Moduli cosmology in four-dimensional effective supergravity and its implications for string model building

有効的４次元超重力模型に基づくモジュライ宇宙論と弦模型への示唆

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Waseda University,
Graduate School of Advanced Science and Engineering,
Department of Pure and Applied Physics, Research on Theoretical Particle Physics

Hajime OTSUKA

大塚　啓

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Chapter 1

Overview

The standard model of particle physics is the most successful model that describes the strong and electroweak (EW) interactions among elementary particles. It is established by the discovery of Higgs field at the LHC experiment. However, some ultraviolet completion beyond the standard model (SM) seems to be required from several observations of its theoretical and phenomenological/cosmological problems. The most severe problem is that the gravitational interaction, which is not included in the SM, is not renormalizable despite the expectation that the gravity universally couples to all the elementary particles. For the past decades, string theory has been expected as the unified theory of all the elementary forces, because the gauge and gravitational interactions are simultaneously realized among the string interaction. Moreover, most important predictions of typical string theories are the existence of the "extra dimensions" and "supersymmetry" (SUSY). These two concepts are expected to play important roles for solving the theoretical and phenomenological/cosmological problems in the SM.

From the cosmological point of view, the current observational data demand the existence of cold dark matter whose relic abundance occupies about twenty-five percent of an energy density of our present universe. In supersymmetric standard models, that is, the supersymmetric extensions of the standard model, the certain superpartner of ordinary particle becomes a plausible candidate for dark matter, consistent with the cosmological observations as well as the collider experiments. Furthermore, we believe the existence of initial era of accelerating universe, i.e., the cosmic inflation [1, 2, 3], because the inflation scenario can not only solve the flatness and horizon problems, but also reproduce the current cosmic microwave background (CMB) data. The inflation mechanism is realized by the vacuum energy density of Lorentz scalar field called inflaton field which is consistent with our isotropic universe. The vacuum energy density of the inflaton field leads to an expanding universe and its quantum fluctuations produce the origin of density perturbation of our universe. However, its potential terms as well as the kinetic term are severely constrained in order to be consistent with CMB observations. From the theoretical point of view, it is required to specify the origin of such an inflaton field itself, otherwise these parameters will not be restricted.

In the higher-dimensional theory including the string theory, a lot of four-dimensional scalar fields (moduli fields) appear associated with extra-dimensional components in vector and tensor fields in higher-dimensional spacetime with the compactified extra dimensions. In particular, within the framework of string theory, the vacuum expectation values of closed string moduli fields determine the size and the shape of extra-dimensional space, whereas those of open string moduli fields give the positions of D-branes [4] (solitonic objects in the string theory) and Wilson-lines of the gauge potential induced in them. Moduli fields are ubiquitous in the string theory in the sense that the number of closed string moduli is typically of order of one hundred in the string landscape. Since the parameters in modulus scalar potential are constrained by the higher-dimensional Lorentz and gauge symmetries in preference to those of matter fields, moduli inflation scenario has a high prophetic instinct for the cosmological observations.

We next mention about the standard model including the extra-dimension and SUSY from the phenomenological point of view. Supersymmetric models are attractive scenarios, e.g., the minimal supersymmetric standard model (MSSM) predicts not only a gauge coupling unification at a high energy scale, so-called grand unification scale (GUT scale), around 2.1×10^{16} GeV, but also a radiative electroweak symmetry breaking through the renormalization group effects whereas the breaking is an assumption for the SM. Furthermore, the local supersymmetric models, i.e., supergravity models necessarily contain the gauge and gravitational interactions among elementary particles. Although most phenomenological/cosmological models are formulated in the four-dimensional supergravity (4D SUGRA), some of them would be derived via the compactification of certain superstring theory. Finally, we show the theoretical problems in the SM. The observed hierarchical structure of quark and lepton mass matrices is not explained at all in the SM itself and Yukawa couplings are treated just as parameters. In the higher-dimensional model with compactified extra dimensions, the size and flavor structure of Yukawa couplings of quarks, leptons and Higgs bosons are determined by the overlap integrals of their wavefunction, which can be localized in the extra-dimensional spaces. Then, the hierarchical structure of Yukawa coupling can be dynamically generated by the quasi-localization of the wavefunction of quarks, leptons and Higgs fields. (For more details, see, e.g., Ref. [5] for the five-dimensional cases.) Moreover, since the matter wavefunction depends on the certain moduli fields appearing from the dimensional reduction of the extra dimensions, the moduli stabilization mechanism is also quite important for the particle phenomenology.

In order to solve the theoretical and phenomenological/cosmological problems of the standard model of particle physics, in this thesis, we take the following two approaches. One of them is called as "bottom-up approach", in which we try to explain these problems by minimally extending the SM. In the first part of this thesis from Chapter 2 to Chapter 4 (part I), we consider a five-dimensional supergravity model (5D SUGRA) that is a minimal extension of the SM to include the extra dimension and SUSY. As explained above, these two concepts are indicated from the string theory. In the second part of this thesis, part II, we take another approach called "top-down approach", in which we study a certain ultraviolet theory and the standard model would be realized as its low energy effective theory. In particular, we focus on the cosmological aspects of string theory from Chapter 5 to Chapter 7. In both approaches, we discuss implications, from the future high-energy experiments and cosmological observations, for the string model building.

The moduli cosmology in 5D supergravity model

As a bottom-up approach, we consider the 5D SUGRA, especially, a 5D SUGRA model compactified on S^1/Z_2, which is the simplest but a workable theory, where the 4D chiral matters arise from the orbifolding of the fifth dimension.

The first part of this thesis, part I, includes the explanation of the cosmological and phenomenological aspects of the five-dimensional supergravity model with multiple moduli fields. First, we show the simple supersymmetric moduli inflation and the moduli stabilization in Chapter 2, where one can consider both small- and large-field inflations by employing the wavefunction localization of matter fields in the extra dimension [6]. The small-field inflation obtained is similar to the Starobinsky model [1], whereas the large-field inflation is categorized into a natural inflation. On the basis of the successful moduli inflation, we discuss particle phenomenology in the MSSM with low-scale SUSY-breaking in Chapter 3 and high-scale SUSY-breaking in Chapter 4, respectively. In the low-scale SUSY-breaking scenario, the gravitino becomes a viable dark matter candidate without contradicting to the Planck and LHC data, even if the SUSY-breaking effects are communicated by the gravity mediation [7]. On the other hand, in the high-scale SUSY-breaking scenario, sparticle spectra are similar to those of split SUSY [8], spread SUSY [9] and pure gravity mediation [10]. Then, the dark matter is considered as wino-like neutralino, whose relic abundance is originated from the nonthermal decay of gravitino [11].

In general, when the other fields oscillate after the inflation, they behave like matter fields and would dominate the universe. Since the moduli fields gravitationally couple to the matter fields in the standard model, their decay would occur after the epoch of Big-Bang Nucleosynthesis (BBN). Its problem is known as a cosmological moduli problem [12], because the successful BBN is violated by the moduli decay. However, in both scenarios, the cosmological moduli and gravitino problems can be solved simultaneously thanks to the structure of supersymmetric moduli inflation and stabilization.

The axion inflation and its cosmology in string theory

As a top-down approach, we have studied the string theory which is expected as a consistent theory of quantum gravity. In particular, we have focused on the type IIB string theory and heterotic string theory [13, 14] with the emphasis on the axion inflation and its cosmological consequences in each model in the second part of this thesis, part II.

When we start from the string theory, a lot of moduli fields and the axions appear in the low-energy effective theory, through a compactification of extra dimensions. The axions are defined as the imaginary part of moduli fields associated with the higher-dimensional tensor fields. First of all, we will briefly review the string axions and the possible inflation mechanisms based on them in the framework of the superstring effective action in Chapter 5. In the string setup, the axion potential is perturbatively prohibited by the gauge symmetry of the higher-dimensional tensor fields. One can generate the axion potential at the non-perturbative level, in which the continuous gauge symmetry is broken into the discrete one. Then, the axion potential is typically of the form of natural inflation. However, in this case, its decay constant should be much larger than the Planck scale to be consistent with the Planck data [15, 16]. It is difficult to realize such a trans-Planckian axion decay constant, because the string scale is typically lower than the Planck scale.

In order to overcome the difficulty about the trans-Planckian axion decay constant, we discuss the enhancement mechanism of axion decay constant by employing the (gauge) threshold corrections for the heterotic string theory in Chapter 6 and type IIB string theory in Chapter 7. Contrary to the previous studies, our inflation scenario predicts the modulation terms in the inflaton potential, which give sizable effects for the cosmological observables. Hence, it is possible to pursue our inflation scenario in the near-future cosmological observations. Finally, we summarize this thesis in Chapter 8.

Part I

Particle phenomenology and cosmology in five-dimensional supergravity models

Chapter 2

Moduli inflation and stabilization in 5D SUGRA

In this chapter, we investigate the moduli cosmology based on Ref. [6] within the framework of the off-shell supergravity. In this framework, one can write down the exact form of moduli kinetic terms and their scalar potential as well as their couplings to matter fields in the SM. In particular, the 5D supergravity, known as a minimal extension of the SM with the local SUSY and an extra dimension, has an off-shell formulation [17, 18] in the language of a local superconformal symmetry. This method allows us a systematic study for the 4D effective action of moduli and matter fields. Indeed, as shown in Ref. [19], the effective action obtained by the dimensional reduction keeps the off-shell structure of the 4D supergravity, which is written in terms of 4D $\mathcal{N}=1$ superspace [20, 21]. Therefore, after the gauge fixing of 4D superconformal symmetry, one can obtain the on-shell action. It is the starting point for the study of moduli inflation and its cosmology as will be discussed in Part I of this thesis. Before going to the detail of the moduli inflation, first we review the matter contents in the 5D off-shell supergravity action compactified on orbifold S^1/Z_2 with two fixed points.

2.1 Elements of 5D SUGRA on S^1/Z_2

In this section, we review the moduli effective action obtained by compactifying 5D off-shell supergravity on orbifold S^1/Z_2. Since the minimal spinor in 5D has eight real components (Dirac spinor), it corresponds to the four-dimensional $N=2$ supersymmetric theory. However, one has to break this $N=2$ SUSY to the $N=1$ to obtain chiral theory. The simple set up to carry out it is the S^1/Z_2 orbifold in fifth dimension that corresponds to an explicit breaking of one of the $N=2$ SUSY and Lorentz symmetry in five dimensions. In general, the 5D background metric preserving a 4D flatness is given by

$$ds^2 = G_{MN}dx^M dx^N = e^{-2f(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{2.1}$$

where $M,N=0,1,2,3,4$ are 5D spacetime indices, whereas $\mu,\nu=0,1,2,3$ are 4D spacetime indices with $\eta_{\mu\nu}$ =diag$(1,-1,-1,-1)$ being the metric of 4D Minkowski spacetime. The warp factor appearing in front of the metric of 4D Minkowski spacetime, $f(y)$ depends on the fifth coordinate y. The fundamental region of y is constrained within the range $0 \leq y \leq L$ under the Z_2 orbifold projection $y \to -y$. Furthermore, points on a circle are identified as $y \simeq y + L$ with L being the length of orbifold segment and $y = 0, L$ correspond to the fixed points. Such the orbifold projection restricts not only the background geometry but also the fields propagating the bulk in addition to the periodic condition $g(x, y + L) = g(x, y)$, where $g(x, y)$ represents an arbitrary 5D field. Indeed, any fields $g(x, y)$ are categorized in one of the two classes, such as Z_2-even and -odd fields, satisfying the Z_2 transformations $g(x, -y) = g(x, y)$ and $g(x, -y) = -g(x, y)$, respectively. Then, only Z_2-even fields have zero-modes which can appear in the low-energy effective theory below the compactification scale, whereas Z_2-odd fields do not have zero-modes as shown later.

Let us show an explicit example for the boundary condition under Z_2-orbifold. We first consider a complex scalar field with the Lagrangian in flat 5D spacetime,

$$S = \int d^4x dy \, \frac{1}{2} \left(\partial_\mu \phi^* \partial^\mu \phi + \partial_y \phi^* \partial^y \phi \right).$$

After the Kaluza-Klein (KK) expansion of this scalar field,

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu) e^{2\pi i n y/L},$$

we obtain

$$S = \frac{L}{2} \int d^4x \sum_{n=-\infty}^{\infty} \left(\partial_\mu \phi^{(n)*} \partial^\mu \phi^{(n)} - \left(\frac{2\pi n}{L} \right)^2 \phi^{(n)*} \phi^{(n)} \right).$$

Thus, we have an infinite tower of scalar fields $\phi^{(n)}$ with the mass-squared $(2\pi n/L)^2$. In particular, the massless complex scalar field $\phi^{(0)}$ is called a *zero-mode*. As a solution of breaking $N = 2$ SUSY, we introduce S^1/Z_2 orbifold by identifying points on a circle under the action, $y \to -y$. Under this Z_2 parity transformation, there are two types of scalar fields whose transformations are different from each other,

$$\begin{aligned} \phi(x, -y) = \phi(x, y) \Rightarrow \phi(x, y) &= \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu) \cos \left(\frac{2\pi n y}{L} \right) \quad (Z_2 - \text{even field}), \\ \phi(x, -y) = -\phi(x, y) \Rightarrow \phi(x, y) &= \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu) \sin \left(\frac{2\pi n y}{L} \right) \quad (Z_2 - \text{odd field}). \end{aligned} \tag{2.2}$$

The above arguments are also applied to fermions and gauge bosons.

2.1.1 The matter contents

First of all, we list the relevant matter contents of 5D SUGRA. From the structure of $\mathcal{N} = 2$ SUSY in 5D theory, there are three types of 5D supermultiplets such as Weyl multiplet (including gravity), hypermultiplets (including matter contents) and vector multiplets (including gauge fields). These supermultiplets can be decomposed into the supermultiplets in terms of 4D $\mathcal{N}=1$ SUSY owing to the structure of the orbifold.

In the following, we focus on the vector multiplets $\boldsymbol{V}^I$ with $I=1,2,\ldots,n_V$ and hypermultiplets $\boldsymbol{\Phi}_\alpha$ with $\alpha=1,2,\cdots,n_\Phi+n_C$ where n_C is the number of compensator hypermultiplets that play a role of gauge fixing local superconformal symmetry. After imposing the orbifold projection, these supermultiplets are decomposed into 4D vector multiplets V^I and three types of chiral multiplets Σ^I, Φ_α and Φ_α^C, that is, $\boldsymbol{V}^I=\{V^I,\Sigma^I\}$ and $\boldsymbol{\Phi}_\alpha=\{\Phi_\alpha,\Phi_\alpha^C\}$ in the language of 4D $\mathcal{N}=1$ SUSY. In addition to the usual Z_2-even vector multiplets V^I involving the vector multiplets in the SM, we introduce multiple Z_2-odd vector multiplets $\boldsymbol{V}^{I'}$ with $I'=1,2,\ldots,n'_V$. In this thesis, we identify the Z_2-odd vector fields $A_M^{I'}$ in $\boldsymbol{V}^{I'}$ as gauge fields of extra $U(1)_{I'}$ symmetries in addition to the SM gauge symmetries, for simplicity. Although extra $U(1)_{I'}$ symmetries are broken by orbifolding, Z_2-even chiral multiplets $\Sigma^{I'}$ have zero-modes where we denote these massless chiral multiplets as moduli chiral multiplets $T^{I'}$. The potential terms for such moduli are prohibited by the hidden $U(1)_{I'}$ gauge symmetry. For $n'_V=1$, the single modulus $T^{I'=1}$ is called as radion chiral multiplet satisfying $\langle \mathrm{Re}\, T^{I'=1}\rangle = L/\pi$, whereas, in the case of multi moduli $n'_V>1$, the radion is identified as a linear combination of $T^{I'}$s determined by their cubic polynomial function as presented later. Moreover, one of these moduli becomes an inflaton field as shown later. For the hypermultiplets $\boldsymbol{\Phi}_\alpha=\{\Phi_\alpha,\Phi_\alpha^C\}$, in the following, we define the zero-mode of chiral multiplets Φ_α as Q_α that includes the MSSM chiral multiplet, right-handed neutrino chiral multiplets, SUSY-breaking chiral multiplet and the stabilizer chiral multiplets $\mathcal{H}_i$ $(i=1,2,\ldots,n_H)$. The stabilizer multiplets play a role of generating a desirable moduli potential for the particle cosmology and phenomenology. These hypermultiplets $\boldsymbol{\Phi}_\alpha$ can have $U(1)_{I'}$ charges $c_{I'}^{(\alpha)}$ under the extra $U(1)_{I'}$ symmetries.

2.1.2 Moduli effective action in four-dimensional effective supergravity

First of all, we fix the number of vector and hypermultiplets in the framework of 5D supergravity. In the following, we choose $n_C=1$ for simplicity. The potential of moduli multiplets $T^{I'}$ is generated by introducing the same number of stabilizer hypermultiplets as that of Z_2-odd vector multiplets, that is, $n_H=n'_V$ as shown later. The 5D bulk action is characterized by a cubic polynomial of vector multiplets, so-called "norm function",

$$\mathcal{N}(M)=\sum_{I,J,K=1}^{n_V} C_{I,J,K}M^IM^JM^K, \tag{2.3}$$

with $C_{I,J,K}$ for $I,J,K=1,2,\ldots,n_V$ being real constants. If the 5D supergravity models are derived from the more fundamental theories, such as type IIB string theory on a warped throat or heterotic M-theory on Calabi-Yau (CY) manifold, these coefficients $C_{I,J,K}$ correspond to the intersection numbers of the CY manifolds [22].

So far, we have focused on the bulk configurations in the 5D SUGRA on S^1/Z_2. In addition to these bulk terms, in general, one can introduce the boundary terms at the orbifold fixed points $y=0,L$, where the $\mathcal{N}=2$ SUSY is partially broken down to the $\mathcal{N}=1$ SUSY.

Kähler and superpotential terms are allowed at Along the line of successful modulus stabilization as pointed out in Ref. [23], we consider the following superpotential for the stabilizer chiral multiplets $\mathcal{H}_i$ at the boundary fixed points,

$$\mathcal{W} = J_0^{(i)}\,\mathcal{H}_i\,\delta(y) + J_L^{(i)}\,\mathcal{H}_i\,\delta(y-L), \tag{2.4}$$

with $J_{0,L}^{(i)}$ being the real constants. We now assume that such linear terms of $\mathcal{H}_i$ are dominant [19] in the superpotential $\mathcal{W}$ compared with the other terms. This assumption would be ensured by some symmetries or dynamics. (See, Ref. [24] for a similar moduli ptential in the case of $n_C = 2$.) Even if the higher-order terms of $\mathcal{H}_i$ appear in the superpotential, these terms could be suppressed due to the almost vanishing vacuum expectation values of $\mathcal{H}_i$ as will be shown in the following analysis. It is also supposed that the boundary terms in the Kähler potential are also negligible compared with the bulk terms in the Kähler potential, which can be ensured when the volume of fifth dimension, L/π, is larger enough than the inverse of the mass scales originating from these terms.

Now the supergravity action for moduli multiplets is completely determined. In accordance with Ref. [19], we can integrate over the fifth coordinate y out the Kaluza-Klein (KK) expansion of fields keeping the $\mathcal{N} = 1$ SUSY. As a result, one can extract the following 4D Kähler potential K and the superpotential W of moduli ($T^{I'}$) and zero-mode of stabilizer fields (H_i),

$$\begin{aligned} K &= -\ln \mathcal{N}(\mathrm{Re}\,T) + Z_{i,\bar{i}}(\mathrm{Re}\,T)\,|H_i|^2, \\ W &= \left(J_0^{(i)} + e^{-c_{I'}^{(i)} T^{I'}}\,J_L^{(i)}\right) H_i, \end{aligned} \tag{2.5}$$

where

$$Z_{i,\bar{j}}(\mathrm{Re}\,T) = \frac{1 - e^{-2c_{I'}^{(i)}\mathrm{Re}\,T^{I'}}}{c_{I'}^{(i)}\mathrm{Re}\,T^{I'}}\delta_{ij}, \tag{2.6}$$

is the Kähler metric of 4D zero-modes H_i. It is remarkable that the exponential factors in K and W originate from the wavefunctions' profile of zero-modes H_i in fifth direction [19], $H_i|_{y=L} = e^{-c_{I'}^{(i)} T^{I'}}\,H_i|_{y=0}$. The quasi-localization of H_i is controlled by the $U(1)_{I'}$ charges $c_{I'}^{(i)}$ of H_i corresponding to the 5D bulk mass. The exponential behavior plays important roles of not only realizing a successful moduli inflation and stabilization, but also the hierarchical Yukawa couplings of quarks and leptons as will be discussed in the Chapter 4.

In this way, we now discuss the effective 4D scalar potential V for moduli and stabilizer fields in the framework of 4D SUGRA, in which the scalar potential is provided by*

$$V = e^K\left(K^{m,\bar{n}}\,D_m W\,D_{\bar{n}}\bar{W} - 3|W|^2\right), \tag{2.7}$$

with $W_m = \partial_m W$, $K_m = \partial_m K$ and m, n runs over moduli fields ($I' = 1, 2, \cdots, n'_V$) and stabilizer fields ($i = 1, 2, \cdots, n_H$). $D_m W = W_m + K_m W$ is the Kähler covariant derivative

*Here and in what follows, we employ the reduced Planck unit $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV $= 1$ and the same notation between the fields and their chiral multiplets, unless we specify them.

for the superpotential and $K^{m,\bar{n}}$ denotes the inverse of Kähler metric $K_{m,\bar{n}} = \partial_m \partial_{\bar{n}} K$. The extremal condition of them $\langle \partial_{I'} V \rangle = \langle \partial_i V \rangle = 0$ is satisfied by the supersymmetric conditions $\langle D_{I'} W \rangle = \langle D_i W \rangle = 0$. We find that the expectation values of moduli $T^{I'}$ and stabilizer fields H_i become

$$c_{I'}^{(i)} \langle T^{I'} \rangle = \ln \frac{J_L^{(i)}}{J_0^{(i)}}, \qquad \langle H_i \rangle \;=\; 0, \tag{2.8}$$

which is also discussed in the single modulus case [23]. When there are no moduli mixings in the Kähler metric, $K_{I',\bar{J}'} = 0$ for $I' \neq J'$, their supersymmetric masses are also estimated at a minimum

$$m_{I'i}^2 \simeq \frac{e^{\langle K \rangle} \langle W_{I'i} \rangle^2}{\langle K_{I',\bar{I}'} \rangle \langle K_{i,\bar{i}} \rangle}, \tag{2.9}$$

where $W_{mn} = \partial_m \partial_n W$ with $\langle W_{I'i} \rangle = -c_{I'}^{(i)} e^{-c_{I'}^{(i)} \langle T^{I'} \rangle} J_L^{(i)}$. From the mass formula (2.9), the mass-squared of moduli and stabilizer fields are exponentially suppressed by the factor $e^{-c_{I'}^{(i)} \langle T^{I'} \rangle}$ which is originated in the wavefunction localization in the fifth dimension. Then, it enable us to verify the description of 4D effective theory, because the compactification scale (typically KK mass) can be lower than that of moduli and stabilizer fields. Furthermore, the obtained minimum given in Eq. (2.8) lead to the supersymmetric Minkowski minimum as can be seen in Eqs. $\langle V \rangle = 0$ and $\langle W \rangle = 0$.

Let us comment on the moduli mixing in the Kähler metric. When there is a sizable moduli mixing in the Kähler metric, $K_{I',\bar{J}'} \neq 0$ for $I' \neq J'$, a saddle point or a local maximum would appear in the scalar potential. In order to avoid the destabilization of moduli fields, the coefficients $C_{I',J',K'}$ in the norm function (2.3) are constrained to be an almost diagonal Kähler metric of moduli fields, $K_{I',\bar{J}'} \approx 0$ for $I' \neq J'$. However, the hierarchical supersymmetric masses $|\langle W_{I'i} \rangle| \ll |\langle W_{J'j} \rangle|$ $(\exists I', J', i, j)$ do not lead to the above situation even in the case of a sizable Kähler mixing as utilized in Sec. 2.3.

From a particle phenomenological point of view, no existence of SUSY implies that SUSY should be broken above the TeV scale. When we add a SUSY-breaking sector with almost vanishing cosmological constant $\langle V \rangle \approx 0$, the moduli stabilization will be generically affected by them. For the time being, we assume that the SUSY-breaking scale is much smaller than the supersymmetric mass (2.9) in order not to affect the moduli potential. In this case, the deviation from the supersymmetric Minkowski minimum (2.8) is negligible. The above assumption is verified in Sec. 2.4 by incorporating the moduli potential and SUSY-breaking sector at the same time.

2.2 Moduli inflation (small-field inflation)

From now on, we show that the discussed moduli potential induces a small-field moduli inflation. The prediction of cosmological observables are better fitted with recent Planck data [15, 16].

First of all, we consider the situation where one pair of modulus and stabilizer fields, e.g. $T^{I'=1}$ and $H_{i=1}$, is decoupled from the other pairs $T^{I'\neq 1}$ and $H_{i\neq 1}$. By setting $c_{I'=1}^{(i\neq 1)} = c_{I'\neq 1}^{(i=1)} = 0$ in Eq. (2.5), $\left|c_{I'\neq 1}^{(i\neq 1)}\right| < \left|c_{I'=1}^{(i=1)}\right|$ and $\left|J_{0,L}^{(i=1)}\right| < \left|J_{0,L}^{(i\neq 1)}\right|$ in Eq. (2.9), the other pairs of moduli and stabilizer fields are much heavier than the light pair. Below the heavier mass scale $m_{I'\neq 1, i\neq 1}$, the other pairs ($T^{I'\neq 1}$, $H_{i\neq 1}$) are replaced by their vacuum expectation values given at their supersymmetric minimum (2.8). Thus, one can extract the potential of the lightest pair ($T^{I'=1}$, $H_{i=1}$) with the following effective Kähler potential and superpotential,

$$\begin{aligned} K_{\rm eff}(T^1, H_1) = K(T^{I'}, H_i)\Big|_0 &= -\ln \mathcal{N}({\rm Re}\, T)\Big|_0 + Z_{1,\bar{1}}({\rm Re}\, T)\Big|_0 |H_1|^2, \\ W_{\rm eff}(T^1, H_1) = W(T^{I'}, H_i)\Big|_0 &= \left(J_0^{(1)} + e^{-c_1^{(1)} T^1} J_L^{(1)}\right) H_1, \end{aligned} \tag{2.10}$$

respectively. Here and in what follows, we denote $f(T^{I'}, H_i)\Big|_0 \equiv f(T^{I'}, H_i)\Big|_{\substack{T^{I'\neq 1} = \langle T^{I'\neq 1}\rangle \\ H_{i\neq 1} = \langle H_{i\neq 1}\rangle}}$ for an arbitrary function $f(T^{I'}, H_i)$. The Kähler metric of H_1 is then given by

$$Z_{1,\bar{1}}({\rm Re}\, T)\Big|_0 = \frac{1 - e^{-2c_1^{(1)}{\rm Re}\, T^1}}{c_1^{(1)}{\rm Re}\, T^1}.$$

The effective potential for the light pair ($T^{I'=1}$, $H_{i=1}$) is calculated by means of the effective Kähler potential and superpotential (2.10), where $m, n = \{T^{I'}, H_i\}$ with $I' = 1$ and $i = 1$,

$$V_{\rm eff}(T^1, H_1) = e^{K_{\rm eff}} \left((K_{\rm eff})^{m,\bar{n}}\, D_m W_{\rm eff}\, D_{\bar{n}} \bar{W}_{\rm eff} - 3|W_{\rm eff}|^2\right). \tag{2.11}$$

2.2.1 Moduli potential

Next, we study the detail of effective potential for the modulus T^1 given in Eq. (2.11). On the $H_1 = 0$ hypersurface, it reads as

$$\begin{aligned} V_{\rm eff}(T^1, H_1 = 0) &= e^{K_{\rm eff}} (K_{\rm eff})^{i=1,\bar{i}=\bar{1}}\, |(W_{\rm eff})_{i=1}|^2 \Big|_{H_1=0} \\ &= \frac{c_1^{(1)}{\rm Re}\, T^1}{\mathcal{N}({\rm Re}\, T)\Big|_0} \times \frac{\left|J_0^{(1)}\right|^2 \left|1 + \frac{J_L^{(1)}}{J_0^{(1)}} e^{-c_1^{(1)} T^1}\right|^2}{1 - e^{-2c_1^{(1)}{\rm Re}\, T^1}}, \end{aligned} \tag{2.12}$$

where $(K_{\rm eff})^{i,\bar{i}}\Big|_{H_1=0} = 1/Z_{i,\bar{i}}({\rm Re}\, T)\Big|_0$. The first factor in Eq. (2.12), i.e., the polynomial term is canceled out in the restricted case,

$$\mathcal{N}({\rm Re}\, T)\Big|_0 = \mathcal{P}_0\, {\rm Re}\, T^1, \tag{2.13}$$

where $\mathcal{P}_0$ is independent of T^1. Then, the potential has the exponential form as drawn in Fig. 2.1, in which $V_{\rm eff}(T^1, H_1)/V_\infty$ is plotted on the ${\rm Im}\, T^1 = H_1 = 0$ hypersurface with the following choice of parameters,

$$c_1^{(1)} = 1/10, \qquad J_L^{(1)}/J_0^{(1)} = -3, \qquad J_0^{(1)} = 2.5 \times 10^{-4}, \tag{2.14}$$

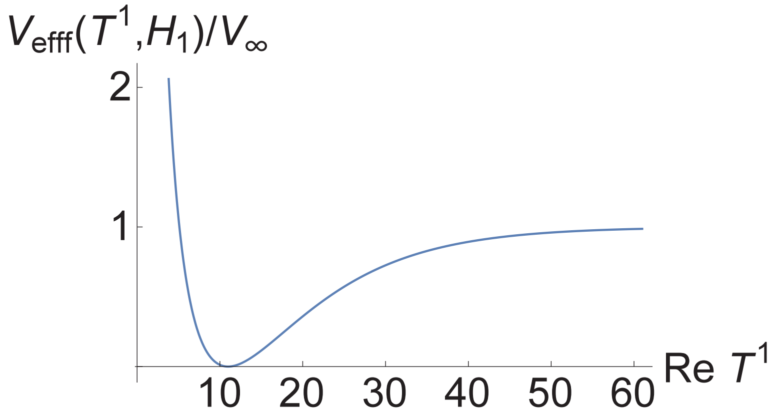

Figure 2.1: The inflaton potential $V_{\rm eff}(T^1, H_1)/V_\infty$ on the $\mathrm{Im}\, T^1 = H_1 = 0$ hypersurface as drawn in Fig. 1 in Ref. [6].

in the reduced Planck unit. In Fig. 2.1, the flat region of the potential is originating from the following structures,

$$\begin{aligned} &\lim_{\mathrm{Re}\, T^1 \to 0} \left| V_{\rm eff}(T^1, H_1 = 0) \right| = \infty, \\ &\lim_{\mathrm{Re}\, T^1 \to \infty} V_{\rm eff}(T^1, H_1 = 0) = c_1^{(1)} \mathcal{P}_0^{-1} \left| J_0^{(1)} \right|^2 \equiv V_\infty, \end{aligned} \tag{2.15}$$

for $J_L^{(1)}/J_0^{(1)} \neq -1$ and $c_1^{(1)} > 0$. The overshooting to negative region $\mathrm{Re}\, T^1 < 0$ is prohibited as can be understood in Eq (2.15).

The obtained potential is identical to that of Starobinsky inflation [1] in the large positive value of $\mathrm{Re}\, T^1$, although the origin of the potential is different. When we identify $\mathrm{Re}\, T^1$ as a inflaton field, it will slowly rolls down to the minimum given by Eq. (2.8) with $i = 1$ from its large positive initial value.

We have analyzed the tree-level Kähler potential. In contrast to the superpotential, the one-loop correction appears in the moduli Kähler potential which is calculated in the large volume limit [25],

$$K = -\ln \mathcal{N} + \mathcal{O}\left(\frac{1}{32\pi^2 \mathcal{N}} \right), \tag{2.16}$$

where the correction terms depend on the number of the charged fields for the gauge fields in Z_2-odd vector multiplets $V^{I'}$. Nevertheless, such contributions in the scalar potential are negligible in the previous analysis, since the moduli fields are stabilized at a minimum of potential independently to the Kähler potential. Furthermore, the one-loop effects do not alter the inflation mechanism in the range $\mathrm{Re}\, T^1 \gg 1$, where the inflaton slowly rolls the potential. We confirm the effects of one-loop corrections in the following numerical analysis.

Finally, we remark a necessary and sufficient condition for the existence of flatness of the potential in the large $\mathrm{Re}\, T^1$ region. As the most general form of the norm function satisfying the assumption (2.13), we find that

$$\mathcal{N}(M) = \mathcal{P}(M)\, M^1, \tag{2.17}$$

where

$$\mathcal{P}(M) = \sum_{J',K'\neq 1}^{n'_V} C_{1,J',K'} M^{J'} M^{K'}, \tag{2.18}$$

is a quadratic polynomial of fields $M^{I'\neq 1}$ in Z_2-odd vector multiplets $\boldsymbol{V}^{I'\neq 1}$ apart from $\boldsymbol{V}^{I'=1}$, up to the fields $M^{I''}$ in Z_2-even vector multiplets $\boldsymbol{V}^{I''} = \{V^{I''}, \Sigma^{I''}\}$ with $I'' = n'_V+1, n'_V+2, \ldots, n_V$. Z_2-odd chiral multiplets $\Sigma^{I''}$ are different from $\Sigma^{I''}$ carrying the moduli fields. When the other fields are larger enough than T^1, the coefficient $\mathcal{P}_0$ of $\mathrm{Re}\, T^1$ in Eq. (2.13) is repressed by their vacuum expectation values as shown in Eq. (2.18), i.e., $\mathcal{P}_0 = \mathcal{P}(\mathrm{Re}\, T)\Big|_0$. Following the above argument, the suitable flat region for the slow-roll inflation is achieved in a moduli potential generated by the superpotential (2.4) and the peculiar form of the norm function (2.19) in 5D SUGRA on S^1/Z_2. Note that, a cubic polynomial norm function $\mathcal{N}(M)$ prohibit the condition (2.19) for the single modulus case $n'_V = 1$.

Although the form of the norm function is undermined at the level of 5D SUGRA, the norm function is directly related to the topology of internal manifold, if the 5D SUGRA is taken as an effective description of certain ultraviolet theory such as heterotic M-theory on Calabi-Yau three-fold. Then, norm function is identical to the $\mathcal{N} = 2$ prepotential and their coefficients are related to the intersection numbers of Calabi-Yau three-fold [22]. The moduli would be identified as the closed string moduli.

2.2.2 The inflation dynamics and cosmological observables

In this section, we show the details of inflaton dynamics by identifying the real part of the lightest modulus, $\mathrm{Re}\, T^1$, as the inflaton field. One can consider any number of moduli fields in Z_2-odd vector multiplets $\boldsymbol{V}^{I'}$ $n'_V \geq 2$. In the following, we choose $n'_V = 3$ in the light of particle cosmology and phenomenology as will be shown in Chapters 3 and 4. The form of norm function

$$\mathcal{N}(\mathrm{Re}\, T) = \mathrm{Re}\, T^1 \mathrm{Re}\, T^2 \mathrm{Re}\, T^3, \tag{2.19}$$

is chosen in order to realize a diagonal Kähler metric of moduli fields, for simplicity. Its choice corresponds to the choice $\mathcal{P}_0 = \mathrm{Re}\, T^2 \mathrm{Re}\, T^3$ in Eq. (2.18). As mentioned in the previous section, one pair of moduli and stabilizer fields (T^1, H_1) can be lighter enough than other pairs $(T^{I'\neq 1}, H_{i\neq 1})$. Furthermore, in the following, we assume that the oscillations of the other light fields $\mathrm{Im}\, T^1$, $\mathrm{Re}\, H_1$ and $\mathrm{Im}\, H_1$ around their expectation values $\langle \mathrm{Im}\, T^1 \rangle = \langle \mathrm{Re}\, H_1 \rangle = \langle \mathrm{Im}\, H_1 \rangle = 0$ are negligible during and after the inflation. For $\mathrm{Re}\, H_1$ and $\mathrm{Im}\, H_1$, their minima of the scalar potential are fixed around the origin by the Hubble-induced masses and their supersymmetric masses during and after the inflation, respectively. $\mathrm{Im}\, T^1$ is also stabilized at the origin by its supersymmetric mass. The fluctuations of these fields are negligible to the inflaton dynamics as explicitly shown in Appendix A.

On the $\mathrm{Im}\, T^1 = \mathrm{Re}\, H_1 = \mathrm{Im}\, H_1 = 0$ hypersurface of the field space, the dynamics of single field $\sigma \equiv \mathrm{Re}\, T^1$ is given by its equation of motion

$$\ddot{\sigma} + 3H\dot{\sigma} + \Gamma^{\sigma}{}_{\sigma\sigma}\dot{\sigma}^2 + g^{\sigma\sigma}\frac{\partial V_{\mathrm{eff}}}{\partial \sigma} = 0, \tag{2.20}$$

where the dot denotes the derivative with respect to a cosmic time t, $g_{\sigma\sigma} = 2(K_{\mathrm{eff}})_{I'=1,J'=1}$, $g^{\sigma\sigma} = g_{\sigma\sigma}^{-1}$ and $\Gamma^{\sigma}{}_{\sigma\sigma} = -1/\sigma$ is the Christoffel symbol constructed by the metric $g_{\sigma\sigma}$. The Hubble parameter H is defined in terms of the scale factor of 4D spacetime,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{6}g_{\sigma\sigma}\dot{\sigma}^2 + \frac{V_{\mathrm{eff}}}{3}, \tag{2.21}$$

where V_{eff} is the effective potential (2.11).

For a computational reason, we change the variable from the cosmic time to the e-folding number $N \equiv \ln a(t)$ in Eq. (2.20). By substituting the explicit form of H and $\Gamma^{\sigma}{}_{\sigma\sigma}$ into the Eq. (2.20), the equation of motion of σ is rewritten as

$$\sigma'' = -\left(1 - \frac{g_{\sigma\sigma}(\sigma')^2}{6}\right)\left(3\sigma' + 6\sigma^2\frac{V'_{\mathrm{eff}}}{V_{\mathrm{eff}}}\right) + \frac{(\sigma')^2}{\sigma}, \tag{2.22}$$

where the prime denotes the derivative with respect to N.

The numerical values of parameters for the light pair of fields (T^1, H_1) and heavy pairs of fields ($T^{2,3}$, $H_{2,3}$) are chosen as (2.14) and

$$c_2^{(2)} = c_3^{(3)} = \frac{1}{50}, \qquad J_0^{(2)} = J_0^{(3)} = -\frac{1}{9}, \qquad J_L^{(2)} = J_L^{(3)} = 1, \tag{2.23}$$

respectively. By inserting these values into Eq. (2.8), their vacuum expectation values are numerically estimated as

$$\langle T^1 \rangle \simeq 11, \qquad \langle T^2 \rangle = \langle T^3 \rangle \simeq 110, \qquad \langle H_1 \rangle = \langle H_2 \rangle = \langle H_3 \rangle = 0, \tag{2.24}$$

which lead to the supersymmetric mass-squared (2.9) of the light pair (T^1, H_1) and heavy pairs ($T^{2,3}$, $H_{2,3}$),

$$m^2_{I'=1,i=1} \simeq \left(4 \times 10^{12}\ \mathrm{GeV}\right)^2, \qquad m^2_{I'=2,i=2} = m^2_{I'=3,i=3} \simeq \left(4.8 \times 10^{15}\ \mathrm{GeV}\right)^2. \tag{2.25}$$

On the other hand, the inflation scale is characterized by the Hubble parameter

$$H_{\mathrm{inf}} \equiv (V_\infty/3)^{1/2} \simeq 1.0 \times 10^{12}\ \mathrm{GeV}, \tag{2.26}$$

with V_∞ given in Eq. (2.15). Because all of these scales $m_{I'=1,i=1}$, $m_{I'=2,3,i=2,3}$ and H_{inf} are below the compactification scale (typical Kaluza-Klein mass scale)

$$M_C \equiv \frac{\pi}{L} \simeq \frac{\pi}{\langle \mathcal{N}^{1/2} \rangle} \simeq 2.1 \times 10^{16}\mathrm{GeV},$$

the 4D effective-theory description is ensured in the present choice of parameters. The Kaluza-Klein scale is reduced to the grand unification theory (GUT) scale owing to the mild large volume of the fifth dimension, $\langle \mathcal{N}^{1/2} \rangle \simeq 364$. As shown in the next Chapter 3, such a mild large volume plays important roles of gauge coupling unification at GUT scale in the framework of MSSM and suppression of undermined boundary Kähler potential. It is also confirmed that the heavy pairs of fields $(T^{2,3},\ H_{2,3})$ are decoupled from the inflation dynamics because of $m_{I'=1,i=1} \sim H_{\rm inf} \ll m_{I'=2,i=2}, m_{I'=3,i=3}$. Hence, their oscillations can be neglected and they are fixed by their own superpotential in Eq. (2.5).

From now on, we discuss whether a small-field inflation is consistent with the recent Planck data or not. To estimate the cosmological observables for the CMB, we first define the generalized slow-roll parameters for the inflaton with its non-canonical kinetic term [26, 27],

$$\epsilon \equiv \frac{M_{Pl}^2}{2}\frac{\partial_\sigma V_{\rm eff} g^{\sigma\sigma}\partial_\sigma V_{\rm eff}}{V_{\rm eff}^2} \sim (2c_1^{(1)}\sigma)^2 \left(\frac{J_L^{(1)}}{J_0^{(1)}} e^{-c_1^{(1)}\sigma}\right)^2,$$
$$\eta \equiv \frac{\nabla^\sigma \nabla_\sigma V_{\rm eff}}{V_{\rm eff}} = \frac{g^{\sigma\sigma}\partial_\sigma^2 V_{\rm eff} - g^{\sigma\sigma}\Gamma^\sigma_{\sigma\sigma}\partial_\sigma V_{\rm eff}}{V_{\rm eff}} \sim -(2c_1^{(1)}\sigma)^2 \frac{J_L^{(1)}}{J_0^{(1)}} e^{-c_1^{(1)}\sigma}, \tag{2.27}$$

where ∇_σ is the Kähler covariant derivative with respect to the field σ. By employing these generalized slow-roll parameters, the observed quantities such as the power spectrum of adiabatic curvature perturbation, its spectral tilt and the tensor-to-scalar ratio are brought into the following form,

$$P_\xi(k) = \frac{1}{24\pi^2}\frac{V}{\epsilon M_{\rm Pl}^4},$$
$$n_s = 1 + \frac{d\ln P_\xi(k)}{d\ln k} \simeq 1 - 6\,\epsilon + 2\,\eta,$$
$$r = 16\,\epsilon. \tag{2.28}$$

With the initial conditions $\sigma = 114$ and $\sigma' = 0$ at $N = 0$, we numerically solve Eq. (2.22). From the trajectory of σ in Fig. 2.2, we find that the inflation ends and oscillates around $N_{\rm end} \simeq 72.3$ where the slow-roll condition is violated as $\max\,\{\epsilon, \eta\} = 1$.

In order to estimate the e-folding number after the pivot scale $k_0 = 0.05$ [Mpc^{-1}], we denote the scalar potential as $V_*^{1/4} \equiv V^{1/4}(\sigma_*)$ with its field value at the pivot scale $\sigma = \sigma_*$. The scalar potential at the end of inflation is defined as $V_{\rm end}^{1/4} \equiv V^{1/4}(\sigma_{\rm end})$. From the simple formula of e-folding number after the pivot scale given by [28], one can realize an enough amount of e-folding number

$$N_{\rm e} \;\equiv\; N_{\rm end} - N_* \simeq 62 + \ln\frac{V_*^{1/4}}{10^{16}\ {\rm GeV}} + \ln\frac{V_*^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\ln\frac{V_{\rm end}^{1/4}}{\rho_R^{1/4}} \;\simeq\; 56. \tag{2.29}$$

It is now assumed that the energy of inflaton is instantaneously converted into radiation. Furthermore, we employ $V_*^{1/4} \simeq V_{\rm end}^{1/4} \simeq 2\times 10^{15}$ GeV and the energy density of the universe at the

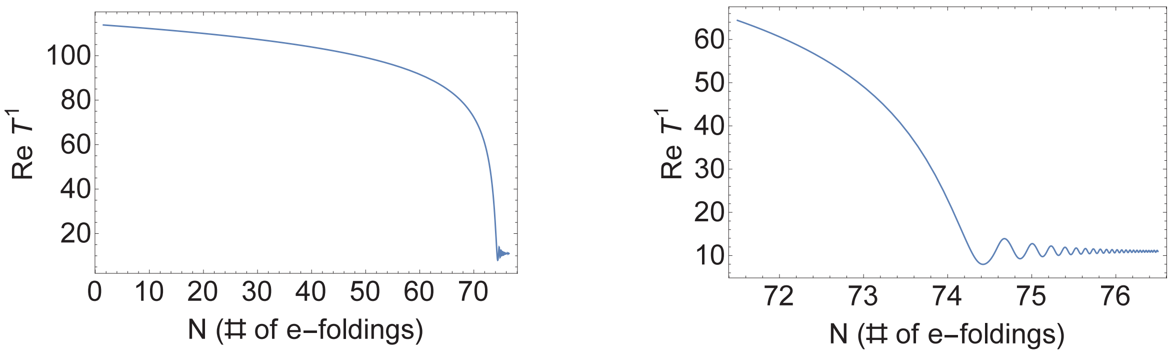

Figure 2.2: The trajectory of inflaton field $\sigma = \mathrm{Re}\, T^1$ as a function of the e-folding number N as drawn in Fig. 2 in Ref. [6].

reheating epoch $\rho_R^{1/4} = (\pi^2 g_*/30) T_R \simeq 1 \times 10^{11}$ GeV. As determined later in Sec. 2.2.3, the effective degrees of freedom of radiation $g_* = 915/4$ at the reheating temperature $T_R \simeq 1.38 \times 10^9$ GeV are fixed by assuming the MSSM matter contents.

By contrast, on the basis of the slow-roll approximation, the same number $N_{\rm e}$ is defined as

$$N_{\rm e} \;=\; -\int_{t_{\rm end}}^{t_*} d\tilde{t}\, H(\tilde{t}) \;\simeq\; \frac{1}{M_{\rm Pl}^2}\int_{\sigma_{\rm end}}^{\sigma_*} d\sigma \frac{V_{\rm eff}}{g^{\sigma\sigma} V'_{\rm eff}}, \tag{2.30}$$

and then the numerical value

$$\sigma_* \simeq 111 \tag{2.31}$$

is determined by equating Eq. (2.29) with Eq. (2.30).

Next, we focus on the Planck normalization on the power spectrum of adiabatic curvature perturbation, $P_\xi(k_0) = 2.20 \pm 0.10 \times 10^{-9}$ [15, 16]. At the pivot scale k_0, the slow-roll parameters ϵ and η are obtained by employing the numerical value (2.31),

$$\begin{aligned} \epsilon &\sim (2c_1^{(1)}\sigma)^2 \left(\frac{J_L^{(1)}}{J_0^{(1)}} e^{-c_1^{(1)}\sigma}\right)^2 \Bigg|_{\sigma=\sigma_*} \simeq \mathcal{O}(10^{-6}), \\ \eta &\sim -(2c_1^{(1)}\sigma)^2 \frac{J_L^{(1)}}{J_0^{(1)}} e^{-c_1^{(1)}\sigma}\Bigg|_{\sigma=\sigma_*} \simeq \mathcal{O}(-0.02). \end{aligned} \tag{2.32}$$

They yield the correct order of the observed power spectrum, $P_\xi(k_0) \sim 2.2 \times 10^{-9}$. Inversely speaking, the parameters $J_{0,L}^{(1)}$ are set as those in Eq. (2.14) in order to realize that resultant $P_\xi(k_0)$ resides within the observed region.

In our model, by employing Eq. (2.28) and Eq. (2.32), we can also realize the correct value of the spectral tilt of curvature perturbation, $n_s \simeq 0.96$ reported by the Planck collaborations $n_s = 0.9655 \pm 0.0062$ [15, 16]. In 4D supergravity inflation models, the slow-roll parameter η is likely to be of order 1, i.e., $|\eta| \simeq 1$ at the pivot scale. It is called as η problem peculiar to the 4D supergravity framework. However, our inflation model is free from such η problem because of the exponential factor and the large value of the inflaton field (real part of modulus field).

We summarize the numerical results of inflation dynamics. With the sample values of parameters (2.14) and (2.23), the numerical values of cosmological observables are estimated as

$$P_\xi = 2.23 \times 10^{-9}, \qquad n_s = 0.96, \qquad r = 1.6 \times 10^{-5}, \tag{2.33}$$

with the enough e-foldings $N_e \simeq 56$. The tiny values of tensor-to-scalar ratio is outside the current sensitivity of Planck,

$$r < 0.11, \tag{2.34}$$

at the scale $k_* = 0.05\,[\mathrm{Mpc}^{-1}]$ [15, 16]. The running of the scalar spectral index is also negligible, relative to the current observational sensitivity. Hence, the analyzed inflation model is consistent with the Planck data [15, 16]. It is remarkable that this inflation mechanism is categorized as the small-field inflation, since the field variable of the canonically normalized inflaton field $\sigma = \mathrm{Re}\,T^1$ is smaller than the reduced Planck scale,

$$\Delta\hat{\sigma} \equiv \hat{\sigma}_* - \hat{\sigma}_{\mathrm{end}} \simeq 0.3 M_{\mathrm{Pl}}, \quad \hat{\sigma} = \frac{1}{2}\log\sigma. \tag{2.35}$$

In this class of small-field inflation, the tensor-to-scalar ratio is suppressed by the tiny slow-roll parameter ϵ as can be seen in Eq. (2.33). Although the obtained predictions are in agreement with the current Planck data, it is hard to detect to them even in the near-future experiments.

