Two Consensus Problems of Multi-agent Systems in accordance with Switching Protocol

スイッチングプロトコルに従ったマルチエージェントシステムの二つの合意問題

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Contents

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Abstract

Control theory could be traced back to the eighteenth century when James Watt designed a centrifugal governor for the speed control of a steam engine. Since then, control theory has attracted more and more attentions, particularly during the Second World War when control theory had been vigorously developed and successfully applied to guidance control and all sorts of electronic equipments. Over the past several decades, modern control theory has evolved from the booming of spacecraft technologies and the large-scale intelligent systems.

Recently more and more attentions have been paid to the multi-agent systems because of its extensive application in various areas, such as cooperative control of unmanned air vehicles, formation control, consensus problems, flocking and tracking, and so on. The consensus problem of multi-agent systems, as one of the most important issues of multi-agent systems, has been investigated by various researchers from different disciplines, such as mathematics, physics, computer sciences and biology, as well as automatic control.

In this dissertation, we study two consensus problems of multi-agent systems in accordance with switching protocol. In real problems, it is very important to take into account channel constraints. However, the problem about the communication channel constraint on signal amplitude has seldom been discussed. One problem discussed in the dissertation is the consensus problem of multi-agent systems with communication channel constraint on signal amplitude. We discuss two types of Laplacians of network topologies in multi-agent systems and give the consensus convergence criterion of system. Some examples and simulations of three agents are provided to verify the rightness of the theoretics.

For a multi-agent system to achieve common group objectives or collectively react to unanticipated external changes, some information state (e.g. moving direction) of all the agents sometimes need to reach a common value, or consensus. The main issue on this topic is to design and analyze the consensus protocol, which is the update law driving the information states to agreement. A fixed consensus value is obtained from a given consensus protocol and initial state. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent system. It is therefore necessary and significant to investigate whether we can design a protocol to change the consensus value of the multi-agent system, and the answer to this question will allow application of multi-agent systems in new fields. Moreover, it seems to be generally complicated and difficult to design an appropriate protocol such that multi-agent systems can converge to any designated point.

To solve such a protocol design problem we pose a new class of consensus problems, called interval consensus problem, and search for a protocol ensuring that the system converges to a point on a specified closed and bounded interval, which is another problem discussed in the dissertation. By introducing two state-dependent switching parameters into the consensus protocol, the system given by the proposed protocol can globally asymptotically converge to a designated point on a special closed and bounded interval. In other words, the system given by the proposed protocol can achieve globally asymptotically interval consensus and then the system can also solve a generalized interval average consensus if the directed graph is balanced. Simulations are presented to demonstrate the effectiveness of our theoretical results.

Keywords: Consensus problem; multi-agent systems; switching protocol

1 Introduction

1.1 Research background

Control theory could be traced back to the eighteenth century when James Watt designed a centrifugal governor for the speed control of a steam engine. Since then, control theory has attracted more and more attentions, particularly during the Second World War when control theory had been vigorously developed and successfully applied to guidance control and all sorts of electronic equipments. The past several decades have witnessed rapid development success of modern control theory owing to the booming of spacecraft technologies and the large-scale intelligent systems.

Autonomous vehicle systems are expected to apply potentially in military actions, search and rescue, environmental monitoring and surveillance, commercial cleaning, material processing, defense and homeland security, and so on. Although single vehicles performing solo duties will yield some benefits, a group of vehicles will benefits much greater from their cooperations. One motivation for the multi-vehicle systems is to derive the same profits for mechanically controlled systems as has been benefited in the distributed computations. Rather than having a single gargantuan and cumbersome (and hence valuable and complex) machine handling affairs, the hope is that many cheap, simple machines, can obtain the same or even more powerful functionality, through collaborative effort.

Recent technological advances in miniaturizing of computing, automation, communication, control and compressed sensing, and actuation have made it practicable to integrate a large number of autonomous agents (air, ground, and water) collaborating with others to achieve goals. Distributed coordination of multiple autonomous agents has potential influence on various civilian and homeland security. Potential civilian applications include monitoring forest fires, oil fields, pipelines, and tracking wildlife. Potential homeland security applications include border patrol and monitoring the perimeter of nuclear power plants. Distributed coordination of multiple autonomous agents has become a hotly discussed research topic due to great benefits could be obtained accordingly, such as robustness, adaptivity, flexibility, and scalability and the ability to perform challenging tasks such as environmental monitoring, target localization, which cannot be achieved by a single agent.

1.2 Previous studies

The research of distributed control of multiple autonomous agents was motivated by the study in distributed computing [1], physics [2], management science [3, 4], and controls society [5, 6]. Recent years have witnessed a lot of research efforts in the study of distributed control of multiple autonomous agents. These research results could be classified as consensus, distributed formation control, distributed optimization, distributed task assignment, distributed estimation and control, and intelligent coordination. In the following, we will briefly introduce them respectively.

As one of the most important issues of multi-agent systems, consensus refers to having agents come to a global agreement on a state value and has been investigated by various researchers from different disciplines [7-41]. Consensus problems have a long history in the field of computer science, particularly in automata theory and distributed computation. The theoretical framework for posing and solving consensus problems for networked dynamic systems was introduced by Olfati-Saber and Murray in [13] and [30] building on the earlier work of Fax and Murray. An interesting topic studied in consensus problem is convergence speed which is used to characterize how fast or slow consensus can be reached [8-12]. Kim and Mesbahi studied the problem of maximizing the second smallest eigenvalues of a state-dependent graph Laplacian and proposed an iterative algorithm for this problem which employed a semidefinite programming solver at each recursive step [8]. Time-delay often appears in control systems and, in many cases, delay is source of instability. Consensus with time delay is also studied in depth [13-22]. Based on the properties of non-negative matrices, Xiao and Wang investigated the state consensus problem for the discrete-time multi-agent systems with changing communication topologies and bounded time-varying communication delays [14]. At the same time, there is an emerging trend to study how an inter-connected group may incorporate or evolve into different sub-groups called clusters. The cluster consensus is referred to such multi-agents systems where different consensus states in different groups are required to be achieved [42-46]. The cluster consensus problem has many potential applications including space-based interferometers; combat, surveillance and reconnaissance systems; hazardous material handing; and distributed reconfigurable sensor networks [43]. It is therefore necessary to study the cluster consensus problem of the multi-agent systems in both theory and practical application. Based on Markov chains and nonnegative matrix analysis, two novel cluster consensus criteria are obtained for multi-agent systems with several different subgroups and with fixed and switching topology respectively [44].

Formations appear in a number of biological systems, such as the well-known V-shape, employed by geese and other large migratory birds that are thought to reduce the drag force on individual birds while ensuring sufficient inter-agent visibility. Distributed formation control can be loosely characterized as geometrical patterns to be realized by a multi-agent team and has been studied in the controls society [47-58]. Pavone and Frazzoli have proposed a decentralized strategy aimed to achieve symmetric formations and shown that a group of agents, every one seeking its leading neighbor along the line of sight that was rotated by a common offset angle, finally converge to a single point, a circle or a logarithmic spiral pattern, which depended on the value of the angle [47]. Zhang and Leonard proposed a method which used the relative arc-length between particles instead of phase angle differences to measure the relative position between agents on a closed curve. Their steering control laws were proved stable by using a Lyapunov function which converged to its critical point along the controlled dynamics [49]. Ceccarelli et al. presented a decentralized control law for a group of nonholonomic vehicles, whose aim was to achieve collective circular motion around a virtual reference beacon [55]. Mastellone et al. studied the problem of formation control and trajectory tracking in a singular perturbation framework [57]. By a simple linear transformation, they could rewrite the dynamics for the group as two coupled systems represented by dynamics of the center of mass and dynamics of the formation. Furthermore, by imposing configuration constraints on the shape system, they obtained a locked system which behaves as a unique rigid body whose center of mass is then driven accordingly to follow a desired trajectory.

Optimization, whose main objective is to find optimal strategies under some given cost function, has been paid many attention because of the important roles which plays in both theoretical studies and practical applications [59-69]. As is known to us, control theory came into being from such a practical application. One problem studied in distributed optimization is convergence speed as introduced previously in [8-12]. Another problem is about cost functions which include individual cost functions and global cost functions. Johansson et al.

propose a negotiation algorithm that computes an optimal consensus point for agents modeled as linear control systems subject to convex input constraints and linear state constraints [61]. By employing a formal definition from shape analysis for formation representation and reposing the motion planning problem to one of changing (or maintaining) the shape of the formation, Derenick and Spletzer investigated convex optimization strategies for coordinating a large-scale team of fully actuated mobile robots, and showed that optimal solutions, minimizing either the total distance or minimax distance the nodes must travel, could be achieved through second-order cone programming techniques [66]. Scutari et al. proposed a new decomposition framework for the distributed optimization of general nonconvex sum-utility functions that arise naturally in the system design of wireless multi-user interfering systems [67].

In many problems, multiple agents are required to interact through a sequence of interdependent tasks. For example, in a manufacturing facility perhaps three processing stations and four material transport systems must be scheduled to process five streams of raw materials into six intermediate parts and one final product. Typically, such tasks and their interactions can be described by a timed sequence of activities for each agent that must be executed according to a prescribed schedule with a prescribed allocation of tasks to resources. Distributed task assignment is to address of task assignment of a team of agents in the way of a distributed manner, which could be approximately classified as coverage control, scheduling, and surveillance [70-81]. Hussein and Stipanovic formulated a coverage control problem that addresses a wide variety of multi-agent system applications [71]. They proposed a control law that ensures that the coverage error converges to zero for both communication structures. A collision avoidance component was appended to the control law and the closed loop system was shown to achieve full coverage of the mission space safely. Ben-Asher et al. studied a new distributed algorithm for task assignment, coordination, and communication of multiple unmanned aerial vehicles that engaged multiple targets and conceived an ad hoc routing algorithm for synchronization of target lists which utilized a distributed computing topology [79]. Kim et al. presented the resource welfare based task allocation framework based on social welfare function for multi-robot systems [80]. They showed that a robot team operating in dynamic and uncertain environments should keep an appropriate level of preparedness in order to respond immediately and smoothly to unpredictable dynamic events. The proposed algorithm enables a robot team to distribute workload in a balanced way considering resource welfare and therefore be well-prepared for future events.

Estimation is a rich discipline with a wide range of applications in signal processing and control. Owing to the absence of global information which could be used for achieving group coordination, distributed estimation and control has attracted great attentions recently [82-87]. The first problem of the distributed estimation and control is to design distributed local estimators such that some global information can be estimated in finite/infinite time. Based on the local estimator, distributed local controllers are designed in order that the closed-loop system is stable, which is the second problem of the distributed estimation and control. Nestinger and Demetriou presented a collaborative adaptive system parameter estimation strategy for multi-agent systems comprised of identical agents with full connectivity [84]. Mourikis and Roumeliotis presented an in-depth study of the localization performance of heterogeneous robotic teams with arbitrary and potentially dynamic relative position measurement graph topologies [87]. Their theoretical analysis allowed the prediction of the magnitude of the cooperative localization position errors when the topology of the relative position measurement graph changed or when the size of the robot team varied over time (e.g., when robots were located out of measurement/communication range or they failed temporarily or permanently).

In some applications, e.g. interferometry, it is very important for spacecrafts to maintain relative alignment during formation maneuvers. This requires that the spacecrafts should reorient about the same axis. Distributed coordination has been investigated because of its broad applying foreground [88-95]. By proposing a distributed control approach called local interactions with local coordinate systems, Cao et al. studied the multi-robot hunting tasks in unknown environments, where a team of mobile robots hunted a target called evader, which would actively try to escape with a safety strategy [89]. Vrancx et al. study the problem of learning Markov games with independent agents that only have knowledge of their own payoff, reward, and the current state. They propose a model based on learning automata and analyze the setting from different perspectives and show that under common ergodic assumptions, the proposed model converges to a pure equilibrium point [92]. Mei et al have studied the issues associated with the distributed coordination for second-order multi-agent systems using only relative position measurements [95]. For the second-order multi-agent systems with intrinsic nonlinear dynamics, they have proposed and analyzed distributed control algorithms combined with distributed filters for both the leaderless consensus problem and the coordinated tracking problem with a dynamic leader under an undirected graph. For multi-agent systems described by double integrators, they have presented a necessary and sufficient condition on the leaderless consensus under a directed graph using only relative position measurements between neighboring agents.

1.3 Our contributions and structure

In this dissertation, we study two consensus problems of multi-agent systems in accordance with switching protocol. This dissertation consists of seven chapters, which are summarized as follows.

In chapter 1, the background of this study is described. Previous work, related studies are explained. Outline and contributions of this dissertation are described in more detail.

In chapter 2, first of all, we review certain basic background from algebra and matrix theory. Relevant concepts and results are given, although we omit proofs. Secondly, we introduce some basic notions in graph theory that are used in modeling and analysis of the multi-agent systems in this dissertation. We also introduce the algebraic theory of graphs, with particular emphasis on the matrix objects associated with graphs, such as the adjacency and Laplacian matrices. Thirdly, we introduce linear and nonlinear system theory background.

In chapter 3, consensus problem of multi-agent systems is introduced. We overview the convergence analysis of a consensus protocol for a network of integrators with directed information flow and fixed topology discussed in previous literature. Some famous theorems and corollaries are introduced.

In real problems, it is very important to take into account channel constraints. However, the problem about the communication channel constraint on signal amplitude has seldom been discussed. In chapter 4, we address consensus problem in multi-agent systems with communication channel constraint on signal amplitude. We explore conditions for consensus problem of multi-agent systems with communication channel constraint on signal amplitude. We discuss two types of Laplacians of network topologies in multi-agent systems and give the consensus convergence criterion of system. Finally, some examples and simulation of three agents are provided to verify the rightness of the theoretics.

For a multi-agent system to achieve common group objectives or collectively react to unexpected external changes, some information state (e.g. moving direction) of all the agents sometimes need to reach a common value, or consensus. The main issue on this topic is to design and analyze the consensus protocol, which is the update law driving the information states to agreement. A fixed consensus value is obtained from a given consensus protocol and initial state. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent system. It is therefore necessary and significant to investigate whether we can design a protocol to change the consensus value of the multi-agent system, and the answer to this question will allow application of multi-agent systems in new fields. Moreover, it seems to be generally complicated and difficult to design an appropriate protocol such that multi-agent systems can converge to any designated point.

To solve such a protocol design problem we pose a new class of consensus problems in chapter 5, called interval consensus problem, and search for a protocol ensuring that the system converges to a point on a specified closed and bounded interval. By introducing two state-dependent switching parameters into the consensus protocol, which is motivated by the results of chapter 4, the system given by the proposed protocol can globally asymptotically converge to a designated point on a special closed and bounded interval. In other words, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average consensus if the directed graph is balanced. Simulations are presented to demonstrate the effectiveness of our theoretical results. It is worth mentioning that the two parameters introduced into the consensus protocol play an important role in our discussion. One role is to change the consensus value which is ideal or meets the quality that we require from the multi-agent system, and the other one is to change the time and speed of convergence of consensus protocols.

Time-delay often appears in control systems and, in many cases, delay is source of instability. Time-delays can sometimes be used to model the effect of propagation of state information between interacting agents, but they are many times neglected to facilitate analysis. The effect of communication delays in multi-agent consensus protocols is worth investigating.

In chapter 6, we discuss interval consensus problem of multi-agent systems with two types of time-delays, i.e., communication delay and input delay. Our work shows that the communication delay does not affect the consensus while the input delay does. For communication delay, the system given by the proposed protocol can reach globally asymptotically interval consensus with any time delay. As for bounded input delay, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average-consensus if the directed graph is balanced. Simulations are provided to demonstrate the effectiveness of our theoretical results.

In chapter 7, the conclusion is summarized and future work is described.

This dissertation solves two consensus problems of multi-agent systems in accordance with switching protocol. There are several directions and possible related research areas in which we can carry out future work. The primary aim of future work is to discuss the convergence towards an interval consensus for second-order multi-agent systems with directed graphs.

2 Preliminaries

This chapter introduces a brief review of relevant concepts in the areas of algebra and matrix theory, graph theory, and linear and nonlinear system theory.

2.1 Algebra and matrix theory background

We need the following definitions, lemmas, and theorems from algebra and matrix theory.

An $m \times n$ matrix consists of mn real numbers arranged in m rows and n columns. The entry in row i and column j of the matrix A is denoted by a_{ij}. An $m \times 1$ matrix is called a column vector of order m; similarly, a $1 \times n$ matrix is a row vector of order n. An $m \times n$ matrix is called a square matrix if $m = n$. The transpose of the $m \times n$ matrix A is denoted by A' or A^T.

A diagonal matrix is a square matrix A such that $a_{ij} = 0$, $i \neq j$. The matrix A is upper triangular if $a_{ij} = 0$, $i > j$. The transpose of an upper triangular matrix is lower triangular.

Let A be an $n \times n$ matrix. The determinant $\det(A - \lambda I)$ is a polynomial in the (complex) variable λ of degree n and is called the characteristic polynomial of A. The equation

$$\det(A - \lambda I) = 0$$

is called the characteristic equation of A. By the fundamental theorem of algebra the equation has n complex roots and these roots are called the eigenvalues of A.

We may factor the characteristic polynomial as

$$\det(A - \lambda I) = (\lambda_1 - \lambda) \cdots (\lambda_n - \lambda).$$

The geometric multiplicity of the eigenvalue λ of A is defined to be the dimension of the null space of $A - \lambda I$. The geometric multiplicity of an eigenvalue does not exceed its algebraic multiplicity.

A square matrix A is called symmetric if $A = A'$. The eigenvalues of a symmetric matrix are real. Furthermore, if A is a symmetric $n \times n$ matrix, then according to the spectral theorem there exists an orthogonal matrix P such that

$$PAP' = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}.$$

An $n \times n$ matrix A is said to be positive definite if it is symmetric and if for any nonzero vector x, $x'Ax > 0$. A symmetric matrix A is called positive semidefinite if $x'Ax \geq 0$ for any x.

For a vector $x_{n\times 1}$, the Euclidean norm of x is defined to be

(i) $\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2} = \sqrt{x^T x}$ whenever $x \in R^{n\times 1}$,

(ii) $\|x\| = \left(\sum_{i=1}^{n} |x_i|^2\right)^{1/2} = \sqrt{\bar{x}x}$ whenever $x \in C^{n\times 1}$.

When $n = 1$, we denote $\|x\| = |x|$.

2.2 Graph theory background

Graphs are frequently used to model a binary relationship between the objects in some domain, for example, the node set may represent computers in a network, with adjacent nodes representing pairs of computers that are physically linked.

In this section, we introduce some basic concepts of consensus and notations of algebraic graph theory that are often used in consensus problems of multi-agent systems and related to our later discussion. More details can be found in [37], [110].

Graph: A finite, undirected, simple graph $G = (V, E)$ -or graph for short- is built upon two finite sets, that is, the sets that have a finite number of elements. We refer to the first set as the node set and denote it by $V(G)$; each element of $V(G)$ is then a node of the graph. When the node set $V(G)$ has n elements, it is represented as $V(G) = \{v_1, v_2, \cdots, v_n\}$. We refer to the second set as the edge set and denote it by $E(G) \subseteq V \times V$, where an edge is an unordered pair of distinct nodes of the graph. This set consists of elements of the form (v_i, v_j) such that $i, j = 1, 2, \ldots, n$ and $i \neq j$. We often denote $V(G)$ and $E(G)$ simply by V and E, respectively, and simplify our notation for an edge (v_i, v_j) by sometimes denoting it as e_{ij} or $v_i v_j$ or even ij.

Edge: An edge is denoted by $e_{ij} = (v_i, v_j)$ if and only if the node v_i receives information from the node v_j. If e_{ij} is an edge, then we say that v_i and v_j are adjacent or that v_j is a neighbour of v_i.

Adjacency matrix: The adjacency matrix of G, denoted $A(G)$, is the $n \times n$ matrix defined as follows. The rows and the columns of $A(G)$ are indexed by V. If $i \neq j$ then the (i, j)-entry of $A(G)$, i.e. a_{ij}, is 0 for nodes v_i and v_j nonadjacent, and the (i, j)-entry of $A(G)$, i.e. a_{ij}, is 1 for nodes v_i and v_j adjacent. The (i, i)-entry of $A(G)$, i.e. a_{ii}, is 0 for $i = 1, \ldots, n$. We often denote $A(G)$ simply by A.

