オンラインのモデリングと
最適化に基づく制御構成法の研究

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まえがき

モデルに基づいて設計される制御の性能は，制御対象を記述しているモデルの正確さに強く左右される．しかしながら制御対象の忠実なモデルを得ることは一般に困難である．本研究では，事前に得られるモデルは不十分なものであるという前提に立ち，オンラインで刻々得られるデータに基づいてモデルを入手／更新し，入手／更新されたモデルを用いて予測される未来の振舞いを最適化することによって制御入力を刻々計算／更新する新しい制御構成法について検討した．不正確なモデルについてもう少し具体化して述べると，制御対象の入手／更新されるモデルは（状態）空間的にも時間的にも局所的なモデルであるという前提に立って，刻々得られるオンラインデータによって更新される局所的モデルに基づいて最適な制御入力を設計する方法を提案し，設計される制御がどのような性能を持つのかを明らかにすることを狙いとした．時間的な局所性のみに注目したものが従来のモデル予測制御であり，空間的な局所性に注目したものが従来のＬＰＶモデルに基づくスケジューリング制御である．事前のモデルとオンラインで得られるデータの構造を既知の情報として，どのようにモデルの更新を行うのか，どのような最適基準でどのような最適化手法を用いるのか，実際に用いる制御入力は何か，オンラインで得られるデータや構成される入力に入ってくる情報遅れにどのように対処すればよいのか，どのような安定性が保証できるのか，どのような制御性能が保証されるのかを検討することにより，理論的な基盤のある実際的な制御構成法を提案した．

本研究は以下のような手順で実施された．最初に，最適制御問題にオンラインモデリングを組み込んだ制御問題を定式化した．どのようなオンラインモデリングを行うのか，すなわちモデルの更新方式の問題，そして組み込み方式の問題を検討した．モデル予測制御における逐次決定の方式を発展させた組み込み方式，ＬＰＶシステムにおけるスケジューリングパラメータの役割を発展させたモデル更新方式を検討し，検討結果を踏まえて制御問題を定式化した．次に，問題の解法と制御構成法を検討し，定式化された問題の解法と解法アルゴリズムの提案を行った．ここでは，本研究者によって確立されている無限次元線形行列不等式に基づく制御系の解析設計法を応用した．最後に，構成された制御の性能検討をおこなった．実際の性能検討のためには制御対象の正確なモデルが必要であるが，局所的なモデルしか入手できな

いという前提に立つ本研究では，まず，中間的な性能評価用モデルを導入してシミュレーションによる検討を行い，その後，実機による検証をおこなった．特に，本研究の後半では，実際の構成において現れる情報遅れの問題の検討に重点を置き，提案法の理論的な可能性，限界を明らかにした．

本研究の前半での成果は次のように要約される．非線形性や不確かさのため正確なモデリングが困難な電気油圧サーボ系の最適制御設計を目標にして，いくつかの動作点の近傍での局所的なモデルを作成し，負荷変動をパラメータとする LPV システムを用いて，最悪外乱を想定した最適速度制御および最適推力制御を構成した．実機試験により提案手法の有効性を確認できた．一方，入出力の大量データベースに基づくオンラインモデリング手法の提案と単純最適化モデル予測制御理論を提案し，シミュレーションによる検証をおこなった．この成果については２００２年度自動制御連合講演会で途中経過の発表はおこなったが，現在，実プラントでの実証に着手しており，その成果も含めて発表予定である．

本研究の後半での成果はつぎのように要約される．局所モデルによる全体システムの制御を構築する際に問題となる状態変数における遅れ，入出力変数における情報遅れの克服を検討した．制御器構成のための最悪外乱に対する最適制御構成のための有限次元アルゴリズムの提案と有効性の実証，入力変数の情報遅れを考慮した場合の最悪外乱に対する最適制御制の予測・オブザーバ構造の解明をおこなうことができた．

研究組織

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目　次

論　文

超磁歪材アクチュエータを有する直動型サーボ弁のモデリングとゲインスケジューリングによるスプール位置制御*

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Modeling of Direct-Drive Servovalve which Has Giant Magnetrostrictive Material and Spool Position Control by Gain Scheduling*

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Because of the reasons of favorable features such as high driving force, fast response, fine environmental durability and big distortion about 10 times more than that of PZT, GMM (Giant Magnetrostrictive Material) is suitable for the actuation structure of servovalve. But, GMM distortion depends on the magnitude of coil current and the output-gain declines according to decreasing current magnitude. Then, the dead-zone appears in small current range. One of the important characteristics of servovalve is the fine tracking performance to the reference signal. So far, we attempted to apply linear controllers, such as PI or H^∞ controller. However GMM tracking performance is inferior to the traditional driving device for reference signal around null area, and this flaw remains as an important matter.

In this paper, adopting LPV (Linear Parameter-Varying) system modeling in which we regard the magnitude of input signal as a scheduling parameter. we design a gain scheduling controller and attempt to solve the problem that is caused by the nonlinearities mentioned above. Since it is hard to implement the controller which uses the information on the varying rate of scheduling parameters, we propose a new approach for output-feedback controller design. First, we describe the plant modeling as an LPV system, then present an outline of the controller design synthesis without any information on the varying rate of scheduling parameter and the design process. Lastly, we show the usefulness of the proposed controller by experimental results.

1. はじめに

高出力・高応答を特徴とする電気油圧サーボシステムにおいて，電気油圧変換のインタフェースとなるサーボ弁には，応答性と高い分解能が求めらる．近年の省資源とランニングコスト低減要求などに対し，サーボ弁スプールの新しい駆動方式の導入が行われている [1,2].

超磁歪材は磁界中で歪む性質を持ち，高推力，高応答，耐環境性に優れ，歪み率は圧電素子より一桁程度大きい．磁石やコイルなどで発生する磁界中へ超磁歪材を配置するだけで歪みを取り出せるので，スプール駆動機構が簡潔になるなどの利点から，超磁歪材駆動によるサーボ弁の開発を行っている．

サーボ弁に求められる特性は目標信号への良好な追従性であり，これまでに代表的動作点に基づくPI制御や H^∞ 制御の適用を試みたが，満足のいく結果は得られていない．これは超磁歪材の歪み量がコイル電流の大きさに依存し，電流値が小さくなるに従い出力ゲインが低減する性質を持つこと，微小電流領域で不感帯が現われるなどによる．したがって一つの動作点に対する固定・線形な制御器を実装しても全動作領域に対して十分な制御性が達成されず，サーボ弁性能で重要な中立点付近での制御性が従来の駆動方式より劣り問題となっている．このような制御対象に対しても，より高い制御性を達成するために，従来その影響が十分小さいとされてきた非線形特性を考慮した制御器設計手法が求められる．

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Key Words: gain scheduling, servovalve, gaint magnetrostrictive material.

本論文では，ゲインスケジューリングにより非線形性を考慮した制御器設計を行い，この問題の克服を試みる．具体的には操作量の大きさをスケジューリングパラメータとし，これが与えられるごとに制御器パラメータを変更するゲインスケジューリング制御を適用することで小振幅応答の改善を行う．ゲインスケジューリング制御器の構成は，制御対象のLPVモデルに基づく方法[3–5]を採用する．

本論文の制御問題では状態変数を測定することが困難であるため，出力フィードバック制御の構成が必要である．標準的な出力フィードバックの構成法を与える参考文献[6]に従えば，スケジューリングパラメータの変化量の測定と測定値を用いた制御器をオンラインで算出する必要があり，本制御問題の性質からこれも困難である．そこで本論文では，スケジューリングパラメータの変化量を必要としない(変化量の最大値は制御器設計の評価に必要)出力フィードバック制御の構成法を新たに展開し，その構成法を用いた制御器の設計を行う．

最初に制御対象のLPVシステムとしてのモデリングを述べ，次にスケジューリングパラメータの変化量を必要としない出力フィードバック制御構成法の概要と設計手順を示す．最後に実機試験結果より本手法の有効性を示す．

2. 直動型サーボ弁概要とモデリング

2.1 サーボ弁構造と特性

Fig. 1に超磁歪材を有するサーボ弁の構造図を示す．超磁歪材のまわりに，コイルと永久磁石が配置されている．コイル電流を印加することで歪みを発生させ，永久磁石によりバイアスを与えている．小型化のため超磁歪材は円周上に配置されている．同時に曲げモーメントの回避，熱膨張による中立点変動を低減する構造となっている．スプール端には，位置検出用の差動トランス(LVDT)が取り付けられている．

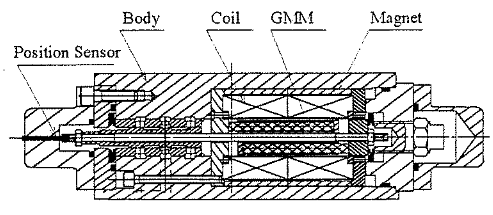

Fig. 1 Servovalve structure with GMM

Fig. 2に開ループの入出力特性を示した．横軸は電流増幅器への入力電圧，縦軸はスプール位置信号である．出力飽和が顕著に表れない範囲で極力大きな変位が得られる領域を利用し，ここでは入力電圧換算で ±1.25[V]，スプール位置電圧で ±5[V] を定格値としている．図からは，制御対象がヒステリシス特性を持つこと，入力が小さくなるに従い出力ゲインが低減し，ヒステリシスループの内側ではこの傾向が顕著に現れること，入力信号換算でおよそ1 %以下では不感帯を持つことなどの特性が読み取られる．

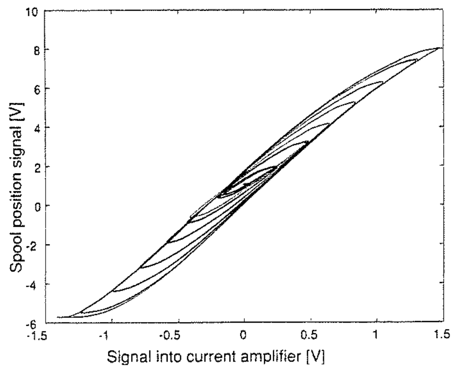

Fig. 2 GMM actuator input-output characteristic

3. システム同定実験と制御対象モデル

このような制御対象をLPVシステムとして数式表現するため，幾通りか大きさの異なる入力を加え同定実験を行う．同定実験では，次のような条件を設定する．

- 同定入力は周期63ステップのM系列信号．
- データ採取サンプリング時間は0.02[ms]．
- 同定入力信号の大きさは定格の±2，3，5，10，20，30，40，50，60，70，80，90，100%．

実験後，一括処理同定アルゴリズムにより制御対象のパラメータを求めるが，次のような前処理を行っている．

- 入出力信号からバイアスやトレンドの影響を取り除くため平均値をデータから差し引く．
- 採取データから4ステップごとに抽出したものにARMAXモデル

$$A(z)y(k) = B(z)u(k) + C(z)w(k)$$

  を採用．ここで $u(k)$ は同定入力，$w(k)$ は雑音，$y(k)$ は出力である．

いくつかの試行錯誤の結果，制御対象の離散時間モデル次数は5，雑音のモデル次数を2，むだ時間は1ステップとする．

連続時間モデルへの変換では，逆応答が小さくなるよう次のような方法を用いる．まず離散時間モデルからむだ時間を除き，周波数応答線図を計算する．注目している周波数領域においてゲイン曲線を近似する4次の連続時間伝達関数をカーブフィットにより求める．最後にむだ時間要素を1次パーデ近似し，カーブフィットで得た伝達関数へ加えて連続時間モデルとする．ここでは分子式の最高次係数が十分に小さいことを確かめたうえで，この項を取り除き厳密にプロパーなモデルとしている．

このようにして得られた13通りのモデルに対して，伝達関数の各係数がスケジューリングパラメータ θ の3次式多項式で表せるものと仮定して，近似誤差を最小とする係数を求めると Table 1 に示す値となる．これより制御対象の伝達関数表現は，次式のようになる．

Table 1　Coefficients of polynomial

$b_{s,i}(\theta)$	b_{i0}	b_{i1}	b_{i2}	b_{i3}
$b_{s,4}(\theta)$	0.0332	−2.4263	4.5279	−3.2769
$b_{s,3}(\theta)$	−0.4526	−26.6971	57.1425	−28.2858
$b_{s,2}(\theta)$	6.7079	−16.8666	16.1603	−13.5530
$b_{s,1}(\theta)$	9.1597	142.4606	−266.5099	146.0958
$b_{s,0}(\theta)$	0.4913	12.7101	−27.1006	16.0988
$a_{s,i}(\theta)$	a_{i0}	a_{i1}	a_{i2}	a_{i3}
$a_{s,4}(\theta)$	4.8881	−2.7046	2.7018	−0.8250
$a_{s,3}(\theta)$	13.2543	−16.3460	15.0593	−3.9602
$a_{s,2}(\theta)$	22.8389	−14.3179	−5.3850	11.7323
$a_{s,1}(\theta)$	9.1647	27.3741	−71.8940	44.8869
$a_{s,0}(\theta)$	· 0.3126	2.1733	−5.5099	3.4476

注) ここでは時間軸を 9425 でスケーリングしている.

$$F_p(s)=\frac{b_{s,4}(\theta)s^4+\ldots+b_{s,1}(\theta)s+b_{s,0}(\theta)}{s^5+a_{s,4}(\theta)s^4+\ldots+a_{s,1}(\theta)s+a_{s,0}(\theta)} \quad (1)$$

ここで,

$$a_{s,i}(\theta)=\sum_{j=0}^{3}a_{ij}\theta^j,\ \ b_{s,i}(\theta)=\sum_{j=0}^{3}b_{ij}\theta^j$$
$$(i=0,1,2,3,4)$$

Figs. 3, 4 に同定から得た連続時間モデルのパラメータ値と 3 次多項式近似曲線とのばらつきの様子を示す.