In Sec. 2.3, we will show a large-field inflation which is one of the few candidates to generate a detectable tensor-to-scalar ratio on the basis of similar moduli potential.

2.2.3 Reheating process

Before going to the detail of large-field inflation, we discuss the reheating process. After the end of inflation, the coherent oscillation of inflaton field dominates the energy density of the universe and releases the entropy. When inflaton decays into the particles in the supersymmetric standard model, the universe is thermalized. For the matter content of the visible sector in the 4D effective theory, we consider that of MSSM. Although the inflaton decay is expected to be model-dependent, the main decay channel is the inflaton into gauge-boson pairs through the gauge kinetic function as confirmed in Chapters 3 and 4. We also discuss the other decay channels later. Thus, the reheating temperature is roughly estimated from this main decay channel.

The moduli fields couple to gauge fields through the following Lagrangian,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\mathrm{Re}\,f_r F^r_{\mu\nu}F^{a\mu\nu} \\ &= -\frac{1}{4}\langle \mathrm{Re}\,f_r\rangle F^r_{\mu\nu}F^{r\mu\nu} - \frac{1}{4}\left\langle \frac{\partial\,\mathrm{Re}\,f_r}{\partial\hat{\sigma}}\right\rangle \delta\hat{\sigma} F^r_{\mu\nu}F^{r\mu\nu},\end{aligned} \tag{2.36}$$

where $f_r = \sum_{I'=1}^{3}\xi^r_{I'}T^{I'}$ with $\xi^r_{I'} \equiv C_{I',J''=r,K''=r}$ is the bulk gauge kinetic function $f_r(T)$. The index $r = 1, 2, 3$ stands for the gauge groups in the standard model, $U(1)_Y$, $SU(2)_L$,

$SU(3)_C$ respectively. The total decay width from the canonically normalized inflaton $\hat{\sigma}$ into the gauge-bosons ($g^{(r)}$) is calculated as

$$\Gamma \simeq \sum_{r=1}^{3} \Gamma(\hat{\sigma} \to g^{(r)} + g^{(r)}) = \sum_{r=1}^{3} \frac{N_G^r}{128\pi} \left\langle \frac{\xi_r^1}{\sqrt{(K_{\rm eff})_{T^1 T^1}} {\rm Re}\, f_r} \right\rangle^2 \frac{m_{\hat{\sigma}}^3}{M_{Pl}^2} \simeq 3.95\,{\rm GeV}, \tag{2.37}$$

where $\{N_G^1, N_G^2, N_G^3\} = \{1, 3, 8\}$ is the number of the gauge bosons in the MSSM. The coefficients in the gauge kinetic function are the free parameters in 5D SUGRA. We chose them as $\xi_1^1 = \xi_1^2 = \xi_1^2 = 0.22$ and otherwise zero so that the gauge coupling unification at the GUT scale is realized,

$$ {\rm Re}\, f_a(\langle T \rangle) = \left(\frac{1}{g_a} \right)^2 \simeq 3.73. \tag{2.38}$$

When the inflaton decays into gauge-boson pairs instantaneously, the reheating temperature is roughly estimated by equating its total decay width with the expansion rate of the universe,

$$\begin{aligned} \Gamma &\simeq H(T_R), \\ \Leftrightarrow T_R &= \left(\frac{\pi^2 g_*}{90} \right)^{-1/4} \sqrt{\Gamma\, M_{\rm Pl}} \simeq 1.38 \times 10^9\,{\rm GeV}, \end{aligned} \tag{2.39}$$

with $g_* = 915/4$ at the reheating in the MSSM. As will be verified in Sec. 2.4, other heavy fields do not oscillate so much and do not dominate the energy density of the universe.

2.3 Moduli inflation (large-field inflation)

In this section, we show the large-field inflation on the basis of the 4D effective theory. In contrast to the small-field inflation, the large-field inflation produces a sizable gravitational wave which could be checked by the near-future observations. In the following, we focus on two light pairs of modulus and stabilizer fields, e.g., $(T^{I'},\, H_i)$ with $I' = i = 1, 2$. The other pairs $T^{I' \neq 1,2}$ and $H_{i \neq 1,2}$ can be heavier enough than the light pairs under the following choice of parameters $c_{I'=1,2}^{(i\neq 1,2)} = c_{I'\neq 1,2}^{(i=1,2)} = 0$ in Eq. (2.5), $\left| c_{I'\neq 1,2}^{(i\neq 1,2)} \right| < \left| c_{I'=1,2}^{(i=1,2)} \right|$ in Eq. (2.9), $\left| J_0^{i=1,2} \right| < \left| J_0^{i\neq 1,2} \right|$ and $\left| J_L^{i=1,2} \right| < \left| J_L^{i\neq 1,2} \right|$ in Eq. (2.9). One can replace the heavy pairs by their expectation values below the heavier mass scale $m_{I'\neq 1,2\, i\neq 1,2}$. Then, the effective Kähler potential and superpotential for the light pairs $(T^{I'},\, H_i)$ with $I' = i = 1, 2$ are described by

$$\begin{aligned} K_{\rm eff}(T^1, H_1, T^2, H_2) &= -\ln \mathcal{N}({\rm Re}\, T) \Big|_0 + Z_{1,\bar{1}}({\rm Re}\, T) \Big|_0 |H_1|^2 + Z_{2,\bar{2}}({\rm Re}\, T) \Big|_0 |H_2|^2, \\ W_{\rm eff}(T^1, H_1, T^2, H_2) &= \left(J_0^{(1)} + e^{-c_{I'}^{(1)} T^{I'}} J_L^{(1)} \right) H_1 + \left(J_0^{(2)} + e^{-c_{I'}^{(2)} T^{I'}} J_L^{(2)} \right) H_2, \end{aligned} \tag{2.40}$$

where $f(T^{I'}, H_i) \Big|_0 \equiv f(T^{I'}, H_i) \Big|_{\substack{T^{I'\neq 1,2} = \langle T^{I'\neq 1,2} \rangle \\ H_{i\neq 1,2} = \langle H_{i\neq 1,2} \rangle}}$ for an arbitrary function of moduli and stabilizer fields $f(T^{I'}, H_i)$. Here, we implicitly assume that H_1 (H_2) has only $U(1)_1$ ($U(1)_2$) charge and their Kähler metrics are given by

$$Z_{1,\bar{1}}(\mathrm{Re}\,T)\Big|_0 = \frac{1-e^{-2c^{(1)}_{I'}\mathrm{Re}\,T^{I'}}}{c^{(1)}_{I'}\mathrm{Re}\,T^{I'}},\quad Z_{2,\bar{2}}(\mathrm{Re}\,T)\Big|_0 = \frac{1-e^{-2c^{(2)}_{I'}\mathrm{Re}\,T^{I'}}}{c^{(2)}_{I'}\mathrm{Re}\,T^{I'}}. \tag{2.41}$$

Thus, the effective scalar potential of light pairs is obtained in terms of Kähler potential and superpotential (2.40),

$$V_{\mathrm{eff}}(T^1,H_1,T^2,H_2) \;=\; e^{K_{\mathrm{eff}}}\left((K_{\mathrm{eff}})^{m,\bar{n}}\,D_mW_{\mathrm{eff}}\,D_{\bar{n}}\bar{W}_{\mathrm{eff}} - 3|W_{\mathrm{eff}}|^2\right), \tag{2.42}$$

where $m,n=\{T^{I'},H_i\}$ with $I'=1,2$ and $i=1,2$.

To complete our discussions, we have to specify the form of the norm function. Its most general form carrying two light moduli T^1 and T^2 is

$$\mathcal{N}(\mathrm{Re}T)\Big|_0 \;=\; C_{1,1,1}(\mathrm{Re}T^1)^3 + C_{1,1,2}(\mathrm{Re}T^1)^2(\mathrm{Re}T^2) + C_{1,2,2}(\mathrm{Re}T^1)(\mathrm{Re}T^2)^2 + C_{2,2,2}(\mathrm{Re}T^2)^3, \tag{2.43}$$

up to the heavy-moduli-dependent parts. Here, we omit the couplings between the heavy fields $T^{I'}$ with $I'=3,4,\cdots$ and the lighter fields T^1 and T^2 for simplicity. To simplify the scalar potential (2.42), we redefine the moduli fields as

$$\hat{T}^1 \equiv \frac{c^{(1)}_1T^1 + c^{(1)}_2T^2}{c},\quad \hat{T}^2 \equiv \frac{c^{(2)}_1T^1 + c^{(2)}_2T^2}{d}. \tag{2.44}$$

Correspondingly, the stabilizer fields H_1 and H_2 have the $U(1)$ charges, c and d, for a linear combination of the Z_2-odd vector fields $A^{I'}_M$ in $\mathbf{V}^{I'}$ with $I'=1,2$. In this field base ($\hat{T}^1$, $\hat{T}^2$), there are no mixing terms between $\hat{T}^1$ and $\hat{T}^2$ in the superpotential (2.40) as can be seen from

$$W_{\mathrm{eff}}(\hat{T}^1,H_1,\hat{T}^2,H_2) \;=\; \left(J^{(1)}_0 + e^{-c\hat{T}^1}J^{(1)}_L\right)H_1 + \left(J^{(2)}_0 + e^{-d\hat{T}^2}J^{(2)}_L\right)H_2. \tag{2.45}$$

Since each of $\hat{T}^1$ and $\hat{T}^2$ has the independent superpotential, the vacuum expectation values of moduli $\hat{T}^1,\hat{T}^2$ and stabilizer fields H_1,H_2 are determined in a similar way as in the case of small-field inflation (2.8). Thus, the minimization conditions of moduli and stabilizer fields in the scalar potential (2.42) $\langle V_{\hat{I}'}\rangle = \langle V_i\rangle = 0$, $\hat{I}',i=1,2$ with $V_{I'}=\partial_{I'}V$ and $V_i=\partial_iV$ are satisfied under

$$\langle\hat{T}^1\rangle \;=\; \frac{1}{c}\ln\frac{J^{(1)}_L}{J^{(1)}_0},\qquad \langle\hat{T}^2\rangle \;=\; \frac{1}{d}\ln\frac{J^{(2)}_L}{J^{(2)}_0},\qquad \langle H_1\rangle \;=\; \langle H_2\rangle \;=\; 0. \tag{2.46}$$

Then, one can obtain the supersymmetric Minkowski minimum $=\langle V\rangle = 0$ because $\langle D_{\hat{I}'}W\rangle = \langle D_iW\rangle = \langle W\rangle = 0$ at the minimum given by Eq. (2.46).

To obtain the successful large-field inflation, we assume that the coefficients $C_{I',J',K'}$ in the norm function and the $U(1)$ charges $c^{(i)}_{I'}$ for $I',J',K'=1,2$ and $i=1,2$ are chosen so that the norm function has the following form in the hatted field base,

$$\mathcal{N}(\mathrm{Re}T)\Big|_0 = a(\mathrm{Re}\hat{T}^1)(\mathrm{Re}\hat{T}^2 - b\,\mathrm{Re}\hat{T}^1)^2, \tag{2.47}$$

up to the heavy-moduli-dependent parts, which is omitted in the following analysis for simplicity. Here, a and b are positive real constants fixed by values of $C_{I',J',K'}$ and $c^{(i)}_{I'}$ as, e.g., $a = c^{(1)}_1(d)^2/c\,(c^{(2)}_2)^2, b = c\,c^{(2)}_1/d\,c^{(1)}_1$ for $c^{(1)}_2 = C_{1,1,1} = C_{1,1,2} = C_{2,2,2} = 0$ and $C_{1,2,2} = 1$. The above specific form of norm function (2.47) yields a moduli mixing in the Kähler metric, $K_{\hat{I}',\bar{\hat{J}}'} \neq 0$ for $\hat{I}' \neq \hat{J}'$, which will plays an important role of stabilizing the $\mathrm{Re}\,\hat{T}^1$ as will be shown in Sec. 2.3.1.

Next, we analyze the mass-squared matrix given by the scalar potential (2.42). It reduces to a block-diagonal form with two nonvanishing blocks at the vacuum, because the mixing terms are absent as follows,

$$\begin{aligned}\langle V_{\hat{T}^1\bar{\hat{T}}^2}\rangle = \langle V_{\hat{T}^1\bar{H}_1}\rangle = \langle V_{\hat{T}^1\bar{H}_2}\rangle = \langle V_{\hat{T}^2\bar{H}_1}\rangle = \langle V_{\hat{T}^2\bar{H}_2}\rangle = 0,\\ \langle (K_{\rm eff})^{\hat{T}^1\bar{H}_1}\rangle = \langle (K_{\rm eff})^{\hat{T}^1\bar{H}_2}\rangle = \langle (K_{\rm eff})^{\hat{T}^2\bar{H}_1}\rangle = \langle (K_{\rm eff})^{\hat{T}^2\bar{H}_2}\rangle = 0,\end{aligned} \tag{2.48}$$

for $V_{mn} = \partial_n\partial_m V$ with $m,n = \{T^{I'}, H_i\}$ with $I', i = 1,2$. Thus, one can analyze the mass matrices of the moduli $\hat{T}^{I'}$ and the stabilizer fields H_i independently.

First of all, let us analyze the mass-squared matrix of the moduli m_t^2, where the canonically normalized moduli fields $(t^{I'})$ are given by

$$t^{I'} = \sum_{J'=1}^{2} \sqrt{2(K_{\hat{T}})_{I'}}\,U_{I',J'}\hat{T}^{J'}, \tag{2.49}$$

with $(K_{\hat{T}})_1$ and $(K_{\hat{T}})_2$ being the eigenvalues of kinetic terms of moduli diagonalized by the matrix U. Their explicit form are given by[†]

$$\begin{aligned}(K_{\hat{T}})_1 &= \frac{(2+3b^2)(\sigma^1)^2 - 2b\,\sigma^1\sigma^2 + (\sigma^2)^2}{8(\sigma^1)^2(\sigma^2 - b\,\sigma^1)^2} + \frac{\sqrt{\mathcal{A}(\sigma^1,\sigma^2)}}{8(\sigma^1)^2(\sigma^2 - b\,\sigma^1)^2},\\ (K_{\hat{T}})_2 &= \frac{(2+3b^2)(\sigma^1)^2 - 2b\,\sigma^1\sigma^2 + (\sigma^2)^2}{8(\sigma^1)^2(\sigma^2 - b\,\sigma^1)^2} - \frac{\sqrt{\mathcal{A}(\sigma^1,\sigma^2)}}{8(\sigma^1)^2(\sigma^2 - b\,\sigma^1)^2},\\ \mathcal{A}(\sigma^1,\sigma^2) &\equiv (4+4b^2+9b^4)(\sigma^1)^4 + 4b(2-3b^2)(\sigma^1)^3\sigma^2\\ &\quad + 2(5b^2-2)(\sigma^1)^2(\sigma^2)^2 - 4b\,\sigma^1\,(\sigma^2)^3 + (\sigma^2)^4,\end{aligned} \tag{2.50}$$

where $\sigma^{I'} = \mathrm{Re}\hat{T}^{I'}$ and

$$\begin{aligned}U &= \begin{pmatrix} U_{1,1} & 1\\ U_{2,1} & 1\end{pmatrix},\\ U_{1,1} &= \frac{(2-3b^2)(\sigma^1)^2 + 2b\,\sigma^1\sigma^2 - (\sigma^2)^2}{4b\,(\sigma^1)^2} - \frac{\sqrt{\mathcal{A}(\sigma^1,\sigma^2)}}{4b\,(\sigma^1)^2},\\ U_{2,1} &= \frac{(2-3b^2)(\sigma^1)^2 + 2b\,\sigma^1\sigma^2 - (\sigma^2)^2}{4b\,(\sigma^1)^2} + \frac{\sqrt{\mathcal{A}(\sigma^1,\sigma^2)}}{4b\,(\sigma^1)^2}.\end{aligned} \tag{2.51}$$

[†]Although we omitted such a diagonalizing matrix U in the imaginary part of $\hat{T}^{J'}$ in Ref. [6], the following discussion and conclusion are the same expect for the supersymmetric masses scale of $\mathrm{Im}\,t^{I'}$ given by Eq. (2.67) and the discussion of reheating process in Sec. 2.3.2.

In this basis, their mass-squared matrix is estimated as

$$m_t^2 = \begin{pmatrix} \sqrt{\frac{1}{(K_{\hat{T}})_1}} & 0 \\ 0 & \sqrt{\frac{1}{(K_{\hat{T}})_2}} \end{pmatrix} U \begin{pmatrix} V_{\hat{T}^1\bar{\hat{T}}^1} & 0 \\ 0 & V_{\hat{T}^2\bar{\hat{T}}^2} \end{pmatrix} U^{-1} \begin{pmatrix} \sqrt{\frac{1}{(K_{\hat{T}})_1}} & 0 \\ 0 & \sqrt{\frac{1}{(K_{\hat{T}})_2}} \end{pmatrix}, \tag{2.52}$$

where $V_{\hat{T}^{I'}\bar{\hat{T}}^{J'}} = \langle e^{K_{\rm eff}}(K_{\rm eff})^{H_i\bar{H}_j} W_{\hat{T}^{I'}H_i}\overline{W_{\hat{T}^{J'}H_j}}\rangle$.

Second, canonically normalized stabilizer fields h_i are given by

$$h_i = \sqrt{2(K_H)_i}H_i, \tag{2.53}$$

where $(K_H)_1 = \langle (K_{\rm eff})_{H_1\bar{H}_1}\rangle$ and $(K_H)_2 = \langle (K_{\rm eff})_{H_2\bar{H}_2}\rangle$ are the eigenvalues of the diagonalized Kähler metric of the stabilizer fields. In this basis, the mass-squared matrix of the stabilizer field is estimated as

$$m_h^2 = \begin{pmatrix} \sqrt{\frac{1}{(K_H)_1}} & 0 \\ 0 & \sqrt{\frac{1}{(K_H)_2}} \end{pmatrix} \begin{pmatrix} V_{H_1\bar{H}_1} & V_{H_1\bar{H}_2} \\ V_{H_2\bar{H}_1} & V_{H_2\bar{H}_2} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{(K_H)_1}} & 0 \\ 0 & \sqrt{\frac{1}{(K_H)_2}} \end{pmatrix}, \tag{2.54}$$

where $V_{H_i\bar{H}_j} = \langle e^{K_{\rm eff}}(K_{\rm eff})^{\hat{T}^{I'}\bar{\hat{T}}^{J'}} W_{\hat{T}^{I'}H_i}\overline{W_{\hat{T}^{J'}H_j}}\rangle$.

Consequently, we find the supersymmetric masses of the canonically normalized moduli $t^{I'}$ and stabilizer fields h_i as

$$\begin{aligned}
m^2_{{\rm Re}\,t^1} = m^2_{{\rm Im}\,t^1} &\simeq \frac{e^{\langle K_{\rm eff}\rangle}\langle (K_{\rm eff})^{H_1\bar{H}_1}\rangle\langle W_{\hat{T}^1H_1}\rangle^2}{\langle (K_{\rm eff})_{\hat{T}_1\bar{\hat{T}}_1}\rangle}, \\
m^2_{{\rm Re}\,t^2} = m^2_{{\rm Im}\,t^2} &\simeq e^{\langle K_{\rm eff}\rangle}\langle (K_{\rm eff})^{H_2H_2}\rangle\langle (K_{\rm eff})^{\hat{T}^2\bar{\hat{T}}^2}\rangle\langle W_{\hat{T}^2H_2}\rangle^2, \\
m^2_{{\rm Re}\,h_1} = m^2_{{\rm Im}\,h_1} &\simeq \frac{e^{\langle K_{\rm eff}\rangle}\langle (K_{\rm eff})^{\hat{T}^1\bar{\hat{T}}^1}\rangle\langle W_{\hat{T}^1H_1}\rangle^2}{\langle (K_{\rm eff})_{H_1\bar{H}_1}\rangle}, \\
m^2_{{\rm Re}\,h_2} = m^2_{{\rm Im}\,h_2} &\simeq \frac{e^{\langle K_{\rm eff}\rangle}\langle (K_{\rm eff})^{\hat{T}^2\bar{\hat{T}}^2}\rangle\langle W_{\hat{T}^2H_2}\rangle^2}{\langle (K_{\rm eff})_{H_2\bar{H}_2}\rangle}.
\end{aligned} \tag{2.55}$$

Thus, their supersymmetric mass-squared are all positive at the vacuum under the limit of $\langle W_{\hat{T}^1H_1}\rangle \ll \langle W_{\hat{T}^2H_2}\rangle$. As mentioned in Sec. 2.1.2, if there is no hierarchy between $\langle W_{\hat{T}^1H_1}\rangle$ and $\langle W_{\hat{T}^2H_2}\rangle$, sizable Kähler mixings may spoil the stability of the vacuum.

As can be seen in Eqs. (2.46) and (2.55), two lighter pairs of modulus and stabilizer ($T^{I'}$, H_i) with $I' = i = 1, 2$ have totally independent vacuum expectation values and supersymmetric masses to each other. By assuming a hierarchy between the parameters in the superpotential such as $|J_0^1| < |J_0^2|$ and $|J_L^1| < |J_L^2|$, we can further integrate out the second pair $(\hat{T}^2, H_2)$ in addition to the heavy pairs of moduli and stabilizers. The effective potential for the first pair $(\hat{T}^1, H_1)$ is then extracted as

$$V_{\rm eff}(\hat{T}^1, H_1) = e^{K_{\rm eff}}\left((K_{\rm eff})^{m,\bar{n}} D_m W_{\rm eff}\, D_{\bar{n}}\bar{W}_{\rm eff} - 3|W_{\rm eff}|^2\right), \tag{2.56}$$

where $m, n = \{\hat{T}^{I'}, i\}$ with $\hat{I}' = 1$ and $i = 1$. In the notation $f(T^{I'}, H_i)\Big|_0 \equiv f(T^{I'}, H_i)\Big|_{\substack{T^{I'\neq 1,2} = \langle T^{I'\neq 1,2}\rangle \\ \hat{T}^{I'\neq 2} = \langle \hat{T}^{I'\neq 2}\rangle \\ H_{i\neq 1} = \langle H_{i\neq 1}\rangle}}$ for an arbitrary function $f(T^{I'}, H_i)$, the above effective Kähler potential $K_{\rm eff}$ and superpotential $W_{\rm eff}$ are expressed as

$$
\begin{aligned}
K_{\rm eff}(\hat{T}^1, H_1) &= -\ln \mathcal{N}(\mathrm{Re}\,\hat{T})\Big|_0 + Z_{1,\bar{1}}(\mathrm{Re}\,\hat{T})\Big|_0 |H_1|^2, \\
W_{\rm eff}(\hat{T}^1, H_1) &= \left(J_0^{(1)} + e^{-c\hat{T}^1} J_L^{(1)}\right) H_1.
\end{aligned} \tag{2.57}
$$

2.3.1 Moduli potential

From now on, we analyze the dynamics of light pair of modulus and stabilizer field appearing in the above effective potential (2.56). On the $H_1 = 0$ hypersurface, the scalar potential of the modulus $\hat{T}^1 = \sigma + i\tau$ is given, by setting the parameter in the norm function (2.47) as $a = 1$, by

$$
\begin{aligned}
V_{\rm eff}(\hat{T}^1, H_1 = 0) &= e^{K_{\rm eff}} (K_{\rm eff})^{i=1,\bar{i}=\bar{1}} \left|(W_{\rm eff})_{i=1}\right|^2 \Big|_{H_1=0} \\
&= \Lambda^4(1 - \lambda \cos(c\,\tau)),
\end{aligned} \tag{2.58}
$$

where

$$
\begin{aligned}
\Lambda^4 &\equiv \frac{c}{(\langle \mathrm{Re}\,\hat{T}^2\rangle - b\,\sigma)^2} \frac{J_{01}^2 + J_{L1}^2 e^{-2c\,\sigma}}{1 - e^{-2c\,\sigma}}, \\
\lambda &\equiv 2\frac{J_{01} J_{L1} e^{-c\,\sigma}}{J_{01}^2 + J_{L1}^2 e^{-2c\,\sigma}}.
\end{aligned} \tag{2.59}
$$

With the following choice of parameters

$$
c = 1/10, \qquad J_L^{(1)} = -4.7 \times 10^{-3}, \qquad J_0^{(1)} = 4.25 \times 10^{-3}, \qquad b = 15, \tag{2.60}
$$

in the reduced Planck unit, the scalar potential on the (σ, τ)-plane is drawn in Fig. 2.3. Here and in what follows, we employ the different numerical values of parameters from those in Ref. [6] in order to be better fitted by the Planck data [15, 16]. It is found that the potential has a periodic property in the imaginary direction τ and τ will be stabilized at the origin as can be seen in Eq. (2.58). To show the behavior of the real direction σ, we draw the potential on the hypersurfaces $\tau = 10$ (dotted line), $\tau = 5$ (dot-dashed line) and $\tau = 0$ (thick line) in Fig. 2.4. Equation (2.58) shows that the negative region of $\sigma < 0$ is unphysical one, because the volume of fifth direction can be negative. Indeed, Λ in Eq. (2.58) diverges in the limit of $\sigma \to 0$. On the other hand, the overshooting to a large-field region, $\sigma > \langle \mathrm{Re}\,\hat{T}^2\rangle/b$, is also prohibited as shown in Eq. (2.58) which is coming from the negative sign in the norm function (2.47). We then find the following property of the potential (2.58)

$$
\lim_{\mathrm{Re}\,\hat{T}^1 \to \langle \mathrm{Re}\,\hat{T}^2\rangle/b} \left| V_{\rm eff}(\hat{T}^1, H_1 = 0)\right| = \infty. \tag{2.61}
$$

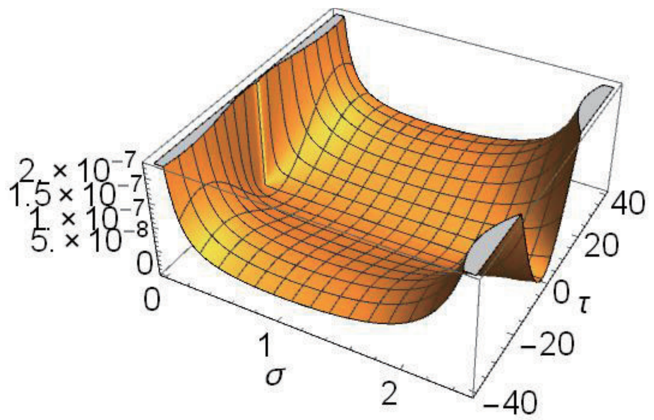

Figure 2.3: The inflaton potential (2.58) on the $H_1 = 0$ hypersurface as drawn in Fig. 5 in Ref. [6].

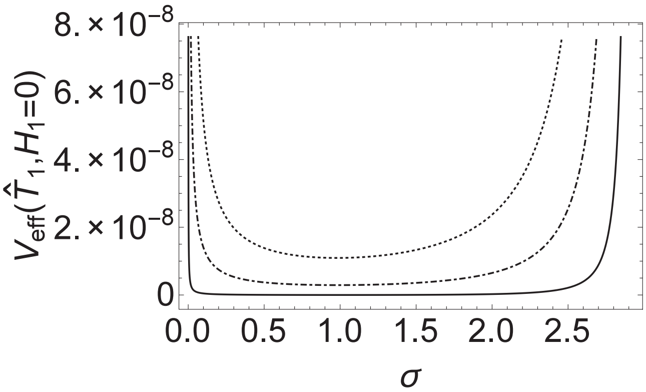

Figure 2.4: The dotted, dot-dashed, and thick lines represent the inflaton potential (2.58) on the hypersurfaces $\tau = 10$, $\tau = 5$ and $\tau = 0$, respectively as drawn in Fig. 6 in Ref. [6].

Now, we suppose that $\mathrm{Re}\,\hat{T}^2$ is already stabilized at the supersymmetric minimum of the potential given by Eq. (2.46).

From the above argument, the potential of natural inflation [29] is effectively realized when we identify $\tau = \mathrm{Im}\,\hat{T}^1$ as the inflaton field. The real direction σ will effectively stabilized at a value without depending on the field value of τ and will not be destabilized during the inflationary era. After the inflation, σ rolls down to the true minimum (2.46) and oscillates around it. The above statements is justified by solving the equations of motion for two fields σ and τ. The stabilizer fields $\mathrm{Re}\,H_1$ and $\mathrm{Im}\,H_1$ are stabilized at the origin by the vacuum energy, i.e., Hubble-induced mass,

$$\frac{\partial^2 V_{\mathrm{eff}}}{\partial H_1 \partial \bar{H}_1} \simeq (K_{\mathrm{eff}})_{H_1\bar{H}_1} V_{\mathrm{eff}}, \tag{2.62}$$

where $(K_{\mathrm{eff}})_{H_1\bar{H}_1} = \partial_{H_1}\partial_{\bar{H}_1} K_{\mathrm{eff}} = Z_{1,\bar{1}}(\mathrm{Re}\,\hat{T})$ is the Kähler metric of stabilizer field H_1. Therefore, in the following analysis, we omit the fluctuations of the stabilizer fields, $\mathrm{Re}\,H_1$ and $\mathrm{Im}\,H_1$, around their vacuum expectation values $\langle \mathrm{Re}\, H_1 \rangle = \langle \mathrm{Im}\, H_1 \rangle = 0$ during and after the inflation. On the $\langle \mathrm{Re}\, H_1 \rangle = \langle \mathrm{Im}\, H_1 \rangle = 0$ hypersurface, the equations of motion for σ and τ reduce to

$$
\begin{aligned}
\sigma^{''} &= -\left(1-\mathcal{L}_{\mathrm{kin}}\right)\left(3\sigma^{'}+6\frac{\sigma^2(\langle \hat{T}^2\rangle - b\sigma)^2}{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}\frac{\partial_\sigma V_{\mathrm{eff}}}{V_{\mathrm{eff}}}\right)+\frac{(\sigma^{'})^2-(\tau^{'})^2}{\sigma(\langle \hat{T}^2\rangle - b\sigma)}\left(\frac{(\langle \hat{T}^2\rangle - b\sigma)^3-2b^3\sigma^3}{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}\right),\\
\tau^{''} &= -\left(1-\mathcal{L}_{\mathrm{kin}}\right)\left(3\tau^{'}+6\frac{\sigma^2(\langle \hat{T}^2\rangle - b\sigma)^2}{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}\frac{\partial_\tau V_{\mathrm{eff}}}{V_{\mathrm{eff}}}\right)+\frac{2\sigma^{'}\tau^{'}}{\sigma(\langle \hat{T}^2\rangle - b\sigma)}\left(\frac{(\langle \hat{T}^2\rangle - b\sigma)^3-2b^3\sigma^3}{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}\right),\\
\mathcal{L}_{\mathrm{kin}} &\equiv \frac{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}{2\sigma^2(\langle \hat{T}^2\rangle - b\sigma)^2}\left((\sigma^{'})^2+(\tau^{'})^2\right),
\end{aligned}
\tag{2.63}
$$

where the prime denotes the derivative with respect to the e-folding number N, and the Christoffel symbol for the target space is conduced by the metric $g_{\sigma\sigma} = g_{\tau\tau} = \frac{(\langle \hat{T}^2\rangle - b\sigma)^2+2b^2\sigma^2}{2\sigma^2(\langle \hat{T}^2\rangle - b\sigma)^2}$.

First of all, we set the numerical values of parameters as those (2.60) for the light fields ($\hat{T}^1$, H_1) and

$$
d = 1/20, \qquad J_L^{(2)}/J_0^{(2)} = -9, \qquad J_0^{(0)} = 10^{-1}, \tag{2.64}
$$

for the heavier fields $\hat{T}^2$ and H_2, respectively. Eq. (2.46) allows us to estimate the vacuum expectation values of fields,

$$
\langle \hat{T}^1 \rangle \simeq 1, \qquad \langle \hat{T}^2 \rangle = 43.94, \qquad \langle H_1 \rangle = \langle H_2 \rangle = 0, \tag{2.65}
$$

which are also translated into those of the canonically normalized fields defined in Eqs. (2.49) and (2.53),

$$
\langle \mathrm{Re}\, t^1 \rangle \simeq 2.29, \qquad \langle \mathrm{Re}\, t^2 \rangle \simeq 1.22, \qquad \langle \phi^1 \rangle = \langle \phi^2 \rangle = \langle h_1 \rangle = \langle h_2 \rangle = 0. \tag{2.66}
$$

By inserting them into Eq. (2.55), one can estimate the supersymmetric masses of moduli and stabilizer fields as

$$
\begin{aligned}
(m_{\mathrm{Re}\, t^1})^2 \simeq (m_{\mathrm{Im}\, t^1})^2 \simeq (m_{\mathrm{Re}\, h_1})^2 \simeq (m_{\mathrm{Im}\, h_1})^2 \simeq \left(6\times 10^{13}\,\mathrm{GeV}\right)^2,\\
(m_{\mathrm{Re}\, t^2})^2 \simeq (m_{\mathrm{Im}\, t^2})^2 \simeq (m_{\mathrm{Re}\, h_2})^2 \simeq (m_{\mathrm{Im}\, h_2})^2 \simeq \left(4\times 10^{16}\,\mathrm{GeV}\right)^2,
\end{aligned}
\tag{2.67}
$$

which ensures that second pair $(\hat{T}^2, H_2)$ are heavier enough than the first pair $(\hat{T}^1, H_1)$ as mentioned before. The numerical value of Hubble scale becomes

$$
H_{\mathrm{inf}} = \left(V_{\mathrm{inf}}/3M_{\mathrm{Pl}}^2\right)^{1/2} \simeq 2\times 10^{14}\mathrm{GeV}, \tag{2.68}
$$

where $V_{\mathrm{inf}} \sim \Lambda^4$ is given in Eq. (2.58). Thus, it is found that the masses of moduli and stabilizers (2.67) and Hubble scale (2.68) are well below the compactification scale $M_C \simeq \pi M_{\mathrm{Pl}}/\langle \mathcal{N}(\mathrm{Re}\,\hat{T})\rangle^{1/2} \simeq 2.6\times 10^{17}\mathrm{GeV}$, and the 4D effective theory description is valid. We again remark that the pair $(\hat{T}^2, H_2)$ playing a role of stabilizing $\mathrm{Re}\,\hat{T}^1$ is stabilized at ($\hat{T}^1$, H_1)-independent minimum and they are decoupled from the inflaton dynamics, i.e., $H_{\mathrm{inf}}^2 \ll (m_{t^2}^2)$, $(m_{\phi^2}^2)$, $(m_{\mathrm{Re}\, h^2}^2)$, $(m_{\mathrm{Im}\, h^2}^2)$. Then, one can treat only the inflaton dynamics.

Next, in order to estimate the cosmological observables constrained by Planck, we define the general slow-roll parameters in the case of two fields [26, 27],

$$\epsilon = \frac{g^{\sigma\sigma}}{2}\left(\frac{\partial_\sigma V_{\rm eff}}{V_{\rm eff}}\right)^2 + \frac{g^{\tau\tau}}{2}\left(\frac{\partial_\tau V_{\rm eff}}{V_{\rm eff}}\right)^2,$$

$$\eta = \text{ minimum eigenvalue of } \left\{\frac{1}{V_{\rm eff}}\begin{pmatrix}\nabla^i\nabla_j V_{\rm eff} & \nabla^i\nabla_{\bar{j}} V_{\rm eff}\\ \nabla^{\bar{i}}\nabla_j V_{\rm eff} & \nabla^{\bar{i}}\nabla_{\bar{j}} V_{\rm eff}\end{pmatrix}\right\}$$

$$=\frac{g^{\sigma\sigma}}{2}\left(\frac{\partial_\sigma\partial_\sigma V_{\rm eff}}{V_{\rm eff}} + \frac{\partial_\tau\partial_\tau V_{\rm eff}}{V_{\rm eff}} - \sqrt{\left(\frac{\partial_\sigma\partial_\sigma V_{\rm eff}}{V_{\rm eff}} - \frac{\partial_\tau\partial_\tau V_{\rm eff}}{V_{\rm eff}} - 2\Gamma^\sigma_{\sigma\sigma}\frac{\partial_\sigma V_{\rm eff}}{V_{\rm eff}}\right)^2 + 4\left(\frac{\partial_\sigma\partial_\tau V_{\rm eff}}{V_{\rm eff}} - \Gamma^\sigma_{\sigma\sigma}\frac{\partial_\tau V_{\rm eff}}{V_{\rm eff}}\right)^2}\right), \tag{2.69}$$

with $i, j = \hat{T}^1$. In a similar way to the single field case, the cosmological observables are defined in terms of these slow-roll parameters as shown in Eq. (2.28).

Let us numerically solve their equations of motion (2.63) as a function of e-folding number N. By setting the initial conditions $(\sigma, \tau) = (1, 20)$ and $(\sigma', \tau') = (0, 0)$ at $N = 0$, σ and τ evolve as drawn in Fig. 2.5. The violation of slow-roll condition, $\max\{\epsilon, \eta\} = 1$, occurs at about $N_{\rm end} \simeq 76$ e-folds. As confirmed in Fig. 2.5, in the inflationary era, the real part of the light modulus, σ, is stabilized at a field value different from that at the true vacuum. After the end of inflation, it rolls down to the minimum and oscillates around the vacuum.

Such a phenomenon is understood from the mass term of σ and the equation of motion (2.63). The mass-squared of σ consists of two parts. First one is the Hubble-induced mass $H_{\rm inf}$ defined by Eq. (2.68) and second one is the supersymmetric mass term $m_{\rm SUSY} \sim m_{t^1} \sim \mathcal{O}(10^{13})$ GeV in the superpotential (2.40). The mass term is thus given by

$$\partial_\sigma\partial_\sigma V_{\rm eff} \simeq 3k(\sigma)H_{\rm inf}^2 + m_{\rm SUSY}^2. \tag{2.70}$$

Here, $k(\sigma)$ represents a certain function of σ whose numerical value is of $\mathcal{O}(1)$ during and after the inflation. Since the Hubble parameter has an almost constant value during the inflation, it controls the mass-squared of σ in Eq. (2.70). Then, in the inflationary era, the real part σ is effectively stabilized at a different point away from its vacuum determined by the second term in Eq. (2.70). After the inflation, the later part in Eq. (2.70) dominates the mass term of σ compared with the Hubble-induced one.

Next, we analytically estimate the "stabilized" value of σ during the inflation. In the slow-roll regime, $\sigma' \ll 1$ and $\tau' \ll 1$, the equation of motion for σ given in Eq. (2.63) is simplified by dropping the mixing term proportional to $\sigma'\tau'$

$$\begin{aligned}\sigma' &= -g^{\sigma\sigma}\frac{V_\sigma}{V}\\ &= -g^{\sigma\sigma}\left(\frac{2}{\langle\hat{T}^2\rangle/b - \sigma} - \frac{2c\,e^{-2c\,\sigma}}{1 - e^{-2c\,\sigma}} - c\right) + \frac{V_{\rm vac}(\sigma)}{V},\end{aligned} \tag{2.71}$$

where $V_{\rm vac}(\sigma) = e^{K_{\rm eff}}(K_{\rm eff})^{H_1\bar{H}_1}c\left(\left|J_0^{(1)}\right|^2 - \left|J_L^{(1)}\right|^2 e^{-2c\,\sigma}\right)/\mathcal{N}$. When the terms in the first

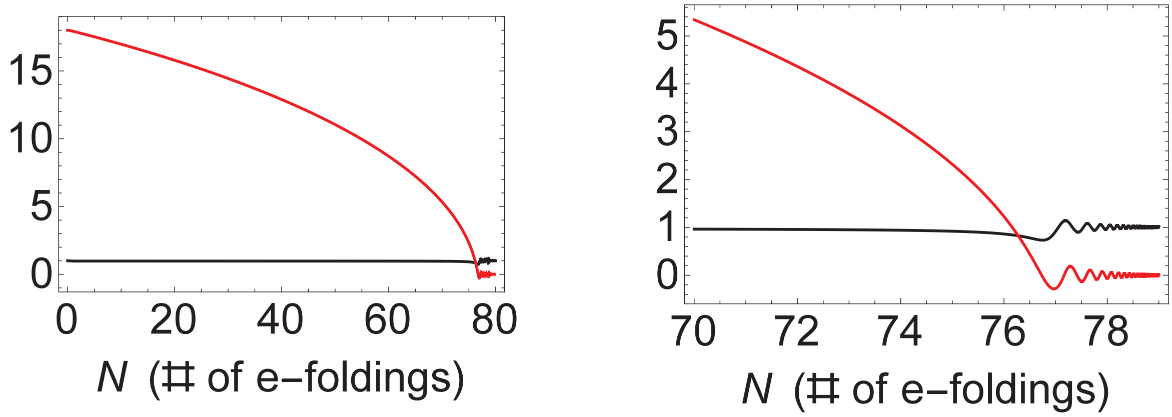

Figure 2.5: The trajectories of $\sigma = \mathrm{Re}\,\hat{T}^1$ (black curves) and $\tau = \mathrm{Im}\,\hat{T}^1$ (red curves) as a function of the e-folding number N drawn in Fig. 7 in Ref. [6].

parentheses of Eq. (2.71) vanish at the field value $\sigma = \sigma_{\rm inf}$, $\sigma_{\rm inf}$ satisfies the following relation,

$$\frac{2}{\langle \hat{T}^2 \rangle / b - \sigma_{\rm inf}} - \frac{2c\, e^{-2c\,\sigma_{\rm inf}}}{1 - e^{-2c\,\sigma_{\rm inf}}} - c = 0,$$
$$\Leftrightarrow \frac{\langle \hat{T}^2 \rangle}{b} = \frac{-2 + c\,\sigma_{\rm inf} + e^{2c\,\sigma_{\rm inf}}(2 + c\,\sigma_{\rm inf})}{c\,(1 + e^{2c\,\sigma_{\rm inf}})}. \tag{2.72}$$

When the value of $\sigma_{\rm inf}$ is chosen as that at the vacuum $\langle \sigma \rangle$ given by Eq. (2.46), the second term in the right handed side of Eq. (2.71) is almost vanishing simultaneously. From this perspective, the parameters of the heavier modulus $\hat{T}^2$ are constrained to satisfy

$$\frac{\langle \hat{T}^2 \rangle}{b} \simeq \frac{-2 + c\,\langle \sigma \rangle + e^{2c\,\langle \sigma \rangle}(2 + c\,\langle \sigma \rangle)}{c\,(1 + e^{2c\,\langle \sigma \rangle})}, \tag{2.73}$$

which holds in the our numerical analysis.

As a result, the real part σ is effectively stabilized at $\sigma_{\rm inf}$ and the inflaton dynamics is dominated by the imaginary part τ. One can then consider the single-field inflation model. In contrast to the multi-field inflation model, sizable isocurvature perturbations caused by the real part σ can be suppressed. Around the end of inflation, one cannot capture the dynamics of σ through Eq. (2.71), because the slow-roll conditions are violated. Therefore, we numerically solve the full equations of motion (2.63) and find the trajectories of σ and τ. The inflationary trajectory is drawn on the $(\tau,\, \sigma)$-plane in Fig. 2.6, where the equation of motion of inflaton is approximated in Eq. (2.71) on the black dotted curve, whereas on the red solid curve, the equations of motion (2.63) are numerically solved.

From the observational point of view, the inflaton potential is categorized into that of natural inflation [29]. With our choice of parameter settings (2.60) to realize $\sigma_{\rm inf} \sim \langle \sigma \rangle$, the value of λ appearing in the scalar potential (2.59) is almost equal to 1. For the canonically normalized field $\phi^1 \equiv k\tau$, the effective potential becomes

$$V_{\rm eff} = \Lambda^4 (1 - \lambda \cos(\hat{c}\, \phi^1)), \tag{2.74}$$

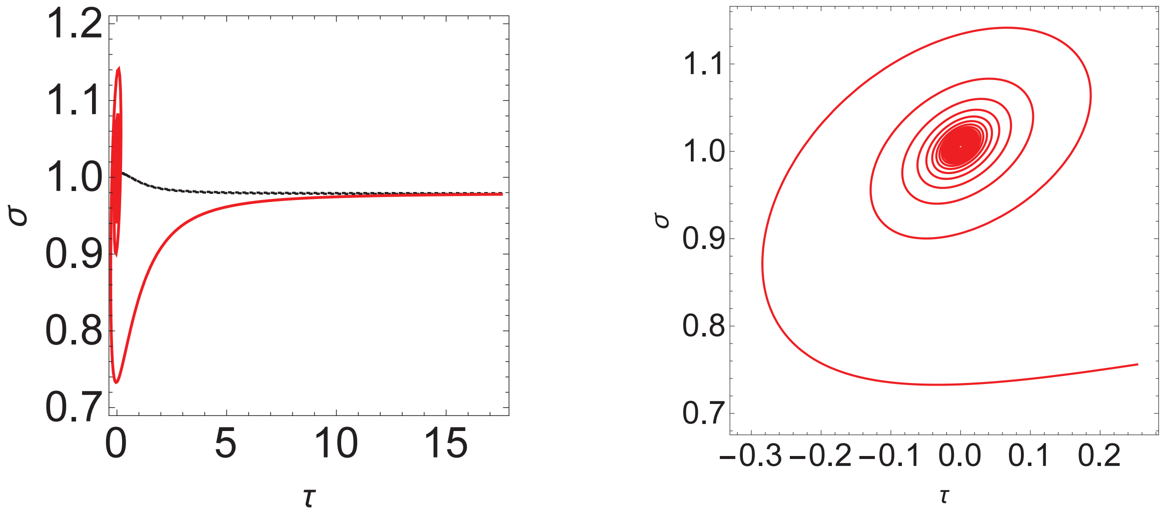

Figure 2.6: The inflaton trajectory on the field space of τ and σ as drawn in Fig. 8 in Ref. [6]. The equation of motion of inflaton is approximated in Eq. (2.71) on the black dotted curve, whereas on the red solid curve, the full equations of motion (2.63) are numerically solved.

where $\hat{c} \equiv c/k$ and $k \equiv \sqrt{2(K_{\rm eff})_{\hat{T}^1\bar{\hat{T}}^1}} = \sqrt{\frac{(\langle T^2\rangle - b\,\sigma)^2 + 2b^2\,\sigma^2}{2\sigma^2(\langle T^2\rangle - b\,\sigma)^2}}$. Correspondingly, the slow-roll parameters are obtained as

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{\partial_{\phi^1}V}{V}\right)^2 = \frac{(\hat{c}\,M_{\rm Pl})^2}{2}\lambda^2\frac{1-\cos^2(\hat{c}\,\phi^1)}{(1-\lambda\cos(\hat{c}\,\phi^1))^2},$$
$$\eta = M_{\rm Pl}^2\frac{\partial_{\phi^1}\partial_{\phi^1}V}{V} = (\hat{c}\,M_{\rm Pl})^2\lambda\frac{\cos(\hat{c}\,\phi^1)}{1-\lambda\cos(\hat{c}\,\phi^1)},$$
$$\xi^2 = M_{\rm Pl}^4\frac{V'V'''}{V^2} = -2(\hat{c}\,M_{\rm Pl})^2\epsilon, \tag{2.75}$$

and hereafter, the tensor-to-scalar ratio, spectral tilt of curvature perturbation and its running are yielded as in Eq. (2.28), up to a leading order,

$$r = 16\,\epsilon,$$
$$n_s = 1 - 2\eta + 6\epsilon,$$
$$\mathrm{d}n_s/\mathrm{d}\ln k = -24\epsilon^2 + 16\epsilon\,\eta - 2\xi^2. \tag{2.76}$$

In our setup, the inflaton field τ is the zero-mode of fifth component of the $U(1)_{I'=1}$ gauge field, $A_y^{I'=1}$, i.e., axion. The Kähler potential has a shift symmetry originating from the $U(1)_{I'=1}$ gauge symmetry, whereas in the superpotential, the continuous shift symmetry is broken to the discrete one. Such a discrete symmetry is controlled by its decay constant $f_{\phi^1} = \hat{c}^{-1}$, as shown in the potential (2.74). The recent Planck data [15, 16] requires the large axion decay constant $f_{\phi^1} \geq 5M_{\rm Pl}$ which is achieved by the small $U(1)_{I'=1}$ charge c in Eq. (2.60). Moreover, the η-problem peculiar to the 4D supergravity models is prevented in our framework. However, the η-problem is solved in a different way to the case of small-field inflation in Sec. 2.2. Since the

Kähler potential does not contain the axion field τ because of the $U(1)_{I'=1}$ gauge symmetry, the slow-roll parameter η can be taken smaller than 1.