Neighbor: If an edge $(v_i, v_j) \in E$, then the node i is a neighbor of the node j. The set of neighbors of the node i is denoted as N_i.

Undirected graph: A graph $G = (V, E, A)$ is undirected if and only if for any $v_i, v_j \in V$, $(v_i, v_j) \in E$ implies $(v_j, v_i) \in E$, i.e., each edge in E is undirected.

Empty graph: A graph with no edges (but at least on node) is called empty.

Null graph: The graph with no nodes and no edges is the null graph.

Path: A directed path in an undirected graph is simply called path. A path from the node v_i to the node v_j (or from v_j to v_i) is also called a path between or connecting nodes v_i

and v_j.

Complete graph: A graph is called complete if every pair of nodes is adjacent.

Cycle: A cycle is a directed path that starts and ends at the same node.

Connected undirected graph: An undirected graph is connected if there is an undirected path between every pair of distinct nodes.

Laplacian matrix: The Laplacian matrix of a graph G, denoted by $L(G)=\left[l_{ij}\right]\in R^{n\times n}$, is the $n\times n$ matrix defined as follows,

$$l_{ij}=\begin{cases}\sum_{k=1}^{n}a_{ik},\ j=i\\ -a_{ij},\ j\neq i\end{cases}.$$

We often denote $L(G)$ simply by L.

Graphs as we have defined them above are sometimes referred to as simple graph, because there are some useful generalizations of this definition. For instance, there are many occasions when we wish to use a graph to model an asymmetric relation. In this situation we define a directed graph $G=(V,E,A)$, where an arc, or directed edge, is an ordered pair of distinct nodes. In a drawing of a directed graph, the direction of a directed edge is indicated with an arrow. Most graph-theoretical concepts have intuitive analogues for directed graphs. Indeed, for many applications a simple graph can equally well be viewed as a directed graph where (v_i,v_j) is a directed edge whenever (v_j,v_i) is a directed edge.

Directed graph: A directed graph $G=(V,E,A)$ consists of a node set $V=\{v_1,v_2,\cdots,v_n\}$, and an edge set $E\subseteq V\times V$, and a weighted adjacency matrix $A=\left[a_{ij}\right]\in R^{n\times n}$ with $a_{ii}=0$.

Directed path: A directed path, with length $n-1$, from the node v_i to the node v_j is a sequence of directed edges in a directed graph of the form (v_1,v_2), (v_2,v_3), $\cdots$, (v_{n-2},v_{n-1}), (v_{n-1},v_n), where $n\geq 2$ and v_1, $\cdots$, v_n are distinct.

Spanning tree: A directed graph is said to have a spanning tree if and only if there exists a node $v_i \in V$, called root, such that there is a directed path from v_i to any other node.

Strongly connected graph: A directed graph is strongly connected if there is a directed path from every node to every other node.

Balanced graph: A graph is balanced if $\sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} a_{ji}$, for all i. For an undirected graph, A is symmetrical, and thus every undirected graph is balanced.

2.3 Linear and nonlinear system theory background

In the following we introduce some definitions, lemmas and theorems from linear and nonlinear system theory.

Consider a linear time-invariant system given by

$$\dot{x} = Ax + Bu \tag{2.1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $A \in R^{n \times n}$, and $B \in R^{n \times m}$. The solution to (2.1) is given by

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau .$$

Letting $t_0 = kT$ and $t = (k+1)T$, where k is the discrete-time index and T is the sampling period; we can obtain the exact discrete-time model as

$$x(kT+T) = e^{AT}x(kT) + \int_{kT}^{kT+T} e^{A(kT+T-\tau)}Bu(\tau)d\tau .$$

With zero-order hold, the control input becomes $u(t) = u(kT)$, $kT \le t < (k+1)T$. It then follows that

$$x[k+1] = e^{AT}x[k] + \left(\int_0^T e^{A\sigma}d\sigma\right)Bu[k],$$

where $x[k] \triangleq x(kT)$ and $u[k] \triangleq u(kT)$.

Definition 2.1. A function $f: R^n \to R^m$ is locally Lipschitz if for each $x_0 \in A$, there exist constants $M > 0$ and $\delta_0 > 0$, such that

$$\|x-x_0\|<\delta_0 \Rightarrow \|f(x)-f(x_0)\| \le M\|x-x_0\|.$$

Theorem 2.1 [100]. Consider the autonomous system

$$\dot{x}=f(x), \tag{2.2}$$

where $f:D\to R^n$ is a locally Lipschitz map from a domain $D\subset R^n$ into R^n. Then the equilibrium point $x=0$ of (2.2) is

(i) Stable if, for any $\varepsilon>0$, there exists $\delta=\delta(\varepsilon)>0$ such that

$$\|x(0)\|<\delta \Rightarrow \|x(t)\|<\varepsilon, \ \forall t\ge 0.$$

(ii) Unstable if it is not stable.

(iii) Asymptotically stable if it is stable and $\delta>0$ can be chosen such that

$$\|x(0)\|<\delta \Rightarrow \lim_{t\to\infty} x(t)=0.$$

Theorem 2.2 [100]. Let $x=0$ be an equilibrium point for (2.2) and $D\subset R^n$ be a domain containing $x=0$. Let $V:D\to R$ be a continuously differentiable function such that

$$V(0)=0 \text{ and } V(x)>0 \text{ in } D-\{0\}, \text{and } \dot{V}(x)\le 0 \text{ in } D,$$

Then, $x=0$ is stable. Moreover, if

$$\dot{V}(x)<0 \text{ in } D-\{0\},$$

then $x=0$ is asymptotically stable.

Lemma 2.1 [100]. Let $x=0$ be an equilibrium point for (2.2). Let $V:R^n\to R$ be a continuously differentiable function such that

(i) $V(0)=0$ and $V(x)>0, \ \forall x\ne 0$,

(ii) $\|x\|\to\infty \Rightarrow V(x)\to\infty$,

(iii) $\dot{V}(x)<0, \ \forall x\ne 0$.

Then $x=0$ is globally asymptotically stable.

Theorem 2.3 [100]. Consider the nonautonomous system

$$\dot{x}=f(t,x), \tag{2.3}$$

where $f:[0,\infty)\times D\to R^n$ is piecewise continuous in t and locally Lipschitz in x on $[0,\infty)\times D$, and $D\subset R^n$ is a domain that contains the origin $x=0$. The origin is an equilibrium point for (2.3) if

$$f(t,0)=0,\ \forall t\geq 0.$$

Then the equilibrium point $x=0$ of (2.3) is

(i) Stable if, for any $\varepsilon>0$, there exists $\delta=\delta(\varepsilon,t_0)>0$ such that

$$\|x(t_0)\|<\delta\Rightarrow\|x(t)\|<\varepsilon,\ \ \forall\, t\geq t_0\geq 0. \tag{2.4}$$

(ii) Uniformly stable if, for each $\varepsilon>0$, there is $\delta=\delta(\varepsilon)>0$, independent of t_0, such that (2.4) is satisfied.

(iii) Unstable if it is not stable.

(iv) Asymptotically stable if it is stable and there is a positive constant $c=c(t_0)$ such that $x(t)\to 0$ as $t\to\infty$, for all $\|x(t_0)\|<c$.

(v) Uniformly asymptotically stable if it is uniformly stable and there is a positive constant c, independent of t_0, such that for all $\|x(t_0)\|<c$, $x(t)\to 0$ as $t\to\infty$, uniformly in t_0; that is, for each $\eta>0$, there is $T=T(\eta)>0$ such that

$$\|x(t)\|<\eta,\ \ \forall\, t\geq t_0+T(\eta),\ \ \forall\ \|x(t_0)\|<c.$$

(vi) Globally uniformly asymptotically stable if it is uniformly stable, $\delta(\varepsilon)$ can be chosen to satisfy $\lim_{\varepsilon\to\infty}\delta(\varepsilon)=\infty$, and for each pair of positive numbers η and c, there is $T=T(\eta,c)>0$ such that

$$\|x(t)\|<\eta,\ \ \forall\, t\geq t_0+T(\eta,c),\ \ \forall\ \|x(t_0)\|<c.$$

3 Consensus problem of multi-agent systems

In this chapter, we introduce consensus problem of multi-agent systems and overview fundamental consensus algorithms and present some famous results discussed in previous literature.

3.1 Introduction

Recent technological advances in miniaturizing of computing, automation, communication, control and compressed sensing, and actuation have made it practicable to integrate a large number of autonomous agents (air, ground, and water) collaborating with others to achieve goals. Cooperative control of multiple agent systems has potential influence on various civilian, homeland security, and military actions. Potential civilian applications include monitoring forest fires, oil fields, pipelines, and tracking wildlife. Potential homeland security applications include border patrol and monitoring the perimeter of nuclear power plants. For the military, applications include surveillance, reconnaissance, and battle damage assessment. These applications are difficult or impossible for an individual agent to solve.

Recently more and more attentions have been paid to the multi-agent systems because of its extensive application in various areas, such as cooperative control of unmanned air vehicles, formation control, consensus problems, flocking and tracking, and so on. The consensus problem of multi-agent systems, as one of the most important issues of multi-agent systems, has been investigated by various researchers from different disciplines, such as mathematics, physics, computer sciences and biology, as well as automatic control.

Consensus problem is such a problem of information consensus, where a team of agents must communicate with its neighbors to agree on important pieces of information that makes them work cooperatively in a coordinated way. This problem is more challenging because communication channels have limited range and experience fading and dropout. The research of information flow and information sharing among multiple agents in a group plays a major role in understanding and analyzing the coordinated activities of these agents. Consequently, it is essential for cooperative control is to design an appropriate distributed algorithm such that the group of agents could achieve consensus on the shared information even though there exist limited and unreliable information exchange and dynamically changing interaction topologies.

In the following, we give a classic example coming from a famous book [35] to illustrate what consensus is. Consider such a meet-for-dinner problem: a team of friends intend to meet for dinner at a special hotel but they cannot specify an accurate time to meet. On the afternoon of the dinner of the appointment, everyone finds that he or she is unsure about the time when they will meet. The coordination variable in this example is the time when the team will meet to have dinner. A distributed solution to this problem would be for each person to call, one at a time, a subset of the team. Given his or her current estimate of the meeting time, i.e., his or her instantiation of the coordination variable, the person might update his or her own estimate of the time of the meeting to be a weighted average of his or her current meeting time and that of the person with whom he or she is conversing. The question is to determine under what conditions this strategy will make the entire team to converge to a consistent meeting time.

To illustrate this meet-for-dinner example better, we assume that ten persons compose the team who communicate with exactly one other person who is chosen randomly from the team, for a random length of time. After the communication has come to a close, the process is repeated. The evolution of the dinner times is shown in Fig. 3.1 in the distributed approach mentioned above, where the initial state of each person is uniformly assigned. From Fig. 3.1 we note that the entire team converges to a consistent meeting time under switching communication topologies.

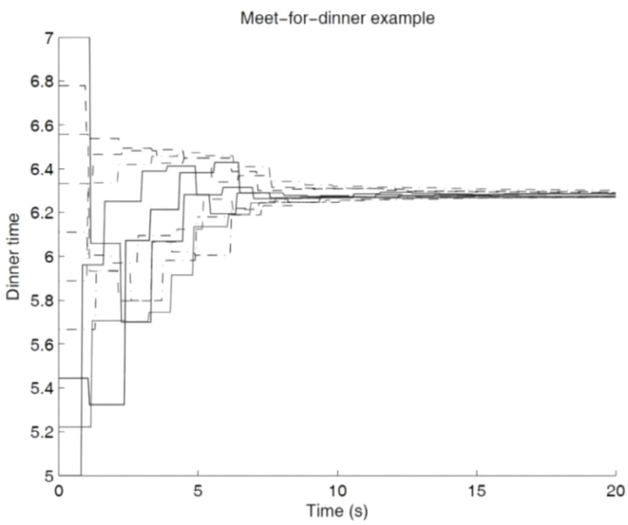

Fig. 3.1. The simulation of discrete-time meet-for-dinner.

3.2 Consensus problem of multi-agent systems

Suppose that there are n agents in the team. The team's communication topology can be represented by a directed graph $G=(V,E,A)$, where $V=\{v_1,v_2,\cdots,v_n\}$ is the node set and $E\subseteq V\times V$ is the edge set, and a weighted adjacency matrix $A=\left[a_{ij}\right]\in R^{n\times n}$ with nonnegative adjacency elements a_{ij}. The node indexes belong to a finite index set $I=\{1,2,\ldots,n\}$. An edge of G is denoted by $e_{ij}=\left(v_i,v_j\right)$. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij}\in E\Leftrightarrow a_{ij}>0$. Moreover, we assume $a_{ii}=0$ for all $i\in I$ (see 2.2 for graph theory notations). The set of neighbors of the node v_i is denoted by $N_i=\left\{v_j\in V:\left(v_i,v_j\right)\in E\right\}$.

For convenience, we also let $I=\{1,2,\ldots,n\}$ represent a set of cooperative agents with the total number n. Assume that the communications among these agents are directed. So we define as $G=(V,E)$ a directed graph where the n nodes represent n agents labeled as $1,2,\ldots,n$. The agent i receives the information of its neighbor agent j, if there is an edge (i,j) connecting the two nodes.

Throughout this dissertation, the following simplest single-integrator dynamics is used to express the dynamics of the agent i,

$$\dot{x}_i=u_i \quad i=1,2,\ldots,n, \tag{3.1}$$

where $x_i\in R$ denotes the information state of the ith vehicle which might represent physical quantities including attitude, position, temperature, voltage, and so on, and $u_i\in R$ is the information control input of the ith vehicle. We should note that (3.1) is standard dynamics form for each agent in this dissertation.

It is said that the node v_i and the node v_j achieve a consensus in a network if $x_i=x_j$. It is said that the network achieve a consensus if $x_i=x_j$ for all $i,j\in I$, $i\neq j$. Whenever the states of the network are all in a consensus, the common value of information states is called as the group decision value or the consensus value.

Definition 3.1 [107]. The set of agents I is said to reach global consensus asymptotically if for any $x_i(0)$, $i=1,2,\cdots,n$, $|x_i(t)-x_j(t)|\rightarrow 0$ as $t\rightarrow\infty$ for each (i,j), $i,j=1,2,\ldots,n$.

In what follows we illustrate consensus problem of the multi-agent system by formulating the velocity consensus problem of the multi-vehicle system.

Velocity Consensus Problem of Multi-Vehicle System

Suppose that there are n vehicles in a team running in the same direction. Let z_i denote the position of the ith vehicle and satisfy the following dynamics,

$$m_i\ddot{z}_i+\mu_i\dot{z}_i+F_i(\dot{z}_i,z_i)=U_{i\text{total}}, \tag{3.2}$$

where m_i is the total mass(including passengers) of the vehicle i, $\mu_i\dot{z}_i$ is the frictional force generated by the contact of the wheels with the road, μ_i is the friction coefficient, $F_i(\dot{z}_i,z_i)$ is the other applied force on the vehicle i and $U_{i\text{total}}$ is the control input. Next we consider the velocity consensus problem of all vehicles by designing the control input $U_{i\text{total}}$, that is, for all $\dot{z}_i(0)$ and all $i,j=1,\ldots,n$, $|\dot{z}_i(t)-\dot{z}_j(t)|\rightarrow 0$ as $t\rightarrow\infty$. In order to solve this problem, we first decompose $U_{i\text{total}}$ into the sum of local feedback control of the control input and control giving consideration to other vehicles' information, which is described as follows,

$$U_{i\text{total}}=U_{i\text{FB}}+U_i. \tag{3.3}$$

We choose $\mu_i\dot{z}_i+F_i(\dot{z}_i,z_i)$ as local feedback control $U_{i\text{FB}}$, i.e.,

$$U_{i\text{FB}}=\mu_i\dot{z}_i+F_i(\dot{z}_i,z_i), \tag{3.4}$$

and then (3.2) can be re-written as follows,

$$m_i \ddot{z}_i = U_i, \tag{3.5}$$

or

$$\ddot{z}_i = \frac{1}{m_i} U_i. \tag{3.6}$$

Here, we take $\dot{z}_i$, the velocity of the i th vehicle, as the information state of the i th vehicle, and set $x_i = \dot{z}_i$. By substituting $\frac{1}{m_i} U_i = u_i$ into (3.6), we obtain (3.1) i.e., the standard dynamics form in this dissertation. Furthermore, the velocity consensus $\left|\dot{z}_i(t) - \dot{z}_j(t)\right| \to 0$ turns into the information consensus $\left|x_i(t) - x_j(t)\right| \to 0$. By this means, the standard consensus problem of multi-agent system discussed in this dissertation can be formulated by way of illustration of the velocity consensus problem of multi-vehicle system.

A well-known consensus protocol to reach a consensus with respect to the states of n integrator agents (3.1) can be expressed as

$$u_i = -\sum_{j \in N_i} a_{ij} \left(x_i(t) - x_j(t)\right), \quad x_i(0) \in R, \tag{3.7}$$

where a_{ij} is the (i, j) th entry of the adjacency matrix of the associated communication graph at time t, and N_i is the set of agents whose information is available to the agent i at time t, and $x_i(0)$ denote the initial state of the agent i.

By applying the protocol (3.7), we can rewrite (3.1) into a single-integrator linear system on a graph,

$$\dot{x}(t) = -Lx(t), \tag{3.8}$$

where $x = (x_1, x_2, \cdots, x_n)^T$ and $L = \left[l_{ij}\right] \in R^{n \times n}$ is the graph Laplacian of the network, whose eigenvalues location determines the stability properties of system (3.8) and its elements are defined as follows:

$$l_{ij} = \begin{cases} \sum_{k=1}^{n} a_{ik}, & j = i \\ -a_{ij}, & j \neq i \end{cases}.$$

The χ-consensus problem in a dynamic graph is a distributed way to calculate $\chi(x(0))$ by applying inputs u_i that only depend on the information state of the ith agent and its neighbors. It is said that protocol (3.7) asymptotically solves the χ-consensus problem if and only if there exists an asymptotically stable equilibrium x^* satisfying $x_i^* = \chi(x(0))$ for all $i \in I$. The special cases with $\chi(x) = \text{Ave}(x(0)) = 1/n\left(\sum_{i=1}^{n} x_i(0)\right)$, $\chi(x) = \max_i x_i(0) = \max\{x_1(0), \cdots, x_n(0)\}$, and $\chi(x) = \min_i x_i(0) = \min\{x_1(0), \cdots, x_n(0)\}$ are called average-consensus, max-consensus, and min-consensus, respectively, which are widely applied to distributed decision-making for the multi-agent systems.

In the following we introduce some famous lemmas and theorems.

Theorem 3.1 [13] (Spectral Localization). Let $G = (V, E, A)$ be a digraph with the Laplacian L. Denote the maximum node out-degree of the digraph G by $d_{\max}(G) = \max_i \deg_{out}(v_i)$. Then, all the eigenvalues of $L = L(G)$ are located in the following disk:

$$D(G) = \{z \in \mathbb{C} : |z - d_{\max}(G)| \le d_{\max}(G)\}$$

centered at $z = d_{\max}(G) + 0j$ in the complex plane (see Fig. 3.2).

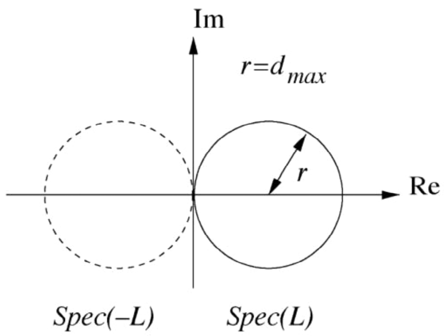

Fig. 3.2. Demonstration of Gersgorin Theorem applied to graph Laplacian.

Lemma 3.1 [13]. Consider a network of integrators $\dot{x}_i = u_i$ where each node applies protocol (3.7). Assume G is a strongly connected digraph. Then, protocol (3.7) globally asymptotically solves a consensus problem.

Denote the right and left eigenvectors of the Laplacian matrix L associated with $\lambda_1 = 0$ by w_r and w_l, respectively.

Theorem 3.2 [13]. Assume G is a strongly connected digraph with Laplacian L satisfying $Lw_r = 0$, $w_l^T L = 0$, and $w_l^T w_r = 1$. Then

$$R = \lim_{t \to +\infty} \exp(-Lt) = w_r w_l^T \in M_n .$$

Theorem 3.3 [13]. Consider a network of integrators with a fixed topology $G = (V, E, A)$ that is a strongly connected digraph. Then, protocol (3.7) globally asymptotically solves the average consensus problem if and only if G is balanced.