スケジューリングパラメータ θ による状態空間表現は (2) 式から

$$\Sigma_p(\theta):\begin{cases}\dot{x}_p(t)=A_p(\theta)x_p(t)+B_pu(t)\\ y_p(t)=C_p(\theta)x_p(t)\end{cases} \quad (2)$$

ここで $A_p(\theta)=\sum_{i=0}^{3}A_{p,i}\theta^i$, $C_p(\theta)=\sum_{i=0}^{3}C_{p,i}\theta^i$ と表され, 上の (2) 式のようにスケジューリングパラメータが固定されると線形システムとなる. なお, スケジュリングパラメータを定格値で規格化し, θ は仮定

1) $\theta\in[0\ \ 1]$, $\forall t\in[0\ \infty)$
2) $\theta\in\mathcal{C}^1$
3) $|\dot{\theta}|\le v_{max}$, $\forall t\in[0\ \infty)$

を満たすものとする. v_{max} については **5.1** で触れる.

ここで前述したモデリングの過程を Figs. 5〜8 に示す. Fig. 5 は ±80% ステップ応答で, Figs. 6〜8 は ±50%, ±10%, ±2% 応答である. 各図は上から順に

1. 同定実験によるアクチュエータ位置信号
2. 離散系制御対象の応答
3. 連続時間系モデルによる応答
4. θ の 3 次多項式モデルによる応答

となっている. これらモデルの応答計算から, 離散系モデルは実応答をよく再現しており, 連続時間系モデルでは振動的な様子が低減するものの立ち上がりから静定までの全体的な応答は良好であること, 3 次多項で表されるモデルも制御器設計に十分なものであることがわかる.

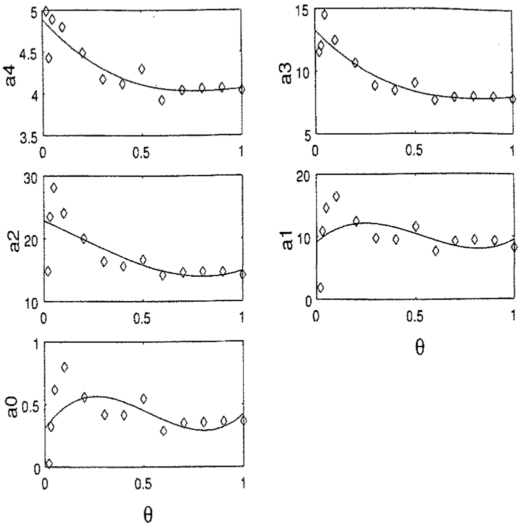

Fig. 3　Coefficients of denominator

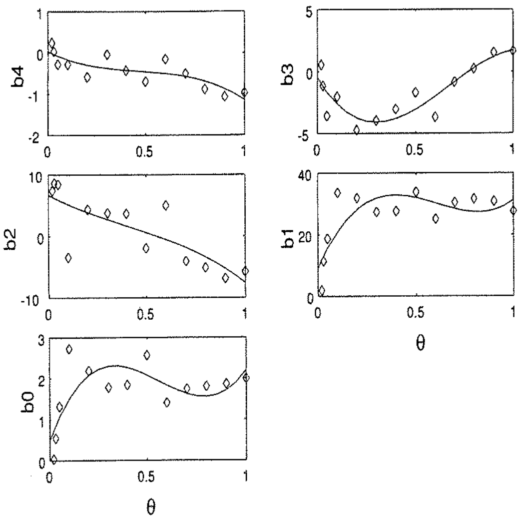

Fig. 4　Coefficients of numerator

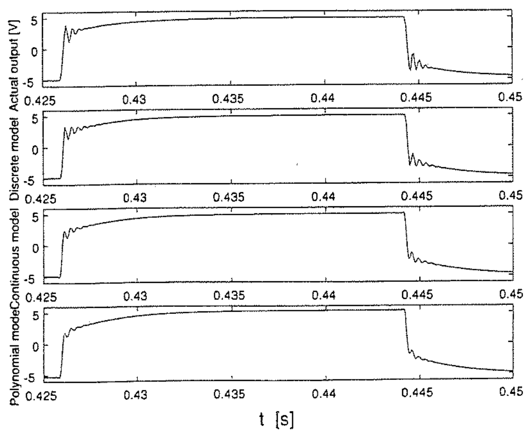

Fig. 5　±80% Step response

4. ゲインスケジューリング制御器設計

LPV 制御対象へ設計仕様を表す重み関数を加えたスケジューリングパラメータ θ を持つ一般化システムの状

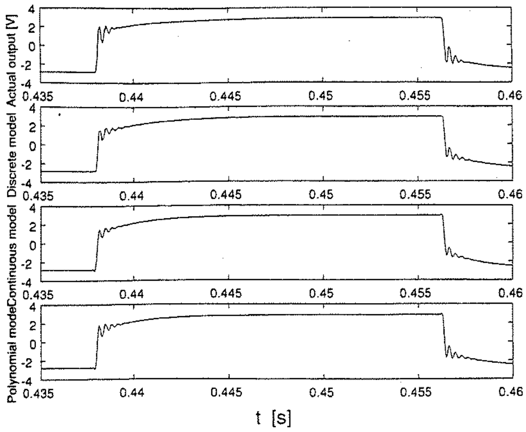

Fig. 6 ±50% Step response

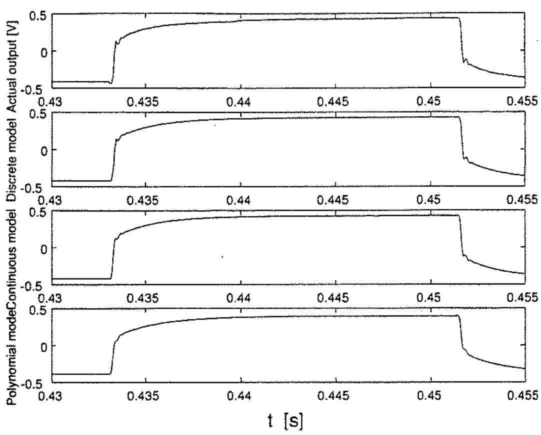

Fig. 7 ±10% Step response

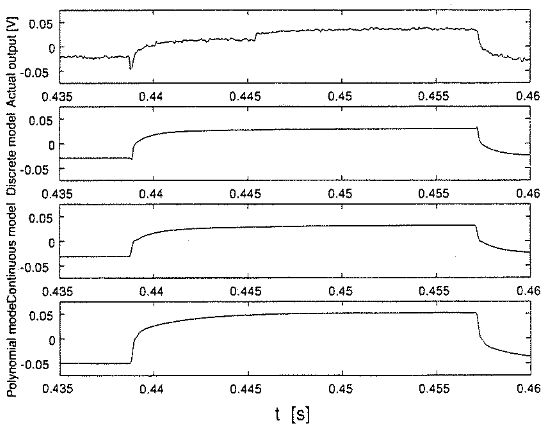

Fig. 8 ±2% Step response

態変数表現を

$$\Sigma_g(\theta):\\ \begin{cases} \dot{x}(t)=A(\theta)x(t)+B_1(\theta)w(t)+B_2(\theta)u(t) \\ z(t)=C_1(\theta)x(t)+D_{11}(\theta)w(t)+D_{12}(\theta)u(t) \\ y(t)=C_2(\theta)x(t)+D_{21}(\theta)w(t) \end{cases} \quad (3)$$

で表す．$x(t)\in\mathcal{R}^n$ は状態，$u(t)\in\mathcal{R}^m$ は操作量，$w(t)\in\mathcal{R}^{m_w}$ は外乱入力，$z(t)\in\mathcal{R}^l$ は被制御量，$y(t)\in\mathcal{R}^{p_y}$ は観測量である．ここで (3) 式の行列の要素は θ の多項式で表される連続な関数とする．

システム $\Sigma_g(\theta)$ が内部安定のとき，このシステムの L_2 ゲイン G を次のように定義する．

$$G=\sup_{w\in L_2, w\neq 0}\frac{\|z\|_2}{\|w\|_2} \quad (4)$$

一般化 LPV システム $\Sigma_g(\theta)$ は，スケジューリングパラメータ θ を固定すると一つの線形システムとなるから，この線形システムを制御するためには θ に依存して決まる線形制御器を考える．制御器の状態変数表現を

$$\Sigma_K:\\ \begin{cases} \dot{x}_K(t)=A_K(\theta)x_K(t)+B_K(\theta)y(t),\ x_K(0)=0 \\ u(t)=C_K(\theta)x_K(t)+D_K(\theta)y(t) \end{cases} \quad (5)$$

で表す．ゲインスケジューリング制御器設計問題は，パラメータ θ の変化しうるすべての領域で閉ループ系を内部安定とし，L_2 ゲインをその上限 γ 以下とする制御器 Σ_K を求めるものである．なお本論文では $A(\theta)$ と $A_K(\theta)$ の次数が同じフルオーダ制御器を考える．

4.1 制御器の可解条件

一般化 LPV システム (3) 式を考え，仮定 1)，2)，3) が満たされるものとする．この時，制御器設計問題は定数行列 K_B，K_C と連続で正定対称行列 $Y(\theta)$，$X(\theta)$ が存在し，これらが次の LMI 制約式 (6)，(7)，(8) と BMI 制約式 (9) を満足すれば可解という．

$$\begin{aligned} &\mathcal{A}'Y(\theta)+Y(\theta)\mathcal{A}+C_2'(\theta)K_B+(C_2'(\theta)K_B)' \\ &-v_{max}\frac{dY(\theta)}{d\theta}+\Big(Y(\theta)\mathcal{B}+K_B'D_{21}(\theta)\ \ \mathcal{C}'\Big)\triangle_{cl}^{-1} \\ &\times\begin{pmatrix}(Y(\theta)\mathcal{B}+K_B'D_{21}(\theta))' \\ \mathcal{C}\end{pmatrix}<0 \end{aligned} \quad (6)$$

$$\begin{aligned} &X(\theta)\mathcal{A}'+\mathcal{A}X(\theta)+B_2(\theta)K_C+(B_2(\theta)K_C)' \\ &+v_{max}\frac{dX(\theta)}{d\theta}+\Big(\mathcal{B}\ \ (\mathcal{C}X(\theta)+D_{12}(\theta)K_C)'\Big)\triangle_{cl}^{-1} \\ &\times\begin{pmatrix}\mathcal{B}' \\ \mathcal{C}X(\theta)+D_{12}(\theta)K_C\end{pmatrix}<0 \end{aligned} \quad (7)$$

$$\begin{pmatrix} Y(\theta) & I \\ I & X(\theta)\end{pmatrix}>0 \quad (8)$$

$$\begin{pmatrix} \dfrac{dY(\theta)}{d\theta} & -X^{-1}(\theta)\dfrac{dX(\theta)}{d\theta} \\ -\dfrac{dX(\theta)}{d\theta}X^{-1}(\theta) & -\dfrac{dX(\theta)}{d\theta}\end{pmatrix}\leq 0 \quad (9)$$

ここで，

$$\mathcal{A}:=A(\theta)+B_2(\theta)D_K(\theta)C_2(\theta) \quad (10)$$

$$\mathcal{B}:=B_1(\theta)+B_2(\theta)D_K(\theta)D_{21}(\theta) \quad (11)$$

$$\mathcal{C}:=C_1(\theta)+D_{12}(\theta)D_K(\theta)C_2(\theta) \quad (12)$$

$$\triangle_{cl} := \begin{pmatrix} \gamma I & -D'_{cl}(\theta) \\ -D_{cl}(\theta) & \gamma I \end{pmatrix} \quad (13)$$
$$D_{cl}(\theta) = D_{11}(\theta) + D_{12}(\theta) D_K(\theta) D_{21}(\theta)$$

また D_K は $\triangle_{cl} > 0$ を満たすように選ぶものとする．

4.2 制御器の算出

前述の可解条件を満たす $Y(\theta)$，$X(\theta)$，K_B，K_C が与えられれば，一般化 LPV システム (3) 式を内部安定化し，L_2 ゲインを γ 以下にする出力フィードバック制御器の一つは以下に示す式から算出される．B_K，C_K は

$$B_K = Z(\theta)^{-1} K'_B \quad (14)$$
$$C_K = -K_C X(\theta)^{-1} \quad (15)$$

で与えられる．ここで，$Z(\theta) = Y(\theta) - X(\theta)^{-1}$ である．このとき A_K は

$$\begin{aligned} A_K = Z^{-1}(\theta) \Big[& \mathcal{A}' + Y(\theta)\mathcal{A}X(\theta) + K'_B C_2(\theta) X(\theta) \\ & + Y(\theta) B_2(\theta) K_C + v_{max} X^{-1}(\theta) \frac{dX(\theta)}{d\theta} \\ & + \Big(Y(\theta)\mathcal{B} + K'_B D_{21}(\theta) \ \mathcal{C}' \Big) \triangle_{cl}^{-1} \\ & \times \begin{pmatrix} \mathcal{B}' \\ \mathcal{C}X + D_{12}(\theta) K_C \end{pmatrix} \Big] X^{-1}(\theta) \end{aligned} \quad (16)$$

で与えられる．出力フィードバック制御器の構成法としては参考文献 [6] の方法が知られているが $\dot{\theta}$ の値が必要である．本構成法の特徴は $\dot{\theta}$ の情報を必要とせずに，その上限 v_{max} を用いて制御器を構成する点にある (導出の詳細は参考文献 [7] 参照)．