Along the same step outlined in Sec. 2.2, we define the e-folding number $N = N_*$ and the scalar potential $V_* \equiv V(\sigma_*, \tau_*)$ at the field values $(\sigma, \tau) = (\sigma_*, \tau_*)$ corresponding to the pivot scale, whereas $N_{\rm end}$ and $V_{\rm end} \equiv V(\sigma_{\rm end}, \tau_{\rm end})$ are evaluated at the field values $(\sigma_{\rm end}, \tau_{\rm end})$ corresponding to the end of inflation. Then, an amount of e-folding number $N_{\rm e} \equiv N_{\rm end} - N_*$ can be written in terms of them [28],

$$N_{\rm e} \simeq 62 + \ln\frac{V_*^{1/4}}{10^{16}\,{\rm GeV}} + \ln\frac{V_*^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\ln\frac{V_{\rm end}^{1/4}}{\rho_R^{1/4}}, \tag{2.77}$$

with $V_{\rm end}^{1/4} \simeq 4\times 10^{14}\,{\rm GeV}$ and $\rho_R^{1/4} = (\pi^2 g_*/30)T_R \simeq 2.6\times 10^9\,{\rm GeV}$. The reheating temperature $T_R \simeq 8.8\times 10^8\,{\rm GeV}$ will be given later in Sec. 2.3.2. Since the same amount of e-folding number $N_{\rm e}$ is also defined by

$$N_{\rm e} = -\int_{t_{\rm end}}^{t_*} H(\tilde{t})\, d\tilde{t}, \tag{2.78}$$

it is found that the numerical values of moduli fields σ_*, τ_* and the scalar potential $V_*^{1/4}$ are obtained by equating Eq. (2.77) with Eq. (2.78),

$$\sigma_* \simeq 0.98, \qquad \tau_* \simeq 16, \qquad V_*^{1/4} \simeq 3\times 10^{16}\,{\rm GeV}, \qquad N_* = 16, \qquad N_{\rm e} = 60. \tag{2.79}$$

At these field values corresponding to the pivot scale, we extract the numerical values of cosmological observables such as the power spectrum of curvature perturbation P_ξ, its spectral index n_s, the running of its spectral index $dn_s/d\ln k$ and the tensor-to-scalar ratio r as follows,

$$P_\xi \simeq 2\times 10^{-9}, \;\; n_s = 0.963, \;\; r \simeq 0.09, \tag{2.80}$$

which are consistent with those of usual natural inflation [30]. This natural inflation is categorized into the large-field inflation, because the field variable of the canonically normalized inflaton ϕ^1 is provided by

$$\Delta\phi^1 = \phi_*^1 - \phi_{\rm end}^1 \simeq 13.3\, M_{\rm Pl}. \tag{2.81}$$

We comment on the details of natural inflation with single axion and multiple axions on the basis of string theory in Chapter 5. In the following subsections 2.3.2 and 2.4, we focus on the oscillations of moduli and stabilizers after the inflation.

2.3.2 Reheating process

After the inflation, the field t^1 and the axion-inflaton ϕ^1 oscillate at the same time. Since the axion-inflaton ϕ^1 is not included in the Kähler potential because of a shift symmetry, that is a $U(1)$ symmetry, ϕ^1 cannot decay into the matter chiral multiplets Q_α originating in the hypermultiplet $\mathbf{\Phi}_\alpha$ through the Kähler potential. A possible decay channel is only coming from the superpotential, in particular, the Yukawa coupling in the superpotential. However, these decay widths are suppressed by the masses of matter fields. Although the couplings among the modulus ϕ^1 and the gauge bosons are dimension-five operators, ϕ^1 mainly decays into the gauge-boson pairs.

Let us consider the 4D effective theory as MSSM. The reheating process after the end of inflation is almost the same as discussed in Sec. 2.2. Around the vacuum, the relevant terms in the Lagrangian is expanded as

$$\begin{aligned}\mathcal{L} &\supset -\frac{1}{8}\mathrm{Im}\, f_r \epsilon^{\mu\nu\rho\sigma} F^r_{\mu\nu} F^r_{\rho\sigma} \\ &= -\frac{1}{8}\langle \mathrm{Im}\, f_r\rangle \epsilon^{\mu\nu\rho\sigma} F^r_{\mu\nu} F^r_{\rho\sigma} - \frac{1}{8}\left\langle \frac{\partial\, \mathrm{Im}\, f_r}{\partial \phi^1}\right\rangle \delta\phi^1\, \epsilon^{\mu\nu\rho\sigma} F^r_{\mu\nu} F^r_{\rho\sigma}. \end{aligned} \tag{2.82}$$

Then the total decay width from the field ϕ into the gauge bosons ($g^{(r)}$) is calculated as

$$\begin{aligned}\Gamma^{\phi^1} &\simeq \sum_{r=1}^{3} \Gamma(\phi^1 \to g^{(r)} + g^{(r)}) \simeq \sum_{r=1}^{3} \frac{N_G^r}{64\pi} \left\langle \frac{\xi_r^1}{\mathrm{Re}\, f_r}\right\rangle^2 \left\langle \frac{U_{2,2}}{\sqrt{2(K_{\hat{T}})_1}(U_{1,1}U_{2,2} - U_{1,2}U_{2,1})}\right\rangle^2 \frac{m_{t^1}^3}{M_{\mathrm{Pl}}^2} \\ &\simeq 22 \text{ GeV}, \end{aligned} \tag{2.83}$$

where N_G^r is the number of the gauge bosons in the MSSM. With the numerical values of moduli fields (2.66), the eigenvalues and diagonalized matrix of Kähler metric are yielded as $\langle\sqrt{2(K_{\hat{T}})_1}\rangle \simeq 0.76$, $\langle U_{1,1}\rangle \simeq -42.34$, $\langle U_{2,1}\rangle \simeq 0.024$, $\langle U_{1,2}\rangle = \langle U_{2,2}\rangle = 1$ and $m_{\phi^1} \simeq 6\times 10^{13}$ GeV. In addition, we set $\xi_r^1 = 3.72$ and otherwise 0 to realize the correct size of gauge coupling unification $\langle f_r\rangle = 1/(g_r)^2 \simeq 3.73$ at the GUT scale ($\simeq 2.0\times 10^{16}$[GeV]).

By employing the sudden-decay approximation, the reheating temperature is estimated by equating the total decay width of inflaton with Hubble parameter,

$$\begin{aligned}&\Gamma^{\phi^1} \simeq H(T_R), \\ \Leftrightarrow\ & T_R = \left(\frac{\pi^2 g_*}{90}\right)^{-1/4} \sqrt{\Gamma^{\phi^1} M_{\mathrm{Pl}}} \simeq 8.8\times 10^8 \text{ GeV}, \end{aligned} \tag{2.84}$$

where $g_* = 915/4$ is the effective degrees of freedom at the reheating in the MSSM. Although the real part σ also oscillates at the end of inflation, its decay time is almost the same as that given by Eq. (2.84).

2.4 Cosmological moduli problems

In this section, we take into account the oscillations of other moduli and stabilizer fields. In general, when the other fields oscillate after the inflation, they behave like matter fields and would dominate the universe. Since the moduli fields gravitationally couple to the matter fields in the standard model, its decay time would occur after the epoch of Big-Bang Nucleosynthesis (BBN). This problem is known as a cosmological moduli problem [12], because the successful BBN is violated by the moduli decay. Moreover, the nonthermally generated gravitinos produced by the moduli decay are severely constrained in order not to induce the huge amount of dark matter and violate the successful BBN. This is called moduli-induced gravitino problem [31].

In our both inflation models, the moduli and stabilizers do not break the supersymmetry and the decay into the gravitino is suppressed. Thus, one can solve the moduli-induced gravitino problem. Even if we add a source of the SUSY-breaking, these moduli and stabilizers would have large supersymmetric masses and do not obtain the F-term. In the following, we add the SUSY-breaking effects on the inflaton superpotential $W_{\rm eff}$ in Eq. (2.57) or[‡],

$$W = W_{\rm eff} + \Delta W(\hat{T}^1), \tag{2.85}$$

where $\Delta W(\hat{T}^1)$ denotes the SUSY-breaking sector, that generically involves the inflaton multiplet $\hat{T}^1$. Here, it is assumed that the other fields ($\hat{T}^{I'}$, H_i) with $I', i \neq 1$ are stabilized and do not oscillate around their supersymmetric minimum thanks to their large supersymmetric masses.

When the SUSY-breaking scale is larger enough than the inflation scale, one has to analyze the full scalar potential in general. From now on, we focus on the situation which the SUSY-breaking is smaller than the inflation scale. By assuming $\langle \Delta W\rangle \sim \langle \partial_{\hat{T}^1}(\Delta W)\rangle \ll 1$ in the reduced Planck unit, the deviation from the supersymmetric Minkowski minimum (2.8) is evaluated by employing the reference point method [32].

The reference point is chosen as the minimum as close to the true minimum as possible. In our model, the reference point is set to satisfy

$$\begin{aligned}
&D_{H_1}W|_{\rm ref} = W_{H_1} + (K_{\rm eff})_{H_1}W = 0,\\
&\qquad\qquad \leftrightarrow c\hat{T}^1|_{\rm ref} = \ln\frac{J_L^{(1)}}{J_0^{(1)}} \ \text{and} \ H_1|_{\rm ref} = 0,\\
&D_{\hat{T}^1}W|_{\rm ref} = (K_{\rm eff})_{\hat{T}^1}\Delta W,
\end{aligned} \tag{2.86}$$

where one can choose the effective Kähler potential $K_{\rm eff}$ given in Eqs. (2.10) or (2.57) for each scenario. As explicitly shown in Appendix A, the deviations $\delta\varphi$ from the reference point $\varphi|_{\rm ref}$ for $\varphi = \hat{T}^1, H_1$ are found as,

$$\delta\hat{T}^1 = \mathcal{O}\left(\frac{|\Delta W|^2}{W_{\hat{T}^1H_1}}\right), \quad \delta H_1 = -\frac{(K_{\rm eff})_{\hat{T}^1}\Delta W}{W_{\hat{T}^1H_1}} + \mathcal{O}(|\Delta W|^2). \tag{2.87}$$

The scalar potential is minimized by these variations at the first order of $\delta\hat{T}^1$ and δH_1. Thus, our reference point method is justified only if the the supersymmetric masses of the moduli and stabilizers are larger than the supersymmetry breaking scale, that is, $\langle W_{\hat{T}^1H_1}\rangle \gg \langle \Delta W\rangle$ in the reduced Planck unit. In terms of them, the F-terms of $\hat{T}^1$ and H_1 are evaluated at the

‡In the following analysis, one can both consider the inflation scenarios.

SUSY-breaking minimum as,

$$\sqrt{(K_{\rm eff})_{\hat{T}^1\bar{\hat{T}}^1}}F^{\hat{T}^1} = -e^{K_{\rm eff}/2}\sqrt{(K_{\rm eff})_{\hat{T}^1\bar{\hat{T}}^1}}(K_{\rm eff})^{\hat{T}^1\bar{J}}\,\overline{D_J W} \simeq \mathcal{O}\left(\frac{(m_{3/2})^3}{(m_{{\rm Re}\,t^1})^2}\right),$$
$$\sqrt{(K_{\rm eff})_{H_1\bar{H}_1}}F^{H_1} = -e^{K_{\rm eff}/2}\sqrt{(K_{\rm eff})_{H_1\bar{H}_1}}(K_{\rm eff})^{H_1\bar{J}}\,\overline{D_J W} \simeq \mathcal{O}\left(\frac{(m_{3/2})^3}{(m_{{\rm Re}\,h_1})^2}\right), \quad (2.88)$$

where $(m_{t^1})^2$ and $(m_{h_1})^2$ are the supersymmetric masses of moduli and stabilizers given in Eq. (2.55).

We conclude that the SUSY-breaking sector does not alter the inflaton dynamics if the size of SUSY-breaking scale is much smaller than the inflation scale. Since the light fields $\hat{T}^1$ and H_1 have almost vanishing F-terms, their decay channels into the gravitino are enough suppressed. There is no moduli-induced gravitino problem in our framework. In Chapter 3, we construct the detailed SUSY-breaking sector and evaluate the amount of gravitino quantitatively. In addition to the above issues, we check whether the field oscillations are suppressed or not. If the fields other than the inflaton oscillate and dominate the universe after the inflation, they drastically change the thermal history of the universe.

In each inflation scenario, even if we add the source of small SUSY-breaking scale compared with the supersymmetric mass term in the superpotential, the stabilizer field H_1 and the inflaton obtain the supersymmetric masses of the same order at the SUSY-breaking minimum. However, in the inflationary era, H_1 has the Hubble-induced mass proportional to $H_{\rm inf}$ shown in Eqs. (2.26) and (2.68) in each inflation scenario proposed in Sec. 2.2 and Sec. 2.3, respectively. Then, its oscillation amplitude ΔH_1 becomes

$$\Delta H_1 \simeq \delta H_1|_{\rm inf} - \delta H_1|_{\rm vac} \simeq \mathcal{O}\left(\frac{\Delta W}{H_{\rm inf}}\right) - \mathcal{O}\left(\frac{\Delta W}{m_{h_1}}\right), \quad (2.89)$$

where $\delta H_1|_{\rm inf}$ ($\delta H_1|_{\rm vac}$) is the deviation of H_1 from the supersymmetric Minkowski minimum (2.8) during the inflation (at the SUSY-breaking minimum). As a result, in each inflation scenario, H_1 is strictly fixed at the origin during inflation and at the SUSY-breaking minimum after inflation, respectively. The oscillation of H_1 is enough suppressed so that it does not dominate the universe. By a similar argument as in the case of small-field inflation discussed in Sec. 2.2, the imaginary part of modulus ${\rm Im}\,T^1$ is also strictly fixed at the origin as well if its initial position is located at the value close to the origin. Since the Kähler potential has a shift symmetry orginating from the gauge symmetry for the imaginary part, the inflationary dynamics is irrelevant to the it.

Finally, we check whether the one-loop corrections affect the inflaton dynamics or not. As calculated in Ref. [25], the moduli Kähler potential given by Eq. (6.10) receives the following one-loop correction,

$$e^{K_{\rm eff}} \simeq \frac{e^{1/(32\pi^2\mathcal{N})}}{\mathcal{N}} \to \infty, \quad (2.90)$$

In the large field inflation scenario, the ${\rm Re}\,\hat{T}^1$-dependence of Kähler potential is the same as that of the tree-level one. Thus the scalar potential also diverges in the limit $b\,{\rm Re}\,\hat{T}^1 \to {\rm Re}\,\hat{T}^2$ and ${\rm Re}\,\hat{T}^1$ is effectively stabilized in the same way as discussed in Sec. 2.3. Also, in the case of small field inflation, the inflaton rolls down to the minimum from the large field value of $\mathrm{Re}\,T^1$. In this region, such a correction is suppressed and inflaton dynamics is not altered.

2.5 Implication for string model building

It would be possible to derive the 5D supergravity studied in this chapter from a more fundamental theory such as type IIB superstring theory in ten-dimensions and the M-theory in eleven-dimensions [22].

The discussed moduli inflation highly depends on the form of norm function. If the 5D supergravity model is embedded into the heterotic M-theory on CY manifold, the norm function coefficients $C_{I,J,K}$ correspond to the intersection numbers of CY manifold. In particular, in the case of large-field inflation, the negative sign in the norm function is important to stabilize other fields than the inflaton. Such a structure can be also seen in the Kähler moduli stabilization in type IIB string theory on "Swiss-Cheese" manifold [33].

On the other hand, the 5D supergravity background is also realized in type IIB string theory on a warped throat represented by Klebanov-Witten model [34]. When a large number of D3-branes exist at the same point in the internal space such as toroidal background, the effective 5D spacetime appears through their backreaction [35]. Moreover, the $U(1)_{I'}$ symmetries discussed above might have an origin in more higher-dimensional local symmetries and then the moduli multiplets would be identified as closed or open string moduli fields. Thus, the cosmological and phenomenological features of 5D SUGRA would be governed by the structure of internal manifold behind it.

2.6 Summary

In this chapter, we have discussed the cosmological feature of 5D supergravity models compactified on S^1/Z_2, in particular, successful two types of moduli inflations, small-field inflation in Sec 2.2 and large-field inflation in Sec. 2.3. Through the compactification of the fifth direction, the inflaton is identified as a linear combination of moduli fields that correpond to the zero-mode of Z_2-even chiral multiplets included in five-dimensional extra $U(1)$ vector multiplets.

When the real part of the lightest modulus plays a role of inflaton field, one can realize the small-field inflation whose potential is similar to the one in Starobinsky model [1]. The exponential behavior in the inflaton potential is originated from the exponentially localized wavefunction of the stabilizer field in the fifth dimension. Thus, when a linear-type superpotential term for the stabilizer field H_1 at the boundary fixed point is dominant, the obtained inflaton potential is consistent with Planck data [15, 16].

We further presented a different type of inflation scenario within the same framework of 5D supergravity model as the previous scenario. In contrast to the previous small-field inflation, the inflaton field is considered as the imaginary part of the lightest modulus and then the obtained potential takes the form of natural inflation [29]. When it rolls down in the scalar potential, the corresponding real part of modulus will be destabilized in general, since a runaway direction will appear in the potential. To realize the situation where the real part of lightest modulus is stabilized during the inflation, we introduced the two light pairs of moduli and stabilizer fields $(\hat{T}^{I'}, H_i)$ with $I', i = 1, 2$. Then, the real part of the heavier modulus produces the potential barrier for the real part of lightest modulus through the couplings in the Kähler potential. During and after the inflation, the stabilizer fields are also fixed at the origin respectively by the Hubble-induced and their own supersymmetric masses. These moduli potential are generated by the superpotential of stabilizer fields in a similar way to the previous small-field inflation. The near-future cosmological observations have a chance to detect the gravitational wave predicted by a class of large-field inflation. We stress that both the inflation scenarios do not suffer the η problem which is the generic feature of inflation models on the basis of the 4D $\mathcal{N} = 1$ supergravity models.

After the end of inflation, the inflaton field oscillates and dominates the universe. When the inflaton reheats the universe, the reheating temperature is mainly determined by the inflaton decay into the gauge-boson pairs, provided that the couplings among them are not suppressed. Moreover, both the proposed inflation scenarios are irrelevant to the dynamics of supersymmetry breaking, if the inflation scale is much larger than the SUSY-breaking scale. This is ensured by the fact that the large masses of inflaton and stabilizer fields are provided by their supersymmetric masses in the superpotential. From this perspective, the decay width of inflaton into the gravitino(s) is suppressed by almost vanishing F-term of inflaton field. We leave the detail of further phenomenological aspects of the 5D SUGRA associated with these two inflation scenarios to the following Chapters, in which we set the concrete matter sectors and SUSY-breaking sector at the same time.

So far, we have concentrated on the moduli dynamics within the framework of 5D supergravity. It turns out that the moduli potential is strictly constrained by the symmetries in higher-dimensional spacetime such as Lorentz and gauge symmetries. In our scenario, extra $U(1)_{I'}$ symmetries enabled us to generate the quasi-localized wavefunctions of charged stabilizer fields and the desirable moduli potential in the inflation scenario. The form of norm function also played essential roles to determine the shape of potential. It would be possible that the 5D supergravity model is derived from, e.g., the low-energy effective theory of heterotic M-theory in eleven-dimensions [22] compactified on CY manifold. In such a case, the norm function is identical to the $\mathcal{N} = 2$ prepotential and its coefficient $C_{I,J,K}$ is related to the intersection numbers of CY manifold. The $U(1)_{I'}$ symmetries might then have an origin in the local symmetries on heterotic five-branes or M5-branes. We conclude that our 5D models are attractive scenario from the observational as well as theoretical points of view.

Chapter 3

Moduli rolling to a natural MSSM vacuum with gravitino dark matter

In this chapter and next chapter 4, we further study the phenomenological and cosmological aspects of 5D SUGRA on S^1/Z_2 where the 4D chirality is caused by the partial breaking of $\mathcal{N} = 2$ SUSY. So far, several systematic studies have been performed by focusing separately on the particle cosmology [36, 6] or phenomenology [37] within the framework of the off-shell formulation of 5D SUGRA [17, 18]. However, it would be important to take both of them into account simultaneously. For this reason, in this chapter and next chapter 4, we propose the phenomenological models consistent with the ongoing LHC experiments [38] and the successful moduli inflation discussed in Chapter 2.

From the phenomenological point of view, the low-scale supersymmetric model is an attractive scenario which not only protects the mass of the Higgs boson from the large radiative corrections but also gives the plausible dark matter candidates. MSSM also gives rise to the successful gauge coupling unification at the GUT scale. In addition to it, the existence of SUSY is also partially motivated in the string theory where the SUSY guarantees the absence of tachyons.

The observed Higgs boson mass [39] indicates that the large radiative corrections are required within the MSSM. The high-scale SUSY-breaking scenario is then discussed as one of the solutions to raise the Higgs boson mass in the MSSM. This is because the dangerous SUSY flavor and CP interactions can be suppressed by the heavy supersymmetric particles. However, such a high-scale SUSY-breaking scenario suffers from a fine-tuning problem, because the large radiative corrections appear in the mass of the Higgs boson. On the other hand, one can enhance the Higgs boson mass by the different approach which can be applied in the case of low-scale SUSY-breaking scenario. The authors of Refs. [40] pointed out that the nonuniversal gaugino masses at the GUT scale not only enhance the Higgs boson mass, but also relax the degree of tuning to realize the EW symmetry breaking. In this chapter, we focus on this low-scale SUSY-breaking scenario on the basis of Ref. [7], whereas in the next Chapter 4, we treat the high-scale SUSY-breaking scenario on the basis of Ref. [11].

3.1 SUSY-breaking scenarios

There is no hint of SUSY at the scale below TeV scale from the collider experiments. It implies that the supersymmetric particle should be heavier than such a scale due to the effect of SUSY-breaking. It is well known that the mechanisms for communicating the breaking effects to the MSSM sector are classified into the gravity mediation [41], gauge mediation [42], the anomaly mediation [43] and their mixed one. As mentioned in Chapter 2, the cosmological problems, in particular, the cosmological gravitino problem [44] are captured by the gravitino mass. Indeed, in the case of unstable gravitino, the mass of the gravitino should be larger than $\mathcal{O}(10)$ TeV in order not to contradict with the successful BBN data, although it depends on the reheating temperature after the inflation. (See, for more details, Refs. [44, 45, 46, 47, 48].)* Thus, before going to details of our model, we summarize the typical features of several SUSY-breaking scenarios in the light of gravitino mass.

The gauge mediated SUSY-breaking scenario predicts the ultralight gravitino with mass $m_{3/2} \ll \mathcal{O}(1)$ GeV in the low-scale SUSY-breaking scenario. Note that when the gravitino mass is larger than GeV scale, the gravitational interactions would spoil the dynamics of the SUSY-breaking sector and change the sparticle spectrum. In the pure anomaly mediated SUSY-breaking scenario, the wino-like neutralino becomes the dark matter candidate through the renormalization group effects in the MSSM [50]. From the recent results of the LHC experiments [51], the gluino mass is constrained to be larger than TeV scale that is in turn in one-to-one correspondence with the gravitino $m_{3/2} \simeq \mathcal{O}(100)$ TeV in its framework. In the mirage mediation [52], which is the mixture of moduli mediation and anomaly mediation, the mixed neutralino will correspond to the dark matter candidate. In order to avoid the cosmological gravitino problem, the gravitino mass has to be larger than $\mathcal{O}(10)$ TeV. One may dilute such a gravitino abundance by moduli decay or the topological defects. Finally, in the gravity mediation, the neutralino becomes the plausible dark matter candidate. However, the large gravitino mass above $\mathcal{O}(10)$ TeV is required not to contradict with flavor experiments. Since the gravitational interactions are flavor dependent interactions, the dangerous SUSY flavor violations will generically appear in this class of model.

In this chapter, we focus on the gravity-mediated SUSY-breaking scenario. In contrast to the previous studies, our model is compatible with the low-scale SUSY and observed Higgs boson mass and is free from the cosmological gravitino and SUSY flavor problems. Since the soft SUSY-breaking terms is originating from the gravitational interactions, the scale of the gravitino mass is correlated with the masses of supersymmetric particles in general. In this way, it would be difficult to combine the low-scale SUSY-breaking and solution of the cosmological gravitino problem. The following sections are organized as follows. To overcome this problem, we propose one of the solutions to generate the mass hierarchies between the gravitino and the other supersymmetric particles on the basis of a general four-dimensional $\mathcal{N} = 1$ supergravity (4D $\mathcal{N} = 1$ SUGRA) in the rest of this chapter. In particular, we focus on a scenario where the gravitino is the lightest supersymmetric particle (LSP) so as not to contradict with the BBN data. Its mass is typically of O(100) GeV which is outside the mass range expected by the gauge mediated SUSY-breaking scenario. That would be a typical feature of the sparticle

*The light gravitino is possible in the extension of the MSSM, e.g., Ref. [49].

spectrum in the gravity mediation. There are several directions to study the cosmological implications of gravitino dark matter by assuming the certain sparticle spectrum and it turns out that the next-to-the-lightest supersymmetric particle (NLSP) is severely constrained in order not to contradict with BBN data. (See, for more details, e.g., Refs. [48, 53, 54, 55].) However, the sparticle spectrum is sensitive to the relevant higher-dimensional operators in 4D $\mathcal{N} = 1$ SUGRA, which requires the knowledge of a fundamental theory. In terms of 5D SUGRA, we can specify these higher-dimensional operators and predict the sparticle spectrum controlled by the $U(1)$ charges of matter fields in the MSSM. Moreover, one can estimate the relic abundance of dark matter on the basis of the successful moduli inflation and stabilization as suggested in Chapter 2. We find that the relic abundance of the gravitino is sensitive to the moduli dynamics, e.g., the moduli decay into the gravitino.

3.2 The gravitino dark matter in 4D SUGRA

To combine the low-scale SUSY-breaking and the solution of the cosmological gravitino problem, we show the mechanism to generate the mass hierarchies between the gravitino and other supersymmetric particles on the basis of 4D $\mathcal{N} = 1$ SUGRA. The scalar potential in 4D $\mathcal{N} = 1$ SUGRA is described by

$$\begin{aligned} V &= e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) \\ &= K_{I\bar{J}} F^I F^{\bar{J}} - 3e^{K/2}|W|^2, \end{aligned} \tag{3.1}$$

where $F^I = -e^{K/2} K^{I\bar{J}} D_{\bar{J}} \bar{W}$ are the F-terms of chiral superfields Q^I. The almost vanishing cosmological constant $\langle V \rangle \simeq 0$ relates their F-terms and gravitino mass ($m_{3/2} = e^{\langle K \rangle/2} \langle W \rangle$) through

$$m_{3/2}^2 = \frac{1}{3} \langle K_{X\bar{X}} F^X F^{\bar{X}} \rangle, \tag{3.2}$$

where the SUSY is assumed to be broken by a single chiral superfield X. One can apply the following discussion to multiple SUSY-breaking fields. On the other hand, along the line of Ref. [56, 57], the soft SUSY-breaking masses of the gauginos (M_r) and scalar components of the chiral superfields Q^I ($m_{Q^I}^2$) are estimated as

$$\begin{aligned} M_r &= \langle F^X \partial_X \ln(\mathrm{Re} f_r) \rangle, \\ m_{Q^I}^2 &= -\langle F^X \bar{F}^{\bar{X}} \partial_X \partial_{\bar{X}} \ln Y_{Q^I} \rangle, \end{aligned} \tag{3.3}$$

where f_r, $r = U(1)_Y, SU(2)_L, SU(3)_C$ represent the gauge kinetic functions of the standard model gauge groups. The kinetic term of Q^I, Y_{Q^I}, is severely constrained by the flavor structure of elementary particles as shown in a concrete model later. By comparing the above Eqs. (3.2) and (3.3), there are two possibilities induced by the nontrivial Kähler metric of the SUSY-breaking field X:

- The gravitino dark matter:

  Let us consider the small Kähler metric of the SUSY-breaking field $\langle K_{X\bar{X}}\rangle \ll 1$. When the value of Kähler metric of the SUSY-breaking field X at the tree-level is smaller than the loop and higher derivative corrections for them, we would have to take into account them. Under this condition, the mass scale of gravitino can be chosen smaller than those of other supersymmetric particles without depending on the value of F-term of SUSY-breaking field $\langle F^X\rangle$. It is expected that the gravitino would become the dark matter candidate even in the gravity mediated SUSY-breaking scenario with TeV scale gauginos and sparticles. The above situation is realized when the derivatives of the gauge kinetic function $\partial_X \text{Re} f_r$ and the kinetic term of Φ^I, $\partial_X \partial_{\bar{X}} \ln Y_{\Phi^I}$ satisfy the certain conditions. This is because the sparticle spectrum receives the sizable quantum corrections through the renormalization group (RG) effects, that can be found in the case of constrained MSSM (CMSSM) [58].

  Furthermore, in order to consider the realistic stable gravitino dark matter consistent with the BBN data, the abundance and decays of NLSP should be restricted by the success of BBN [48, 53, 54, 55]. Also, the relic abundance of gravitino is constrained within the data reported by the Planck Collaboration [15, 16]. We remark that, in the case of stable gravitino, the F-term of the SUSY-breaking field can be considered as usual low-scale SUSY-breaking scenario and it is favored from the perspective of naturalness.

- Other dark matter candidates:

  For the opposite case $\langle K_{X\bar{X}}\rangle \gg 1$, the gravitino would be heavier than the other sparticles for any value of the F-term $\langle F^X\rangle$ when $\partial_X \text{Re} f_r$ and $\partial_X \partial_{\bar{X}} \ln Y_{\Phi^I}$ are of order of unity. Then, the cosmological gravitino problem can be also relaxed even in the low-scale SUSY-breaking scenario. The gravitino mass should be larger than O(10) TeV, otherwise the electronic and hadronic showers produced by the gravitino decay threaten to spoil the success of BBN. Although we do not pursue such a possibility in this thesis, it is interesting to work in this direction.

3.3 4D effective Lagrangian of matter fields

First of all, we again remark the relevant matter contents of 5D SUGRA. The structure of the orbifold breaks 5D SUSY into the 4D $\mathcal{N}=1$ SUSY. In addition to the moduli multiplets $T^{I'}$ and stabilizer multiplets H_i as discussed in the previous Chapter 2, we consider the following zero-modes of chiral multiplets included in the hypermultiplets $\mathbf{\Phi}_\alpha$ and Z_2-even vector multiplets $\boldsymbol{V}^I$ involving the standard model gauge fields,

$$\begin{aligned} (V^1, V^2, V^3) \quad &: \quad \text{gauge vector multiplets}, \\ (\mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i) \quad &: \quad \text{quark chiral multiplets}, \\ (\mathcal{L}_i, \mathcal{E}_i, N_i) \quad &: \quad \text{lepton chiral multiplets}, \\ (\mathcal{H}_u, \mathcal{H}_d) \quad &: \quad \text{Higgs chiral multiplets}, \\ (X) \quad &: \quad \text{SUSY-breaking chiral multiplet}. \end{aligned} \tag{3.4}$$

The index $i = 1, 2, 3$ denotes the number of generation and X is the gauge singlet chiral multiplet under the standard model gauge groups. The above chiral multiplets have representations of the standard model gauge groups and extra $U(1)_{I'}$ gauge groups with their gauge fields A^I_M and $A^{I'}_M$ in vector multiplets $\boldsymbol{V}^I$ with $I = 1, 2, 3$ and $\boldsymbol{V}^{I'}$ with $I' = 1, 2, \cdots, n'_V$, respectively. In particular, $U(1)_{I'}$ charges $c^{(\alpha)}_{I'}$ are assigned to the hypermultiplets $\boldsymbol{\Phi}_\alpha$. In summary, the visible sector consists of the MSSM plus right-handed (s)neutrinos, whereas the SUSY-breaking is induced by the SUSY-breaking chiral multiplet.

Next, we summarize the 4D effective action after the off-shell dimensional reduction of the 5D off-shell supergravity [17, 18] along the line of Refs. [59, 60, 19]. As shown in Refs. [20, 21], the 4D effective Lagrangian is described in the 4D $\mathcal{N} = 1$ superspace† as follows

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left[\int d^2\theta \ \sum_r f_r(X,T){\rm tr}(\mathcal{W}^r\mathcal{W}^r) + {\rm h.c.}\right] + \int d^4\theta \ \ |\phi|^2\Omega_{\rm eff}(|Q|^2, {\rm Re}T)$$
$$+ \left[\int d^2\theta \ \ \phi^3 W(Q,T) + {\rm h.c.}\right], \tag{3.5}$$

where ϕ is the compensator multiplet fixing the 4D conformal symmetry, $\mathcal{W}^r$ are the field strength supermultiplets for a massless 4D vector multiplets V^r with $r = U(1)_Y, SU(2)_L, SU(3)_C$ in the 5D Z_2-even multiplets $\boldsymbol{V}^I$, Q_α are all the 4D chiral multiplets, and $T^{I'}$ are the moduli chiral multiplets.

The gauge kinetic functions $f_r(X,T)$ in Eq. (3.5) are extracted as

$$f_r(X,T) = \xi^r_X X + \sum_{I'=1}^{n'_V} \xi^r_{I'} T^{I'}, \tag{3.6}$$

where the first term on the right-handed side denotes the gauge kinetic function at the orbifold fixed point $y = 0$ with ξ^r_X being the real constants. The others on the right-handed side are the bulk gauge kinetic functions with $\xi^r_{I'}$ being the real constants determined by the real coefficients $C_{I',J,K}$ in the norm function. We will no go into the detail of the gauge kinetic functions at the orbifold fixed points which is dependent on the dynamics of the SUSY-breaking sector, but come back to it later.

In addition, the effective Kähler potential in Eq. (3.5) becomes

$$\Omega_{\rm eff}(|Q|^2, {\rm Re}T)$$
$$= -3\mathcal{N}^{1/3}({\rm Re}T)\left[1 - \frac{2}{3}\sum_\alpha Y(c^{(\alpha)}\cdot T)|Q_\alpha|^2 + \sum_{\alpha,\beta}\tilde{\Omega}^{(4)}_{\alpha,\beta}({\rm Re}T)|Q_\alpha|^2|Q_\beta|^2 + \mathcal{O}\Big(|Q|^6\Big)\right], \tag{3.7}$$

up to a boundary Kähler potential at the orbifold fixed points $y = 0, L$. The kinetic terms of Q_α, $Y(z)$ $(z = c^{(\alpha)}\cdot T)$, are obtained by solving their equation of motion in the fifth direction,

$$Y(z) \equiv \frac{1 - e^{-2{\rm Re}z}}{2{\rm Re}z}, \tag{3.8}$$

†It is generalized by Refs. [61, 62] including the Z_2-odd fields.

where the above exponential factor is a consequence of the localized wavefunctions in the fifth dimension. The bulk mass, i.e., $U(1)_{I'}$ charges control the wavefunction profile of charged fields in our framework. Furthermore, the four-point couplings between chiral multiplets Q_α, $\tilde{\Omega}^{(4)}_{\alpha,\beta}$ also depends on the $U(1)_{I'}$ charges,

$$\tilde{\Omega}^{(4)}_{\alpha,\beta} \equiv \frac{(c^{(\alpha)} \cdot \mathcal{P}a^{-1} \cdot c^{(\beta)})\{Y((c^{(\alpha)} + c^{(\beta)}) \cdot T) - Y(c^{(\alpha)} \cdot T)Y(c^{(\beta)} \cdot T)\}}{3(c^{(\alpha)} \cdot \mathrm{Re}T)(c^{(\beta)} \cdot \mathrm{Re}T)} - \frac{Y((c^{(\alpha)} + c^{(\beta)}) \cdot T)}{9}, \tag{3.9}$$

where $a_{IJ} \equiv -(\mathcal{N}_{IJ} - \mathcal{N}_I\mathcal{N}_J/\mathcal{N})/(2\mathcal{N})$ and $\mathcal{P}^I{}_J \equiv \delta^I{}_J - \mathcal{X}^I\mathcal{N}_J/3\mathcal{N}$ is the projection operator from the moduli multiplet out the radion multiplet, which is a single modulus $T^{I'=1}$ with $n'_V = 1$. Finally, as in the moduli superpotential discussed in Chapter 2, Z_2-orbifolding enables us to write the superpotential $W(Q,T)$ at the orbifold fixed points $y = 0, L$ where the $\mathcal{N} = 2$ SUSY reduces to $\mathcal{N} = 1$ SUSY. Its explicit form is shown later.

3.4 Gravitino dark matter in 5D SUGRA

We now proceed to discussion where one can realize the idea in Sec. 3.2 on the basis of 5D SUGRA on S^1/Z_2 or not. From Eq. (3.7), the bulk Kähler potential is rewritten in terms of $\Omega_{\mathrm{eff}} = -e^{-K_{\mathrm{bulk}}/3}$,

$$K_{\mathrm{bulk}} = -\ln\mathcal{N}(\mathrm{Re}T) + \sum_a Z_{Q_\alpha}(\mathrm{Re}T)|Q_\alpha|^2 + Z_X(\mathrm{Re}T)|X|^2 + \mathcal{O}(|Q|^4), \tag{3.10}$$

where $Z_X(\mathrm{Re}T) = K_{X\bar{X}}$ ($Z_{Q_\alpha}(\mathrm{Re}T) = K_{Q_\alpha\bar{Q}_\alpha}$) is the Kähler metric of SUSY-breaking field X (matter chiral multiplet Q_α),

$$\begin{aligned} Z_X(\mathrm{Re}T) &= \frac{1 - e^{-2c_X\cdot\mathrm{Re}T}}{c_X \cdot \mathrm{Re}T} \\ &\simeq \begin{cases} \frac{1}{c_X\cdot\mathrm{Re}T}, & c_X \cdot \mathrm{Re}T > 0, \\ \frac{1}{|c_X\cdot\mathrm{Re}T|}|\exp(2|c_X \cdot \mathrm{Re}T|) & c_X \cdot \mathrm{Re}T < 0. \end{cases} \end{aligned} \tag{3.11}$$

The moduli dependence in the Kähler metric appears only when the fields have the $U(1)_{I'}$ charges for the Z_2-odd vector multiplets $\boldsymbol{V}^{I'}$. When the volume of the fifth dimension is large, $L \simeq \mathcal{N}^{1/2}(\langle\mathrm{Re}T\rangle) \gg 1$, the vacuum expectation value of the Kähler metric is smaller than $\mathcal{O}(1)$ in the case of positive $U(1)_{I'}$ charges, i.e., $\langle K_{X\bar{X}}\rangle \ll 1$. Thus, one can obtain the desired situation so that the gravitino would be lighter than the other sparticles.

The couplings between the SUSY-breaking multiplet X and Q_α appear through the four-point couplings $\tilde{\Omega}^{(4)}_{\alpha,X}$ in Eq. (3.9). When $\langle F^X\rangle \neq 0$, the soft SUSY-breaking masses for the scalar components of Q_α are generated. As will be shown in the concrete model in Sec. 3.7, the soft SUSY-breaking masses of Q_α are typically larger than the gravitino mass for $U(1)_{I'}$ charge assignments of Q_α which are determined to reproduce the realistic Yukawa couplings of quarks and leptons. Furthermore, by substituting the gauge kinetic functions in Eq. (3.6) into the formula (3.3), the gaugino masses at the cutoff scale (compactification scale in our scenario) are given by

$$M_r = \frac{\langle F^X \rangle}{g_r^2}\xi_X^r + \sum_{I'=1}^{n'_V} \frac{\langle F^{T^{I'}} \rangle}{g_r^2}\xi_{I'}^r. \tag{3.12}$$

When the compactification scale (such as KK scale) is set as the GUT scale, the gaugino masses at the EW scale ($M_a(M_{\rm EW})$) are written in terms of those at GUT scale ($M_a(M_{,\rm GUT})$) by considering the one-loop RG equations from the GUT scale to the EW scale,

$$M_1(M_{\rm EW}) \simeq 0.4\, M_1(M_{\rm GUT}),\ \ M_2(M_{\rm EW}) \simeq 0.8\, M_2(M_{\rm GUT}),\ \ M_3(M_{\rm EW}) \simeq 2.9\, M_3(M_{\rm GUT}). \tag{3.13}$$

When these gaugino masses at the EW scale and Higgsino mass are larger than the gravitino mass, gravitino becomes LSP. This is because Higgsino is likely to be light in the low-scale SUSY-breaking scenario. The parameters ξ_X^a and $\xi_{I'}^a$ in Eq. (3.12) are constrained to realize the gauge coupling unification at GUT scale. In this way, the gravitino becomes the dark matter candidate in the 5D supergravity model for any values of F-term as mentioned in Sec. 3.2. Since the abundance of gravitino is thermally and nonthermally generated by the moduli and/or inflaton decay, we discuss its abundance by taking into account the concrete successful moduli inflation and stabilization scenarios in the next section 3.5.

We comment on the Kähler potential at the orbifold fixed points $y = 0, L$. In our setup, it can be taken small compared with the bulk Kähler potential (3.10) owing to the suppression from the mild large volume of fifth dimension $\mathcal{N}^{1/3} \gg 1$ as can be seen from its explicit form,

$$K_{\rm boundary} = \mathcal{N}^{-1/3}\left(K^{(0)}(|X|^2) + K^{(L)}(e^{-c_X(T+\bar{T})}|X|^2) + \cdots\right). \tag{3.14}$$

The mild large volume of the fifth dimension also suppresses the one-loop corrections in the moduli Kähler potential (2.90).

On the other hand, when the $U(1)_{I'}$ charges of SUSY-breaking field are chosen as negative, the vacuum expectation value of the Kähler metric becomes larger than $\mathcal{O}(1)$, i.e., $\langle K_{X\bar{X}} \rangle \gg 1$. The mild large volume of fifth dimension contributes to enhance it. According to it, it is possible to consider a scenario that gravitino is much heavier than the sparticles for any vacuum expectation values of F-term of the SUSY-breaking field from the general mass formula of gravitino and sparticles given by Eqs. (3.2) and (3.3). It would gives rise to one of the solutions to the cosmological gravitino problem and fine-tuning problems.

3.5 The SUSY-breaking effects

To complete our discussion, we focus on the moduli inflation and stabilization scenario as those discussed in Sec. 2.2, in which one of three moduli multiplets is relevant to the slow-roll inflation.

So far, we do not specify the SUSY-breaking sector and its matter couplings. In the following, we take into account the O'Raifeartaigh model [63] with the simplified Kähler potential and superpotential,

$$K = Z_X(\mathrm{Re}T^2, \mathrm{Re}T^3)|X|^2 - \frac{1}{\Lambda^2}|X|^4,$$
$$W = w + \nu X, \tag{3.15}$$

where w and ν are the real parameters determined in the SUSY-breaking sector. Here we assume that the SUSY-breaking field X has no $U(1)_1$ charge for Z_2-odd vector multiplets carrying the inflaton field. This assignment prohibits the inflaton decay into the SUSY-breaking field as will be discussed later. In addition, there are loop corrections to the Kähler potential after integrating out the heavy modes in the SUSY-breaking sector and this mass scale is given by Λ [64].

Such SUSY-breaking effects deviate the supersymmetric minimum of the moduli and stabilizer fields from their supersymmetric minimum. As mentioned in Sec. 2.4, their F-terms are sensitive to the cosmological moduli problems as well as the sparticle masses. To estimate them, we adopt the reference point method [32] for the Kähler and superpotential (3.15) in addition to those of Eq. (2.5) with $I', i = 1, 2, 3$. The reference points for the moduli and stabilizer fields are chosen as those satisfying their global supersymmetric minimum,

$$D_{H_i}W|_{\mathrm{ref}} = W_{H_i} + K_{H_i}W = 0,$$
$$\leftrightarrow c\hat{T}^{I'=i}|_{\mathrm{ref}} = \ln\frac{J_L^{(i)}}{J_0^{(i)}} \text{ and } H_i|_{\mathrm{ref}} = 0,$$
$$D_{T^{I'}}W|_{\mathrm{ref}} = K_{T^{I'}}w. \tag{3.16}$$

The reference point of SUSY-breaking field X is chosen as that satisfying its extremal condition:

$$\begin{aligned} e^{-K}V_X|_{\mathrm{ref}} &= \partial_X\Big(\sum_{I'} K^{T^{I'}\bar{T}^{\bar{I}'}}|D_{T^{I'}}W|^2 + K^{X\bar{X}}|D_XW|^2 - 3|W|^2\Big) \\ &\simeq 3W_X\bar{W} + \partial_X(K^{X\bar{X}})|W_X|^2 + K^{X\bar{X}}W_XK_{X\bar{X}}\bar{W} - 3W_X\bar{W} \\ &\simeq \frac{4|W_X|^2}{\Lambda^2(Z_X)^2}\bar{X} + W_X\bar{W} = 0, \end{aligned} \tag{3.17}$$

in the limit $w \ll 1$, where $V_X = \partial_X V$. Then, we find its minimum,

$$X|_{\mathrm{ref}} = -\frac{\Lambda^2(Z_X)^2}{4}\left(\frac{W}{W_X}\right) \simeq -\frac{\Lambda^2(Z_X)^2 w}{4\nu}. \tag{3.18}$$

Around their reference points, we search for the true vacua of moduli, stabilizer and SUSY-breaking fields after expanding the total F-term potential under the expansion of fields, $\phi \to \phi|_{\mathrm{ref}} + \delta\phi$, $\phi = T^{I'}, H_i, X$ with $I', i = 1, 2, 3$,

$$V = V|_{\mathrm{ref}} + V_I|_{\mathrm{ref}}\delta\phi^I + V_{\bar{I}}|_{\mathrm{ref}}\delta\bar{\phi}^I + V_{IJ}|_{\mathrm{ref}}\delta\phi^I\delta\phi^J + V_{I\bar{J}}|_{\mathrm{ref}}\delta\phi^I\delta\bar{\phi}^J + V_{\bar{I}\bar{J}}|_{\mathrm{ref}}\delta\bar{\phi}^I\delta\bar{\phi}^J + O(\delta\phi^3), \tag{3.19}$$

where $V_I = \partial_I V$ and $V_{IJ} = \partial_I \partial_J V$ are the derivatives with respect to the relevant fields ϕ. Their corrections deviated from the reference points given by Eqs. (3.16) and (3.18) are evaluated as those satisfying

$$\left| V_I|_{\rm ref}\delta\phi^I + V_{\bar{I}}|_{\rm ref}\delta\bar{\phi}^I \right| \gg \left| V_{IJ}|_{\rm ref}\delta\phi^I\delta\phi^J + V_{I\bar{J}}|_{\rm ref}\delta\phi^I\delta\bar{\phi}^J + V_{\bar{I}\bar{J}}|_{\rm ref}\delta\bar{\phi}^I\delta\bar{\phi}^J \right|. \tag{3.20}$$

Thus, it is found that $\phi|_{\rm ref} + \delta\phi$ are considered as the vacuum expectation values of relevant fields. Note that these perturbations are relied on the fact that the SUSY-breaking scale is smaller than those of supersymmetric masses given by Eq. (2.9). We follow the details of the their evaluations in Appendix A. The perturbations of moduli, stabilizers and SUSY-breaking field around their reference points (3.16), (3.18) become

$$\delta H_i \simeq \frac{w}{2{\rm Re}T^{I'=i}W_{T^{I'=i}H_i}}, \quad \delta T^{I'=i} \simeq \left(\frac{w}{W_{T^{I'=i}H_i}}\right)^2, \quad \delta X \simeq \left(\frac{\Lambda^2 Z_X^2}{4w^2}\right) 5wW_X, \tag{3.21}$$

which lead to their F-terms and squared masses of moduli, stabilizer, and SUSY-breaking fields,

$$\begin{aligned}
&\sqrt{K_{T^{I'}\bar{T}^{I'}}}F^{T^{I'}} \simeq O\left(\frac{w^3}{m_{T^{I'}}^2}\right), \quad \sqrt{K_{H_i\bar{H}_i}}F^{H_i} \simeq O\left(\frac{w^3}{m_{H_i}^2}\right), \quad \sqrt{K_{X\bar{X}}}F^X \simeq \frac{-\nu}{\mathcal{N}^{1/2}Z_X^{1/2}}, \\
&m_{T^{I'}}^2 \simeq m_{H_i}^2 \simeq \frac{e^K W_{T^{I'}H_i}^2}{K_{T^{I'}\bar{T}^{I'}}K_{H_i\bar{H}_i}} \ (I' = i), \quad m_X^2 \simeq \frac{e^K}{K_{X\bar{X}}}\frac{4w^2}{\Lambda^2 Z_X^2},
\end{aligned} \tag{3.22}$$

at their vacuum, $\phi = \phi|_{\rm ref} + \delta\phi$. We now denote the mass-squared of real and imaginary parts of moduli, stabilizer, and SUSY-breaking fields as $m_{T^{I'}}^2$, $m_{H_i}^2$ and m_X^2. Although the mass differences between $m_{T^{I'=i}}$ and m_{H_i} are determined by the SUSY-breaking scale, that is, the gravitino mass $m_{3/2} = e^{\langle K\rangle}\langle W\rangle$, in our choice of parameters, SUSY-breaking effects are parametrically negligible as shown later. Thus, the real and imaginary parts of fields are almost the same as each other.