Theorem 3.4 [13]. Consider a network of integrator agents with a fixed topology $G = (V, E, A)$ that is a strongly connected digraph. Then, protocol (3.7) globally asymptotically solves the average-consensus problem if and only if $1^T L = 0$.

The following lemma gives the consensus value for arbitrary digraphs including unbalanced digraphs.

Lemma 3.2 [13]. Assume all the conditions in Theorem 3.4 hold. Suppose L has a nonnegative left eigenvector $\gamma = (\gamma_1, \ldots, \gamma_n)^T$ associated with $\lambda = 0$ that satisfies $\sum_i \gamma_i > 0$. Then, after reaching a consensus, the group decision value is

$$\alpha = \frac{\sum_i \gamma_i x_i(0)}{\sum_i \gamma_i}$$

i.e., the decision value belongs to the convex hull of the initial values.

3.3 Consensus problem of multi-agent systems with time delays

We note that the consensus protocol (3.7) assumes that each agent can get the states of its neighbors without any time delay. This assumption gives birth to an obvious limitation because time delay often appears in every practical system and, therefore, deserves consideration in the consensus problem of the multi-agent systems. In particular, two types of time delays, i.e., communication delay and input delay, have been considered in the existing literature.

When there exists communication delay, the protocol (3.7) becomes

$$u_i = -\sum_{j\in N_i} a_{ij}\left[x_i(t) - x_j(t-\tau)\right],\ \ x_i(0)\in R, \tag{3.9}$$

where τ is the communication delay from the j th agent to the i th agent.

When there exists input delay, the protocol (3.7) becomes

$$u_i = -\sum_{j\in N_i} a_{ij}\left[x_i(t-\tau) - x_j(t-\tau)\right],\ \ x_i(0)\in R, \tag{3.10}$$

where τ is the input delay for information communicated from the j th agent to the i th agent.

The following theorems are about communication delay and input delay, respectively.

Definition 3.2 [111]. A real $n\times n$ matrix M is a Metzler matrix if $m_{ij}\geq 0$ for all $i\neq j$. In other words, M is a Metzler matrix if all nondiagonal elements are nonnegative.

Definition 3.3 [28]. Consider an $n\times n$ Metzler matrix M with zero row sums. The δ -digraph $(\delta\geq 0)$ associated to M is a digraph with the node set $\{1,2,\ldots,n\}$ and with an arc from l to k $(l\neq k)$ if and only if the element of M on the k th row and the l th column is strictly greater than δ.

Theorem 3.5 [28]. Consider the linear system

$$\dot{x}(t) = \text{diag}\left(K(t)\right)x(t) + \left(K(t) - \text{diag}\left(K(t)\right)\right)x(t-\tau)$$

with $\tau > 0$. Assume that the system matrix $K(t)$ is a bounded and piecewise continuous function of time. Assume that, for every time t, the system matrix is Metzler with zero row sums. If there is $k\in\{1,\ldots,n\}$, $\delta>0$ and $T>0$ such that for all $t\in R$ the δ -digraph associated to

$$\int_t^{t+T} K(s)ds$$

has the property that all nodes may be reached from the node k, then the equilibrium set of consensus states is uniformly exponentially stable. In particular, all components of any solution $x(t)$ of this linear system converge to a common value as $t\to\infty$. Here $\text{diag}\left(K(t)\right)$

is the obvious notation for the diagonal matrix obtained from $K(t)$ by setting all off-diagonal entries equal to zero.

Remark 3.1. Observe that the delay τ is only featuring in those terms that correspond to the off-diagonal elements of $K(t)$. Here and in the remainder of the chapter τ is a fixed positive real number.

Theorem 3.6 [13]. Consider a network of integrator agents with equal input time-delay $\tau > 0$ in all edges. Assume the network topology G is fixed and connected. Then, protocol (3.10) with τ globally asymptotically solves the consensus problem if and only if either of the following equivalent conditions are satisfied.

i) $\tau \in (0, \tau^*)$ with $\tau^* = \pi/2\lambda_n$, $\lambda_n = \lambda_{\max}(L)$.

ii) The Nyquist plot of $\Gamma(s) = e^{-\tau s}/s$ has a zero encirclement around $-1/\lambda_k$, $\forall k > 1$.

Moreover, for $\tau = \tau^*$ the system has a globally asymptotically stable oscillatory solution with frequency $\omega = \lambda_n$.

4 Consensus problem in multi-agent systems with communication channel constraint on signal amplitude

In recent years, as a new field of research, consensus problems of multi-agent systems have drawn substantial attention from various fields such as vehicle formations, attitude alignment, rendezvous problem, flocking, and so on. In real problems, it is very important to take into account channel constraints. As a result, research on time-delay systems and their control has been active in the last decade. However, the problem about the communication channel constraint on signal amplitude has seldom been discussed.

The purpose of this chapter is to explore conditions for consensus problem of multi-agent systems with communication channel constraint on signal amplitude. We discuss two types of Laplacians of network topologies in multi-agent systems. Then the consensus convergence criterion of system is proposed. Finally, some examples and simulation of three agents verify the rightness of the theoretics.

The Laplacian introduced in this chapter defines a graph with specific network topologies that may change as the agent states proceed. From the state dependence of network topology, we can say that the graph is a special type of general “dynamic graph.” However, to the best of the authors’ knowledge, the proposed type of dynamic graph has not been discussed so far; in the literature on dynamic graphs (see, e.g., Mesbahi and Egerstedt [37], and the references therein), most research considers the case where network topology depends on relative states, whereas the network topologies in this chapter depend on absolute states coming from the control parameter of the consensus value on the absolute value of the transmitted state signal.

4.1 Preliminaries

It is said that the node of a digraph $G=(V,E,A)$ is balanced if and only if its in-degree and out-degree are equal, i.e. $\deg_{\text{out}}(v_i)=\deg_{\text{in}}(v_i)$. A graph $G=(V,E,A)$ is called balanced if and only if all of its node s are balanced.

Lemma 4.1 [35]. Suppose that $L=\left[l_{ij}\right]\in R^{n\times n}$ satisfies that $l_{ij}\leq 0$, $i\neq j$, $\sum_{j=1}^{n} l_{ij}=0$, $i=1,2,\cdots,n$, and denote $\vec{1}_n=(1,\ldots,1)^T\in R^n$. Then the following five conditions are equivalent,

(i) L has a simple zero eigenvalue with an associated $\vec{1}_n$ and all other eigenvalues have positive real parts;

(ii) $Lx=0$, $x=\left(x_1,x_2,\cdots,x_n\right)^T$ implies $x_1=\cdots=x_n$;

(iii) Global consensus is reached asymptotically for the system (3.8);

(iv) The directed graph with L as the Laplacian has a directed spanning tree;

(v) $Rank\left(L\right)=n-1$.

Theorem 4.1 [104]. Suppose G is a strongly connected digraph. Then,

(i) Global consensus is asymptotically reached for the system (3.8);

(ii) If the digraph is balanced, an average-consensus is asymptotically reached.

Theorem 4.2 [28]. Consider the linear system

$$\dot{x}\left(t\right)=-L\left(t\right)x\left(t\right). \tag{4.1}$$

Assume that the system matrix is a bounded and piecewise continuous function of time. Assume that, for every time t, the system matrix is Metzler with zero row sums. If there is an index $k\in\left\{1,\ldots,n\right\}$, a threshold value $\delta>0$ and an interval length $T>0$ such that for all $t\in R$ the δ- digraph associated to

$$\int_t^{t+T}-L(s)ds\,,$$

has the property that all nodes may be reached from the node k, then the equilibrium set of consensus states is uniformly exponentially stable. In particular, all components of any solution $x\left(t\right)$ of (4.1) converge to a common value as $t\rightarrow\infty$.

Remark 4.1. In the proof of Theorem 4.2 in [28], we have the following results on the condition of Theorem 4.2: $x_{\max}\left(t\right)=\max\left\{x_1\left(t\right),\ldots,x_n\left(t\right)\right\}$ is a non-increasing function, and $x_{\min}\left(t\right)=\min\left\{x_1\left(t\right),\ldots,x_n\left(t\right)\right\}$ is a non-decreasing function. These two results are the foundation of our proofs later.

Lemma 4.2 [37]. Let $M = \left[m_{ij} \right]$ be an $n \times n$ real matrix. Then all eigenvalues of M are located in $\bigcup_i \left\{ z \in C \middle| \left| z - m_{ii} \right| \le \sum_{j=1,\ldots,n; j \ne i} \left| m_{ij} \right| \right\}$.

4.2 Consensus with communication channel constraint on signal amplitude

In this section, we introduce some concepts about the communication channel constraint in multi-agent systems.

The information states with single-integrator dynamics are given by

$$\dot{x}_i = u_i, \quad x_i(0) \in R, \quad i = 1, 2, \ldots, n, \tag{4.2}$$

where $x_i \in R$ denotes the information state of the i th agent and $u_i \in R$ is the information control input of the i th agent.

As to the information acquisition system for each agent, we assume that the agent i has a sensor system to identify its own information state x_i, and receives output signals y_{ij} from the agent j through communication channel with constraints on signal amplitude which are described as follows:

$$y_{ij} = \begin{cases} x_j, & \left| x_j \right| \le b_{ij} \\ \phi, & \left| x_j \right| > b_{ij} \end{cases}, \quad i, j = 1, 2, \ldots, n, \tag{4.3}$$

where ϕ denotes the agent i receives no information from the agent j, and b_{ij} is the amplitude constraint parameter of the communication channel from the agent j to the agent i. Of course, those constraints are caused by physical conditions of communication channels. If x_j denotes the velocity of the motion of the agent j, the channel constraint (4.3) implies that the agent i cannot measure the velocity x_j faster than b_{ij}.

A consensus protocol to reach a consensus with respect to the states of n integrator agents (4.2) can be expressed as

$$u_i = -\sum\nolimits_{j \in N_i} a_{ij}\left(y_{ij} \right)\left(x_i - y_{ij} \right), \quad x_i(0) \in R, \tag{4.4}$$

where a_{ij} is the (i,j)th entry of the adjacency matrix of the associated communication graph, and N_i represents the set of agents whose information is available to agent i. Note a_{ij} may depend on y_{ij}.

Remark 4.2. Note that the communication channel constraint (4.3) is the condition of the 'absolute' states. In the research area on dynamic graphs (see, e.g., [37]), the consensus problem under the constraint $|x_i - x_j| \le b_{ij}$, which is the condition of the 'relative' states to represent the sensor range limit, is solved successfully, but does not cover our problem under the constraint (4.3) as a special case.

4.3 Two consensus protocols

In this section, we present two consensus protocols that solve consensus problems in a network of continuous-time integrator agents. The first linear consensus protocol is defined as follows:

$$u_i = -\sum_{j\in N_i} a_{ij}\left(x_i - \sigma_{ij}x_j\right),\ \left(a_{ij} > 0\right),\ x_i(0) \in R, \tag{4.5}$$

and the following is the second linear consensus protocol:

$$u_i = -\sum_{j\in N_i} \sigma_{ij}a_{ij}\left(x_i - x_j\right),\ \left(a_{ij} > 0\right),\ x_i(0) \in R, \tag{4.6}$$

where σ_{ij} is defined as follows:

$$\sigma_{ij} = \begin{cases} 1, & y_{ij} = x_j \left(\Leftrightarrow |x_j| \le b_{ij}\right) \\ 0, & y_{ij} = \phi\left(\Leftrightarrow |x_j| > b_{ij}\right) \end{cases},$$

where b_{ij} is the amplitude constraint parameter of the communication channel from the agent j to the agent i.

To make the difference between the protocol (4.5) and the protocol (4.6) clear, we present an example: Consider a case of three agents $I = \{1,2,3\}$ and focus on the control input of the agent 1 to show the difference between control structures given by the protocol

(4.5) and the protocol (4.6). If the agent 1 receives the information $y_{12} = x_2$ and $y_{13} = x_3$ (i.e., $|x_2| \le b_{12}$ and $|x_3| \le b_{13}$), the protocol (4.5) and the protocol (4.6) provides the same control $u_1 = -a_{12}(x_1 - x_2) - a_{13}(x_1 - x_3)$; if the agent 1 receives only the information $y_{12} = x_2$ (i.e., $|x_2| \le b_{12}$ and $|x_3| > b_{13}$), the protocol (4.5) provides the control input $u_1 = -a_{12}(x_1 - x_2) - a_{13}x_1$ and the protocol (4.6) provides the control input $u_1 = -a_{12}(x_1 - x_2)$; if the agent does not receives any information from other agents (i.e., $|x_2| > b_{12}$ and $|x_3| > b_{13}$), the protocol (4.5) provides $u_1 = -(a_{12} + a_{13})x_1$ and the protocol (4.6) provides $u_1 = 0$.

Remark 4.3. If for all i, j, $b_{ij} = \infty$, it is obvious that for all i, j, $\sigma_{ij} = 1$ and at this time the consensus protocol (4.4) falls into the form of the consensus protocol (3.7). Then it follows form Theorem 4.1 that the consensus protocol (4.4) can be asymptotically reached for all initial states.

By applying the protocol (4.5) or (4.6), we can rewrite (4.2) into

$$\dot{x}(t) = -L^{\sigma(t)}x(t), \tag{4.7}$$

where L^{σ} is defined as the following L_1^{σ} and L_2^{σ}, for the consensus protocol (4.5) and (4.6) respectively.

$$L_1^{\sigma} = \left[l_{ij}^{\sigma}\right] = \begin{bmatrix} \sum_j a_{1j} & -\sigma_{12}a_{12} & \cdots & -\sigma_{1n}a_{1n} \\ -\sigma_{21}a_{21} & \sum_j a_{2j} & \cdots & -\sigma_{2n}a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n1}a_{n1} & -\sigma_{n2}a_{n2} & \cdots & \sum_j a_{nj} \end{bmatrix},$$

$$L_2^\sigma = \left[l_{ij}^\sigma \right] = \begin{bmatrix} \sum_j \sigma_{1j} a_{1j} & -\sigma_{12} a_{12} & \cdots & -\sigma_{1n} a_{1n} \\ -\sigma_{21} a_{21} & \sum_j \sigma_{2j} a_{2j} & \cdots & -\sigma_{2n} a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n1} a_{n1} & -\sigma_{n2} a_{n2} & \cdots & \sum_j \sigma_{nj} a_{nj} \end{bmatrix},$$

where $a_{ij} > 0$ for all i, j.

Next are the main results that we present in this section.

Lemma 4.3. The Laplacian matrix is assumed to be L^σ,

$$L^\sigma = \left[l_{ij}^\sigma \right] = \begin{bmatrix} \sum_j a_{1j} & -\sigma_{12} a_{12} & \cdots & -\sigma_{1n} a_{1n} \\ -\sigma_{21} a_{21} & \sum_j a_{2j} & \cdots & -\sigma_{2n} a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n1} a_{n1} & -\sigma_{n2} a_{n2} & \cdots & \sum_j a_{nj} \end{bmatrix},$$

where $\sum_j a_{ij} > 0$ for all i. The Laplacian matrix L^σ has the following properties:

(i) If for all i, j, $\sigma_{ij} = 1$, then $L^\sigma \vec{1}_n = 0$;

(ii) If for all i, there exists j such that $\sigma_{ij} = 0$ and $a_{ij} > 0$, then all eigenvalues of $-L^\sigma$ have negative real parts.

Proof: (i) It is obvious from the definition of L^σ and Lemma 4.1.

(ii) Based on Lemma 4.2, all the eigenvalues of $L^\sigma = \left[l_{ij}^\sigma \right]$ are located in the union of the following disks:

$$D_i^\sigma = \left\{ z \in \mathrm{C} \,\middle|\, \left| z - l_{ii}^\sigma \right| \le \sum_{j=1,\ldots,n;\, j \ne i} \left| l_{ij}^\sigma \right| \right\},$$

$l_{ii}^\sigma = \sum_j a_{ij}$, $l_{ij}^\sigma = -\sigma_{ij} a_{ij}$. If for all i, there exists j such that $a_{ij} > 0$ and $\sigma_{ij} = 0$, then we will have $0 < l_{ii}^\sigma - \sum_{j=1,\ldots,n;\, j \ne i} \left| l_{ij}^\sigma \right| \le z \le l_{ii}^\sigma + \sum_{j=1,\ldots,n;\, j \ne i} \left| l_{ij}^\sigma \right|$. From this we know all eigenvalues of $-L^\sigma$ have negative real parts.

Theorem 4.3. Consider a network of integrators with a fixed topology $G = (V, E, A)$ that is a digraph and satisfies $\sum_j a_{ij} > 0$ for all i; constraint parameters $\left[b_{ij}\right]$ of the communication channels in the network are identical one such as $b_{ij} = b$ for all i, j. Then, along the trajectory $x(t)$, $t \geq 0$ of the system (4.7) given by the protocol (4.5) for any initial state $x(0)$ such that $|x_i(0)| > b$ for some i, $-L^{\sigma(t)}$ is stable at each time t in a finite time interval $[0, t_c)$.

Proof: Let $\{\sigma\}$ be the set of all $2^{n\times n}$ matrices $\sigma = \left[\sigma_{ij}\right] \in R^{n\times n}$ where $\sigma_{ij} \in \{0,1\}$. Denote by L_1^σ the matrix L^σ defined in Lemma 4.3, and by $\{\sigma\}_{S1}$ the subset of $\{\sigma\}$ whose element σ makes $-L_1^\sigma$ stable. Let $x(t)$, $t \geq 0$ be a trajectory of the system (4.7) with the protocol (4.5) for any fixed initial state $x(0)$, and $t_c \geq 0$ be the first time when $|x_j(t_c)| \leq b_{ij} = b$, that is, $\sigma_{ij}(t_c) = 1$ holds for all i, j. As shown later, t_c is finite for any initial state $x(0)$.

We first show that $\sigma(t) \in \{\sigma\}_{S1}$ for each t in $[0, t_c)$ along the trajectory $x(t)$, $t \geq 0$. From the definition of t_c, we have $\sigma(t) \in \{\sigma\} / \{\sigma_{ij} = 1, \forall\, i, j\}$ on $[0, t_c)$. At any fixed time t in $[0, t_c)$, if for any i there exists j such that $\sigma_{ij}(t) = 0$ and $a_{ij} > 0$, then $\sigma(t) \in \{\sigma\}_{S1}$ follows from (ii) of Lemma 4.3. Contrary, if there exists i such that $\sigma_{ij}(t) = 1$ for all j $(j \neq i)$ (or $a_{ij} = 0$ for all j $(j \neq i)$, which are not possible because of the assumption: $\sum_j a_{ij} > 0$ for all i), we see $|x_j(t)| \leq b_{ij} = b$ for all j $(j \neq i)$; then, we are lead to the following two alternatives:

(i) $|x_i(t)| \leq b_{ji} = b$, i.e., $\sigma_{ji}(t) = 1$ for all j $(j \neq i)$;

(ii) $|x_i(t)| > b_{ji} = b$, i.e., $\sigma_{ji}(t) = 0$ for all j $(j \neq i)$;

The case (i), however, does not occur, since, in the case (i), it follows that $\sigma_{ij}(t)=1$ for all i,j, but this contradicts the fact that the time t is in $[0,t_c)$. Thus, the case (ii) is possible, and in the case (ii), $-L_1^\sigma$ is a stable matrix, that is, $\sigma(t)\in\{\sigma\}_{S1}$, which is shown by observing that , if we take $i=1$ for example, L_1^σ is expressed as

$$L_1^\sigma = \begin{bmatrix} \sum_j a_{1j} & -a_{12} & \cdots & -a_{1n} \\ 0 & & & \\ \vdots & & \left(L_1^\sigma\right)_{n-1} & \\ 0 & & & \end{bmatrix},$$

where the sub-matrix $\left(L_1^\sigma\right)_{n-1}$ with $(n-1)\times(n-1)$ dimension is given by

$$\left(L_1^\sigma\right)_{n-1} = \begin{bmatrix} \sum_j a_{2j} & -\sigma_{23}a_{23} & \cdots & -\sigma_{2n}a_{2n} \\ -\sigma_{32}a_{32} & \sum_j a_{3j} & \cdots & -\sigma_{3n}a_{3n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n2}a_{n2} & -\sigma_{n3}a_{n3} & \cdots & \sum_j a_{nj} \end{bmatrix},$$

and using the assumption $\sum_j a_{1j}>0$ and the fact that $-\left(L_1^\sigma\right)_{n-1}$ is a stable matrix. The stability of $-\left(L_1^\sigma\right)_{n-1}$ is shown just by modifying the proof for (ii) of Lemma 4.3. To sum up the above arguments, at each moment $t\in[0,t_c)$, $\sigma(t)\in\{\sigma\}_{S1}$ holds, that is, $-L_1^\sigma$ is always a stable matrix along the trajectory of the system (4.7) with the protocol (4.5) and the given initial state.