しかしながら本構成法では，BMI 制約式 (9) が新たに加わる．この制約式は簡単に解くことができない．そこで，実際の構成では制約式 (9) をゆるめて，次の制約を用いることにする．いま $dX(\theta)/d\theta > 0$ を仮定すれば，(9) 式から $dY(\theta)/d\theta < 0$ がいえる．これら二つの制約式と他の LMI 制約式 (6)，(7)，(8) ともに解の候補を求めた後，それらが BMI 制約式 (9) を満たすかを調べることにする．

5. 制御器設計と実機試験

5.1 一般化 LPV システムと制御器

Fig. 9 に一般化 LPV システム $\Sigma_g(\theta)$ の構成を示す．制御対象 $\Sigma_p(\theta)$ には，入力側に定常偏差を低減する目的で積分器を挿入している．$W_s(s)$ は閉ループ系のバンド幅が 400Hz 以上となるように，$W_t(s)$ は同定モデルとスケジューリングパラメータ依存の 3 次多項式モデルの誤差を考慮するとともに，閉ループ系ステップ応答のオーバシュートが極力小さくなるように選んだ重み関数で，ここでは

$$W_s(s) = \frac{s + 1.5}{2(s + 0.001)}, \quad W_t(s) = \frac{2s}{s+2}$$

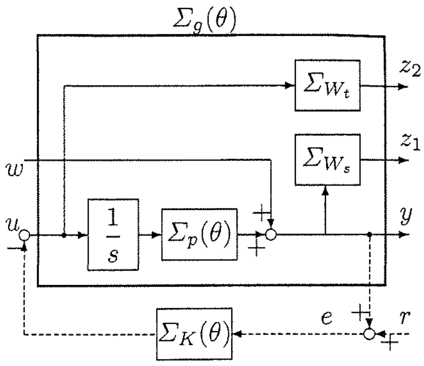

Fig. 9 Generalized LPV system

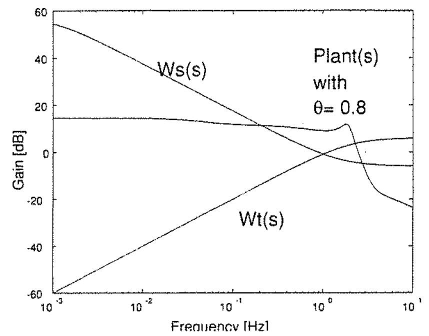

Fig. 10 Bode plots of weighting function

のように決める．また，これらの状態変数表示を

$$\Sigma_{W_s} : \begin{cases} \dot{x}_s(t) = A_s x_s(t) + B_s u_s(t) \\ y_s(t) = C_s x_s(t) + D_s u_s(t) \end{cases}$$

$$\Sigma_{W_t} : \begin{cases} \dot{x}_t(t) = A_t x_t(t) + B_t u_t(t) \\ y_t(t) = C_t x_t(t) + D_t u_t(t) \end{cases}$$

とする．Fig. 10 に $\theta = 0.8$ 時の制御対象と周波数重みのゲイン曲線を示す．また (3) 式に対応する Fig. 9 の一般化 LPV システムの状態変数表示は

$$\Sigma_g(\theta) : \left\{ \begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_i(t) \\ \dot{x}_s(t) \\ \dot{x}_t(t) \\ z_1(t) \\ z_2(t) \\ y(t) \end{bmatrix} = \left(\begin{array}{cccc|c|c} A_p(\theta) & 0 & 0 & 0 & 0 & -B_p \\ C_p(\theta) & 0 & 0 & 0 & 0 & -D_p \\ 0 & B_s & A_s & 0 & B_s & 0 \\ 0 & 0 & 0 & A_t & 0 & -B_t \\ \hline 0 & D_s & C_s & 0 & D_s & 0 \\ 0 & 0 & 0 & C_t & 0 & -D_t \\ \hline 0 & I & 0 & 0 & I & 0 \end{array} \right) \begin{bmatrix} x_p(t) \\ x_i(t) \\ x_s(t) \\ x_t(t) \\ w(t) \\ u(t) \end{bmatrix} \right.$$
$$= \left(\begin{array}{c|c|c} A(\theta) & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array} \right) \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix} \quad (17)$$

のようになる．ただし積分器は

$$\Sigma_{W_i} : \begin{cases} \dot{x}_i(t) = 0 \cdot x_i(t) + u_i(t) \\ y_i(t) = x_i(t) + 0 \cdot u_i(t) \end{cases}$$

とした．

制御器算出では，制御器の保守性を低減する目的で $X(\theta)$ と $Y(\theta)$ も θ に依存する3次多項式とする．v_{max} の値は，つぎのようにして決める．製品への実装時には，高速性の観点から制御器をアナログ素子により構成することを想定し，その操作信号範囲から v_{max} の候補として15を選ぶ．これを基に1.5，15と150の3通りの候補に対する制御器を求めた後，v_{max} が指定した値内に留まるかを調べる．v_{max} が1.5の場合は，条件が満たされない．ここでは，より安定な範囲を得るとの考えから150を採用する．この時 γ の値は6.89を得る．

先に述べたLMI制約は，連続なスケジューリングパラメータ θ に依存しており，解を得るには無限個のLMIを解く必要がある．この問題に対し有限個のLMI制約に帰着させる方法が提案されており，ここでは参考文献[8]による方法を用いている．

このような準備の基で前述のLMIを解き，考慮されなかった制約式(9)を解が満足することを固有値計算から確認し，その後に**4.2**で述べた手順で制御器を算出する．可解条件式(6)，(7)，(8)と(9)を満たす解を用い，θ による制御器の違いを比較するため $\theta = 0.01, 0.5, 1.0$ に対する出力フィードバック制御器を算出し，それらのゲインと位相特性をFigs. 11, 12に示す．ただし，制御器の特徴を示すため積分要素は除いている．

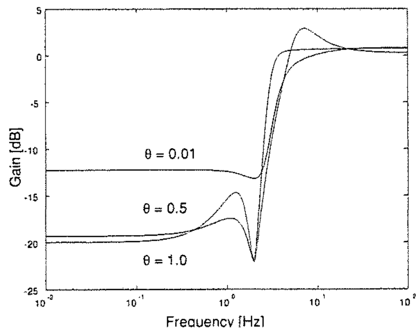

Fig. 11 Controller gain curve

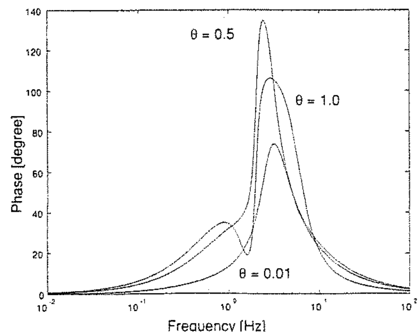

Fig. 12 Controller phase curve

5.2 実装と試験結果

実機における試験条件を示す．

- サンプリング周波数50MHzのDSPと変換速度10μsのAD変換器を持つコントローラを使用．
- 演算時間の問題から，スケジュリングパラメータによりオンラインで制御器のパラメータを算出できないのでTable 2に示すように9通りの制御器を準備し，これらをスケジュリングパラメータで切り換える．

Table 2 Division of θ

range of θ	corresponding θ
0 $\leq$ 0.015	0.01
0.015 $\leq$ 0.025	0.02
0.025 $\leq$ 0.035	0.03
0.035 $\leq$ 0.055	0.05
0.055 $\leq$ 0.15	0.1
0.15 $\leq$ 0.25	0.2
0.25 $\leq$ 0.45	0.4
0.45 $\leq$ 0.65	0.6
0.65 $\leq$ 1.0	0.8

5.2.1 直線性と微小目標信号応答

閉ループ系によって，入出力特性が直線化されている様子をFig. 13に示す．Fig. 14は $0.2\%_{p-p}$ の微小サイン波信号に対する応答である．本制御器により閉ループ系が線形化され，高い分解能を持つことが確認される．

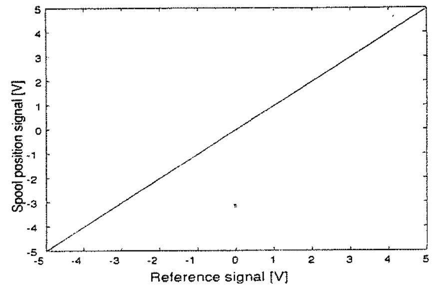

Fig. 13 Closed loop input-output characteristic

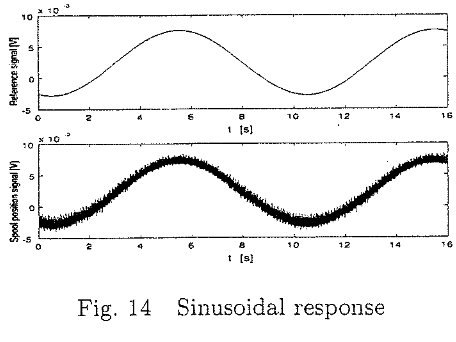

Fig. 14 Sinusoidal response

5.3 ステップ応答と周波数応答

Fig. 15にステップ応答，Fig. 16に周波数応答を示す．いずれも従来の代表的な一つの動作点に対し設計された固定・線形な制御器を用いた時に比べ，5%以下の小振幅ステップ応答の立上がり時間が半分に短縮され，周波数応答ではバンド幅が1.7倍程度改善されている．

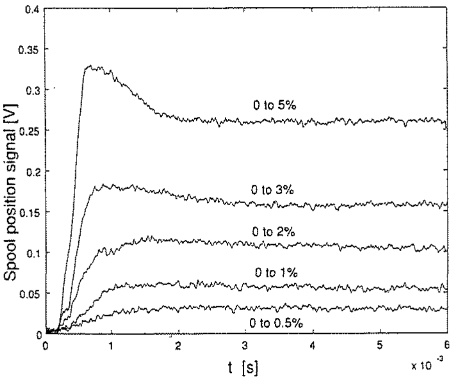

Fig. 15 Closed loop step response

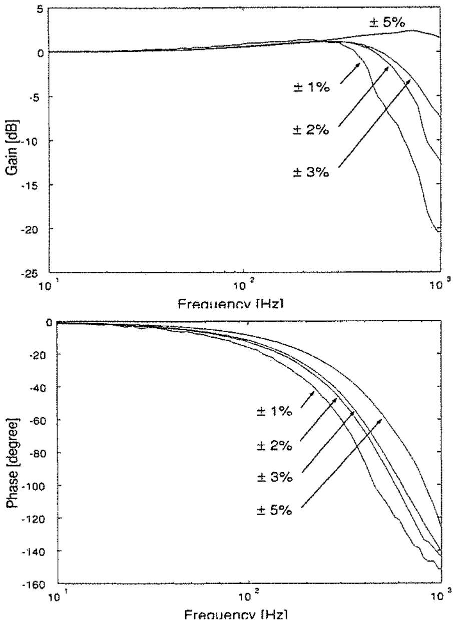

Fig. 16 Closed lood bode plot

6. おわりに

サーボ弁の重要な特性の一つである小振幅応答の追従特性改善に関し，本手法の有効性が実機による微小信号応答，ステップ応答，周波数応答試験結果から確認できた．また，制御器の実装にあたり問題となるスケジューリングパラメータの変化量情報を必要としない，出力フィードバック制御器の設計法を新たに提案した．なお有限個のLMI条件へ帰着させる過程で生じる制御器の保守性に対する問題とワインドアップ現象問題が今後の課題である．

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Gain Scheduled Velocity and Force Controllers for the Electro-hydraulic Servosystem

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Abstract

The load variation of the electro-hydraulic servosystem causes degradation of the control characteristic. This is because the flow characteristic of the flow control valve depends on the load condition. Here, we propose the flow calculation formula to have continues flow between turbulent and laminar flow so that compose the linear plant model with a scheduling parameter. Then, design gain scheduled controllers for the velocity and hydraulic force control.

1 Introduction

Hydraulic control system is used in various industrial applications for the power in size, high durability. The electro-hydraulic servosystem is applied to obtain the accurate position, velocity or force control, means the high reproducibility and the fine dynamic performance in many of cases. However, the hydraulic plant is nonlinear, caused by the relationship between the load pressure and the effective flow from the control valve and asymmetric cylinder, so on. Addition to these, the plant parameters are not sure by the factor such as oil bulk modulus, viscosity and temperature. Because of the electro-hydraulic servosystem capability and the usefulness, mentioned above, a number of studies have investigated for nonlinear features. Recent approaches to design controller for the electro-hydraulic servosystem are robust control design by H_∞ framework[1] and adaptive control[2, 3]. In the controller design, which is based on the linear plant model, only the local stability and performance can be guaranteed. One of the reasons to make the situation difficult is the discontinuity of the control flow calculation formula around the null. In this paper, propose a formula, which interpolates the turbulent and the laminar flow in the flow control valve so that the linear model becomes continuous at the boundary for the precise force control. Then, compose the linear plant model as an LPV(linear parameter varying) system with a scheduling parameter which depends on load force so that design the gain scheduled controller for the velocity and hydraulic force control. Simulation outputs and experiment results of the both velocity and force control loop are presented in the Injection Molding Machine application.