Indeed, the numerical values of their masses are summarized as

$$m_{T^1} \simeq m_{T^2} \simeq m_{H_1} \simeq m_{H_2} \simeq 4.8 \times 10^{15}\,{\rm GeV}, \quad m_{T^3} \simeq m_{H_3} \simeq 4 \times 10^{12}\,{\rm GeV}, \tag{3.23}$$

and their vacuum expectation values of F-terms are also estimated as

$$\langle F^{T^1}\rangle \simeq \langle F^{T^2}\rangle \simeq \langle F^{H_1}\rangle \simeq \langle F^{H_2}\rangle \simeq 1 \times 10^{-42}, \langle F^{T^3}\rangle \simeq \langle F^{H_3}\rangle \simeq 1.6 \times 10^{-36}, \tag{3.24}$$

in the reduced Planck unit. The tiny cosmological constant can be realized by properly setting the parameters w and ν at the vacuum $\phi = \phi|_{\rm ref} + \delta\phi$,

$$\begin{aligned}
e^{-K}V|_{\phi=\phi_{\rm ref}+\delta\phi} &\simeq K^{K\bar{X}}|D_X W|^2 - 3|W|^2 \\
&\simeq \frac{\nu^2}{Z_X} - 3w^2 \simeq 0.
\end{aligned} \tag{3.25}$$

We stress that the moduli and stabilizer fields have almost vanishing $\langle F \rangle$-terms compared with the SUSY-breaking field X. With the following choice of parameters in the Kähler and superpotential (3.15) for the SUSY-breaking sector,

$$\nu \simeq -1.567 \times 10^{-14},\ w = -6 \times 10^{-14},\ \Lambda = 10^{-4}, c_X^{(1)} = \frac{3}{10},\ c_X^{(2)} = \frac{1}{10},\ c_X^{(3)} = 0, \tag{3.26}$$

one can estimate the masses of the gravitino and X and its F-term as

$$m_{3/2} \simeq 395\,\mathrm{GeV},\ \ m_X \simeq 6 \times 10^8\,\mathrm{GeV},\ \ \frac{F^X}{M_{\mathrm{Pl}}} \simeq 4541\,\mathrm{GeV}, \tag{3.27}$$

which shows that the SUSY is mainly broken by SUSY-breaking X. The tiny cosmological constant is achieved by properly choosing ν. The above parameters suppress the Kähler metric of the SUSY-breaking field, $K_{X\bar{X}} \simeq 1/(c_X \cdot \mathrm{Re}T) \simeq 0.023$ as discussed in Sec. 3.4. We confirm that the obtained small gravitino mass is smaller than the other sparticle spectra in Sec. 3.7.3.

3.6 Moduli-induced gravitino problem

Following the above arguments, the moduli and stabilizer fields are much heavier than the inflaton field and it is expected that they decay into the particles in the MSSM before the BBN. However, as commented in Sec. 2.4, one should study the cosmological moduli problem, in particular, the moduli-induced gravitino problem [31, 65], even if the masses of heavy modes are larger than $\mathcal{O}(100)$ TeV.

First of all, we show the relevant couplings between moduli and gravitinos in the unitary gauge,

$$\mathcal{L}_{3/2} = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\tilde{D}_\rho\psi_\sigma - e^{K/2}W^*\psi_\mu\sigma^{\mu\nu}\psi_\nu - e^{K/2}W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu, \tag{3.28}$$

where $\tilde{D}_\rho\psi_\sigma = \partial_\rho\psi_\sigma + \frac{1}{4}(K_J\partial_\rho\phi^J - K_{\bar{J}}\partial_\rho\bar{\phi}^{\bar{J}})$ are the covariant derivatives with respect to the gravitino ψ_μ in two-component formalism. (See, for more details, e.g., Refs. [65].) Under a field-dependent chiral transformation,

$$\psi_\mu \to \left(\frac{W}{\overline{W}}\right)^{-1/4}\psi_\mu, \tag{3.29}$$

the Lagrangian (3.28) is brought into the following form

$$\mathcal{L}_{3/2} = \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\partial_\rho\psi_\sigma + \frac{\epsilon^{\mu\nu\rho\sigma}}{4}(G_J\partial_\rho\phi^J - G_{\bar{J}}\partial_\rho\bar{\phi}^{\bar{J}})\bar{\psi}_\mu\bar{\sigma}_\nu\psi_\sigma - e^{G/2}(\psi_\mu\sigma^{\mu\nu}\psi_\nu + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu), \tag{3.30}$$

with $G = K + \ln|W|^2$ and $G_J = \partial_J G$. Around the vacuum expectation values of moduli $T^{I'}$ given by (3.16) and (3.21), the Lagrangian (3.28) is expanded in terms of the four-component formalism of the gravitino Ψ_μ,

$$\mathcal{L}_{3/2} = -\frac{\epsilon^{\mu\nu\rho\sigma}}{2}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\rho\Psi_\sigma + \frac{\epsilon^{\mu\nu\rho\sigma}}{8}(\langle G_{T^{J'}}\rangle\partial_\rho\delta T^{J'} - \langle G_{\overline{T}^{J'}}\rangle\partial_\rho\delta\overline{T}^{\bar{J}'})\bar{\Psi}_\mu\gamma_\nu\Psi_\sigma$$
$$-\frac{1}{4}\langle e^{G/2}\rangle\bar{\Psi}_\mu[\gamma^\mu,\gamma^\nu]\Psi_\nu - \frac{1}{8}\langle e^{G/2}\rangle(\langle G_{T^{J'}}\rangle\delta T^{J'} + \langle G_{\overline{T}^{\bar{J}'}}\rangle\delta\overline{T}^{\bar{J}'})\bar{\Psi}_\mu[\gamma^\mu,\gamma^\nu]\Psi_\nu. \tag{3.31}$$

From the mass-squared (2.25) of the moduli and stabilizer fields, the pair (T^1, H_1) is lighter enough than other pairs. Thus, the dynamics of heavy pairs are irrelevant to the thermal history of the universe after the inflation and in what follows, we analyze the decay processes of T^1, H_1, and SUSY-breaking field X.

3.6.1 The decay of inflaton field

Let us estimate the decay width from inflaton into gravitino pair. According to the equivalence theorem [66], the gravitino wavefunction is written in terms of its helicity $\pm 1/2$ components, i.e., goldstinos at a high-energy limit. With the F-term of modulus (3.22) and the mass of light modulus field given by Eq. (2.25), we find that the canonically normalized inflaton (σ^1) decay width into the gravitino pair ($\Psi_{3/2} + \Psi_{3/2}$) is

$$\Gamma(\sigma^1 \to \Psi_{3/2}\Psi_{3/2}) \simeq \frac{1}{288\pi\langle K_{T^1\bar{T}^1}\rangle}\left|\left\langle\frac{D_{T^1}W}{W}\right\rangle\right|^2\frac{m_{T^1}^5}{m_{3/2}^2 M_{\rm Pl}^2} \simeq \frac{1}{288\pi\langle K_{T^1\bar{T}^1}\mathcal{N}\rangle}\frac{m_{T^1}m_{3/2}^2}{M_{\rm Pl}^2}$$
$$\simeq 1.6\times 10^{-18}\left(\frac{m_{3/2}}{400\,{\rm GeV}}\right)^2\ {\rm GeV}, \tag{3.32}$$

which is calculated from the couplings in Eq. (3.31). The factor $m_{3/2}^{-2}$ originating from the longitudinal mode of the gravitino enhances the decay width. It would produce an enough amount of gravitinos and threaten to spoil the successful BBN. However, the above direct decay can be neglected thanks to the almost vanishing $\langle F\rangle$-term of the inflaton as shown in Eq. (3.22). The main decay channel of inflaton is derived from the following Lagrangian,

$$\mathcal{L}_{Tgg} = -\frac{1}{4(g_r)^2}F^r_{\mu\nu}F^{r\mu\nu} - \frac{1}{4}\xi^r_{J'}\delta T_R^{J'}F^r_{\mu\nu}F^{r\mu\nu} - \frac{1}{8}\xi^r_{J'}\delta T_I^{J'}\epsilon^{\mu\nu\rho\sigma}F^r_{\mu\nu}F^r_{\rho\sigma}, \tag{3.33}$$

with $T_R^{J'} = {\rm Re}\,T^{J'}$ and $T_I^{J'} = {\rm Im}\,T^{J'}$. The gauge kinetic functions $f_r(X,T)$ are now considered as in Eq. (3.6), where the R-symmetry breaking term such as $\xi^r_X X$ is assumed in the gauge kinetic function. Note that the R-symmetry is also explicitly broken by the existence of constant superpotential w in Eq. (3.15).

The main decay width from the inflaton into the gauge-boson pairs ($g^{(r)} + g^{(r)}$) is estimated as

$$\sum_{r=1}^{3}\Gamma(\sigma^1 \to g^{(r)}g^{(r)}) \simeq \sum_{r=1}^{3}\frac{N_G^r}{128\pi}\left\langle\frac{\xi_3^r}{\sqrt{2K_{T^1\bar{T}^1}}}\right\rangle^2\frac{m_{T^1}^3}{M_{\rm Pl}^2} \simeq 3.95\ \ {\rm GeV}, \tag{3.34}$$

with the numerical values of mass and vacuum expectation value of inflaton (2.8) and (2.25). As shown in Sec. 2.2.3, the number of gauge bosons N_G^r is that of MSSM and the coefficients in the gauge kinetic function (free parameters in 5D SUGRA) are chosen as $\xi_1^1 = \xi_1^2 = \xi_1^2 = 0.22$ otherwise zero so that the gauge coupling unification at the GUT scale is realized.

Although the inflaton also decays into the gauginos λ^r with the following interactions,

$$\mathcal{L}_{T\lambda\lambda} = -\frac{i}{2}\sum_r \mathrm{Re}\, f_r(\lambda^r\sigma^\mu D_\mu\bar{\lambda}^r + (\mathrm{H.c.})) + \frac{i}{2}\sum_r \mathrm{Im}\, f_r D_\mu(\lambda^r\sigma^\mu\bar{\lambda}^r) + \sum_r \left(\frac{1}{4}\frac{\partial f_r}{\partial T^{I'}} F^{T^{I'}}\lambda^r\lambda^r + (\mathrm{H.c.})\right), \tag{3.35}$$

with $D_\mu\lambda^r$ being the covariant derivative with respect to gaugino, the above decay channels are suppressed by the gaugino masses and almost vanishing F-term of inflaton such as

$$\sum_{r=1}^{3}\Gamma(\sigma^1 \to \tilde{\lambda^r}\tilde{\lambda^r}) \simeq \sum_{r=1}^{3}\frac{m_{T^1}}{16\pi}\frac{(\xi_1^r)^2 m_{\lambda^r}^2}{M_{\mathrm{Pl}}^2} \simeq 1.5\times 10^{-21}\sum_{r=1}^{3}(\xi_1^r)^2\left(\frac{m_{\lambda^r}}{1.5\,\mathrm{TeV}}\right)^2 \mathrm{GeV}, \tag{3.36}$$

where $\tilde{\lambda^r}$ are the canonically normalized gauginos. Now, the derivative of F-term for the inflaton is estimated as,

$$\left\langle \frac{\partial F^{T^1}}{\partial T^1}\right\rangle = \left\langle \frac{\partial}{\partial T^1} e^{K/2}\left(K^{T^1\bar{T}^1}|D_{T^1}W|^2 + K^{T^1\bar{H}_1}D_{T^1}W\overline{D_{H_1}W}\right)\right\rangle \sim O\left(\frac{m_{3/2}^4}{m_{T^1}^2}\right). \tag{3.37}$$

Finally, we comment on the other decay processes. When the masses of sfermions are much smaller than that of the inflaton field, the inflaton decay into the sfermions is suppressed by the factor, $m_{\mathrm{sfermion}}/m_{T^1}$. The inflaton decay into the fermion pairs and quark-quark-gluon are also negligible compared with the main decay channel owing to their small masses and phase factors, as pointed out in Ref. [65]. In our setup, the inflaton decay process via a μ-term is irrelevant in our estimation as will be shown in Sec. 3.7.1. This is because μ-term is smaller enough than the inflaton mass m_{T^1}. Since SUSY is broken at the vacuum, the inflaton can decay into superpartner of inflaton field, so-called the modulino, and gravitino. The SUSY-breaking, in particular, the gravitino mass determine not only the mass difference between the inflaton and modulino, but also the mixing terms between T^1 and H_1 in their mass-squared matrices. One can then find that the inflaton decay width into the modulino $\tilde{\sigma}^1$ and gravitino is given by

$$\Gamma(\sigma^1 \to \tilde{\sigma}^1\Psi_{3/2}) \simeq \frac{1}{48\pi}\left(\frac{m_{T^1}}{M_{\mathrm{Pl}}}\right)^2\left(\frac{m_{3/2}}{m_{T^1}}\right)m_{3/2} \simeq 7.2\times 10^{-22}\left(\frac{m_{3/2}}{400\,\mathrm{GeV}}\right)^2 \mathrm{GeV}, \tag{3.38}$$

with $m_{T^1} \simeq 4\times 10^{12}\,\mathrm{GeV}$ given by Eq. (3.23). By a similar argument as in the case of other decay channels, the phase factor $m_{3/2}/m_{T^1}$ leads to the small amount of gravitino. As mentioned before, we take the ansatz that X does not have a $U(1)_1$ charge for the 5D Z_2-odd vector multiplet carrying the inflaton field. Thus, the inflaton decay into the SUSY-breaking field X is also suppressed, because of the vanishing tree-level interaction between X and T^1.

In summary, the total decay width and the branching ratios of the inflaton decaying into the gravitino(s) are

$$\Gamma^{\sigma^1}_{\rm all} \equiv \Gamma(\sigma^1 \to {\rm all}) \simeq \sum_{r=1}^{3} \Gamma(\sigma^1 \to g^{(r)} g^{(r)}) \simeq 3.95 \ \ {\rm GeV},$$
$$ {\rm Br}(\sigma^1 \to \Psi_{3/2}\Psi_{3/2}) \simeq \frac{\Gamma(\sigma^1 \to \Psi_{3/2}\Psi_{3/2})}{\Gamma^{\sigma^1}_{\rm all}} \simeq 1.4 \times 10^{-20},$$
$$ {\rm Br}(\sigma^1 \to \tilde{\sigma}^1\Psi_{3/2}) \simeq \frac{\Gamma(\sigma^1 \to \tilde{\sigma}^1\Psi_{3/2})}{\Gamma^{\sigma^1}_{\rm all}} \simeq 1.8 \times 10^{-22}, \tag{3.39}$$

in the case of $m_{3/2} = 395\,{\rm GeV}$. As discussed in Sec. 2.2.3, the reheating temperature $T_R \simeq 1.38 \times 10^9\,{\rm GeV}$ is estimated in Eq. (2.39) by equating the total decay width of inflaton $\Gamma^{\sigma^1}_{\rm all}$ with the Hubble parameter H_R. The tiny branching ratio of the inflaton decay into the gravitino(s) reduces the nonthermally generated gravitino yield $Y_{3/2}$,

$$Y_{3/2} = \frac{n_{3/2}}{s} \simeq {\rm Br}(\sigma^1 \to \tilde{\sigma}^1\Psi_{3/2})\frac{3T_R}{4m_{T^1}} \simeq 3.8 \times 10^{-24}, \tag{3.40}$$

with $m_{3/2} = 395\,{\rm GeV}$, $T_R = 1.38 \times 10^9\,{\rm GeV}$ and $s = 4\rho/3T$. $n_{3/2}$, s, and ρ represent the number density of the gravitino, entropy and energy densities of the Universe, respectively. In the previous calculation, we assume that the oscillating energy of inflaton field dominates that of the Universe after the inflation and the energy densities of other fields will be analyzed in the next section.

We stress that the gravitino nonthermally generated through the inflaton decay can be suppressed by the structure of the supersymmetric moduli stabilization and inflation. Thus, it is expected that supersymmetric moduli inflation would give the solution to the cosmological moduli problem, in particular, the moduli-induced gravitino problem. We show an another gravitino production through the stabilizer fields, the SUSY-breaking field, and the thermal bath in the next section.

3.6.2 The decay of stabilizer and SUSY-breaking fields

Next, we discuss the decay of stabilizer field H_1 and SUSY-breaking field X. Their imaginary components ${\rm Im}\,H_1$ and ${\rm Im}\,X$ stay at the origin during and after the inflation. Hence, they do not oscillate as explicitly shown in Appendix A. By contrast, the real parts of stabilizer field $h_1 = {\rm Re}\,H_1$ and SUSY-breaking field $x = {\rm Re}\,X$ are stabilized at the values close to the origin by the Hubble induced masses during the inflation. When the Hubble parameters are comparable to the mass scales of h_1 and x after the inflation, they oscillate around their SUSY-breaking minima of potential (3.21) evaluated by the reference point method (3.16).

The results of Appendix A allow us to write down the amplitudes of both fields

$$\Delta h_1 \simeq \frac{m_{3/2}}{m_{H_1}}, \quad \Delta x \simeq \left(\frac{m_{3/2}}{m_X}\right)^2. \tag{3.41}$$

First of all, we take into account the dynamics of h_1. From the mass of h_1 given in Eq. (3.23) and the reheating temperature (2.39), h_1 stars to oscillate before the reheating process, i.e., $H^{h_1}_{\rm osc} \simeq m_{h_1} \gg H(T_R)$, where $H_{\rm osc}$ is the Hubble parameter at the beginning of its oscillation. When there are no interactions between H_1 and fields in the MSSM, H_1 mainly decays into the gravitino pairs ‡,

$$\Gamma^{h_1}_{\rm all} \equiv \Gamma(h_1 \to \Psi_{3/2}\Psi_{3/2}) \simeq \frac{1}{288\pi K_{H_1\bar{H}_1}} \left| \left\langle \frac{D_{H_1}W}{W} \right\rangle \right|^2 \frac{m^5_{H_1}}{m^2_{3/2}M^2_{Pl.}} \simeq \frac{1}{288\pi \langle K_{H_1\bar{H}_1}\mathcal{N}\rangle} \frac{m^3_{H_1}}{M^2_{\rm Pl}}$$
$$\simeq 0.02 \;\; {\rm GeV}, \qquad (3.42)$$

from which, h_1 decays after the coherent oscillation of h_1 and reheating process, i.e., $H^{h_1}_{\rm osc} > H(T_R) > H^{h_1}_{\rm dec}$, with $H^{h_1}_{\rm osc} \simeq m_{h_1}$ and $H^{h_1}_{\rm dec} \simeq \Gamma(h_1 \to \Psi_{3/2}\Psi_{3/2})$. Here and in what follows, we represent $H_{\rm R}$, $H^{\Phi}_{\rm osc}$, and $H^{\Phi}_{\rm dec}$ as the Hubble parameters at the moment of reheating, beginning of oscillation of relevant fields Φ, and decay of Φ, respectively. Correspondingly, the scale factors of 4D spacetime a_R, $a^{\Phi}_{\rm osc}$, and $a^{\Phi}_{\rm dec}$ are also defined in a similar fashion to the Hubble parameters presented above.

When h_1 oscillates after the inflation around its SUSY-breaking minimum, its oscillating energy density is represented as

$$\rho_{h_1} \simeq \frac{1}{2}m^2_{H_1}(\Delta h_1)^2 \simeq \frac{1}{2}m^2_{3/2}\left(\frac{a}{a^{h_1}_{\rm osc}}\right)^{-3}, \qquad (3.43)$$

which is converted into the gravitino yield,

$$Y^{h_1}_{3/2} = \frac{2\rho_{h_1}}{m_{H_1}s} \simeq \frac{1}{4}\frac{m^2_{3/2}T_R}{m^3_{H_1}} \simeq 8.2\times 10^{-25}\left(\frac{m_{3/2}}{400\,{\rm GeV}}\right)^2, \qquad (3.44)$$

with $T_R = 1.38\times 10^9\,{\rm GeV}$, $m_{H_1} = 4\times 10^{12}\,{\rm GeV}$ and s is the entropy density of the Universe. As can be seen in the following inequality, h_1 does not dominate the Universe and release a sizable amount of entropy,

$$1 \gg \left.\frac{\rho_{h_1}}{\rho_R}\right|_{T=T^{h_1}_{\rm dec}} = \left.\frac{\rho_{h_1}}{\rho}\right|_{\rm end}\left(\frac{T_R}{T_{t^{h_1}_{\rm dec}}}\right) \simeq \frac{m^2_{3/2}M^2_{\rm Pl}}{2V_{\rm inf}}\left(\frac{T_R}{T_{t^{h_1}_{\rm dec}}}\right), \qquad (3.45)$$

where ρ_{h_1} (ρ_R) is the energy density of h_1 (radiation). In addition, $\rho|_{\rm end} = V_{\rm inf} \simeq \mathcal{O}(10^{-13})$ stands for the energy density at the end of inflation and $T^{h_1}_{\rm dec}$ corresponds to the decay temperature of h_1,

$$T^{h_1}_{\rm dec} = \left(\frac{\pi^2 g_*}{90}\right)^{-1/4}\sqrt{\Gamma^{h_1}_{\rm all}M_{\rm Pl}} \simeq 8.6\times 10^7 \;\; {\rm GeV}. \qquad (3.46)$$

The tiny mass of the gravitino contributes to the suppression of the amplitude of h_1 and its energy density.

‡Even if H_1 couples to the standard model field, the conclusion is the same as that of the following discussion.

Next, we discuss the dynamics of SUSY-breaking field x. The production of gravitinos through x decay is evaluated from the following dominant decay width,

$$\Gamma(x \to \Psi_{3/2}\Psi_{3/2}) \simeq \frac{1}{288\pi\langle K_{X\bar{X}}\rangle}\left|\left\langle\frac{D_X W}{W}\right\rangle\right|^2 \frac{m_X^5}{m_{3/2}^2 M_{\rm Pl}^2} \simeq \frac{1}{288\pi\langle K_{X\bar{X}}\rangle}\left|\frac{\nu}{w}\right|^2 \frac{m_X^5}{m_{3/2}^2 M_{\rm Pl}^2}. \quad (3.47)$$

which is enhanced by the factor $m_{3/2}^{-2}$ originating from the longitudinal mode of the gravitino. By employing the parameters (3.26), the VEVs of moduli (2.8), and masses of gravitino $\Psi_{3/2}$ and SUSY-breaking field X (3.27), the total decay width of x is estimated as

$$\Gamma_{\rm all}^x \equiv \Gamma(x \to \Psi_{3/2}\Psi_{3/2}) \simeq 3.7 \times 10^8 \ \ {\rm GeV}. \quad (3.48)$$

Thus, the reheating process starts after the oscillation and decay of $x = {\rm Re}\, X$, that is, $H_{\rm osc}^x > H_{\rm dec}^x \gg H(T_R)$, with $H_{\rm osc}^x \simeq m_X$ and $H_{\rm dec}^x \simeq \Gamma(x \to \Psi_{3/2}\Psi_{3/2})$. The energy density of the coherent oscillation x at the reheating

$$\rho_x \simeq \frac{1}{2}m_x^2(\Delta x)^2 \simeq \frac{1}{2}\left(\frac{m_{3/2}^4}{m_x^2}\right)\left(\frac{a_{\rm dec}^x}{a_{\rm osc}^x}\right)^{-3}\left(\frac{a_R}{a_{\rm dec}^x}\right)^{-4}, \quad (3.49)$$

is converted into gravitino yield which is relativistic at the production,

$$Y_{3/2}^x = \frac{2\rho_x}{m_x s} \simeq \frac{3}{2}\frac{T_R}{m_X}\left(\frac{m_{3/2}}{m_X}\right)^{16/3}\left(\frac{\Gamma_{\rm all}^{\sigma^3}}{\Gamma_{\rm all}^x}\right)^{2/3} \simeq 2 \times 10^{-32}\left(\frac{m_{3/2}}{400\,{\rm GeV}}\right)^{16/3}. \quad (3.50)$$

Now, the scale factors are related as,

$$\frac{a_R}{a_{\rm osc}^{\sigma^1}} = \left(\frac{\sqrt{6}\Gamma_{\rm all}^{\sigma^1}}{m_{T^1}}\right)^{-2/3}, \quad \frac{a_{\rm osc}^x}{a_{\rm osc}^{\sigma^3}} = \left(\frac{\sqrt{6}m_X}{m_{T^1}}\right)^{-2/3}, \quad \frac{a_{\rm dec}^x}{a_{\rm osc}^{\sigma^3}} = \left(\frac{6(\Gamma_{\rm all}^x\, m_X)^2}{m_{3/2}^4}\right)^{-1/3}, \quad (3.51)$$

and the numerical values are given in Eqs. (3.27), (3.39), (2.39), and (3.48). Since x decaying into the gravitino is negligible because of the tiny mass of the gravitino, it implies that the gravitino production from x decay is not the dominant source for the relic abundance of gravitino reported by Planck. At the same time, x does not produce a sizable entropy in a way similar to the case of h_1. Our results are consistent with the studies in Ref. [67], which suggest that the gravitino production is significantly relaxed under the condition $m_{3/2} \ll m_{\rm X} \ll m_{T^1} \leq \Lambda$. When Λ is smaller than the inflaton mass, i.e., $m_{T^1} \geq \Lambda$, the inflaton would decays into the fields in the hidden sector which also produce the gravitino fields. In the next section, we show the thermal production of gravitino, which will explain the current relic abundance of gravitino.

3.7 The Higgs boson mass, gravitino dark matter and sparticle spectra

3.7.1 Yukawa couplings and naturalness

Before going to the detail of the relic abundance of gravitino, we analyze the Yukawa couplings and μ-term included in the superpotential of MSSM. The holomorphic superpotential can be only introduced at the orbifold fixed points $y = 0, L$, where the $\mathcal{N} = 2$ SUSY is partially broken down to the $\mathcal{N} = 1$ SUSY. First, the Yukawa couplings among the MSSM chiral multiplets and (s)neutrinos chiral multiplets involved in the hypermultiplets Φ_α are written as

$$W_{\text{Yukawa}} = \lambda^u_{ij}\mathcal{Q}_i\mathcal{H}_u\mathcal{U}_j + \lambda^d_{ij}\mathcal{Q}_i\mathcal{H}_d\mathcal{D}_j + \lambda^e_{ij}\mathcal{L}_i\mathcal{H}_d\mathcal{E}_j + \lambda^n_{ij}\mathcal{L}_i\mathcal{H}_u N_j, \tag{3.52}$$

where $\lambda^{u,d,e,n}_{ij}$ are the holomorphic Yukawa coupling constants. We choose the size of $\lambda^{u,d,e,n}_{ij}$ as of $\mathcal{O}(1)$. Furthermore, the R-charge of the chiral multiplets in Eq. (3.4) (R_ϕ) is assigned as $R_X = R_{H_a} = 2$, $R_{\mathcal{Q}_i} = R_{\mathcal{U}_i} = R_{\mathcal{D}_i} = R_{\mathcal{L}_i} = R_{\mathcal{E}_i} = R_{N_i} = 1$, $R_{\mathcal{H}_u} = R_{\mathcal{H}_d} = 0$. After canonically normalizing their matter chiral multiplets, the physical Yukawa couplings are obtained as

$$y^u_{ij} = \frac{\lambda^u_{ij}}{\sqrt{\langle Y_{\mathcal{Q}_i} Y_{\mathcal{H}_u} Y_{\mathcal{U}_j}\rangle}}, \quad y^d_{ij} = \frac{\lambda^d_{ij}}{\sqrt{\langle Y_{\mathcal{Q}_i} Y_{\mathcal{H}_d} Y_{\mathcal{D}_j}\rangle}}, \quad y^e_{ij} = \frac{\lambda^e_{ij}}{\sqrt{\langle Y_{\mathcal{L}_i} Y_{\mathcal{H}_d} Y_{\mathcal{E}_j}\rangle}}, \quad y^n_{ij} = \frac{\lambda^n_{ij}}{\sqrt{\langle Y_{\mathcal{L}_i} Y_{\mathcal{H}_u} Y_{N_j}\rangle}}, \tag{3.53}$$

where

$$Y_a \equiv 2\mathcal{N}^{1/3}(\text{Re}T)\left\{Y(c_a\cdot T) + \tilde{\Omega}^{(4)}_{a,X}(\text{Re}T)|X|^2 + \mathcal{O}(|X|^4)\right\} \simeq 2\mathcal{N}^{1/3}(\text{Re}T)Y(c_a\cdot T), \tag{3.54}$$

with an almost vanishing vacuum expectation value of X. The function $Y(z)$ is always positive, and is approximated as

$$Y(z) \equiv \frac{1-e^{-2\text{Re}z}}{2\text{Re}z} \simeq \begin{cases} \frac{1}{2\text{Re}\,z} & \text{Re}\,z > 0, \\ \frac{1}{2|\text{Re}z|}\exp(2|\text{Re}z|) & \text{Re}\,z < 0, \end{cases} \tag{3.55}$$

which reflects the fact that the wavefunctions of fields in the fifth direction are localized toward $y = 0$ ($y = L$) when $c_a \cdot \langle \text{Re}T\rangle$ is positive (negative). When all the relevant fields are localized toward $y = 0$, the size of Yukawa couplings $y^{u,d,e,n}_{ij}$ are of $\mathcal{O}(1)$. On the other hand, in the localized case toward $y = L$, the Yukawa coupling constants among relevant matter fields are exponentially small. It is thus possible to realize the hierarchical Yukawa couplings of quarks and leptons, and the tiny Yukawa couplings of neutrinos, even in the case of Dirac neutrinos. As shown in Tables 3.1 and 3.2, we find that the observed masses and mixing angles of quarks and leptons at the EW scale are obtained by setting the proper $U(1)_{I'}$ charges of matter fields and the holomorphic Yukawa couplings $\lambda^{u,d,e,n}_{i,j}$ with of $\mathcal{O}(1)$. Their numerical values are obtained by solving the full one-loop RG group equations of the MSSM from the GUT scale to EW scale.

Interestingly, the $U(1)_{I'}$ charge assignments of matter fields give rise not only to the observed hierarchical masses and mixing angles among the elementary particles, but also to the flavor structure of the soft SUSY-breaking terms which are derived from the four-point couplings between the SUSY-breaking field and the matter multiplets in the effective Kähler potential (3.9). The soft SUSY-breaking terms are characterized by the Lagrangian,

$$\mathcal{L}_{\text{soft}} = -\sum_\alpha m^2_{S_\alpha}|S_\alpha|^2 - \left(\frac{1}{2}\sum_{r=1}^{3} M_r\lambda^r\lambda^r + \frac{1}{6}\sum_{\alpha,\beta,\gamma} a_{\alpha\beta\gamma}S_\alpha S_\beta S_\gamma + B\mu h_u h_d + \text{h.c.}\right), \tag{3.56}$$

$c^{I'=3}_{\mathcal{Q}_i} = (0.1,\ 0.1,\ 1.1)$ $c^{I'=2}_{\mathcal{Q}_i} = (-0.1, -0.1, 0.8)$ $c^{I'=1}_{\mathcal{Q}_i} = (0.1,\ 0.4,\ 1)$	$c^{I'=3}_{\mathcal{L}_i} = (0.1,\ 0.1,\ 1.6)$ $c^{I'=2}_{\mathcal{L}_i} = (-0.1, -0.1,\ 0)$ $c^{I'=1}_{\mathcal{L}_i} = (0.1,\ 0.5,\ 0)$	$c^{I'=3}_{\mathcal{H}_u} = 0$ $c^{I'=2}_{\mathcal{H}_u} = 0.1$ $c^{I'=1}_{\mathcal{H}_u} = -0.9$
$c^{I'=3}_{\mathcal{U}_i} = (0.1,\ 0.1,\ 0.6)$ $c^{I'=2}_{\mathcal{U}_i} = (-0.1, -0.1, 0.3)$ $c^{I'=1}_{\mathcal{U}_i} = (-0.2,\ 0.2,\ 1)$	$c^{I'=3}_{\mathcal{E}_i} = (0.1,\ 0.2,\ 0.2)$ $c^{I'=2}_{\mathcal{E}_i} = (-0.1, -0.1, 0)$ $c^{I'=1}_{\mathcal{E}_i} = (-0.2,\ 0,\ -0.5)$	$c^{I'=3}_{\mathcal{H}_d} = 0$ $c^{I'=2}_{\mathcal{H}_d} = 0$ $c^{I'=1}_{\mathcal{H}_d} = -0.1$
$c^{I'=3}_{\mathcal{D}_i} = (0.1,\ 0.1,\ 0.2)$ $c^{I'=2}_{\mathcal{D}_i} = (-0.1, -0.1, 0)$ $c^{I'=1}_{\mathcal{D}_i} = (0.3,\ 0.2,\ -0.5)$	$c^{I'=3}_{N_i} = (0.1,\ 0.1,\ 0.1)$ $c^{I'=2}_{N_i} = (-0.3, -0.3, -0.3)$ $c^{I'=1}_{N_i} = (-0.7,\ -0.7,\ -0.7)$	

Table 3.1: $U(1)_{I'}$ charge assignments of the quarks, leptons, and Higgs for the Z_2-odd vector multiplets $\boldsymbol{V}^{I'}$ in Ref. [7].

$\lvert\lambda^u_{ij}\rvert$	$\lvert\lambda^d_{ij}\rvert$
$\begin{pmatrix} 0.32 & 0.35 & 0.95 \\ 0.22 & 0.42 & 0.33 \\ 0.51 & 0.48 & 1.5 \end{pmatrix}$	$\begin{pmatrix} 0.45 & 0.5 & 0.59 \\ 0.28 & 0.24 & 0.38 \\ 1.03 & 1.02 & 0.81 \end{pmatrix}$
$\lvert\lambda^e_{ij}\rvert$	$\lvert\lambda^n_{ij}\rvert$
$\begin{pmatrix} 0.28 & 0.22 & 0.52 \\ 0.4 & 1.15 & 0.31 \\ 0.8 & 1.02 & 1.05 \end{pmatrix}$	$\begin{pmatrix} 0.77 & 0.85 & 0.69 \\ 0.25 & 0.98 & 0.58 \\ 0.34 & 0.26 & 1.03 \end{pmatrix}$

Table 3.2: The holomorphic Yukawa coupling constants $\lambda^{u,d,e,n}_{ij}$ in the superpotential (3.52) [7].

where $S_\alpha = h_u, h_d, \tilde{q}_i, \tilde{u}_i, \tilde{d}_i, \tilde{l}_i, \tilde{e}_i, \tilde{\nu}_i$ are the scalar components of $\mathcal{H}_u, \mathcal{H}_d, \mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i, L_i, \mathcal{E}_i, N_i$ and λ^r $(r = 1, 2, 3)$ are the gauginos. The soft SUSY-breaking masses and trilinear scalar couplings (A-terms) are determined as [56, 57],

$$m^2_{S_\alpha} = -\langle F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \ln(\hat{Y}_{S_\alpha})\rangle, \quad a_{\alpha\beta\gamma} = y_{\alpha\beta\gamma}\langle F^I \partial_J \ln(\hat{Y}_{S_\alpha}\hat{Y}_{S_\beta}\hat{Y}_{S_\gamma})\rangle, \tag{3.57}$$

where the indices I and J run over all the chiral multiplets. As a counterpart of μ-term in the MSSM, we consider the following superpotential,

$$W_\mu = \sum_{i=1}^{3} \kappa_i H_i \mathcal{H}_u \mathcal{H}_d, \tag{3.58}$$

with κ_i being the $\mathcal{O}(1)$ dimensionless couplings. The R-charge is assigned to the stabilizer chiral multiplets as 2 and Higgs chiral multiplets as 0, respectively. Even if we add the above cubic

Sparticles	Mass [GeV]	(S)Particles	Mass [GeV]
$m_{\tilde{\mathcal{Q}}_1}$	1682	$m_{\tilde{\mathcal{L}}_3}$	2834
$m_{\tilde{\mathcal{Q}}_2}$	1530	$m_{\tilde{\mathcal{E}}_1}$	1157
$m_{\tilde{\mathcal{Q}}_3}$	581	$m_{\tilde{\mathcal{E}}_2}$	2390
$m_{\tilde{\mathcal{U}}_1}$	1157	$m_{\tilde{\mathcal{E}}_3}$	2298
$m_{\tilde{\mathcal{U}}_2}$	1698	$m_{\tilde{N}_1}$	414.5
$m_{\tilde{\mathcal{U}}_3}$	799	$m_{\tilde{N}_2}$	414.5
$m_{\tilde{\mathcal{D}}_1}$	1636	$m_{\tilde{N}_3}$	414.5
$m_{\tilde{\mathcal{D}}_2}$	1698	$M_{\mathcal{H}_u}$	1100
$m_{\tilde{\mathcal{D}}_3}$	2298	$M_{\mathcal{H}_d}$	298.5
$m_{\tilde{\mathcal{L}}_1}$	1682	M_3	550
$m_{\tilde{\mathcal{L}}_2}$	1396		

Table 3.3: The soft scalar masses $m_{\tilde{Q}_\alpha}$, the up-and down-type Higgs boson masses $M_{\mathcal{H}_{u,d}}$, and the gluino mass M_3 at the GUT scale in Ref. [7]. The subscripts $\tilde{Q}_\alpha$ denote the mass eigenstates for the three-generations of left-handed $\tilde{\mathcal{Q}}_i$, up-type right-handed $\tilde{\mathcal{U}}_i$, down-type right-handed $\tilde{\mathcal{D}}_i$ squarks, left-handed $\tilde{\mathcal{L}}_i$, right-handed $\tilde{\mathcal{E}}_i$ charged sleptons, and right-handed sneutrinos $\tilde{N}_i$ with $i = 1, 2, 3$.

interactions between the stabilizer fields and Higgs fields, the moduli stabilization and inflation scenario in Sec. 2.2 are irrelevant to their dynamics. This is because their vacuum expectation values are almost vanishing at the vacuum. Thus, the effective μ-term at the GUT scale is generated by the existence of nonvanishing vacuum expectation values of H_i. It is expressed as

$$\mu = \sum_{i=1}^{3} \frac{\kappa_i \langle H_i \rangle}{\langle Y_{H_i} Y_{\mathcal{H}_u} Y_{\mathcal{H}_d} \rangle}, \tag{3.59}$$

where the relevant fields are canonically normalized. The above μ-term could be chosen as TeV scale, e.g., in the case of $\kappa_1 = \kappa_2 = 0$, $\kappa_3 = 2/3$, $m_{3/2} = 395\,\mathrm{GeV}$, $m_{H_3} \simeq 4.8 \times 10^{15}\,\mathrm{GeV}$, and $\langle H_3 \rangle \simeq m_{3/2}/m_{H_1}$ given by Eq. (3.21),

$$\mu \simeq 3.8 \times 10^{-3} \frac{m_{3/2}}{m_{H_3}} M_{\mathrm{Pl}} \simeq \mathcal{O}(m_{3/2}). \tag{3.60}$$

The suppression factor 3.8×10^{-3} is related to the mild large volume of fifth dimension. One can also consider the origin of μ- and $B\mu$-terms as the Giudice-Masiero terms [68] in the boundary Kähler potential at $y = 0$,

$$K^{(0)} = c\mathcal{H}_u\mathcal{H}_d + \mathrm{h.c.}, \tag{3.61}$$

with c being a constant, that leads to μ-and $B\mu$-terms of the order of gravitino mass,

$$\mu \simeq c m_{3/2}, \quad B\mu \simeq -c m_{3/2}^2. \tag{3.62}$$

These holomorphic terms do not have an origin in the bulk Kähler potential (3.7) in the nature of $\mathcal{N}=2$ SUSY.

As a result, one can estimate the masses and coupling constants of supersymmetric particles. In Table 3.3, the soft scalar masses and gluino masses at the GUT scale are summarized by employing the $U(1)_{I'}$ charges in Table 3.1 and the F-term of the SUSY-breaking field X given by Eq. (3.27). On the other hand, the tiny F-terms of moduli give rise to the almost vanishing A-terms. In the light of relic abundance of gravitino and Higgs boson mass, we parametrize the ratios of gaugino masses at the GUT scale as

$$r_1 = \frac{M_1(M_{\rm GUT})}{M_3(M_{\rm GUT})}, \quad r_2 = \frac{M_2(M_{\rm GUT})}{M_3(M_{\rm GUT})}, \tag{3.63}$$

where $M_r(M_{\rm GUT})$, $r=1,2,3$ represent the bino, wino, and gluino masses at the GUT scale. Their ratios can be changed by the parameters ξ_X^a in the gauge kinetic function (3.6) without spoiling the gauge coupling unification at the GUT scale. This is because the SUSY-breaking field X is irrelevant to the size of gauge couplings owing to its tiny vacuum expectation value.

After the EW symmetry breaking, the Z-boson mass m_Z and soft SUSY-breaking masses of the up-type Higgs $m_{\mathcal{H}_u}$ satisfies the following relation at the EW scale,

$$\frac{m_Z^2}{2} \simeq -m_{H_u}^2(M_{\rm EW}) - |\mu(M_{\rm EW})|^2 + O\left(\frac{1}{\tan^2\beta}\right), \tag{3.64}$$

where $\mu(M_{\rm EW})$ and $m_{H_u}(M_{\rm EW})$ are the μ-term and m_{H_u} at the EW scale, respectively. Here, we take the limit of large value of $\tan\beta$ to realize the correct Higgs boson mass in the low-scale SUSY-breaking scenario. In order to obtain the observed Z-boson mass, the μ-term and up-type Higgs boson mass are typically of the order of the Z-boson mass $m_Z = 91.2$ GeV, otherwise we have to properly tune both terms. As a measure of degree of tuning of μ-term at the GUT scale, we introduce the so-called Barbieri-Giudice parameter [69], $100\times|\Delta_\mu^{-1}|$ % with

$$\Delta_\mu = \frac{1}{2}\frac{\partial \ln m_Z^2}{\partial \ln|\mu|}. \tag{3.65}$$

In the following, we explore the parameter regions on the place of the ratio of gaugino masses to obtain the observed Higgs boson mass and correct relic abundance of gravitino without demanding the fine-tunings.

We remark about the origin of neutrino masses and related phenomenology. Although we focus on the Dirac neutrinos, one can introduce the mass terms of Majorana neutrinos under a different ansatz of R-charges of matter fields that also explain the tiny masses of the neutrinos by employing the see-saw mechanism [70]. In such a case, these terms would be inserted in the superpotential at the other boundary $y=L$ to obtain the correct mass scale of Majorana neutrino. In terms of $\mathcal{O}(1)$ parameters κ_{ij} and the $U(1)_{I'=i}$ charges of the Majorana neutrino chiral multiplets N_i for the $U(1)_{I'=i}$ vector multiplets $V^{I'}$, the superpotential is obtained as $W = \kappa_{ij} e^{-2c_{I'}^{N_i}\hat{T}^{I'}} N_i N_j$. As discussed in Ref. [71], the inflaton decays into the Majorana neutrinos, i.e., $T^1 \to N_1 N_1$ after the inflation. When the Majorana neutrino is not thermalized, the nonthermally generated Majorana neutrinos induce the lepton-number violations and hereafter baryon asymmetry is generated by the sphaleron process [72, 73, 74], known as the nonthermal leptogenesis [75].

3.7.2 Relic abundance of gravitino dark matter

In this section, we estimate the relic abundance of the gravitino by employing the obtained sparticle spectrum. The discussion in Sec. 3.6 indicates that the gravitino is not efficiently produced by the decay of inflaton, heavy moduli, stabilizer, and SUSY-breaking fields after the inflation. However, for the thermal bath that is occupied of the relativistic particles after the reheating process, the gravitinos are thermally produced by the particles in the MSSM. The light supersymmetric particles would also produce the gravitino at their decoupling time.

First, we study the thermal production of gravitino from the thermal bath. The dominant production channel comes from the scattering between gauginos and gravitinos, provided that the couplings among them are not suppressed. On the other hand, other production channels through the sparticles are more suppressed than those of gauginos as discussed in Refs. [76, 77]. The authors of Refs. [76, 77] show that the abundance of gravitino is written in terms of numerical parameters $w_r = (11, 27, 72)$ and $k_r = (1.266, 1.312, 1.271)$ in Ref. [77],

$$\Omega^{TP}_{3/2}h^2 = \sum_{r=1}^{3}\left(1+\frac{M_r(T_R)^2}{3m^2_{3/2}}\right) w_r g_r(T_R)^2 \ln\left(\frac{k_r}{g_r(T_R)}\right)\left(\frac{m_{3/2}}{100\text{GeV}}\right)\left(\frac{T_R}{10^{10}\text{GeV}}\right), \qquad (3.66)$$

where h is dimensionless Hubble parameter. We plot the thermal abundance of gravitino in Fig. 3.1 as a function of the ratios of gaugino masses at the GUT scale $M_{\rm GUT}$, $r_1 = M_1(M_{\rm GUT})/M_3(M_{\rm GUT})$ and $r_2 = M_2(M_{\rm GUT})/M_3(M_{\rm GUT})$ with $M_3(M_{\rm GUT}) = 550$ GeV. Here, we suppose that the gaugino masses at the reheating temperature $M_r(T_R)$ are given by those at GUT scale $M_a(M_{\rm GUT})$ by employing the one-loop RG equations in the MSSM. In Fig. 3.1, the dotted curves correspond to the upper and lower limits of dark matter abundance $0.1175 \leq \Omega^{TP}_{3/2}h^2 \leq 0.1219$ reported by the Planck Collaboration [15, 16]. It is now assumed that the relic abundance of dark matter only consists of the thermally produced gravitino.

NNLSP(Higgsino-like neutralino)	mass[GeV]
$\tilde{\chi}^0_1$	441
NLSPs(right-handed sneutrinos)	mass[GeV]
$\tilde{\nu}_{e_2}$	415
$\tilde{\nu}_{\mu_2}$	415
$\tilde{\nu}_{\tau_2}$	415
LSP(gravitino)	mass[GeV]
$\Psi_{3/2}$	395

Table 3.4: The masses of NNLSP, NLSPs, and the gravitino at the EW scale for the reference point $(r_1, r_2) = (6, 3.5)$ in Ref. [7]. The subscripts denote the mass eigenstates for the sneutrinos ($\tilde{\nu}$) and the Higgsino-like neutralino ($\tilde{\chi}$).

Next, we show the nonthermal productions of gravitino from the decay of the light supersymmetric particles such as the NLSP and/or next-to-next-to-lightest supersymmetric particle (NNLSP). Let us take the ratios of gaugino masses $(r_1, r_2) = (6, 3.5)$ in a consistent way with

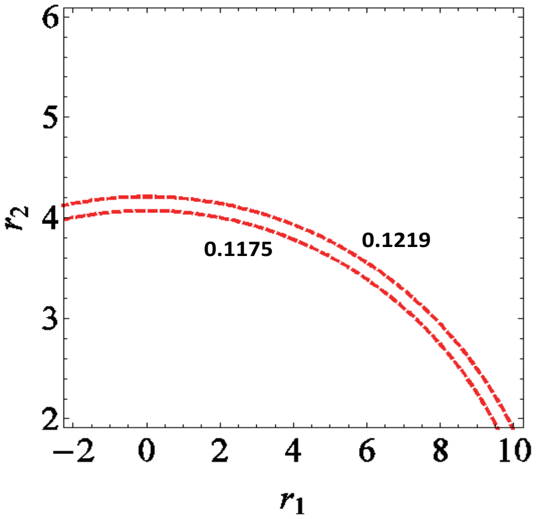

Figure 3.1: The thermal abundance of gravitino within the range of Planck data [15, 16], $0.1175 \leq \Omega^{TP}_{3/2} h^2 \leq 0.1219$. As drawn in Ref. [7], it is dependent on the ratios of gaugino masses r_1 and r_2.

the observed relic abundance of dark matter as drawn in Fig. 3.1. By solving the full one-loop RG equations in the MSSM from GUT to EW scales with $(r_1, r_2) = (6, 3.5)$ and the input parameters in Table 3.3, one can obtain the sparticle spectra. As shown in Table 3.4, the NLSPs and NNLSP are identified as three degenerate sneutrinos and Higgsino-like neutralino. The full sparticle spectra are shown in the next section. Since the tiny Yukawa couplings of Dirac-type neutrinos suppress the couplings between the degenerate sneutrinos and other (s)particles, loop-corrections are negligible for the soft SUSY-breaking masses of right-handed sneutrinos. It implies that the interactions of gravitino and right-handed sneutrinos are so suppressed that they are not thermalized. In this respect, the abundance of nonthermal gravitino produced by the higgsino-like neutralino is roughly estimated in terms of mass and the thermal abundance of the Higgsino-like neutralino $\tilde{\chi}^0_1$, $m_{\tilde{\chi}^0_1}$ and $\Omega_{\tilde{\chi}^0_1}$,

$$\Omega^{NTP}_{3/2} h^2 = \frac{m_{3/2}}{m_{\tilde{\chi}^0_1}} \Omega_{\tilde{\chi}^0_1} h^2. \tag{3.67}$$

In the low-scale SUSY-breaking scenario, that is, $\mu < \mathcal{O}(500)$ GeV, the thermal abundance of Higgsino-like neutralino is suppressed by the large annihilation cross section which originates from the nature of degenerated chargino and Higgsino-like neutralino. Then, the decoupled time of both chargino and Higgsino-like neutralino are almost the same as each other. As a result, the abundance of nonthermal gravitino is outside the current sensitivity of Planck,

$$\Omega^{NTP}_{3/2} h^2 \ll 0.11, \tag{3.68}$$

and then the relic abundance of gravitino is given by its thermal abundance as mentioned before,

$$\Omega_{3/2} h^2 \simeq \Omega^{TP}_{3/2} h^2. \tag{3.69}$$

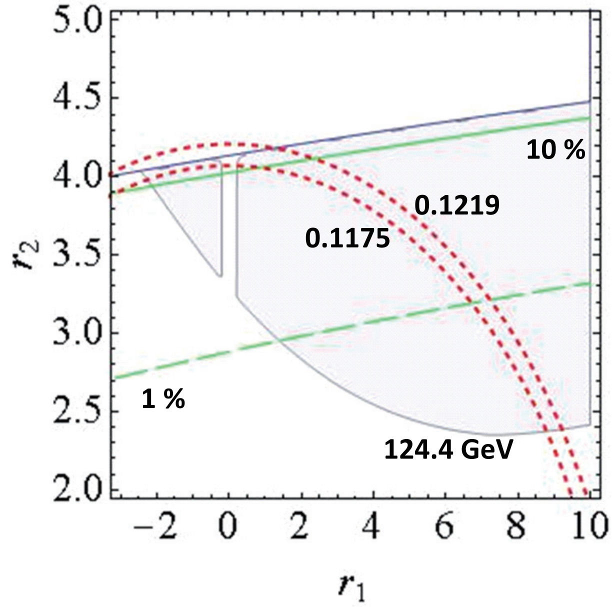

Figure 3.2: The blue shaded regions correspond to the Higgs boson mass resides in the allowed range, $124.4 \leq m_h \leq 126.8\,\mathrm{GeV}$ [39]. The green dashed (solid) line represents the 1% (10%) tuning the μ-term, $|\Delta_\mu| \times 100(\%)$. The relic abundance of the gravitino $\Omega_{3/2}h^2$ corresponds to the red dashed curves constrained within the ranges $0.1179 \leq \Omega_{3/2}h^2 \leq 0.1215$ by the Planck [15, 16]. This figure is shown in Ref. [7].

