many criteria cannot be measured in ratios, it is a formidable task of the decision maker to determine the values to be assigned to the ARI weight bounds. Allen et al. (1997) also point that the key difficulty in using this weight restriction method is the estimation of the appropriate values for the constants in the restrictions that compatible with the value judgements to be reflected in the efficiency assessments.

### 3.1.2 Transitivity property

When handling data set with several type of index numbers, comparison of the observation requires consistency which is "transitivity" (Coelli, et al., 2005). The transitivity is an operational constraint preserving internal consistency and is an extremely important property to be satisfied when the data is computed for pairs in the sample. It is a property of relationships in which objects of a similar nature may stand to each other, and is also a key property of both partial order relations and equivalence relations. In term of mathematical technique, relation of the transitive can be defined as

$$\forall a, b, c \in X : (aRb \wedge bRc) \implies aRc \tag{3.1}$$

where $R$ is particular relation, and $a$, $b$, $c$ are variables. This means a binary relation $R$ over a set $X$ is transitive iff for all element $a$, $b$, $c$ in a set $X$, $a$ is related to $b$, and $b$ is in turn related to $c$, then $a$ is also related to $c$. The "$a \geq b$ and $b \geq c$, then also $a \geq c$" is an example of transitive relation which means if $a$ is greater than or equal to $b$ and $b$ is greater than or equal to $c$, then $a$ is greater than or equal to $c$. This can be illustrated in Figure 3.2 showing an example of transitivity and non-transitivity of the three elements. If the comparison of three elements is transitive, no circular path exists. The circular path in the red arrows shown in the right side of the figure exists when the relation is not transitive.

Considering when there are much more elements or criteria in an evaluation, it is likely that non-transitivity will easily occur. Figure 3.3 shows an example of transitivity and non-transitivity when there are eighteen criteria. For the ARI constraints, the value of bounds are in the form of ratio of weights which is comparisons across a number of criteria thus these comparisons need to be internally consistent, i.e. to satisfy the property of transitivity. According to all these concerned issues, the key difficulty in ARI technique is to consistently determine the values of the bounds in ARI inequality equation that can hold transitivity property and reflect the information obtained from expert opinion.


**Figure 3.2** Transitivity and Non-Transitivity of Three Elements

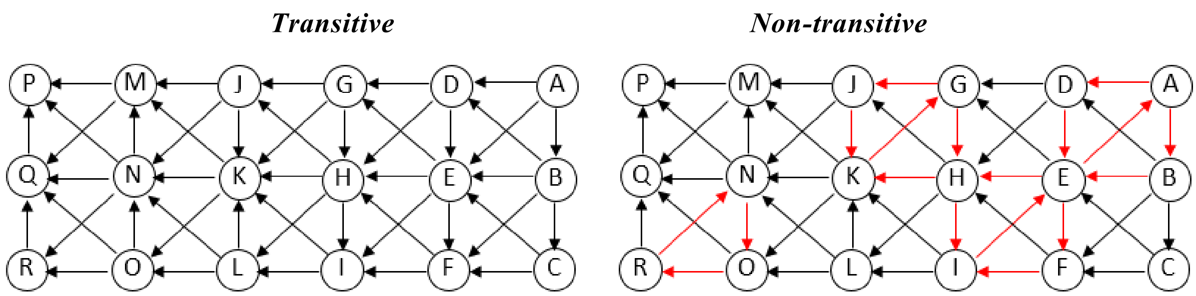



**Figure 3.3** Transitivity and Non-Transitivity of Eighteen Elements

**Example of non-transitivity**

The following example explains the problem of non-transitive that occurs due to the inconsistency in determining bounds of the ARI to incorporate to the conventional DEA model, which eventually leads to infeasible solutions. Suppose that the evaluation contains six alternatives or DMUs, i.e. A, B, C, D, E, and F with four decision criteria, which three are inputs and one is output. The output value is equal to one for each DMU. The efficiency of DMU A is evaluated by solving the original linear programming problem below:

$$\max \ h_A = u \tag{3.2}$$

subject to

$$v_1 x_{1A} + v_2 x_{2A} + v_3 x_{3A} = 1$$
$$v_1 x_{1A} + v_2 x_{2A} + v_3 x_{3A} \geq u$$
$$v_1 x_{1B} + v_2 x_{2B} + v_3 x_{3B} \geq u$$
$$v_1 x_{1C} + v_2 x_{2C} + v_3 x_{3C} \geq u$$
$$v_1 x_{1D} + v_2 x_{2D} + v_3 x_{3D} \geq u$$
$$v_1 x_{1E} + v_2 x_{2E} + v_3 x_{3E} \geq u$$
$$v_1 x_{1F} + v_2 x_{2F} + v_3 x_{3F} \geq u$$
$$v_1, \ v_2, \ v_3, \ u \ \geq 0$$

Suppose that the decision maker gives value judgement in the form of ARI constraint as

$$1 \leq v_2 / v_1 \leq 2 \tag{3.3a}$$
$$2 \leq v_3 / v_2 \leq 3 \tag{3.3b}$$
$$1 \leq v_1 / v_3 \leq 3 \tag{3.3c}$$

These three constraints can be written as inequality equations as

$$v_1 \leq v_2 \leq 2v_1 \quad \Longrightarrow \quad 2v_1 \leq 2v_2 \leq 4v_1 \tag{3.4a}$$
$$2v_2 \leq v_3 \leq 3v_2 \tag{3.4b}$$
$$v_3 \leq v_1 \leq 3v_3 \tag{3.4c}$$

From the left hand side of (3.4a) to (3.4c), the above equations can be rearranged to one inequality equation in a form of linear constraint as

$$2v_1 \leq 2v_2 \leq v_3 \leq v_1 \tag{3.5}$$

It can be seen that inequality equation (3.5) is not transitive because the variable $v_1$ is in circular. It is also not true that $v_1$ can be greater than or equal to twice amount of itself, i.e. $2v_1 \leq v_1$, unless the value of $v_1$ has to be equal to zero (0). When $v_1 = 0$, then values of $v_2$ and $v_3$ have to be equal to zero, i.e. $v_2 = 0$, $v_3 = 0$. In consequent, this linear problem is impossible to have feasibility of solution.

Infeasibility is a potential problem for the approach of imposing weight restrictions. In fact, the infeasibility frequently occurs and is not easily anticipated by the decision maker. Estellita Lins et al. (2007) state that most researches only mention the possibility of infeasibility but so far none of them have developed a strategy for dealing with infeasibility. The objective of this chapter is to illustrate a methodology to determine the values of bounds on the ARI weight restriction constraints which try to resolve an infeasibility problem and make ease in setting bounds for decision maker.

Also in order to demonstrate the problem in coping with MAMC, an example of decision making on facility location problem containing (many) nineteen alternatives and (many) thirteen criteria is calculated by applying the methods of the original DEA and the DEA with ARI weight restrictions. An illustrative example of how a DMU can take advantage of total weights flexibility to appear efficient in the DEA can be seen in the next chapter. Table 3.1 below shows the results of MAMC. The DEA method has a problem of lack of discrimination in the result since sixteen out of nineteen alternatives are determined as efficient, while the technique of the DEA with ARI cannot give feasible solution during calculation of efficiency due to the difficulties and problems in setting weight bound constraints. These two methods fail to provide solution for MAMC, therefore it is necessary to develop tool for supporting the decision making.

**Table 3.1** Result from Solving MAMC

| | **MAMC**<br>(Many Alternatives Many Criteria) |
|---|---|
| No. of Alternative | 19 (many) |
| No. of Criteria | 13 (many) |

| **Method** | **No. of selected alternative** |
|---|---|
| DEA | 16 (19) |
| DEA/ARI | N/A |

The following section proposes a methodology for determining the values of the ARI weight bound constraints to be incorporated in the DEA model. The technique is based on pairwise comparison of Analytic Hierarchy Process (AHP) method in which criteria are compared in pairs to judge which of each criterion is preferred from an opinion of expert. Concerning that MAMC contains a large number of criteria thus instead of making comparisons directly on each pair of input or output criteria, grade system is developed in order to use in specifying score of importance of each criterion. The grades will subsequently be paired comparison. This is to avoid complication in analyzing relative importance in case of having a large number of criteria in MAMC.

### 3.2 Proposed Method in Determining Bounds

Figure 3.4 shows a flowchart summarized an execution of the DEA with ARI weight restrictions along with an application of the proposed method. A general process of applying the ARI approach integrated with the original DEA method is shown on the left side of the flowchart while the right side shows a procedure of the proposed technique for determining values of the weight bounds to be incorporated into the ARI weight restriction constraints. The procedure composes five steps where Step 2, Step 3, and Step 4 require an involvement of the decision maker to give opinion or make judgement during an analysis.

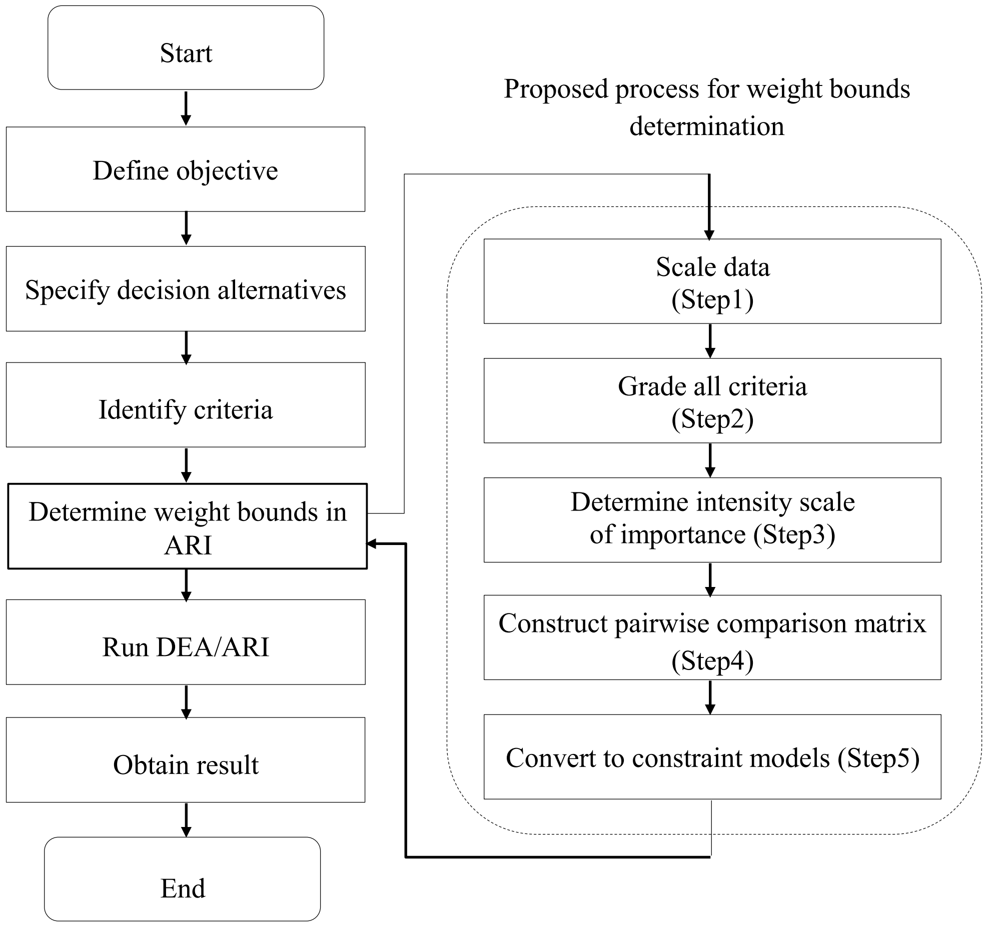



**Figure 3.4** Execution Flowchart of Proposed Method

After a problem or objective of an analysis is correctly defined and a set of alternatives and a set of decision criteria that the alternatives need to be evaluated with are developed, the proposed method is executed. The detail is explained as follows:

*Step 1: Scale data*

The DEA method can accommodate multiple inputs and multiple outputs which can be in different units of measurement. This will affect setting ARI bounds because the bound values are in a form of ratios of inputs and ratio of outputs. The first step, therefore, is to adjust the actual observed input and output data measured on different scale to a notionally common scale in order to avoid the differences in units of measurement of various criteria. Each original input data $x_{ij}$ and output data $y_{rj}$ are to be transformed into $\bar{x}_{ij}$ and $\bar{y}_{rj}$ in a scale of 1.0 respectively. This is done by dividing the original data of each criterion by the maximum value of that criterion as

$$\bar{x}_{ij} = \frac{x_{ij}}{\max x_{ij}} \qquad j \in \{1,\ldots,n\}$$

$$\bar{y}_{rj} = \frac{y_{rj}}{\max y_{rj}} \qquad j \in \{1,\ldots,n\} \tag{3.6}$$

where $x_{ij}$ and $y_{rj}$ are input $i$th and output $r$th of DMU$_j$, and $n$ is a number of DMUs. $x_{ij}$ and $y_{rj}$ are normalized data of the input $i$ consumed by DMU$_j$, $i = 1,\ldots, m$ and output $r$ produced from DMU$_j$, $r = 1,\ldots, s$.

*Step 2: Grade all criteria*

The decision making usually consists of large diversified type of criteria which the level of importance of each criterion to an objective is not the same. Since the decision maker usually has in some contexts value judgments on the criteria that can be formalized a priori, and therefore should be taken into account in the assessment. These value judgments can reflect known information about how the criteria used by the DMUs behave or influent to an objective. As stated in Chapter 1 that clearly defined objective or problem is the most important thing in making decision, so this step is to ask questions of the type like "how much important is criteria $x_1$ regarding to the objective of an analysis".