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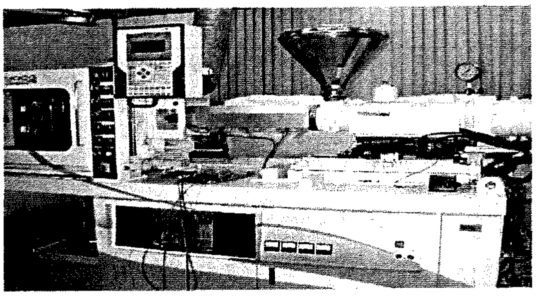
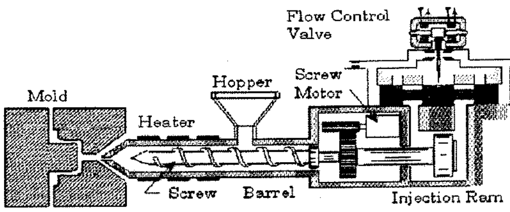
2 Injection Molding Machine Application

In the Injection Molding process, there is a velocity control mode of the injection speed and are force control modes for the holding and back pressure control. The velocity profile is generated in accordance with the mold shape so that the velocity between the melt plastic and mold surface become constant. The force control of the holding pressure mode makes the plastic stress uniform in order to minimize the deformation of the product. The electro-hydraulic servosystem is adapted to both the velocity and force control for the high power, fast response and fine reproducibity. The appearance and the structure scheme of the Injection Molding Machine, which is used in this paper is shown in Figure 1. The main components of the electro-hydraulic servosystem are the hydraulic injection ram cylinder, the flow control valve and transducers for the velocity and force. In this experiment, we adopt the rated velocity is 200 mm/s and the rated force is 160 kN for the specific product. The actual load force in the velocity control mode becomes 40 to 50 kN. In the force control mode, it comes up to 60 kN. The purpose to design the gain scheduled controller here, is to have stable and good performance according to the load force change.



(a) Appearance of the machine



(b) Machine scheme

Figure 1: Test equipment

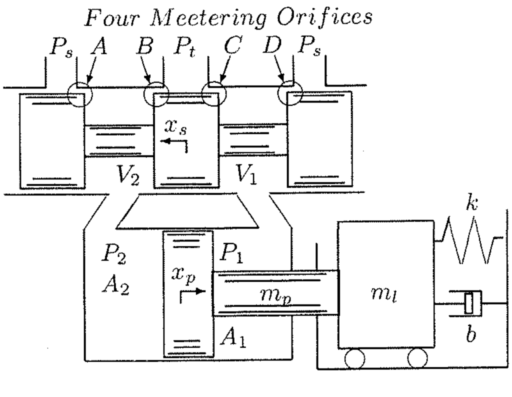


Figure 2: Configuration of the plant

3 Flow Calculation Formulas

In the flow control valve, which has a sleeve and a spool, there are two typical flow conditions, called the turbulent flow at the metering orifice opening and laminar flow in the clearance between sleeve and spool. Now, let think about the flow at the metering orifice A out of four orifices in Figure 2, as an example. There are commonly used flow calculation formulas (1), (3) for each conditions. However, these two equations are not continuous at the boundary. Hence, we adopt the proposed equation (2), which interpolates both of the flow and deviations of the turbulent and the laminar flow conditions. Then, we have continuous plant model in whole operating range.

$$q_{2int} = K_t\sqrt{(x_s-l_a)^2+c_r^2}\sqrt{P_s-P_2} \;\ldots\; l_a \le x_s \quad (1)$$

$$q_{2ini} = \left(\frac{3}{K_l}\right)^3\left(\frac{K_t c_r}{4}\right)^4\frac{(x_s-l_a)^3}{P_s-P_2} + K_t c_r\sqrt{P_s-P_2} \;\ldots\; l_a + x_{sa} \le x_s < l_a \quad (2)$$

$$q_{2inl} = K_l\frac{P_s-P_2}{-(x_s-l_a)} \;\ldots\; x_s < l_a + x_{sa} \quad (3)$$

here, $x_{sa} = -4C_r\sqrt{Ps-P_2}/(3K_tK_l)$

Looking at equations (1),(2) and (3), we understand that the flow from control valve highly depends on load pressure(force) and nonlinear. Because of such a characteristics, a fixed controller at the one operating condition can not satisfy the performances in the over all operating range.

$P_2(t)$, $P_1(t)$ are bore and rod side pressure, A_2, A_1 are effective piston areas of the actuator. m_p and m_l are masses and $x_p(t)$ is piston displacement. K_t, K_l are coefficients of the turburant and the laminar flow. Figure 3 shows lap l_a and c_r is clearance of the spool and sleeve.

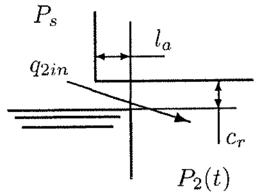


Figure 3: Meetring orifice A

Linearization of the equations (1),(2) and (3) at the arbitrary operating point (x_{s0}, P_{20}). We have

$$\delta q_{2int} = K_t\left(\frac{x_{s0}-l_a}{\sqrt{(x_{s0}-l_a)^2+c_r^2}}\sqrt{P_s-P_{20}}\,\delta x_s - \frac{\sqrt{(x_{s0}-l_a)^2+c_r^2}}{2\sqrt{(P_s-P_{20})}}\delta P_2\right) \quad (4)$$

$$\delta q_{2ini} = \left(\frac{3}{K_l}\right)^3\left(\frac{K_t c_r}{4}\right)^4\frac{3(x_s-l_a)^2}{P_s-P_{20}}\delta x_s + \left(\left(\frac{3}{K_l}\right)^3\left(\frac{K_t c_r}{4}\right)^4\frac{(x_s-l_a)^3}{P_s-P_{20}} - \frac{K_t c_r}{2\sqrt{P_s-P_{20}}}\right)\delta P_2 \quad (5)$$

$$\delta q_{2inl} = K_l\left(\frac{P_s-P_{20}}{-(x_{s0}-l_a)}\delta x_s - \frac{1}{-(x_{s0}-l_a)}\delta P_2\right) \quad (6)$$

When, we choose the corresponding equation from (4), (5) or (6) according to spool displacement $x_s(t)$, we yield simple presentation as equation (7).

$$\delta q_{2in} = K_{2inxs}\delta x_s + K_{2inp2}\delta P_2 \quad (7)$$

In the same manner as mentioned above, linearization of the equations at the metering orifice B, C and D are described below, respectively.

$$\delta q_{2out} = K_{2outxs}\delta x_s + K_{2outp2}\delta P_2 \quad (8)$$

$$\delta q_{1out} = K_{1outxs}\delta x_s + K_{1outp1}\delta P_1 \quad (9)$$

$$\delta q_{1in} = K_{1inxs}\delta x_s + K_{1inp1}\delta P_1 \quad (10)$$

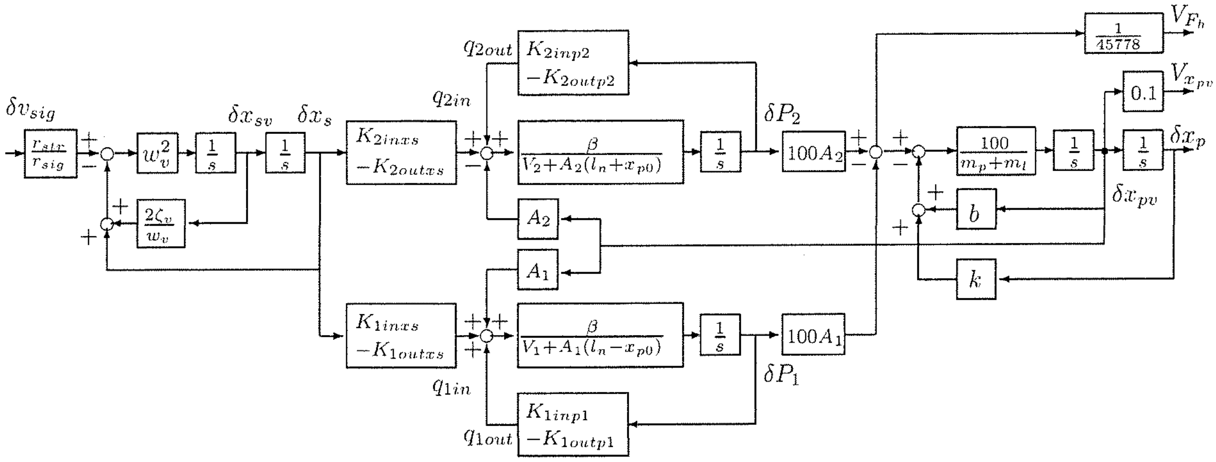


Figure 4: Block diagram of the linearlized plant

4 Modeling

4.1 Linear System

Figure 4 presents the linear block diagram from the input δv_{sig}, which is applied to the flow control valve, to the measured variable hydraulic force V_{F_h} and piston velocity $V_{x_{pv}}$. v_{sig} is the control variable. r_{sig} is the rated signal and r_{str} is the rated spool displacement. w_v and ζ_v represents the control valve dynamics as the second order transfer function. The pressure change in the actuator is calculated from the hydraulic compressibility, called as bulk modulus β, and the effective volume change caused by flow in and out from pressure chamber, plus piston displacement. Masses, an equivalent damper and a spring compose the load. In the injection process, the damper and spring rate change by the various operating conditions, such as velocity and force control mode. Here, we take state vector δx as,

$$\delta x = (\delta x_{sv},\ \delta x_s,\ \delta P_2,\ \delta P_1,\ \delta x_{pv},\ \delta x_p)^T \qquad (11)$$

From the linear block diagram in Figure 4, the linear state space equation is represented in equation (12) and (13). However, some of the matrix elements, such as $a_p(3,2)$ is not constant.

$$\delta\dot{x} = \begin{pmatrix} -2\zeta_v\omega_v & -\omega_v^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_p(3,2) & a_p(3,3) & 0 & a_p(3,5) & 0 \\ 0 & a_p(4,2) & 0 & a_p(4,4) & a_p(4,5) & 0 \\ 0 & 0 & \frac{10^4 A_2}{mp+m_l} & -\frac{10^4 A_1}{mp+m_l} & \frac{10^2 b}{mp+m_l} & \frac{10^2 k}{mp+m_l} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \delta x + \begin{pmatrix} \frac{r_{str}}{r_{sig}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \delta v_{sig} \qquad (12)$$

$$\begin{pmatrix} V_{Fh} \\ V_{x_{pv}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{A_2}{457.78} & -\frac{A_1}{457.78} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \end{pmatrix} \delta x \qquad (13)$$

Here,

$$a_p(3,2) = \beta \frac{K_{2inxs} - K_{2outxs}}{V_2 + A_2(L_n + x_{p0})}$$
$$a_p(3,3) = \beta \frac{K_{2inP_2} - K_{2outP_2}}{V_2 + A_2(L_n + x_{p0})}$$
$$a_p(3,5) = -\beta \frac{A_2}{V_2 + A_2(L_n + x_{p0})}$$
$$a_p(4,2) = \beta \frac{K_{1inxs} - K_{outxs}}{V_1 + A_1(L_n - x_{p0})}$$
$$a_p(4,4) = \beta \frac{K_{1inP_1} - K_{1outP_1}}{V_1 + A_1(L_n - x_{p0})}$$
$$a_p(4,5) = \beta \frac{A_1}{V_1 + A_1(L_n - x_{p0})}$$

L_n shows a half of the total piston stroke.

4.2 LPV System

As mentioned before, some of the elements in the state space equation vary according to the operating conditions. To have linear state space equation, we apply the LPV system presentation, which has the parameter that is the function of the load force. Equation 12 has the form as

$$\delta\dot{x} = \frac{\partial}{\partial x} f(x_0)\delta x + B_p \delta v_{sig} \qquad (14)$$
$$\delta y = C_p \delta x \qquad (15)$$

The matrix B_p and C_p are constant and all elements in these matrices are decided from the mechanical specifications. x_0 is a state at the equilibrium point. But, it is difficult to have the solution x_0 from the implicit function of $f(x_0) + B_p v_{sig0} = 0$. Now, here suppose that x_0 is given and then y_0 is calculated by $y_0 = C_p x_0$. Addition to this, suppose that the scheduling parameter θ is a smooth function of the y_0, such as $\theta = \varphi(y_0)$.

The linear state space equation, which depends on the scheduling parameter θ, represents the plant behavior in the neighborhood of the equilibrium x_0 [4, 5]. By the way, $a_p(3,2)$, $a_p(4,4)$ and so on, are decided when x_{s0}, P_{20}, P_{10} and x_{p0} are fixed. But, as explained already, it is hard to decide these values from the related equations. Therefore, we adopt the means which use the value of the variable from the simulation result. For the closed loop simulation, we use PI velocity and force control in the nonlinear plant model. Then, apply relatively slow enough ramp velocity and force reference signal ($\delta v_{sig} = 0$) so that we are able to assume the state variables are close enough to the equilibrium states. Now, we have a set of the equilibrium points for the specified operating range in both velocity and force control. Using the values of the entries in the equilibrium set, we figure out the elements, such as $a_p(3,2)$ so on, with respect to corresponding scheduling parameter which is described as equation (19) or (20). Then, we represent these elements by the approximate 3rd polynomial, as follows

$$a_p(\theta) = a_{p0} + a_{p1}\theta + a_{p2}\theta^2 + a_{p3}\theta^3 \qquad (16)$$

The state space equation (12), (13) yields to equation (17), (18) with the scheduling parameter.

$$\delta\dot{x}_p = A_p(\theta)\delta x_p + B_p\delta v_{sig} \qquad (17)$$

$$\delta y_p = C_p\delta x_p \qquad (18)$$

Considering the flow characteristic which depends on $\sqrt{\triangle P}$, define the scheduling parameter θ_v for the velocity and θ_n for the force control.

$$\theta_v = \frac{\dfrac{1}{\sqrt{1-\frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1-\frac{F_{v\cdot rated}}{F_{max}}}} - 1}, \quad [0 \le \theta_v \le 1] \qquad (19)$$

$$\theta_n = \frac{\dfrac{1}{\sqrt{1-\frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1-\frac{F_{n\cdot rated}}{F_{max}}}} - 1}, \quad [0 \le \theta_n \le 1] \qquad (20)$$

F_{max} is maximum force, $F_{v\cdot rated}$ and $F_{n\cdot rated}$ are the rated force in the velocity and force control mode, F_x is actual force which is measured as V_{F_h}. Figure 5 shows the plant bode plot that depends on the value of the scheduling parameter θ_v in the velocity mode. And, Figure 6 presents bode plot in the force mode.