However, the neutralino and sneutrinos decay into the gravitino dark matter after the BBN. The electronic and hadronic showers released by the their decay threaten to spoil the successful BBN. Although the right-handed neutrinos are produced associated with the sneutrino decay into the gravitino, their abundance is suppressed and harmless for the BBN.

For the Higgsino-like neutralino decay, one can relax the constraints from the BBN when the NLSP occupies the Dirac-type right-handed sneutrinos [78]. The result of Ref. [78] in the case of bino-like neutralino NNLSP is also applied in our setup, because the sparticle spectrum is almost the same. Thus, the small thermal abundance of Higgsino-like neutralino reduces the constraints from BBN data. The bino-like neutralino can be also considered in our scenario by choosing the small value of $|r_1|$ in Fig. 3.1. However, the nonthermal production of gravitino enhanced by the large thermal abundance of bino-like neutralino would break the successful BBN as discussed in Refs. [48, 55, 78].

3.7.3 Results

In the previous section, it is found that the relic abundance of gravitino constrains the ratios of gaugino masses at the GUT scale, r_1 and r_2 as illustrated in Fig. 3.1. On top of that, we show that the mass of Higgs boson gives the further constraints on r_1 and r_2 in this section. One of the two Higgs bosons in the MSSM, in particular, the lightest *CP*-even Higgs boson becomes a plausible candidate of that of standard model. The low-scale SUSY-breaking scenario in the MSSM is difficult to achieve the realistic Higgs boson mass. Since it is of the order of the Z-boson mass at the tree-level, the loop corrections play an important role for raising the Higgs boson mass to be consistent with the LHC experiment [79]. Moreover, the recent LHC data implies that such loop-corrections have to be maximally enhanced that can be realized by the maximal mixing of left-and right-handed top squarks. When the mass eigenstates of these top squarks are nearly degenerated, it could be difficult to realize the observed lightest CP-even Higgs boson mass. As pointed out in Ref. [40], the certain nonuniversal gaugino masses are relevant to realize such an enhancement through the renomralization group effects and at the same time, the degree of tuning the μ-parameter can be relaxed. This is because the up-type Higgs boson mass appearing in Eq. (3.64) can be suppressed by the certain ratios of gaugino masses.

In this regard, we analyze the full one-loop renormalization group equations of the MSSM from GUT scale to the EW scale by scanning the ratio of nonuniversal gaugino masses r_1 and r_2. The numerical calculation of Higgs boson mass and the degree of tuning the μ-parameter, $|\Delta_\mu| \times 100\%$, are drawn in Fig. 3.2. It is then found that the blue-colored region resides in the range of $124.4 \leq m_h \leq 126.8$ [39], whereas the green dashed and solid lines correspond to the degree of tuning the μ-parameter 1% and 10%, respectively. Thus, there are parameter regions which lead to the correct relic abundance of gravitino and the observed Higgs boson mass simultaneously without a severe fine-tuning.

In Tables 3.4, 3.5, and 3.6, we show a typical sparticle spectrum, the Higgs boson mass m_h, and the degree of tuning the μ-parameter $|\Delta_\mu| \times 100(\%)$ for the benchmark point $(r_1, r_2) = (6, 3.5)$. Our results are consistent with the experimental lower bounds for all the sparticle masses reported by the LHC experiments in Refs. [51] and [80]. In the gravity-mediated SUSY-breaking scenario, the flavor dependent interactions in general cause the dangerous SUSY flavor violations. However, in our setup, the flavor dependent soft SUSY-breaking terms induced by the A-terms are suppressed by the vanishing F-terms of moduli fields. Even if the moduli fields have the sizable F-terms at the vacuum, the SUSY flavor violating interactions are controlled by the $U(1)_{I'}$ charge assignments of matter fields [81]. As a result, dangerous SUSY flavor-violating interactions in the decay channels such as $\mu \to e\gamma$ and $b \to s\gamma$ are not discriminated by the present sensitivity of experiments [82, 83]. In the next chapter, we discuss the high-scale SUSY-breaking scenario considered as another simple solution to raise the Higgs boson mass and avoid the SUSY flavor violations by allowing some demanding the tuning of parameters to obtain the EW vacuum.

3.8 Summary

In an former part of this chapter, we have discussed the several SUSY-breaking scenarios on the basis of the 4D $\mathcal{N} = 1$ SUGRA in the light of gravitino mass. In particular, we focus on the gravity-mediated SUSY-breaking scenario where the gravitino is the lightest supersymmetric particle, i.e, the gravitino dark matter that is triggered by the nontrivial Kähler metric of the SUSY-breaking field. With the small Kähler metric of SUSY-breaking field, the gravitino mass becomes small compared with those of other supersymmetric particles without relying on the

Sparticles	Mass[GeV]	Sparticles	Mass[GeV]
$m_{\tilde{u}_1}$	2618	$m_{\tilde{e}_1}$	3241
$m_{\tilde{u}_2}$	2359	$m_{\tilde{e}_2}$	2525
$m_{\tilde{c}_1}$	2520	$m_{\tilde{\mu}_1}$	2421
$m_{\tilde{c}_2}$	2011	$m_{\tilde{\mu}_2}$	2331
$m_{\tilde{t}_1}$	1735	$m_{\tilde{\tau}_1}$	2133
$m_{\tilde{t}_2}$	974	$m_{\tilde{\tau}_2}$	1447
$m_{\tilde{d}_1}$	2625	$m_{\tilde{\nu}_{e_1}}$	3240
$m_{\tilde{d}_2}$	2620	$m_{\tilde{\nu}_{e_2}}$	415
$m_{\tilde{s}_1}$	2522	$m_{\tilde{\nu}_{\mu_1}}$	2330
$m_{\tilde{s}_2}$	2189	$m_{\tilde{\nu}_{\mu_2}}$	415
$m_{\tilde{b}_1}$	2117	$m_{\tilde{\nu}_{\tau_1}}$	2132
$m_{\tilde{b}_2}$	1724	$m_{\tilde{\nu}_{\tau_2}}$	415
$m_{\tilde{\chi}_4^0}$	1723	$m_{\tilde{\chi}_1^\pm}$	444
$m_{\tilde{\chi}_3^0}$	1135	$m_{\tilde{\chi}_2^\pm}$	1723
$m_{\tilde{\chi}_2^0}$	448		
$m_{\tilde{\chi}_1^0}$	441		

Table 3.5: The mass eigenvalues of sparticle spectra at the EW scale for the reference point, $(r_1, r_2) = (6, 3.5)$ in Ref. [7]. The subscripts denote the mass eigenstates of sparticles such as up ($\tilde{u}$), charm ($\tilde{c}$), top ($\tilde{t}$), down ($\tilde{d}$), strange ($\tilde{s}$), bottom ($\tilde{b}$) squarks, the scalar electron ($\tilde{e}$), muon ($\tilde{\mu}$), tauon ($\tilde{\tau}$), neutrino ($\tilde{\nu}$), the neutralino ($\tilde{\chi}$), and the chargino ($\tilde{\chi}^\pm$).

details of SUSY breaking mechanism. Thus, one can consider the gravitino dark matter with low-scale SUSY-breaking scenario where the cosmological gravitino problem is solved if NLSP decays do not spoil the success of BBN. Note that here the gauge kinetic functions and the kinetic terms of the matter fields should satisfy certain conditions as discussed in the case of CMSSM [58].

Since the abundance of gravitino depends on the moduli and inflaton decays, as a concrete model, we have considered the 4D $\mathcal{N} = 1$ SUGRA derived from 5D SUGRA model on S^1/Z_2 as discussed in Chapter 2. The gravitinos produced by the moduli decays would threaten to spoil the success of BBN when these moduli have sizable F-terms. One of the solutions to avoid such problems is that these moduli F-terms are suppressed. Such a situation can be realized in our supersymmetric moduli stabilization and inflation shown in Chapter 2. The moduli, inflaton, and stabilizer fields have supersymmetric masses at their supersymmetric vacuum. We found that, even after introducing the source of SUSY-breaking, their minima are not so deviated from their supersymmetric one and then their F-terms are suppressed by the gravitino mass. Although the couplings between matter fields in the MSSM and SUSY-breaking field are controlled by their $U(1)$ charge assignments for 5D Z_2-odd vector multiplets, they are almost uniquely determined to realize the hierarchical Yukawa couplings among the elementary particles. In our setup, the NLSP and NNLSP are then uniquely fixed as the

m_h[GeV]	m_H[GeV]	m_A[GeV]	$m_{H\pm}$[GeV]
125.4	1423	1423	1425
$\Delta_\mu^{-1} \times 100(\%)$	$M_1(m_Z)$[GeV]	$M_2(m_Z)$[GeV]	$M_3(m_Z)$[GeV]
2.1	1133	1719	1575

Table 3.6: The masses of lightest CP-even Higgs boson m_h and charged Higgs bosons m_H, m_A, and $m_{H_\pm}$, the degree of tuning of the μ-parameter, $|\Delta_\mu| \times 100(\%)$, and the gaugino masses at the EW scale for the reference point $(r_1, r_2) = (6, 3.5)$ in Ref. [7].

sneutrino and Higgsino-like neutralino, respectively. The small thermal abundance of Higgsino-like neutralino leads to a negligible nonthermal productions of the gravitino. The sneutrionos are not thermalized due to tiny Yukawa couplings of Dirac-type neutrinos. One can relax the constraints from the BBN in these sparticle spectrum as discussed in Refs. [48, 55, 78], and at the same time, the produced right-handed neutrinos via the sneutrino decay are irrelevant to the BBN. Thus, the total relic abundance of gravitino dark matter is dominated by its thermal production after the inflation, that is controlled by the gaugino masses. The authors of Refs. [40] show that the certain ratios of gaugino masses also play an important role for raising the Higgs boson mass in the MSSM without a severe fine-tuning. Taking these issues into account, we find that certain ratios of gaugino masses in our model lead to both the correct relic abundance of gravitino reported by Planck [15, 16] and observed Higgs boson mass in LHC data [39].

So far, we concentrated on the gravitino dark matter in 5D SUGRA as one of concrete models describing particle phenomenology and cosmology both at the same time. The suppressed Kähler metric of the SUSY-breaking field is the key ingredient to generate the mass hierarchies between the gravitino and other supersymmetric particles. If the 5D SUGRA model is derived from type IIB string theory on a warped throat and/or heterotic M-theory on the Calabi-Yau manifold [22], the SUSY-breaking sector could be originated from the gauge theory living on D-branes and/or NS5-branes. For the case of type IIB string, the visible and SUSY-breaking sectors can be constructed from the different types of D-branes wrapping the different cycles in the internal manifold. The different sizes of internal cycles would lead to the hierarchical Kähler metric between the SUSY-breaking field and matter fields in the visible sector as in the 5D model discussed here.

Chapter 4

Moduli rolling to a MSSM vacuum with wino dark matter

In contrast to the previous chapter, we study the 5D SUGRA with high-scale SUSY on the basis of Ref. [11]. Before going to its details, we summarize the current experimental results of the LHC. The LHC Run I results show that there is no hint of supersymmetric particles up to a scale of $\mathcal{O}(1)$ TeV [38], and as we mentioned in Chapter 3, certain mechanism is required to realize the observed Higgs boson mass [39] in the framework of MSSM with a low-scale SUSY-breaking. Even in the case of low-scale SUSY-breaking scenario, one can achieve the observed value of Higgs boson mass as discussed in Chapter 3. When the SUSY-breaking scale is larger than $\mathcal{O}(10)$ TeV, the observed Higgs boson mass is achieved in the MSSM with split SUSY [8], spread SUSY [9] or pure gravity mediation [10]. These high-scale SUSY-breaking models predict the small gaugino masses compared with those of the other supersymmetric particles. In particular, in the pure gravity mediation scenario, the gaugino masses are generated by the anomaly mediation [43, 84], whereas the other sparticle masses are induced by the gravity-mediated SUSY-breaking. Thus, in such a scenario, the lightest supersymmetric particle is the wino-like neutralino that will be the dark matter candidate [43, 84]. However, one cannot determine the relic abundance of wino-like neutralino unless we specify the thermal history of the universe after the inflation. In addition to thermally produced wino-like neutralino, there is a nonthermal production from the gravitino decay [85], moduli decay [50] and Q-ball [86]. In this respect, one has to specify a concrete model to discuss the high-scale SUSY-breaking scenario with the wino dark matter. In the following, we construct a phenomenologically successful model derived from the 5D SUGRA on S^1/Z_2 with successful moduli inflation and stabilization proposed in Chapter 2.

4.1 High-scale SUSY-breaking scenario and sparticle spectra

In addition to the low-scale SUSY-breaking model in Chapter 3, we discuss a high-scale SUSY-breaking scenario of 5D SUGRA on the basis of the previous moduli inflation and stabilization in Chapter. 2. The SUSY-breaking sector is set as that in the low-scale SUSY-breaking scenario in Chapter 3. Even in this situation, one can discuss the high-scale SUSY-breaking scenario without contradicting to the moduli inflation and the moduli stabilization, as far as the SUSY-breaking scale is much lower than the inflation scale. In contrast to the low-scale SUSY-breaking scenario, we do not consider the R-parity violating operator in the gauge kinetic function, represented by $f^{(0)} = \xi_x X$ with ξ_x being the real parameter. Thus, the sizable gaugino masses are not generated from the tree-level gauge kinetic function,

$$f_r(T) = \sum_{I'=1}^{n_V'} \xi_{I'}^r T^{I'}. \tag{4.1}$$

due to the almost vanishing F-terms of moduli fields. The anomaly mediation generates the gaugino masses [43, 84],

$$M_r = \frac{b_r g_r^2}{16\pi^2} m_{3/2}, \tag{4.2}$$

where $b_r = (33/5, 1, -3)$ are the beta-function coefficients in the MSSM. Now, the relevant vacuum expectation values of conformal compensator, $\langle F^\phi \rangle / \langle \phi \rangle$ are replaced by the gravitino mass $m_{3/2}$, since the moduli and the stabilizer fields have almost vanishing F-terms (3.22) and the vacuum expectation value of SUSY-breaking field X is much smaller than reduced Planck scale (3.18).

Furthermore, there are other one-loop threshold corrections to the gaugino masses. When the μ-term is of the order of the gravitino mass $m_{3/2}$, the heavy higgsino contributes to the gaugino masses as calculated in Refs. [84, 85],

$$M_r = \frac{b_r c_r g_r^2}{16\pi^2} \mu \sin 2\beta \frac{m_A^2}{|\mu|^2 - m_A^2} \ln \frac{|\mu|^2}{m_A^2}, \tag{4.3}$$

where $c_r = (1/11, 1, 0)$ and $\tan\beta = v_d/v_u$ with $v_{u,d}$ being the vacuum expectation values of up-and down-type Higgs fields. Here, we take the limit, $m_W \ll \mu, m_A$ with m_W and m_A being the masses of W-boson and CP-odd heavy Higgs boson, respectively. We explain the origin of μ- and $B\mu$-terms later.

The soft SUSY-breaking masses of supersymmetric particles are derived from the four-point couplings between X and the matter multiplets S_α in the MSSM (3.9) that depend on the $U(1)_{I'}$ charges of S_α for 5D Z_2-odd vector multiplets. As shown later, typical sparticle masses are of the order of the gravitino mass. It then turns out that these sparticle spectra are similar to those predicted by the pure gravity mediation [10], where the gaugino masses are lighter than other sparticle masses by one-loop factor (4.2). Then, the wino-like neutralino corresponds to the LSP through the renormalization group effects as can be seen in Eq. (3.13). The thermal abundance of wino LSP is determined by the wino mass that is converted into the gravitino mass from Eq. (4.2) after solving the Boltzmann equations. On the other hand, the nonthermal abundance of wino is highly model-dependent. Hence, on the basis of moduli inflation as discussed in Chapter 2, we proceed to study these nonthermal abundances produced by the decays of gravitino, moduli and stabilizer fields.

The $U(1)$ charges of the matter chiral multiplets in the MSSM and (s)neutrinos yield the flavor structure of them as well as the hierarchical structure of the mass matrices of quarks and leptons as shown in Chapter 3. In the high-scale SUSY-breaking scenario, we set the same $U(1)$ charges of matter fields as in the case of low-scale SUSY-breaking scenario. In addition, we also consider the origin of μ-term as in Eq. (3.58), which leads to the μ-term of the order of the gravitino mass (3.60). $B\mu$-term is obtained from the Giudice-Masiero terms [68] in Eq. (3.61). From the soft SUSY-breaking terms defined in Eq. (3.63), the soft scalar masses are of the order of the gravitino mass and they can be extracted from the four-point couplings between the SUSY-breaking field X and the matter multiplets in Eq. (3.9). On the other hand, the gaugino masses and the A-terms are almost vanishing, because the moduli and the stabilizer fields have almost vanishing F-terms. Therefore, one-loop anomaly mediated effects are the leading contribution to them, which yields Eq. (4.2) for gaugino masses and

$$a_{\alpha\beta\gamma} = -\left(\gamma_\alpha^\zeta y_{\zeta\beta\gamma} + \gamma_\beta^\zeta y_{\alpha\zeta\gamma} + \gamma_\gamma^\zeta y_{\alpha\beta\zeta}\right) m_{3/2}, \tag{4.4}$$

for A-terms, where $\gamma_\alpha^\beta = \frac{1}{16\pi^2}\left(\frac{1}{2}y^{\alpha\beta\gamma}y^*_{\alpha\beta\gamma} - 2g_r^2 C_r(S_\alpha)\delta_\beta^\alpha\right)$ are the anomalous dimension with S_α being the fields in the MSSM and $C_r(S_\alpha)$ are the quadratic Casimir invariants. These anomaly mediated effects for the soft SUSY-breaking masses can be negligible compared with the F-term of SUSY-breaking field X, that is, gravity-mediated effects. We stress that the wino mass is constrained by the LHC experiments. The unstable gravitino must decays before the BBN, otherwise the electronic and hadronic showers associated with the gravitino decay would spoil the successful BBN. Thus, from Eqs. (3.13) and (4.2), the wino mass is constrained to $M_2 > 200 - 250\,\text{GeV}$. The LHC experiments searching for the disappearing tracks [87] also give the lower bound for the wino mass $M_2 > 270\,\text{GeV}$.

4.2 Relic abundance of wino dark matter

In the following, we explore the relic abundance of the wino LSP by a similar argument as in the case of low-scale SUSY-breaking scenario in Chapter 3. Their main contributions are categorized into two types of decay channels. First, the inflaton ($\sigma^1 = \mathrm{Re}\,T^1$), the real part of stabilizer ($h_1 = \mathrm{Re}\,H_1$) and SUSY-breaking fields ($x = \mathrm{Re}\,X$) decay into the gravitino pair after the inflation and hereafter the nonthermally produced gravitinos decay into the wino LSP at its decoupling time. On the other hand, their imaginary parts of fields, $\mathrm{Im}\,T^1$, $\mathrm{Im}\,H_1$ and $\mathrm{Im}\,X$ do not oscillate around their minimum in the inflationary era and the other heavy moduli and stabilizer fields are also irrelevant to the inflaton dynamics. Second, the thermally produced gravitinos in the thermal plasma during the era of radiation domination decay into the wino LSP.

First of all, we estimate the nonthermal production of gravitinos given through the decays of σ^1, h_1 and x after the inflation shown in Ref. [7]. For the inflaton decays into the gravitino pairs, the gravitino yield $Y_{3/2} = n_{3/2}/s$ written in terms of the number density of gravitino $n_{3/2}$

and the entropy density of Universe s is given by

$$Y_{3/2}^{\sigma^1} \simeq \mathrm{Br}(\sigma^1 \to \Psi_{3/2}\Psi_{3/2})\frac{3T_R}{4m_{T^1}} \simeq \frac{1}{288\pi\langle K_{T^1\bar{T}^1}\rangle \Gamma_{\mathrm{all}}^{\sigma^1}}\frac{3m_{3/2}^2 T_R}{4M_{\mathrm{Pl}}^2}$$
$$\simeq 2\times 10^{-19}\left(\frac{m_{3/2}}{10^5\,\mathrm{GeV}}\right)^2\left(\frac{T_R}{10^9\,\mathrm{GeV}}\right), \tag{4.5}$$

where $\Gamma_{\mathrm{all}}^{\sigma^1} \simeq 3.95\,\mathrm{GeV}$ is the total decay width of inflaton and $T_R \simeq 1.38\times 10^9$ GeV is the reheating temperature, which are calculated in Sec. 3.6.

Next, the gravitino yield produced by the stabilizer field $h_1 = \mathrm{Re}\,H_1$ becomes

$$Y_{3/2}^{h_1} = \frac{2\rho_{h_1}}{m_{H_1}s} \simeq \frac{1}{4}\frac{m_{3/2}^2 T_R}{m_{H_1}^3} = 2.5\times 10^{-18}\left(\frac{m_{3/2}}{10^5\,\mathrm{GeV}}\right)^2\left(\frac{10^{12}\,\mathrm{GeV}}{m_{H_1}}\right)^3\left(\frac{T_R}{10^9\,\mathrm{GeV}}\right), \tag{4.6}$$

with ρ_{h_1} being the energy density of field h_1. Finally, the following amount of gravitino is yielded by the decay of the SUSY-breaking field $x = \mathrm{Re}\,X$,

$$Y_{3/2}^{x} \simeq \frac{3}{2}\frac{T_R}{m_X}\left(\frac{m_{3/2}}{m_X}\right)^{16/3}\left(\frac{\Gamma_{\mathrm{all}}^{\sigma^1}}{\Gamma_{\mathrm{all}}^{x}}\right)^{2/3} \simeq 1.2\times 10^{-18}\left(\frac{m_{3/2}}{10^5\,\mathrm{GeV}}\right)^{20/3}\left(\frac{10^9\,\mathrm{GeV}}{m_X}\right)^{29/3}\left(\frac{T_R}{10^9\,\mathrm{GeV}}\right), \tag{4.7}$$

where $\Gamma_{\mathrm{all}}^{x}$ is the total decay width of SUSY-breaking field x,

$$\Gamma_{\mathrm{all}}^{x} \simeq \Gamma(x \to \Psi_{3/2}\Psi_{3/2}) \simeq \frac{1}{96\pi}\frac{m_X^5}{m_{3/2}^2 M_{\mathrm{Pl}}^2}. \tag{4.8}$$

We conclude that the gravitino yield from the decays of σ^1 and h_1 is suppressed as a consequence of almost vanishing F-terms of T^1 and H_1 . However, for x decay, the gravitino yield is sensitive to the masses of gravitino $m_{3/2}$ and SUSY-breaking field m_x, which is also pointed out in Ref. [88]. Fig. 4.1 shows that the particular values of m_X and $m_{3/2}$ lead to a sufficient abundance of gravitino. Throughout the above calculations, it is supposed that there is no matter dominated era for the fields expect the inflaton field and thus inflaton releases the entropy and reheats the universe at its decay after the inflation.

In addition, the gravitino is thermally produced in the radiation-dominated era. As discussed in Sec. 3.6, the thermal abundance of gravitino is calculated in Refs. [76, 47, 77, 89] in terms of dimensionless Hubble parameter h, and the numerical parameters $y_r/10^{-12} = (0.653, 1.604, 4.276)$ and $k_r = (1.266, 1.312, 1.271)$ defined in Ref. [89],

$$Y_{3/2}^{\mathrm{th}} = \sum_{r=1}^{3} y_r g_r(T_R)^2\left(1 + \frac{M_r(T_R)^2}{3m_{3/2}^2}\right)\ln\left(\frac{k_r}{g_r(T_R)}\right)\times\left(\frac{T_R}{10^{10}\mathrm{GeV}}\right), \tag{4.9}$$

where $M_r(T_R)$ and $g_r(T_R)$ are the gaugino masses and gauge couplings in the MSSM at the reheating temperature T_R. After solving the Boltzmann equation for wino LSP ($\tilde{\chi}_1^0$), its non-thermal abundance is estimated as [50, 86],

$$Y_{\tilde{\chi}_1^0}^{\mathrm{nth}} \simeq \min\left[Y_{3/2}^{\mathrm{th}} + Y_{3/2}^{\sigma^1} + Y_{3/2}^{h_1} + Y_{3/2}^{x}, \sqrt{\frac{45}{8\pi^2 g_*(T_{3/2})}}\frac{1}{M_{\mathrm{Pl}}T_{3/2}\langle\sigma_{\mathrm{ann}}v\rangle}\right], \tag{4.10}$$

where the entropy released from the gravitino decay is neglected and $g_*(T_{3/2}) \simeq 10.75$ is the effective degrees of freedom in the MSSM at decay temperature of gravitino,

$$T_{3/2} = \left(\frac{10}{\pi^2 g_*(T_{3/2})} M_{\rm Pl}^2 \Gamma_{3/2}^2 \right)^{1/4}. \tag{4.11}$$

The gravitino decay width ($\Gamma_{3/2}$) is determined by the gravitino decays into the gauginos in the case of our sparticle spectrum as can be seen in Tab. 4.1. Since the other sparticles are heavier than the gravitino, $\Gamma_{3/2}$ is simplified as

$$\Gamma_{3/2} = \frac{3}{8\pi} \frac{m_{3/2}^3}{M_{\rm Pl}^2}. \tag{4.12}$$

The thermally averaged annihilation cross section of the wino LSP ($\langle \sigma_{\rm ann} v \rangle$) in Eq. (4.10) is roughly estimated by the annihilation cross section between wino and W-boson,

$$\langle \sigma_{\rm ann} v \rangle = \frac{3(g_2(M_{\rm EW}))^4}{16\pi M_2^2}, \tag{4.13}$$

where it is given in the limit of $m_W \ll M_2$ and $g_2(M_{\rm EW})$ is the $SU(2)_L$ gauge coupling at the EW scale. For more details of estimating the annihilation cross section of wino LSP, refer to, e.g., Ref. [90]. The annihilation of the nonthermally produced wino LSP can be negligible due to its large cross section. In our setup, the wino yield produced nonthermally is then approximately given by $Y_{\tilde{\chi}_1^0}^{\rm nth} \simeq Y_{3/2}^{\rm th} + Y_{3/2}^{\sigma^1} + Y_{3/2}^{h_1} + Y_{3/2}^{x}$.

Second, we estimate the thermal production of wino LSP given in the thermal plasma and it is also roughly estimated by solving its Boltzmann equation,

$$Y_{\tilde{\chi}_1^0}^{\rm th} \simeq \left(\sqrt{\frac{8\pi^2 g_*(T_{\tilde{\chi}_1^0})}{45}} \langle \sigma_{\rm ann} v \rangle M_{\rm Pl} T_{\tilde{\chi}_1^0} \right)^{-1}, \tag{4.14}$$

with $g_*(T_{\tilde{\chi}_1^0}) \simeq 80$ being the effective degree of freedom in the MSSM at the freeze-out temperature of wino LSP $T_{\tilde{\chi}_1^0}$. When we consider the non-perturbative effects for the estimation of thermal abundance of wino LSP, the wino yield is affected in the mass region above $m_{\tilde{\chi}_1^0} > \mathcal{O}(1)$ TeV [91]. In such a case, the observed dark matter density is achieved for $m_{\tilde{\chi}_1^0} \simeq 2.7$ TeV in the case of pure wino produced thermally.

Therefore, the sum of its thermal and nonthermal abundances contribute to the relic density of wino LSP, i.e., $Y_{\tilde{\chi}_1^0} \simeq Y_{\tilde{\chi}_1^0}^{\rm th} + Y_{\tilde{\chi}_1^0}^{\rm nth}$. Since $Y_{\tilde{\chi}_1^0}$ becomes constant at the time of low enough temperature, the total relic density of wino LSP becomes

$$\Omega_{\tilde{\chi}_1^0} = \Omega_{\tilde{\chi}_1^0}^{\rm th} + \Omega_{\tilde{\chi}_1^0}^{\rm nth}, \tag{4.15}$$

where

$$\Omega_{\tilde{\chi}_1^0}^{\rm th(nth)} = m_{\tilde{\chi}_1^0} Y_{\tilde{\chi}_1^0}^{\rm th(nth)} \frac{s_{\rm now}}{\rho_{\rm cr}}. \tag{4.16}$$

The ratio of critical density of Universe $\rho_{\rm cr}$ and the present entropy density of Universe $s_{\rm now}$ are evaluated as $\rho_{\rm cr}/s_{\rm now} \simeq 3.6\, h^2 \times 10^{-9}$ GeV [16], where $h \simeq 0.673$ is the present Hubble constant in units of 100km/sec/Mpc [16]. We plot the total relic abundance of wino LSP as functions of $m_{3/2}$ and m_X in Fig. 4.1. The dotted, dashed and solid contours denote the $\Omega_{\tilde{\chi}_1^0} h^2 = 0.1, 0.11, 0.12$, respectively. The results are consistent within the range $0.1175 \leq \Omega_{\tilde{\chi}_1^0} h^2 \leq 0.1219$ reported by the Planck collaboration [15, 16]. Although, in our estimation, we do not take into account the non-perturbative effect known as Sommerfeld effect for the annihilation cross sections of wino [91], it is expected that the obtained result is reliable for $m_{\tilde{\chi}_1^0} \leq 1$ TeV. Even if the non-perturbative effect reduces an amount of thermally produced wino in the mass region above $\mathcal{O}(1)$ TeV, the nonthermal production of wino compensates its total relic abundance.

In particular, with the gravitino mass $m_{3/2} = 1.4 \times 10^5$ GeV and mass of SUSY-breaking field $m_X = 2.9 \times 10^8$ GeV, the total relic abundance of the wino is within the range reported by the Planck [15, 16]. At this point, the sparticle spectra and the Higgs boson mass are summarized in Table 4.1 by setting $\tan\beta = 4$ and the $U(1)_{I'}$ charges of matter fields given in Tab. 3.1. The above numerical masses of sparticles are given by employing the one-loop RG equations of MSSM from the SUSY-breaking scale $m_{3/2}$ to the compactification scale $M_C \simeq 2.1 \times 10^{16}$ GeV, whereas we evaluate the Higgs boson mass on the basis of the formula shown in Ref. [92]. The sparticle spectrum in Tab. 4.1 implies that there are no dangerous SUSY flavor and *CP*-problems peculiar to the supergravity models, that is a common feature in high-scale SUSY-breaking scenario. Note that it is now supposed that the parameters w, ν, Λ in the superpotential (3.15) are chosen so as to realize the above masses of gravitino $m_{3/2} = e^{\langle K/2 \rangle} \langle W \rangle$ and SUSY-breaking field through Eq. (3.22).

In this chapter, we have studied the particle phenomenology and cosmology on the same footing with an emphasis on the relic abundance of wino LSP. It is possible to check these wino dark matter scenario by the ongoing LHC 14 TeV data [93] and cosmological observations represented by the cosmic rays from the Fermi Gamma-Ray Space Telescope [94] and AMS-02 experiment [10, 95, 96]. When the large mass of wino dark matter is excluded by collider experiments and cosmological observations, the nonthermal production of wino LSP explicitly shown in this chapter is quite important to realize its correct relic abundance.

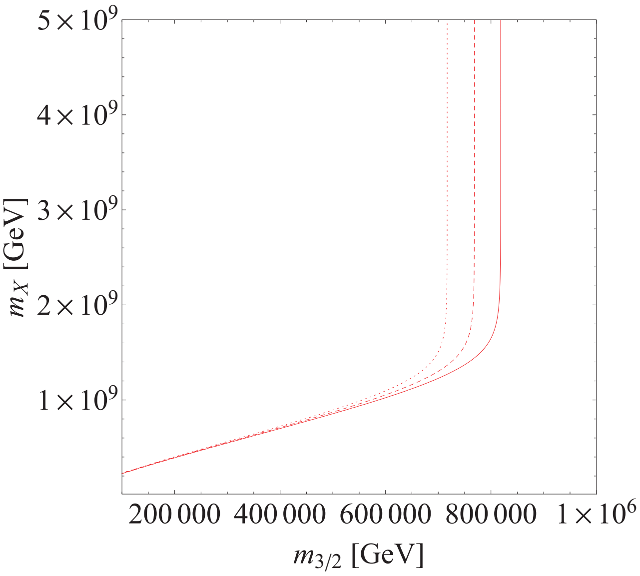

Figure 4.1: The relic abundance of the wino-like neutralino as functions of the gravitino mass $m_{3/2}$ and the mass of SUSY-breaking field m_X in Ref. [11]. The dotted, dashed and solid contours denote $\Omega_{\tilde{\chi}_1^0}h^2 = 0.1$, $\Omega_{\tilde{\chi}_1^0}h^2 = 0.11$ and $\Omega_{\tilde{\chi}_1^0}h^2 = 0.12$, respectively.

	mass[GeV]		mass[GeV]
$m_{\tilde{u}_1}$	1.3×10^6	$m_{\tilde{e}_1}$	1.2×10^6
$m_{\tilde{u}_2}$	1.2×10^6	$m_{\tilde{e}_2}$	9.3×10^5
$m_{\tilde{c}_1}$	9.1×10^5	$m_{\tilde{\mu}_1}$	1.0×10^6
$m_{\tilde{c}_2}$	8.9×10^5	$m_{\tilde{\mu}_2}$	9.0×10^5
$m_{\tilde{t}_1}$	7.0×10^5	$m_{\tilde{\tau}_1}$	9.0×10^5
$m_{\tilde{t}_2}$	4.1×10^5	$m_{\tilde{\tau}_2}$	8.6×10^5
$m_{\tilde{d}_1}$	1.3×10^6	$m_{\tilde{\nu}_{e_1}}$	1.2×10^6
$m_{\tilde{d}_2}$	1.3×10^6	$m_{\tilde{\nu}_{e_2}}$	1.4×10^5
$m_{\tilde{s}_1}$	1.3×10^6	$m_{\tilde{\nu}_{\mu_1}}$	1.0×10^6
$m_{\tilde{s}_2}$	8.9×10^5	$m_{\tilde{\nu}_{\mu_2}}$	1.4×10^5
$m_{\tilde{b}_1}$	6.7×10^5	$m_{\tilde{\nu}_{\tau_1}}$	8.6×10^5
$m_{\tilde{b}_2}$	4.1×10^5	$m_{\tilde{\nu}_{\tau_2}}$	1.4×10^5
$m_{\tilde{\chi}_4^0}$	7.3×10^5	$m_{\tilde{\chi}_1^\pm}$	377
$m_{\tilde{\chi}_3^0}$	7.3×10^5	$m_{\tilde{\chi}_2^\pm}$	7.3×10^5
$m_{\tilde{\chi}_2^0}$	1227	$m_{3/2}$	1.4×10^5
$m_{\tilde{\chi}_1^0}$	377	m_h	125.5
M_3	3896		

Table 4.1: The masses of sparctiles, the Higgs boson mass m_h, the gravitino mass $m_{3/2}$, and the gluino mass M_3 are evaluated at the EW scale in Ref. [11]. The subscripts of sparticles denote such as up ($\tilde{u}$), charm ($\tilde{c}$), top ($\tilde{t}$), down ($\tilde{d}$), strange ($\tilde{s}$), bottom ($\tilde{b}$) squarks, the scalar electron ($\tilde{e}$), muon ($\tilde{\mu}$), tauon ($\tilde{\tau}$), neutrino ($\tilde{\nu}$), the neutralino ($\tilde{\chi}$) and the chargino ($\tilde{\chi}^\pm$).

4.3 Summary

We have investigated the 5D SUGRA compactified on S^1/Z_2 in a high-scale SUSY-breaking scenario. In our setup, the systematic analysis for the particle phenomenology and cosmology can be performed on the same footing. In general, the nonthermal abundance of dark matter is highly model-dependent and one has to study them in a concrete model. We adopted the successful scenario for moduli inflation and stabilization explained in Chapter 2 to estimate the relic abundance of dark matter.

The supersymmetric moduli stabilization and inflation do not lead to the gaugino masses at the tree-level, unless the R-parity violating term is introduced in the gauge kinetic function as discussed in Chapter 3. Thus, in our model, after introducing the SUSY-breaking sector, the gaugino masses are dominated by the anomaly mediation and/or threshold corrections, whereas the gravity mediation via the SUSY-breaking field dominates the other soft SUSY-breaking terms such as sparticle masses. As commented in Chapter 2, the flavor structure of soft SUSY-breaking terms is governed by the extra $U(1)$ symmetries. Thus, the sparticle spectra in this high-scale SUSY-breaking model are almost the same as those predicted by pure gravity mediation [10] and the lightest supersymmetric particle corresponds to the wino-like neutralino, which would be dark matter candidate. Its relic abundance arises from the sum of thermal and nonthermal processes after the inflation. The inflaton and moduli are supersymmetrically stabilized at the minimum of potential so that the gravitino production from their decays is suppressed by their negligible F-terms. As a result, the small amount of nonthermally produced wino-like neutralino via the gravitino decay is irrelevant to the Planck data. On the other hand, the SUSY-breaking field and thermal bath generated after the inflation produce the significant amount of gravitino that decays into the wino LSP. The amount of wino LSP depends on the masses of gravitino and SUSY-breaking field as shown in Fig. 4.1. The wino-like neutralino is also produced by the thermal process in the thermal bath.

In summary, it is found that the relic abundance of wino dark matter is approximately given by its nonthermal abundances when the mass of wino-like neutralino is smaller than $\mathcal{O}(1)$ TeV. Thus, the nonthermal process is quite important to realize the correct relic abundance reported by Planck [15, 16] that would be checked by ongoing collider experiments and cosmological observations. If the large mass of wino dark matter is excluded by them, the scenario of moduli inflation and stabilization shown in Chapter 2 is one of possibilities to realize the wino dark matter scenario.

Part II

Axion inflation in string theory

Chapter 5

String axions

Axion is the light scalar field introduced by the Pececi-Quinn as the solution to the strong CP problem [97]. In QCD (quantum chromodynamics), one can generically write down the CP violating term,

$$\mathcal{L} = \frac{\theta}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \tag{5.1}$$

where G and $\tilde{G}$ are the field strength of gluons and its dual one, respectively. The no-observation of the electric dipole moment of the neutron [98] and and 199Hg [99] show that the size of θ should be smaller than 10^{-10}. It is no reason why the theta is so small within the framework of QCD (strong CP problem). As the solution to the strong CP problem, Peccei-Quinn introduced the global symmetry called as PQ symmetry. After the spontaneous PQ symmetry breaking, the Pseudo-Nambu-Goldstone boson appeared through the following coupling

$$\mathcal{L} = \frac{a}{f_a} G\tilde{G}, \tag{5.2}$$

a is the axion and f_a denote the scale of PQ symmetry breaking. When the axion obtain the vacuum expectation value, the theta term is canceled out.

In contrast to QCD axion, axions are defined in string theory in a different way. The string axion is the extra-dimensional components of several higher-form fields through the dimensional reduction of the obtained effective supergravity action. In this chapter, we briefly review the cosmology of string axion on the basis of the 10D low-energy effective supergravity action of several string theory. We also show the possible string inflation with axion in the light of its axion decay constants. .

5.1 Elements of weakly coupled heterotic string theory

First of all, we begin with the heterotic string theory [13] in which the closed string is only propagating. The anomaly-and tachyon-free heterotic string theory allows the existence of gauge fields whose gauge groups are only $SO(16) \times SO(16)$ or $SO(32)$ or $E_8 \times E_8$. For our purpose, we restrict ourselves to the $SO(32)$ and $E_8 \times E_8$ heterotic string theory in which the SUSY exists.*

The matter contents in the bosonic sector are dilaton (ϕ_{10}), metric tensor (g_{MN}) with $M, N = 1, 2, ..., 10$, antisymmetric tensor (B_{MN}), adjoint gauge field (A_M^I) with $I = 1, 2, ..., 16$ being the Cartan indices of $SO(32)$ or $E_8 \times E_8$. Also, their superpartners appear as massless modes. The low-energy effective action of the heterotic string theory in string frame is described by $10D$ supergravity action whose bosonic part is given in the notation of [101],

$$S_{\rm bulk}^{(\rm hetero)} \supset \frac{1}{2\kappa_{10}^2}\int_{M^{(10)}} e^{-2\phi_{10}}\left[R + 4d\phi_{10}\wedge *d\phi_{10} - \frac{1}{2}H_3^{(\rm h)}\wedge *H_3^{(\rm h)}\right] - \frac{1}{2g_{10}^2}\int_{M^{(10)}} e^{-2\phi_{10}}{\rm tr}(F\wedge *F), \tag{5.3}$$

which can be determined by the relevant scattering amplitudes on the worldsheet up to of order $\mathcal{O}(\alpha')$ with α' being the regge slope. The gravitational and Yang-Mills couplings are defined as $2\kappa_{10}^2 = (2\pi)^7(\alpha^{'})^4$, $g_{10}^2 = 2(2\pi)^7(\alpha^{'})^3$ and the vacuum expectation value of the ten-dimensional dilaton ϕ_{10} leads to the string coupling, $g_s = e^{\langle\phi_{10}\rangle}$. F represents for the field-strength of $SO(32)$ or $E_8 \times E_8$ gauge groups and it has the index of vector-representation, normalized as ${\rm tr}_v(T^aT^b) = 2\delta^{ab}$. In what follows, "tr" and "Tr" represent for the trace in the vector and adjoint representation of the $SO(32)$ gauge group, respectively. In addition, H denotes the heterotic three-form field strength defined by

$$H_3^{(\rm h)} = dB_2 - \frac{\alpha^{'}}{4}(w_{\rm YM} - w_L), \tag{5.4}$$

where $w_{\rm YM}$ and w_L are the gauge and gravitational Chern-Simons three-forms, respectively.

One can extract the kinetic term of the B-field from the three-form field strength given in Eq. (5.3),

$$\begin{aligned} S_{\rm kin} + S_{\rm WZ} &= -\frac{1}{4\kappa_{10}^2}\int_{M^{(10)}} dB_2\wedge *dB_2 - \sum_a N_aT_5\int_{\Gamma_a} B_6 \\ &= -\frac{1}{4\kappa_{10}^2}\int_{M^{(10)}} dB_2\wedge *dB_2 - \sum_a N_aT_5\int_{M^{(10)}} B_6\wedge\delta(\Gamma_a), \end{aligned} \tag{5.5}$$

where the latter part denote the magnetic sources for the Kalb-Ramond field B_6, so-called Wess-Zumino term. Such sources correspond to the non-perturbative objects, i.e., the stacks of N_a five-branes with their tensions being $T_5 = ((2\pi)^5(\alpha^{'})^3)^{-1}$, where $N_a = \pm 1$ correspond to the single heterotic and anti-heterotic five-branes, respectively. To preserve the SUSY, five-branes wrap the holomorophic two-cycles Γ_a and their Poincáre dual four-forms are represented by $\delta(\Gamma_a)$. The Kalb-Ramond two-form B_2 and six-form B_6 are connected with the ten-dimensional Hodge duality,

$$*dB_2 = e^{2\phi_{10}}dB_6, \tag{5.6}$$

*Although it is difficult to discuss the consistency condition and five-branes in $SO(16)\times SO(16)$ heterotic string theory for lack of SUSY, there are several studies in this direction, see for Refs. [100].

and then the action of Kalb-Ramond two-form in Eq. (5.5) are rewritten in terms of six-form B_6 as

$$S_{\rm kin} + S_{\rm WZ} = -\frac{1}{4\kappa_{10}^2}\int_{M^{(10)}} e^{2\phi_{10}} dB_6 \wedge *dB_6 + \frac{\alpha\,'}{8\kappa_{10}^2}\int_{M^{(10)}} B_6 \wedge \left(\mathrm{tr}F^2 - \mathrm{tr}R^2 - 4(2\pi)^2 \sum_a N_a \delta(\Gamma_a)\right). \tag{5.7}$$

The following tadpole condition of the NS-NS fluxes results from the equation of motion of B_6,

$$d(e^{2\phi_{10}} * dB_6) = -\frac{\alpha^{'}}{4}\left(\mathrm{tr}\bar{F}^2 - \mathrm{tr}\bar{R}^2 - 4(2\pi)^2 \sum_a N_a \delta(\Gamma_a)\right) = [0], \tag{5.8}$$

in cohomology and where $\bar{R}$ and $\bar{F}$ represent the field strengths of the extra-dimensional components of curvature on internal manifold and gauge fields whose gauge groups are embedded in $SO(32)$ or $E_8 \times E_8$. When the five-branes do not exist, the tadpole condition must be canceled by the contributions between the geometrical part and magnetic fluxes, otherwise the non-Abelian gauge and gravitational anomalies arise in the system. If the heterotic five-branes exist, such anomalies are canceled by themselves at the non-perturbative level. On the worldsheet, these heterotic five-branes also recover the modular invariance [102, 103] which can be shown in the case of heterotic orbifold [104]. However, in such a case, we must take into account the gauge anomaly and the global Witten anomaly on heterotic five-branes where the partition function is vanished in the even number of chiral fermions on the heterotic five-branes [105, 106].

At the field theoretical approach, the cancellation of Abelian gauge and gravitational anomalies are captured by taking into account the following one-loop Green-Schwarz term at the string frame [107, 108],

$$S_{\rm GS} = \frac{1}{24(2\pi)^5\alpha'}\int B_2 \wedge X_8, \tag{5.9}$$

which can be extracted from the S-dual type I theory up to normalization factors as shown in Ref. [109] and the eight-form X_8 is yielded as

$$X_8 = \frac{1}{24}\mathrm{Tr}F^4 - \frac{1}{7200}(\mathrm{Tr}F^2)^2 - \frac{1}{240}(\mathrm{Tr}F^2)(\mathrm{tr}R^2) + \frac{1}{8}\mathrm{tr}R^4 + \frac{1}{32}(\mathrm{tr}R^2)^2. \tag{5.10}$$

For more details of the cancellation of Abelian gauge anomalies through Green-Schwarz mechanism, see Refs. [110]. Hence, the abelian and non-abelian gauge anomalies and gravitational anomalies are canceled by the above Green-Schwarz mechanism (5.9) and the tadpole condition (5.8) as discussed in Refs. [111, 110]. These conditions severely constrain the phenomenological models in the framework of heterotic string theory. It is possible to achieve these anomaly cancellation by constructing just the three-generation standard-like models. However, even if there are no Abelian gauge anomalies, the Abelian gauge bosons may become massive by absorbing the axions appearing through the Green-Schwarz coupling given by Eq. (5.9) [110]. One should take care of the couplings among hypercharge gauge boson and axions in order to avoid such situations.

5.2 Axions in heterotic string theory

Let us dimensionally reduce the above effective action of the heterotic string theory on a general complex 6D manifold $\mathcal{M}$. Along the line of Refs. [110], we expand the B-field in the basis of Kähler form w_k with $k = 1, 2, \cdots, h^{1,1}$ and its hodge dual four-form $\hat{w}_k$,

$$\begin{aligned} B_2 &= b_0^{(2)} + l_s^2 \sum_{k=1}^{h^{1,1}} b_k^{(0)} w_k, \\ B_6 &= l_s^6 b_0^{(0)} \mathrm{vol}_6 + l_s^4 \sum_{k=1}^{h^{1,1}} b_k^{(2)} \hat{w}_k, \end{aligned} \tag{5.11}$$

where $l_s = 2\pi\sqrt{\alpha'}$ is the string length and $\mathrm{vol}_6 = w_1 \wedge w_2 \wedge w_3$ is the volume form of internal manifold normalized by $\int_{\mathcal{M}} \mathrm{vol}_6 = 1$. The two-form B_2 and six-form B_6 are related by the Hodge duality, $*_{10} dB_2 = e^{2\phi_{10}} dB_6$. Note that the hodge dual four-form satisfies the following relation

$$\int_{\mathcal{M}} w_k \wedge \hat{w}_{k'} = \delta_{kk'}. \tag{5.12}$$

We call the $b_0^{(0)}$ and $b_k^{(0)}$ as the universal axion pairing with the dilaton and the Kähler axion pairing with the Kähler moduli, respectively. These axions have continuous shift symmetry originating from the gauge symmetry of Kalb-Ramond $B_{2,6}$ field, where B_2 (B_6) couple to the fundamental string (a five-dimensional object called as heterotic five-branes). The explicit computation showed that such a shift symmetry is preserved to all orders in the σ-model perturbation theory [112].

When the heterotic five-branes appear in the low-energy effective theory, one cannot ensure the approximation of weakness of string coupling which arises from the fact that the tension of heterotic five-branes is of order g_s^{-2}. In order to study these objects, it is required to start from the strong coupling limit of heterotic string theory. The strong coupling limit of $SO(32)$ heterotic string is described by its S-dual theory, that is, type I string theory, in which the heterotic five-branes become D5-branes. On the other hand, Horava and Witten pointed out the existence of the strong coupling limit of $E_8 \times E_8$ heterotic string theory in Ref. [113] and the low-energy phenomena is captured by eleven-dimensional supergravity on S^1/Z_2 background.

In the strongly coupled $E_8 \times E_8$ heterotic string theory, the heterotic five brane is considered as the M5-brane compactified on eleventh direction S^1/Z_2, where the self-dual tensor field $\tilde{B}$ exist. In the same way as in the case of weakly coupled heterotic string theory, we can define the axion and dual two-form under the following convention

$$\tilde{B} = \tilde{b}_0^{(2)} + l_s^2 \sum_{k=1}^{h^{1,1}} \tilde{b}_k^{(0)} w_k, \tag{5.13}$$

where w_k denote the two-cycle wrapped by the M5-brane. Thus, existence of M5-brane implies that the extra axions ($\tilde{b}_k^{(0)}$) and two-form field ($\tilde{b}_0^{(2)}$) appear in the low-energy effective theory.