This second step requires the decision maker to set and give particular values or grades in order to indicate level of importance of the criteria regarding to the objective of an analysis. Let $g_i$, $g_r = k$ and $k \in \{1,\ldots, m\}$ where $k$ is a set of grades specified by the decision maker and $m$ is a number of grades. This set of grade $k$ could be identified in alphabet such as A, B, C, etc. After the set of grade is specified, the decision maker then assigns each specific grade to each input $i$ and output $r$ according to his/her opinion. Each grade associated with each input criterion, $g_i$, and output criterion, $g_r$, represents the rate of importance or priority that relates to the mission and strategy of the organization or objectives of the assessment in perspective of the decision maker.

Step 2 needs thoroughly consideration from the decision maker when giving a grade to each of criterion, especially when there are many criteria in an analysis. The grades given to criteria centers on an objective of the analysis and ultimately affects the solution.

*Step 3: Determine intensity scale of importance*

The proposed method applies pairwise comparison technique which are quantified by using a scale, so this step requires the decision maker to determine a numerical scale of intensity of importance. The scale is relative importance values between two grades and is used when comparing the grade of Step 2 in a pairwise comparison matrix of Step 4.

The scale can be similar to scale of rating introduced by Saaty (1980) as part of the AHP in which it matches a discrete set of linguistic choices available to the decision maker and a discrete set of numbers which represent the importance, or weight, of the previous linguistic choices in one-to-one manner (Triantaphyllou and Mann, 1995). For example in the AHP, the available values of rating scale for pairwise comparisons are 1 to 9. A value of 1 indicates that the comparison of the two criteria are identical, whereas a value of 9 indicates that one of the pair is "absolutely more important", and following Saaty a score of 3 indicates "weakly more important", 5 suggests "strongly more important" and 7 "very strongly more important".

*Step 4: Construct pairwise comparison matrix*

Pairwise comparison plays an important role in decision making problems and often provides an effective and efficient manner for eliciting qualitative information from the decision maker. After determining scale of intensity of importance, this step requires the decision maker to construct a pairwise comparison matrix $p$, which is $m\ x\ m$ matrix. The values scale $p_{kk'}$ is used in translating the decision maker's qualitative evaluations of the relative importance between two grades defined in Step 2 by comparing them one grade to another at a time. Each entry $p_{kk'}$ of the matrix represents the importance of the $k$th grade relative to the $k'$th grade. If $p_{kk'} > 1$, the $k$th grade is more important than the $k'$th grade. If $p_{kk'} < 1$, then the $k$th grade is less important than the $k'$th grade. The entry $p_{kk'}$ is 1 when two grades have the same importance.

*Step 5: Convert to restriction constraint models*

The final step is to use the pairwise comparison matrix of the grades in Step 4 to convert the differences of grades which are assigned to each criterion in Step 2 into additional ARI constraint formulas.

$$
\begin{aligned}
&p_{kk'} \leq \frac{v_i}{v_{i'}} \qquad \text{if } g_i \neq g_{i'} \qquad \text{for input bounds} \\
&p_{kk'} \leq \frac{u_r}{u_{r'}} \qquad \text{if } g_i \neq g_{i'} \qquad \text{for output bounds}
\end{aligned}
\tag{3.7}
$$

The efficiency score of each DMU is obtained by applying the DEA with additional ARI restriction constraints obtained from the proposed method to the transformed data in Step 1.

In addition, an infinitesimal constant might be assigned to the input and output weights as additional constraints. A very small positive value $\varepsilon > 0$ can be assigned to the input and output weights by inequalities $v_i \geq \varepsilon$ and $u_r \geq \varepsilon$ in the DEA model, where $\varepsilon$ is a mathematical infinitesimal to prevent the criteria from being omitted from an assessment. This condition is used to guarantee that the solution will be positive in these variables which means no zero weight is assigned to any input and output criteria. However, the value of $\varepsilon$ must not be too large otherwise it will eliminate the weight flexibility in an assessment and could further lead to the infeasibility in some DMUs.

### 3.3 Advantage of Proposed Method

Since a number of values of the bounds on ratios between criteria weight in the ARI weight restrictions are determined based on judgement of decision maker, an inconsistency usually occurs which leads to infeasible solution during the calculation of the DEA with ARI weight restriction constraints. The key importance of the proposed technique is that the bounds extracted from the application of pairwise comparison satisfy transitivity property so the considerable set of additional ARI constraints can give feasibility of solution.

Moreover, making comparison on each pair of criteria usually a difficult task for the decision maker especially when many criteria are involved in an analysis. The proposed technique introduces grading system in which the importance of each criterion is individually evaluated considering the relation to the objective. This is easy for the decision maker to give judgements because he/she only has to consider the importance of each single criterion in relation to the objective of an analysis while the comparison between criteria are not required. By doing so, the importance of criteria can directly reflect the objective of the decision. The number of paired comparison is also reduced by applying grade. In addition, the use of pairwise comparisons provides an effective and efficient manner for eliciting qualitative information from the decision maker so the overall technique of the DEA with ARI weight restrictions is a combination of the two intuitive and reasoning.

### 3.4 Proof of Transitivity

The section aims to clarify how the proposed method satisfies transitivity property which leads to the possibility of a linear program with feasible solutions. According to procedure Step 2 of the proposed method, suppose that the decision maker decides to use three grades A, B, and C and these grades are assigned to three criteria as

| **Criteria** | **Grade** |
|---|---|
| $x_1$ | C |
| $x_2$ | B |
| $x_3$ | A |

Then the grades A, B, and C are paired comparison according to procedure Step 4 as

| | A | B | C |
|---|---|---|---|
| A | 1 | $p$ | $r$ |
| B | $1/p$ | 1 | $q$ |
| C | $1/r$ | $1/q$ | 1 |

Converting grade of each criterion in Step 2 by using the relationship of grades from pairwise comparison matrix constructed in Step 4 above, three inequality equations which are in the form of the ARI weight restriction constraints can be obtain as

$$q \le \frac{v_2}{v_1} \quad \Rightarrow \quad v_2 \ge qv_1 \tag{3.8}$$

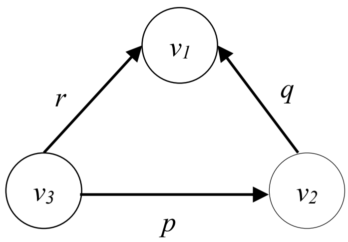
$$r \le \frac{v_3}{v_1} \quad \Rightarrow \quad v_3 \ge rv_1 \tag{3.9}$$

$$p \le \frac{v_3}{v_2} \quad \Rightarrow \quad v_3 \ge pv_2 \tag{3.10}$$

from (3.8) and (3.10); $\because$ $$v_3 \ge pqv_1 \tag{3.11}$$

from (3.9) and (3.11); $\therefore$ $$\boxed{\begin{array}{l} v_3 \ge rv_1 \\ v_3 \ge pqv_1 \end{array}}$$ ***Transitivity***

The inequality equation (3.9) and (3.11) can guarantee the transitivity of all comparison judgements. The figure below also proves transitivity along circular.




The transitivity ultimately leads to feasible solution of linear programs of the DEA with ARI weight restriction constraints.

# 4 NUMERICAL APPLICATIONS

This chapter presents a framework of the methodology developed for dealing with MAMC, in particular the decision making problem that contains many alternatives and many criteria. This is done by considering a case study of facility location problem which has long been recognized as one strategic decision making problem entailing multiple alternatives and criteria. The aims of this chapter are to demonstrate how the developed methodology for MAMC is utilized, and to show the capability of the proposed method for setting bounds of ARI constraints in order to deal with this large scale problem. The drawbacks of the conventional DEA model due to its property of weight flexibility are also exemplified in the case study. This chapter is divided into two main sections. Section 4.1 shows the application of the proposed method based on the DEA and ARI techniques to solve MAMC. Section 4.2 shows the application of AHP to select one best alternative from FAMC.

### 4.1 MAMC to FAMC (Few Alternative Many Criteria)

Facility location is one of critical issues in strategic logistics planning of supply chain in all industries, regardless of the size of company or the type of operation that it is planning to establish. It is a part of the company's strategy and is one of the most important decisions a company makes because facility location requires large investment that cannot be recovered and commits the organization to long-term execution which directly has an effect not only on cost of doing business but also the company's income and its competitive capacity. Not only operations but all areas of the company such as finance, human resources, are also affected by location decision. A good location could further give a strategic advantage against competitors.

Selection of appropriate location requires joint consideration of multiple alternatives which are various locations for a new facility and several evaluation criteria that influence the location decision. So an expert or decision maker may require a large amount of data to be effectively assessed by considering conflicting tangible and intangible criteria. Regarding its importance and difficulty, facility location problem has been tackled by many researchers using several different types of multiple criteria decision making techniques.

### 4.1.1 Selection alternatives and decision criteria

The example shows a case of Japanese manufacturing company considering shifting its production site to international location. The objective is to minimize cost of production operations, and reduce transportation time. One optimal location has to be selected among candidate locations. The company considers nineteen potential location alternatives locating in ten countries in Asia as shown in Table 4.1.

**Table 4.1** Alternative Locations (DMUs)

| Country | Location | DMU |
|---|---|---|
| India | Chennai | DMU 1 |
| | New Delhi | DMU 2 |
| | Bangalore | DMU 3 |
| Indonesia | Jakarta | DMU 4 |
| | Batam | DMU 5 |
| Cambodia | Phnom Penh | DMU 6 |
| Thailand | Bangkok | DMU 7 |
| Philippines | Cebu | DMU 8 |
| | Manila | DMU 9 |
| Vietnam | Da Nang | DMU 10 |
| | Hanoi | DMU 11 |
| | Ho Chi Minh | DMU 12 |
| Malaysia | Kuala Lumpur | DMU 13 |
| Myanmar | Yangon | DMU 14 |
| Lao PDR | Vientiane | DMU 15 |
| China | Guangzhou | DMU 16 |
| | Shanghai | DMU 17 |
| | Qingdao | DMU 18 |
| | Wuhan | DMU 19 |

The criteria important for the location selection analysis of nineteen locations listed in a complete hierarchy of location criteria is shown in Figure 4.1. The effective international location strategy of the company is analyzed based on thirteen criteria that influence manufacturing plant location planning. Ten input criteria related to cost, time, economic and environment, and governance are considered a comprehensive set of resources necessary for the company to start up and run a business, and three criteria relate to economy and quality of living aspects are considered as outputs. An explanation of each criterion is summarized below. The detail description is also illustrated in Appendix A.

| | |
|---|---|
| Input $x_1$ | Capital cost: cost of business start-up procedure in percentage of gross national income per capita |
| Input $x_2$ | Land cost: a monthly rental of land in industrial park per square meter |
| Input $x_3$ | Labour cost: a monthly wage of worker per person |
| Input $x_4$ | Transportation cost: cost of transporting a 40ft container to Japan |
| Input $x_5$ | Proximity to customer: lead time from shipment point to port of loading |
| Input $x_6$ | Proximity to supplier: lead time from port of discharge to arrival at the consignee |
| Input $x_7$ | Tax: a tax on corporate profits or net income |
| Input $x_8$ | Inflation: annual percentage change in cost to average consumer of acquiring goods and services |
| Input $x_9$ | Natural hazard: risk that communities are exposed to natural hazards and degree of vulnerability |
| Input $x_{10}$ | Governance: perceptions of the degree of corruption as seen by business people and country analysts |
| Output $y_1$ | Industry value added: net output of a sector after adding up all outputs and subtracting intermediate inputs |
| Output $y_2$ | Net flow: the net inflows of investment to acquire a lasting management interest |
| Output $y_3$ | Quality of life: workers' remittances and compensation of employees |

Taking a look at the selected indicators, it can be noted that all of them can be presented by quantitative or numerical data. Values of a real data set of observed inputs and outputs on nineteen location alternatives are obtained from Japan External Trade Organization (JETRO), World Bank, and Asian Development Bank (ADB). Table 4.2 presents the observed input and output data of nineteen alternative locations used in the analysis.