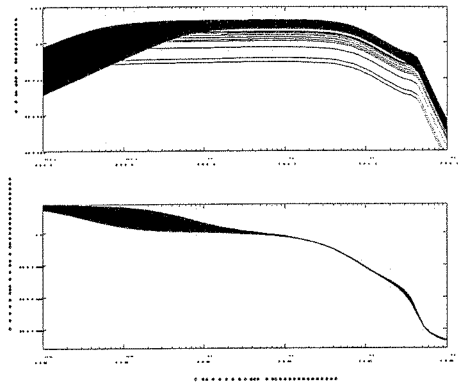


Figure 5: Plant Varying Range in the Velocity Mode

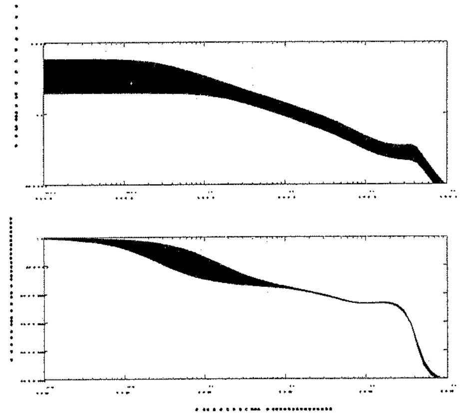


Figure 6: Plant Varying Range in the Force Mode

5 Gain Scheduled Controllers

When we design the gain scheduled controller, we take following issues in consideration. In the velocity loop, the rise time is within 15ms to the step reference and minimizes steady state error.

In the force controller design, the force follows to the ramp reference signal, which reaches to the rated force with 15 ms and zero steady state error. In order to construct the generalized plant for the H_∞ controller design framework, two weighting functions, $W_s(s)$ for the sensitivity function and $W_a(s)$ for the additive uncertainty at the input of the plant, are specified after the several try and error. For the velocity controller, we use

$$W_{sv}(s) = \frac{0.0025s + 68.75}{s + 0.005}, \quad W_{av}(s) = \frac{5(s+1)}{s+375}$$

and at the force controller design

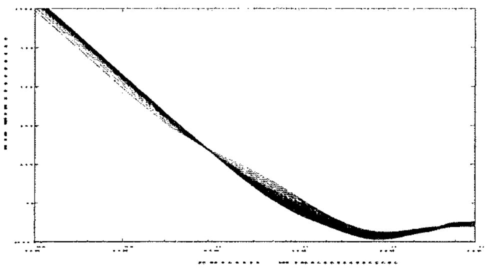
$$W_{sn}(s) = \frac{0.4s + 5}{s + 0.01}, \quad W_{an}(s) = \frac{25(s+1)}{s+75}$$

Beside these weighting functions, $(0.1s + 0.015)/s$ is added in series to the plant in the velocity control.

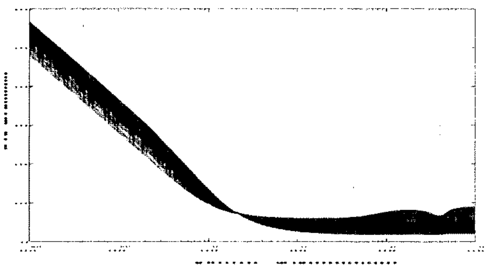
Also, $(50s+1)/(s+0.01)$ is in series to force plant to improve the response performance. When solving H_∞ controller design problem with LMI formulation, the two positive definite matrices, in many cases described as $\mathcal{X}$ and $\mathcal{Y}$, also let the function of the scheduling parameter, such as

$$\mathcal{X}_v(\theta_v) = \mathcal{X}_{v0} + \mathcal{X}_{v1}\theta_v + \mathcal{X}_{v2}{\theta_v}^2 + \mathcal{X}_{v3}{\theta_v}^3$$

so that minimize the conservatives of the controller (for the details, see[6, 7]). As the results, the generalized plant becomes function of the continuous scheduling parameter and has to solve infinite number of LMIs. For reducing to finite number of constraints a technique, that proposed by Azuma et al[8] to construct a convex hull that covers the model, is introduced. Figure 7-(a), (b) are gain plots of the gain scheduled controller for the velocity and force control.



(a) Velocity Controller

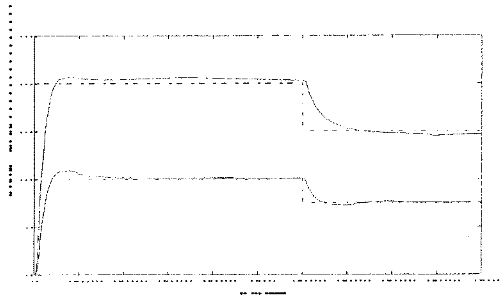


(b) Force Controller

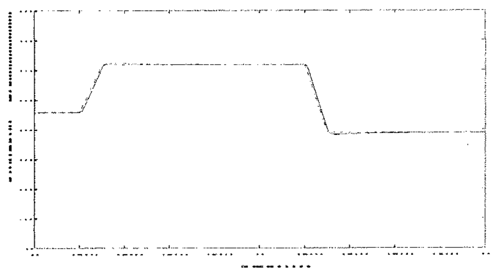
Figure 7: Gain Plot of Gain Scjeduled Controller

6 Simulation and Test Results

The simulation results, applying the designed velocity controller to the nonlinear plant model, are shown in Figure 8-(a) for the 0 to 10 cm/s and 0 to 20 cm/s step references. These responses satisfy the requirements quite well. Figure 8-(b) shows force control response. We obtained good tracking performance in force control, too. The test results in velocity control is shown in Figure 9-(a), (b).

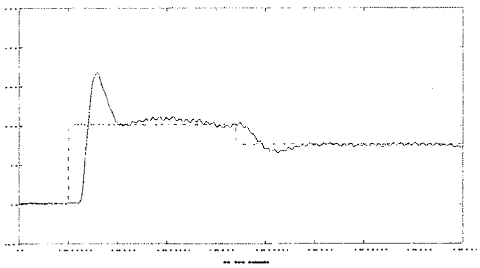


(a) Velocity Control

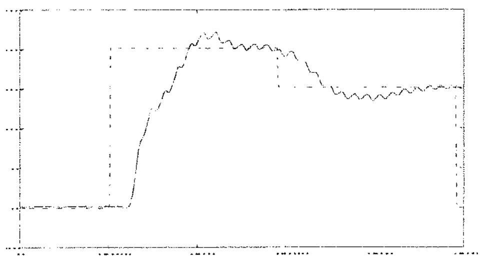


(b) Force Control

Figure 8: Simulation Results



(a) 0 to 10 cm/s



(b) 0 to 20 cm/s

Figure 9: Test Results for Veloacity Control

There is a quite big over shoot in the 10 cm/s response and observed 9 to 10 ms dead time. In the 0 to 20 cm/s response, to minimize the over shoot, controller out put is saturated in adequate level. This leads to the slow response and big differences between simulation results and experimental results. To examine the cause for the over shoot, run the velocity simulation again with 3.5 ms dead time at the flow control valve response. The results from this condition are shown in Figure 10 and we observe the quite big over shoot, same as the results from the experiment.

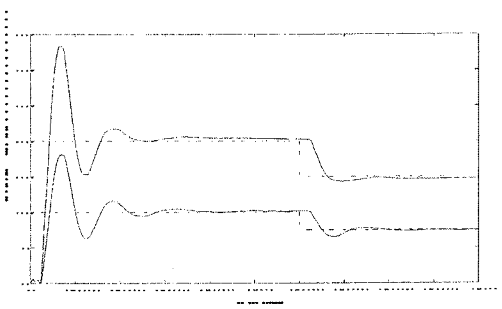


Figure 10: Simulation Results in Veloacity Control with Dead Time

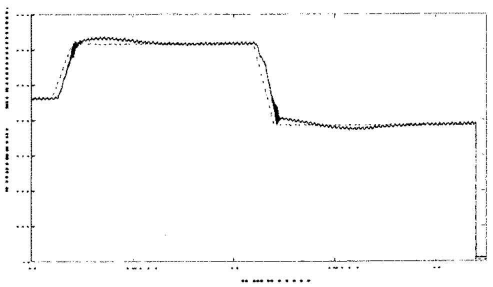


Figure 11: Test Result in the Force Control

On the other hand, as for the pressure control, the approximately expected result is obtained as Figure11. In this case, the force control is not as sensitive to the dead time as the velocity control.

7 Conclusion

In this paper, propose the flow calculation formula to have continues flow between turbulent and laminar flow so that compose the linear plant model with the scheduling parameter. Then, obtained the values of the system variables by the simulation in order to design the gain scheduled controller for the velocity and force control in the electro-hydraulic servosystem. The designed controller usefulness is confirmed by simulation. In the force control, we have reasonable results, so far. However, in velocity control, because of the dead time, we have the not satisfied response. The design to have considered the dead time is a subject in the future.

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Adaptive Identification as Gain Scheduling

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Abstract

In this paper, we consider the Lyapunov approach to adaptive identification for systems with unknown time invariant parameter. We describe the adaptive identification problem as stability analysis for linear parameter varying system, and then discuss the convergence of identification error and its convergence rate based on parameter dependent Lyapunov function.

keywords : gain scheduling, adaptive identification, parameter dependent Lyapunov function, linear parameter varying system

1 Introduction

Adaptive identification[1] plays an important role in adaptive control system, especially in model reference adaptive control system(MRACS)[4][5], as an on-line identification technique for systems with unknown parameter. The objective of the MRACS is to track the output of the system with unknown parameter to the output of the reference model. The MRACS achieves this objective by combining the adaptive identification with model reference control that is a technique to track the output of the known system to the output of the reference model.

While Morse[5] etc. provide essential results on the stability of the MRACS, which comprehend solution to the problem on the asymptotic convergence of the identification error in the adaptive identification. These results on the adaptive identification problem in the conventional adaptive control theory are based on Lyapunov's 2nd method[9]. However, these results require another discussion[5] to guarantee the asymptotic convergence of the identification error.

Almost all the discussions on the convergence of the identification error in the conventional adaptive control theory are summarized as follows.

1) For adaptive identification system, calculate the state space representation whose state includes identification error. Call this state space representation an error system.

2) Prove the state of the error system is bounded for all the bounded initial states.

3) Prove the bounded state of the error system converges to zero.

In the discussion of conventional adaptive control theory, Lyapunov's 2nd method is applied in the step of 2). Namely a quadratic Lyapunov function defined by a positive definite matrix is introduced for the analysis in that step. For this quadratic Lyapunov function V, it is shown that V satisfies the condition below.

$$V > 0, \quad \dot{V} \leq 0. \tag{1}$$

As is well known, this condition guarantees the boundedness of the identification error. However, this condition does not guarantee that the identification error converges to zero, so the step of 3), called the "another discussion" in the preceding paragraph, is required[5].
In this paper we consider the adaptive identification problem from a viewpoint that is different from that of the conventional adaptive control theory and show that the problem can be regarded as the stability analysis problem for LPV(Linear Parameter Varying) system treated in modern

gain scheduling[6]. Then based on this consideration, we introduce parameter dependent Lyapunov function[7][8] for the stability analysis. For this parameter dependent Lyapunov function V, we show V satisfies the condition below. (Cf. (1))

$$V > 0, \quad \dot{V} < 0.$$

By this approach, we can guarantee that the identification error converges to zero without the discussion in the step of 3). In addition we can discuss convergence rate of identification error.

This paper is organized as follows. In section 2, first we state the description of a system with unknown parameter and assumptions for the system. Then, we formulate the adaptive identification problem. In section 3, we discuss the convergence of identification error. First we point out that the adaptive identification problem can be regarded as stability analysis for LPV system. Then we show that the identification error converges to zero exponentially under some condition based on parameter dependent Lyapunov function.

Notation: $\|x\|$ is the Euclidean norm of a vector x. P' is the transpose of a matrix P. $P > (<)0$ denotes a symmetric matrix P is positive(negative) definite and $P \geq (\leq)0$ denotes a symmetric matrix P is positive(negative) semi-definite. $\mathcal{E}(P)$ denotes the following domain defined by a matrix $P > 0$.

$$\mathcal{E}(P) = \{\zeta |\, \zeta' P \zeta \leq 1\}.$$

2 Adaptive Identification Problem

2.1 System Description

Consider a linear time invariant system $\Sigma(\theta)$ of the state space form

$$\frac{d}{dt}x = A_0 x + \theta A_1 x + Bw(t), \quad x(0) = 0, \qquad (2)$$

where $x \in R^n$ and $w(t) \in R^m$ denote the state and the input of the system $\Sigma(\theta)$, respectively. θ is a time invariant unknown parameter that satisfies the condition below.

$$\theta \in [0, 1].$$

We assume that the system $\Sigma(\theta)$ satisfies the following four conditions.

A-1 : The state x of the system $\Sigma(\theta)$ is measurable.

A-2 : The input $w(t)$ of the system $\Sigma(\theta)$ is restricted in the bounded domain $\mathcal{D} \in R^m$ as follows.

$$w(t) \in \mathcal{D}, \quad \forall t \in [0, \infty). \qquad (3)$$

A-3 : There exists a matrix $P > 0$ such that

$$A_0' P + P A_0 < 0. \qquad (4)$$

A-4 : The state reachable set[3] of the system $\Sigma(\theta)$ is bounded for all $\theta \in [0, 1]$.