In addition, in the strongly coupled $SO(32)$ heterotic string theory, the heterotic five brane is considered as the S-dual of D5-brane where the symplectic gauge fields live on. The axion is also defined as the four-dimensional component of Ramond-Ramond field living on D5-brane.

5.3 Elements of type II string theory

Next, we discuss the type IIA and IIB string theory, in which both the closed and open strings are propagating. Before going to define the axions in type II string theory, let us first summarize their matter contents and low-energy effective actions on the basis of Ref. [114].

In type IIA superstring theory, massless fields in the bosonic sector are dilaton (ϕ_{10}), metric tensor (g_{MN}), antisymmetric tensor (B_{MN}) for Neveu-Schwarz (NS) sector and Ramond-Ramond (RR) fields such as a one-form C_1 and a three-form C_3. Other RR-fields are related to above them by hodge duality. Also, their superpartners appear as massless modes. Its low-energy effective action in string frame is described by 10D supergravity action whose bosonic part is given by

$$S^{(\mathrm{IIA})}_{\mathrm{bulk}} = S_{\mathrm{NS}} + S^{(\mathrm{IIA})}_{\mathrm{R}} + S^{(\mathrm{IIA})}_{\mathrm{CS}}, \tag{5.14}$$

where

$$\begin{aligned} S_{\mathrm{NS}} &= \frac{1}{2\kappa_{10}^2}\int_{M^{(10)}} e^{-2\phi_{10}}\left[R + 4d\phi_{10}\wedge *d\phi_{10} - \frac{1}{2}H_3\wedge *H_3\right], \\ S^{(\mathrm{IIA})}_{\mathrm{R}} &= -\frac{1}{4\kappa^2}\int_{M^{(10)}}\left[F_2\wedge *F_2 + \tilde{F}_4 * \wedge\tilde{F}_4\right], \\ S^{(\mathrm{IIA})}_{\mathrm{CS}} &= -\frac{1}{4\kappa^2}\int_{M^{(10)}} B_2\wedge F_4\wedge F_4, \end{aligned} \tag{5.15}$$

which can be calculated by the relevant scattering amplitudes on the worldsheet up to of order $\mathcal{O}(\alpha')$. The gravitational and Yang-Mills couplings are defined as $2\kappa_{10}^2 = (2\pi)^7(\alpha^{'})^4$, $g_{10}^2 = 2(2\pi)^7(\alpha^{'})^3$ and the vacuum expectation value of the ten-dimensional dilaton ϕ_{10} leads to the string coupling, $g_s = e^{\langle\phi_{10}\rangle}$ in the notation of [101]. In addition, H_3 and $F_{2,4}$ denote the NS and R-R field strengths defined by

$$\begin{aligned} H_3 &= dB_2, \\ F_p &= dC_{p-1}, \\ \tilde{F}_4 &= F_4 + C_1\wedge H_3. \end{aligned} \tag{5.16}$$

By contrast, in type IIB string theory, massless fields in the bosonic sector are dilaton (ϕ_{10}), metric tensor (g_{MN}), antisymmetric tensor (B_{MN}) for Neveu-Schwarz sector and several Ramond-Ramond fields such as a scalar C_0, a two-form C_2 and a four-form C_4. Other R-R fields are related to above them by hodge duality. Also, their superpartners appear as massless modes. Its low-energy effective action in string frame is described by $10D$ supergravity action whose bosonic part is given by

$$S^{(\mathrm{IIB})}_{\mathrm{bulk}} = S_{\mathrm{NS}} + S^{(\mathrm{IIB})}_{\mathrm{R}} + S^{(\mathrm{IIB})}_{\mathrm{CS}}, \tag{5.17}$$

where

$$S_{\rm R}^{\rm (IIB)} = -\frac{1}{4\kappa^2}\int \left[F_1\wedge *F_1 + \tilde{F}_3\wedge *\tilde{F}_3 + \frac{1}{2}\tilde{F}_5\wedge *\tilde{F}_5\right],$$
$$S_{\rm CS}^{\rm (IIB)} = -\frac{1}{4\kappa^2}\int C_4\wedge H_3\wedge F_3, \tag{5.18}$$

with $F_p = dC_{p-1}$, $\tilde{F}_3 = F_3 - C_0\wedge H_3$ and self-dual field strength $\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$ satisfying the constraint $\tilde{F}_5 = *\tilde{F}_5$.

String theory is not only the theory of string, but also describes the solitonic objects such as D-branes where the open strings have the Dirichlet (Neumann) boundary conditions in the directions transverse (longitudinal) to the brane. From the quantization of the open string living on a D-brane, one finds the massless gauge field and its superpartner. The low-energy action of Dp-brane with p being spatial dimension is calculated by computing scattering amplitude between open strings and closed strings. The explicit form of it is obeyed by the *Dirac-Born-Infeld* (DBI) action in a supergravity background and Chern-Simons term,

$$S_{\rm DBI} = -\int_M d^{p+1}\xi e^{(p-3)\phi/4}\sqrt{-\det(g^E_{ij} + e^{-\phi/2}(F_{ij} - B_{ij}))},$$
$$S_{\rm WZ} = \int_M e^{F-B}\wedge C, \tag{5.19}$$

where ξ^i denote the coordinate of world volume $\mathcal{M}$ mapped by the that of 10D spacetime and g^E_{ij} is the pullback of the metric at the Einstein frame. The metric at the string frame g^S_{ij} is transformed under $SL(2,\mathbf{R})$ symmetry of worldsheet which is related to the $SL(2,\mathbf{R})$-invariant metric at the Einstein frame $g^E_{ij} = e^{-\phi/2}g^S_{ij}$.

5.4 Axions in type II string theory

Let us perform Kaluza-Klein reduction of the type II string effective action on CY threefold. For completeness, we restrict ourselves to the type IIB string theory.† In the same way as the case of heterotic string theory, we expand the Kähler form J, B-field and R-R form in the base

†An extension to the case for type IIA string theory is straightforward.

of Kähler form w_k with $k = 1, 2, \cdots, h^{1,1}$ and its hodge dual four-form $\hat{w}_k$ of CY threefold,

$$J = l_s^2 \sum_{I=1}^{h^{1,1}} t_I w_I,$$

$$B_2 = b_0^{(2)} + l_s^2 \sum_{I=1}^{h^{1,1}} b_I^{(0)} w_I,$$

$$C_2 = c_0^{(2)} + l_s^2 \sum_{I=1}^{h^{1,1}} c_I^{(0)} w_I,$$

$$C_4 = l_s^4 \sum_{I=1}^{h^{1,1}} \theta_I^{(0)} \hat{w}_I, \tag{5.20}$$

where t^I, $b_I^{(0)}$, $c_I^{(0)}$ and $\theta_I^{(0)}$ are $h^{1,1}$ four-dimensional scalar fields. $b_I^{(2)}$ and $c_I^{(2)}$ four-dimensional two-form fields. Up to now, there is a unbroken $\mathcal{N} = 2$ SUSY in four-dimensional effective theory derived from type IIB string theory on CY manifold. In order to obtain chiral spectrum, it is required to break $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$ SUSY. That situation is realized in the presence of non-dynamical objects, i.e., orientifold planes. It plays an important role for tadpole cancellation induced by certain D-branes including the standard model sector.

We briefly review the CY orientifold with O3/O7-planes on the basis of Ref. [115]. In the presence of O3/O7-planes, the basis of Kähler form is decomposed as the orientifold-even and-odd bases w^i, $i = 1, \cdots, h_+^{1,1}$ and w^α, $\alpha = 1, \cdots, h_-^{1,1}$, respectively. Since only orientifold invariant fields appear in the four-dimensional effective theory, the orientifold-even fields (R-R 0-form C_0, dilaton Φ, $\theta_I^{(0)}$, t^I) appear in the four-dimensional effective theory when they are expanded in the basis of w^i. On the other hand, the orientifold-odd fields ($b_I^{(0)}$, $c_I^{(0)}$, $b_0^{(2)}$ and $c_0^{(2)}$) appear in the four-dimensional effective theory when they are expanded in the basis of w^α. Thus, the orientifold invariant fields are the axiondilaton τ, Kähler moduli T_i, two-form scalars G_α summarized as

$$\begin{aligned}
&\tau = C_0 + ie^{-\Phi}, \\
&T_i = \frac{1}{2}\kappa_{ijk} t^j t^k + i\theta_i^{(0)} + \frac{1}{4} e^{\Phi} \kappa_{i\alpha\beta} G^\alpha (G - \bar{G})^\beta, \\
&G_\alpha = c_\alpha - \tau b_\alpha.
\end{aligned} \tag{5.21}$$

The orientifold-odd complex structure moduli ζ^α also appear through the complex structure deformations of CY metric,

$$\delta g_{ij} = \frac{6i}{\Omega_{klm}\bar{\Omega}^{klm}} \zeta^\alpha (\xi_\alpha)_{i\bar{i}\bar{j}} \Omega_j^{\bar{i}\bar{j}}, \tag{5.22}$$

where Ω is the holomorphic three-form of CY manifold. Note that $b_0^{(0)}$ and $b_k^{(0)}$ as the universal axion pairing with the dilaton and the Kähler axion pairing with the Kähler moduli, respectively. These axions have continuous shift symmetry originating from the gauge symmetry of Kalb-Ramond B-field.

5.5 Inflation with axions

The cosmic inflation is a most successful scenario which not only generates an origin of the current temperature fluctuation, but also provides testable predictions for the cosmological observables. Especially, the primordial gravitational wave is one of the main target for the current and future cosmological observations. According to the size of the tensor-to-scalar ratio, the slow-roll inflation scenarios are mostly classified into two types of them. The small-field inflation predicts the small tensor-to-scalar ratio, whereas the other scenario is the large-field inflation which predicts a measurable tensor-to-scalar ratio. These large field inflation would be testable for the near-future cosmological observations. Thus, it has substantial implications for the fundamental theory such as string theory. For the detectable tensor-to-scalar-ratio, $r = \mathcal{O}(0.01 - 0.1)$, a so-called Lyth bound [116] suggested that the inflaton is realized in the region of the trans-Planckian value of inflaton field. However, in this field regime, one cannot neglect the Planck suppressed operators and correspondingly description of effective field theory is lost. In the case of large-field inflation models, we always encounter such problems how to treat the trans-Planckian field values. In this section, we show the treatment of the trans-Planckian values on the basis of the string theory.

5.5.1 Natural inflation

When we consider string axions as the candidate of inflaton, axion potential is mainly classified into two types of models. One is the natural inflation scenario where the inflaton is identified as the pseudo-Nambu Goldstone boson [29] with the following Lagrangian,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right], \tag{5.23}$$

where ϕ is the axion with f being its decay constant. In the string set up, the axion shift symmetry is spontaneously broken down to the discrete one by nonperturbative effect such as D-brane instanton effects and gaugino-condensation living on the hidden D-brane expect for the D-brane including the standard model. In any cases, their dynamical scales are characterized by Λ. However, the natural inflation compatible with the observed Planck data [15, 16] requires the trans-Planckian axion decay constant. (See, for more details, e.g., Ref. [30].) It is in general hard to realize such the axion decay constant beyond the Planck scale in the 4D effective theory as well as the higher-dimensional theory, since the scale of axion decay constant is connected to the volume of internal manifold and the cut-off scale of higher-dimensional theory. In particular, the string theory shows that the typical fundamental axion decay constants are below the scale 10^{17}GeV as pointed out in [117].

To overcome such a problem, there are several approaches to realize the natural inflation with trans-Planckian axion decay constant. Along the line of Ref [118], let us consider the scenario where two axion fields ϕ_1 and ϕ_2 couple to two non-Abelian gauge groups G_1 and G_2. When these gauge theories are confined at the dynamical scales Λ_1 and Λ_2, the axion potential becomes

$$\mathcal{L} = \Lambda_1^4 \left[1 - \cos\left(\frac{\phi_1}{f_1} + \frac{\phi_2}{f_2}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\phi_1}{g_1} + \frac{\phi_2}{g_2}\right)\right], \tag{5.24}$$

where $f_{1,2}$ and $g_{1,2}$ are axion decay constants. In the case of $\Lambda_2^4 \gg \Lambda_1^4$, there appears a heavy and a light linear combination of axions,

$$\begin{aligned} \tilde{\phi}_{\rm heavy} &= \frac{1}{\sqrt{g_1^2 + g_2^2}} \left(\frac{\phi_1}{g_1} + \frac{\phi_2}{g_2}\right), \\ \tilde{\phi}_{\rm light} &= \frac{1}{\sqrt{g_1^2 + g_2^2}} \left(\frac{\phi_1}{g_1} - \frac{\phi_2}{g_2}\right). \end{aligned} \tag{5.25}$$

After integrating out $\tilde{\phi}_{\rm heavy}$, the effective potential for $\tilde{\phi}_{\rm light}$ becomes

$$\mathcal{L} = \Lambda_1^4 \left[1 - \cos\left(\frac{\tilde{\phi}_{\rm light}}{f}\right)\right], \tag{5.26}$$

with

$$f = \sqrt{g_1^2 + g_2^2} \left(\frac{f_1 f_2}{g_1 g_2}\right) \left(\frac{f_2}{g_2} - \frac{f_1}{g_1}\right)^{-1}. \tag{5.27}$$

As pointed out in Ref [118], under the condition

$$\frac{f_2}{g_2} \simeq \frac{f_1}{g_1}, \tag{5.28}$$

the effective axion decay constant for $\tilde{\phi}_{\rm light}$ can be chosen as trans-Planckian size. This alignment mechanism is realized even for the sub-Planckian axion decay constants $f_{1,2}, g_{1,2} \ll M_{\rm Pl}$. It is generalized to many axion cases in Ref. [119]. Also, in the case of many axions, one can realize the super-Planckian decay constant called as N-flation [120]. (See, a similar proposal for assisted inflation [121] and M-flation [122].) On the other hand, in the single axion case, its decay constant can be enhanced by a small five-dimensional gauge coupling as pointed out in the five-dimensional theory [123, 6]. In the next Chapter, we show an another approach to enhance the axion decay constant in the single axion case on the basis of Ref. [124].

5.5.2 Axion monodromy inflation

Let us focus on the another popular axion inflation, i.e., the axion monodromy inflation [125, 126] in string theory. In contrast to the natural inflation, the axion shift symmetry is explicitly broken down by the existence of D-brane. As a result, the inflaton potential has a structure of monodromy and the field range of inflaton during the inflation becomes larger than its fundamental period determined by the axion decay constant. In this scenario, we do not need the trans-Planckian axion decay constant.

The axion monodromy inflation is characterized by the following Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \mu_1^{4-p}\phi^p, \tag{5.29}$$

where ϕ is the axion associated with the higher-dimensional form fields and μ_1 denotes the energy scale as shown later. p is the model-dependent fractional number. (For more details, see Ref. [114] and references therein.)

We proceed to explain the details of monodromy inflation by taking into account the spacetime filling D5-brane in type IIB string theory [126]. Now, we assume that the D5-brane wraps the 4D spacetime and certain internal two-cycle Σ_2 in the 6D internal space. The action of D5-brane is given by the unmagnetized DBI action,

$$S_{D5} = \frac{1}{(2\pi)^5 g_s(\alpha')^3}\int d^6\sigma\sqrt{-\det(G_{ab}+B_{ab})}, \tag{5.30}$$

where G_{ab}, $a, b = 0, 1, 2, 3, 4, 5$ denotes the pullback of the metric of the target space and an extra-dimensional component of the Kalb-Ramond field B_{ab} corresponds to the axion $b = \int_{\Sigma_2} B_2$ where B_2 is the Kalb-Ramond two-form.

After dimensional reduction of DBI action along the cycle Σ_2 with its volume l in $\alpha' = 1$, the axion potential is extracted as

$$V_{\rm eff} \simeq \frac{\mathcal{T}}{(2\pi)^5 g_s(\alpha')^2}\sqrt{l^4+b^2}, \tag{5.31}$$

where $\mathcal{T}$ is some unknown warp factor. In the large field regime of axion $b \gg l^2$, one can obtain a linear-type potential term,

$$V_{\rm eff} \simeq \frac{\mathcal{T}}{(2\pi)^5 g_s(\alpha')^2} b. \tag{5.32}$$

The relevant Lagrangian of the canonically normalized axion $\phi = b$ is given by

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \mu_1^3\phi, \tag{5.33}$$

where $\mu_1^3 = \frac{\mathcal{T}}{f(2\pi)^5 g_s(\alpha')^2}$ with f being the axion decay constant for ϕ. We stress that the axion exhibits not a shift symmetry but monodromy in its scalar potential. Furthermore, the above discussion is also applied for its S-dual NS5-brane. The axion living on an NS5-brane wrapping the cycle Σ_2 is defined as a four-dimensional component of RR field C_2. The other types of monodromy inflation in Eq. (5.29) with $p = 2/3$ [125] have been proposed for the D4-brane in type IIA string theory on a twisted torus. When we consider a coupling between NS-NS two-form and the R-R field strength, the axion monodromy inflation becomes the form of Eq. (5.29) with $p = 4/3, 3$ [127]. Also, the other types of axion monodromy inflation with $p = 2$ is discussed in terms of the seven-branes [128] or a four-form field strength [129].

Chapter 6

Natural inflation in weakly coupled heterotic string theory

In this chapter, we take into account the weakly coupled heterotic string theory with an emphasis on axion dynamics based on Ref. [124]. When the anomalous $U(1)$ symmetries exist in the system, the moduli fields contribute to the anomaly cancellation through the Stückelberg couplings between the axions and gauge bosons. Such axions appearing in the Stückelberg couplings are then absorbed by the anomalous $U(1)$ gauge bosons and become massive. Those mass scales are typically the string scale. Thus, certain axion fields are decoupled from the system. In particular, when the anomalous $U(1)$s are included in the Cartan direction of $E_8 \times E_8$ heterotic string theory, it is possible to absorb the linear combinations of the universal axion $b_0^{(0)}$ and Kähler axions $b_i^{(0)}$. In this chapter, we identify one of the unabsorbed Kähler axion as the inflaton. Also, the real parts of moduli fields such as dilaton and Kähler moduli should be stabilized and heavier enough than the inflaton field, otherwise they would be destabilized and generate non-adiabatic curvature perturbations constrained by Planck data. In order to overcome this problem, we consider that the certain non-perturbative corrections lead to stabilize the dilaton and real parts of Kähler moduli. Note that, the complex structure moduli can be stabilized at the minimum by the flux-induced superpotential at the tree-level. When the internal manifold is chosen as "Swiss-Cheese" CY manifold, its geometrical structure leads to stabilize a linear combination of Kähler moduli without relying on a lot of non-perturbative effects to the superpotential. These CY manifold is extensively studied in the string phenomenology and cosmology on the basis of the Type IIB string theory [33], F-theory [130] and heterotic string theory [131]. The remaining Chapter is organized as follows. First of all, we review the heterotic string theory on general CY manifold with multiple anomalous $U(1)$ symmetry in Sec. 6.1. In Secs. 6.2.1 and 6.2.2, we propose two successful inflation models. We summarize this chapter in Sec. 6.3.

6.1 Effective action of heterotic string with $U(1)$ magnetic fluxes

We restrict ourselves to the $E_8^{\rm vis}\times E_8^{\rm hid}$ heterotic string.* In contrast to the standard embedding where the $SU(3)$ gauge fields in $E_8^{\rm vis}$ are identified as the spin connection of CY manifold, we consider other class of model by employing the $U(1)$ magnetic fluxes (line bundles) called as non-standard embedding. In such a model, the visible $E_8^{\rm vis}$ gauge group decomposes into the product group of $G_{\rm vis}$ (certain GUT or just the SM gauge groups) and multiple $U(1)$s whose rank depends on that of $G_{\rm vis}$. Such a decomposition of the gauge group is induced by multiple $U(1)$ magnetic fluxes (multiple line bundles), in other words, the constant extra-dimensional components of multiple $U(1)$ field strengths. Although we do not know the consistent quantization condition of string in the presence of magnetic fluxes in contrast to type II string theory, we assume that they are treated as those in the field theoretical approach. These $U(1)$ fluxes play phenomenologically important role of realizing the 4D SM model gauge groups and chiral matters. Their possibilities were pointed out in Refs. [111, 109] and the type IIB string theory [132]. Throughout this chapter, we further assume that the charged scalar fields under the multiple $U(1)$s and bundle moduli (Wilson line moduli in type II string theory) do not affect our following discussion. It would be ensured by the remaining gauge symmetries and certain dynamics.

Let us carry out the KK reduction of 10D $E_8^{\rm vis}\times E_8^{\rm hid}$ heterotic supergravity on the general CY manifold. Effective tree-level Kähler potential becomes

$$\mathcal{K} = -\,M_{\rm Pl}^2\ln\left(S+\bar{S}\right) - M_{\rm Pl}^2\ln(\mathcal{V}), \tag{6.1}$$

where $M_{\rm Pl}^2=\frac{e^{-2\phi_{10}}\mathcal{V}}{\kappa_{10}^2}$. First part is the the Kähler potential of dilaton and the latter is that of Kähler moduli t_i whose size determines the internal two-cycle of CY manifold with its volume being $\mathcal{V}=\frac{1}{6}\int_{\rm CY} J\wedge J\wedge J$ written in terms of Kähler form $J=l_s^2\sum_i t_i w_i$. The dilaton and Kähler moduli are defined as

$$\begin{aligned} S &= \frac{1}{4\pi}\left[\frac{e^{-2\phi_{10}}\mathcal{V}}{l_s^6}+ib_0^{(0)}\right],\\ T_i &= t_i + ib_i^{(0)}, \end{aligned} \tag{6.2}$$

where their imaginary parts correspond to the dilaton axion $b_0^{(0)}$ and Kähler axion $b_i^{(0)}$ for $i=1,2,\cdots h^{1,1}$, respectively.

Let us take a closer look at these axionic couplings descended from 10D kinetic terms of H in Eq. (5.3) and one-loop GS counter term [108] in Eq. (5.9). When $U(1)$ magnetic fluxes (extra-dimensional components of constant field strength) are inserted into the Cartan direction of $E_8^{\rm vis}$, the axions have the following couplings of the $U(1)^m$ gauge bosons A_m, $m=1,\cdots,8-{\rm rank}(G_{\rm vis})$ in the action,

$$S_{\rm axion} = \sum_m \frac{\mathcal{Q}_S^m}{4l_s^2}\int_{R^{1,3}} b_0^{(2)}\wedge F_m + \sum_{i,m}\frac{\mathcal{Q}_{T_i}^m}{2l_s^2}\int_{R^{1,3}} b_i^{(2)}\wedge F_m, \tag{6.3}$$

*It is straightforward to extend the following analysis to $SO(32)$ heterotic string theory.

where

$$\mathcal{Q}_S^m \equiv \mathrm{tr}(T^m T^m)\int_{\rm CY}\frac{\mathrm{tr}\bar{F}_m}{2\pi}\wedge\frac{1}{16\pi^2}\left(\mathrm{tr}\bar{F}^2-\frac{1}{2}\mathrm{tr}\bar{R}^2\right),\ \ \mathcal{Q}_{T_i}^m \equiv \mathrm{tr}(T^m T^m)\int_{T_i}\frac{\mathrm{tr}\bar{F}_m}{2\pi}. \tag{6.4}$$

Here, F_m and $\bar{F}_m$ represent the four-dimensional and extra-dimensional $U(1)^m$ field strengths with their generators T^m. $\bar{F}$ is the extra-dimensional components of field strengths of $E_8^{\rm vis}$ symmetry. The above Stückelberg couplings indicate that the dilaton and Kähler moduli have $U(1)^m$ charges, $\mathcal{Q}_S^m$ and $\mathcal{Q}_{T_i}^m$ $i=1,\cdots h^{1,1}$, respectively.

Hence, 4D $U(1)$ gauge symmetries restrict the form of tree-level Kähler potential as

$$\begin{aligned}\mathcal{K}=&-M_{\rm Pl}^2\Biggl[\ln\left(S+\bar{S}-\sum_m\frac{\mathcal{Q}_S^m}{16\pi^2}V_m\right)\\&+\ln\Biggl\{\frac{\kappa_{ijk}}{48}\left(T_i+\bar{T}_i-\sum_m\frac{\mathcal{Q}_{T_i}^m}{2\pi}V_m\right)\left(T_j+\bar{T}_j-\sum_m\frac{\mathcal{Q}_{T_j}^m}{2\pi}V_m\right)\left(T_k+\bar{T}_k-\sum_m\frac{\mathcal{Q}_{T_k}^m}{2\pi}V_m\right)\Biggr\}\Biggr],\end{aligned} \tag{6.5}$$

where κ_{ijk} stands for the intersection number of CY manifold. $U(1)$ vector multiplets V_m, $m=1,\cdots,8-\mathrm{rank}(G_{\rm vis})$, respect the shift symmetry (gauge symmetry) of dilaton axion $b_0^{(0)}$ and Kähler axion $b_i^{(0)}$ for $i=1,2,\cdots h^{1,1}$. As mentioned before, we focus on so-called "Swiss-cheese" CY manifolds with the following form of Kähler potential,

$$\mathcal{K}=-\ln\Bigl\{k_1(T_1+\bar{T}_1)^3-\sum_{i=2}^{h^{1,1}}k_i(T_i+\bar{T}_i)^3\Bigr\}, \tag{6.6}$$

with $k_1,k_i>0$. In the next section, such a negative sign in the moduli Kähler potential plays an important role of stabilizing the Kähler moduli.

After expanding the Kähler potential to second order on the vector multiplets, the mass terms of the $U(1)$ gauge bosons are obtained as

$$S_{\rm mass}=-\sum_{m,n}\frac{M_{\rm Pl}^2}{4}\left(\frac{K_{S\bar{S}}\mathcal{Q}_S^m\mathcal{Q}_S^n}{(16\pi^2)^2}+\sum_{i,j}\frac{K_{T_i\bar{T}_j}\mathcal{Q}_{T_i}^m\mathcal{Q}_{T_j}^n}{(2\pi)^2}\right)\int_{R^{1,3}}A_m\wedge *_4A_n, \tag{6.7}$$

which is typically of the order of the string scale $M_s^2=1/l_s^2$ with $l_s=2\pi\sqrt{\alpha'}$.† The $U(1)$ invariant Kähler potential with $U(1)$ magnetic fluxes in Eq. (6.5) also generate the moduli-dependent Fayet-Iliopoulos terms [135],

$$\xi_m=\left.\frac{\partial\mathcal{K}}{\partial V_m}\right|_{V_m=0}=-\frac{\mathcal{Q}_S^m}{16\pi^2}K_S-\sum_{i=1}^{h^{1,1}}\frac{\mathcal{Q}_{T_i}^m}{2\pi}K_{T_i}, \tag{6.8}$$

where $K_I=\partial K/\partial Z^I$ for $Z^I=S,T_1,\cdots,T_{h^{1,1}}$.

†See for more details, e.g., Refs. [110, 133] for $E_8\times E_8$ and [133, 134] for $SO(32)$ heterotic string theories.

For our purpose, we show the gauge threshold correction for gauge kinetic function of the non-abelian gauge groups embedded in $E_8^{(\mathrm{vis})} \times E_8^{(\mathrm{hid})}$ gauge group. The existence of $U(1)$ magnetic fluxes induce the one-loop corrections originating from the one-loop GS term in Eq. (5.9),

$$f_{\mathrm{vis}} = S + \beta_i T_i, \\ f_{\mathrm{hid}} = S - \beta_i T_i, \tag{6.9}$$

where

$$\beta_i \equiv \frac{1}{8\pi} \int_{\mathrm{CY}} \frac{1}{16\pi^2} \left(\mathrm{tr}\bar{F}^2 - \frac{1}{2}\mathrm{tr}\bar{R}^2 \right) \wedge \hat{w}_i. \tag{6.10}$$

As shown in Eq. (5.8), the gauge threshold corrections in both visible and hidden sectors are correlated each other. It is originated from the tadpole cancellation condition of $E_8^{(\mathrm{vis})} \times E_8^{(\mathrm{hid})}$ heterotic string theory. On the other hand, the gauge kinetic functions of non-abelian gauge groups embedded in $SO(32)$ heterotic string theory have the nonuniversality due to the structure of SO group.

6.2 Natural inflation and moduli stabilization

Based on the weakly coupled heterotic string theory on "Swiss-Cheese" Calabi-Yau manifold, we propose two moduli stabilization scenarios in Secs. 6.2.1 and 6.2.2. In both cases, the lightest Kähler axion have the trans-Planckian axion decay constant that induces a successful natural inflation. Its axion decay constant can be enhanced by the one-loop corrections to the gauge kinetic function of the hidden E_8^{hid} groups. That situation is different from the usually considered aligned natural inflation scenario in the case of two axions with sub-Planckian axion decay constants [118].

The other moduli fields expect for the inflaton should be heavier than the inflaton field, otherwise they would be destabilized. In our setup, the multiple $U(1)$ gauge bosons absorb the linear combination of the other Kähler axions. These axionic couplings (Stückelberg couplings) are originated from multiple $U(1)$ magnetic fluxes. The real part of dilaton is assumed to be stabilized at its minimum by the contributions from the non-perturbative effects. In Sec. 6.2.1, the dilaton potential is generated from the Kähler potential in Sec. 6.2.1 and gaugino-condensation effects in Sec. 6.2.2. Furthermore, the world-sheet instanton effect gives rise to stabilize one of the real parts of Kähler moduli and then the structure of "Swiss-Cheese" Calabi-Yau manifold contributes to the stabilization of other real parts of Kähler moduli.

6.2.1 Single gaugino condensation

Let us take a closer look at the detail of moduli stabilization. In our setup, the multiple $U(1)$ gauge bosons absorb the universal and Kähler axions except for the axion-inflaton as seen in Eq. (6.7). After that, the non-perturbative effects stabilize the dilaton and all the real parts of Kähler moduli at the SUSY-breaking minimum. At this level, one of the Kähler axion remains massless. Finally, after considering the Kähler axion as the inflaton, one can obtain its effective scalar potential.

Setup

For complete our discussion, we adopt the following "Swiss-Cheese" CY manifold with five Kähler moduli ($h^{1,1} = 5$) and four anomalous $U(1)^m$ symmetries ($m = 1, 2, 3, 4$) in the reduced Planck unit,

$$\begin{aligned}\mathcal{K} =& K\left(S+\bar{S}, V^1, V^2, V^3\right) \\ &- \ln\left\{k_1(T_1+\bar{T}_1)^3 - k_2\left(T_2+\bar{T}_2 - \sum_{n=1}^{3} q_{T_2}^n V^n\right)^3 - k_3\left(T_3+\bar{T}_3 - \sum_{n=1}^{3} q_{T_3}^n V^n\right)^3\right. \\ &\left. - k_4\left(T_4+\bar{T}_4 - q_{T_4}^4 V^4\right)^3 - k_5\left(T_5+\bar{T}_5 - q_{T_5}^4 V^4\right)^3\right\},\end{aligned} \tag{6.11}$$

where $q_{T_i}^m \equiv \mathcal{Q}_{T_i}^m/2\pi$ and k_i, $i = 1, 2, 3, 4, 5$ are the positive constants in correspondence with the intersection numbers of "Swiss-Cheese" CY manifold, $d_{t_1t_1t_1}$, $d_{t_2t_2t_2}$, $d_{t_3t_3t_3}$, $d_{t_4t_4t_4}$,$d_{t_5t_5t_5}$. We will come back to the reason why we consider the model with five Kähler moduli and four anomalous $U(1)$s in the next section. The dilaton Kähler potential is given in terms of its $U(1)_n$ charges $q_s^n = \mathcal{Q}_S^n/16\pi^2$, $n = 1, 2, 3$,

$$K^0 = -\ln\left(S + \bar{S} - \sum_{n=1}^{3} q_S^n V^n\right), \tag{6.12}$$

at the tree-level.

Moreover, we consider the following $U(1)$ invariant superpotential,

$$W = W_0 + A\, e^{-\frac{8\pi^2}{a}(S-\beta_2T_2-\beta_3T_3-\beta_4T_4-\beta_5T_5)} + B\, e^{-\mu_1 T_1}, \tag{6.13}$$

where W_0 is the constant term as a consequence of the moduli stabilization of complex structure moduli of CY manifold. In a similar way to the case of type IIB string theory, the flux induced superpotential could be tuned to small values when we consider the vacuum away from large complex structure limit. In such a case, it is expected that the backreaction from the three-form flux to the CY manifold is the sub-leading order. When the hidden sector gaugino condensate, the second term in Eq. (6.13) appears in the superpotential. It encodes the contribution of one-loop threshold corrections in Eq. (6.10) to the gauge kinetic function at the hidden sector. The third term in Eq. (6.13) corresponds to the non-perturbative potential for T_1 as a consequence of world-sheet instanton effects on its cycle.

Moduli stabilization at the perturbative level

Next, we explore the moduli stabilization at the perturbative level. With the help of NS-NS three-form flux as shown in Eq. (6.13), the complex structure moduli associated with the "Swiss-Cheese" CY manifold can be stabilized.

The certain linear combinations of imaginary components of the dilaton and the Kähler moduli, i.e., axions are also stabilized (absorbed) by the anomalous $U(1)^m$ gauge bosons included in the vector multiplets V^m, $m = 1, 2, 3, 4$. The mass-squared of $U(1)^m$ vector multiplets depend on $U(1)^m$ magnetic fluxes as shown in Eq. (6.7). It is then turned out that the following linear combination of the canonically normalized axions,

$$X^1 = \frac{1}{N^1}\left(\frac{\mathrm{Im}\, S}{q_S^1\sqrt{K_{S\bar{S}}}} + \frac{\mathrm{Im}\, T_2}{q_{T_2}^1\sqrt{K_{T_2\bar{T}_2}}} + \frac{\mathrm{Im}\, T_3}{q_{T_3}^1\sqrt{K_{T_3\bar{T}_3}}}\right),$$
$$X^2 = \frac{1}{N^2}\left(\frac{\mathrm{Im}\, S}{q_S^2\sqrt{K_{S\bar{S}}}} + \frac{\mathrm{Im}\, T_2}{q_{T_2}^2\sqrt{K_{T_2\bar{T}_2}}} + \frac{\mathrm{Im}\, T_3}{q_{T_3}^2\sqrt{K_{T_3\bar{T}_3}}}\right),$$
$$X^3 = \frac{1}{N^3}\left(\frac{\mathrm{Im}\, S}{q_S^3\sqrt{K_{S\bar{S}}}} + \frac{\mathrm{Im}\, T_2}{q_{T_2}^3\sqrt{K_{T_2\bar{T}_2}}} + \frac{\mathrm{Im}\, T_3}{q_{T_3}^3\sqrt{K_{T_3\bar{T}_3}}}\right), \tag{6.14}$$

are absorbed by $U(1)^n$ (n=1,2,3) gauge bosons. The overall normalization factor is denoted as

$$N^n = \sqrt{(1/q_S^n\sqrt{K_{S\bar{S}}})^2 + (1/q_{T_2}^n\sqrt{K_{T_2\bar{T}_2}})^2 + (1/q_{T_3}^n\sqrt{K_{T_3\bar{T}_3}})^2}, \tag{6.15}$$

for $n = 1, 2, 3$. The details of Kähler metrics of axions are summarized in Appendix B. Thus, after canonically normalizing the $U(1)$ gauge bosons, we obtain their mass-squared matrices

$$M_{m,n}^2 \simeq \frac{M_{\mathrm{Pl}}^2}{4\sqrt{\langle \mathrm{Re}\, f_{m,m}\rangle}\sqrt{\langle \mathrm{Re}\, f_{n,n}\rangle}}\left(K_{S\bar{S}}q_S^m q_S^n + \sum_{i,j} K_{T_i\bar{T}_j}q_{T_i}^m q_{T_j}^n\right), \tag{6.16}$$

for $m, n = 1, 2, 3$. The gauge kinetic functions of $U(1)$s, $f_{m,n}$ are also obtained by compactifying the internal manifold, $f_{m,n} = \mathrm{tr}(T^m T^n) S\delta_{m,n} + \mathcal{O}(\beta T)$. We stress that $U(1)^n$ gauge invariance of the superpotential (6.13) requires the following relation between these $U(1)$ charges of the moduli S, T_2 and T_3,

$$q_S^1 = q_{T_2}^1\beta_2 + q_{T_3}^1\beta_3,\ \ q_S^2 = q_{T_2}^2\beta_2 + q_{T_3}^2\beta_3,\ \ q_S^3 = q_{T_2}^3\beta_2 + q_{T_3}^3\beta_3. \tag{6.17}$$

Since the $U(1)$ gauge invariance condition (6.17) restricts the form of mass-squared matrices (6.16), one can achieve the full-rank mass matrices (6.16) iff the number of $U(1)$s is bigger than three. As a result, the imaginary components of S, T_2 and T_3 are completely absorbed by the $U(1)^{1,2,3}$ gauge bosons. From the superpotential (6.13), the decay constant of universal axion is much smaller than the Planck scale and it cannot be identified as the inflaton.

In a similar fasion, the following linear combination of axion is absorbed by $U(1)^4$ gauge boson,

$$X^4 = \frac{1}{N^4}\left(\frac{\mathrm{Im}\, T_4}{q_{T_4}^4\sqrt{K_{T_4\bar{T}_4}}} + \frac{\mathrm{Im}\, T_5}{q_{T_5}^4\sqrt{K_{T_5\bar{T}_5}}}\right), \tag{6.18}$$

where $N^4 = \sqrt{(1/q_{T_4}^4\sqrt{K_{T_4\bar{T}_4}})^2 + (1/q_{T_5}^4\sqrt{K_{T_5\bar{T}_5}})^2}$, and the orthogonal direction of X_4

$$Y^4 = \frac{1}{N^4}\left(-\frac{\mathrm{Im}\, T_4}{q_{T_5}^4\sqrt{K_{T_5\bar{T}_5}}} + \frac{\mathrm{Im}\, T_5}{q_{T_4}^4\sqrt{K_{T_4\bar{T}_4}}}\right), \tag{6.19}$$

remains massless at this stage. Its massless axion is identified as the inflaton later. Following the above arguments, one can stabilize the complex structure moduli and the four imaginary parts of the moduli X^m, $m = 1, 2, 3, 4$ at the perturbative level in the presence of four anomalous $U(1)$ vector multiplets.

Moduli stabilization at the non-perturbative level

Next, we discuss the stabilization mechanism of moduli fields at the non-perturbative level.

In order to stabilize the dilaton field, we assume the following form of non-perturbative correction to the dilaton Kähler potential in addition to its tree-level one (6.12),

$$K^{\rm np} = d\, g^{-p} e^{-b/g}, \tag{6.20}$$

where b, p, and d are the real unknown constants. The gauge coupling in the hidden sector is expressed as $g = ({\rm Re}\, S - \sum_{i\neq 1} \beta_i {\rm Re}\, T_i)^{-1/2}$ as given in the superpotential (6.9). The authors of Refs. [136, 137, 138] proposed that the two ansatzs of the non-perturbative correction to the Kähler potential $K^{\rm np}$ which are written by‡

$$K = K^0 + K^{\rm np} \quad {\rm or} \quad K = \ln\left(e^{K^0} + e^{K^{\rm np}}\right). \tag{6.21}$$

Along the line of Ref. [139], the dilaton is assumed to be stabilized at the minimum in the presence of such corrections to the Kähler potential. However, our following inflation mechanism does not rely on the detailed form of non-perturbative effects.

Then, one can write down the F-term scalar potential in terms of the Kähler potential (6.11) and the superpotential (6.13). To simplify the analysis, we redefine the linear combination of the dilaton and the Kähler moduli as,

$$\Phi = S - \beta_2 T_2 - \beta_3 T_3 - \beta_4 T_4 - \beta_5 T_5, \tag{6.22}$$

which leads to the following form of the Kähler potential (6.11) and superpotential (6.13),

$$\begin{aligned}
\mathcal{K} =& K\left(\Phi + \bar{\Phi}, T_2 + \bar{T}_2, T_3 + \bar{T}_3, T_4 + \bar{T}_4, T_5 + \bar{T}_5, V^1, V^2, V^3\right) \\
&- \ln\left\{ k_1 (T_1 + \bar{T}_1)^3 - k_2 \left(T_2 + \bar{T}_2 - \sum_{n=1}^{3} q^n_{T_2} V^n\right)^3 - k_3 \left(T_3 + \bar{T}_3 - \sum_{n=1}^{3} q^n_{T_3} V^n\right)^3 \right. \\
&\left. \quad - k_4 \left(T_4 + \bar{T}_4 - q^4_{T_4} V^4\right)^3 - k_5 \left(T_5 + \bar{T}_5 - q^4_{T_5} V^4\right)^3 \right\}, \\
W =& W_0 + A\, e^{-\frac{8\pi^2}{a}\Phi} + B\, e^{-\mu_1 T_1}.
\end{aligned} \tag{6.23}$$

In what follows, we suppose that the gaugino condensation term in Eq. (6.23) is enough small compared with the others in Eq. (6.23), i.e., W_0, $B\, e^{-\mu_1 T_1} \gg A e^{-\frac{8\pi^2}{a}\Phi}$. In this way, it enables

‡The non-perturbative correction to dilaton Kähler potential was also addressed in the approaches of effective field theory [137].

us to ignore the gaugino condensation term that generates the inflaton potential as discussed later.

In these field bases, the moduli T_1, $\mathrm{Re}\, T_2$, $\mathrm{Re}\, T_3$, $\mathrm{Re}\, T_4$, $\mathrm{Re}\, T_5$ and $\mathrm{Re}\, \Phi$ can be stabilized at the minimum satisfying the supersymmetric conditions,

$$\begin{aligned}&D_{T_1}W = 0,\\ &D_{T_2}W = K_{T_2}W = 0,\ D_{T_3}W = K_{T_3}W = 0,\ D_{T_4}W = K_{T_4}W = 0,\ D_{T_5}W = K_{T_5}W = 0,\\ &D_{\Phi}W = K_{\Phi}W = 0,\end{aligned} \tag{6.24}$$

where the non-perturbative correction to the dilaton in Eq. (6.21) is importanto to stabilize $\mathrm{Re}\, \Phi$. The real parts of moduli T_j, $j = 2, 3, 4, 5$, are also stabilized under the following conditions,

$$K_{T_j} \simeq \frac{3k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^3} + \frac{\partial K^0}{\partial T_j} \simeq \frac{3k_j(T_j + \bar{T}_j)^2}{k_1(T_1 + \bar{T}_1)^3} - \frac{\beta_j}{\Phi + \bar{\Phi}} + \mathcal{O}\left(\beta_j \frac{\sum_{k=2}^{5} \beta_k \mathrm{Re}\, T_k}{\mathrm{Re}\, \Phi}\right) = 0, \tag{6.25}$$

in the limit of $\mathrm{Re}\, T_1 > \mathrm{Re}\, T_i$ and $\mathrm{Re}\, S > \mathrm{Re}\, T_i$. The dilaton Kähler potential is now approximated as its tree-level one K^0 in Eq. (6.21). After solving the Eq. (6.25) for $\mathrm{Re}\, \Phi$, $\mathrm{Re}\, \Phi$ is written in terms of Kähler moduli,

$$\mathrm{Re}\, S \simeq \mathrm{Re}\, \Phi \simeq \frac{k_1(\mathrm{Re}\, T_1)^3}{3k_j \mathrm{Re}\, T_j^2}\beta_j \gg \beta_j \mathrm{Re}\, T_j, \tag{6.26}$$

for $j = 2, 3, 4, 5$. Therefore, as shown in Eq. (6.9), the gauge kinetic function is dominated by its tree-level part rather than its one-loop corrections. It implies that the perturbative expansion is valid in our setup. This structure is coming from the condition $\mathrm{Re}\, T_1 > \mathrm{Re}\, T_j$ ($j \neq 1$) and negative signs in the volume of "Swiss-Cheese' CY manifold (6.11). Since the above stabilization mechanism cannot be realized without these negative sign, it is an important feature of the "Swiss-Cheese" Calabi-Yau manifold. Finally, we comment on the D-term potential provided by the anomalous $U(1)$ symmetries included in Kähler potential (6.11). In our setup, the D-term potentials are automatically vanished thanks to the supersymmetric conditions for moduli fields, $K_{T_2} = K_{T_3} = K_{T_4} = K_{T_5} = K_{\Phi} = 0$,

At the minimum of the relevant moduli fields given by Eq. (6.24), the scalar potential of 4D $N = 1$ supergravity,

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\right), \tag{6.27}$$

has the supersymmetric AdS minimum,

$$\langle V \rangle = -3e^K |W|^2. \tag{6.28}$$

Although several approaches to uplift such an AdS vacuum have been addressed in various papers such as the F-terms with dynamical SUSY-breaking sector [140, 32, 64, 141] or D-terms with anti-heterotic five branes [142], in the following, the SUSY is assumed to be broken by the dynamical SUSY-breaking sector as discussed in Chapter. 3. Their Kähler potential and superpotential are described by

$$\Delta K = |X|^2 - \frac{|X|^4}{\Lambda^2},$$
$$\Delta W = \nu X. \tag{6.29}$$

Here, we suppose that the dynamical SUSY-breaking sector is originating from the non-abelian gauge theory living on the heterotic five-branes and Λ is its dynamical SUSY-breaking scale. X is a gauge singlet chiral superfield under $E_8^{\rm vis} \times E_8^{\rm hid}$ symmetry. Furthermore, we omit the moduli dependence of X for simplicity. By setting the parameter ν as also discussed in 5D SUGRA in Chapter 3,

$$\langle V \rangle + \Delta V \simeq e^{\langle K \rangle}\left(-3|\langle W \rangle|^2 + K^{X\bar{X}}|\nu|^2\right) = 0 \Leftrightarrow |\nu|^2 = 3|\langle W \rangle|^2, \tag{6.30}$$

the Minkowski minimum can be realized.

In the following discussion, we take into account the term $Ae^{-\frac{8\pi^2}{a}\Phi}$ omitted in the superpotential (6.23). Even when such a term is included in the full scalar potential, the moduli $\mathrm{Re}\,\Phi$, T_1, $\mathrm{Re}\,T_2$, $\mathrm{Re}\,T_3$, $\mathrm{Re}\,T_4$ and $\mathrm{Re}\,T_5$ could remain to stay at the minimum close to the values given by Eq. (6.24). This is because it is supposed that the gaugino condensation term is much small compared with the others in the superpotential (6.23) at the minimum. Thus, the inflaton mass appearing through the gaugino condensation term can be parametrically lower than those of heavy moduli fields which are stabilized by the flux-induced constant term and the world-sheet instanton effect in Eq. (6.23), and the D-term contribution in Eq. (6.11). It will be shown in the following numerical analysis. The explicit form of their mass matrices are summarized in the Appendix B. We again remark that the four $U(1)$ gauge bosons absorb the same number of axions except for the axion-inflaton Y^4 at the string scale and then, they are decoupled from the inflaton dynamics below the string and compactification scale.

Inflaton potential and its dynamics

Let us write down the inflation potential. Along the above stabilization procedures, one can integrate out the heavy moduli and substitute their field values given by Eq. (6.24). First, we canonically normalize the light moduli Y^4 (the orthogonal direction of absorbed axion in Eq. (6.19)) as

$$\hat{Y}^4 \simeq \frac{1}{N^4}\sqrt{2\left(\frac{K_{T_4\bar{T}_4}}{(q^4_{T_5})^2 K_{T_5\bar{T}_5}} + \frac{K_{T_5\bar{T}_5}}{(q^4_{T_4})^2 K_{T_4\bar{T}_4}}\right)}\, Y^4 \equiv \hat{N}^4\, Y^4 \tag{6.31}$$

by employing the following relation,

$$\mathrm{Im}\, T_4 = \frac{1}{N^4}\left(\frac{X^4}{q^4_{T_4}\sqrt{K_{T_4\bar{T}_4}}} - \frac{Y^4}{q^4_{T_5}\sqrt{K_{T_5\bar{T}_5}}}\right),$$
$$\mathrm{Im}\, T_5 = \frac{1}{N^4}\left(\frac{X^4}{q^4_{T_5}\sqrt{K_{T_5\bar{T}_5}}} - \frac{Y^4}{q^4_{T_4}\sqrt{K_{T_4\bar{T}_4}}}\right). \tag{6.32}$$

Note that $U(1)^4$ gauge invariance of the superpotential (6.23) requires

$$q^4_{T_4}\,\beta_4 + q^4_{T_5}\,\beta_5 = 0. \tag{6.33}$$

Then, one can extract the effective scalar potential for $\hat{Y}^4$

$$V_{\rm eff} \simeq \Lambda^4(1-\cos{(\beta\,\hat{Y}^4)}), \tag{6.34}$$

in the limit of $Ae^{-\frac{8\pi^2}{a}\langle {\rm Re}\Phi\rangle} \ll W_0, Be^{-\mu_1\langle T_1\rangle}$. The overall scale of scalar potential (inflation scale) Λ^4 and the inverse of its axion decay constant β are given by

$$\begin{aligned}\Lambda^4 &\equiv 6\,e^K e^{-\frac{8\pi^2}{a}\,{\rm Re}\Phi} A(W_0 + Be^{-\mu_1 T_1}),\\ \beta &\equiv \frac{8\pi^2}{a\,N^4\hat{N}^4}\left(\frac{\beta_5}{q^4_{T_4}\sqrt{K_{T_4\bar{T}_4}}} - \frac{\beta_4}{q^4_{T_5}\sqrt{K_{T_5\bar{T}_5}}}\right).\end{aligned} \tag{6.35}$$

The obtained scalar potential (6.34) is that of natural inflation by identifying the axion $\hat{Y}^4$ as the inflaton. The observed power spectrum of curvature perturbation is achieved by setting the parameter in Eq. (6.35) satisfying $\Lambda^4 \sim \mathcal{O}(10^{-9})$ in the reduced Planck unit. The trans-Planckian axion decay constant β^{-1} is realized by the enhancement of one-loop factor compared with the dilaton-axion as shown in Eq. (6.35). We expect that the spectral index of curvature perturbation and the tensor-to-scalar ratio can be consistent with the Planck data.