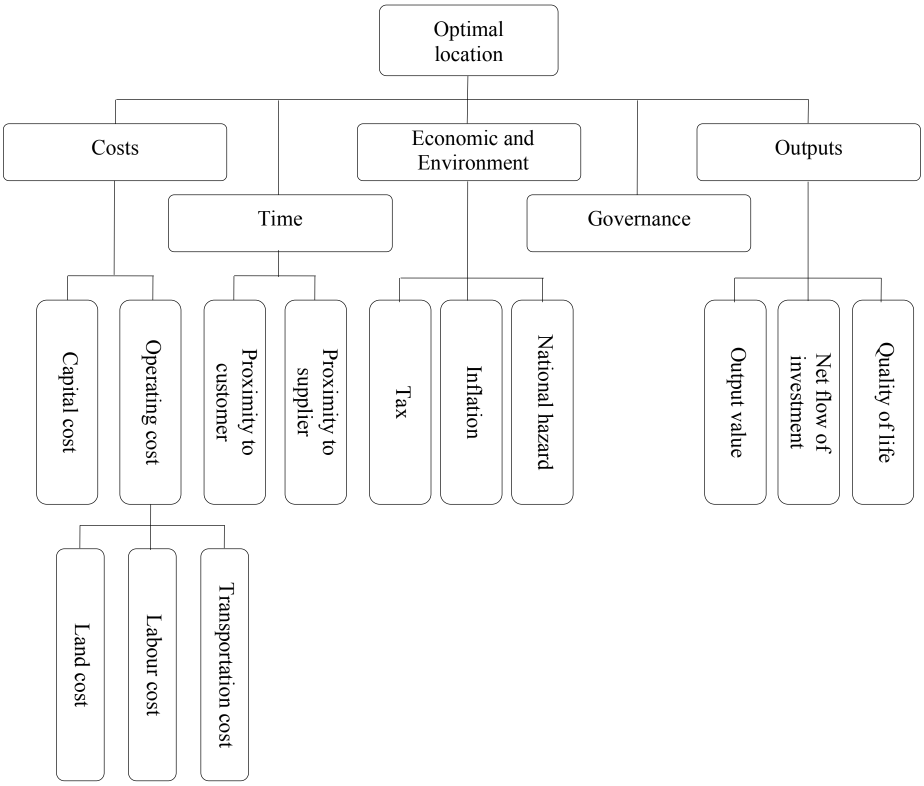



**Figure 4.1** A Hierarchy of Criteria for Evaluating Facility Location

**Table 4.2** Data Set of the Location Selection

| DMU | Inputs | | | | | | | | | | Outputs | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $y_1$ | $y_2$ | $y_3$ |
| 1 | 49.8 | 5.29 | 324 | 979 | 3 | 3 | 30 | 9.3 | 7.68 | 3.1 | 26.40 | 32,190,000,000 | 3.35509 |
| 2 | 49.8 | 3.93 | 276 | 1,566 | 3 | 3 | 30 | 9.3 | 7.68 | 3.1 | 26.40 | 32,190,000,000 | 3.35509 |
| 3 | 49.8 | 5.29 | 398 | 1,901 | 3 | 3 | 30 | 9.3 | 7.68 | 3.1 | 26.40 | 32,190,000,000 | 3.35509 |
| 4 | 22.7 | 8.13 | 239 | 800 | 2 | 3 | 25 | 4.3 | 11.70 | 3.0 | 47.15 | 19,852,569,230 | 0.81763 |
| 5 | 22.7 | 8.13 | 177 | 1,850 | 2 | 3 | 25 | 4.3 | 11.70 | 3.0 | 47.15 | 19,852,569,230 | 0.81763 |
| 6 | 100.5 | 0.11 | 74 | 1,500 | 2 | 2 | 20 | 2.9 | 16.60 | 2.1 | 23.50 | 901,668,591 | 2.75867 |
| 7 | 6.7 | 7.22 | 345 | 1,162 | 2 | 1 | 20 | 3.0 | 6.86 | 3.4 | 40.10 | 8,616,301,338 | 1.08043 |
| 8 | 18.1 | 3.50 | 218 | 1,276 | 3 | 4 | 30 | 3.2 | 24.30 | 2.6 | 31.54 | 2,797,000,000 | 10.22086 |
| 9 | 18.1 | 6.60 | 301 | 850 | 3 | 4 | 30 | 3.2 | 24.30 | 2.6 | 31.54 | 2,797,000,000 | 10.22086 |
| 10 | 8.7 | 0.12 | 107 | 2,500 | 2 | 2 | 25 | 9.1 | 11.20 | 2.9 | 40.25 | 7,430,000,000 | 6.95122 |
| 11 | 8.7 | 0.17 | 145 | 2,000 | 2 | 2 | 25 | 9.1 | 11.20 | 2.9 | 40.25 | 7,430,000,000 | 6.95122 |
| 12 | 8.7 | 0.28 | 148 | 500 | 2 | 2 | 25 | 9.1 | 11.20 | 2.9 | 40.25 | 7,430,000,000 | 6.95122 |
| 13 | 15.1 | 8.13 | 344 | 643 | 3 | 2 | 25 | 1.7 | 6.69 | 4.3 | 40.67 | 12,000,756,384 | 0.42881 |
| 14 | 6.7 | 0.50 | 53 | 1,600 | 1 | 1 | 25 | 5.0 | 8.54 | 1.5 | 26.00 | 1,000,557,266 | 0.00160 |
| 15 | 7.1 | 0.06 | 132 | 1105 | 2 | 2 | 24 | 4.3 | 5.80 | 2.2 | 27.68 | 300,743,507 | 0.61038 |
| 16 | 2.1 | 6.36 | 395 | 650 | 3 | 4 | 25 | 2.7 | 6.36 | 6.1 | 46.80 | 253,474,944,300 | 0.13131 |
| 17 | 2.1 | 3.58 | 449 | 564 | 3 | 4 | 25 | 2.7 | 6.36 | 6.1 | 46.80 | 253,474,944,300 | 0.13131 |
| 18 | 2.1 | 1.40 | 281 | 198 | 3 | 4 | 25 | 2.7 | 6.36 | 6.1 | 46.80 | 253,474,944,300 | 0.13131 |
| 19 | 2.1 | 3.18 | 308 | 1,068 | 3 | 4 | 25 | 2.7 | 6.36 | 6.1 | 46.80 | 253,474,944,300 | 0.13131 |

### 4.1.2 Application of DEA to facility location problem

The original DEA model is applied in order to evaluate performance or efficiency of a set of alternative locations to solve the facility location problem. An example of formulation to calculate an efficiency of DMU 1 can be seen in Appendix B. The result of efficiency of all locations and the corresponding optimal weights are shown in Table 4.3.

**Table 4.3** Result from Original DEA Calculation

| **DMUs** | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $u_1$ | $u_2$ | $u_3$ | **Efficiency** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1302 | 0 | 0.019784 | 0 | 0.095887 | 0.830162 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1302 | 0 | 0.019784 | 0 | 0.095887 | 0.830162 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.1302 | 0 | 0.019784 | 0 | 0.095887 | 0.830162 |
| 4 | 0 | 0 | 0 | 0.0005 | 0 | 0 | 0.0255 | 0 | 0 | 0 | 0.021685 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0.0036 | 7E-05 | 0 | 0 | 0.0072 | 0.0101 | 0 | 0 | 0.020768 | 0 | 0 | 1 |
| 6 | 0 | 0.0352 | 0.0052 | 0.0001 | 0 | 0 | 0.0137 | 0.0584 | 0 | 0 | 0.042538 | 0 | 0 | 1 |
| 7 | 0.0058 | 0 | 0 | 0.0003 | 0 | 0 | 0.033 | 0 | 0 | 0 | 0.025182 | 0 | 0 | 1 |
| 8 | 0 | 0 | 0.0027 | 6E-05 | 0 | 0 | 0 | 0.0472 | 0 | 0.0686 | 0.019913 | 0 | 0.035959 | 1 |
| 9 | 0 | 0 | 0 | 0.001 | 0 | 0 | 0 | 0.0573 | 0 | 0 | 0.017688 | 0 | 0.043777 | 1 |
| 10 | 0 | 0 | 0.005 | 0.0001 | 0 | 0 | 0.0084 | 0 | 0 | 0 | 0.024489 | 0 | 0 | 1 |
| 11 | 0.0144 | 2E-13 | 0 | 0 | 0 | 0 | 0.0074 | 0.0072 | 0 | 0.2155 | 0.024552 | 0 | 0 | 1 |
| 12 | 0 | 0 | 0 | 0.002 | 0 | 0 | 0 | 0 | 0 | 0 | 0.025421 | 0 | 0 | 1 |
| 13 | 0 | 0 | 0 | 0.0015 | 0 | 0 | 0 | 0.0291 | 0 | 0 | 0.025543 | 0 | 0 | 1 |
| 14 | 0 | 0 | 0.0096 | 0.0003 | 0 | 0 | 0 | 0 | 0 | 0 | 0.038316 | 0 | 0 | 1 |
| 15 | 0 | 0.0007 | 0.0034 | 7E-05 | 0 | 0 | 0 | 0 | 0.0806 | 0 | 0.036127 | 0 | 0 | 1 |
| 16 | 0 | 0 | 0 | 1E-17 | 0 | 0 | 0.04 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 17 | 0 | 0 | 1E-17 | 4E-19 | 0 | 0 | 0.04 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 18 | 0 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 19 | 0 | 0 | 1E-18 | 0 | 0 | 0 | 0.04 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

In this case, the DEA model is unable to provide an optimum solution for the analysis since sixteen out of nineteen locations are evaluated as efficient. This low discrimination of the model can probably occur when a number of DMUs are small in comparison with total number of criteria. And the DEA property of total weight flexibility allowing an assessed DMU to assign zero weights to a number of criteria, which means that some criteria are not included in the analysis, is another factor apparently contributing to the low discrimination.

For example, DMU 19 achieves 100% efficiency by being assigned weights on only two criteria and the zero weights are assigned to the rest of all criteria. This is equivalent to leaving out there criteria from the analysis. As explained in Chapter 2, an assessed DMU in the DEA formulation can freely choose the weight values to be assigned to each input and output in a way that maximizes its efficiency. A DMU that is superior to all other units in any single output/input ratio will therefore receive an efficiency score equal to one, and will consequently be considered efficient. The DEA also allows this DMU to assign very high weights to the criteria for which the unit is particularly efficient and very low weights to all the other criteria and some of them may be ignored by giving weight of zero. This aspect of the conventional DEA is one of the main problems of the technique. It is unacceptable given the fact all criteria are thoroughly selected and they are relevant to the efficiency assessment. If they are not important, then why were they included in the analysis in the first place? In addition, the efficiency of a DMU may not really reflect its performance with respect to the inputs and outputs taken as a whole.

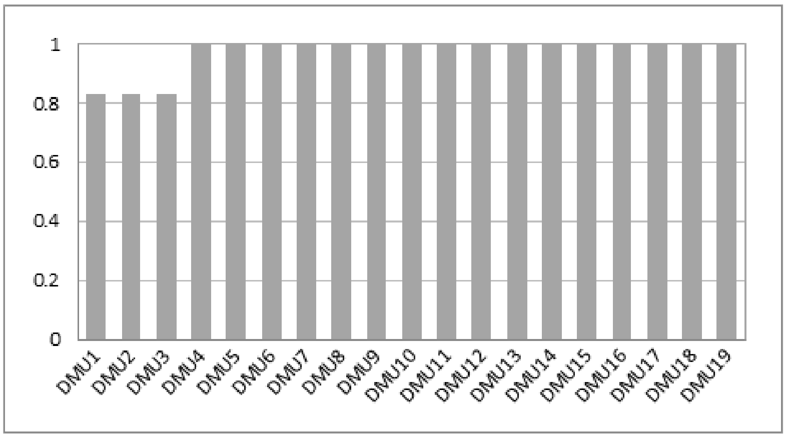



**Figure 4.2** Efficiency of Each Location

### 4.1.3 Application of proposed method

This section illustrates an application of the proposed method for determining weight bounds on the ARI with the previous facility location problem.

*Step 1: Scale Data*

All input and output data which encompass several units of measurement such as US dollar for labour cost, number of days for transportation lead time are converted in to scale of one. For example, all data of nineteen locations of input variable $x_1$, from $x_{1,1}$ to $x_{1,19}$, are divided by $x_{1,6}$ which is the maximum value of $x_1$, e.g. $\overline{x}_{1,1}$ = 49.8/100.5 = 0.50. Table 4.4 shows the new data set which is transformed from the original data set in Table 6.2 into a scale of one.

**Table 4.4** Data Transformation

| ***DMU*** | DEA inputs | | | | | | | | | | DEA outputs | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $y_1$ | $y_2$ | $y_3$ |
| 1 | 0.50 | 0.65 | 0.72 | 0.39 | 1.00 | 0.75 | 1.00 | 1.00 | 0.32 | 0.51 | 0.56 | 0.13 | 0.33 |
| 2 | 0.50 | 0.48 | 0.61 | 0.63 | 1.00 | 0.75 | 1.00 | 1.00 | 0.32 | 0.51 | 0.56 | 0.13 | 0.33 |
| 3 | 0.50 | 0.65 | 0.89 | 0.76 | 1.00 | 0.75 | 1.00 | 1.00 | 0.32 | 0.51 | 0.56 | 0.13 | 0.33 |
| 4 | 0.23 | 1.00 | 0.53 | 0.32 | 0.67 | 0.75 | 0.83 | 0.46 | 0.48 | 0.49 | 1.00 | 0.08 | 0.08 |
| 5 | 0.23 | 1.00 | 0.39 | 0.74 | 0.67 | 0.75 | 0.83 | 0.46 | 0.48 | 0.49 | 1.00 | 0.08 | 0.08 |
| 6 | 1.00 | 0.01 | 0.16 | 0.60 | 0.67 | 0.50 | 0.67 | 0.31 | 0.68 | 0.34 | 0.50 | 0.00 | 0.27 |
| 7 | 0.07 | 0.89 | 0.77 | 0.46 | 0.67 | 0.25 | 0.67 | 0.32 | 0.28 | 0.56 | 0.85 | 0.03 | 0.11 |
| 8 | 0.18 | 0.43 | 0.49 | 0.51 | 1.00 | 1.00 | 1.00 | 0.34 | 1.00 | 0.43 | 0.67 | 0.01 | 1.00 |
| 9 | 0.18 | 0.81 | 0.67 | 0.34 | 1.00 | 1.00 | 1.00 | 0.34 | 1.00 | 0.43 | 0.67 | 0.01 | 1.00 |
| 10 | 0.09 | 0.01 | 0.24 | 1.00 | 0.67 | 0.50 | 0.83 | 0.98 | 0.46 | 0.48 | 0.85 | 0.03 | 0.68 |
| 11 | 0.09 | 0.02 | 0.32 | 0.80 | 0.67 | 0.50 | 0.83 | 0.98 | 0.46 | 0.48 | 0.85 | 0.03 | 0.68 |
| 12 | 0.09 | 0.03 | 0.33 | 0.20 | 0.67 | 0.50 | 0.83 | 0.98 | 0.46 | 0.48 | 0.85 | 0.03 | 0.68 |
| 13 | 0.15 | 1.00 | 0.77 | 0.26 | 1.00 | 0.50 | 0.83 | 0.18 | 0.28 | 0.70 | 0.86 | 0.05 | 0.04 |
| 14 | 0.07 | 0.06 | 0.12 | 0.64 | 0.33 | 0.25 | 0.83 | 0.54 | 0.35 | 0.25 | 0.55 | 0.00 | 0.00 |
| 15 | 0.07 | 0.01 | 0.29 | 0.44 | 0.67 | 0.50 | 0.80 | 0.46 | 0.24 | 0.36 | 0.59 | 0.00 | 0.06 |
| 16 | 0.02 | 0.78 | 0.88 | 0.26 | 1.00 | 1.00 | 0.83 | 0.29 | 0.26 | 1.00 | 0.99 | 1.00 | 0.01 |
| 17 | 0.02 | 0.44 | 1.00 | 0.23 | 1.00 | 1.00 | 0.83 | 0.29 | 0.26 | 1.00 | 0.99 | 1.00 | 0.01 |
| 18 | 0.02 | 0.17 | 0.63 | 0.08 | 1.00 | 1.00 | 0.83 | 0.29 | 0.26 | 1.00 | 0.99 | 1.00 | 0.01 |
| 19 | 0.02 | 0.39 | 0.69 | 0.43 | 1.00 | 1.00 | 0.83 | 0.29 | 0.26 | 1.00 | 0.99 | 1.00 | 0.01 |

*Step 2: Grade all criteria*

In this case study, the decision maker decides to use three grades as a particular level of importance of criteria regarding to the objective of the analysis. A set of grades $g_i$, $g_r$ = {A, B, C} is specified. Then these grades are assigned to each criterion in accordance with the management judgment.