Lemma 4.4. The Laplacian matrix is assumed to be L_2^σ. The Laplacian matrix L_2^σ has the following properties:

(i) If for all i,j, $\sigma_{ij}=1$, then $L_2^\sigma \bar{1}_n = 0$;

(ii) If there exists j, such that for all i, $\sigma_{ij}=1$, then $-L_2^\sigma$ has a simple zero eigenvalue and all non-zero eigenvalues of $-L_2^{\sigma}$ have negative real parts.

Proof: (i) It is obvious from Lemma 4.1.

(ii) If there exists j, such that for all i, $\sigma_{ij}=1$, then it follows from $a_{ij}>0$ for all i,j that, for the digraph G^{σ} with L_2^{σ} as its Laplacian, there should be always an edge from the node $i\left(i\neq j\right)$ to the node j. Then we can get a directed spanning tree with the root node j. According to Lemma 4.1, $-L_2^{\sigma}$ has a simple zero eigenvalue and all non-zero eigenvalues of $-L_2^{\sigma}$ have negative real parts.

Now we can state the main results: Theorem 4.4 for the protocol (4.5) and Theorem 4.5 for the protocol (4.6).

Theorem 4.4. Consider a network of integrators with a fixed topology $G=\left(V,E,A\right)$ that is a complete digraph and satisfies $\sum_j a_{ij}>0$ for all i; constraint parameters $\left[b_{ij}\right]$ of the communication channels in the network are identical one such as $b_{ij}=b$ for all i,j. Then,

(i) There exists a time $t_c>0$, such that $\sigma_{ij}\left(t_c\right)=1, i,j=1,2,\ldots,n$, and the system (4.7) given by the protocol (4.5) can solve the global consensus problem asymptotically.

(ii) The protocol (4.5) globally asymptotically achieves the following average-consensus: $x\left(t\right)\rightarrow\frac{1}{n}\sum_i x_i\left(t_c\right)$ as $t\rightarrow\infty$, if G is balanced.

Proof: Let the set I_1 denote the subset of the cooperative agents set I whose elements' initial states are less than or equal to the identical communication channel constraint b. The set I_2 is the complementary set of the set I_1 in I, that is, the elements' initial states of the set I_2 are greater than the identical communication channel constraint b. It is known that if the agent j belongs to I_1, there exists c such that $\left|x_j\right|\leq c<b$ and $\sigma_{ij}=1$ for all i.

We suppose that the agent i belongs to I_2, i.e., $\left|x_i\right|>b_{ji}=b$ and $\sigma_{ji}=0$ for all j.

$$
\begin{aligned}
\dot{x}_i &= -\sum_{j\in N_i} a_{ij}\left(x_i - \sigma_{ij} x_j\right) \\
&= -\sum_{j\in N_i} a_{ij} x_i + \sum_{j\in I_1} a_{ij}\cdot\sigma_{ij}\cdot x_j + \sum_{j\in I_2} a_{ij}\cdot\sigma_{ij}\cdot x_j \qquad (\sigma_{ij}=1 \text{ for } j\in I_1) \\
&= -\sum_{j\in N_i} a_{ij} x_i + \sum_{j\in I_1} a_{ij}\cdot 1\cdot x_j + \sum_{j\in I_2} a_{ij}\cdot 0\cdot x_j \qquad (\sigma_{ij}=0 \text{ for } j\in I_2) \\
&\le -\sum_{j\in N_i} a_{ij} x_i + \sum_{j\in I_1} a_{ij}\cdot 1\cdot c + \sum_{j\in I_2} a_{ij}\cdot 1\cdot c \qquad (a_{ij}>0 \text{ and } c>0) \\
&= -\sum_{j\in N_i} a_{ij}\left(x_i - c\right) \ .
\end{aligned}
$$

Then we also have

$$\frac{d}{dt}\left(x_i - c\right) \le -\sum_{j\in N_i} a_{ij}\left(x_i - c\right).$$

By the Gronwall's inequality, we have

$$x_i(t) - c \le \left(x_i(t_0) - c\right) e^{-\sum_{j\in N_i} a_{ij}(t-t_0)},$$

$$x_i(t) \le c + \left(x_i(t_0) - c\right) e^{-\sum_{j\in N_i} a_{ij}(t-t_0)}. \tag{4.8}$$

The above evaluation of (4.8) assures the existence of a finite time t_1 ($t_1 \ge t_0$), such that $\left|x_i(t_1)\right| = b$. Since then, we know $i \in I_2$ and $x_i(t_1)$ ($\left|x_i(t_1)\right| = b$) becomes a new initial state of the agent i. Further, using again the evaluation of (4.8), we have that at the time $t_1+\varepsilon$ (ε is a small enough positive number), $\left|x_i(t_1+\varepsilon)\right| < b$ holds, because on the closed interval $\left[t_0, t_1+\varepsilon\right]$, there is no agent moving from I_1 to I_2 in accordance with Remark 4.1.

Similarly, for other elements in I_2, there also exists $t_2, t_3, \cdots$, such that all other elements in I_2 fall into I_1. Eventually, there must exists a time t_c such that for any time $t \ge t_c$, all agents belong to I_1 and $\sigma_{ij}(t_c) = 1$ for all i, j. At that time, L_1^{σ} turns into L as in the usual case, that is, the system (4.7) is reduced to the system (3.8); at this time, $x(t_c)$ as a new initial state and the protocol (4.5) globally asymptotically solves the consensus problem according to Theorem 4.1. Thus, the proof of (i) is obtained. Because the directed complete graph G is a strongly connected and balanced graph, the proof of (ii) can be obviously obtained from Theorem 4.1.

Remark 4.4. Clearly $\left|\frac{1}{n}\sum_i x_i\left(t_c\right)\right| \le b$. If we generalize the signal amplitude constraint $\left|x_i\left(t\right)\right| \le b$ to $\left|x_i\left(t\right)-x^*\right| \le b$ for $x^* \in R^n$, then we have $\left|\frac{1}{n}\sum_i x_i\left(t_c\right)-x^*\right| \le b$. Thus, b and x^* can be used as design parameters for specifying the consensus value.

Theorem 4.5. Consider a network of integrators with a fixed topology $G=\left(V,E,A\right)$ that is a complete and bi-directed graph; constraint parameters $\left[b_{ij}\right]$ of communication channels in the network are specified as $b_{ij}=b_j$ for all i. Then, the system (4.7) given by the consensus protocol (4.6) reaches consensus asymptotically for any initial state $x(0)$ that has at least one element, say $x_j(0)$, satisfying $\left|x_j(0)\right| \le b_j$.

Proof: Let $\{\sigma\}$ be the set of all $2^{n\times n}$ matrices $\sigma=\left[\sigma_{ij}\right]\in R^{n\times n}$ where $\sigma_{ij}\in\{0,1\}$. Denote by $\{\sigma\}_{S2}$ the subset of $\{\sigma\}$ whose element σ makes $-L_2^\sigma$ have a simple zero eigenvalue and negative real-part eigenvalues. Let $x(t)$, $t \ge 0$ be a trajectory of the system (4.7) with the protocol (4.6) and any given initial state $x(0)$ satisfying $\left|x_j(0)\right| \le b_j$. We first show that $\sigma(t)\in\{\sigma\}_{S2}$ for each t along the trajectory (Step 1), and prove that the system (4.7) with this switching parameter $\sigma(t)$ achieves asymptotic consensus (Step 2). In the following, we use a simplified notation $\sigma_{ij}=\sigma_j$ for all i, which is justified by the assumption $b_{ij}=b_j$ for all i.

(Step 1) First note that $\left|x_j(t)\right| \le b_j$ implies $\sigma_j(t)=1$ at each time $t \ge 0$. Hence, $\sigma(0)\in\{\sigma\}_{S2}$ follows from the assumption $\left|x_j(0)\right| \le b_j$ and Lemma 4.4, and further it follows from Lemma 4.4 that $\sigma(t)\in\{\sigma\}_{S2}$ for any time $t>0$ as long as $\left|x_j(t)\right| \le b_j$ holds. If there exists a time $t_a \ge 0$ and a small number $\bar{\varepsilon}>0$ such that $\left|x_j(t_a)\right| = b_j$ and

$\left|x_j(t_a+\varepsilon)\right|>b_j$ for all ε in $(0,\bar{\varepsilon}]$, where

$$x_j(t_a+\varepsilon)-x_j(t_a)=-\int_{t_a}^{t_a+\varepsilon}\sum_k \sigma_k(\tau)a_{jk}[x_j(\tau)-x_k(\tau)]d\tau \tag{4.9}$$

then the identity (4.9) enables us to find a subscript k where $k\neq j$ and a small number $\bar{\eta}$ $(0<\bar{\eta}\le\varepsilon)$ such that $\sigma_k(t_a+\eta)=1$ for all η in $(0,\bar{\eta}]$; thus, in this case, Lemma 4.4 again assures that $\sigma(t_a+\eta)\in\{\sigma\}_{S2}$ for all η in $(0,\bar{\eta}]$. (Contrary, if $\left|x_j(t_a+\varepsilon)\right|\le b_j$ for all $\varepsilon>0$, it is a matter of course that $\sigma(t_a+\varepsilon)\in\{\sigma\}_{S2}$ for all $\varepsilon>0$.) Repeating the above argument, we can conclude that $\sigma(t)\in\{\sigma\}_{S2}$ for all $t\ge 0$ along all the trajectory satisfying the assumption on the initial state $\left|x_j(0)\right|\le b_j$.

(Step 2) From Step 1, for the system (4.7) given by the consensus protocol (4.6), we know that the Laplacian $L_2^{\sigma(t)}$ is Metzler with zero row sums and the digraph with $L_2^{\sigma(t)}$ as its Laplacian always has a directed spanning tree.

From what is stated above, the conditions of Theorem 4.2 are satisfied and then following from this theorem the global consensus can be reached asymptotically.

Remark 4.5. In Theorem 4.5, assume that σ_{ij} takes 0 or c (instead of 1), $i,j=1,2,\ldots,n$, the consensus can be more quickly reached asymptotically if $c>1$ (see Example 4.3), or be more slowly reached asymptotically if $0<c<1$.

4.4 Examples and simulation results

This section presents some illustrative examples to describe the theoretical results in this chapter. The following directed graphs with different weights are needed in the analysis of this section.

Example 4.1. Figure 4.1 is a complete digraph with order $n=3$. $I=\{1,2,3\}$ represents cooperative agents at the nodes. By simulation studying, we investigate the consensus convergence character of the multi-agent systems and verify the proposed Theorem 4.4 in this example.

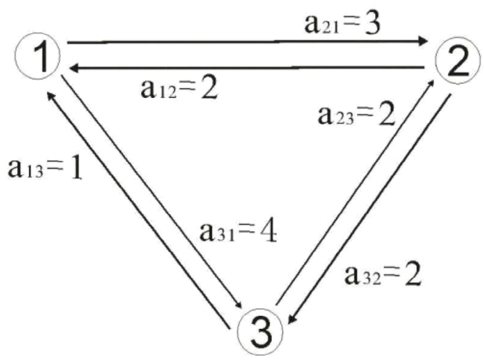

Fig. 4.1. The communication topology of three agents.

We suppose the initial states of three agents are $x(0)=(3.6,1.6,2)^T$. The identical constraint of communication channel is $\left[b_{ij}=b=2\right]$. Based on Theorem 4.4, the consensus can be reached with $x(t_c)$ as a new initial state $(t_c=0.317)$ and the simulation of these three agents is in Fig. 4.2. The usual consensus $\left[b_{ij}=b=\infty\right]$ is like in Fig. 4.3.

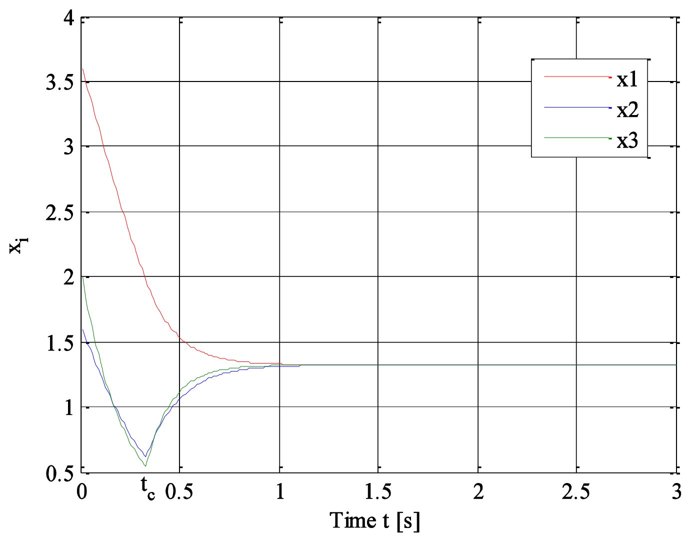

Fig.4.2. The consensus of three agents based on Theorem 4.4.

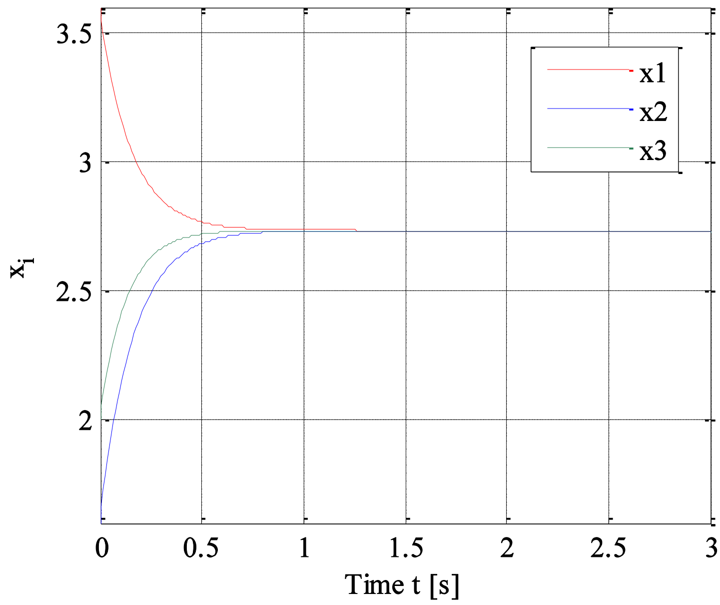

Fig. 4.3. The usual consensus of these three agents.

Example 4.2. We consider the system which is the same as that stated in Example 4.1 except for having a balanced complete digraph with $a_{ij} = 2$ for all $i, j = 1,2,3\,(i \neq j)$. By simulation study, we investigate the generalized average-consensus convergence character of the multi-agent systems and verify the second statement proposed in Theorem 4.4. By Theorem 4.1, a usual average consensus is reached as shown in Fig. 4.4. By Theorem 4.4, the consensus can be reached with $x(t_c)$ as a new initial state $(t_c = 0.294)$ and the simulation of these three agents is shown in Fig. 4.5. Furthermore, we notice that $x(t_c) = (2, 0.966, 1.034)^T$ and the consensus value is equal to $\frac{1}{3}\sum_{i=1}^{3} x_i(t_c) = 1.3333$, which is what we call the generalized average consensus.

Example 4.3. We suppose the initial states of three agents are $x(0) = (3.6, 2.4, 1.6)^T$ in Fig. 4.1. The identical constraint of communication channel is $[b_{ij} = b = 2]$. Based on Theorem 4.5, the consensus can be reached with $x(t_a)$ as a new initial state $(t_a = 0.9722)$ and the simulation of these three agents is shown in Fig. 4.6. The usual consensus is like in Fig. 4.7.

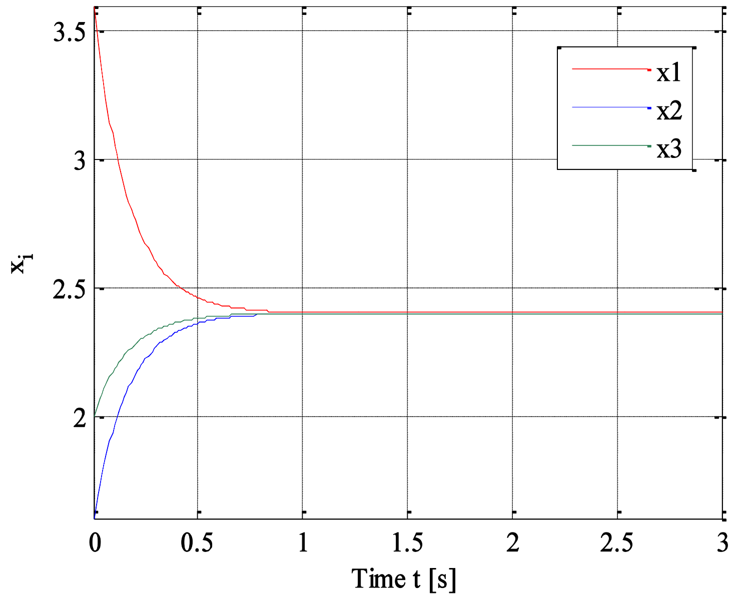

Fig. 4.4. The usual average consensus of three agents.

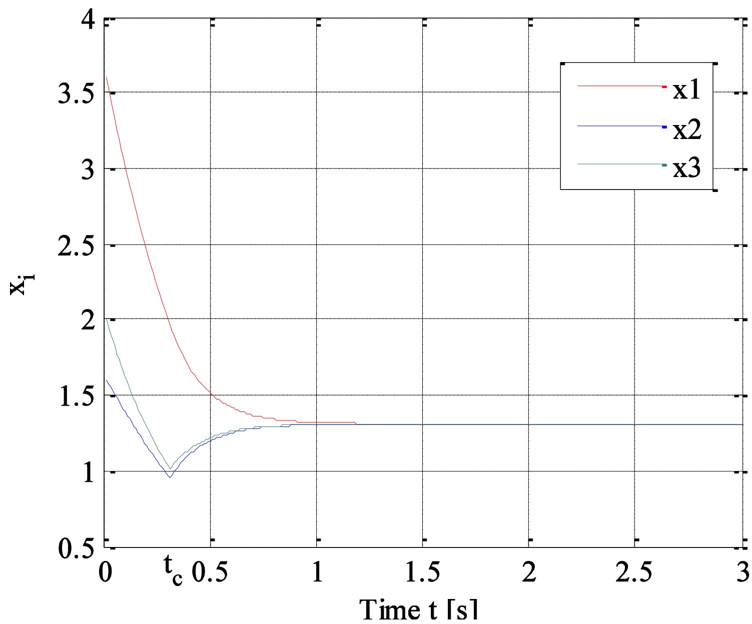

Fig. 4.5. The generalized average consensus of three agents.

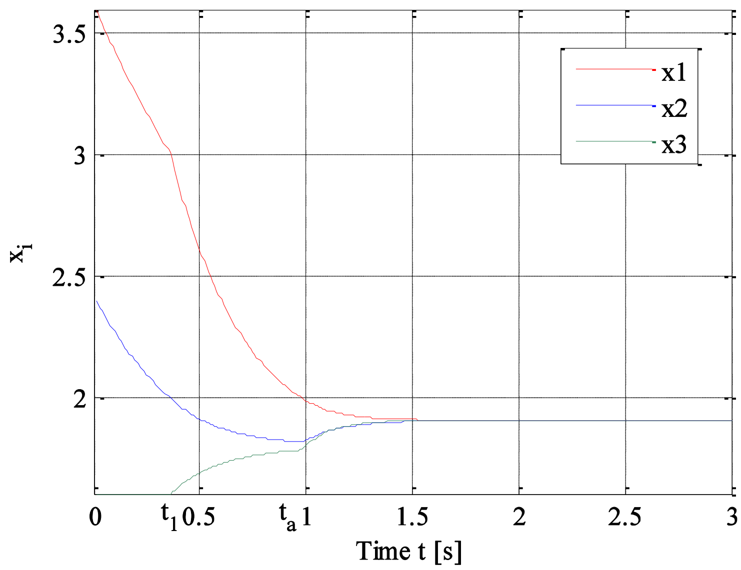

Fig. 4.6. The consensus of three agents based on Theorem 4.5.