To justify our expectation, we numerically estimate the cosmological quantities constrained by the cosmological observations. As a matter of convenience, we take the dilaton Kähler potential as $K^0 + K^{\rm np}$ given by Eqs. (6.11) and (6.20). Their parameters are chosen as

$$\begin{aligned}&k_1 = \frac{1}{6},\ k_2 = k_3 = k_4 = k_5 = 4,\\ &d = 7,\ b = 1,\ p = 2,\end{aligned} \tag{6.36}$$

in the Kähler potential and

$$\begin{aligned}&A = \frac{1}{300},\ \ a = 30,\ \ B = -\frac{1}{2},\ \ \mu_1 = 2\pi,\ \ W_0 = 6\times 10^{-4},\ \ \mu \simeq 1\times 10^{-3},\\ &\beta_2 \simeq \beta_3 \simeq \beta_4 \simeq \beta_5 \simeq \frac{1}{8\pi},\end{aligned} \tag{6.37}$$

in the superpotential given by Eqs. (6.13) and (6.29), respectively. Here, we consider $\mathcal{O}(1)$ $U(1)$ charges of moduli fields.

By setting these parameters, the vacuum expectation values of the moduli are obtained as

$$T_1 \simeq 1.3,\ \ T_2 \simeq T_3 \simeq T_4 \simeq T_5 \simeq 0.025,\ \ S \simeq \Phi \simeq 1.9, \tag{6.38}$$

which yield the typical Kaluza-Klein scale

$$M_{KK} \simeq \frac{M_s}{\mathcal{V}^{1/6}} \simeq 1.2\times 10^{17}\,{\rm GeV}, \tag{6.39}$$

with

$$M_s = \frac{M_{\rm Pl}}{\sqrt{4\pi\alpha^{-1}}} \simeq 1.4 \times 10^{17}\,{\rm GeV}. \tag{6.40}$$

This size of gauge coupling of visible gauge group $G_{\rm vis}\alpha^{-1} \simeq 24$ is mainly determined by the vacuum expectation value of dilaton and it is consistent with that predicted by the gauge coupling unification at the string scale . On the other hand, one would suspect that the effective field description is violated because the vacuum expectation values of Kähler moduli given in Eq. (6.38) are smaller than 1 in string unit. In such a case, the effective field theory generically receives the stringy and higher derivative corrections that translates into the correction terms to the CY volume ($\Delta\mathcal{V}$) in the Kähler potential, although we do not its explicit form in the CY manifold. However, in our setup, the CY moduli volume is larger than the correction term $\Delta\mathcal{V}$ at the vacuum given in Eq. (6.25) and then moduli T_j $(j = 2, 3, 4, 5)$ will still be stabilized at the vacuum close to that in Eq. (6.25). In any rate, our proposed idea to enhance the axion decay constant is irrelevant to the detail of moduli stabilization.

The input parameters given by Eqs. (6.36) and (6.37) are set to reproduce the CMB scale

$$\Lambda^4 \simeq 3.22 \times 10^{-9}, \tag{6.41}$$

and the desired trans-Planckian axion decay constant,

$$\beta^{-1} \simeq 6.1, \tag{6.42}$$

in the reduced Planck unit.

As discussed in Chapter 2, in terms of the slow-roll parameters,

$$\begin{aligned}
\epsilon &\equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{\partial_{\hat{Y}^4} V_{\rm eff}}{V_{\rm eff}}\right)^2, \\
\eta &\equiv M_{\rm Pl}^2 \frac{\partial^2_{\hat{Y}^4} V_{\rm eff}}{V_{\rm eff}}, \\
\xi^2 &\equiv M_{\rm Pl}^4 \frac{\partial_{\hat{Y}^4} V_{\rm eff}\,\partial^3_{\hat{Y}^4} V_{\rm eff}}{V_{\rm eff}^2},
\end{aligned} \tag{6.43}$$

an amount of e-folding from the time t_* at the pivot scale to the inflation end $t_{\rm end}$ is estimated as

$$N_e = -\int_{t_{\rm end}}^{t_*} dt H(t) \simeq \frac{1}{M_{\rm Pl}} \int_{\hat{Y}^4_*}^{\hat{Y}^4_{\rm end}} \frac{d\hat{Y}^4}{\sqrt{2\epsilon}}, \tag{6.44}$$

where $H(t)$ is the Hubble parameter, $\hat{Y}^4_*$ and $\hat{Y}^4_{\rm end}$ represent for the field values of the inflaton $\hat{Y}^4$ at the time t_* and $t_{\rm end}$, respectively. The end of inflation corresponds to the violation of slow-roll conditions, $\max\{|\epsilon|, |\eta|\} = 1$. From the cosmological observables written in terms of the slow-roll parameters given by Eq. (2.28), it turns out that their numerical values and the e-folding number at the field value $\hat{Y}^4_* \simeq 13 M_{\rm Pl}$,

$$P_\xi \simeq 2.2 \times 10^{-9}, \;\; n_s \simeq 0.961, \;\; r \simeq 0.05, \;\; N_e \simeq 62, \tag{6.45}$$

are consistent with the Planck data [15, 16],

$$P_\xi = 2.196^{+0.051}_{-0.060} \times 10^{-9}, \;\; n_s = 0.9655 \pm 0.0062, \;\; r < 0.11, \tag{6.46}$$

at the pivot scale $k_* = 0.05\mathrm{Mpc}^{-1}$. The dual Coxeter number $a = 30$ of hidden $E_8^{\rm hid}$ gauge group and factors $\beta_3 \simeq \beta_4 \simeq \beta_5 \simeq 1/8\pi$ are important to enhance the axion decay constant of inflaton field.

6.2.2 Double gaugino condensations

Contrary to the moduli stabilization in the previous section 6.2.1, in this section, we derive the inflaton potential along the different type of the Kähelr potential and superpotential. In the previous model in Sec. 6.2.1, the dilaton is assumed to be stabilized at the minimum by the Kähler potential including the non-perturbative corrections (6.21), whereas in the model of this Sec. 6.2.2, the dilaton is stabilized by the inclusion of the gaugino condensation terms. That is the the main difference between the model 1 and the model 2. In a similar way to the model in Sec. 6.2.1, the one-loop threshold corrections enhance the axion decay constant which appear in the gauge kinetic function of the hidden gauge group. Guided by the results in model in Sec. 6.2.1, the volume form of "Swiss-Cheese" CY manifold also gives rise to stabilize the real parts of moduli.

Setup

For complete our discussion, we adopt the "Swiss-Cheese" CY manifold with three Kähler moduli ($h^{1,1} = 3$) and one anomalous $U(1)$ symmetry,

$$\mathcal{K} = -\ln\left(S + \bar{S}\right) - \ln\left(k_b(T_b + \bar{T}_b)^3 - k_s\left(T_s + \bar{T}_s - \frac{\mathcal{Q}_s}{2\pi}V_s\right)^3 - k_s^\prime\left(T_s^\prime + \bar{T}_s^\prime - \frac{\mathcal{Q}_s^\prime}{2\pi}V_s\right)^3\right), \tag{6.47}$$

where $k_b,\ k_s,\ k_s^\prime$ are positive constants in correspondence with the intersection numbers of "Swiss-Cheese" CY manifold $d_{t_b t_b t_b}$, $d_{t_s t_s t_s}$, $d_{t_s^\prime t_s^\prime t_s^\prime}$. Since only two moduli T_s and $T_s^\prime$ have $U(1)_s$ charges under an anomalous $U(1)_s$ vector multiplet V_s, the single linear combination of Kähler axions $b_s^{(0)}$ and $b_s^{\prime(0)}$ are absorbed by it. By contrast, its orthogonal direction remains massless and is identified as the inflaton later. Differently from the previous model in Sec. 6.2.1, we do not consider the non-perturbative corrections to the dilaton Kähler potential.

Moreover, the $U(1)_s$ invariant superpotential is chosen as,

$$W = w_0 + A_2\, e^{-\frac{8\pi^2}{a_2}(S-\beta_s^{(1)}T_s-\beta_s^{\prime(1)}T_s^\prime)} + B_2\, e^{-\frac{8\pi^2}{b_2}(S-\beta_s^{(2)}T_s-\beta_s^{\prime(2)}T_s^\prime)} + C_2\, e^{-\mu_b T_b}, \tag{6.48}$$

where w_0 is the constant term as a consequence of the complex structure moduli of the CY manifold. When the gauginos condensate at two hidden sectors, the second and third terms in Eq. (6.48) appear in the superpotential, where these gaugino condensation scales are determined by ranks of the two hidden gauge groups characterized by a_2 and b_2, respectively. On the other hand, the fourth term of the (r.h.s.) denotes the non-perturbative potential for T_b as a consequence of the world-sheet instanton effect on its two-cycle.

Moduli stabilization

Next, we begin with the moduli stabilization at the perturbative level. As with the case of model in Sec. 6.2.1, the complex structure moduli associated with "Swiss-Cheese" CY manifold are stabilized with the help of fluxes, whereas one linear combinations of the Kähler axions are stabilized by anomalous $U(1)_s$ symmetry.

Indeed, $U(1)_s$ vector multiplet absorbs the following linear combination of the canonically normalized axions Im T_s and Im T_s' as,

$$X_s = \frac{1}{N_s}\left(\frac{\mathrm{Im}\,T_s}{q_s\sqrt{K_{T_s\bar{T}_s}}} + \frac{\mathrm{Im}\,T_s'}{q_s'\sqrt{K_{T_s'\bar{T}_s'}}}\right), \tag{6.49}$$

where $N_s = \sqrt{(1/q_s\sqrt{K_{T_s\bar{T}_s}})^2 + (1/q_s'\sqrt{K_{T_s'\bar{T}_s'}})^2}$ with $q_s = \mathcal{Q}_s/2\pi$ and $q_s' = \mathcal{Q}_s'/2\pi$. Now the Kähler mixing of two Kähler moduli are neglected due to the fact that their stabilization is also the same as model 1 in Sec. 6.2.1. Its orthogonal direction

$$Y_s = \frac{1}{N_s}\left(-\frac{\mathrm{Im}\,T_s}{q_s'\sqrt{K_{T_s'\bar{T}_s'}}} + \frac{\mathrm{Im}\,T_s'}{q_s\sqrt{K_{T_s\bar{T}_s}}}\right), \tag{6.50}$$

remains massless and it is identified as the inflaton later. Note that the $U(1)_s$ gauge invariance of the superpotential (6.48) requires that the $U(1)_s$ charges of the moduli are related as

$$\begin{aligned} &q_s\,\beta_s^{(1)} + q_s'\,\beta_s'^{(1)} = 0,\\ &q_s\,\beta_s^{(2)} + q_s'\,\beta_s'^{(2)} = 0. \end{aligned} \tag{6.51}$$

To simplify our analysis of F-term scalar potential, we change the field basis as

$$\Phi = S - \beta_s^{(1)}T_s - \beta_s'^{(1)}T_s', \tag{6.52}$$

which leads to the following form of the Kähler poential and superpotential,

$$\begin{aligned} \mathcal{K} =& -\ln\left(\Phi + \bar{\Phi} + \beta_s^{(1)}(T_s + \bar{T}_s) + \beta_s'^{(1)}(T_s' + \bar{T}_s')\right)\\ &- \ln\left(k_b(T_b + \bar{T}_b)^3 - k_s\left(T_s + \bar{T}_s - \frac{\mathcal{Q}_s}{2\pi}V_s\right)^3 - k_s'\left(T_s' + \bar{T}_s' - \frac{\mathcal{Q}_s'}{2\pi}V_s\right)^3\right),\\ W =& w_0 + A_2\,e^{-\frac{8\pi^2}{a_2}\Phi} + B_2\,e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T_s')} + C_2\,e^{-\mu_b T_b}. \end{aligned} \tag{6.53}$$

In what follows, we suppose that the third term of the (r.h.s.) in Eq. (7.9) is much smaller than the other terms in Eq. (7.9), i.e., w_0, $A_2\,e^{-\frac{8\pi^2}{a_2}\Phi}$, $C_2\,e^{-\mu_b T_b} \gg B_2\,e^{-\frac{8\pi^2}{b_2}(\Phi + (\beta_s^{(1)} - \beta_s^{(2)})T_s + (\beta_s'^{(1)} - \beta_s'^{(2)})T_s')}$. In particular, such a hierarchical structure among two gaugino condensations result from the different hidden gauge groups. In this way, it enables us to ignore the third term in the superpotential (7.9) that generates the inflaton potential as discussed later.

Next, we show the moduli stabilization at the non-perturbative level. At this stage, the dilaton (Φ) and Kähler moduli (T_b, $\mathrm{Re}\, T_s$, $\mathrm{Re}\, T_s^\prime$) are stabilized at the minimum satisfying the supersymmetric conditions. They are written by

$$\begin{aligned} &D_\Phi W = 0, \\ &D_{T_b} W = 0, \\ &K_{T_s} = K_{T_s^\prime} = 0. \end{aligned} \tag{6.54}$$

Since T_b is stabilized at its own superpotential, T_s and $T_s^\prime$ are also stabilized under the following conditions,

$$K_{T_i} \simeq \frac{3k_i (T_i + \bar{T}_i)^2}{k_b (T_b + \bar{T}_b)^3} - \frac{\beta_j}{\Phi + \bar{\Phi}} = 0, \tag{6.55}$$

in the limit of $\mathrm{Re}\, T_1 > \mathrm{Re}\, T_s, \mathrm{Re}\, T_s^\prime$ and $\mathrm{Re}\, S > \mathrm{Re}\, T_s, \mathrm{Re}\, T_s^\prime$. After solving the above Eq. (6.55) for $\mathrm{Re}\, \Phi$, $\mathrm{Re}\, \Phi$ is written in terms of Kähler moduli,

$$\mathrm{Re}\, S \simeq \mathrm{Re}\, \Phi \simeq \frac{k_b (\mathrm{Re}\, T_b)^3}{3k_i \mathrm{Re}\, T_i^2} \beta_j \gg \beta_j \mathrm{Re}\, T_j, \tag{6.56}$$

Therefore, the gauge kinetic function in Eq. (6.9) is dominated by its the tree-level part compared with its one-loop corrections when $\mathrm{Re}\, T_1 > \mathrm{Re}\, T_i$ $(j \neq 1)$ is satisfied. It implies that the perturbative expansion is valid in our setup. This structure is coming from the negative signs in the volume of "Swiss-Cheese" CY manifold (6.47). It is an important feature of "Swiss-Cheese" CY manifold. Although we focus on the F-term potential until now, the D-terms induced from the Kähler potential (7.9) is vanished due to the supersymmeric conditions (6.54).

At the minimum of relevant moduli fields given by Eq. (6.54), the scalar potential of 4D $N = 1$ supergravity has the the supersymmetric AdS minimum at the minimum,

$$\langle V \rangle = -3 e^K |W|^2. \tag{6.57}$$

In the same way as model 1 in the Sec. 6.2.1, we suppose that the dynamical SUSY-breaking sector associated with the heterotic five-brane uplift the AdS minimum. The Kähler potential and superpotential of SUSY-breaking field X in the dynamical SUSY-breaking sector become

$$\begin{aligned} \Delta K &= |X|^2 - \frac{|X|^4}{\Lambda^2}, \\ \Delta W &= \nu X, \end{aligned} \tag{6.58}$$

where X do not have some charges under $E_8^{\mathrm{vis}} \times E_8^{\mathrm{hid}}$ symmetry and Λ is the dynamical SUSY-breaking scale. Here, we omit the moduli dependence in the Kähler potential of X which is irrelevant to the moduli stabilization, for simplicity. By setting the parameter ν as

$$\langle V \rangle + \Delta V \simeq e^K \left(-3|W|^2 + K^{X\bar{X}} |\nu|^2 \right) = 0, \;\; \Leftrightarrow |\nu|^2 = 3 |\langle W \rangle|^2, \tag{6.59}$$

one can obtain the Minkowski minimum.

In the following discussion, we take into account the term $B_2\, e^{-\frac{8\pi^2}{b_2}(\Phi+(\beta_s^{(1)}-\beta_s^{(2)})T_s+(\beta_s'^{(1)}-\beta_s'^{(2)})T_s')}$ omitted in the superpotential (7.9). Even when such a term is included in the full scalar potential, the moduli Φ, T_b, ReT_s, ReT_s' are stabilized at the values close to the minimum given by Eq. (6.54). This is because it is supposed that one of the gaugino condensation term is much small compared with the others in the superpotential (7.9) at the minimum. Thus, the inflaon mass appearing through the other gaugino condensation term can be lower than those of heavy moduli. As ReT_s and ReT_s', their masses also include the D-term contributions from the Kähler potential (7.9).

Inflaton potential

Let us write down the inflaton potential. Along the above stabilization procedures, one can integrate out these heavy moduli and they could be replaced with the vacuum expectation values given by Eq. (6.54). First of all, we canonically normalize the lightest moduli Y_s (the linear combination of $\mathrm{Im}\, T_s$ and $\mathrm{Im}\, T_s'$ given by Eq. (6.50)) as

$$\hat{Y}_s \simeq \frac{1}{N_s}\sqrt{2\left(\frac{K_{T_s\bar{T}_s}}{(q_s')^2 K_{T_s'\bar{T}_s'}} + \frac{K_{T_s'\bar{T}_s'}}{(q_s)^2 K_{T_s\bar{T}_s}}\right)}\, Y_s \equiv \hat{N}_s\, Y_s, \tag{6.60}$$

by employing the following relation, moduli,

$$\begin{aligned}
\mathrm{Im}\, T_s &= \frac{1}{N_s}\left(\frac{X_s}{q_s\sqrt{K_{T_s\bar{T}_s}}} - \frac{Y_s}{q_s'\sqrt{K_{T_s'\bar{T}_s'}}}\right), \\
\mathrm{Im}\, T_s' &= \frac{1}{N_s}\left(\frac{X_s}{q_s'\sqrt{K_{T_s'\bar{T}_s'}}} + \frac{Y_s}{q_s\sqrt{K_{T_s\bar{T}_s}}}\right),
\end{aligned} \tag{6.61}$$

We stress that $U(1)_s$ gauge invariance of the superpotential (6.48) requires

$$\begin{aligned}
q_s\, \beta_s^{(1)} + q_s'\, \beta_s'^{(1)} &= 0, \\
q_s\, \beta_s^{(2)} + q_s'\, \beta_s'^{(2)} &= 0.
\end{aligned} \tag{6.62}$$

Thus, for the canonically normalized field $\hat{Y}_s$, one can obtain its effective scalar potential,

$$V_{\rm eff} \simeq \Lambda_s^4 (1 - \cos(\beta_s \hat{Y}_s)), \tag{6.63}$$

in the limit of $B_2 e^{-\frac{8\pi^2}{b_2}\langle \mathrm{Re}\Phi\rangle} \ll w_0, A_2 e^{-\frac{8\pi^2}{a_2}\langle \mathrm{Re}\Phi\rangle}, C_2 e^{-\mu_b\langle T_b\rangle}$. The overall scale of scalar potential (inflation scale) Λ_s^4 the inverse of its decay constant β are given by

$$\Lambda_s^4 \equiv 6e^K e^{-\frac{8\pi^2}{b_2}\mathrm{Re}\Phi} B_2 (w_0 + A_2\, e^{-\frac{8\pi^2}{a_2}\Phi} + C_2\, e^{-\mu_b T_b}), \beta_s \equiv \frac{8\pi^2}{b_2 N_s \hat{N}_s}\left(-\frac{\beta_s^{(1)} - \beta_s^{(2)}}{q_s'\sqrt{K_{T_s'\bar{T}_s'}}} + \frac{\beta_s'^{(1)} - \beta_s'^{(2)}}{q_s\sqrt{K_{T_s\bar{T}_s}}}\right). \tag{6.64}$$

The obtained scalar potential is that of natural inflation when we identify the axion $\hat{Y}_s$ as the inflaton. The correct power spectrum of the curvature perturbation is achieved by setting the parameter in Eq. (6.64) satisfying

$$\Lambda_s^4 \sim \mathcal{O}(10^{-9}), \tag{6.65}$$

in the reduced Planck unit. The enhancement of axion decay constant β_s^{-1} originates from the loop-correction in the gauge kinetic function which do not arise in the dilaton-axion as shown in Eq. (6.64). As a result, the spectral index of curvature perturbation and the tensor-to-scalar ratio are expected to be consistent with the Planck data. By solving equation of motion of inflaton field, we find that the predictions of this model are same as those obtained in the previous model in Sec. 6.2.1. However, it is difficult to consider two gaugino condensation terms originating from $E_8 \times E_8$ or $SO(32)$ heterotic string theories. This is because the rank of $E_8 \times E_8$ or $SO(32)$ gauge groups should incorporate the rank 4 SM gauge groups in addition to two gaugino condensation sectors. In such a case, it is required to tune some parameters to realize the correct inflation scale. Therefore, in this section, we focus on the situation that at least one of the gaugino condensation sector is derived from the gauge theory living on the heterotic five-branes.

6.3 Summary

In this chapter, we proposed the natural inflation scenario in the framework of weakly coupled $E_8^{\rm vis} \times E_8^{\rm hid}$ or $SO(32)$ heterotic string theory on the "Swiss-Cheese" CY manifold. The $E_8^{\rm vis}$ gauge group are decomposed into the SM (GUT) and extra $U(1)$ gauge groups, where the magnetic fluxes are inserted into this Cartan direction. Recent Planck data indicate that the axion-inflaton should have the trans-Planckian decay constant, otherwise the obtained natural inflation is inconsistent with the Planck data. So far, there are several approaches to attack this problem. The most familiar scenario to obtain the trans-Planckian axion decay is alignment mechanism by employing two axions with sub-Planckian decay constants [118]. In this chapter, in order to overcome such a problem, we focused on the one-loop corrections to the gauge coupling in the hidden gauge group. These corrections result from the dimensional reduction of one-loop Green-Schwarz term [107, 108] which is the typical feature of the weakly coupled heterotic string theory. In particular, these threshold corrections are not relevant to a dilaton axion but influential to Kähler axions. Thus, we considered the inflaton as one of Kähler axions associated with the two-cycles of the CY manifold. The axion potential is generated by gaugino condensation terms in the hidden gauge group. As a result, the decay constant of axion-inflaton reaches the trans-Planckian value which leads to the successful natural inflation.

In order to extract the potential of single axion, we must take into account the moduli stabilization of other fields so as not to destabilize them. Their mass scales are then constrained to be larger than the inflation scale and one can avoid the cosmological moduli problem. Hence, we discussed two moduli stabilization scenarios in Secs. 6.2.1 and 6.2.2. In Sec. 6.2.1, the non-perturbative corrections to the dilaton Kähler potential lead to stabilization of the real part of dilaton. On the other hand, in Sec. 6.2.2, the dilaton is stabilized by the gaugino condensation term. In both cases, the volume moduli can be stabilized by the world-sheet instanton effect. The structure of "Swiss-Cheese" Calabi-Yau manifold plays an essential role to stabilize the other real parts of Kähler moduli. Furthermore, the anomalous $U(1)$ gauge bosons absorb the imaginary parts of the moduli except for the axion-inflaton, and they become massive. As a result, one can stabilize the moduli fields expect for the inflaton and the obtained scalar potential is the form of natural inflation with trans-Planckian axion decay constant. In the next chapter, we discuss the natural inflation in type II string theory.

Chapter 7

Natural inflation with and without modulations in type IIB string theory

In this chapter, we take into account the type IIB string theory with an emphasis on axion dynamics, in particular, the axion inflation based on Ref. [143]. In the same way as in the case of heterotic string theory, we show that the decay constant of axion associated with the complex structure moduli is enhanced to the trans-Planckian value due to the moduli-dependent gauge threshold corrections. Thus, it is expected to identify the inflaton as the imaginary part of the complex structure moduli. As a simple setup to study such threshold corrections, we consider the internal space as toroidal orientifold or orbifold.

This chapter is organized as follows. First of all, we briefly review the moduli-dependent gauge threshold corrections in type IIB string theory. The open strings stretched between D-branes in the $\mathcal{N} = 2$ bulk induces the moduli-dependent corrections for the gauge coupling. In Sec. 7.2, we show the moduli stabilization except for the inflaton sector. In contrast to the heterotic string theory, the three-form flux-induced potential generates the potential of dilaton and complex structure moduli. However, it depends on the quanta of three-form fluxes. In our setup, certain linear combinations of dilaton and the complex structure moduli remain massless at the perturbative level.

Next, we consider the remaining moduli stabilization by the non-perturbative effects such as the racetrack scenario [144] adopted in Secs. 7.2.1, 7.2.2 and the Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [145] adopted in Sec. 7.2.2. Then, one can stabilize the linear combination of dilaton and complex structure modulus, and Kähler modulus. At the same time, the real part of complex structure moduli paired with the axion-inflaton can be also stabilized due to the nonvanishing superpotential terms.

Finally, from the other non-perturbative effects, one can extract the effective scalar potential of remaining massless axion. The obtained axion potential is a type of natural inflation in the large complex structure moduli limit, where the vacuum expectation values of complex structure moduli are much larger than 1 in the string unit. Since its decay constant is enhanced by the loop-corrections, the successful natural inflation is realized to be discussed in Sec. 7.3. On the other hand, in the region away from the large complex structure moduli limit, it is found that there are the modulation terms to the original scalar potential of natural inflation as shown in

Sec. 7.4. Finally, we summarize these contents in Sec. 7.5.

7.1 Moduli-dependent threshold corrections

Along the line of Refs. [146, 147], we summarize the fact that the gauge couplings on D-branes receive the one-loop gauge threshold corrections. Let us consider the non-abelian gauge theory with the gauge group G_a living on D-branes. At the scale μ below the string scale M_s, the running gauge coupling ($g_a(\mu)$) on D-branes is given in terms of 4D gauge coupling at the string scale M_s (g_a) and the beta-function coefficient of the gauge group G_a,

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2} + b_a \ln\left(\frac{M_s^2}{\mu^2}\right) + \frac{\Delta_a}{16\pi^2}. \tag{7.1}$$

In type II string theory, Δ_a stands for the one-loop corrections originated from the open strings between two stacks of D-branes or O-planes. Such correctoins are mostly moduli-dependent [148].

In particular, the stringy threshold corrections are explicitly calculated by means of CFT technique. The authors of [146] calculated their detailed form in type IIA string theory on toroidal orientifold or orbifold in the existence of O-planes and D6-branes which wrap a certain supersymmetric three-cycle of tori. There is some studies on the local cycle in type IIB/F-theory in the existence of fractional D-branes [149, 150]. As he T-dual system of type IIA string theory, one can two brane systems depend on the choice of T-duality. One is the D5/D9-branes system and other is D3/D7-branes system that will be concentrated in this chapter. Since there are O3/O7-planes in addition to the D3/D7-branes, the gauge coupling on D7-branes with the gauge group G_a receives the threshold corrections from the $\mathcal{N}=2$ SUSY sector in this system. Its explicit form is provided by

$$\Delta_a = -\sum_c b_{ac}^{N=2}\left[\ln|\eta(i\,U^k)|^4 + \ln\left(\mathrm{Re}\,U^k\frac{|p_a^k + i\,q_a^k \mathrm{Re}\,T^k|^2}{\mathrm{Re}\,T^k}\right)\right], \tag{7.2}$$

up to the regularization constant, where $\eta(i\,U^k)$, $(k = 1,2,3)$ is the Dedekind eta-function depends on not the Kähler moduli (T^k), but three complex structure moduli U^k. (p_a^k, q_a^k) denote the wrapping numbers on three two-tori and $b_{ac}^{N=2}$ are the beta-function coefficients labeled by the a-stack of D7-branes and the other c-stack of D-branes. The contribution from the charged massive open strings stretched between both branes is counted by the summation over c.

Since the gauge kinetic function is a holomorphic function, correspondingly only holomorphic corrections appear in the gauge kinetic function on D7-branes [151]. Thus, the first term on the right-handed side of Eq. (7.2) contributes to it,

$$f_a^{1-\mathrm{loop}} = -\frac{1}{4\pi^2}\sum_c b_{ac}^{N=2}\ln\left(\eta(i\,U^k)\right). \tag{7.3}$$

In particular, the Dedekind eta-function in Eq. (7.3) is approximately given in the large complex structure moduli limit ($|U^k| \gg 1,\ k=1,2,3$),

$$\eta(i\,U^k) \to e^{-\frac{\pi}{12}U^k}. \tag{7.4}$$

Note that in such a large complex structure moduli limit, the instanton effects for the Kähler and superpotential are suppressed. In the case away from the large complex structure limit, we have to include a more general form of Dedekind eta-function in Eq. (7.2) that is discussed later. In summary, the gauge kinetic function on D7-branes is expressed by including the threshold correction,

$$f_a \simeq \sum_i \frac{T^i}{4\pi} + \sum_j \frac{b^j}{48\pi} U^j, \tag{7.5}$$

where the summations of Kähler and complex structure moduli depends on the wrapping cycle of D7-branes and b^j characterize the effects of charged massive string. Here we consider the gauge kinetic function without dilaton dependence. When the two-form fluxes inserted along the wrapping cycle of D7-branes, the gauge kinetic function has a dilaton dependence. However, we omit such a dilaton dependent term, because it is irrelevant in our moduli stabilization and inflation as shown later.

7.2 Moduli stabilization on toroidal orientifold or orbifold

Let us proceed to the details of moduli stabilization on the basis of type IIB string theory on toroidal orientifold or orbifold such as T^2/Z_2 or $T^2/(Z_2 \times Z_2)$ with D3/D7-branes.

Contrary to the heterotic string theory on Calabi-Yau three-fold in Chapter 6, in the type IIB string theory, there are R-R three-form flux F_3 and NS-NS three-form flux H_3. These three-form fluxes generate the dilaton S and complex structure moduli U^k dependent potential in the 4D effective theory. This kind of flux-induced potential is formulated in the language of superfield by Gukov-Vafa-Witten superpotential [152],

$$W_{\rm flux} = \int_{\rm CY} G_3 \wedge \Omega, \tag{7.6}$$

where Ω is the holomorphic three-form of the CY manifold and $G_3 = F_3 - i\,SH_3$ is the three-form flux. In general, such fluxes yield the potential of the dilaton and all complex structure moduli and they could be stabilized at the perturbative level [153].

As an example, when we take into account the extra-dimensional space as T^2/Z_2 or $T^2/(Z_2\times Z_2)$, moduli fields are categorized into dilaton S, three complex structure moduli U^k and Kähler moduli T^k with $k=1,2,3$. In what follow, we study the overall Kähler modulus $T=T^1=T^2=T^3$ for simplicity. Our following analysis can be applied to the more general case of three Kähler moduli. In the next section, we show the detail of their moduli stabilization.

7.2.1 Moduli stabilization at the perturbative level

First of all, we begin with the stabilization of the dilaton (S) and complex structure moduli (U^1, U^2, U^3) in terms of the three-form flux (7.6). By dimensionally reducing the system, their Kähler potential and superpotential is written as

$$\begin{aligned} K &= -\ln(S+\bar{S}) - \sum_{i=1}^{3} \ln(U^i + \bar{U}^i), \\ W_{\rm flux} &= w_1 + iw_2\,(U^1 - U^2) + iw_3\,U^3 + iw_4\,S + w_5 U^3\,(U^1-U^2) + w_6 S\,U^3 + w_7 S\,(U^1-U^2) \\ &\quad + iw_8 S U^3 (U^1 - U^2), \end{aligned} \tag{7.7}$$

in the reduced Planck unit, $M_{\rm Pl} = 1$, where the three-form fluxes are specified in Eq. (7.7) with w_m ($m = 1, 2, \ldots, 8$) being integers originated from the quantization conditions of the R-R and NS-NS fluxes. The above ansatz plays an important role for the moduli stabilization as discussed later.

Before going to discuss the potential with the Kähler and superpotential (7.7) we redefine one of the complex structure modulus as

$$U^4 = U^1 - U^2, \tag{7.8}$$

which leads to following Kähler potential and superpotential,

$$\begin{aligned} K &= -\ln(S+\bar{S}) - \ln(U^2+\bar{U}^2) - \ln(U^3+\bar{U}^3) - \ln(U^4+\bar{U}^4+U^2+\bar{U}^2), \\ W_{\rm flux} &= w_1 + iw_2\,U^4 + iw_3\,U^3 + iw_4\,S + w_5 U^3\,U^4 + w_6 S\,U^3 + w_7 S\,U^4 + iw_8 S U^3 U^4. \end{aligned} \tag{7.9}$$

in the field base S, U^2, U^3 and U^4.

These moduli fields are stabilized at the minimum satisfying the supersymmetric condition, i.e., the extremal condition,

$$D_I W = 0, \tag{7.10}$$

for $\Phi^I = S, U^2, U^3$ and U^4. Moreover, we search for the supersymmetric Minkowski minimum $W = 0$. Then, the above stabilization condition (7.10) can be summarized as

$$W_S = W_{U^3} = W_{U^4} = W = 0. \tag{7.11}$$

Under the following ansatz of R-R and NS-NS fluxes

$$w_1 = w_2\,w_6,\;\; w_3 = -w_5\,w_6,\;\; w_4 = -w_6\,w_7,\;\; w_8 = 1, \tag{7.12}$$

the expectation values of S, U^3 and U^4 are obtained so as to satisfy the Eq. (7.11),

$${\rm Re}\,U^3{\rm Re}\,S = -(w_2 + w_5\,w_7),\;\; {\rm Re}U^4 = 0,\;\; {\rm Im}\,U^3 = w_7,\;\; {\rm Im}\,U^4 = w_6,\;\; {\rm Im}\,S = w_5. \tag{7.13}$$

At this supersymmetric Minkowski minimum, we find that their mass-squared matrices in the field basis (U^4, U^3, S) become

$$m_S^2 = \begin{pmatrix} K^{U^3\bar{U}^3}|W_{U^3U^4}|^2 + K^{S\bar{S}}|W_{SU^4}|^2 & 0 & 0 \\ 0 & K^{U^4\bar{U}^4}|W_{U^3U^4}|^2 & K^{U^4\bar{U}^4}W_{U^3U^4}\bar{W}_{\bar{S}\bar{U}^4} \\ 0 & K^{U^4\bar{U}^4}\bar{W}_{\bar{U}^3\bar{U}^4}W_{SU^4} & K^{U^4\bar{U}^4}|W_{SU^4}|^2 \end{pmatrix}, \tag{7.14}$$

whose rank is 2. Thus, U^4 and the linear combination of S and U^3 are stabilized at the supersymmetric Minkowski minimum with some appropriate choices of the integers w_m. For the supersymmetry breaking minimum, the authors of Ref. [154] pointed out the possibilities of one massless axion in the light of F-term axion monodromy inflation. So far, we do not canonically normalize these moduli field. These canonically normalization is summarized in the Appendix C.

We comment on the tadpole condition which severely constrains the string model building. Although, with the above choice of R-R and NS-NS fluxes, one would not realize the tadpole condition among themselves, the following moduli inflation scenario is irrelevant to the structure of tadpole condition.

7.2.2 Moduli stabilization at the non-perturbative level

Next, we concentrate on the remaining massless moduli. They are complex structure modulus U^2, the linear combination of S and U^3 and overall Kähler modulus T. The Kähler potential of single overall Kähler modulus T is described in the large volume limit,

$$K = -3\ln(T + \bar{T}). \tag{7.15}$$

where we do not consider the world-sheet instanton effects, that possibilities are discussed in a separate paper. The non-perturbative corrections to the superpotential such as the gaugino condensation on D3/D7-branes are introduced to stabilize the Kähler modulus T, complex structure modulus $\mathrm{Re}\,U^2$ and dilaton S,

$$W_{\mathrm{non}} = A(U)e^{-\frac{8\pi^2 f_1}{N_1}} - B(U)\,e^{-\frac{8\pi^2 f_2}{N_2}} + C(U)e^{-\frac{8\pi^2 f_3}{N_3}} - D(U)\,e^{-\frac{8\pi^2 f_4}{N_4}}, \tag{7.16}$$

where f_1 (f_2) represents the gauge kinetic function of pure $SU(N_1)$ ($SU(N_2)$) gauge theory on two stacks of D7-branes,

$$f_1 = f_2 = \frac{T}{4\pi} + \frac{b^2 U^2}{48\pi}. \tag{7.17}$$

To simplify our discussion, it is then assumed that both gauge kinetic functions receive the same U^2-dependent threshold corrections characterized by beta-function coefficient b^2. On the other hand, the latter parts are originated from pure $SU(N_3) \times SU(N_4))$ gauge theories on two stacks of D3-branes at the orbifold fixed points with the following gauge kinetic functions f_3 and f_4,

$$f_3 = f_4 = \frac{S}{4\pi}. \tag{7.18}$$

Even if the prefactors in the gaugino condensation terms ($A(U)$, $B(U)$, $C(U)$, $D(U)$) depend on the heavy complex structure moduli stabilized at the tree-level (7.6), there will no sizable influences for the following discussion of moduli stabilization. Then, these prefactors are treated as constants and neglect the fluctuations of these heavy complex structure moduli around their minimum (7.13).

To brighten the outlook for stabilizing the moduli fields, we first redefine the Kähler modulus as

$$\tilde{T} = T + \frac{b^2}{12} U^2. \tag{7.19}$$

In these field basis ($\tilde{T}$, S), the Kähler modulus $\tilde{T}$ and dilaton S are stabilized along a similar step to the the racetrack scenario [144]. Their supersyummetric conditions

$$\begin{aligned} D_{\tilde{T}} W_{\text{non}} &= (W_{\text{non}})_{\tilde{T}} + K_{\tilde{T}} W_{\text{non}} = 0, \\ D_S W_{\text{non}} &= (W_{\text{non}})_S + K_S W_{\text{non}} = 0, \end{aligned} \tag{7.20}$$

enable us to estimate the expectation values of Kähler modulus and dilaton,

$$\langle \tilde{T} \rangle \simeq \frac{N_1 N_2}{2\pi(N_2 - N_1)} \ln \frac{N_2\, A}{N_1\, B}, \quad \langle S \rangle \simeq \frac{N_3 N_4}{2\pi(N_4 - N_3)} \ln \frac{N_4\, C}{N_3\, D}, \tag{7.21}$$

at the racetrack minimum under the condition that $\langle W_{\text{non}} \rangle \ll \langle (W_{\text{non}})_{\tilde{T}} \rangle, \langle (W_{\text{non}})_S \rangle$. Such a relationship is satisfied in the following numerical analysis. The racetrack superpotential for S stabilizes the certain linear combination of S and U^3, whereas the orthogonal direction is already stabilized by the flux-induced superpotential as discussed in Sec. 7.2.1. Now, we choose $w_5 = 0$ in the superpotential (7.9) so that the stabilized point of $\text{Im}\, S$ is the same as that given in Eq. (7.13).

Next, we focus on the massless complex structure modulus U^2. Since we rotate the field basis of Kähler modulus as in Eq. (7.19), the Kähler potential is also changed as

$$K = -\ln(U^2 + \bar{U}^2) - \ln(U^4 + \bar{U}^4 + U^2 + \bar{U}^2) - 3\ln(\tilde{T} + \bar{\tilde{T}} - \frac{b^2}{12}(U^2 + \bar{U}^2)). \tag{7.22}$$

,whereas the superpotential do not include the potential of U^2. Therefore, the extremal condition $V_{U^2} = \partial V / \partial U^2 = 0$ is satisfied under the following condition:

$$K_{U^2} = -\frac{1}{U^2 + \bar{U}^2} - \frac{1}{U^4 + \bar{U}^4 + U^2 + \bar{U}^2} + \frac{b^2}{4} \frac{1}{\tilde{T} + \bar{\tilde{T}} - \frac{b^2}{12}(U^2 + \bar{U}^2)} = 0, \tag{7.23}$$

which leads to the expectation value of $\mathrm{Re}\, U^2$ by employing $\langle \mathrm{Re}\, U^4 \rangle = 0$,

$$\mathrm{Re}\, U^2 = \frac{24 \langle \mathrm{Re}\, \tilde{T} \rangle}{5b^2}. \tag{7.24}$$

In the above analysis, we assume that U^4 and the linear combination of U^3 and S are replaced by constants given by the supersymmetric conditions $D_{U^3} W = D_{U^4} W = D_S W = 0$, when we evaluate the stabilization of light moduli $\tilde{T}$, S and $\mathrm{Re}\, U^2$. Furthermore, we neglect the deviations and fluctuations of heavy moduli from the minimum given by Eq. (7.13). This assumption is ensured when the gaugino condensation scale is small compared with the mass scales of heavy moduli in Sec. 7.2.1.

The stabilization of $\mathrm{Re}\, U^2$ is checked by evaluating the rank of the mass matrices for U^2, U^3, U^4, S and $\tilde{T}$. It turns out that the squared mass of $\mathrm{Re}\, U^2$ is positive at the uplifted vacuum as shown later. This is because the gaugino condensation scale determined by the superpotential (7.16) is suppressed from the mass scales of heavy moduli such as U^4 and the linear combination of U^3 and S. They are summarized in Appendix C.

So far, the non-vanishing superpotential $\langle W \rangle \neq 0$ gives rise to the negative vacuum energy at the supersymmetric minimum $D_I W = 0$ for $I = U^2, U^3, U^4, S$ and $\tilde{T}$. In a similar way to the case of heterotic string theory, we assume that the AdS vacuum is uplifted to the Minkowski minimum by some uplifting sector as suggested in Ref. [145],

$$V = V_F + V_{\mathrm{up}}, \tag{7.25}$$

where V_F denotes the F-term scalar potential obtained in the above analysis. It is possible to construct the uplifting potential V_{up} in type IIB string theory by setting anti-D3-branes [145] on warped throat and some dynamical SUSY-breaking sectors [140, 32, 64, 141]. In the next section, we move on to the dynamics of the light axion $\mathrm{Im}\, U^2$ that corresponds to the inflaton field.

Finally, we show an another possibility to stabilize the Kähler modulus T. Until now, we perform the racetrack scenario [144] for the stabilization of Kähler modulus, whereas, for the case of KKLT scenario [145], its stabilization is achieved by single non-perturbative correction and a tiny constant value in the superpotential. In such a case, one can also derive the same inflaton potential for $\mathrm{Im}\, U^2$ as will be shown in the next section, since the obtained inflaton potential is irrelevant to the detail of Kähler moduli stabilization. One way to realize the tiny constant value of superpotential in the KKLT scenario is to tune R-R and NS-NS fluxes in such a way that the tiny expectation value of superpotential $\langle W \rangle < 10^{-2}$ is realized so as to be compatible with the large-volume limit of T as shown in Eq. (7.15). Furthermore, the energies of these fluxes may spoil the background solution and the product of three-tori is modified to complicated geometry such as non-Kähler manifold so as to keep the stability of system. Even when the supersymmetric fluxes are turn on, the D-branes and O-planes required from the tadpole cancellation [153] may give the backreactions to the geometry. From this perspective, these backreactions induced by the three-form fluxes and the source of branes are further assumed to be negligible in the relevant sector of moduli stabilization.

We again remark that the stabilization of other Kähler moduli. Although we have taken int account the single overall Kähler modulus T, the other Kähler moduli T_i $(i = 1, 2)$ could be stabilized by the similar step as in the case of T. It can be realized by the another non-perturbative corrections to the superpotential such as the gaugino condensation on D7-branes wrapping the irrelevant cycle associated with the modulus T. Let us demonstrate the above statements. When the gauginos of $SU(M_1^{(i)})$ and $SU(M_2^{(i)})$ gauge theories condensate at a scale heavier than the those for the modulus T, the superpotential is generated as

$$W = \sum_i A_i(U) e^{-\frac{8\pi^2 f_1^{(i)}}{M_1^{(i)}}} - B_i(U)\, e^{-\frac{8\pi^2 f_2^{(i)}}{M_2^{(i)}}}, \tag{7.26}$$

where $f_1^{(i)} = a_1 T_i$ and $f_2^{(i)} = a_2 T_i$ with a_1, a_2 being constants and these prefactors $A_i(U)$ and $B_i(U)$ depend on only the heavy complex structure moduli stabilized at the tree-level (7.6). Then, T_1 and T_2 are also stabilized at the racetrack minimum.

7.3 Natural inflation without modulations

Let us proceed to discuss the inflaton potential. First of all, we set an another $SU(L)$ gaugino-condensation term on D7-brane wrapping the cycle associated with T,

$$W \supset E(\langle U\rangle) e^{-\frac{2\pi}{L}\langle T\rangle - \frac{b\pi}{6L}\langle \mathrm{Re}\, U^2\rangle - i\frac{b\pi}{6L}\mathrm{Im}\, U^2}, \tag{7.27}$$

where the gauge coupling on $SU(L)$ gauge theory receives the U^2-dependent threshold corrections. The other threshold corrections from the heavy complex structure moduli, U^3 and U^4 are included in the prefactor of gaugino condensation term $E(\langle U\rangle)$. Here, the rank of the $SU(L)$ and $SU(N_i)$ $(i = 1, 2, 3, 4)$ gauge theories are chosen as $L < N_i$ with $i = 1, 2, 3, 4$ to ensure the reliability of our calculation. In this setup, one can analyze only the dynamics of $\mathrm{Im}\, U^2$, because all the other moduli fields are much heavier than $\mathrm{Im}\, U^2$ due to the flux-induced superpotential (7.6) and the high-scale gaugino-condensation terms (7.16). Under $L \simeq N_i$, the other moduli discussed in the previous section would not be fixed at the minimum of potential given by Eqs. (7.13) and (7.21) and one cannot neglect their fluctuations around their vacuum expectation values.

As a result, one can extract the effective scalar potential for $\mathrm{Im}\, U^2$ from Eq. (7.27) by setting certain uplifting sector (7.25),

$$V_{\mathrm{eff}} = \Lambda_1 \left(1 - \cos\left(\lambda_1 \phi\right)\right), \tag{7.28}$$

where $\Lambda_1 \simeq 6 e^{\langle K\rangle}\langle W_{\mathrm{non}}\rangle E(\langle U\rangle) e^{-\frac{2\pi}{L}\langle T\rangle - \frac{b\pi}{6L}\mathrm{Re}\langle U^2\rangle}$ and $\lambda_1 = b\pi/6d\, L$. Now, we canonically normalize the axion as $\phi = d\, \mathrm{Im}\, U^2$ where the canonical normalization factor $d \simeq 1/\langle \mathrm{Re}\, U^2\rangle$ depends on the relevant complex structure moduli. It is summarized in Appendix C. Although there are kinetic mixing between U^2, U^4 and $\tilde{T}$ as can be seen in the Kähler potential (7.22), their mixing terms are negligible on the inflaton dynamics. This is because other fields expect for $\mathrm{Im}\, U^2$ are already fixed at their minmum.

By identifying the inflaton as ϕ, the axion potential is just that of natural inflation as mention before in Chapter 5. The desired trans-Planckian axion decay constant can be obtained by the enhancement factor as seen in the scalar potential (7.28). In addition to the inverse of one-loop factor, the ratio b/L and the vacuum expectation value $\langle \mathrm{Re}\, U^2 \rangle$ are dominant sources to enhance the axion decay constant that leads to the successful cosmological observables reported by Planck [15, 16]. We stress that the beta-function coefficient b in $\mathcal{N} = 2$ sector is irrelevant to the dynamics of $SU(L)$ gauge sector.

The cosmological observables such as the power spectrum of the curvature perturbation P_ξ, its spectral tilt n_s, running of the n_s $dn_s/d\ln k$ and the tensor-to-scalar ratio r are are estimated by employing the slow-roll parameters for the inflaton ϕ,

$$\begin{aligned}\epsilon &= \frac{1}{2}\left(\frac{\partial_\phi V_{\rm eff}}{V_{\rm eff}}\right)^2 = \frac{(\lambda_1)^2}{2}\frac{1-\cos^2(\lambda_1\,\phi)}{(1-\cos(\lambda_1\,\phi))^2},\\ \eta &= \frac{\partial_\phi\partial_\phi V_{\rm eff}}{V_{\rm eff}} = (\lambda_1)^2\frac{\cos(\lambda_1\,\phi)}{1-\cos(\lambda_1\,\phi)},\\ \xi &= \frac{\partial_\phi V_{\rm eff}\partial_\phi\partial_\phi\partial_\phi V_{\rm eff}}{V_{\rm eff}^2} = -(\lambda_1)^4\frac{1-\cos^2(\lambda_1\,\phi)}{(1-\cos(\lambda_1\,\phi))^2},\end{aligned} \tag{7.29}$$

An amount of e-folding is also evaluated from

$$N_e = \int_{\phi_{\rm end}}^{\phi_*}\frac{V_{\rm eff}}{\partial_\phi V_{\rm eff}}d\phi, \tag{7.30}$$

where ϕ_* and $\phi_{\rm end}$ stand for the field values at the pivot scale and the end of inflation, respectively. The inflation ends when the slow-roll condition is violated, $\max\{\epsilon,\eta,\xi\} = 1$.