For example, company considers transportation cost ($x_4$), proximity to customer ($x_5$), proximity to supplier ($x_6$) of input, and industry value added ($y_1$) of output the most importance criteria in selecting a location for its manufacturing plant. Therefore, grade A's are assigned to these criteria, i.e. $g_{x4}$, $g_{x5}$, $g_{x6}$, $g_{y1}$ = A. Other criteria are examined in descending order of their importance from decision maker's point of view. The judgement of grades to all criteria of facility location problem is shown in Table 4.5.

**Table 4.5** Grading the Criteria

| Criteria | Grade |
|---|---|
| $x_1$ | B |
| $x_2$ | B |
| $x_3$ | B |
| $x_4$ | A |
| $x_5$ | A |
| $x_6$ | A |
| $x_7$ | B |
| $x_8$ | B |
| $x_9$ | B |
| $x_{10}$ | C |
| $y_1$ | A |
| $y_2$ | B |
| $y_3$ | C |

*Step 3: Determine intensity scale of importance*

Since the grade in Step 2 are deiced to be three grades, the values scale that indicate the relative importance of the grades also have to be three values. Suppose that the decision maker decides to use numerical scale of intensity of importance of 1, 3, and 5 as shown in Table 4.6. This scale is flexible and can be adjusted by increasing or reducing according to viewpoint of the decision maker. The higher amount the more importance when each pair of criteria is compared.

**Table 4.6** Scale of Importance

| Intensity of Importance | Definition |
|---|---|
| 1 | Equal importance |
| 3 | Much more importance |
| 5 | Very importance |

*Step 4: Construct pairwise comparison matrix*

A pairwise comparison matrix of the grades is constructed by using judgement of the decision maker as shown in Figure 4.3. For example in this case, the decision maker considers grade A is much more importance than grade B, so $p_{AB} = 3$ is assigned to the comparison matrix.

| **Importance Score** | A | B | C |
|---|---|---|---|
| A | 1 | 3 | 5 |
| B | 1/3 | 1 | 1.7 |
| C | 1/5 | 1/1.7 | 1 |

**Figure 4.3** Pairwise Comparison for Importance Score

*Step 5: Convert to restriction constraint models*

The last step is to convert the grade score of each criterion in Table 6.5 into weight restriction constraint models by using the relative importance between grades in pairwise comparison matrix decided by the decision maker in Figure 4.3. The following are additional ARI formulations for weight restrictions that will be incorporated in the original DEA model.

$$3v_1 - v_4 \leq 0$$

$$3v_1 - v_5 \leq 0$$

$$3v_1 - v_6 \leq 0$$

$$1.7v_{10} - v_1 \leq 0$$

$$3v_2 - v_4 \leq 0$$

$$3v_2 - v_5 \leq 0$$

$$3v_2 - v_6 \leq 0$$

$$1.7v_{10} - v_2 \leq 0$$

$$3v_3 - v_4 \leq 0$$

$$3v_3 - v_5 \leq 0$$

$$3v_3 - v_6 \leq 0$$

$$1.7v_{10} - v_3 \leq 0$$

$$3v_7 - v_4 \leq 0$$

$$3v_7 - v_5 \leq 0$$

$$3v_7 - v_6 \leq 0$$

$$1.7v_{10} - v_7 \leq 0$$

$$3v_8 - v_4 \leq 0$$

$$3v_8 - v_5 \leq 0$$

$$3v_8 - v_6 \leq 0$$

$$1.7v_{10} - v_8 \leq 0$$

$$3v_9 - v_4 \leq 0$$

$$3v_9 - v_5 \leq 0$$

$$3v_9 - v_6 \leq 0$$

$$1.7v_{10} - v_9 \leq 0 \tag{4.1}$$

**4.1.4 Result of the analysis**

Table 4.7 gives result of the efficiency scores of all alternative locations and the corresponding weight value of each criterion which is solve by the DEA model with ARI additional weight restriction constraints acquired from proposed method. The value of weights assigned to each criterion are more in line with the relative importance of criteria from viewpoint of the decision maker. The efficiency is also generally reduced in value, e.g. DMU 5 becomes inefficient in Table 4.7 whereas it was efficient in Table 4.3. This reduction of efficiency score is traceable to the change in weights resulting from the proposed method to set the bounds in ARI constrains. This also leads to an improvement of discrimination among the DMUs due to the fact that the weight bounds reduce the region of choosing weight to the specified ranges, thus possibly reducing the efficiency of the DMUs. To portray the situation graphically, Figure 4.4 provides a comparison of efficiency scores of all nineteen location alternatives obtained from the original DEA model and the proposed method.

**Table 4.7** Weights and Efficiency Scores

| ***DMU*** | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ | $v_{10}$ | $u_1$ | $u_2$ | $u_3$ | **Efficiency** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0.3487854 | 0.3487854 | 0.4239594 | 0.1162618 | 0.0436847 | 0.1162618 | 0 | 0.7078463 | 0.2359488 | 0.1415693 | 0.4728952 |
| 2 | 3.07E-16 | 0 | 0.048618 | 0.1458541 | 0.5835542 | 0.2434776 | 0.048618 | 0.048618 | 0.048618 | 0 | 0.7053945 | 0.2351315 | 0.0843805 | 0.4526455 |
| 3 | 8.98E-18 | 3.469E-18 | 0 | 0.0665937 | 0.7347411 | 0.2176205 | 0.0221979 | 0.0221979 | 0.0221979 | 0 | 0.6877661 | 0.2292554 | 0.1007895 | 0.4474121 |
| 4 | 0 | 0.0699658 | 0 | 0.2098975 | 0.8688425 | 0.3505469 | 0 | 0.0448313 | 0 | 0 | 0.9771957 | 0.2911619 | 0 | 1 |
| 5 | 0 | 0 | 0.0170474 | 0.0511422 | 1.3301929 | 0.0511422 | 0.0170474 | 0.0170474 | 0.0170474 | 0 | 0.9412419 | 0.3137473 | 0.1882484 | 0.9808743 |
| 6 | 0 | 0.0726895 | 0.0726895 | 0.2180685 | 0.8909228 | 0.3822411 | 0.0726895 | 0.0726895 | 0 | 0 | 1.1559767 | 7.286E-17 | 0 | 0.5762401 |
| 7 | 0.1528191 | 0.1528191 | 0.0344531 | 0.4584572 | 0.4584572 | 0.4584572 | 0.1528191 | 0.1528191 | 0.1528191 | 0 | 1.1757517 | -6.939E-18 | 0 | 1 |
| 8 | 0 | 0 | 0.0828454 | 0.2485362 | 0.4730362 | 0.2485362 | 0.0828454 | 0.0828454 | 0 | 0 | 0.6670874 | 0.2223625 | 0.1334175 | 0.5821467 |
| 9 | 0 | 0 | 0 | 0.2358121 | 0.5783611 | 0.2358121 | 0.078604 | 0.078604 | 0 | 0 | 0.6934186 | 0.2311395 | 0.1386837 | 0.6051251 |
| 10 | 0.0225458 | 0.0225458 | 0.0225458 | 0.0676374 | 1.2498212 | 0.0676374 | 0.0225458 | 0.0225458 | 0.0225458 | 0.0135275 | 0.9488837 | 0.3162946 | 0.1897767 | 0.9483925 |
| 11 | 0.0224012 | 0.0224012 | 0 | 0.0672035 | 1.2816415 | 0.0769787 | 0.0224012 | 0.0224012 | 0.0224012 | 0 | 0.9604782 | 0.3201594 | 0.1920956 | 0.959981 |
| 12 | 0 | 0.0250189 | 0 | 0.0933991 | 1.3549432 | 0.0933991 | 0 | 0.031133 | 0 | 0 | 1.000518 | 0.333506 | 0.2001036 | 1 |
| 13 | 0 | 0 | 0 | 0.9133968 | 0.3596271 | 0.5013248 | 0.1198757 | 0.1198757 | 0.1198757 | 0 | 1.1075931 | 0 | 0 | 0.9555322 |
| 14 | 0 | 0 | 0 | 0.1942588 | 2.6270231 | 0 | 0 | 0 | 0 | 0 | 1.8135115 | 0 | 0 | 1 |
| 15 | 0.1446077 | 0.1446077 | 0.1446077 | 0.4338232 | 0.4338232 | 0.4338232 | 0.1446077 | 0.1446077 | 0.1446077 | 0.0867646 | 1.2041408 | 1.527E-16 | 0 | 0.7070486 |
| 16 | 0.0461787 | 0 | 0 | 0.138536 | 0.678716 | 0.2203344 | 0.0461787 | 0.0461787 | 0.0461787 | 0 | 0.7353234 | 0.2451078 | 0 | 0.9749527 |
| 17 | 0.0463998 | 3.469E-18 | 0 | 0.1391994 | 0.681966 | 0.2213895 | 0.0463998 | 0.0463998 | 0.0463998 | 0 | 0.7388445 | 0.2462815 | 0 | 0.9796212 |
| 18 | 0.044769 | 0.044769 | 0.044769 | 0.1563494 | 0.8166474 | 0.134307 | 0 | 0 | 0 | 0 | 0.7542144 | 0.2514048 | 0 | 1 |
| 19 | 0.0310779 | 0.0304529 | 0 | 0.0932336 | 0.6875874 | 0.2169743 | 0.0310779 | 0.0310779 | 0.0310779 | 0 | 0.7247151 | 0.2415717 | 0 | 0.9608873 |

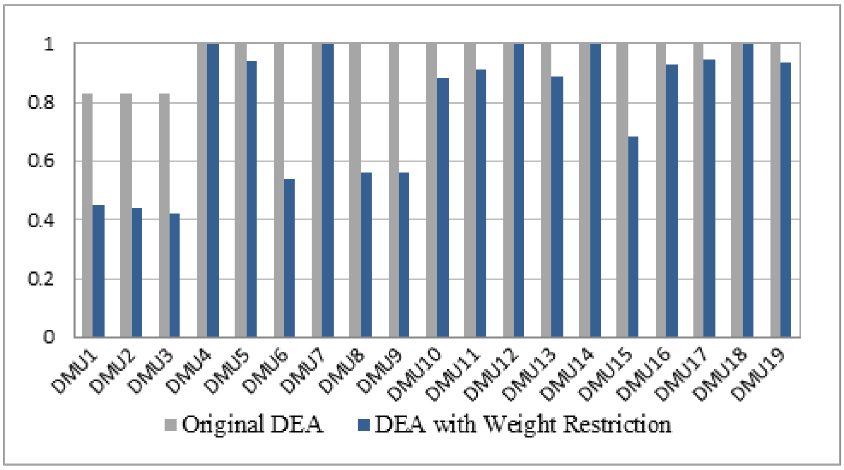



**Figure 4.4** Comparison of Results

From this numerical application of facility location problem which contains nineteen alternatives and thirteen criteria, it can be seen that a result from the application of the original DEA itself has very low discrimination because sixteen alternatives are selected as efficient locations. So it cannot give solution for the decision making problem. When combining the technique of ARI in the analysis, the models give no feasible solution. This situation occurs due to the problem of intransitivity of setting weight bounds. Applying proposed method not only offers the ease to the decision maker to set the bounds in ARI weight restriction constraints, but also helps solving MAMC. There is a significantly reduction of the number of efficient locations since five out of nineteen alternatives are selected as efficiency. Table 4.8 summarizes the number of selected locations with each method.

**Table 4.8** Summarize Number of Selected Alternatives from Different Methods

| **Method** | **No. of selected alternative** |
|---|---|
| DEA | 16 (19) |
| DEA/ARI | N/A |
| Proposed method | 3 – 5 (19) |

For the problem of decision making where only one alternative can be selected like this example of facility location problem, the selected five alternatives from MAMC are considered and can be treated as a few alternatives in decision making problem. Thus, the problem is moved from MAMC to FAMC. Next step depends on the decision maker whether to use intuition and instinct, or other multi-criteria decision making techniques to solve FAMC in order to get one best solution. This thesis suggests that the Analytics Hierarchy Process is suitable and should be applied for purpose of dealing with FAMC. An application of AHP will later be demonstrated in Section 4.2.