Remark The state reachable set in the condition A-4 denotes the reachable domain of the state x in the state space by the inputs $w(t)$ that satisfies the condition A-2. In the case where $\mathcal{D}$ is an ellipsoidal domain $\mathcal{E}(W)$ defined by a matrix $W > 0$, the following condition is known as a sufficient condition for that the system $\Sigma(\theta)$ satisfies the condition A-4[2].

There exist a matrix $Q > 0$ and a real number $\beta \geq 0$ such that

$$\begin{bmatrix} \begin{pmatrix} [A_0 + \theta A_1]' Q \\ + Q[A_0 + \theta A_1] + \beta Q \end{pmatrix} & QB \\ B'Q & -\beta W \end{bmatrix} \leq 0,$$
$$\forall \theta \in [0, 1]. \qquad (5)$$

See appendix for detail of the state reachable set.

Remark If the following condition is assumed instead of the condition A-3,

A-3' : There exists a matrix $P > 0$such that

$$(A_0 + \theta A_1)' P + P(A_0 + \theta A_1) < 0, \quad \forall t \in [0, 1],$$

then the condition A-4 holds automatically.

2.2 Problem Formulation

Framework of the adaptive identification in this paper is shown in Fig.1. In Fig.1, θ is a time invariant unknown parameter, $K(x, w)$ is a parameter identification law, $\bar{\theta}$ is a parameter estimate, and $e_\theta = \bar{\theta} - \theta$ is an identification error. We describe the adaptive identification problem in this paper as follows.

Problem Find the parameter identification law $K(x, w)$ that achieves

$$\lim_{t \to \infty} e_\theta = 0.$$

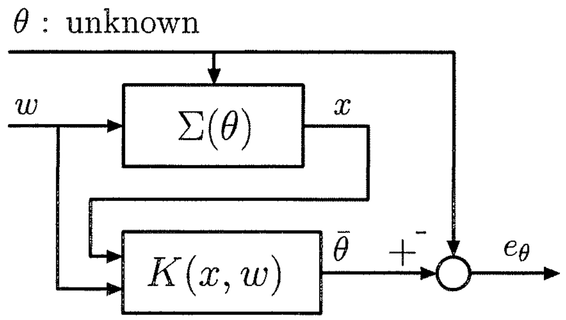

Figure 1: Framework for Adaptive Identification

3 Convergence of Parameter Estimate

3.1 Parameter Identification Law

We consider the parameter identification law $K(x,w)$ defined by the two differential equations below,

$$\frac{d}{dt}\bar{x} = A_0\bar{x} + \bar{\theta}A_1x(t) + Bw(t), \quad \bar{x}(0) = 0, \quad (6)$$

$$\frac{d}{dt}\bar{\theta} = -\frac{1}{\alpha}[A_1x(t)]'P(\bar{x} - x(t)), \quad \bar{\theta}(0) \in [0,1], \quad (7)$$

where $\alpha > 0$ is a real number, and $P > 0$ is a matrix that satisfies the condition A-3.

Remark The parameter identification law $K(x,w)$ given by (6) and (7) is essentially equivalent to that of [4][5].

3.2 Equivalent Representation

Then we consider the equivalent representation of the block diagram in Fig.1. From (2), (6), and (7), the state equations of the block diagram in Fig.1 are given as follows.

$$\frac{d}{dt}x = (A_0 + \theta A_1)x + Bw(t), \quad (8)$$

$$\frac{d}{dt}e_x = A_0e_x + [A_1x]e_\theta, \quad (9)$$

$$\frac{d}{dt}e_\theta = -\frac{1}{\alpha}[A_1x]'Pe_x. \quad (10)$$

Where e_x is defined as follows,

$$e_x = \bar{x} - x,$$

and we should note that the following equation holds since the unknown parameter θ is time invariant.

$$\frac{d}{dt}e_\theta = \frac{d}{dt}\bar{\theta}.$$

Then we describe (9) and (10) as follows.

$$\frac{d}{dt}\begin{bmatrix} e_x \\ e_\theta \end{bmatrix} = \begin{bmatrix} A_0 & G(x(t)) \\ F(x(t)) & 0 \end{bmatrix}\begin{bmatrix} e_x \\ e_\theta \end{bmatrix}. \quad (11)$$

We call this differential equation a system $\Gamma(x)$. In (11), $G(x)$ and $F(x)$ are given as follows.

$$G(x) = A_1x,$$
$$F(x) = -\frac{1}{\alpha}[A_1x]'P = -\frac{1}{\alpha}G'(x)P$$

Since the equation (8) is the state equation of the system $\Sigma(\theta)$, the block diagram in Fig.1 can be represented by the block diagram in Fig.2.

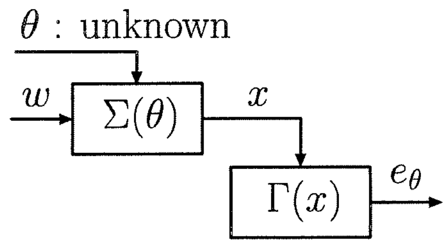

Figure 2: Equivalent Representation

Remark By the condition A-1 and the block diagram of Fig.2, the state x is measurable signal determined by the unknown parameter θ, the initial state $x(0)$ and the input $w(t)$. In addition, by the condition A-4, the state x has a common peak value for all $w(t)$ that satisfies the condition A-2.

Remark Since the signal x in Fig.2 is measurable and bounded, it is possible to regard the system $\Gamma(x)$ as LPV (Linear Parameter Varying) system[6] with scheduling parameter x.

Remark The block diagram in Fig.2 is nonlinear system that consists of two linear systems, linear time-invariant system $\Sigma(\theta)$ and the LPV system $\Gamma(x)$.

3.3 Analysis of Convergence

Then we discuss the convergence of the parameter estimate in the framework of Fig.2. First we note that the role of the system $\Sigma(\theta)$ in Fig.2 is only a generator of the scheduling parameter x for the LPV system $\Gamma(x)$. Then we note that the identification error e_θ is a part of the state of the LPV system $\Gamma(x)$. These suggest that we don't need to analyze the nonlinear system shown in Fig.2 to discuss the adaptive identification problem in this paper, it is enough to discuss the asymptotic stability of the LPV system $\Gamma(x)$ in Fig.2. In this case, we can apply the technique of modern gain scheduling, especially the analysis technique based on parameter dependent Lyapunov function[7][8] since the system $\Gamma(x)$ is LPV system.

Theorem 1 Suppose that the state $x(t)$ of the system $\Sigma(\theta)$ satisfies the following condition for some real number $\gamma > 0$.

$$\|A_1 x(t)\| \geq \gamma, \quad \forall t \in [0, \infty). \tag{12}$$

Then there exist real numbers $K > 0$ and $\mu > 0$ such that the parameter identification law $K(x, w)$ defined by (6) and (7) achieves the condition below.

$$|\bar{\theta}(t) - \theta| \leq K e^{-\mu t}. \tag{13}$$

Remark The condition (12) corresponds to the PE (persistently exciting) condition in the adaptive control theory.

Remark Theorem 1 guarantees that the error e_θ of identification converges to zero exponentially. While the results of adaptive control theory[4][5] do not mention the convergence rate.

Proof

Step 1: For $e_x \in R^n$, $e_\theta \in R$, and $x(t) \in R^n$, we define a function $V(e_x, e_\theta, x(t))$ as follows.

$$V(e_x, e_\theta, x(t)) = \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}' \begin{bmatrix} P & P_1(x(t)) \\ P_1'(x(t)) & \alpha \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}.$$

The definition of the function $P_1(x)$ will be given in the next step. Then we define the function $\dot{V}(e_x, e_\theta, x(t))$ for the function $V(e_x, e_\theta, x(t))$ as follows[9].

$$\dot{V}(e_x, e_\theta, x(t)) = \begin{bmatrix} \frac{\partial V}{\partial e_x} & \frac{\partial V}{\partial e_\theta} \end{bmatrix} \times \begin{bmatrix} A_0 & G(x(t)) \\ F(x(t)) & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \end{bmatrix} + \frac{\partial V}{\partial x} \frac{dx}{dt}(t).$$

Next we define the functions $L_1(x)$, $L_2(x, \dot{x})$, and $L_3(x)$ for $x \in R^n$ and $\dot{x} \in R^n$ as follows.

$$L_1(x) = A_0'P + PA_0 + F'(x)P_1'(x) + P_1(x)F(x)$$

$$L_2(x, \dot{x}) = F'(x)\alpha + A_0'P_1(x) + PG(x) + \sum_{j=1}^{n} \frac{\partial P_1}{\partial x_j}(x)\dot{x}_j$$

$$L_3(x) = G'(x)P_1(x) + P_1'(x)G(x).$$

Then the function $\dot{V}(e_x, e_\theta, x(t))$ is represented by $L_1(x)$, $L_2(x, \dot{x})$, and $L_3(x)$ as follows.

$$\dot{V}(e_x, e_\theta, x(t)) = \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}' \times \begin{bmatrix} L_1(x(t)) & L_2(x(t), \dot{x}(t)) \\ L_2'(x(t), \dot{x}(t)) & L_3(x(t)) \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}.$$

Step 2: For a real number $\delta > 0$, we define the function $P_1(x)$ as follows.

$$P_1(x) = -\delta G(x) = -\delta A_1 x.$$

Where δ is a design parameter. For this function $P_1(x)$, $L_1(x)$, $L_2(x, \dot{x})$, and $L_3(x)$ is computed as follows.

$$L_1(x) = A_0'P + PA_0 - \delta \left[F'(x)G'(x) - G(x)F(x)\right]$$

$$L_2(x, \dot{x}) = F'(x)\alpha + PG(x) - \delta \left[A_0'G(x) + G(\dot{x})\right]$$

$$L_3(x) = -2\delta G'(x)G(x).$$

Since $F(x)$ is given by

$$F(x) = -\frac{1}{\alpha}G'(x)P,$$

$L_2(x, \dot{x})$ is computed as follows.

$$L_2(x, \dot{x}) = -\delta \left[A_0'G(x) + G(\dot{x})\right].$$

Step 3: We define $\hat{L}_1(x)$, $\hat{L}_2(x, \dot{x})$, and $\hat{L}_3(x)$ for $L_1(x)$, $L_2(x, \dot{x})$, and $L_3(x)$ as follows.

$$\begin{bmatrix} \hat{L}_1(x) & \hat{L}_2(x, \dot{x}) \\ \hat{L}_2'(x, \dot{x}) & \hat{L}_3(x) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1/\sqrt{\delta} \end{bmatrix} \times \begin{bmatrix} L_1(x) & L_2(x, \dot{x}) \\ L_2'(x, \dot{x}) & L_3(x) \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 1/\sqrt{\delta} \end{bmatrix}. \tag{14}$$

$\hat{L}_1(x)$, $\hat{L}_2(x,\dot{x})$, and $\hat{L}_3(x)$ is computed as follows.

$$\begin{aligned}\hat{L}_1(x) &= A_0'P + PA_0 \\ &\qquad - \delta\left[F'(x)G'(x) - G(x)F(x)\right] \\ \hat{L}_2(x,\dot{x}) &= -\sqrt{\delta}\left[A_0'G(x) + G(\dot{x})\right] \\ \hat{L}_3(x) &= -2G'(x)G(x).\end{aligned}$$

At this point we note that the following two inequalities are equivalent conditions since the design parameter δ is a positive real number.

$$\begin{bmatrix} L_1(x) & L_2(x,\dot{x}) \\ L_2'(x,\dot{x}) & L_3(x) \end{bmatrix} <0,$$

$$\begin{bmatrix} \hat{L}_1(x) & \hat{L}_2(x,\dot{x}) \\ \hat{L}_2'(x,\dot{x}) & \hat{L}_3(x) \end{bmatrix} <0.$$

Step 4: By the condition A-3 for the system $\Sigma(\theta)$, there exists a matrix $P>0$ such that

$$A_0'P + PA_0 < 0.$$

While the peak values of the system $\Sigma(\theta)$'s state $x(t)$ and its derivative $\dot{x}(t)$ are bounded by the condition A-4 for the $\Sigma(\theta)$. In addition, from the condition (12) of the theorem 1, the state $x(t)$ of the system $\Sigma(\theta)$ satisfies the following condition for some real number $\gamma > 0$.

$$G'(x)G(x) = x'A_1'A_1x \geq \gamma^2.$$

Therefore if we chose the design parameter $\delta > 0$ sufficiently small, then the next inequality holds.

$$\begin{bmatrix} P & P_1(x(t)) \\ P_1'(x(t)) & \alpha \end{bmatrix} = \begin{bmatrix} P & -\delta A_1 x(t) \\ -[\delta A_1 x(t)]' & \alpha \end{bmatrix} >0. \qquad (15)$$

Similarly next inequality holds for sufficiently small $\delta > 0$.

$$\begin{bmatrix} \hat{L}_1(x(t)) & \hat{L}_2(x(t),\dot{x}(t)) \\ \hat{L}_2'(x(t),\dot{x}(t)) & \hat{L}_3(x(t)) \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} A_0'P + PA_0 \\ -\delta[F'(x)G'(x) \\ -G(x)F(x)] \end{pmatrix} & -\sqrt{\delta}\left[A_0'G(x) + G(\dot{x})\right] \\ -\sqrt{\delta}\left[G'(x)A_0 + G'(\dot{x})\right] & -2G'(x)G(x) \end{bmatrix} < 0. \qquad (16)$$

Thus the following condition holds for all realizable state $x(t)$ of the system $\Sigma(\theta)$ that satisfies (12).

$$V(e_x, e_\theta, x(t)) > 0, \quad \dot{V}(e_x, e_\theta, x(t)) < 0.$$

Where inequality sign denotes the function is positive or negative definite[9]. Since the system $\Gamma(x)$ is linear and its initial state is bounded, we can conclude that there exist real numbers $K > 0$ and $\mu > 0$ such that the state of the system $\Gamma(x)$ satisfies the condition below[9].

$$\left\| \begin{bmatrix} e_x(t) \\ e_\theta(t) \end{bmatrix} \right\| \leq Ke^{-\mu t}.$$

Q.E.D.