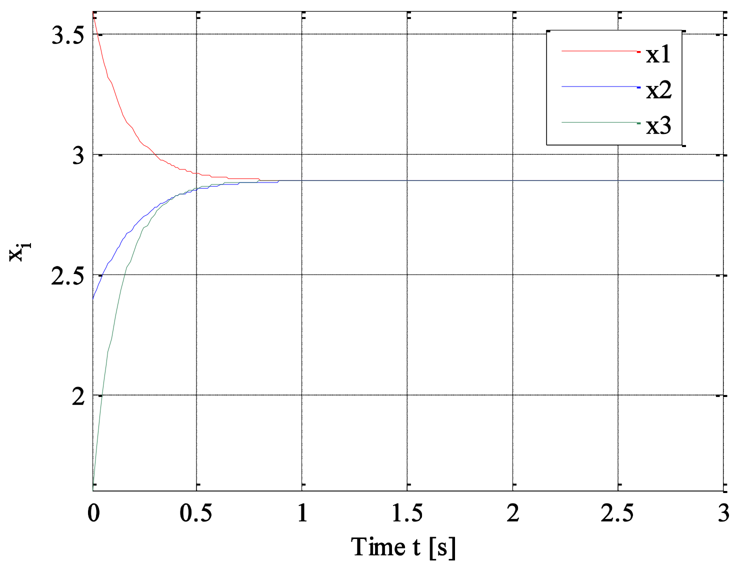

Fig. 4.7. The usual consensus of three agents in Example 4.3.

Example 4.4. In Example 4.3, we took $\sigma_{ij} = 0 \text{ or } 1$ and here we assume $\sigma_{ij} = 0 \text{ or } 5$ as stated in Remark 4.5 and by Theorem 4.5, the consensus can be reached with $x(t_a)$ as a new initial state $(t_a = 0.1944)$ and the simulation of these three agents is shown in Fig. 4.8. It can be seen from Fig. 4.8 and Fig. 4.6 that the consensus can be more quickly reached asymptotically in Fig. 4.8. Furthermore, we notice that the time t_a in Fig. 4.6 is five times as great as another one in Fig. 4.8.

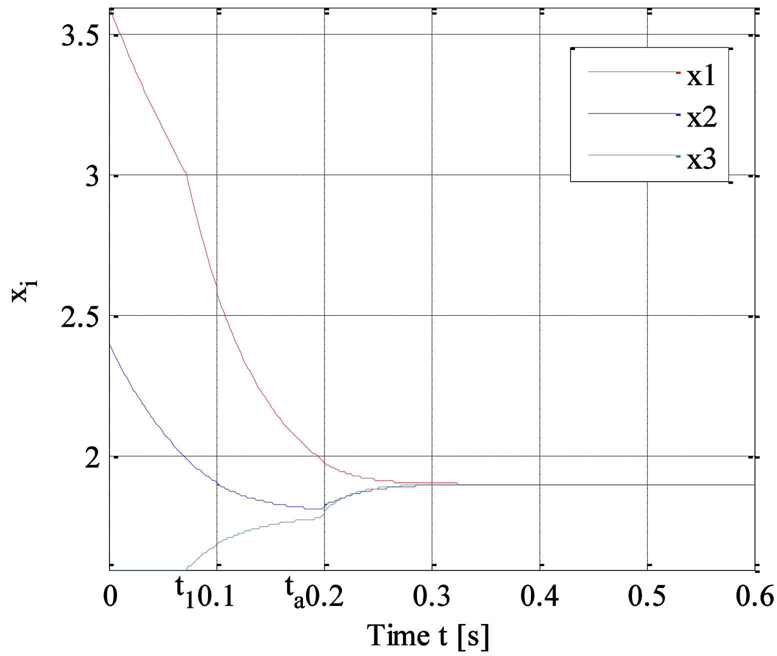

Fig. 4.8. The consensus of three agents based on Remark 4.5.

Example 4.5. In this example, we consider the case that the multi-agent systems have different communication channel constraints for different agents. For example, the constraint of communication channel is $b_{21} = b_{31} = 3$ and $b_{12} = b_{32} = b_{13} = b_{23} = 1.5$. The initial state of the multi-agent systems is $x(0) = (2.6, 2, 1.7)^T$. The topological structure of the multi-agent systems is the same as Example 4.1. Finally, based on Theorem 4.5, the consensus can be reached and the simulation of these three agents is in Fig. 4.9. Note the trajectory of the first agent x_1 coincides with the top line, and the value of x_1 is equal to the consensus value.

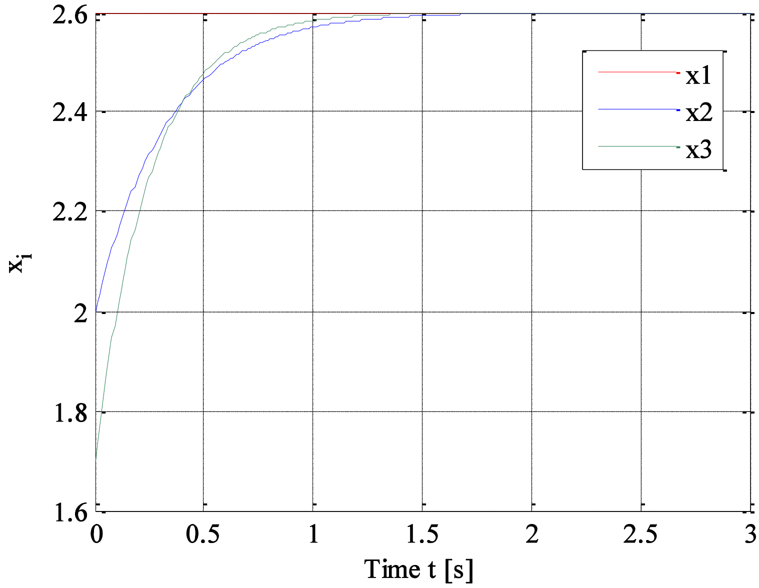

Fig. 4.9. The consensus of three agents in Example 4.5.

4.5. Chapter summary

In this chapter, the consensus problem in the multi-agent systems with the communication channel constraint has been investigated by providing special Laplacians representing the topological structure of the multi-agent systems. We introduce the two types of the protocols using state-dependent switching parameters. Our work shows that those two protocols obtain the global consensus as long as some conditions on the graph topology and the channel constraint are satisfied. Examples have been proposed to illustrate the effectiveness of the methods.

5 Interval consensus problem of multi-agent systems in accordance with switching protocol

5.1 Introduction

Multi-agent systems have recently received increased attention due to their broad applications in various fields, including computer science [1-2]; vehicle systems, unmanned aerial vehicles, and vehicle formations [33, 36, 38, 112-114]; flocking and tracking [39, 115, 116, 117]; and others [13, 28, 29, 30, 31, 32, 40, 41, 103-109, 118, 120, 121, 123-130].

As one of the most important issues in the coordinated control of multi-agent systems, the consensus problem requires that the output of several spatially distributed agents reach a common value which depends on the state of all agents. A fixed consensus value is obtained from a given consensus protocol and initial state. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent system. It is therefore necessary and significant to investigate whether we can design a protocol to change the consensus value of the multi-agent system, and the answer to this question will allow application of multi-agent systems in new fields. Moreover, it seems to be generally complicated and difficult to design an appropriate protocol such that multi-agent systems can converge to any designated point.

To solve such a protocol design problem we pose a new class of consensus problems, called interval consensus problem, and search for a protocol ensuring that the system converges to a point on a specified closed and bounded interval. By introducing two state-dependent switching parameters into the consensus protocol, which is motivated by the results of chapter 4, the system given by the proposed protocol can globally asymptotically converge to a designated point on a special closed and bounded interval. In other words, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average consensus if the directed graph is balanced. Simulations are presented to demonstrate the effectiveness of our theoretical results. It is worth mentioning that the two parameters introduced into the consensus protocol play an important role in our discussion. One role is to change the consensus value which is ideal or meets the quality that we require from the multi-agent system, and the other one is to change the time and speed of convergence of consensus protocols.

The Laplacian introduced in this chapter defines a graph with specific network topologies that may change as the agent states proceed. From the state dependence of network topology, we can say that the graph is a special type of general “dynamic graph.” However, to the best of the authors’ knowledge, the proposed type of dynamic graph has not been discussed so far; in the literature on dynamic graphs (see, e.g., Mesbahi and Egerstedt [37], and the references therein), most research considers the case where network topology depends on relative states, whereas the network topologies in this chapter depend on absolute states coming from the control parameter of the consensus value on the absolute value of the transmitted state signal.

5.2 The consensus protocol

The information states with single-integrator dynamics are given by

$$\dot{x}_i = u_i, \quad x_i(0) \in R, \quad i = 1, 2, \ldots, n, \tag{5.1}$$

where $x_i \in R$ denotes the information state of the ith agent and $u_i \in R$ is the information control input of the ith agent.

Let d in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$ be the control parameter of the consensus value of the multi-agent system that prompts the system to converge to the desired consensus value on the closed and bounded interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$. By introducing into the consensus protocol two state-dependent switching parameters σ_{ij} and δ_{ij}, which are defined by d, the multi-agent system given by the proposed protocol can globally asymptotically converge to a point on the closed and bounded interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$, which is what we call the interval consensus problem of multi-agent systems in accordance with the switching protocol. Denote by d^* in the interval

$\left[\min_i x_i(0), \max_i x_i(0)\right]$ the parameter that makes the multi-agent system converge to a point x^* in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$ correspondingly. The most important point we discuss in this chapter is that we can make the system converge to a point x^* in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$ as long as we take d^*, the control parameter of the consensus value, in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$.

Let x_{mid} be the usual consensus value of the multi-agent system without the control parameter of the consensus value or the usual average consensus value without the control parameter of the consensus value, if the digraph G is balanced. Next we will introduce the state-dependent switching parameters σ_{ij} into the first consensus protocol in the interval $\left[\min_i x_i(0), x_{mid}\right]$ and the state-dependent switching parameters δ_{ij} into the second consensus protocol in the interval $\left[x_{mid}, \max_i x_i(0)\right]$, respectively.

5.2.1 The first consensus protocol

We present the first consensus protocol that solves the interval consensus problem of the multi-agent system in the interval $\left[\min_i x_i(0), x_{mid}\right]$ as follows:

$$u_i = -\sum\nolimits_{j \in N_i} \sigma_{ij} a_{ij} \left(x_i - x_j\right), \ \left(a_{ij} > 0\right), \ x_i(0) \in R. \tag{5.2}$$

Here, a_{ij} is the (i, j)th entry of the adjacency matrix of the associated communication graph, and N_i is the set of agents whose information is available to the agent i, and σ_{ij} is defined as

$$\sigma_{ij} = \begin{cases} 1 & x_j \le d \\ 0 & x_j > d \end{cases}, \tag{5.3}$$

where x_j denotes the information state of the jth agent, and d is the control parameter of the consensus value of the multi-agent system and d is in the interval $\left[\min_i x_i(0), x_{mid}\right]$, whose role is to prompt the multi-agent system to converge to the desired consensus value in the interval $\left[\min_i x_i(0), x_{mid}\right]$.

By applying the protocol (5.2), we can rewrite (3.8) as

$$\dot{x}(t) = -L^{\sigma} x(t), \tag{5.4}$$

where L^{σ} is defined as L_2^{σ} in Lemma 4.4 as follows,

$$L^{\sigma} = \left[l_{ij}^{\sigma}\right] = \begin{bmatrix} \sum_j \sigma_{1j} a_{1j} & -\sigma_{12} a_{12} & \cdots & -\sigma_{1n} a_{1n} \\ -\sigma_{21} a_{21} & \sum_j \sigma_{2j} a_{2j} & \cdots & -\sigma_{2n} a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n1} a_{n1} & -\sigma_{n2} a_{n2} & \cdots & \sum_j \sigma_{nj} a_{nj} \end{bmatrix}.$$

Remark 5.1. If $d = \max_i x_i(0)$, it is obvious that $\sigma_{ij} = 1$ for all i, j, and the consensus protocol (5.2) falls into the form of the consensus protocol (3.7). Then it follows from Theorem 4.1 that the consensus protocol (5.2) can be asymptotically reached for all initial states.

Next is Theorem 5.1 for the protocol (5.2) discussed in chapter 4. From a different perspective, it is an essential theoretical foundation related to the problem we are addressing in this chapter.

Theorem 5.1. Consider a network of integrators with a fixed topology $G = (V, E)$ that is a complete digraph; the control parameter of the consensus value is d. Then,

(i) There exists a time $t_c > 0$, such that $\sigma_{ij}(t_c) = 1$, $i, j = 1, 2, \ldots, n$, and the system (5.4) given by the consensus protocol (5.2) reaches consensus asymptotically for any initial state $x(0)$ that has at least one element, say $x_j(0)$, satisfying $x_j(0) \le d$; and

(ii) The protocol (5.2) globally asymptotically achieves the following average consensus: $x(t) \to \frac{1}{n} \sum_i x_i(t_c)$ as $t \to \infty$, if the digraph G is balanced.

Proof: See the proofs of the Theorem 4.4 and Theorem 4.5 in chapter 4.

As seen from Theorem 5.1, the multi-agent system can converge to $\min_i x_i(0) = \min\{x_1(0), \ldots, x_n(0)\}$ if the control parameter of the consensus value is taken as $d = \min\{x_1(0), \ldots, x_n(0)\}$, which we call the min-consensus. In other words, consensus protocol (5.2) can asymptotically solve the min-consensus problem.

It can be seen from Theorem 5.1 that the consensus value of the multi-agent system varies with the control parameter of the consensus value d. In more specific terms, the consensus value of the multi-agent system decreases with the decrease of the control parameter of the consensus value d.

5.2.2 The second consensus protocol

Next we present the second consensus protocol that solves the interval consensus problem of the multi-agent system in the interval $[x_{mid}, \max_i x_i(0)]$ as

$$u_i = -\sum_{j \in N_i} \delta_{ij} a_{ij} (x_i - x_j), \ (a_{ij} > 0), \ x_i(0) \in R. \tag{5.5}$$

Here, a_{ij} is the (i, j)th entry of the adjacency matrix of the associated communication graph, N_i represents the set of agents whose information is available to agent i, and δ_{ij} is defined as

$$\delta_{ij} = \begin{cases} 1 & x_j \geq d \\ 0 & x_j < d \end{cases}, \tag{5.6}$$

where x_j denotes the information state of the jth agent, and d is the control parameter of the consensus value of the multi-agent system and d is in the interval $[x_{mid}, \max_i x_i(0)]$, whose role is to prompt the multi-agent system to converge to the desired consensus value in the interval $[x_{mid}, \max_i x_i(0)]$.

By applying the protocol (5.5), we can rewrite (3.8) as

$$\dot{x}(t) = -L^{\delta(t)} x(t), \tag{5.7}$$

where L^{δ} is similar to L_2^{σ} in Lemma 4.4 in definition as follows and of course has the same properties as L_2^{σ},

$$L^{\delta}=\left[l_{ij}^{\delta}\right]=\begin{bmatrix}\sum_j \delta_{1j}a_{1j} & -\delta_{12}a_{12} & \cdots & -\delta_{1n}a_{1n} \\ -\delta_{21}a_{21} & \sum_j \delta_{2j}a_{2j} & \cdots & -\delta_{2n}a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\delta_{n1}a_{n1} & -\delta_{n2}a_{n2} & \cdots & \sum_j \delta_{nj}a_{nj}\end{bmatrix}.$$

Remark 5.2. If $d=\min_i x_i(0)$, it is obvious that $\delta_{ij}=1$ for all i,j, and the consensus protocol (5.5) falls into the form of the consensus protocol (3.7). Then it follows from Theorem 4.1 that the consensus protocol (5.5) can be asymptotically reached for all initial states.

Theorem 5.2. Consider a network of integrators with a fixed topology $G=(V,E)$ that is a complete digraph; the control parameter of the consensus value is d. Then,

(i) There exists a time $t_a>0$, such that $\delta_{ij}(t_a)=1$, $i,j=1,2,\ldots,n$, and the system (5.7) given by the consensus protocol (5.5) reaches consensus asymptotically for any initial state $x(0)$ that has at least one element, say $x_j(0)$, satisfying $x_j(0)\geq d$; and

(ii) The protocol (5.5) globally asymptotically achieves the following average consensus: $x(t)\rightarrow\frac{1}{n}\sum_i x_i(t_a)$, as $t\rightarrow\infty$, if the digraph G is balanced.

Proof: Let $\{\delta\}$ be the set of all $2^{n\times n}$ matrices $\delta=\left[\delta_{ij}\right]\in R^{n\times n}$ where $\delta_{ij}\in\{0,1\}$. Denote by $\{\delta\}_S$ the subset of $\{\delta\}$ whose element δ makes $-L^{\delta}$ have a simple zero eigenvalue and negative real-part eigenvalues. Let $x(t)$, $t\geq 0$ be a trajectory of the system (5.7) with the protocol (5.5) and any given initial state $x(0)$ satisfying $x_j(0)\geq d$. We first show that $\delta(t)\in\{\delta\}_S$ for each t along the trajectory (Step 1), and prove that the system (5.7) with this switching parameter $\delta(t)$ achieves asymptotic consensus (Step 2). In the following, we use a simplified notation $\delta_{ij}=\delta_j$ for all i, which is justified by d for all i.

(Step 1) First note that $x_j(t) \ge d$ implies $\delta_j(t) = 1$ at each time $t \ge 0$. Hence, $\delta(0) \in \{\delta\}_S$ follows from the assumption $x_j(0) \ge d$ and Lemma 4.4, and further it follows from Lemma 4.4 that $\delta(t) \in \{\delta\}_S$ for any time $t > 0$ as long as $x_j(t) \ge d$ holds. If there exists a time $t_a \ge 0$ and a small number $\bar{\varepsilon} > 0$ such that $x_j(t_a) = d$ and $x_j(t_a + \varepsilon) < d$ for all ε in $(0, \bar{\varepsilon}]$, where

$$x_j(t_a + \varepsilon) - x_j(t_a) = -\int_{t_a}^{t_a+\varepsilon} \sum_k \delta_k(\tau) a_{jk} [x_j(\tau) - x_k(\tau)] d\tau , \tag{5.8}$$

then identity (5.8) enables us to find a subscript k where $k \ne j$ and a small number $\bar{\eta}$ $\left(0 < \bar{\eta} \le \varepsilon\right)$ such that $\delta_k(t_a + \eta) = 1$ for all η in $(0, \bar{\eta}]$. Thus, in this case, Lemma 4.4 assures that $\delta(t_a + \eta) \in \{\delta\}_S$ for all η in $(0, \bar{\eta}]$. (Contrarily, if $x_j(t_a + \varepsilon) \ge d$ for all $\varepsilon > 0$, then $\delta(t_a + \varepsilon) \in \{\delta\}_S$ for all $\varepsilon > 0$). Repeating the above argument, we can conclude that $\delta(t) \in \{\delta\}_S$ for all $t \ge 0$ along all the trajectory satisfying the assumption on the initial state $x_j(0) \ge d$.

(Step 2) From Step 1, for the system (5.7) given by the consensus protocol (5.5), we know that the Laplacian $L^{\delta(t)}$ is Metzler with zero row sums and the digraph with $L^{\delta(t)}$ as its Laplacian always has a directed spanning tree.

(Step 3) Next we prove that there exists a time $t_a > 0$, such that $\delta_{ij}\left(t_a\right) = 1$, $i, j = 1, 2, \ldots, n$. If $x_j(0) = d$ for $j = 1, 2, \ldots, n$, then $\delta_{ij} = 1$, $i, j = 1, 2, \ldots, n$. It is obvious that there exists a time $t_a > 0$, such that $\delta_{ij}\left(t_a\right) = 1$, $i, j = 1, 2, \ldots, n$. In the following, we assume that all of the initial states of the agents are not equal to d.

Let the set I_1 denote the subset of the cooperative agents set I whose elements' initial states are larger than or equal to d. The set I_2 is the complementary set of the set I_1 in I, that is, the elements' initial states of the set I_2 are less than d. It is known that if the agent

j belongs to I_1, there exists c such that $x_j \geq c > d$ and $\delta_{ij} = 1$ for all i.