### 4.1.5 Sensitivity test

Since the procedure Step 2, 3, and 4 of the proposed method rely on uncertainty judgement of the decision maker, this section considers the sensitivity test for generating a solution. Step 2, 3, and 4 are relevant to each other, i.e. the number of grades (Step 2) are direct variation to the number of numerical scale of intensity of importance (Step 3) which is used in a pairwise comparison matrix (Step 4). For example, if the decision maker decides to use four grades A, B, C, D instead of just A, B, C, then the number of scale of intensity of importance also have to be increased. In order to perform a sensitivity test, an ideal pairwise comparison matrix is constructed as in Figure 4.5 to compare the grades in relation to the intensity of importance. $k$ is variable value used in adjusting the scale of importance. The value of $k$ will be increased from one in order to check the sensitivity.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 1 | $a_{aa}+k$ | $a_{ab}+k$ | $a_{ac}+k$ | $a_{ad}+k$ |
| B | $1/a_{ab}$ | 1 | $a_{ac}/a_{ab}$ | $a_{ad}/a_{ab}$ | $a_{ae}/a_{ab}$ |
| C | $1/a_{ac}$ | $1/a_{bc}$ | 1 | $a_{ad}/a_{ac}$ | $a_{ae}/a_{ac}$ |
| D | $1/a_{ad}$ | $1/a_{bd}$ | $1/a_{cd}$ | 1 | $a_{ae}/a_{ad}$ |
| E | $1/a_{ae}$ | $1/a_{be}$ | $1/a_{ce}$ | $1/a_{de}$ | 1 |

**Figure 4.5** Pairwise Comparison Matrix for Sensitivity Test

1. Sensitivity of intensity scale of importance: when the scale of importance which is used for comparing the grade is varied by increasing the value used on scale, i.e. expand the scale of importance, the number of

efficient DMUs or selected alternatives tend to decrease which means the discrimination is improved. For example in Table 4.9 which three grades, i.e. A, B, C, are used in the analysis, enlarging scale of importance by varying $k$ from one to three can reduce the number of efficient DMUs from nine to five respectively.

**Table 4.9** Number of Selected DMUs when Expanding Scale of Importance

| $k$ | Number of efficient DMUs | DMU |
|---|---|---|
| 1 | 9 | 4, 7, 10, 11, 12, 13, 14, 17, 18 |
| 2 | 5 | 4, 7, 12, 14, 18 |
| 3 | 5 | 4, 7, 12, 14, 18 |

2. Sensitivity of the number of grades: when the number of grades, $m$, which are used to indicate level of importance of the criteria are increased, the number of efficient DMUs also tend to decrease. For example, varying $m$ = 3, 4, 5 and suppose that the decision maker gives a grade to each criterion as show in the Table 4.10, and $k$ is fixed to 1. The number of efficient DMUs reduces from nine to four as shown in Table 4.11.

**Table 4.10** Grading when $m$ = 3, 4, 5

| Criteria | Number of Grades | | |
|---|---|---|---|
| | 3 | 4 | 5 |
| $x_1$ | B | D | D |
| $x_2$ | B | C | C |
| $x_3$ | B | B | B |
| $x_4$ | A | A | A |
| $x_5$ | A | A | A |
| $x_6$ | A | B | B |
| $x_7$ | B | D | E |
| $x_8$ | B | B | B |
| $x_9$ | B | D | D |
| $x_{10}$ | C | C | E |
| $y_1$ | A | A | A |
| $y_2$ | B | C | C |
| $y_3$ | C | B | E |

Note that it might cause some difficulty in making comparison when the number of grades is large.

**Table 4.11** Number of Selected DMUs when Increasing Number of Grades

| Number of Grades | Number of efficient DMUs | DMU |
|---|---|---|
| 3 | 9 | 4, 7, 10, 11, 12, 13, 14, 17, 18 |
| 4 | 6 | 4, 10, 11, 12, 14, 18 |
| 5 | 4 | 4, 12, 14, 18 |

3. Sensitivity of an infinitesimal constant: when imposing an infinitesimal constant, $\varepsilon$, to the input and output weights, the result tends to be more discrimination. Table 6.13, 6.14, and 6.15 show the result of the number of selected DMUs when imposing different value of $\varepsilon$ while varying the number of grades and scale of importance $k$. For example from Table 6.13, imposing $\varepsilon = 0.005$ to the input and output weights, so the constraints $v_{10} \geq 0.005$ and $u_3 \geq 0.005$ are included in the calculation. The result shows that three DMUs are selected as efficient when applying five grades with scale of importance $k = 11$.

**Table 4.12** Number of Efficient DMUs when $\varepsilon = 0.005$

| ***k*** | **Number of Grades** | | |
|---|---|---|---|
| | **3** | **4** | **5** |
| 1 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 4 (DMU 4,12,14,18) |
| 11 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 3 (DMU 4,12,18) |
| 20 | 5 (DMU 4,7,12,14,18) | 3 (DMU 4,12,18) | 3 (DMU 4,12,18) |
| 36 | 4 (DMU 4,7,12,18) | 3 (DMU 4,12,18) | - |

**Table 4.13** Number of Efficient DMUs when $\varepsilon = 0.01$

| ***k*** | **Number of Grades** | | |
|---|---|---|---|
| | **3** | **4** | **5** |
| 1 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 4 (DMU 4,12,14,18) |
| 6 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 3 (DMU 4,12,18) |
| 10 | 5 (DMU 4,7,12,14,18) | 3 (DMU 4,12,18) | 3 (DMU 4,12,18) |

**Table 4.14** Number of Efficient DMUs when $\varepsilon = 0.03$

| ***k*** | **Number of Grades** | | |
|---|---|---|---|
| | **3** | **4** | **5** |
| 1 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 4 (DMU 4,12,14,18) |
| 2 | 5 (DMU 4,7,12,14,18) | 4 (DMU 4,12,14,18) | 3 (DMU 4,12,18) |
| 3 | 5 (DMU 4,7,12,14,18) | 3 (DMU 4,12,18) | 3 (DMU 4,12,18) |
| 4 | 4 (DMU 4,7,12,18) | 3 (DMU 4,12,18) | - |

The result of number of selected DMUs is sensitive to judgement of the decision maker on the number of grades, the intensity scale of importance, and the infinitesimal constant. From the result of the sensitivity test, when increasing value of one of these items or a combination of them, the discrimination of the result tends to be improved. The result of selected alternative can be reduced down to three. Moreover, value of grade on each criterion assigned by the decision maker and the actual observed input and output data used in an analysis also have an effect on the result because the DEA calculates efficiency by directly using data of input and output criteria of all DMUs.

### 4.2 Resolution of FAMC

Moving from MAMC to FAMC, this section illustrates the use of the Analytic Hierarchy Process (AHP) to deal with FAMC. Considering that an appropriate route for transport product is one of strategic components and is important especially for exporting organizations. The proper route not only minimizes transportation cost which is a major share of total logistics cost of the company but also increases distribution efficiency. The study, therefore, focuses on a real-world problem of route selection.

In Greater Mekong Subregion (GMS), the Government of member countries have contributed to the development in the form of economic corridor in an attempt to improve intraregional logistics and supply chain benefits. A number of development projects of infrastructure and road transportation across member countries are conducted since road is considered an efficient transport mode in connecting countries in Mekong subregion to each other. The GMS program also tries to promote freer flow of goods and people in order to facilitate trade among the member countries by reducing different regulation and ratifying several transport facilitation agreement such as the reduction of cross-border processes and costs by constructing customs support systems, e.g. Single Window Inspection (SWI) and Single Stop Inspection (SSI) system, and improving information and communication equipment. Tax deduction and exemption are also applied for some products.

With regard to the advantage of the GMS program in terms of the transportation, the company may seek for an opportunity to increase transportation option to export product for its international trade by using the region's new developed roads. The objective is to select potential route for exporting product from the Northeast of Thailand to East Asia markets including America.

### 4.2.1 Selection alternatives and decision criteria

Northeast of Thailand is one of important area for production base of the country. As it is a landlocked area, current export route to East Asia market has to make a detour to ship product from the country's major ports, i.e. via Bangkok port in central and Laem Chabang port in southeastern Thailand. The development of East-West Economic Corridor or so called EWEC, which the route crosses several provinces in northeast Thailand, has emerged a great opportunity for the region to increase transportation options for exporting products via the GMS road networks instead of making a detour to the country's ports. Apart from the EWEC which known as route R9, there are several road construction connecting NE Thailand to the west coast in Vietnam. Route R8 linked to Port of Vinh and route R12 linked to Port of Vung Ang are likely to be other alternative options for the country to distribute product for international trade. All these three routes were chosen as potential candidates by manufacturer and exporter company. Khon Kaen province is chosen an origin of the routes as it is an important production base for industrial product in the region. The alternative routes are shown in Figure 4.6 and following is detail of each route.

1. Route R8: Khon Kaen - Nakhonphanom (Thailand) - Tha Khaek (Lao PDR) - Vinh (Vietnam) - Port of Vinh
2. Route R9: Khon Kaen - Mukdahan (Thailand) - Sawanakhet (Lao PDR) - Dongha - Hue - Danang (Vietnam) - Port of Danang
3. Route R12: Khon Kaen - Nakhonphanom (Thailand) - Tha Khaek (Lao PDR) - Ha Tinh (Vietnam) - Port of Vung Ang


**Figure 4.6** Current and Potential Alternative Routes

Three main criteria namely engineering, economics, and environment and society are used for the route selecting decision. Figure 4.7 shows a five-level hierarchy model of the route selection problem for transport export products from NE Thailand to East Asia market. The hierarchical structure, which contains the decision objective, criteria and sub-criteria for evaluation and the alternatives, is constructed after investigating criteria and its subcriteria by selecting and grouping.

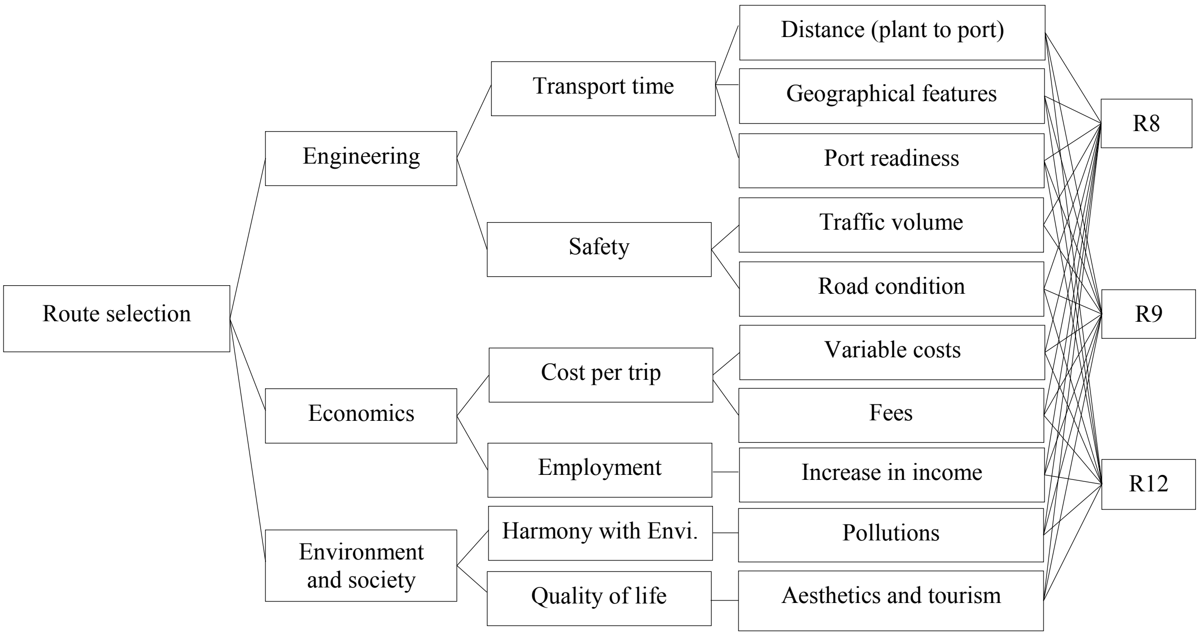



**Figure 4.7** Hierarchical Structure for Route Selection

From theory of the AHP, the first level of the hierarchy model represents decision objective or goal of the problem which is to find the optimal route among potential candidates. In the second level, the objective of the model is divided into three main criteria namely engineering, economics, and environment and society. The third and fourth level are subcriteria level 1 and 2 respectively which are related to the main criteria. The three potential alternative routes are given at the final level of the hierarchical model.

### 4.2.2 Application of AHP to routing selection problem

The elements of each level in the hierarchy structure are pairwise compared with the element in the next higher level, which leads to a number of pairwise comparison matrices (Saaty, 2008). Using the hierarchy model and the criteria previous mentioned, a group AHP consisting of fourteen experts and specialists from different fields individually analyze and pairwise compare all the decision criteria. Then an aggregation of each individual judgments is computed using a geometric mean. Table 4.15 lists organization and expertise of the all experts involve in the analysis.

**Table 4.15** List of Expert

| No. | Organization | Expertise |
|---|---|---|
| 1 | Dept. of Industrial Engineering, Ubon Rachathani University | Transport & traffic engineering |
| 2 | Dept. of Civil Engineering, Khon Kaen University | |
| 3 | International transport companies | International transport in GMS |
| 4 | | |
| 5 | Japanese multinational electronics company | Manufacturer & exporter |
| 6 | | |
| 7 | Bank of Thailand | Economics |
| 8 | Faculty of Management Sciences, Khon Kaen University | |
| 9 | Office of Commercial Affairs Nakhonphanom, Ministry of Commerce Thailand | Economics, trade & investment in GMS |
| 10 | Office of Commercial Affairs Mukdahan, Ministry of Commerce Thailand | |
| 11 | Dept. of Environmental Eng., Khon Kaen University | Environment |
| 12 | | |
| 13 | Faculty of Humanities and Social Sciences, Khon Kaen University | Society |
| 14 | | |

In order to check consistency of the decision, consistency ratio (CR) is calculated separately for each level of hierarchy of criteria. The judgements in the matrix is adjusted when CR value is greater than 0.1 until obtaining a satisfactory consistency of the matrix. The same method is applied to analyzing an importance of alternatives, but CR value of 0.05 is used since there are only three alternatives in the evaluation. The analysis of AHP conducted in this study has no problem with respect to CR value 0.05.