Remark In the proof of theorem 1, the boundedness of the system $\Sigma(\theta)$'s state reachable set plays an essential role. By this property, (15) and (16) hold for sufficiently small $\delta > 0$. In the case where the state reachable set of the system $\Sigma(\theta)$ is not bounded, for instance the input $w(t)$ of the system $\Sigma(\theta)$ is not restricted in the bounded domain, it is impossible to choose such δ.

Remark The typical Lyapunov function V in the conventional adaptive control theory is given as follows.

$$V(e_x, e_\theta) = \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}' \begin{bmatrix} P & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}.$$

For this V, function $\dot{V}$ is computed as follows.

$$\dot{V}(e_x, e_\theta) = \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}' \begin{bmatrix} A_0'P + PA_0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \end{bmatrix}.$$

Clearly, V and $\dot{V}$ satisfies the conditions below.

$$V(e_x, e_\theta) > 0, \quad \dot{V}(e_x, e_\theta) \leq 0,$$

The boundedness of the identification error e_θ derives from this condition but the convergence to zero is not guaranteed[9].

4 Conclusion

In this paper, we have considered the convergence of parameter estimate in the adaptive identification from the viewpoint of modern gain scheduling. The adaptive identification law used in this paper was essentially equivalent to that of conventional adaptive control theory. However, it guaranteed not only that the identification error converges to zero but also the convergence rate.

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Appendix: State Reachable Set

Consider a linear system with unknown time-varying parameter θ of the state space form

$$\frac{d}{dt}x = A(\theta(t))x + Bw(t), \quad x(0) = 0,$$

where $x \in R^n$ and $w(t) \in R^m$ denote a state and an input, respectively. We call this system $\Sigma_x(\theta)$. We assume that the time varying parameter θ satisfies the condition below.

$$\theta(t) \in [0,1], \quad \forall t \in [0,\infty).$$

We also assume that the input $w(t)$ is restricted in the bounded domain $\mathcal{D} \in R^m$ as follows.

$$w(t) \in \mathcal{D}, \quad \forall t \in [0,\infty). \tag{17}$$

Definition[3] $\xi \in R^n$ is said to be reachable from $x(0)$ if there exists $w(t)$ that satisfies the condition (17) and a finite time $T \in [0,\infty)$ such that $x(T) = \xi$.

Definition[3] The state reachable set $\mathcal{R}_x(\Sigma_x(\theta))$ of the system $\Sigma_x(\theta)$ is defined as follows.

$$\mathcal{R}_x(\Sigma_x(\theta)) \\ = \{\xi \in R^n : \xi \text{ is state reachable from } x(0)\}.$$

In the case where the set $\mathcal{D}$ is given by $\mathcal{D} = \mathcal{E}(W)$ for some matrix $W > 0$, the next theorem is known as a result for the evaluation of the state reachable set.

Theorem 2 [2] Suppose that there exist a matrix $Q > 0$ and a real number $\beta \geq 0$ such that

$$\begin{bmatrix} A'(\theta^*)Q + QA(\theta^*) + \beta Q & QB \\ B'Q & -\beta W \end{bmatrix} \leq 0, \quad \theta^* \in [0,1].$$

Then the state reachable set $\mathcal{R}_x(\Sigma_x(\theta))$ of the system $\Sigma_x(\theta)$ satisfies the condition below.

$$\mathcal{R}_x(\Sigma_x(\theta)) \subset \mathcal{E}(Q), \quad \forall \theta \in \Theta.$$

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A Generalized H_∞ Control System Design Attenuating Initial State Uncertainties

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Abstract

This paper deals with a generalized H_∞ control attenuating initial-state uncertainties. An H_∞ control problem, which treats a mixed attenuation of disturbance and initial-state uncertainty for linear time-invariant systems in the infinite-horizon case, is examined. The mixed attenuation supplies H_∞ controls with good transients and assures H_∞ controls of robustness against initial-state uncertainty. We derive a necessary and sufficient condition of the generalized mixed attenuation problem. Furthermore we apply this proposed method to a magnetic suspension system, and evaluate attenuation property of the proposed generalized H_∞ control approach.

keywords: H_∞ Control, DIA Control, Initial-State Uncertainties, Magnetic Suspension Systems

1 Introduction

H_∞ control for linear time-invariant systems attenuates the effect of disturbances on controlled outputs and is originally defined under the assumption that the initial states of the system are zero. Initial states are often uncertain and might be zero or non-zero. If the initial states are non-zero, the system adopting an H_∞ control will present some transients as the effect of the non-zero initial states, to which the H_∞ control is not intrinsically responsible. It is expected that the mixed attenuation of disturbance and initial-state uncertainty in controlled outputs supplies H_∞ controls with some good transients and assures H_∞ controls of robustness against initial-state uncertainty. Recently, hybrid/switching control are actively studied, this method might be one of the reasonable approach to implement them. In the finite-horizon case, a generalized type of H_∞ control problem which formulated and solved by Uchida and Fujita[1] and Khargonekar et al.[2]. This problem was extended to the infinite-horizon case, and a result was derived by Uchida et al.[3](see also Khargonekar et al.[2]). The problem discussed in [3] was, however, limited to time-invariant systems satisfying the orthogonality assumptions [4, 5]. This is an immensely serious problem as a matter of fact, if we apply this problem setup to the real physical control system design. The previous mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case is not sufficient[6] in practice, because time-invariant systems satisfying the orthogonality assumptions restrict the degrees of freedom of the control system design, and have difficulties in regulating control inputs[6].

In this paper, we have formulated an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions. The solution based on [4] is given as a natural but complicated extension of the previous results in [3, 6]. A necessary and sufficient condition for a solution to exist, together with an explicit formula of the solution, is derived. Based on the condition, a robustness property of H_∞ controls against initial-state uncertainty is discussed. Moreover, we apply the proposed approach to a magnetic suspension system, and evaluate the effectiveness of the generalized H_∞ control attenuating initial state uncertainties comparing with the previous results[6].

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2 Problem Statement

Consider the linear time-invariant system which is defined on the time interval $[0, \infty)$ and described by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $x \in R^n$ is the state and x_0 is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. Without loss of generality, we regard x_0 as the initial-state uncertainty, and $x_0 = 0$ as a known initial-state case. The disturbance $w(t)$ is a square integrable function defined on $[0, \infty)$. Note that this system does not have the orthogonality assumptions[5]. A, B_1, B_2, C_1, C_2, D_{12} and D_{21} are constant matrices of appropriate dimensions and satisfies that

- (A, B_1) is controllable and (A, C_1) is observable
- (A, B_2) is controllable and (A, C_2) is observable
- $D_{12}^T D_{12} \in R^{r \times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m \times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by a linear time-invariant system of the form

$$\begin{aligned} u &= J\zeta + Ky \\ \dot{\zeta} &= G\zeta + Hy, \quad \zeta(0) = 0 \end{aligned} \tag{2}$$

which makes the closed-loop system given by (1) and (2) internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension; J, K, G and H are constant matrices of appropriate dimensions.

For the system and the class of admissible controls described above, consider a mixed-attenuation problem stated as below.

Problem 1 DIA Control Problem
Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, z satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

for all $w \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$.

We call such an admissible control the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (**DIA**) control. The weighting matrix N on x_0 is a measure of relative importance of the initial-state uncertainty attenuation to the disturbance attenuation. A larger choice of N in the sense of matrix inequality order means finding an admissible control which attenuates the initial-state uncertainty more. In the special case when the initial state is known, that is $x_0 = 0$, the problem is reduced to finding an admissible control which assures that

$$\|z\|_2^2 < \|w\|_2^2 \tag{4}$$

for all $w \in L^2[0, \infty)$. Then, we call the admissible control the H_∞ control as usual.

3 Mixed attenuation of disturbance and initial-state uncertainty

From the definition, a DIA control should be an H_∞ control when the initial state is known($x_0 = 0$). This implies that, in order to solve the DIA control problem, we require the so-called Riccati equation conditions:

(A1) There exists a solution $M > 0$ to the Riccati equation

$$\begin{aligned} &M(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &+(A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T M \\ &-M(B_2(D_{12}^T D_{12})^{-1} B_2^T - B_1 B_1^T) M \\ &+C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \tag{5}$$

such that

$$\begin{aligned} &A - B_2(D_{12}^T D_{12})^{-1} D_{12}^T C_1 \\ &\quad -B_2(D_{12}^T D_{12})^{-1} B_2^T M + B_1 B_1^T M \end{aligned} \tag{6}$$

is stable.

(A2) There exists a solution $P > 0$ to the Riccati equation

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2) P \\ &+P(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2)^T \\ &-P(C_2^T (D_{21} D_{21}^T)^{-1} C_2 - C_1^T C_1) P \\ &+B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T = 0 \end{aligned} \tag{7}$$

such that

$$A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2$$

$$-PC_2^T(D_{21}D_{21}^T)^{-1}C_2 + PC_1^T C_1 \quad (8)$$

is stable.

(A3) $\rho(PM) < 1$,
where $\rho(X)$ denotes the spectral radius of matrix X, and $\rho(X) = \max|\lambda_i(X)|$.

Then we can obtain the following result.

Lemma 1 *Suppose that the conditions (A1), (A2) and (A3) are satisfied, then the central control satisfies the following inequality.*

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^T P^{-1} x_0 \quad (9)$$

for all $w \in L^2[0,\infty)$, and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$, where the central control is given by

$$\begin{aligned} u &= -(D_{12}^T D_{12})^{-1}(B_2^T M + D_{12}^T C_1)(I - PM)^{-1}\zeta \\ \dot{\zeta} &= A\zeta + B_2 u + PC_1^T(C_1\zeta + D_{12}u) \\ &\quad +(PC_2^T + B_1 D_{21}^T)(D_{21}D_{21}^T)^{-1}(y - C_2\zeta) \\ \zeta(0) &= 0 \end{aligned} \quad (10)$$

and $S := M(I - PM)^{-1}$.

Proof: First note that $S = M(I - PM)^{-1}$ satisfies the Riccati equation

$$\begin{aligned} &S(A + PC_1^T C_1 - (B_2 + PC_1^T D_{12})(D_{12}^T D_{12})^{-1} D_{12}^T C_1) \\ &\quad +(A + PC_1^T C_1 \\ &\qquad - (B_2 + PC_1^T D_{12})(D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T S \\ &\quad -S((B_2 + PC_1^T D_{12})(D_{12}^T D_{12})^{-1}(B_2 + PC_1^T D_{12})^T \\ &\qquad - (PC_2^T + B_1 D_{21}^T)(D_{21}D_{21}^T)^{-1}(PC_2^T + B_1 D_{21}^T)^T)S \\ &\quad +C_1^T C_1 - C_1^T D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T C_1 = 0 \end{aligned} \quad (11)$$

Consider the functional $V(t)$,

$$V(t) := \zeta^T S \zeta + (x - \zeta)^T P^{-1}(x - \zeta) \quad (12)$$

then, differentiating both sides with respect to t, and inserting conditions (A1)-(A3) into the right hand side, we have

$$\begin{aligned} \dot{V}(t) &= -\|z\|^2 + \|w\|^2 \\ &\quad +\|(D_{12}^T D_{12})^{1/2} u + (D_{12}^T D_{12})^{-1/2} \\ &\qquad \times (B_2^T M + D_{12}^T C_1)(I - PM)^{-1}\zeta\|^2 \\ &\quad -\|w - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2 P + D_{21}B_1^T)S\zeta \\ &\qquad - (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2 P + D_{21}B_1^T)) \\ &\qquad \times P^{-1}(x - \zeta)\|^2 \end{aligned} \quad (13)$$

integrating both sides with respect to t over the interval $[0,\infty)$, we obtain the left hand side as

$$V(\infty) - V(0) = -x_0^T P^{-1} x_0$$

implying the control input $u(t)$ as (10), and

$$\begin{aligned} -x_0^T P^{-1} x_0 &= -\|z\|_2^2 + \|w\|_2^2 \\ &\quad -\|w - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2 P + D_{21}B_1^T)S\zeta \\ &\qquad - (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2 P + D_{21}B_1^T)) \\ &\qquad \times P^{-1}(x - \zeta)\|_2^2 \end{aligned} \quad (14)$$

then we finally obtain as

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^T P^{-1} x_0$$

■

This Lemma is concerned with the condition for P, not for N. This conclusion does not solve the infinite horizon DIA control problem, because the inequality (9) does not generally imply the inequality (3). Next, the following condition is assumed.

(A4) $N < P, \quad (N^{-1} > P^{-1})$.

If the condition (A4) holds, the inequality (3) follows from the inequality (9), and the central control (10) is a DIA control.

$$\|z\|_2^2 \leq \|w\|_2^2 + x_0^T P^{-1} x_0 < \|w\|_2^2 + x_0^T N^{-1} x_0 \quad (15)$$

Since N is regarded as a measure of initial state uncertainties, e.g., a variance matrix, we can state that, if the initial state uncertainty is sufficiently small (so that (A4) holds), the central control has robustness against the initial state uncertainty. In view of the discussion above, the condition (A4) seems necessary for the central control to be a DIA control. We will show that this is not true by presenting a necessary and sufficient condition, which is the main result of this paper. In order to state the result, let us introduce the following condition:

(A5) $Q + N^{-1} - P^{-1} > 0$,
where Q is the maximal solution of the Riccati equation

$$\begin{aligned} &Q(A - B_1 D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\ &\quad + (B_1 B_1^T - B_1 D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1}) \\ &\quad +(A - B_1 D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\ &\quad + (B_1 B_1^T - B_1 D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1})^T Q \end{aligned}$$

$$
\begin{aligned}
&-Q(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)^T \\
&\quad \times (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)Q \\
&= 0 \qquad (16)
\end{aligned}
$$

with $L := (I - PM)^{-1}$.