We suppose that the agent i belongs to I_2, i.e., $x_i < d$ and $\delta_{ji} = 0$ for all j.

$$\dot{x}_i = -\sum_{j\in N_i} a_{ij}\delta_{ij}\left(x_i - x_j\right)$$

$$= -\sum_{j\in N_i} a_{ij}\delta_{ij}x_i + \sum_{j\in I_1} a_{ij}\cdot\delta_{ij}\cdot x_j + \sum_{j\in I_2} a_{ij}\cdot\delta_{ij}\cdot x_j \qquad (\delta_{ij} = 1 \text{ for } j \in I_1)$$

$$\geq -\sum_{j\in I_1} a_{ij}x_i + \sum_{j\in I_1} a_{ij}x_j \qquad (\delta_{ij} = 0 \text{ for } j \in I_2)$$

$$= -\sum_{j\in I_1} a_{ij}\left(x_i - x_j\right) \qquad (a_{ij} > 0 \text{ and } x_j \geq c > d)$$

$$\geq -\sum_{j\in I_1} a_{ij}\left(x_i - c\right).$$

Then we also have

$$\frac{d}{dt}\left(c - x_i\right) \leq -\sum_{j\in I_1} a_{ij}\left(c - x_i\right).$$

By the Gronwall's inequality, we have

$$c - x_i\left(t\right) \leq \left(c - x_i\left(t_0\right)\right)e^{-\sum_{j\in I_1} a_{ij}\left(t - t_0\right)},$$

$$x_i\left(t\right) \geq c - \left(c - x_i\left(t_0\right)\right)e^{-\sum_{j\in N_i} a_{ij}\left(t - t_0\right)}. \tag{5.9}$$

The above evaluation of (5.9) assures the existence of a finite time t_1 ($t_1 \geq t_0$), such that $x_i\left(t_1\right) = d$. Since then, we know $i \in I_2$ and $x_i\left(t_1\right)$ ($x_i\left(t_1\right) = d$) becomes a new initial state of the agent i. Further, using again the evaluation of (5.9), we have that at the time $t_1 + \varepsilon$ (ε is a small enough positive number), $x_i\left(t_1 + \varepsilon\right) > d$ holds, because on the closed interval $\left[t_0, t_1 + \varepsilon\right]$, there is no agent moving from I_2 to I_1 in accordance with Remark 4.1.

Similarly, for other elements in I_2, there also exists $t_2, t_3, \cdots$, such that all other elements in I_2 fall into I_1. Eventually, there must exists a time t_a such that for any time $t \geq t_a$, all agents belong to I_1 and $\sigma_{ij}\left(t_a\right) = 1$ for all i, j. At that time, $L^{\delta(t_a)}$ turns into L

as in the usual case, that is, the system (5.7) is reduced to the system (3.8); at this time, $x(t_a)$ as a new initial state and the protocol (5.5) globally asymptotically solves the consensus problem according to Theorem 4.1.

From the above, the conditions of Theorem 4.2 are satisfied and the global consensus can be reached asymptotically. Thus, the proof of (i) is obtained. Because the directed complete graph G is a strongly connected and balanced graph, the proof of (ii) can be easily obtained from Theorem 4.1.

As seen from Theorem 5.2, the multi-agent system can converge to $\max_i x_i(0) = \max\{x_1(0), \ldots, x_n(0)\}$ if the control parameter of the consensus value is taken as $d = \max\{x_1(0), \ldots, x_n(0)\}$, which we call the max-consensus. In other words, the consensus protocol (5.5) can asymptotically solve the max-consensus problem.

It can be seen from Theorem 5.2 that the consensus value of the multi-agent system varies with the control parameter of the consensus value d. In more specific terms, the consensus value of the multi-agent system increases with the increase of the control parameter of the consensus value d.

5.2.3 The combined protocol

By the above arguments, we obtain the following result.

Let x_{mid} be the usual consensus value of the multi-agent system without the control parameter of the consensus value or the usual average consensus value without the control parameter of the consensus value, if the digraph G is balanced.

Theorem 5.3. Assume that the multi-agent system has a complete digraph as its network topology. Then the following are equivalent:

(i) For all x^* on the closed and bounded interval $[\min_i x_i(0), \max_i x_i(0)]$, there exists one control parameter of the consensus value d^* in $[\min_i x_i(0), \max_i x_i(0)]$ such that the system with the protocol (5.2) or (5.5) converges to x^*;

(ii) The system with the protocol (5.2) or (5.5) can converge to a point x^* in the interval

$\left[\min_i x_i(0), \max_i x_i(0)\right]$ as long as d^* is taken in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$; and

(iii) If the system is required to converge to a point x^* in the interval $\left[\min_i x_i(0), x_{mid}\right]$, then the protocol (5.2) is adopted and d^* is also in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$. Conversely, if the system is required to converge to a point x^* in the interval $\left[x_{mid}, \max_i x_i(0)\right]$, then the protocol (5.5) is adopted and d^* is also in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$.

Proof: Here we prove only the second part of (iii), namely, that if the system is required to converge to a point x^* in the interval $\left[x_{mid}, \max_i x_i(0)\right]$, then the protocol (5.5) is adopted and d^* is also in the interval $\left[\min_i x_i(0), \max_i x_i(0)\right]$. The first part of (iii) and other equivalences can be proved similarly. Throughout the proof we shall use properties of sequences. Define $\min_i x_i(0) = a_0$, $\max_i x_i(0) = b_0$, and let $c_0 = \dfrac{a_0 + b_0}{2}$ be the middle point of the interval $[a_0, b_0]$. It follows that the consensus value of the multi-agent system increases with increase of the control parameter of the consensus value, and conversely that the control parameter of the consensus value decreases with decrease of the consensus value of the multi-agent system. When the control parameter of the consensus value is equal to a_0 or b_0, if the system right converges to x^*, then a_0 or b_0 is what we are looking for and the proof ends. Otherwise, we take c_0 as the control parameter of the consensus value. From Theorem 5.2 we know the system converges to x_0^*. There are three possibilities for x_0^*. If $x_0^* = x^*$, c_0 is d^* and the proof ends. If $x_0^* > x^*$, we set $a_1 = a_0$ and $b_1 = c_0$, to consider the left half of the original interval. If $x_0^* < x^*$, let $a_1 = c_0$, $b_1 = b_0$, take the right half of $[a_0, b_0]$ this time.

In either case we have generated a sub-interval $[a_1,b_1]\subset[a_0,b_0]$ such that

$$a_1 < x_0^* < b_1,\ \ a_1 < x^* < b_1 \text{ and } b_1 - a_1 = \frac{b_0 - a_0}{2}.$$

Let $c_1 = \frac{a_1 + b_1}{2}$ be the middle point of the interval $[a_1,b_1]$ and let $d_1^* = c_1$ be a new control parameter of the consensus value. It also follows from Theorem 5.2 that the system converges to x_1^*. Likewise, there are three possibilities for x_1^*. If $x_1^* = x^*$, c_1 is d^* and the proof ends. If $x_1^* > x^*$, we set $a_2 = a_1$ and $b_2 = c_1$, to consider the left half of the original interval. If $x_1^* < x^*$, let $a_2 = c_1$, $b_2 = b_1$, take the right half of $[a_1,b_1]$ this time. In either case we have generated a sub-interval $[a_2,b_2]\subset[a_1,b_1]$ such that

$$a_2 < x_1^* < b_2,\ \ a_2 < x^* < b_2 \text{ and } b_2 - a_2 = \frac{b_0 - a_0}{2^2}.$$

Let $c_2 = \frac{a_2 + b_2}{2}$ be the middle point of the interval $[a_2,b_2]$ and let $d_2^* = c_2$ be a new control parameter of the consensus value. It also follows from Theorem 5.2 that the system converges to x_2^*, … .

Repeating this procedure we either reach x^* after a finite number of steps, or we build a sequence of nested intervals $[a_n,b_n]$ satisfying

$$[a_0,b_0]\supset[a_1,b_1]\supset[a_2,b_2]\supset\ldots\supset[a_n,b_n]\supset\ldots,$$

$$a_n < x_{n-1}^* < b_n, a_n < x^* < b_n \text{ and } c_{n-1} = \frac{a_{n-1} + b_{n-1}}{2},\ b_n - a_n = \frac{b_0 - a_0}{2^n}.$$

In the first situation, $c_{n-1} = \frac{a_{n-1} + b_{n-1}}{2}$ coming into being from Theorem 5.2 is d^*, the consensus parameter of the consensus value that makes the system converge to x^*, and the proof ends.

In this second situation, we claim that there is a point belonging to every interval of the sequence, and this point is x^*. For this, observe that the sequences $\{a_n\}$ and $\{b_n\}$ satisfy

$$a_0 \le a_1 \le \ldots \le a_n \le \ldots \le b_n \le \ldots \le b_1 \le b_0,$$

$$a_1 < x_0^* < b_1, a_1 < x^* < b_1,\ \ a_2 < x_1^* < b_2, a_2 < x^* < b_2, \ldots,\ \ a_n < x_{n-1}^* < b_n, a_n < x^* < b_n, \ldots.$$

Therefore $\{a_n\}$ is monotone increasing and bounded, while $\{b_n\}$ is monotone decreasing and bounded. Clearly, as $n \to \infty$, the left endpoints a_n, the right endpoints b_n and the sequences $\{x_n^*\}$ must approach the same limit, say to x^*. What makes the system converge to x^* is the consensus parameter of the consensus value d^*. In other words, the system can converge to a point x^* in the interval $[\min_i x_i(0), \max_i x_i(0)]$ as long as d^*, the control parameter of the consensus value, is taken in the interval $[\min_i x_i(0), \max_i x_i(0)]$.

It follows from Theorem 5.3 that the system with the protocol (5.2) or (5.5) converges to a consensus value on a closed and bounded interval by choosing the control parameter in the corresponding closed and bounded interval, but this result does not directly supply a design method of the control parameter that assures the convergence to a specified consensus value. Nonetheless, the specified consensus value can be approximately achieved by the concatenation of choices of the control parameter, which is based on the repeated use of the result of Theorem 5.3.

In the proof of Theorem 5.3 we constructed a sequence $\{d_n^*, x_n^*\}$, and showed that $\{x_n^*\}$ converges to the specified consensus value x^* as $n \to \infty$. In the approximated computing process, we would rather solve this problem in a finite number of steps than complete the infinite procedure. By constructing $\{d_n^*\}$, as in the proof of Theorem 5.3, we have $\{x_n^*\}$ such that

$$\left|x^* - x_n^*\right| \le \frac{b_n - a_n}{2} = \frac{b_0 - a_0}{2^{n+1}} = \frac{\max_i x_i(0) - \min_i x_i(0)}{2^{n+1}},$$

as long as n is large enough, then

$$\left|x^* - x_n^*\right| < \varepsilon,$$

where ε is the desired computational accuracy. It will be validated by exemplification in great detail later in this chapter. It is worth noting that in the approximated computing process we do not have to do exactly what we stated in the proof of Theorem 5.3, for we note that the consensus value of the multi-agent system increases with the increase of the control parameter of the consensus value, which will be explained in Example 5.3.

Remark 5.3. In this chapter, we assume that the network topology is a complete digraph to ensure that at each time for the digraph at any switching state there is a spanning tree so that the system can achieve consensus. Extension of our method to the general digraph is our next challenge.

5.2.4 Improvement of consensus speed

Convergence speed is an interesting topic in the study of the consensus problem. Convergence speed is used for characterizing how fast or slow consensus can be achieved.

As we stated earlier, two parameters introduced into the consensus protocol play an important role in our discussion. The first role, to change the consensus value, has been discussed. Next we continue to explain the second role, to change the time and speed of convergence of the consensus protocols.

The second smallest eigenvalue of graph Laplacians matrix, called the algebraic connectivity, quantifies the speed of convergence of consensus protocols. It follows from this that for a given graph Laplacian, the second smallest eigenvalue of graph Laplacians is kept constant, this is to say, the speed of convergence of consensus protocols will not vary. In Theorem 5.1, however, assume that σ_{ij} takes 0 or c (instead of 1), $i, j = 1, 2, \ldots, n$, the consensus can be more quickly reached asymptotically if $c > 1$, or more slowly reached asymptotically if $0 < c < 1$. This is to say that σ_{ij} can change the time and speed of convergence of consensus protocols in the multi-agent system, and σ_{ij} is inversely proportional to the time of convergence and is directly proportional to the speed of convergence.

Similarly, in Theorem 5.2 assume that δ_{ij} takes 0 or c (instead of 1), $i, j = 1, 2, \ldots, n$, the consensus can be more quickly reached asymptotically if $c > 1$, or more slowly reached asymptotically if $0 < c < 1$. This is to say that δ_{ij} can change the time and speed of convergence of consensus protocols of the multi-agent system, and δ_{ij} is inversely proportional to the time of convergence and is directly proportional to the speed of convergence.

5.3 Examples and simulation results

This section presents some illustrative examples to describe the theoretical results in this chapter.

Example 5.1. We consider a multi-agent system involving four agents under the consensus protocol (5.2). By simulation, we investigate the consensus convergence character of the multi-agent system and verify the proposed Theorem 5.1.

We suppose the initial states of the four agents are $x(0)=(4,3,1,2)^T$. From Theorem 4.1, the multi-agent system can reach usual consensus and $x_{mid}=2.5$, as shown in Fig. 5.1. The control parameter of the consensus value is $d=2$. By Theorem 5.1 the consensus can be reached, and Fig. 5.2 shows a simulation of these four agents. Comparing Fig. 5.1 and Fig. 5.2 shows that the consensus values are different. Moreover, in the example protocol (5.2) is adopted, so the system converges to a point in the subinterval $[1, 2.5]$.

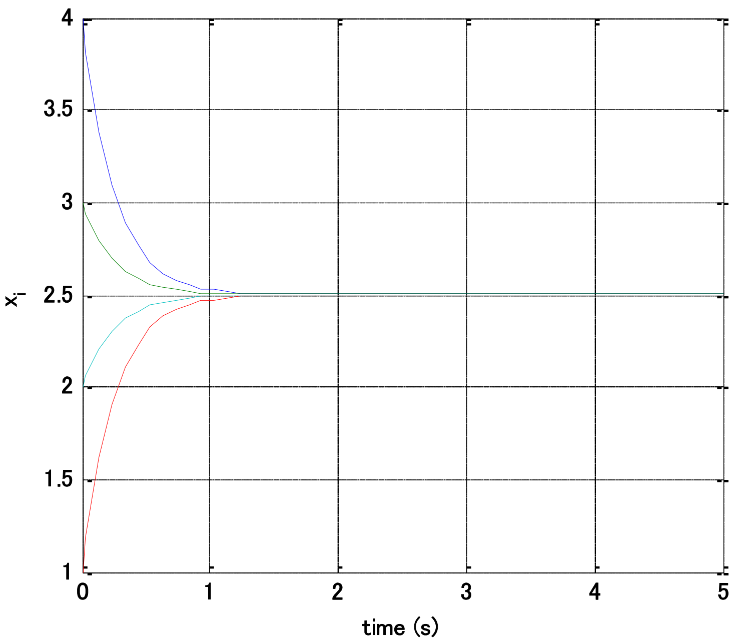

Fig. 5.1. Usual consensus of four agents.

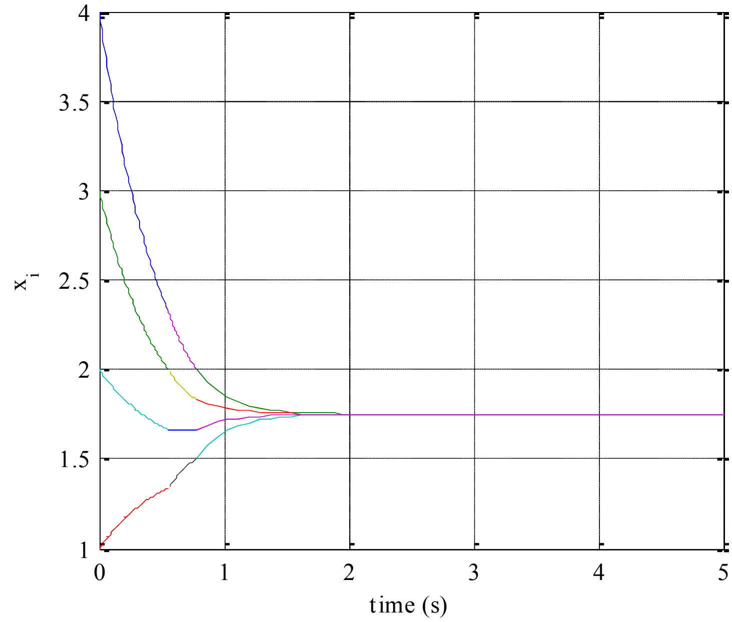

Fig. 5.2. Consensus of four agents based on Theorem 5.1.

From Theorem 5.1, we know that the multi-agent systems can solve the min-consensus if the control parameter of the consensus value is taken as $d = \min\{x_1(0), \ldots, x_n(0)\}$, which is shown in Fig. 5.3 where we take $d = 1$ as the control parameter of the consensus value.

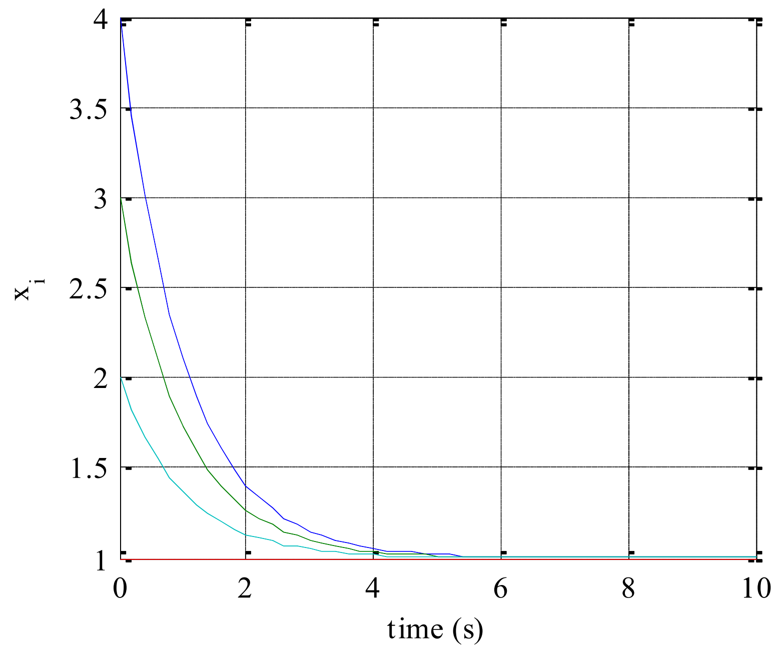

Fig. 5.3. Min-consensus of four agents based on Theorem 5.1.

Example 5.2. We continue to consider the multi-agent system described in Example 5.1. We suppose the initial states of four agents under the consensus protocol (5.5) are again $x(0)=(4,3,1,2)^T$. From Theorem 4.1, the multi-agent system again reaches the usual consensus, as shown in Fig. 5.1. The control parameter of the consensus value is $d=3$. Based on Theorem 5.2, the consensus can be reached and the simulation of these four agents is shown in Fig. 5.4. It can be also seen from Fig. 5.1 and Fig. 5.4 that the consensus value in Fig. 5.1 is different from that in Fig. 5.4. Moreover, in the example the protocol (5.5) is adopted, so the system converges to a point in the subinterval $[2.5,4]$.

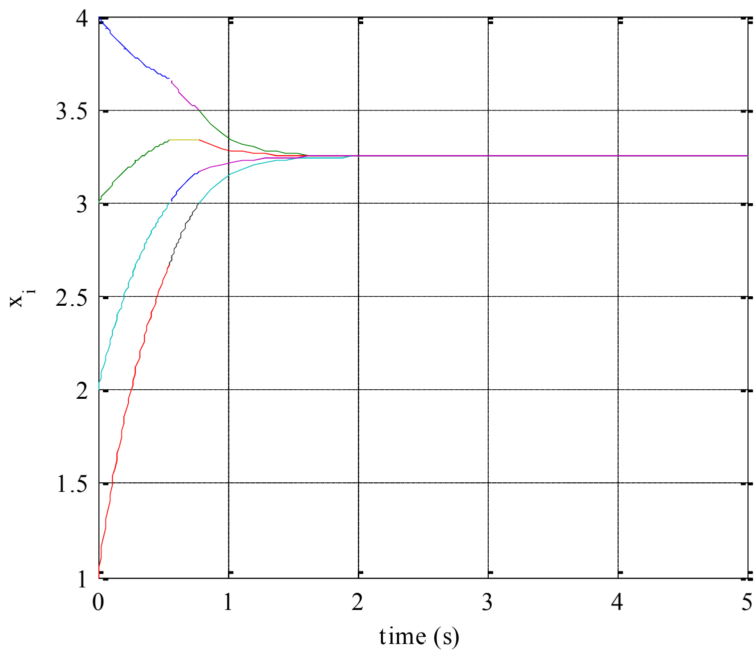

Fig. 5.4. Consensus of four agents based on Theorem 5.2.