**Table 4.16** Criteria and Result of Importance Weights

| **Criteria** | **Weight** |
|---|---|
| 1. Engineering issue | 0.280 |
| 1.1 Time period of transportation | 0.128 |
| - Proximity of plant to destination | 0.050 |
| - Geographical & topographical features | 0.033 |
| - Port readiness | 0.045 |
| 1.2 Safety in transportation | 0.152 |
| - Traffic volume | 0.037 |
| - Road conditions (surface, steep, curve) | 0.115 |
| 2. Economical issue | 0.598 |
| 2.1 Expenses per trip | 0.508 |
| - Variable costs (fuel, maintenance) | 0.346 |
| - Fees (road charge, customs duty, tariff) | 0.162 |
| 2.2 Employment | 0.090 |
| - Increase in income | 0.090 |
| 3. Environmental & Social issue | 0.122 |
| 3.1 Harmony with environment | 0.028 |
| - Pollution (noise, air) | 0.028 |
| 3.2 Quality of life | 0.094 |
| - Decrease in aesthetics and tourism | 0.094 |

The relative importance weight value of all criteria and subcriteria with respect to the higher level criteria is shown in Table 4.16. The economics is the most significant criterion since it has the highest importance weight among the other criteria (59.8%), followed by engineering criterion (28%), and environment and society criterion (12.2%). Table 4.17 presents the result from the AHP analysis. The most appropriate route used for transportation of export product is based on the following ranking: route R12, route R9, and route

R8 respectively. Among the three candidates, route R12, export via Port of Vung Ang, is the optimum option.

**Table 4.17** AHP Result

| Alternatives | Importance weight | Ranking |
|---|---|---|
| Route R8 | 0.1120 | 3 |
| Route R9 (EWEC) | 0.3811 | 2 |
| Route R12 | 0.5069 | 1 |

Sensitivity analysis is performed to survey the criteria weight with respect to determining how they influence the alternative ranking. The weight of an assessed criterion is varied from zero to one in order to investigate the change of optimum route. After analyzing every criterion, the result indicates that variance in weight of criterion B12, which relates to various types of fees and charges for export activity, only sensitively affects the change of optimal route. When increasing the weight of B12 to 0.6, route R12 will be changed from the best alternative to the second rank, and route R9 which is EWEC turns to be the best alternative as shown in Figure 4.8 In other words, the more important of fees, the less favorable of route R12. Meanwhile, optimum route has not been changed when varying the weight of the other criteria.

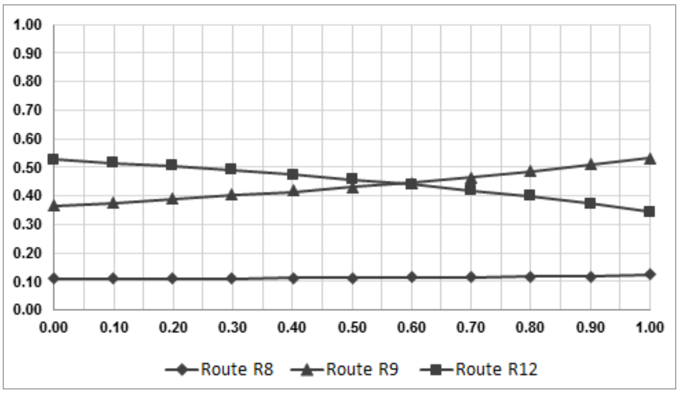



**Figure 4.8** Sensitivity Analysis of Fees (Criterion B12)

For further analysis of the route selection, Figure 4.9 presents multimodal transportation models for international trade via current route and route R12, the optimal alternative resulted from the AHP analysis. The models illustrate relationship between the travel distance and the logistics cost of one TEU of the product transported from its origin to destination. It is obvious that over half of total logistics cost has arisen during inland transportation because road is more costly than ship especially in terms of variable costs such as fuel consumption, road surcharge. In addition, various fees such as customs duty are included along road transport. The current route offers lower cost in that no cross border transport is required. The only expense for export activity is at Bangkok port before shipping oversea. Meanwhile transport via route R12 has to perform the customs processes at the border of Thailand – Lao PDR, and Lao PDR – Vietnam, as well as at the port of Vung Ang. These costs are considerably high and directly have an effect on excessive logistics cost which is a major drawback of route R12.

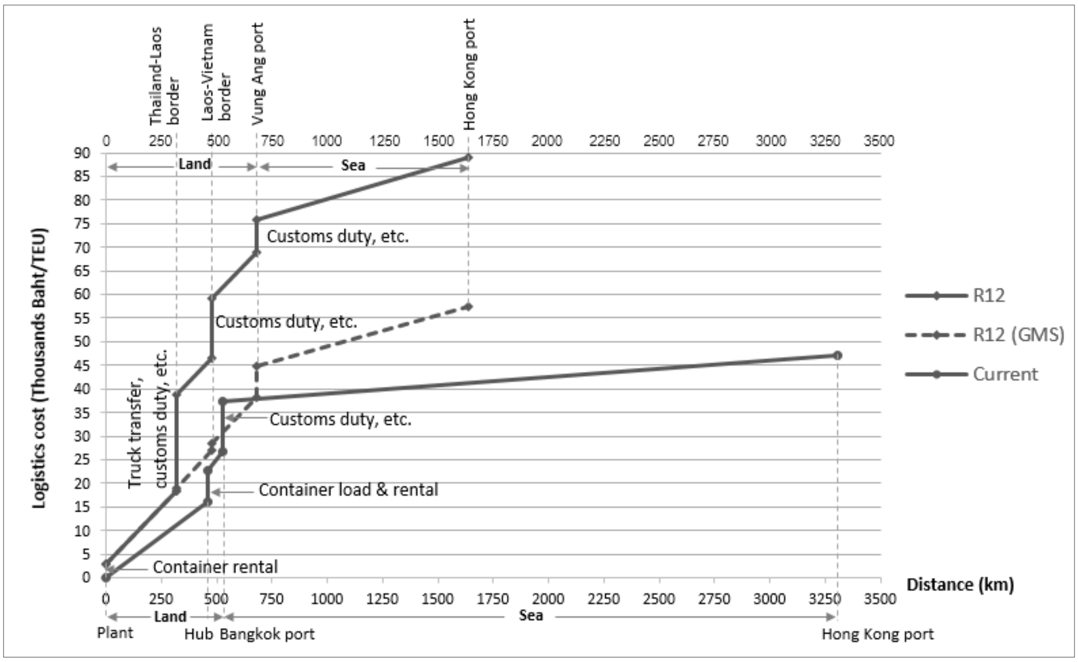



**Figure 4.9** Multimodal Transportation

The dashed line in Figure 4.9 represents the logistics cost of route R12 in the case that it is under the umbrella of the GMS cross border transportation agreement, likewise route R9, exporter will get the benefit from the exemption of fees for customs clearance processes, and the right to operate the transport without transferring to local truck when crossing borders. The process of incoming goods inspection at the borders of Lao PDR and Vietnam would also be decreased from five times to two times owing to the employment of the customs support systems, which will result in the reduction of processing time at customs and immigration to be four hours. Moreover, the total logistics cost would be reduced by 35% in conjunction with the reduction of cost of export activity and cost of storage. The logistics cost of transport via route R12 will be 12% lower than route R9. Consequently, it will definitely be the best alternative for transport option according to the selection by the AHP. In addition, although road R9 is under the GMS agreement which attempts to facilitate transportation between member countries, it still takes an amount of time in processing through customs. This is due to the lack of law and regulation among the GMS member countries to support mutual operation and collaboration between staff at the border of two countries for customs immigration and quarantine. The single window inspection and single stop inspection systems are also not thoroughly applied.

Table 4.18 summarizes the numerical application of route selection problem which is considered as FAMC since only three routes are alternatives to be selected but the problem contain many decision criteria. It can be proved that the application of AHP, which is another useful method for decision making, works very well in solving FAMC. The method can always provide satisfied result in giving one optimal solution.

**Table 4.18** Result from Solving FAMC

| | **FAMC**<br>(Few Alternatives Many Criteria) |
|---|---|
| No. of Alternative | 3 (few) |
| No. of Criteria | 10 (many) |

| **Method** | **No. of selected alternative** |
|---|---|
| AHP | 1 |

# 5 DISCUSSION

The imposition of additional constraints to the original DEA can provide benefits to situations where management or decision maker has opinions or preferences about the relative importance of different criteria, an analysis of the DEA model is unacceptable due to its excessive weight flexibility, and when the DEA fails to discriminate alternative DMUs because of the small number of DMUs comparing to total number of decision criteria in an analysis. These situations happens on most occasions with strategic decision making which usually comprises complicated MAMC problem. The application of ARI technique introducing restriction constraints on the ratios of criteria weights can affect solution obtained from the corresponding linear programming model which generally reduces efficiency scores from the initial unconstrained DEA model. The generality of AR constraints also provides flexibility in employment so the bounds can be tightened or loosened until satisfied solution or a feasible result is obtained. In consequence, the values of lower and upper bounds have to be carefully chosen. The infeasibility is further a potential problem with the ARI approach to control weights as illustrated in the preceding chapters. The method suffers from the possibility that it may lead to a linear program with no feasible solutions. Furthermore, an evaluation process usually consists of complicated inputs and outputs where many criteria cannot be measure in ratios. The key issue in using this technique is consequently the difficulty in determination of the values of weight bounds that can reflect the information obtained from the decision maker and provide feasible solution.

The use of ARI technique can be found in applications of performance measurement and decision making which could somehow guide the reader and decision maker on the practice of determining appropriate values of the bounds. Nevertheless, most of literature are presented as illustrative examination in the applied studies with little attention to illustrate the process in which the values of the weight bounds could be derived and either to analyze the weights used in evaluating the efficiency of DMUs. The issue related to possibility of infeasibility is even scarcely mentioned in the literature on ARI weight restrictions. To the best of this thesis's knowledge, no researches on strategy or method for obtaining feasible solution directly from setting possible bounds in the ARI have been developed.

Setting bounds for weight ratios in ARI constraints has been done in different ways in the literature. A general approach mostly seen in applications is based on determining a lower and upper bound of each criterion, and then restricting the weight ratios according to these bounds. Although determining ranges of each criteria is obviously less complicated than of the ratio of criteria, it is still the key in this approach which is not an easy task for the decision maker. Another common way of setting bounds is to use historical data and opinion of the decision maker on price and cost of the decision criteria, nonetheless these kinds of information is not always available for use.

One simple method to set the bounds is to initially run an unbounded DEA model, and then use the average values for the weights obtained to estimate values of the bounds. This can also be done by using regression analysis. There are also several researches purposed mathematical formulas that work as ARI bounds. However, the bounds from these techniques do not derive from viewpoint of the decision maker, so they cannot reveal real relative importance of the criteria in the eyes of the decision maker. Besides, all of the methods previously mentioned cannot guarantee the feasibility for the resulting linear programming model.

A straightforward restriction as $p_1v_1 \leq p_2v_2 \leq p_3v_3$, where $v$ represents the weight of the criterion which can be either an input or output and $p$ represents any values specified by the decision maker is also used. This method, which can later be translated into a form of ARI constrains, provides transitivity in the inequality equation which can consequently lead to feasible solution. However, this form of restriction will turn to be complicated and very confusing task when a large number of criteria are included in an analysis. Some researches introduce approach to modify the bound in order to avoid the infeasibility problem, yet most of them usually involve complicate mathematic technique which is not easy for the decision maker to comprehend. Moreover, the adjusted bounds are distorted from literal judgement given by the decision maker so the weights obtained are not exact values that reflect the decision maker point of view.

### 5.1 Significant of the Study

The research interest is therefore in examining the ARI technique and its difficulties, and introducing method to determine sensible ARI restriction attached to ratios of criteria weights that can improve the efficiency estimation yielded by the DEA. A significant of the method proposed in Chapter 3 is the theoretical concept of determining the possible values of the ARI weight restriction constraints that could provide feasible solution and reflect value judgement of the decision maker. This thesis makes four theoretical contributions which are discussed as follows.

First, the formulation of ARI enables the model to determine sets of weights that most favorable to assessed DMUs but only within certain common bounds. Imposing these bounds is not complete freedom as the resulting model may become infeasible if the bounds are intransitive. Another issue to be concerned is that weights are sensitive to units of measurement, thus ratios of weights also depend on units of measurement. The proposed method suggests to normalize data so that a fair comparison between weights can be undertaken. The procedure step 2, 3 and 4 of the proposed method provides conditions for developing transitivity in the ARI inequality equations that can guarantee an existence of feasible solution for the linear program, which is the main difficulty in setting the ARI constraints, as proved in Section 3.4 in Chapter 3.

Secondly, the attractive feature of this proposed method is its ease for employment. Imposition of weight restrictions by incorporating value judgements can be a problem for the analysis when dealing with manager who does not necessarily understand DEA. The procedure for setting bounds of the proposed method only requires management or decision maker to express he/her opinions on two elements: the degree of importance of each criterion with respect to the objective of an evaluation or decision problem, and the relative importance between each pair of them. There is no need to make comparisons on the relative importance between criteria weights, regarding the form of ARI constraints, which these complicated criteria are usually difficult to be measured in ratio. The perplexing mathematical model that needs the decision maker to be involved is neither required. The proposed method also provides the ease with which the opinion of the decision maker can be converted into the values of weight bounds in practice.