In the following numerical analysis, we choose the representative parameters in the superpotential given by Eqs. (7.9), (7.21), (7.27) and the Kähler potential given by Eq. (7.22) as follows,

$$\begin{aligned}&w_5 = 0,\;\; w_6 = 1,\;\; w_2 = -8,\;\; w_7 = -3,\;\; N_1 = N_3 = 12,\;\; N_2 = N_4 = 20,\;\; L = 10,\;\; b = 1,\\ &b^2 = 12,\;\; A = -8,\;\; B = -3,\;\; C = 9,\;\; D = 3,\;\; E = \frac{1}{12},\end{aligned} \tag{7.31}$$

and the other parameters in Eq. (7.9) are fixed to satisfy Eq. (7.12). These parameters are fixed such that the correct order of power spectrum of curvature perturbation is generated. Also, these parameters give rise to vacuum expectation values of moduli fields,

$$\begin{aligned}&\langle \mathrm{Re}\, U^1\rangle \simeq \langle \mathrm{Re}\, U^2\rangle \simeq 2.8,\;\; \langle \mathrm{Re}\, U^3\rangle \simeq 1,\;\; \langle \mathrm{Im}\, U^1\rangle \simeq 1,\;\; \langle \mathrm{Im}\, U^2\rangle \simeq 0,\;\; \langle \mathrm{Im}\, U^3\rangle \simeq -3,\\ &\langle \mathrm{Re}\, S\rangle \simeq 7.7,\;\; \langle \mathrm{Im}\, S\rangle \simeq 0,\;\; \langle \hat{T}\rangle \simeq 7.1.\end{aligned} \tag{7.32}$$

By solving the equation of motion of inflaton field with the above set of parameters, we find that the cosmological observables with an enough amount of e-folding $N_e \simeq 61$ are

$$n_s \simeq 0.963,\;\; r \simeq 0.06,\;\; dn_s/d\ln k \simeq -8\times 10^{-4}, \tag{7.33}$$

consistent with WMAP and Planck data [15, 16] as shown in Eq. (6.46).

We again remark that, the gauge coupling receives the moduli-dependent threshold correction induced by the massive open-string. The axon appear through such corrections have the trans-Planckian axion decay constant due to the enhancement of one-loop factor in Eq. (7.18). It is the model-independent feature in string theory and higher-dimensional theory. Thus, when we consider certain non-perturbative effects depending on such the particular axion, it gives rise to a successful natural inflation and one can obtain several tensor-to-scalar ratio which is of $\mathcal{O}(0.01-0.1)$ in our framework.

7.4 Modulated Natural inflation

In the previous analysis, we approximate the Dedekind eta-function in the threshold correction by its leading term in Eq. (7.4). It is an usually considered situation corresponding to the large field limit of complex structure moduli and in this limit, the obtained inflaton potential is just that of usual natural inflation with trans-Planckian decay constant.

In this section, we take a closer look at the next leading term in the Dedekind eta-function,

$$\eta(i\,U^2) \to e^{-\frac{\pi}{12}U^2}\Big[1-e^{-2\pi U^2}-\mathcal{O}(e^{-4\pi U^2})\Big], \tag{7.34}$$

that will deviate the minimum from that given in the large complex-structure limit, the correction terms are suppressed exponentially though. According to it, the inflaton potential receives the following corrections,

$$V_{\rm inf} = V_{\rm eff} + V_{\rm mod}, \tag{7.35}$$

where

$$V_{\rm mod} = \Lambda_2 \cos\left(\lambda_2 \phi\right), \tag{7.36}$$

with $\Lambda_2 = \Lambda_1\, \frac{2b}{L} e^{-\left(2\pi+\frac{b\,\pi}{6L}\right)\langle {\rm Re}\, U^2\rangle}$ and $\lambda_2 = (2\pi + b\,\pi/6L)/d$. Even though it is the next leading term in the scalar potential, the modulations [155, 156, 157] to the leading inflaton potential $V_{\rm eff}$ will appear as a consequence of the correction $V_{\rm mod}$. That effect depends on the vacuum expectation value $\langle {\rm Re}\, U^2\rangle$. In the previous analysis with the numerical values of parameters (7.31), such a correction is suppressed by the large field value of $\langle {\rm Re}\, U^2\rangle \gg 1$.

However, we should take into account the strong CP problem if the relevant axion couples o the QCD sector. When there is a tachyonic potential around the origin, $\phi = 0$, ϕ has the nonvanishing vacuum expectation value, which would lead to the unobservable θ-term. Let us estimate whether its tachyonic potential appears or not. From the axion mass-squared at the origin given by

$$\partial_\phi^2 V_{\rm inf}\Big|_{\phi=0} = (\lambda_1)^2\Lambda_1 - (\lambda_2)^2\Lambda_2, \tag{7.37}$$

its positivity is realized under the following condition,

$$(\lambda_1)^2\Lambda_1 - (\lambda_2)^2\Lambda_2 > 0 \;\leftrightarrow\; \left(\frac{\pi}{6}\right)^2 \frac{b}{L} > 2\left(2\pi + \frac{\pi\, b}{6L}\right)^2 e^{-2\pi\langle {\rm Re}\, U^2\rangle}. \tag{7.38}$$

Thus, in the following analysis, we concentrate on the region where the above inequality is satisfied.

Next, we discuss the cosmological effects, in particular, modulations from the additional scalar potential $V_{\rm mod}$. For the general scalar potential (7.35), the slow-roll parameters are found as

$$\begin{aligned}
\epsilon &= \frac{(\lambda_1\Lambda_1 \sin(\lambda_1\,\phi) - \lambda_2\Lambda_2 \sin(\lambda_2\,\phi))^2}{2\,V_{\rm inf}^2},\\
\eta &= \frac{(\lambda_1)^2\Lambda_1 \cos(\lambda_1\,\phi) - (\lambda_2)^2\Lambda_2 \cos(\lambda_2\,\phi)}{V_{\rm inf}},\\
\xi^2 &= -\frac{\lambda_1\Lambda_1 \sin(\lambda_1\,\phi) - \lambda_2\Lambda_2 \sin(\lambda_2\,\phi)}{V_{\rm inf}} \times \frac{(\lambda_1)^3\Lambda_1 \sin(\lambda_1\,\phi) - (\lambda_2)^3\Lambda_2 \sin(\lambda_2\,\phi)}{V_{\rm inf}},
\end{aligned} \tag{7.39}$$

In addition, the spectral index n_s including the next-leading order of slow-roll approximation are given by

$$n_s = 1 + 2\eta - 6\epsilon + 2\left[-\left(\frac{5}{3} + 12C\right)\epsilon^2 + (8C-1)\epsilon\,\eta + \frac{1}{3}\eta^2 - \left(C - \frac{1}{3}\right)\xi^2\right] + \cdots, \tag{7.40}$$

where $C = -2 + \ln 2 + \gamma$ is the numerical factor written in terms of the Euler-Mascheroni constant $\gamma \simeq 0.577$. The ellipsis includes more higher corrections about slow-roll parameters, e.g., fourth derivative with respect to the inflaton field as discussed in Ref. [26]. Eq. (7.40) implies that ,in some parameter regions, a sizable $\xi^2 = \mathcal{O}(0.01)$ will contribute to the numerical value of spectral tile n_s. On the other hand, the authors of Refs. [158, 159] discuss the higher-order corrections to P_ξ which do not induce sizable effects. It is remarkable that such a modification from the natural inflation is controlled by the field value of complex structure modulus ($\langle {\rm Re}\, U^2\rangle$) and the ratio of beta-function coefficient and rank of gauge group (b/L) in the superpotential (7.9).

In the following numerical analysis, we focus on a particular value $\langle {\rm Re}\, U^2\rangle \simeq 1$ by setting certain numerical values of parameters different from Eq. (7.31). A non-negligible $V_{\rm mod}$ gives a different prediction of cosmological observables from that of natural inflation. In particular, several values of b/L and $\langle {\rm Re}\, U^2\rangle$ drastically change the cosmological observables r, n_s, $dn_s/d\ln k$ as shown later. The power spectrum of curvature perturbation P_ξ can be fixed as the observed value 2.2×10^{-9} by suitably choosing overall scale of inflaton potential, i.e., the gaugino-condensation terms in Eq. (7.21). As drawn in Fig. 7.1, for the universal value of $\langle {\rm Re}\, U^2\rangle = 1$ with $b/L = 1/10, 1/5, 1/4, 1/3, 1/2$, one can predict several size of spectral index n_s and the tensor-to-scalar ratio r within enough amount of e-folding numbers, $50 \leq N_e \leq 60$. The oscillating curves in Fig. 7.1 are originating from the oscillations of slow-roll parameters through the modulation term in the scalar potential. In Fig. 7.2, we set $b/L = 1/5$ $(1/10)$ and $\langle {\rm Re}\, U^2\rangle = 1.2$ (2.4) in the left (right) panel such that the leading scalar potential $V_{\rm eff}$ has the same structure. However, in the left panel with $\langle {\rm Re}\, U^2\rangle \simeq 1$, the next-leading scalar potential $V_{\rm mod}$ provides significant contributions compared with the right panel. With $b/L = 1/5$ and $\langle {\rm Re}\, U^2\rangle = 1.2$, the scalar potential with and without such modulations is shown in Fig. 7.3, although there is no significant difference between them. If we consider the region of $\langle {\rm Re}\, U^2\rangle < 1$, the next-to-next leading order of the Dedekind functions should be add in the potential. In such a parameter space, we should include the instanton correction to the Kähler potential and superpotential. In Tab. 7.1, we summarize the predictions of cosmological observables.

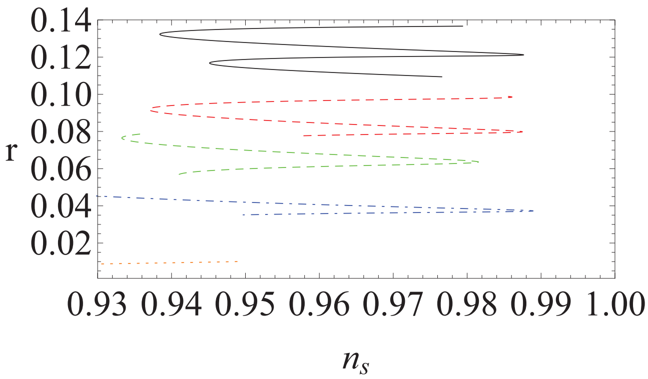

Figure 7.1: Predictions of spectral tilt of curvature perturbation n_s and tensor-to-scalar ratio r within the range of e-folding number, $50 \leq N_e \leq 60$ in Ref. [143]. For the universal value of $\langle \mathrm{Re}\, U^2 \rangle = 1$, black-solid, red-dashed, green-dashed, blue-dot-dashed and orange-dotted curves represent the fixed ratios $b/L = 1/10, 1/5, 1/4, 1/3, 1/2$, respectively.

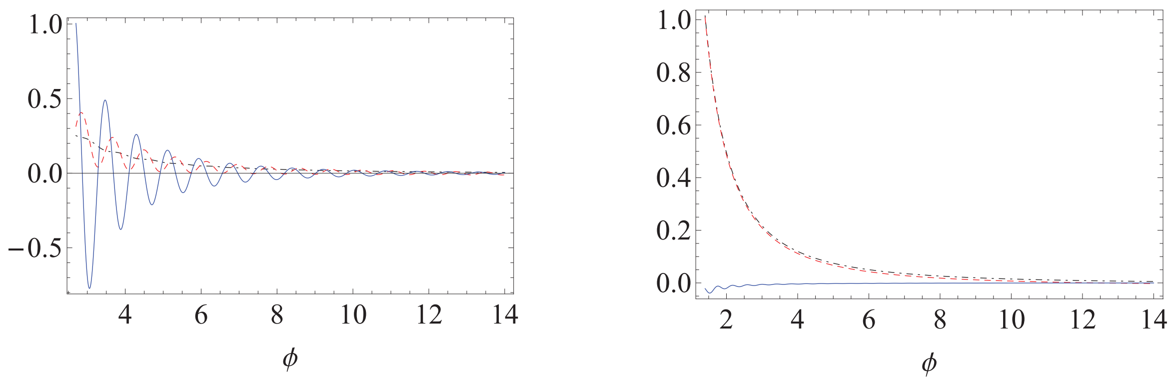

Figure 7.2: The three slow-roll parameters, ϵ, η and ξ^2 as a function of inflaton field ϕ. ϵ, η and ξ^2 are in correspondence with black-dot-dashed, red-dashed and blue-solid curves, respectively [143]. We set the numerical values of parameters as $b/L = 1/5$ $(1/10)$ and $\langle \mathrm{Re}\, U^2 \rangle = 1.2$ (2.4) in the left (right) panel.

As a result, one can obtain several values of the tensor-to-scalar ratio r and allowed spectral index $n_s \simeq 0.96$ reported by Planck which is sensitive to the value of complex-structure modulus, $\langle \mathrm{Re}\, U^2 \rangle$. This intersecting phenomena is seen not in an original natural inflation model [29], but in a modulated natural inflation discussed here. It is governed by the structure of Dedekind eta-function. In the large field limit of complex structure moduli $\langle \mathrm{Re}\, U^2 \rangle \gg 1$, our inflaton potential reduces to the original natural inflation [29], whereas, in $\langle \mathrm{Re}\, U^2 \rangle \simeq 1$, modulation terms appear in the scalar potential. Such modulation effects are also discussed in the multi-natural inflation scenario [71]. In a near-future, it is expected that future precise cosmological observations select certain values of cosmological observables.

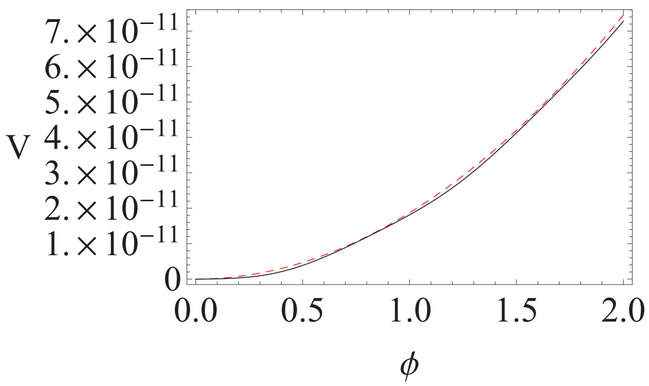

Figure 7.3: The scalar potential V as a function of the inflaton field ϕ shown in Ref. [143]. In a similar way to Fig. 7.2, the black-solid curve denotes the scalar potential (7.35) with modulations by setting the parameters $b/L = 1/5$ and $\langle \mathrm{Re}\, U^2 \rangle = 1.2$, whereas the red-dotted curve denotes the leading scalar potential (7.28) without modulations for the same parameters.

7.5 Summary

In this chapter, we discussed the natural inflation in the framework of type IIB string theory on toroidal orientifold or orbifold. The trans-Planckian decay constant required in the successful natural inflation is achieved by one-loop threshold corrections for the gauge coupling. In type IIB string theory, there is an explicit calculation for such quantities on the toroidal orientifold or orbifold background [146, 147]. In general, such corrections are moduli-dependent. When one of the moduli appearing in the threshold correction is identified as the inflaton, its decay constant can be enhanced by the inverse of one-loop factor. As shown in this chapter, these gauge threshold corrections are phenomenologically important not only for the gauge coupling unification, but also for the cosmic inflation.

On toroidal background, the moduli-dependent correction is characterized by the Dedekind eta-function which respect the modular symmetry of torus. In our model, the axion paired with the complex structure modulus plays the role of inflaton. The other axions and moduli fields are stabilized at the minimum by three-form flux induced potential and racetrack superpotential. Then, they are decoupled from the inflaton dynamics. In the large complex-structure limit, $\langle \mathrm{Re}\, U^2 \rangle > 1$, the obtained form of inflaton potential reduces to that of natural inflation, whereas in $\langle \mathrm{Re}\, U^2 \rangle \simeq 1$, one has to treat the full Dedekind eta-function. Due to the analytic form of the function, cosmological observables such as tensor-to-scalar ratio and spectral tilt of curvature perturbations are better fitted with the Planck data. Indeed, both the small and large tensor-to-scalar ratios can be realized with the fixed spectral tilt of curvature perturbation. It is expected that the modulated spectrum of natural inflation could be detectable in the near-future experiments. Thus, one can obtain suggestive cosmological implications for string model building.

Although it is also interesting to extend our scenario to more general curved background such as Calabi-Yau manifold, we do not know the explicit form of one-loop threshold corrections. It is expected that some (discrete) symmetries would be preserved in the effective theory of

b/L	$\langle \mathrm{Re}\, U^2 \rangle$	N_e	n_s	r	$dn_s/d\ln k$
1/10	1.3	50	0.96	0.14	−0.0008
1/10	1.3	57	0.96	0.12	−0.012
1/5	1.2	55	0.96	0.08	−0.002
1/5	1.2	60	0.96	0.08	−0.001
1/4	1.2	53	0.96	0.07	−0.002
1/4	1.2	58	0.96	0.06	−0.001
1/3	1.1	54	0.96	0.04	−0.002
1/3	1.1	60	0.96	0.04	−0.001
1/2	1.1	50	0.95	0.01	−0.0003

Table 7.1: The input parameters b/L, $\langle \mathrm{Re}\, U^2 \rangle$ and the obtained results such as the e-folding number N_e, spectral tilt of curvature perturbation n_s, its running of spectral index $dn_s/d\ln k$ and tensor-to-scalar ratio r in Ref. [143].

type IIB string theory on curved background. By focusing on such symmetries, one would guess the moduli-dependent function appeared in the threshold correction. We leave it for a future work. If a lot of axions appear in the low-energy effective theory derived from the string theory, they would induce the isocurvature and the cross-correlated perturbations in addition to an adiabatic curvature perturbations induced by the inflaton. In Ref. [160], we have found that Planck analysis on the generally correlated isocurvature perturbations prefers the existence of the correlated isocurvature modes for the axion monodromy inflation in contrast to the situation in the natural inflation.

Chapter 8

Conclusions and discussions

In this thesis, we have taken into account the following two approaches to solve and explain the theoretical and phenomenological/cosmological problems of the standard model of particle physics. One of them is the "bottom-up approach" that explains these problems by minimally extending the standard model. In particular, we adopt the five-dimensional supergravity model compactified on S^1/Z_2 in the light of the extra-dimension and local SUSY. In Chapter 2, we investigated the cosmological aspects of 5D SUGRA with the emphasis on the moduli inflation. The moduli fields appear associated with extra-dimensional components of higher-dimensional vector and tensor fields. In our setup, the moduli fields are represented by chiral multiplets in the 4D effective theory, those are originated from 5D $U(1)$ Z_2-odd vector multiplets. As shown in this chapter, the parameters in modulus scalar potential as well as the kinetic term are severely constrained by the higher-dimensional Lorentz and gauge symmetries.

We have presented two types of successful inflation scenario in the single framework of phenomenological 5D supergravity model. One of them is the small-field inflation that is similar to the Starobinsky one [1], whereas the other one behaves like a natural inflation categorized as the large-field inflation. Both mechanisms are achieved by the potential induced by the $U(1)$ charged stabilizer fields. The stabilizer fields play essential roles of giving not only the desired inflaton potential, but also the supersymmetric moduli stabilization that enables us to integrate out the heavy moduli and stabilizer fields supersymmetrically. Thus, the obtained results are irrelevant to the dynamics of SUSY-breaking field and therefore it turns out that the SUSY-breaking scale is constrained to be lower than the inflation scale.

Based on the successful moduli inflation scenarios, we have studied the phenomenological as well as cosmological aspects of 5D SUGRA in Chapters 3 and 4, where the SUSY-breaking scale is taken as low- and high-scale, respectively. The low-scale SUSY-breaking scenario discussed in Chapter 3 is attractive scenario which not only protects the mass of the Higgs boson from the huge radiative corrections, but also gives the plausible dark matter candidate. By setting the $U(1)$ charge assignments of matter fields in the MSSM to realize the observed hierarchical mass matrices of quarks and leptons, one can explicitly determine the couplings between the SUSY-breaking field and matter fields. It is then possible to consider the stable gravitino in the suppressed Kähler metric of SUSY-breaking field, which is realized by mildly large volume of fifth dimension. The obtained results are consistent with not only the observed Higgs boson mass by LHC experiments, but also the BBN data in the case of sneutrino NLSP and higgsino NNLSP scenario.

In contrast to the discussion in Chapter 3, we also studied the high-scale SUSY-breaking scenario with wino LSP in Chapter 4. The relic abundance of wino-like neutralino is so model-dependent that one cannot determine it unless we specify the whole thermal history of the universe after the inflation. In Chapter 4, we have taken into account both the thermal and nonthermal processes to estimate the relic abundance of wino-like neutralino. As a result, it was found that its relic abundance depends on the gravitino mass. When ongoing LHC experiments and cosmological observations give an hint of the mass of wino LSP, one would judge whether the abundance of wino is originated from the thermal or nonthermal process. One would then extract the constraints on the inflation and SUSY-breaking sector in this scenario.

In Part I of this thesis, we have concentrated on the cosmological and phenomenological aspects of 5D supergravity model. However, when 5D supergravity is derived as a low-energy effective theory of ultraviolet theory such as heterotic M-theory and type IIB string theory on a warped throat, our results and discussions provide insights into such ultraviolet completions. Indeed, the norm function coefficients in the Kähler potential are related to the intersection number of CY manifold and the extra $U(1)$ symmetries can be originated from local symmetries in D-brane or M5-brane configurations. It is interesting to proceed in this direction that will be studied elsewhere.

In the second part of this thesis, we take an another approach called "top-down approach", in which we have studied certain ultraviolet theories towards the standard model. In particular, we have focused on the cosmological aspects of heterotic string theory on CY manifold in Chapter 6 and type IIB string theory on toroidal orientifold or orbifold in Chapter 7, respectively. In both scenarios, we have studied a natural inflation caused by the closed string axion. The recent Planck data suggested that the decay constant of axion-inflaton should be larger than the Planck scale. It is problematic in the string theory, because the fundamental axion decay constants are typically lower than the Planck scale. To overcome this problem, in both scenarios, we have studied the threshold corrections in the gauge coupling. When the axion-inflaton appears in the gauge kinetic function only through such threshold corrections, its decay constant can reach the trans-Planckian value. For the heterotic string theory discussed in Chapter 6, the correction terms are originated from the one-loop Green-Schwarz term that depends on the Kähler axions. We analyzed the system where the gauginos condensate in the hidden gauge group and one of the Kähler axion becomes an inflaton field. We have shown the stabilization of other moduli fields by employing the nature of "'Swiss-cheese" CY manifold.

In the case of type IIB string theory, the threshold corrections are also calculated in a simple toroidal orientifold or orbifold. However, in contrast to the heterotic string theory, the correction terms are written in terms of Dedekind eta-function depending on the complex structure moduli. When we consider the non-perturbative effects in the hidden gauge group, one of the axions in the complex structure moduli have a potential similar to that of natural inflation. Interestingly, the potential has a modulation term stemming from the Dedekind eta function. After performing the numerical calculation including the modulation terms, it turns out that the cosmological observables are more consistent with those predicted by the natural inflation without modulation. The near-future cosmological observations such as BICEP, Keckarray and

Planck have a potential to detect such modulation effects. Even in the case of general curved background, it is expected that some discrete symmetries protect the form of moduli-dependent threshold corrections and we would obtain the similar results.

In Part II of this thesis, we have obtained successful cosmic inflation scenarios within the framework of string theory. However, there are no phenomenologically desired string model, at present compatible with both the collider experiments and cosmological observations. By comparing the results in bottom-up and top-down approaches, we will reveal an underlying nature of the standard model of particle physics and cosmology in the future.

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Appendix A

The SUSY-breaking effects to the moduli inflation in 5D SUGRA

A.1 The fluctuations of fields around the vacuum

Along the line of Ref. [7], we summarize the deviations of moduli, stabilizer, and SUSY-breaking fields from the SUSY-preserving minimum to the SUSY-breaking minimum. The reference point method enables us to extract the fields values at the SUSY-breaking minimum under the expansion of fields around the reference points given in Eqs. (3.16) and (3.18). Then, the superpotential and Kähler potential are expanded as

$$
\begin{aligned}
W &\simeq w + W_X \delta X + \sum_{I'=i}^{3} W_{T^{I'} H_i} \delta T^{I'} \delta H_i, \\
K &\simeq \sum_{I'=1}^{3} \left(-\ln(\mathrm{Re}\, T^{I'}) - \frac{\mathrm{Re}\, \delta T^{I'}}{\mathrm{Re}\, T^{I'}} + \frac{1}{2} \left(\frac{\mathrm{Re}\, \delta T^{I'}}{\mathrm{Re}\, T^{I'}} \right)^2 \right) + \sum_{i=1}^{3} Z_{H_i} |\delta H_i|^2 + Z_X |\delta X|^2,
\end{aligned}
\tag{A.1}
$$

where $\delta\phi$, $\phi = T^{I'}, H_i, X$ with $I', i = 1, 2, 3$, denote the perturbations around the reference points. In the field basis $(T^1, T^2, T^3, H_1, H_2, H_3, X)$, the Kähler metric is obtained as

$$
K_{I\bar{J}} = K^{(0)}_{I\bar{J}} + K^{(1)}_{I\bar{J}}, \tag{A.2}
$$

where

$$
K^{(0)}_{I\bar{J}} = \begin{pmatrix}
1/(2\mathrm{Re}T^1)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/(2\mathrm{Re}T^2)^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/(2\mathrm{Re}T^3)^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z_{H_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z_{H_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & Z_{H_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z_X - 4|X|^2/\Lambda^2
\end{pmatrix}, \tag{A.3}
$$

and

$$K^{(1)}_{I\bar{J}} = \begin{pmatrix} 0 & 0 & 0 & a_{H_1}H_1 & 0 & 0 & a^1_X X \\ 0 & 0 & 0 & 0 & a_{H_2}H_2 & 0 & a^2_X X \\ 0 & 0 & 0 & 0 & 0 & a_{H_3}H_3 & a^3_X X \\ a_{H_1}\bar{H}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{H_2}\bar{H}_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{H_3}\bar{H}_3 & 0 & 0 & 0 & 0 \\ a^1_X\bar{X} & a^2_X\bar{X} & a^3_X X & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.4}$$

with

$$\begin{aligned} a_{H_i} &\equiv \frac{\partial Z_{H_i}}{\partial T^{I'}} = \frac{1}{\mathrm{Re}T^{I'}}\left(e^{-2c_{H_i}\mathrm{Re}T^{I'}} - \frac{Z_{H_i}}{2}\right), \ \ (I'=i), \\ a^i_X &\equiv \frac{\partial Z_X}{\partial T^{I'}} = \frac{c^i_X}{c_X\cdot \mathrm{Re}T}\left(e^{-2c_X\cdot ReT} - \frac{Z_X}{2}\right), \end{aligned} \tag{A.5}$$

The inverse matrix of Kähler metric is given by

$$\begin{aligned} &K^{T^{I'}\bar{T}^{I'}} \simeq (2\mathrm{Re}T^{I'})^2 + 8\mathrm{Re}T^{I'}\mathrm{Re}\,\delta T^{I'}, \ \ K^{T^{I'}\bar{H}_i} \simeq -A_{H_i}\delta H_i, \\ &K^{T^{I'}\bar{X}} \simeq -A^i_X X - A^i_X\delta X, \ \ K^{H_i\bar{H}_i} \simeq \frac{1}{Z_{H_i}} - \frac{2a_{H_i}}{(Z_{H_i})^2}\mathrm{Re}\,\delta T^{I'}, \\ &K^{X\bar{X}} \simeq \frac{1}{Z_X} - \frac{2a^i_X}{(Z_X)^2}\mathrm{Re}\,\delta T^{I'} + \frac{4}{\Lambda^2(Z_X)^2}|\delta X|^2, \end{aligned} \tag{A.6}$$

where

$$A_{H_i} \equiv (2\mathrm{Re}T^{I'})^2\frac{a_{H_i}}{Z_{H_i}}, \ \ A^i_X \equiv (2\mathrm{Re}T^{I'})^2\frac{a^i_X}{Z_X}. \tag{A.7}$$

Thus, the covariant derivatives of superpotential with respect to moduli, stabilizer, and SUSY-breaking fields are given by

$$\begin{aligned} D_{T^{I'}}W \simeq\ & K_{T^{I'}}w, \\ & + W_{T^{I'}H_i}\delta H^i + K_{T^{I'}\bar{T}^{I'}}w(\delta T^{I'} + \delta\bar{T}^{I'}) + K_{T^{I'}}W_X\delta X \\ & + W_{T^{I'}T^{I'}H_i}\delta T^{I'}\delta H_i + \sum_{J'=k} K_{T^{I'}}W_{T^{J'}H_k}\delta T^k\delta H^k, \\ D_{H_i}W \simeq\ & W_{T^{I'}H_i}\delta T^{I'} + K_{H_i\bar{H}_i}w\delta\bar{H}_i + \frac{W_{T^{I'}T^{I'}H_i}}{2}(\delta T^{I'})^2, \\ D_X W \simeq\ & W_X + K_{X\bar{X}}w\delta\bar{X} + \frac{1}{2}\partial_X(K_{X\bar{X}})w(2|\delta X|^2 + (\delta X)^2 + (\delta\bar{X})^2), \end{aligned} \tag{A.8}$$

and the total F-term scalar potential is found at the second order of $\delta\phi$,

$$\begin{aligned}
V \simeq &\frac{W_X^2}{Z_X} - 2wW_X(\delta X + \delta\bar{X}) - \sum_{I'=i}(2\mathrm{Re}T^{I'})wW_{T^{I'}H_i}(\delta H_i + \delta\bar{H}_i) \\
&+ \frac{4w^2}{\Lambda^2 Z_X^2}|\delta X|^2 + \sum_{I'=i}\frac{W^2_{T^{I'}H_i}}{Z_{H_i}}|\delta T^{I'}|^2 + \sum_{I'=i}^{3}(2\mathrm{Re}\,T^{I'})^2 W^2_{T^{I'}H_i}|\delta H_i|^2 \\
&+ \sum_{I'=i}^{3}(-2\mathrm{Re}\,T^{I'}wW_{T^{I'}T^{I'}H_i} + wW_{T^{I'}H_i})(\delta T^{I'}\delta H_i + \delta\bar{T}^{I'}\delta\bar{H}_i) \\
&+ \sum_{I'=i}\frac{A_{H_i}}{2\mathrm{Re}T^{I'}}wW_{T^{I'}H_i}(\delta T^{I'}\delta\bar{H}_i + \delta\bar{T}^{I'}\delta H_i) - \sum_{i=1}^{3} wW_{T^{I'}H_i}(\delta T^{I'} + \delta\bar{T}^{I'})(\delta H_i + \delta\bar{H}_i) \\
&+ \sum_{I'=i}^{3} 2\mathrm{Re}\,T^{I'}W_X W_{T^{I'}H_i}(\delta H_i\delta\bar{X} + \delta\bar{H}_i\delta X) \\
&+ \sum_{I'=i}^{3}\sum_{J'=1}^{3}\frac{T^{I'}+\bar{T}^{I'}}{T^{J'}+\bar{T}^{J'}}wW_{T^{I'}H_i}(\delta T^{J'} + \delta\bar{T}^{J'})(\delta H_i + \delta\bar{H}_i). \qquad \text{(A.9)}
\end{aligned}$$

From the obtained scalar potential, the moduli and stabilizer fields receive the following variations from the SUSY-preserving minmum,

$$\begin{aligned}
\delta H_i &\simeq \frac{w}{2\mathrm{Re}T^{I'}W_{T^{I'}H_i}} \sim \mathcal{O}\left(\frac{m_{3/2}}{m_{H_i}}\right), \quad \delta X \simeq \left(\frac{\Lambda^2 Z_X^2}{4w^2}\right)5wW_X, \\
\delta T^{I'} &\simeq \left(\frac{w}{W_{T^{I'}H_i}}\right)^2 Z_{H_i}\left(\frac{1 + A_{H_i}K_{T^{I'}}}{2\mathrm{Re}T^{I'}} + \frac{W_{T^{I'}T^{I'}H_i}}{W_{T^{I'}H_i}} - \frac{3}{\mathrm{Re}T^{I'}}\right) \sim \mathcal{O}\left(\frac{m_{3/2}}{m_{T^{I'}}}\right). \qquad \text{(A.10)}
\end{aligned}$$

and the F-terms of moduli, stabilizer, and SUSY-breaking fields are

$$\begin{aligned}
\sqrt{K_{T^{I'}\bar{T}^{I'}}}F^{T^{I'}} &= -e^{K/2}\sqrt{K_{T^{I'}\bar{T}^{I'}}}K^{T^{I'}\bar{J}}\overline{D_J W} \sim O\left(\frac{m_{3/2}^3}{m_{T^{I'}}^2}\right), \\
\sqrt{K_{H_i\bar{H}_i}}F^{H_i} &= -e^{K/2}\sqrt{K_{H_i\bar{H}_i}}K^{H_i\bar{J}}\overline{D_J W} \sim O\left(\frac{m_{3/2}^3}{m_{H_i}^2}\right), \\
\sqrt{K_{X\bar{X}}}F^X &\simeq -e^{K/2}\sqrt{K_{X\bar{X}}}K^{X\bar{X}}D_X W \simeq \frac{-W_X}{(\mathrm{Re}T^1\mathrm{Re}T^2\mathrm{Re}T^3)^{1/2}Z_X^{1/2}}, \qquad \text{(A.11)}
\end{aligned}$$

with

$$\begin{aligned}
&D_{T^{I'}}W = \min\left(O\left(\frac{m_{3/2}^3}{m_{T^{I'}}^2}\right), O\left(\frac{m_{3/2}^3}{m_X^2}\right)\right), \quad (I' = 2,3), \\
&D_{T^1}W = O\left(\frac{m_{3/2}^3}{m_{T^1}^2}\right), \quad D_{H_i}W = O\left(\frac{m_{3/2}^2}{m_{H_i}}\right), (i = 1,2,3), \quad D_X W \simeq \nu. \qquad \text{(A.12)}
\end{aligned}$$

The SUSY is mainly broken by the SUSY-breaking field X and the F-terms of moduli and stabilizer fields are suppressed by the gravitino mass.

A.2 The fluctuations of fields during the inflation

Next, we show the deviations of fields during the inflation in contrast to those around the vacuum. The reference point method also allows us to extract them in a similar way to the previous analysis at the vacuum. By expanding the fields around the reference points in Eqs. (3.16) and (3.18), the covariant derivatives of superpotential with respect to $\phi = T^{I'}, \mathrm{Im}\,T^1, H_i, X$ with $I' = 2, 3$ and $i = 1, 2, 3$, are obtained as

$$
\begin{aligned}
D_{T^{I'}}W &\simeq K_{T^{I'}}w + W_{T^{I'}H_i}\delta H_i + K_{T^{I'}\bar{T}^{I'}}w(\delta T^{I'} + \delta\bar{T}^{I'}) + K_{T^{I'}}(W_{H_1}\delta H_1 + W_X\delta X) \\
&\quad + K_{T^{I'}\bar{T}^{I'}}W_{H_1}(\delta T^{I'} + \delta\bar{T}^{I'})\delta H^1 + W_{T^{I'}T^{I'}H_i}\delta T^{I'}\delta H_i + K_{T^{I'}}\sum_{J'=j}^{3} W_{T^{J'}H^j}\delta T^{J'}\delta H^j, \\
D_{H_3}W &\simeq W_{T^3H_3}\delta T^3 + K_{H_3\bar{H}_3}w\delta\bar{H}_3 + K_{H_3\bar{H}_3}W_{H_1}\delta\bar{H}_3\delta H_1, \\
D_{H_2}W &\simeq W_{T^2H_2}\delta T^2 + K_{H_2\bar{H}_2}w\delta\bar{H}_2 + K_{H_2\bar{H}_2}W_{H_1}\delta\bar{H}_2\delta H_1, \\
D_{H_1}W &\simeq W_{H_1} + W_{T^1H_1}\delta T^1 + K_{H_1\bar{H}_1}W\delta\bar{H}_1 + \partial_{T^1}(K_{H_1\bar{H}_1})w(\delta T^1 + \delta\bar{T}^1)\delta\bar{H}_1, \\
&\quad + K_{H_1\bar{H}_1}(W_{H_1}|\delta H_1|^2 + W_X\delta\bar{H}_1\delta X) + \frac{W_{T^1T^1H_1}}{2}(\delta T^1)^2, \\
D_XW &\simeq W_X + K_{X\bar{X}}w\delta\bar{X} + \frac{1}{2}\partial_X(K_{X\bar{X}})w(2|\delta X|^2 + (\delta X)^2 + (\delta\bar{X})^2) + K_{X\bar{X}}W_{H_1}\delta H_1\delta\bar{X},
\end{aligned}
\tag{A.13}
$$

which lead to the variations of relevant fields after solving their the extremal conditions

$$
\begin{aligned}
&\delta\tau^1 = \delta\tau^2 = \delta\tau^3 = \delta k_1 = \delta k_2 = \delta k_3 = \delta y = 0, \\
&\delta\sigma^{I'} \sim O\left(\frac{Z_{H_i}}{W^2_{T^{I'}H_i}\mathrm{Re}\,T^{I'}}\frac{|W_{H_1}|^2}{Z_{H_1}}\right) \simeq O\left(\left(\frac{H_{\mathrm{inf}}}{m_{T^{I'}}}\right)^2\right), \quad (I' = 2, 3), \\
&\delta h_i \sim O\left(\frac{K_{T^{I'}}w}{W_{T^{I'}H_i}(2\mathrm{Re}T^{I'})^2}\right) \simeq O\left(\frac{m_{3/2}}{m_{H_i}}\right), \quad (i = 2, 3), \\
&\delta h_1 \sim O\left(\frac{w}{W_{H_1}}\right) \simeq O\left(\frac{m_{3/2}}{m_{H_1}}\right), \\
&\delta x \sim O\left(\frac{\Lambda^2 Z_X^2}{4W_X^2}W_Xw\right) \simeq O\left(\left(\frac{m_{3/2}}{m_X}\right)^2\right),
\end{aligned}
\tag{A.14}
$$

where

$$
\delta T^{I'} \equiv \delta\sigma^{I'} + i\delta\tau^{I'}, \quad \delta H_i \equiv \delta h_i + i\delta k_i, \quad \delta X \equiv \delta x + i\delta y, \tag{A.15}
$$

with $I', i = 1, 2, 3$.

Appendix B

Mass matrices in the inflationary model of heterotic string theory

Along the line of Ref. [124], we review the mass-squared matrices of dilaton and Kähler moduli in the case of single gaugino condensation with the dilaton Kähler potential being the type of $K = K^0 + K^{\rm np}$ in Eq. (6.21).

Before going to estimate the masses of moduli fields, we canonically normalize the moduli fields whose Kähler metrics are given in the limit of $\mathrm{Re}\, S \gg \beta_j \mathrm{Re}\, T_j$ and $T_1 \gg T_j$ with $j = 2, 3, 4, 5$,

$$\begin{aligned}
K_{\Phi\bar{\Phi}} &\simeq -\frac{b}{16}\frac{2}{(\Phi+\bar{\Phi})^{3/2}}K^{\rm np} + \frac{1}{2}\left(p - b\left(\frac{\Phi+\bar{\Phi}}{2}\right)^{1/2}\right)\frac{1}{(\Phi+\bar{\Phi})^2},\\
K_{\Phi\bar{T}_j} &\simeq \frac{\beta_j}{(\Phi+\bar{\Phi})^2},\\
K_{T_1\bar{T}_1} &\simeq \frac{3}{(T_1+\bar{T}_1)^2},\\
K_{T_1\bar{T}_j} &\simeq \frac{9k_j(T_j+\bar{T}_j)^2}{k_1(T_1+\bar{T}_1)^4},\\
K_{T_j\bar{T}_j} &\simeq \frac{6k_j(T_j+\bar{T}_j)}{k_1(T_1+\bar{T}_1)^3},\\
K_{T_i\bar{T}_j} &\simeq \frac{9k_ik_j(T_i+\bar{T}_i)^2(T_j+\bar{T}_j)^2}{k_1^2(T_1+\bar{T}_1)^6},
\end{aligned} \tag{B.1}$$

where $i \neq j$, $i, j = 2, 3, 4, 5$. These moduli fields are stabilized at the supersymmetric minimum $K_I = 0$ with $I = \Phi, T^2, T^3, T^4, T^5$. Thus, the off-diagonal elements of Kähler metric is negligible compared with the diagonal one, because of small β_j and values of moduli T_j, $j = 2, 3, 4, 5$. The Kähler metric of moduli fields are then dominated by the diagonal components such as

$$K_{I\bar{J}} \simeq K_{I\bar{J}}\delta_{I\bar{J}}, \tag{B.2}$$

with $I, J = \Phi, T_1, T_j$ for $j = 2, 3, 4, 5$.

These mass-squared matrices of moduli fields receive both the F-term and and D-term contributions. First, we show the D-term potential induced by the anomalous $U(1)$ symmetries,

$$\begin{aligned} V_D =& \frac{1}{2f_{U(1)^1}}(q_S^1 K_S + q_{T_2}^1 K_{T_2} + q_{T_3}^1 K_{T_3})^2 + \frac{1}{2f_{U(1)^2}}(q_S^2 K_S + q_{T_2}^2 K_{T_2} + q_{T_3}^2 K_{T_3})^2 \\ &+ \frac{1}{2f_{U(1)^3}}(q_S^3 K_S + q_{T_2}^3 K_{T_2} + q_{T_3}^3 K_{T_3})^2 + \frac{1}{2f_{U(1)^4}}(q_{T_4}^4 K_{T_4} + q_{T_5}^4 K_{T_5})^2, \end{aligned} \tag{B.3}$$

where $f_{U(1)^m}$ denote the gauge kinetic functions of $U(1)^m$, $m = 1, 2, 3, 4$ symmetries and their approximated forms become $f_{U(1)^m} \simeq \mathrm{tr}(T^m T^m)S$. Such D-term contributions are also expanded by the smallness of β_j, $j = 2, 3, 4, 5$ at the supersymmetric minimum of moduli fields,

$$(V_D)_{I\bar{J}} = (V_D)^0_{I\bar{J}} + (V_D)^1_{I\bar{J}}, \tag{B.4}$$

where the leading term is given by

$$\begin{aligned} (V_D)^0_{\Phi\bar{\Phi}} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}}(q_S^n K_{\Phi\bar{\Phi}} + q_{T_2}^n K_{T_2\bar{\Phi}} + q_{T_3}^n K_{T_3\bar{\Phi}})^2, \\ (V_D)^0_{\Phi\bar{T}_2} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}}(q_S^n K_{\Phi\bar{\Phi}} + q_{T_2}^n K_{T_2\bar{\Phi}}) q_{T_2}^n K_{T_2\bar{T}_2}, \\ (V_D)^0_{\Phi\bar{T}_3} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}}(q_S^n K_{\Phi\bar{\Phi}} + q_{T_3}^n K_{T_3\bar{\Phi}}) q_{T_3}^n K_{T_3\bar{T}_3}, \\ (V_D)^0_{T_2\bar{T}_2} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}}(q_{T_2}^n)^2 (K_{T_2\bar{T}_2})^2, \\ (V_D)^0_{T_2\bar{T}_3} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}} q_{T_2}^n q_{T_3}^n K_{T_2\bar{T}_2} K_{T_3\bar{T}_3}, \\ (V_D)^0_{T_3\bar{T}_3} =& \sum_{n=1}^{3} \frac{1}{2f_{U(1)^n}}(q_{T_3}^n)^2 (K_{T_3\bar{T}_3})^2, \\ (V_D)^0_{T_4\bar{T}_4} =& \frac{1}{2f_{U(1)^4}} \left(q_{T_4}^4 K_{T_4\bar{T}_4} + q_{T_5}^4 K_{T_4\bar{T}_5}\right)^2, \\ (V_D)^0_{T_4\bar{T}_5} =& \frac{1}{2f_{U(1)^4}} \left(q_{T_4}^4 K_{T_4\bar{T}_4} + q_{T_5}^4 K_{T_4\bar{T}_5}\right) \left(q_{T_4}^4 K_{T_4\bar{T}_5} + q_{T_5}^4 K_{T_5\bar{T}_5}\right), \\ (V_D)^0_{T_5\bar{T}_5} =& \frac{1}{2f_{U(1)^4}} \left(q_{T_4}^4 K_{T_4\bar{T}_5} + q_{T_5}^4 K_{T_5\bar{T}_5}\right)^2, \end{aligned} \tag{B.5}$$

and otherwise zero. Although these leading terms are of order $(\beta_j)^2$, $j = 2, 3, 4, 5$, the next-leading terms are of order $(\beta_j)^3$. In our choice of parameters, the next-leading terms are smaller than the mass scales originating from the F-term contributions which are summarized as

$$
\begin{aligned}
(V_F)_{\Phi\bar{\Phi}} \simeq & e^K K^{\Phi\bar{\Phi}} |K_{\Phi\bar{\Phi}} W|^2, \\
(V_F)_{T_1\bar{T}_1} \simeq & e^K K^{T_1\bar{T}_1} |W_{T_1}|^2, \\
(V_F)_{T_2\bar{T}_2} \simeq & e^K K^{T_2\bar{T}_2} |K_{T_1\bar{T}_1} W|^2, \\
(V_F)_{T_3\bar{T}_3} \simeq & e^K K^{T_3\bar{T}_3} |K_{T_3\bar{T}_3} W|^2, \\
(V_F)_{T_4\bar{T}_4} \simeq & e^K K^{T_4\bar{T}_4} |K_{T_4\bar{T}_4} W|^2, \\
(V_F)_{T_5\bar{T}_5} \simeq & e^K K^{T_5\bar{T}_5} |K_{T_5\bar{T}_5} W|^2.
\end{aligned} \tag{B.6}
$$

As a result, the total mass-squared matrices are summation of both the D-term and F-term contributions,

$$
(V)_{I\bar{J}} \simeq (V_D)^0_{I\bar{J}} + (V_F)_{I\bar{J}}. \tag{B.7}
$$

Appendix C

The canonical normalization in the inflationary model of type IIB string theory

Following Ref. [143], we summarize the canonical normalization of moduli fields in the case of type IIB string theory.

The Kähler metric of dilaton and complex structure moduli are described as

$$K_{I\bar{J}} = \begin{pmatrix} K_{U^4\bar{U}^4} & K_{U^4\bar{U}^2} & 0 & 0 & 0 \\ K_{U^2\bar{U}^4} & K_{U^2\bar{U}^2} & K_{U^2\bar{\tilde{T}}} & 0 & 0 \\ 0 & K_{\tilde{T}\bar{U}^2} & K_{\tilde{T}\bar{\tilde{T}}} & 0 & 0 \\ 0 & 0 & 0 & K_{U^3\bar{U}^3} & 0 \\ 0 & 0 & 0 & 0 & K_{S\bar{S}} \end{pmatrix}, \tag{C.1}$$

where

$$\begin{aligned}
K_{U^2\bar{U}^2} &= \frac{1}{(U^2+\bar{U}^2)^2} + \frac{1}{(U^4+\bar{U}^4+U^2+\bar{U}^2)^2} + \frac{3(c_2)^2}{(\tilde{T}+\bar{\tilde{T}}-c_2(U^2+\bar{U}^2))^2} = \frac{10}{3}\frac{1}{(U^2+\bar{U}^2)^2}, \\
K_{U^2\bar{U}^4} &= K_{U^4\bar{U}^2} = K_{U^4\bar{U}^4} = \frac{1}{(U^2+\bar{U}^2)^2}, \quad K_{U^2\bar{\tilde{T}}} = K_{\tilde{T}\bar{U}^2} = -\frac{4}{3c_2}\frac{1}{(U^2+\bar{U}^2)^2}, \\
K_{U^3\bar{U}^3} &= \frac{1}{(U^3+\bar{U}^3)^2}, \quad K_{S\bar{S}} = \frac{1}{(S+\bar{S})^2}, \quad K_{T\bar{T}} = \frac{3}{(\tilde{T}+\bar{\tilde{T}}-c_2(U^2+\bar{U}^2))^2} = \frac{4}{3(c_2)^2}\frac{1}{(U^2+\bar{U}^2)^2},
\end{aligned} \tag{C.2}$$

with $c_2 \equiv b^2/12$. For the case of $c_2 = 1$, the numerical values of the eigenvalues $(K_{\rm eig})_I$ and diagonalizing matrix $U_{I\bar{J}}$ are obtained as

$$
\begin{aligned}
&(K_{\rm eig})_{U^4} \simeq \frac{4.3}{(U^2+\bar{U}^2)^2}, \;\; (K_{\rm eig})_{U^2} \simeq \frac{1.1}{(U^2+\bar{U}^2)^2}, \;\; (K_{\rm eig})_{\tilde{T}} = \frac{0.27}{(U^2+\bar{U}^2)^2}, \\
&(K_{\rm eig})_{U^3} = K_{U^3\bar{U}^3}, \;\; (K_{\rm eig})_S = K_{S\bar{S}}, \\
&U_{I\bar{J}} = \begin{pmatrix} -0.67 & -2.19 & 1 & 0 & 0 \\ 1 & 0.14 & 1 & 0 & 0 \\ -1.1 & 0.8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.
\end{aligned}
\tag{C.3}
$$

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氏名　大塚　啓

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種 類 別	題名、　発表・発行掲載誌名、　発表・発行年月、　連名者（申請者含む）
論文	Axion inflation with cross-correlated axion isocurvature perturbations, Journal of Cosmology and Astroparticle Physics, 01 (2016) 044, 2016 年 1 月, Kenji Kadota, Tatsuo Kobayashi and Hajime Otsuka.
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