From Theorem 5.2, we know that the multi-agent systems can solve the max-consensus if the control parameter of the consensus value is taken as $d=\max\{x_1(0),\ldots,x_n(0)\}$, which is shown in Fig. 5.5 where we take $d=4$ as the control parameter of the consensus value.

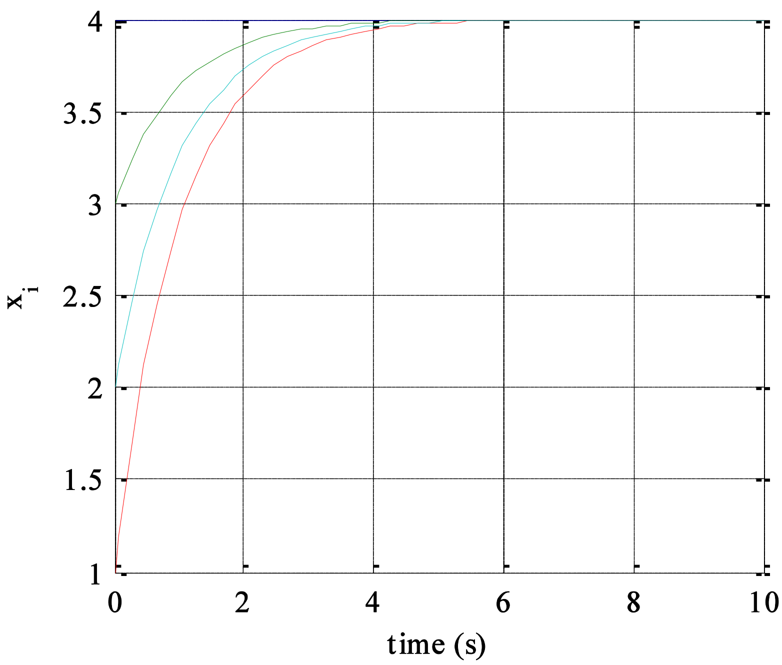

Fig. 5.5. Max-consensus of four agents based on Theorem 5.2.

The above two examples address the case in which for a given consensus parameter of the consensus value d, the multi-agent systems can solve the global consensus problem. The following example is presented to verify the rightness of the theoretical result stated in Theorem 5.3.

Example 5.3. We continue to consider the multi-agent system described in Example 5.1. The system is required to converge to a point $x^* = 3$ in the interval $[1, 4]$. We must find the control parameter of the consensus value d^* that makes the system converge to the point $x^* = 3$ in the interval $[1, 4]$. Figure 5.1 shows that $x^* = 3 > x_{mid} = 2.5$, so we use consensus protocol (5.5) in the interval $[2.5, 4]$. According to Theorem 5.3, although the system does not converge to the point $x^* = 3$ when the control parameter of the consensus value is equal to 2.5 or 4, as shown in Fig. 5.6 the system converges to the point $x_1^* = 2.9745$, very close to $x^* = 3$, when the control parameter of the consensus value is equal to 2.5. We know that the consensus value of the multi-agent system increases together with the control parameter of the consensus value, so we take $d_1^* = 2.6$ and then by Theorem 5.3 the system converges to the point $x_2^* = 3.027$, again very close to $x^* = 3$. This time we take $d_2^* = \frac{2.5+2.6}{2} = 2.55$ and the system converges to $x_3^* = 3.001$ by Theorem 5.3, as shown in Fig. 5.7. For $|x^* - x_3^*| = |3 - 3.001| = 0.001 < 0.01$, the resulting consensus value meets the requirements of the system and $d^* = d_2^* = 2.55$.

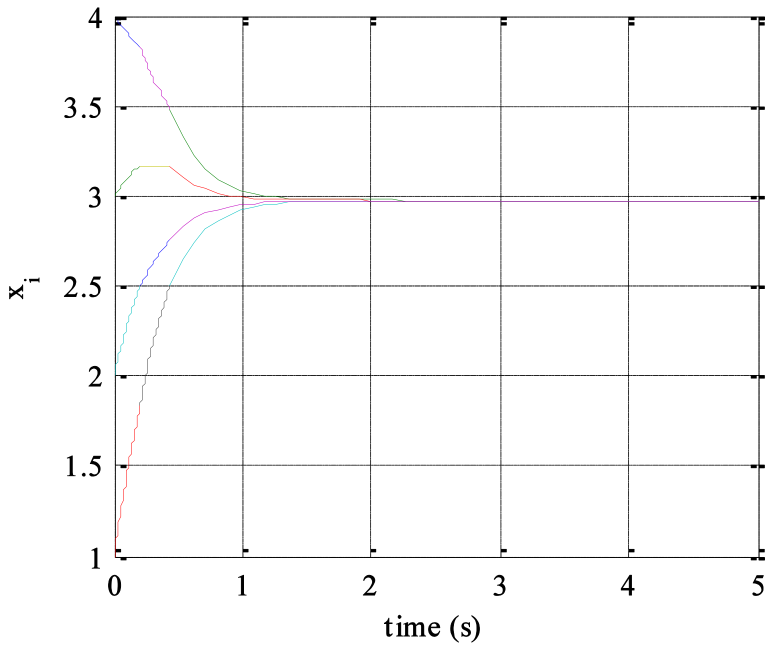

Fig. 5.6. First consensus of four agents based on Theorem 5.3.

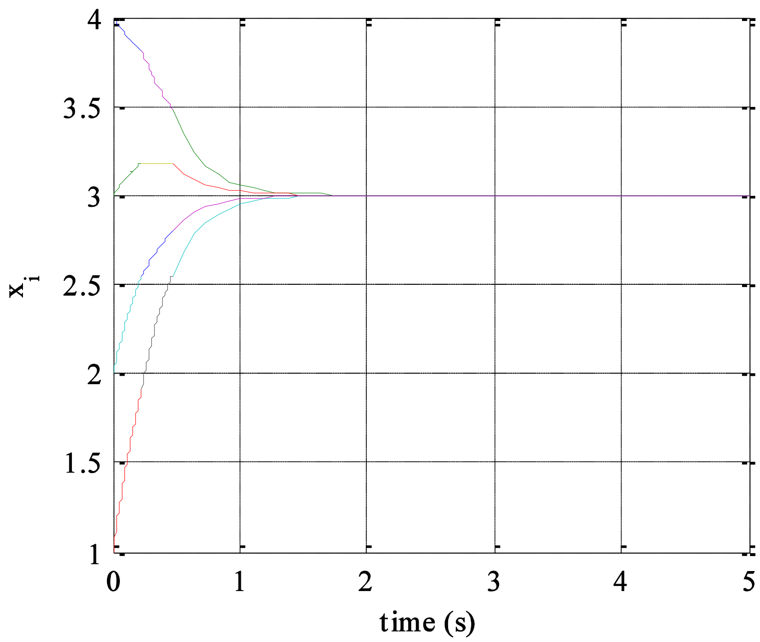

Fig. 5.7. Third consensus of four agents based on Theorem 5.3.

According to Theorem 5.3, by repeating the above procedure we can find a new d^* such that the system converges to the point $x^* = 3$ in a finite number of steps. Regarding the actual computing process of Example 5.3, we note that if the system fails to converge to the point $x^* = 3$ when the control parameter of the consensus value is equal to 2.5 or 4, we do not take $d_1^* = \frac{2.5+4}{2} = 3.25$ as the control parameter of the consensus value but rather $d_1^* = 2.6$, because the consensus value of the multi-agent system increases together with the control parameter of the consensus value and $x_1^* = 2.9745$ is very close to $x^* = 3$, and it is possible to obtain d^* in a few steps.

As stated earlier, the parameters introduced into the consensus protocol play two important roles in our discussion. One role is to change the consensus value that is ideal or meets the quality we require from the multi-agent system, as demonstrated in Example 5.1 and Example 5.2. The other role is to change the time and speed of convergence of consensus protocols. In more specific terms, those two parameters are inversely proportional to the time of convergence of consensus protocols, and are directly proportional to the speed of convergence of consensus protocols. In the following, we take δ_{ij} as an example to illustrate this role.

Example 5.4. In Example 5.2, we took $\delta_{ij} = 0 \text{ or } 1$, but here we assume $\delta_{ij} = 0 \text{ or } 5$. As stated in Section 5.2.4 and by Theorem 5.2, the consensus can be reached and the simulation of these four agents is shown in Fig. 5.8. It can be seen from Fig. 5.4 and Fig. 5.8 that the consensus can be more quickly reached asymptotically.

Next we assume $\delta_{ij} = 0 \text{ or } 0.2$ as stated in Section 5.2.4 and by Theorem 5.2, the consensus can be reached and a simulation of these four agents is shown in Fig. 5.9. It can be seen from Fig. 5.4 and Fig. 5.9 that the consensus can be more slowly reached asymptotically.

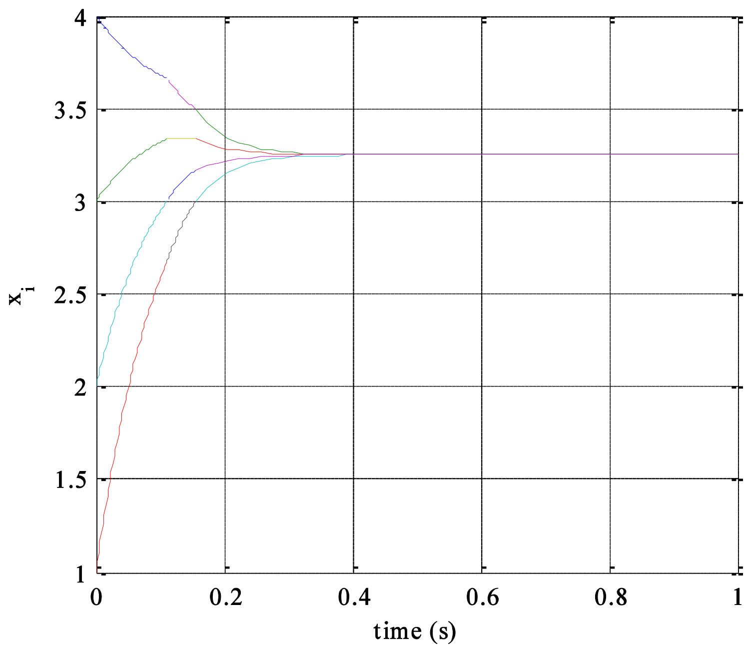

Fig. 5.8. Fast consensus of four agents based on Section 5.2.4.

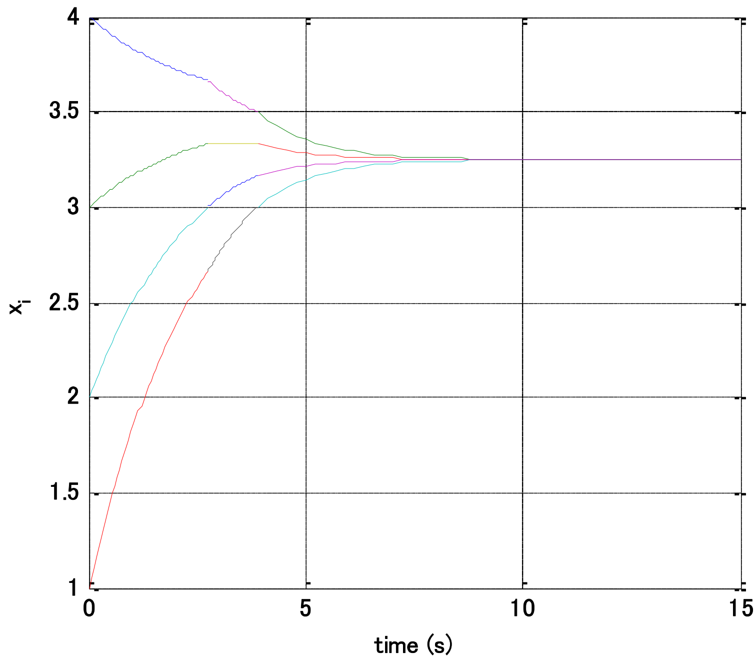

Fig. 5.9. Slow consensus of four agents based on Section 5.2.4.

5.4 Chapter summary

This chapter discussed the interval consensus problem of multi-agent systems by providing a special Laplacian of directed graphs. Generally, for the given consensus protocol and initial states, a fixed consensus value is obtained. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent system. By introducing two state-dependent switching parameters into the consensus protocol and taking algebraic graph theory, matrix theory, and control theory as bases, the system given by the proposed protocol can reach globally asymptotically interval consensus and can also achieve a generalized interval average consensus if the directed graph is balanced. Simulations were provided to demonstrate the effectiveness of our theoretical results.

6 Interval consensus problem of multi-agent systems with time delays

6.1 Introduction

Many benefits can be obtained such as robustness, adaptivity, flexibility, and scalability when replacing a solo complicated system with several simple systems. It is because of this that the control of multiple interconnected systems has received too many attentions [7]. Recently multi-agent systems have received increasing attentions due to its broad applications in various fields, such as computer science, vehicle systems and unmanned air vehicles vehicle formations, flocking and tracking, and so on.

As one type of critical problems for cooperative control of multiple agents, consensus problems concern such case that agents in a network converge to consistent states by designing appropriate protocols and algorithms. Various researchers from different disciplines, such as mathematics, physics, computer sciences and biology, have paid great attention to the consensus of multi-agent systems over the past decade.

In practice, the speeds in transmission between the individual information are limited and it is inevitable that there exist communication delays in the system, so the effort of delay on the collective behavior of the system can not be ignored [131]. In recent years, the multi-agent system with time delay has been widely studied and made lots of achievements. By a linear matrix inequality method, Sun et al. provided an appropriate upper bound for communication delays and proved that all the nodes in the network could solve average consensus asymptotically [17]. By using a new approach based on the tree-type transformation, Sun and Wang not only established necessary and sufficient conditions for consensus in directed networks with dynamically changing topology and nonuniform time-varying delays, but also presented some feasible conditions in terms of linear matrix inequalities to determine the allowable upper bounds of delays [119]. Papachristodoulou et al. considered what effect multiple, non-commensurate (heterogeneous) communication delays can have on the consensus properties of large-scale multi-agent systems endowed with nonlinear dynamics [132].

Generally speaking, for the given consensus protocol and initial states, a fixed consensus value is obtained. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent systems. To solve such a protocol design problem, we pose a new class of consensus problem, called interval consensus problem, and we try to find a protocol ensuring that the system converges to a point on a specified closed and bounded interval in chapter 5. In this chapter, we address interval consensus problem of multi-agent systems with time delay, i.e., communication delay and input delay, based on the results in chapter 5. Our work shows that the communication delay does not affect the consensus while the input delay does. For communication delay, the system given by the proposed protocol can reach globally asymptotically interval consensus with any time delay. As for bounded input delay, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average-consensus if the directed graph is balanced. Examples and simulations are provided to demonstrate the effectiveness of our theoretical results.

6.2 Interval consensus problem of multi-agent systems with time delays

It is well-known that, in general, unmodelled delay effects in a feedback mechanism may destabilize an otherwise stable system. This destabilizing effect of delay has been well documented in the literature. In the present context delay effects may arise naturally, for example, because of the finite transmission speed due to the physical characteristics of the medium transmitting the information (e.g. acoustic wave communication between underwater vehicles).

We note that the consensus protocol (5.2) assumes that each agent can get the states of its neighbors without any time delay. This assumption gives birth to an obvious limitation because time delay often appears in every practical system and, therefore, deserves consideration in the consensus problem of the multi-agent systems. In this section we will consider two types of time delays, i.e., communication delay and input delay, and then give the main results.

When there exists communication delay, the protocol (5.2) becomes

$$u_i = -\sum_{j\in N_i} \sigma_{ij} a_{ij} \left[x_i(t) - x_j(t-\tau) \right], \quad x_i(0) \in R, \tag{6.1}$$

where τ is the communication delay from the j th agent to the i th agent.

We first consider a simple network structure with one single coupling. If we assume that the delay affects only the variable that is actually being transmitted from one system to another then it makes sense to assume that an edge from the agent l to the agent k contributes to the dynamics as follows

$$\dot{x}_k(t) = -\sigma_{kl} a_{kl} \left(x_k(t) - x_l(t-\tau) \right),$$

which makes the real variable x_k evolve towards the delayed variable x_l with a rate of change proportional to difference $x_k(t) - x_l(t-\tau)$. More generally, we consider the delay differential equation

$$\dot{x}_i(t) = -\sum_{j \in N_i} \sigma_{ij} a_{ij} \left[x_i(t) - x_j(t-\tau) \right],$$

or

$$\dot{x}(t) = \text{diag}\left(-L^\sigma\right) x(t) + \left(-L^\sigma - \text{diag}\left(-L^\sigma\right)\right) x(t-\tau), \tag{6.2}$$

where $\text{diag}\left(-L^\sigma\right)$ is the diagonal matrix obtained from $-L^\sigma$ by setting all off-diagonal entries equal to zero and L^σ is defined in (5.4) or as L_2^σ in Lemma 4.4 as follows,

$$L^\sigma = \left[l_{ij}^\sigma \right] = \begin{bmatrix} \sum_j \sigma_{1j} a_{1j} & -\sigma_{12} a_{12} & \cdots & -\sigma_{1n} a_{1n} \\ -\sigma_{21} a_{21} & \sum_j \sigma_{2j} a_{2j} & \cdots & -\sigma_{2n} a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -\sigma_{n1} a_{n1} & -\sigma_{n2} a_{n2} & \cdots & \sum_j \sigma_{nj} a_{nj} \end{bmatrix}.$$

Next we present Theorem 6.1 for the first time-delayed protocol about communication delay.

Theorem 6.1. Consider a network of integrators with a fixed topology $G = (V, E)$ that is a complete digraph; control parameters of the consensus value is d in the interval $\left[\min_i x_i(0), x_{mid}\right]$. Then, the system (6.2) given by the time-delayed protocol (6.1) globally asymptotically solves the consensus problem with any time delay in the interval $\left[\min_i x_i(0), x_{mid}\right]$.

Proof: First of all, the system (5.4) without communication time can reach consensus by Theorem 5.1. Let $-L^{\sigma(t_i)}$ be the ith switching matrix of the system (6.2), and let t_0, t_1, ... be the time sequence corresponding to the times at which $-L^{\sigma(t_i)}$ switches. From that the digraph G is a complete and has, of course, has a spanning tree, and from the definition of Laplacian matrix, we know that $-L^{\sigma(t_i)}$ of every network topology is bounded and piecewise function of time by Lemma 4.1 and Theorem 4.1. Secondly, $-L^{\sigma(t_i)}$ are row-zero-sum sub-blocks and all nondiagonal elements are nonnegative, which can also be seen from the definition of Laplacian matrix. Therefore, $-L^{\sigma(t_i)}$ the system matrix is Metzler with zero row sums. Thirdly, it follows from that the sub-graph associate with $-L^{\sigma(t_i)}$ has a spanning tree, all nodes can be reached from a node, say the node k, so the equilibrium set of consensus states is uniformly exponentially stable according to Lemma 4.1. From Theorem 3.5, we know all components of any solution $x(t)$ of each $-L^{\sigma(t_i)}$ converges to a common value as t goes to the infinity. This is to say, the system (6.2) given by the time-delayed protocol (6.1) is robust with respect to an arbitrary delay in the interval $\left[\min_i x_i(0), x_{mid}\right]$.

In the following, we present the time-delayed protocol with input delay.

When there exists input delay, the protocol (5.2) becomes

$$u_i = -\sum_{j\in N_i} \sigma_{ij} a_{ij} \left[x_i(t-\tau) - x_j(t-\tau)\right], \quad x_i(0) \in R, \tag{6.3}$$

where τ is the input delay for information communicated from the jth agent to the ith agent.

The collective dynamics of the network can be expressed as

$$\dot{x}(t) = -L^{\sigma} x(t-\tau), \tag{6.4}$$

where L^{σ} is the same as in (6.2).

Next we put forward the main result for the second time-delayed protocol about input delay.