The third contribution is the flexibility and capability of the proposed method. The theoretical framework is developed for general purpose to support any decision making problems or efficiency measurements. A large number of criteria can also be included without causing any irritation to the decision maker in setting bounds and the model can still give feasible solution to the analysis. In addition, the number of criteria and DMUs used in an analysis directly has an effect on the discrimination power of DEA models and also with the potential number of zero weights. If the number of criteria is very high compared to the number of alternative DMUs as in the application of facility location problem in Chapter 4, the possibility of an assessed DMU to be evaluated as efficient increases since the DEA will assign weights to at least one criterion on which it performs well and give very low or even try to neglect all other criteria on which its performance is low. The procedure of proposed method allows the decision maker to adjust the numerical scale of intensity of importance, which used to identify relative importance values between two grades, until obtain satisfied solution. When the scale is increased, the narrower the bounds are imposed with an expectation of higher discriminating power.

The decision making usually consists of large diversified type of criteria which the level of importance of each criterion to the objective of an analysis is not the same. The proposed method allows the decision maker to deliver this information since the decision maker normally has viewpoint on criteria in light of the objective. Thus, the weights, that represents the relative values of criteria, assigned to inputs and outputs are more in line with general view of perceived importance and consistent with the objective, which contribute to an evaluation of efficiency of a DMU that reflects its performance on the inputs and outputs taken as a whole. The last contribution is therefore the development of ARI weight restrictions that is carried out systematically within the objective of an analysis. The proposed method is likely to provide the suitable solutions for problem.

The numerical application of facility location problem illustrated in Chapter 4 has shown how the weight assigned by the DEA improve considerably by introducing reasonable restrictions on the weights reflecting the relative importance of each criterion in an analysis. After calculation of efficiency of locations using the proposed method, the result in Table 4.7 shows that the weights are greatly improved. The zero weights are extremely reduced and their values are more consistent with prior knowledge or accepted views on the relative values of the inputs and outputs that relate to the objective of an analysis.

Table 5.1 summarizes and compares values of weights assigned to criterion $x_4$, $x_5$, $x_6$, $y_1$ by original DEA from Table 4.3 and the proposed method from Table 4.7. The decision maker considers three input criteria, i.e. transportation cost ($x_4$), proximity to customer ($x_5$), proximity to supplier ($x_6$), and one output, i.e. industry value added ($y_1$) the most importance in the location selection of manufacturing plant, which is the objective of an analysis, as grade "A" are assigned to these criteria as can be seen in Table 4.5. However, the conventional DEA assigns very low and zero weights to these criteria, and even worse that all zero weights are given to criteria $x_5$ and $x_6$, meaning that these two criteria are totally ignored in the efficiency assessment. This is not in correspondence with the viewpoint of the decision maker since these four criteria are expressed as the most importance so they definitely should not be eliminated from the analysis. The weights selected by the proposed method are improved to be consistent and relate to the importance of the criteria.

**Table 5.1** Comparison of Weights Assigned by DEA and Proposed Method

| DMU | $v_4$ | | $v_5$ | | $v_6$ | | $u_1$ | |
|---|---|---|---|---|---|---|---|---|
| | DEA | Proposed method | DEA | Proposed method | DEA | Proposed method | DEA | Proposed method |
| 1 | 0 | 0.3487854 | 0 | 0.3487854 | 0 | 0.4239594 | 0.019784 | 0.7078463 |
| 2 | 0 | 0.1458541 | 0 | 0.5835542 | 0 | 0.2434776 | 0.019784 | 0.7053945 |
| 3 | 0 | 0.0665937 | 0 | 0.7347411 | 0 | 0.2176205 | 0.019784 | 0.6877661 |
| 4 | 0.0005 | 0.2098975 | 0 | 0.8688425 | 0 | 0.3505469 | 0.021685 | 0.9771957 |
| 5 | 7E-05 | 0.0511422 | 0 | 1.3301929 | 0 | 0.0511422 | 0.020768 | 0.9412419 |
| 6 | 0.0001 | 0.2180685 | 0 | 0.8909228 | 0 | 0.3822411 | 0.042538 | 1.1559767 |
| 7 | 0.0003 | 0.4584572 | 0 | 0.4584572 | 0 | 0.4584572 | 0.025182 | 1.1757517 |
| 8 | 6E-05 | 0.2485362 | 0 | 0.4730362 | 0 | 0.2485362 | 0.019913 | 0.6670874 |
| 9 | 0.001 | 0.2358121 | 0 | 0.5783611 | 0 | 0.2358121 | 0.017688 | 0.6934186 |
| 10 | 0.0001 | 0.0676374 | 0 | 1.2498212 | 0 | 0.0676374 | 0.024489 | 0.9488837 |
| 11 | 0 | 0.0672035 | 0 | 1.2816415 | 0 | 0.0769787 | 0.024552 | 0.9604782 |
| 12 | 0.002 | 0.0933991 | 0 | 1.3549432 | 0 | 0.0933991 | 0.025421 | 1.000518 |
| 13 | 0.0015 | 0.9133968 | 0 | 0.3596271 | 0 | 0.5013248 | 0.025543 | 1.1075931 |
| 14 | 0.0003 | 0.1942588 | 0 | 2.6270231 | 0 | 0 | 0.038316 | 1.8135115 |
| 15 | 7E-05 | 0.4338232 | 0 | 0.4338232 | 0 | 0.4338232 | 0.036127 | 1.2041408 |
| 16 | 1E-17 | 0.138536 | 0 | 0.678716 | 0 | 0.2203344 | 0 | 0.7353234 |
| 17 | 4E-19 | 0.1391994 | 0 | 0.681966 | 0 | 0.2213895 | 0 | 0.7388445 |
| 18 | 0 | 0.1563494 | 0 | 0.8166474 | 0 | 0.134307 | 0 | 0.7542144 |
| 19 | 0 | 0.0932336 | 0 | 0.6875874 | 0 | 0.2169743 | 0 | 0.7247151 |

When weight restrictions are imposed, the DEA still assigns weights that emphasize the best input and output level of an assessed DMU but with subject to satisfying the weight restriction constraints. These weights are different from those in the original DEA without weight restrictions, therefore efficient DMUs are identified different from those demonstrated in pure DEA. The proposed method proves that it not only prevents DMUs from inflating their efficiency scores by means of attaching unreasonable weights to their inputs and/or outputs, but it also can assign legitimate weights which give rise to valuable result and the solution can reach more discrimination among DMUs.

The proposed method tends to determine more accurate bounds which can lead to an ability to achieve solution that is consistent with prior knowledge or accepted views and presents better strategic and decision making tool by decision maker. However, there is no single correct process for determining values of bounds and none of the methods is all-purpose. This thesis believes that the proposed method and its procedure described in Chapter 3 could be alternate option in determining ARI weight restriction constraints since the method is generally applicable and is likely to result in more realistic estimation of efficiency. The procedure provided in the thesis is in intelligible explanation for solving a common decision making problem is simple for the decision maker or reader to follow. Moreover, the analysis process described and practiced in Chapter 4 can provide guidance to the decision maker or user who wish to bring the proposed method to an application.

To an extent of the proposed method in improving discrimination of solution, the number of selected alternative in MAMC can be numerously reduced. As a result, the proposed method shows that it is capable to transpose decision making problem from MAMC to FAMC. In addition to the theoretical contributions this thesis also makes contribution in terms of methodology. It provides a framework for dealing with the whole problem of decision making, from MAMC to FAMC, in order to get one best alternative. After solving MAMC by applying the proposed method to the DEA with ARI, it suggests the use of AHP to solve FAMC. The application of AHP on FAMC illustrated in Chapter 4 shows that the method proves to provide optimum solution. Therefore, any decision making problems either selection of the best alternative or ranking of alternatives mentioned in Chapter 1 can be solved by follow the framework offered in this thesis. This, as a whole, could lead to practical contribution since the proposed method and framework provided in this thesis may be applied as a useful managerial tool helping the decision maker or practitioner to achieve better result for an analysis.

In addition to the application of AHP method which has been widely employed in many other studies, this thesis provides the first effort to adopt the method on the real practice of selection of transportation route for exporting products from Thailand to East Asia markets. The analysis addresses various important issues. Firstly, the alternative routes are explored in a view of regional logistics network with the consideration of several programs under Greater Mekong Subregion (GMS) to improve intraregional logistics and supply chain benefits of member countries in order to facilitate and promote international trade. Secondly, multiple experts who master at different fields are involved in the AHP analysis. This means that the analysis of the best suitable route can be established more reliably. Thirdly, travel distance, travel time, and logistics cost including multimodal transportation and cross border process of each alternative route are calculated and compared with a conventional exporting route. This comparison provides some useful information for the policy makers in terms of agreements and corroborations among GMS member countries in order to improve the performance of the routes.

### 5.2 Limitation and Future Research

Although this thesis makes several contributions in terms of theoretical, methodical, and practical, it encounters limitation which is no exception in any study. The proposed method does not guarantee the capability of handling problems when the number of alternatives is very large. As shown in the numerical application of facility location problem in Chapter 4, the application of proposed method successfully solve the problem with nineteen alternatives. However, in case that an analysis consists of huge sample size, it is difficult to discriminate efficient alternatives from all alternatives due to the nature of DEA that the method will try to find the best combination of multiple weighted inputs and multiple weighted outputs for the purpose of maximizing the efficiency score of assessed alternative. As a result, many alternatives are determined as efficient from the calculation which simply means the problem is still MAMC.

The limitation of the proposed method can provide some direction for further research to improve the method in the way to increase discrimination power when a very large number of alternatives are included in an analysis. The result in this study should also be benchmarked with those of other developing methods with an analysis under the same objective and situation. Therefore, it would be interest to compare the relative outcomes. Another possibility for future research is to develop the method to be available for multiple decision makers since some problems require more than one decision maker to assure fairness of judgement.

# 6 CONCLUSIONS

The decision making is very important for every organization no matter what type of business they are operating. A major part of the decision making involves an analysis of a finite set of alternatives which described in terms of various conflicting evaluative criteria. It is important to properly structure the problem and explicitly evaluate multiple criteria especially when the stakes are high. These decision problems are not only very complex issues involving multiple criteria, but it can become much more complex and are considered very difficult to handle when facing with a set of large number of selection alternatives. The main focus of this thesis is thus in developing a framework to tackle such problems of MAMC, which is considered the most complex problem, in order to find one best solution for the assessment.

This thesis is eclectic in its nature. It not only aims to contribute to the broad area in dealing with decision making problems, but also to the more technical research area of the DEA that incorporates weight restrictions in detail. The whole idea is to introduce an effective way or method to support the decision maker in choosing or evaluating among alternatives. Both aspects of technical and broad area should be dealt with in any of these strategic decision making. Therefore, this thesis is concerned with establishing a theoretical framework in a hybrid essence for solving MAMC based on the development of the DEA model with ARI weight restriction technique for a technical aspect, and the use of proposed method on DEA with ARI in addition with AHP to solve MAMC problems for a broad aspect. Figure 5.1 illustrates the framework to achieve the goal.

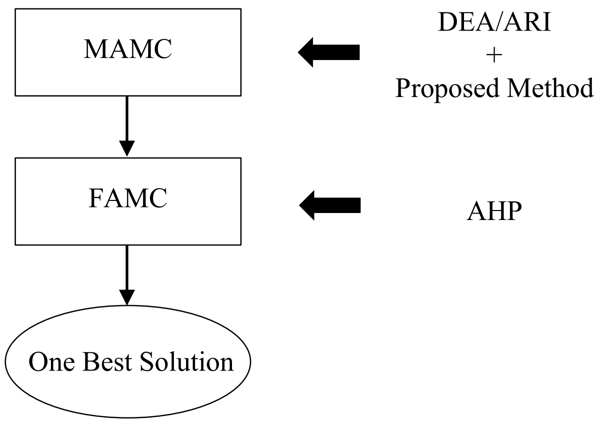



**Figure 6.1** Framework for Solving MAMC

The conventional DEA model is an appropriate decision making method to deal with MAMC since it is capable of handling a large number of alternatives. However, the total flexibility of weights that the DEA assigns to input and output criteria is a main consideration of this method. With an integration of the ARI technique, it allows to bring the perspectives of the decision maker into an analysis so that the decision process includes a combination of both tacit and explicit knowledge. The ARI can also reduce inappropriate estimation of efficiency due to the property of complete weight flexibility of the DEA model. The efficiency score in the corresponding model is worsened by the additional constraints and a DMU previously evaluated as efficient by the original DEA may subsequently be assessed as inefficient after imposing such constraints.

Imposing these limits on weights, however, may possibly cause problems of infeasibility conditions for the DEA linear programs with ARI weight restrictions since the feasible region for the weights is limited by the ARI constraints. Therefore, the main difficulty with the ARI approach is in deciding the values of bounds on weights that could avoid the infeasibility problems. Moreover, the bound values which are represented in the forms of the ratios between each pair of input weights and the ratios between each pair of output weights have to legitimately reflect the judgement or opinion of the decision maker that is expressed during processes of an assessment.

In order to circumvent the problem of infeasibility and the difficulty of bounds in ARI constraints, the thesis develops a methodology to achieve setting weight restrictions in order to obtain feasible solution. The determination of weight bound values in the ARI constrains is simplified by the adoption of proposed method. The proposed method introduces two main techniques which involve the decision maker to give judgement during decision process, i.e. grade system and pairwise comparison. Grade is used to specify the degree of importance of each single criterion with respect to the objective or goal of the analysis. This makes easier for the decision maker to evaluate the importance of each criterion instead of comparing each pair of criteria. And each criterion is directly considered to problem or objective of the decision.