Theorem 6.2. Consider a network of integrators with a fixed topology $G=(V,E)$ that is a complete digraph; control parameters of the consensus value is d. Then, the system (6.4) given by the time-delayed protocol (6.3) with equal input time-delay τ globally asymptotically solves the consensus problem in the interval $\left[\min_i x_i(0), x_{mid}\right]$ if and only if $\tau \in (0, \tau^*)$ with $\tau^* = \min\left\{\pi / 2\lambda_{\max}\left(L^{\sigma(t_1)}\right), \pi / 2\lambda_{\max}\left(L^{\sigma(t_2)}\right), \pi / 2\lambda_{\max}\left(L^{\sigma(t_3)}\right), \ldots\right\}$, and here $\lambda_{\max}\left(L^{\sigma(t_i)}\right)$ denotes the largest eigenvalue of the ith switching Laplacian matrix. And the system (6.4) given by the proposed protocol (6.3) can also achieve a generalized interval ave-consensus if the directed graph is balanced.

Proof: First of all, the system (5.4) without input time can reach consensus by Theorem 5.1. Let $L^{\sigma(t_i)}$ be the ith switching Laplacian matrix of the system, and let t_0, t_1, ... be the time sequence corresponding to the times at which $L^{\sigma(t_i)}$ switches. From that the digraph G is a complete and has, of course, has a spanning tree, it follows that every network topology of with $L^{\sigma(t_i)}$ as its Laplacian matrix is fixed and strongly connected by Lemma 4.1 and Theorem 4.1. For each $L^{\sigma(t_i)}$, the system with equal communication time-delay $\tau > 0$ in all edges with time-delay τ solves the consensus problem in the interval $\left[\min_i x_i(0), x_{mid}\right]$ if and only if time-delay $\tau \in (0, \tau_i)$ with $\tau_i = \pi / 2\lambda_{\max}\left(L^{\sigma(t_i)}\right)$ by Theorem 3.6, here $\lambda_{\max}\left(L^{\sigma(t_i)}\right)$ denotes the largest eigenvalue of the ith switching Laplacian matrix. Therefore, according to Theorem 3.6 and Theorem 5.1, the system (6.4) given by the time-delayed protocol (6.3) with equal input time-delay τ globally asymptotically solves the consensus problem if and only if $\tau \in (0, \tau^*)$ with $\tau^* = \min\left\{\pi / 2\lambda_{\max}\left(L^{\sigma(t_1)}\right), \pi / 2\lambda_{\max}\left(L^{\sigma(t_2)}\right), \pi / 2\lambda_{\max}\left(L^{\sigma(t_3)}\right), \ldots\right\}$. From Theorem 4.1, if the directed graph is balanced, the system (6.4) given by the time-delayed protocol (6.3) can also achieve a generalized interval average-consensus.

6.3 Example and simulation

This section presents some illustrative examples to describe the theoretical results in this chapter.

Example 6.1. We consider the multi-agent systems involving four agents that are under the time-delayed consensus protocol (6.1). By simulation studying, we investigate the consensus convergence character of the multi-agent systems and verify the proposed Theorem 6.1.

We suppose the initial states of four agents are $x(0)=(4,3,1,2)^T$ and the control parameter of the consensus value is $d=2$. The usual consensus of these four agents is shown in Fig. 6.1. The consensus of these four agents under the consensus protocol (5.2) is shown in Fig. 6.2. As seen from Fig. 6.1 and Fig. 6.2, the system given by the proposed protocol converges globally asymptotically to a new point in Fig. 6.2 by introducing a state-dependent switching parameter and the control parameter of the consensus value into the consensus protocol, which is what we call the interval consensus. Based on Theorem 6.1, the consensus under the time-delayed consensus protocol (6.1) can be reached and the simulations of these four agents with $\tau=1$ and $\tau=10$ are shown in Fig. 6.3 and Fig. 6.4, respectively.

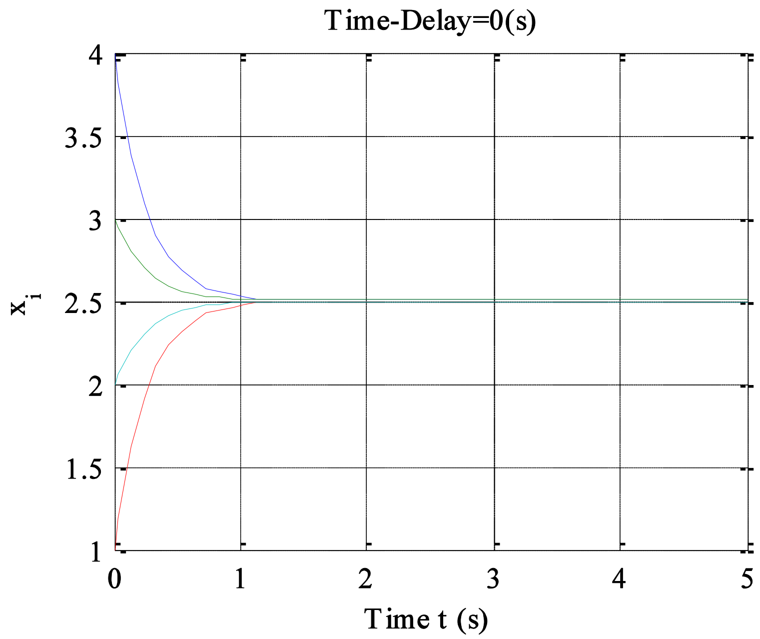

Fig. 6.1. The usual consensus of four agents.

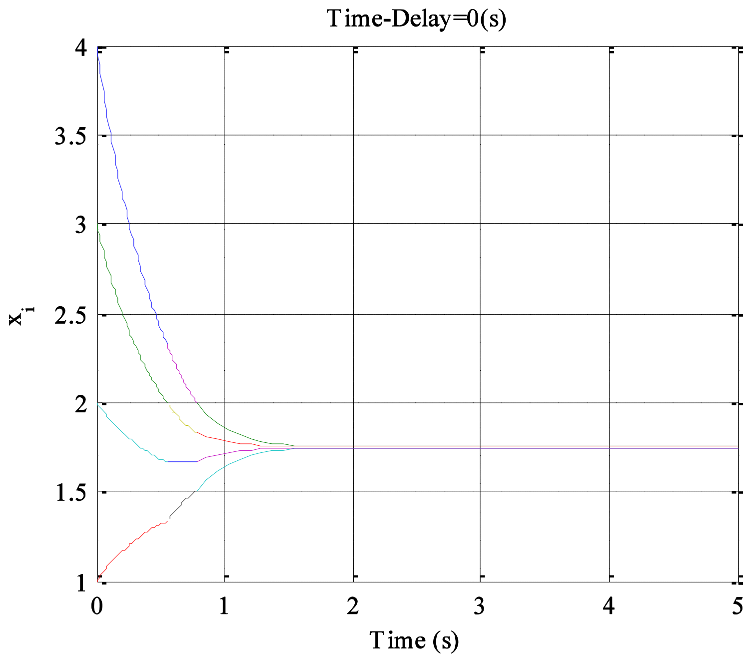

Fig. 6.2. The consensus of four agents under the protocol (5.2).

Example 6.2. In this example, we continue to discuss the multi-agent systems stated in Example 6.1. The usual consensus of these four agents is also shown in Fig. 6.1. The consensus of these four agents under the consensus protocol (5.2) is also shown in Fig. 6.2. Based on Theorem 6.2, the consensus under the time-delayed consensus protocol (6.3) can be reached and the simulations of these four agents with $\tau = 0.1$ and $\tau = 0.35$ are shown in Fig. 6.5 and Fig. 6.6, respectively.

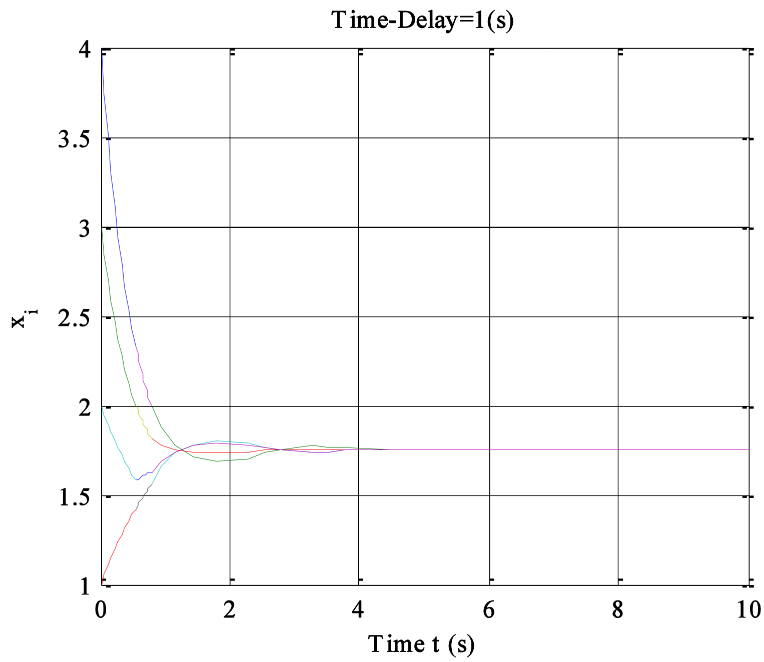

Fig. 6.3. The consensus of four agents with time-delay $\tau = 1$.

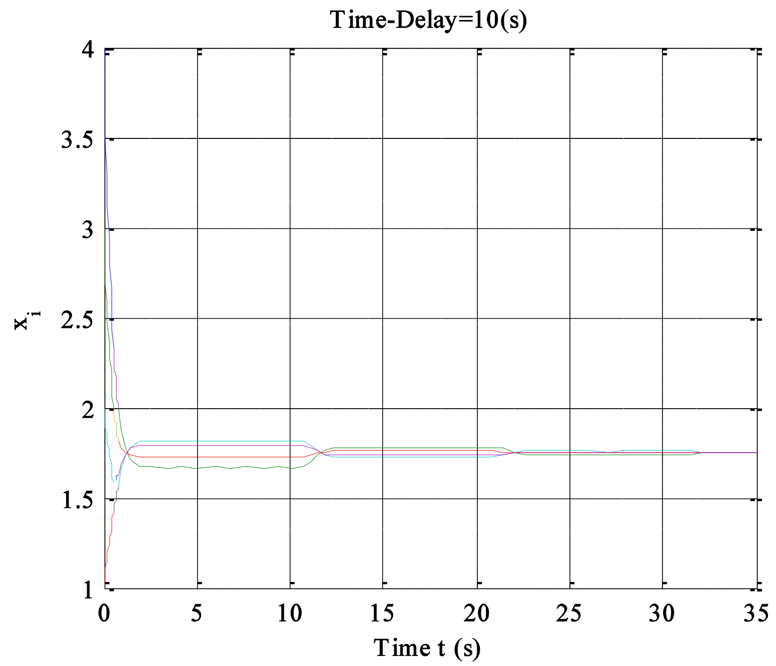

Fig. 6.4. The consensus of four agents with time-delay $\tau = 10$.

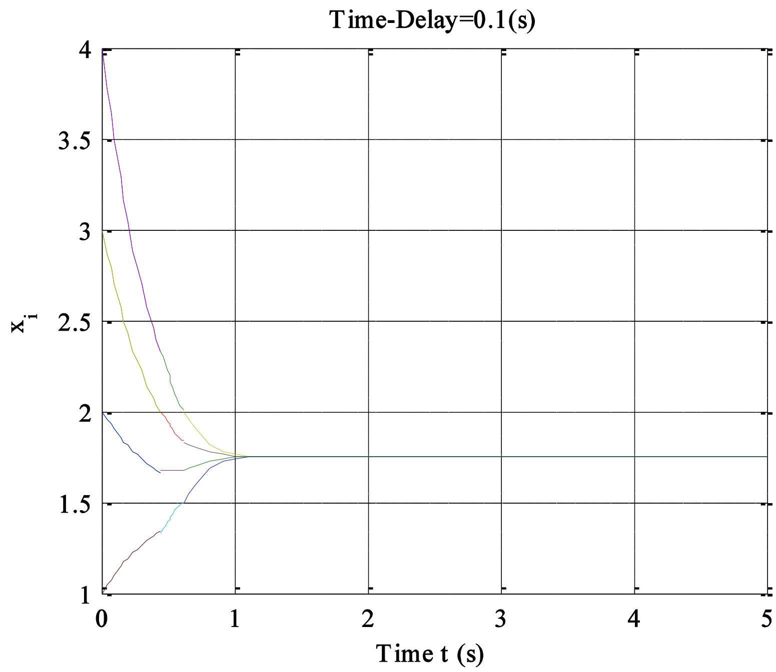

Fig. 6.5. The consensus of four agents with time-delay $\tau = 0.1$.

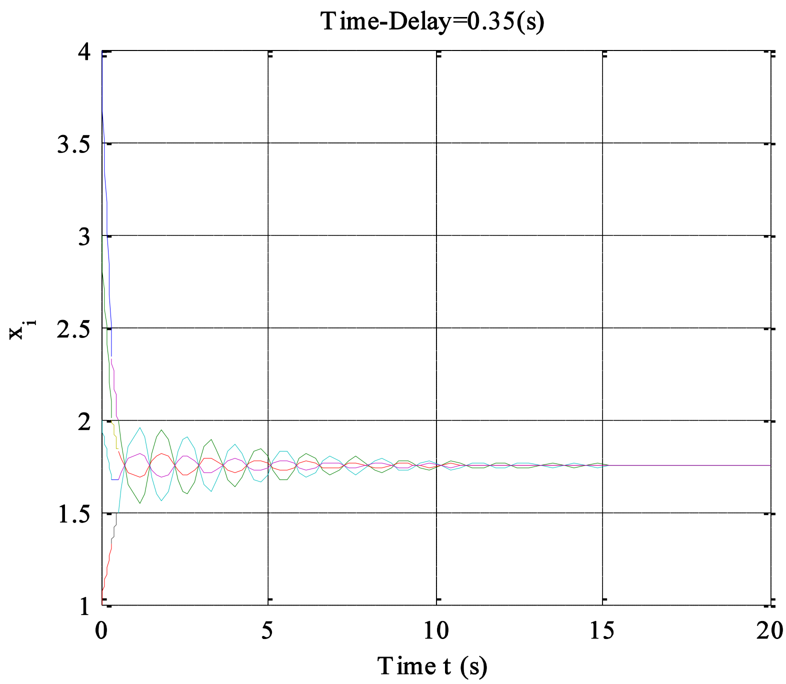

Fig. 6.6. Consensus of four agents with time-delay $\tau = 0.35$.

However, the multi-agent systems may not reach the consensus with the increment of the number of the time delay. For instance, the multi-agent systems cannot achieve the consensus with communication time-delay $\tau = 0.4 > \tau^*$, just as shown in Fig. 6.7.

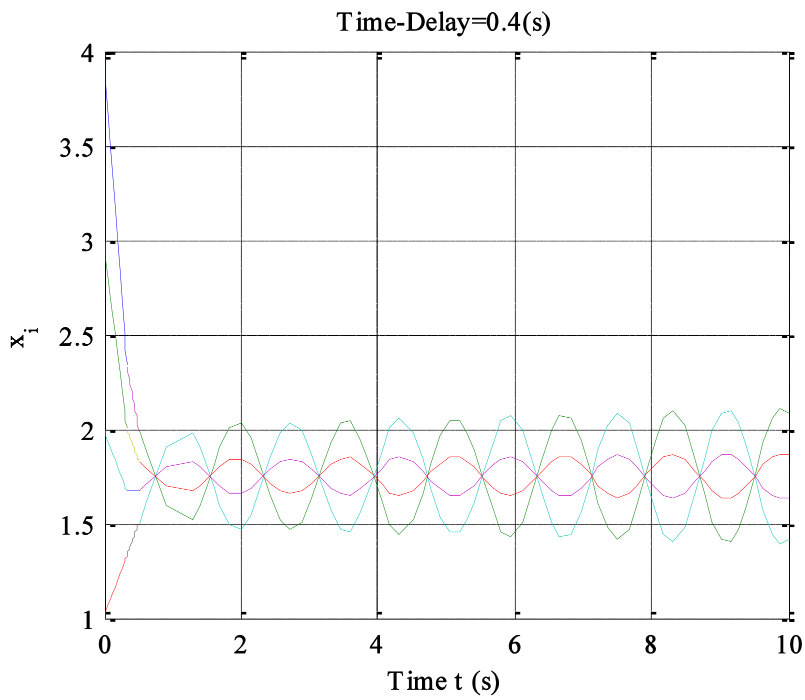

Fig. 6.7. The consensus of four agents with time-delay $\tau = 0.4$.

Furthermore, as stated similarly in [103] or according to Theorem 4.1, the average interval consensus of the multi-agent systems can be reached if the digraph of the network topology is strongly connected and balanced. Of course, the protocol (6.3) with time-delay globally asymptotically solves the average consensus problem in this example, which can be seen from Fig. 6.6 and Fig. 6.7 clearly.

6.4 Chapter summary

This chapter discussed interval consensus problem of multi-agent systems with two types of time-delays, i.e., communication delay and input delay. Our work showed that the communication delay does not affect the consensus while the input delay does. For communication delay, the system given by the proposed protocol can reach globally asymptotically interval consensus with any time delay. As for bounded input delay, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average-consensus if the directed graph is balanced. Simulations have been given to show the effectiveness of our theoretical results.

7 Conclusions and Future Works

7.1 Summary of main contributions

This dissertation solves two consensus problems of multi-agent systems in accordance with switching protocol. One is the problem about the communication channel constraint on signal amplitude. We explore conditions for consensus problem of multi-agent systems with communication channel constraint on signal amplitude and propose the consensus convergence criterion of system. By simulation study, we verify the rightness of the theoretics.

The other is the interval consensus problem of multi-agent systems. Interval consensus problem of multi-agent systems is a new class of consensus problems. For the given consensus protocol and initial states, a fixed consensus value is obtained. The resulting consensus value, however, may not be ideal or meet the quality that we require from the multi-agent system. It is therefore necessary and significant to investigate whether we can design a protocol to change the consensus value of the multi-agent system, and the answer to this question will allow application of multi-agent systems in new fields. Moreover, it seems to be generally complicated and difficult to design an appropriate protocol such that multi-agent systems can converge to any designated point. By introducing two state-dependent switching parameters into the consensus protocol, we make the system given by the proposed protocol globally asymptotically converge to a designated point on a special closed and bounded interval. In other words, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average consensus if the directed graph is balanced. The effectiveness of our theoretical results is also demonstrated by simulations.

In addition, we also discuss interval consensus problem of multi-agent systems with two types of time-delays, i.e., communication delay and input delay. Our work shows that the communication delay does not affect the consensus while the input delay does. For communication delay, the system given by the proposed protocol can reach globally asymptotically interval consensus with any time delay. As for bounded input delay, the system given by the proposed protocol can reach globally asymptotically interval consensus and then the system can also achieve a generalized interval average-consensus if the directed graph is balanced. Examples and simulations are given to demonstrate the effectiveness of our theoretical results.

7.2 Future directions and possible extensions

There are several directions and possible related research areas in which we can carry out future work.

The primary aim of future work is to discuss the convergence towards an interval consensus for second-order multi-agent systems with directed graphs,

$$\dot{x}_i = v_i,\ \dot{v}_i = u_i,\ i = 1,\ldots n.$$

A possible extension is about communication graphs. In this dissertation, we addressed the multi-agent system with a complete digraph, whose every pair of nodes is adjacent, as its communication topology. In future work, we will enlarge the multi-agent system to more ecumenical form system, for instance, to a multi-agent system with a strongly connected graph. In the formulation of the dissertation, we should consider the possibility that all the channels are switched off, so that we assume that the communication topology of the multi-agent system is a complete digraph. In light of this, to relax the assumption of the complete graph, we may generalize the definition of the consensus protocol.

Another interesting problem is studying robust control for interval consensus problem of multi-agent systems consisting of n identical agents with the ith one modeled by the following linear coupling dynamic system subject to external disturbances

$$\dot{x}_i(t) = Ax_i(t) + B_1 w_i(t) + B_2 u_i(t),\ i = 1,\ldots n.$$

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Achievements

Journal Paper

[1] M. H. Wang and K. Uchida, "Consensus problem in multi-agent systems with communication channel constraint on signal amplitude," SICE Journal of Control, Measurement, and system Integration, vol. 6, no. 1, pp. 007-013, 2013.

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