Pairwise comparison is applied to compare the level of importance between each pair of grade. These number of comparisons are reduced which again is simple for the decision maker to give judgement. The main advantage of using grade and pairwise comparison to translate opinion of the decision maker into weight bound value is that it makes the ARI inequality equations transitive. Therefore, the proposed method is likely to establish feasibility conditions for the DEA with ARI weight restrictions. The values of the weight bounds also correspond to viewpoint of the decision maker, thus the result can reflect judgement or opinion of the decision maker.

The proposed method is easy to apply, moreover it improves discrimination power of the result. From the numerical example of facility location problem in Chapter 4, Table 6.1 compares the results of efficiency score of each location alternatives calculated by the original DEA from Table 4.3 and the proposed method from Table 4.7. Many of the DMUs are classified as efficient and are rated near the maximum efficiency score by using the DEA model. This is mostly a consequence of having small number of DMUs compared to a number of criteria. However, the application of proposed method of incorporating ARI weight restrictions shows that it can reduce the flexibility in weights and it generally improves discrimination. Except the five efficient DMUs, the result can also be put in ranking.

Table 6.2 summarizes the result of numerical applications from Table 4.8 and Table 4.18. For MAMC of facility location problem containing nineteen alternatives and thirteen criteria, poor discrimination is found in the assessment with the DEA model since sixteen alternatives are evaluated as efficient. The DEA with ARI method gives no feasible solution due to the problem of intransitivity of setting weight bounds. The proposed method helps improving discrimination among the efficient DMUs and also reduces weights dispersion. Three to five out of nineteen alternatives are selected as efficiency. This can be considered as a few alternatives in decision making problem. So the decision making problem is moved from MAMC to FAMC. The application of AHP can then be used to make the final judgment on FAMC in order to select one best alternative. Note that the number of alternatives involved in MAMC is not limited to nineteen. The proposed technique can be applied to any type of common MAMC where many more of alternatives can be included in an assessment.

**Table 6.1** Result from DEA and Proposed Method

| DMU | DEA | Proposed Method |
|---|---|---|
| DMU 1 | 0.830162 | 0.4728952 |
| DMU 2 | 0.830162 | 0.4526455 |
| DMU 3 | 0.830162 | 0.4474121 |
| DMU 4 | 1 | 1 |
| DMU 5 | 1 | 0.9808743 |
| DMU 6 | 1 | 0.5762401 |
| DMU 7 | 1 | 1 |
| DMU 8 | 1 | 0.5821467 |
| DMU 9 | 1 | 0.6051251 |
| DMU 10 | 1 | 0.9483925 |
| DMU 11 | 1 | 0.959981 |
| DMU 12 | 1 | 1 |
| DMU 13 | 1 | 0.9555322 |
| DMU 14 | 1 | 1 |
| DMU 15 | 1 | 0.7070486 |
| DMU 16 | 1 | 0.9749527 |
| DMU 17 | 1 | 0.9796212 |
| DMU 18 | 1 | 1 |
| DMU 19 | 1 | 0.9608873 |

**Table 6.2** Summarize Result

| | | **MAMC** (Many Alternatives Many Criteria) | **FAMC** (Few Alternatives Many Criteria) |
|---|---|---|---|
| | Example | Facility location problem | Route selection problem |
| | No. of Alternative | 19 (many) | 3 (few) |
| | No. of Criteria | 13 (many) | 10 (many) |
| ***Method*** | **DEA** | 16 (19) | - |
| | **DEA/ARI** | N/A | - |
| | **Proposed method** | 3 – 5 (19) | - |
| | **AHP** | - | 1 (3) |
| | | ***No. of selected alternative*** | |

In summary, the thesis intends to produce a framework for using multi-criteria decision analysis to support strategic decision making so that it can be deployed as an effective strategic decision supporting tool. The focus of attention is on the utility of techniques as they serve to help in resolution of decision making problems which contain many alternatives and many criteria, with an overall intention to improve the quality of decision making. The integration of the DEA with ARI under the application of the proposed technique to determine ARI weight restriction along with additional employment of the AHP is likely to produce favorable result which could contribute to a successful final decision. It is hoped that this thesis will provide significant contributions to both academics and practitioners. For academics, an improvement of methodology to deal with multi-criteria decision making problem using the DEA with ARI is introduced. For practitioners, the proposed technique provided in this thesis serves as useful tool helping the decision maker to achieve better result from an application.

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# APPENDIX A   Description and Attribute of Indicators

| ***DEA attribute*** | ***Indicator*** | ***Description*** |
|---|---|---|
| $x_1$ | Cost of business start-up procedure (% of GNI/capita) | Cost to register a business normalized by presenting it as a percentage of gross national income (GNI) per capita |
| $x_2$ | Industrial park rent ($/square meter/month) | A monthly rental of land in industrial park  per square meter in USD |
| $x_3$ | Industrial workers wage ($/month) | A monthly wage of worker per person in USD |
| $x_4$ | Transport container exports to Japan  ($/40ft container) | Cost of transporting a 40ft container to Japan in USD |
| $x_5$ | Lead time to export, median case (days) | The median time (the value for 50 percent of shipments) from shipment point to port of loading |
| $x_6$ | Lead time to import, median case (days) | The median time (the value for 50 percent of shipments) from port of discharge to arrival at the consignee |
| $x_7$ | Corporate income tax (%) | A tax on corporate profits (net income), which are derived from total income minus the cost associated with generating that income |
| $x_8$ | Inflation, consumer prices (annual %) | Inflation as measured by the consumer price index reflects the annual percentage change in the cost to the average consumer of acquiring a basket of goods and services |
| $x_9$ | World Risk Index | Risk that communities are exposed to natural hazards e.g. droughts, storms or earthquakes, and by their degree of vulnerability, which depend on social factors e.g. public infrastructure, medical services, level of education, the availability of insurance that might help to deal with economic losses in an event, as well as the condition of the environment |

| ***DEA attribute*** | ***Indicator*** | ***Description*** |
|---|---|---|
| $x_{10}$ | Corruption Perceptions Index | Perceptions of the degree of corruption as seen by business people and country analysts |
| $y_1$ | Industry Value Added (% of GDP) | The net output of a sector after adding up all outputs and subtracting intermediate inputs. It is calculated without making deductions for depreciation of fabricated assets or depletion and degradation of natural resources. |
| $y_2$ | Foreign direct investment, net inflows (BoP, $) | The net inflows of investment (new investment inflows less disinvestment) to acquire a lasting management interest in an enterprise operating in an economy other than that of the investor. It is the sum of equity capital, reinvestment of earnings, other long-term capital, and short-term capital as shown in the balance of payments in USD. |
| $y_3$ | Workers' Remittances and Compensation of Employees, Receipts (% of GDP) | Personal transfers consist of all current transfers in cash or in kind made or received by resident households to or from nonresident households. Compensation of employees refers to the income of border, seasonal, and other short-term workers who are employed in an economy where they are not resident and of residents employed by nonresident entities. Data are the sum of two items in USD. |

## APPENDIX B Example of Formulation for Efficiency Assessment of DMU 1

Maximize $26.4091u_1 + 32{,}190{,}000{,}000u_2 + 3.35509u_3$

Subject to :

$49.8v_1 + 5.29v_2 + 324v_3 + 979v_4 + 3v_5 + 3v_6 + 30v_7 + 9.3v_8 + 7.68v_9 + 3.1v_{10} = 1$

$26.4091u_1 + 32{,}190{,}000{,}000u_2 + 3.35509u_3 - 49.8v_1 - 5.29v_2 - 324v_3 - 979v_4 - 3v_5 - 3v_6 - 30v_7 - 9.3v_8 - 7.68v_9 - 3.1v_{10} \leq 0$

$26.4091u_1 + 32{,}190{,}000{,}000u_2 + 3.35509u_3 - 49.8v_1 - 3.93v_2 - 276v_3 - 1566v_4 - 3v_5 - 3v_6 - 30v_7 - 9.3v_8 - 7.68v_9 - 3.1v_{10} \leq 0$

$26.4091u_1 + 32{,}190{,}000{,}000u_2 + 3.35509u_3 - 49.8v_1 - 5.29v_2 - 393v_3 - 1901v_4 - 3v_5 - 3v_6 - 30v_7 - 9.3v_8 - 7.68v_9 - 3.1v_{10} \leq 0$

$47.1513u_1 + 19852569230u_2 + 0.81763u_3 - 22.7v_1 - 8.13v_2 - 239v_3 - 800v_4 - 2v_5 - 3v_6 - 25v_7 - 4.3v_8 - 11.69v_9 - 3v_{10} \leq 0$

$47.1513u_1 + 19852569230u_2 + 0.81763u_3 - 22.7v_1 - 8.13v_2 - 177v_3 - 1850v_4 - 2v_5 - 3v_6 - 25v_7 - 4.3v_8 - 11.69v_9 - 3v_{10} \leq 0$

$23.50434u_1 + 901668591u_2 + 2.75867u_3 - 100.5v_1 - 0.11v_2 - 74v_3 - 1500v_4 - 2v_5 - 2v_6 - 20v_7 - 2.9v_8 - 16.58v_9 - 2.1v_{10} \leq 0$

$40.10311u_1 + 8616301338u_2 + 1.08043u_3 - 6.7v_1 - 7.22v_2 - 345v_3 - 1162v_4 - 2v_5 - 1v_6 - 20v_7 - 3v_8 - 6.86v_9 - 3.4v_{10} \leq 0$

$31.5438u_1 + 2797000000u_2 + 10.22086u_3 - 18.1v_1 - 3.5v_2 - 218v_3 - 1276v_4 - 3v_5 - 4v_6 - 30v_7 - 3.2v_8 - 24.32v_9 - 2.6v_{10} \leq 0$

$31.5438u_1 + 2797000000u_2 + 10.22086u_3 - 18.1v_1 - 6.6v_2 - 301v_3 - 850v_4 - 3v_5 - 4v_6 - 30v_7 - 3.2v_8 - 24.32v_9 - 2.6v_{10} \leq 0$

$40.25265u_1 + 7430000000u_2 + 6.95122u_3 - 8.7v_1 - 0.12v_2 - 107v_3 - 2500v_4 - 2v_5 - 2v_6 - 25v_7 - 9.1v_8 - 11.21v_9 - 2.9v_{10} \leq 0$

$40.25265u_1 + 7430000000u_2 + 6.95122u_3 - 8.7v_1 - 0.17v_2 - 145v_3 - 2500v_4 - 2v_5 - 2v_6 - 25v_7 - 9.1v_8 - 11.21v_9 - 2.9v_{10} \leq 0$

$40.25265u_1 + 7430000000u_2 + 6.95122u_3 - 8.7v_1 - 0.28v_2 - 148v_3 - 500v_4 - 2v_5 - 2v_6 - 25v_7 - 9.1v_8 - 11.21v_9 - 2.9v_{10} \leq 0$

$40.67792u_1 + 12000756384u_2 + 0.42881u_3 - 15.1v_1 - 8.13v_2 - 344v_3 - 643v_4 - 3v_5 - 2v_6 - 25v_7 - 1.7v_8 - 6.69v_9 - 4.3v_{10} \leq 0$

$$26u_1 + 1000557266u_2 + 0.0016u_3 - 6.7v_1 - 0.5v_2 - 53v_3 - 1600v_4 - v_5 - v_6 - 25v_7 - 5v_8 - 8.54v_9 - 1.5v_{10} \leq 0$$

$$27.68635u_1 + 300743507u_2 + 0.61038u_3 - 7.1v_1 - 0.06v_2 - 132v_3 - 1105v_4 - 2v_5 - 2v_6 - 24v_7 - 4.3v_8 - 5.8v_9 - 2.2v_{10} \leq 0$$

$$46.8u_1 + 253474944300u_2 + 0.13131u_3 - 2.1v_1 - 6.36v_2 - 395v_3 - 650v_4 - 3v_5 - 4v_6 - 25v_7 - 2.7v_8 - 6.36v_9 - 6.1v_{10} \leq 0$$

$$46.8u_1 + 253474944300u_2 + 0.13131u_3 - 2.1v_1 - 3.58v_2 - 449v_3 - 564v_4 - 3v_5 - 4v_6 - 25v_7 - 2.7v_8 - 6.36v_9 - 6.1v_{10} \leq 0$$

$$46.8u_1 + 253474944300u_2 + 0.13131u_3 - 2.1v_1 - 1.4v_2 - 281v_3 - 198v_4 - 3v_5 - 4v_6 - 25v_7 - 2.7v_8 - 6.36v_9 - 6.1v_{10} \leq 0$$

$$46.8u_1 + 253474944300u_2 + 0.13131u_3 - 2.1v_1 - 3.18v_2 - 308v_3 - 1068v_4 - 3v_5 - 4v_6 - 25v_7 - 2.7v_8 - 6.36v_9 - 6.1v_{10} \leq 0$$

$$u_r \geq 0$$

$$v_i \geq 0$$

# Lists of research achievements

（As of December, 2015）

| 種 類 別 (By Type) | 題名、　発表・発行掲載誌名、　発表・発行年月、　連名者（申請者含む）(theme, journal name, date & year of publication, name of authors inc. yourself) |
|---|---|
| 学術誌掲載論文 (査読付) | Selection of Export Route Option in GMS Region: Recommendation on Current Situation,<br>Journal of Japan Industrial Management Association, Vol. 67, No. 2E, 2015,<br>Sirawadee ARUNYANART, Shunichi OHMORI, Kazuho YOSHIMOTO |
| 学術誌掲載論文 (査読付) | Pairwise Comparison for Weight Restriction in DEA/ARI,<br>International Journal of Japan Association for Management Systems, Vol. 7, No. 1, pp. 53-60, 2015,<br>Sirawadee ARUNYANART, Shunichi OHMORI, Kazuho YOSHIMOTO |
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