Theorem 1 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The central control (10) is a DIA control if and only if the condition (A5) is satisfied.*

Remark 1 *The Riccati equation (16) has the maximal solution $Q \geq 0$ for any P^{-1}. such that*

$$
\begin{aligned}
&A - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}C_2 \\
&\quad +(B_1B_1^T - B_1D_{21}^T(D_{21}D_{21}^T)^{-1}D_{21}B_1^T)P^{-1} \\
&\quad -(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)^T \\
&\quad (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)L)Q \qquad (17)
\end{aligned}
$$

is stable, since (A, B_1) is stabilizable. Hence, (A4) is a sufficient condition for the condition (A5) to be satisfied.

4 Proof of Theorem 1

We prove Lemma 2 and Lemma 3. Then Theorem 1 follows. Lemma 2 and Lemma 3 require the following condition:

(A6) : For all $(w) \in L^2[0,\infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$, the inequality

$$\|w - w_0\|_2^2 + x_0^T\left(N^{-1} - P^{-1}\right)x_0 > 0 \qquad (18)$$

holds, where w_0 is given by

$$
\begin{aligned}
w_0 &= D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)S\zeta \\
&\quad +(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1}(C_2P + D_{21}B_1^T)) \\
&\quad \times P^{-1}(x - \zeta) \qquad (19) \\
\dot{\zeta} &= (A + PC_1^TC_1 - (B_2 + PC_1^TD_{12})(D_{12}^TD_{12})^{-1} \\
&\quad \times(B_2^TM + D_{12}^TC_1)L \\
&\quad +(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)S)\zeta \\
&\quad +(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}D_{21} \\
&\quad \times (w - w_0) \qquad (20)
\end{aligned}
$$

Lemma 2 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The central control (10) is a DIA control if and only if the condition (A6) is satisfied.*

Proof: Consider the functional $V(t) = \zeta^TS\zeta + (x-\zeta)^TP^{-1}(x-\zeta)$, then, differentiating both sides with respect to t, and inserting (1) and (10) into the right hand side, and integrating both sides with respect to t over the interval $[0,\infty)$, we obtain

$$-x_0^TP^{-1}x_0 = -\|z\|_2^2 + \|w\|_2^2 - \|w - w_0\|_2^2 \qquad (21)$$

Insert (21) into (18), then, we have

$$\|z\|_2^2 < \|w\|_2^2 + x_0^TN^{-1}x_0. \qquad (22)$$

Converse, insert (21) into (3), then, we have

$$\|w - w_0\|_2^2 + x_0^T\left(N^{-1} - P^{-1}\right)x_0 > 0 \qquad (23)$$

■

Lemma 3 *Suppose that the conditions (A1), (A2), and (A3) are satisfied. The condition (A6) is equivalent to the condition (A5)*

Proof: Consider the functional $U(t) := f^TQf$, where $f(t) := x(t) - L\zeta(t)$ is given by

$$
\begin{aligned}
\dot{f}(t) &= (A + B_1(B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times(C_2P + D_{21}B_1^T))P^{-1})f \\
&\quad + (B_1 - L(PC_2^T + B_1D_{21}^T)(D_{21}D_{21}^T)^{-1}D_{21}) \\
&\quad \times(w - w_0), \quad f(0) = x_0. \qquad (24)
\end{aligned}
$$

Differentiating both sides with respect to t and completing the square argument as

$$
\begin{aligned}
\dot{U}(t) &= \|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times (C_2P + D_{21}B_1^T)L)Qf\|^2 \\
&\quad -\|w - w_0\|^2 \qquad (25)
\end{aligned}
$$

then, integrating both sides with respect to t over the interval $[0,\infty)$, we finally obtain

$$
\begin{aligned}
-x_0^TQx_0 &= \|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times(C_2P + D_{21}B_1^T)L)Qf\|_2^2 \\
&\quad -\|w - w_0\|_2^2 \qquad (26)
\end{aligned}
$$

Inserting (26) into the condition (A6), then we have

$$
\begin{aligned}
&\|w - w_0 + (B_1^T - D_{21}^T(D_{21}D_{21}^T)^{-1} \\
&\quad \times(C_2P + D_{21}B_1^T)L)Qf\|_2^2 \\
&\quad + x_0^T\left(Q + N^{-1} - P^{-1}\right)x_0 > 0. \qquad (27)
\end{aligned}
$$

The 1st term in the left hand side are positive, hence

$$Q + N^{-1} - P^{-1} > 0.$$

■

5 Application to Magnetic Suspension Systems

The state-space representation of the magnetic suspension system is given as follows[6, 7].

$$\begin{aligned} \dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\ y_g &= C_g x_g + w_0 \end{aligned} \tag{28}$$

where $x_g := [x \ \ \dot{x} \ \ i]^T$, u_g is a control input, v_0 and w_0 are exogenous disturbance inputs, $x(t)$ is a gap between the electromagnet and a suspended iron ball, and $i(t)$ is a current. First, let us consider the disturbances v_0 and w_0. Since v_0 mainly acts on the plant in a low frequency, and w_0 shows an uncertainty caused via unmodeled dynamics. Hence let v_0 and w_0 be of the form

$$\begin{aligned} v_0 &= W_v(s)\, w_2 && (29) \\ W_v &= \Phi C_w (sI - A_w)^{-1} B_w, \quad \Phi = [1 \ \ 1]^T \\ w_0 &= W_w w_1 && (30) \end{aligned}$$

where W_w is a weighting scalar. Next we consider the variables which we want to regulate. In this study, the gap and the corresponding velocity are chosen. Then, as the error vector, let us define as follows;

$$\begin{aligned} z_g &= F_g x_g, \quad F_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} && (31) \\ z_1 &= \Theta z_g, \ \Theta = \text{diag}\begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}, \ z_2 = \rho u && (32) \end{aligned}$$

where Θ is a weighting matrix on the regulated variables z_g, and ρ is an weighting scalar for the regulation of the control input $u(:= u_g)$. Finally, let $x := [x_g^T \ \ x_w^T]^T$, where x_w denotes the state of $W_v(s)$, and $w := [w_1^T \ \ w_2^T]^T$, $z := [z_1^T \ \ z_2^T]^T$, then we can construct the generalized plant as in (33). Note that this plant does not have the orthogonality assumptions[5].

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{33}$$

$$A = \begin{bmatrix} A_g & D_g C_w \\ 0 & A_w \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 & D_g D_w \\ 0 & B_w \end{bmatrix}, \ B_2 = \begin{bmatrix} B_g \\ 0 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} \Theta F_g & 0 \\ 0 & 0 \end{bmatrix}, \ D_{12} = \begin{bmatrix} 0 \\ \rho \end{bmatrix},$$
$$C_2 = \begin{bmatrix} C_g & 0 \end{bmatrix}, \ D_{21} = \begin{bmatrix} W_w & 0 \end{bmatrix}$$

Now our control problem setup is: find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3).

After some control design iteration, the design parameters; $W_v(s)$, W_w, Θ and ρ are chosen appropriately, and a direct calculations yield the H_∞ DIA controller $K(s)$. The frequency response of the controller $K(s)$ is shown in Fig.1 by a solid line. And the maximum value of the weighting matrix N is $N = 2.7735 \times 10^{-2} \times I$. We designed the standard H_∞ controller denoted as K_∞ [7] via the MATLAB command `hinfsyn.m`. The frequency response of the controller K_∞ and the previous DIA controllers $K_{DIA_1}(s)$ and $K_{DIA_2}(s)$[3, 6] are shown in Fig.1 by a dotted line, a dashed line and a dash-dot line, respectively. Comparing these four controllers, $K(s)$ has a high gain at the low frequency and a good roll-off property at the high frequency, and the comprehensive frequency response looks like a modified PID controller. In the previous DIA design framework, it was difficult to let controllers $K_{DIA_1}(s)$ and $K_{DIA_2}(s)$[3, 6] get hold an integral property.

We have conducted simulations to evaluate properties of $K(s)$. To ascertain transient responses, we input a step reference signal to the system with a nonzero initial state x_0. An initial response for $x_0 = [0.0 \ \ 0.0 \ \ 0.1]^T$ is shown in Fig.2, and a time response for a step reference signal(0.0[mm] $\rightarrow$ 0.1[mm]) is shown in Fig.3, where the signal is added to the system around 1.0[s]. The K_{DIA_1} and K_{DIA_2} show relatively better performance than K for the initial state uncertainty in Fig.2, K has, however, a better transient performance than K_∞. Since our concerns are not only in the attenuation of the initial state uncertainty, but also in the basic control performance of the controllers, we then wonder whether the controller has a good performance for the step reference signal. Controller K shows better and quicker transient response than K_∞. K_{DIA_1} and K_{DIA_2} show pretty quick responses but bigger overshoots around 1.0[s] because of their high gain at the high frequency in Fig.1, however we must give careful attention for steady-state error with those controllers. K_{DIA_1} and K_{DIA_2} leave steady-state errors because of their low gain at the low frequency in Fig.1. In the previous problem setup, the degrees of freedom in the design parameters are limited, so that it is difficult to shape a good controller frequency response[6]. Considering all the factors, we reached the conclusion that K has a pretty good performance for all control requirements, and has a potential ability to be improved by using the degrees of freedom in the design parameters.

6 Conclusion

In this paper, we formulated and solved a generalized H_∞ control problem which considers a mixed attenuation of disturbance and initial-state uncertainty in the infinite-horizon case, without the orthogonality assumptions. The solution was given as a natural but complicated extension of the previous results in [3, 6]. A necessary and sufficient condition for a solution of the generalized H_∞ mixed attenuation problem to exist, together with an explicit formula of the solution, was derived. Based on the condition, a robustness property of H_∞ controls against initial-state uncertainty was discussed.

Moreover, we applied an infinite horizon disturbance and initial state uncertainty attenuation control problem without the orthogonality assumptions to the magnetic suspension system. Comparing the proposed controller with the standard H_∞ controller and the other controllers based on previous results[6], we showed the property and effectiveness of the proposed mixed attenuation controller.

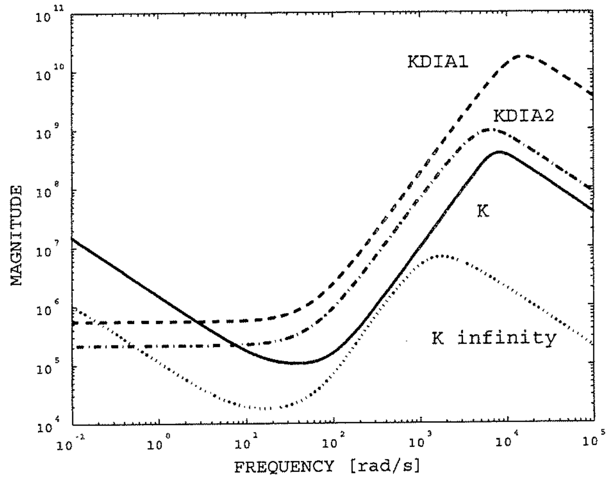

Figure 1: Frequency Response of the controller K with K_{DIA_1} , K_{DIA_2} and K_∞

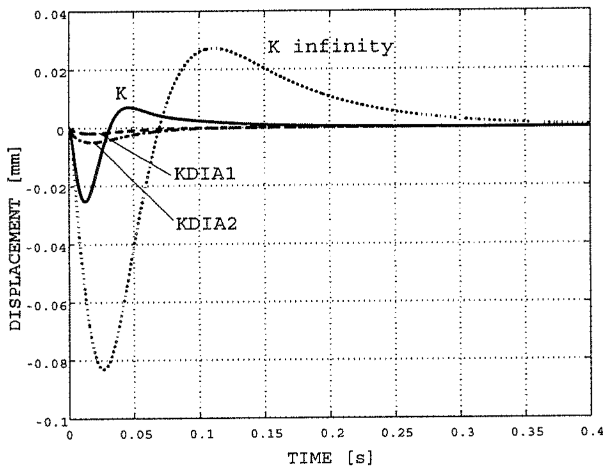

Figure 2: Initial Responses for $x_0 = [0.0 \;\; 0.0 \;\; 0.1]^T$

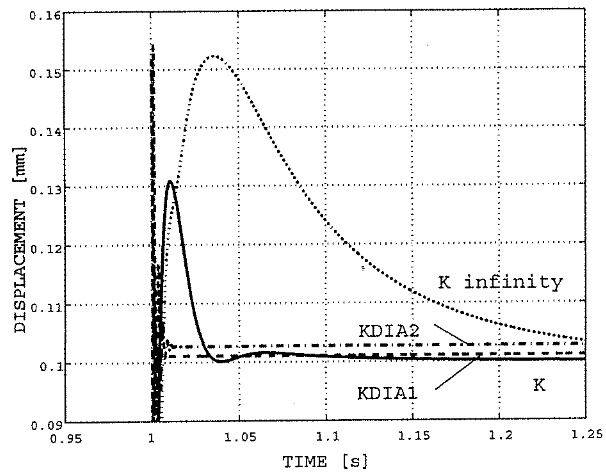

Figure 3: Step Responses

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SWITCHING FROM VELOCITY TO FORCE CONTROL FOR THE ELECTRO-HYDRAULIC SERVOSYSTEM BASED ON LPV SYSTEM MODELING

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Abstract: In the hydraulic system, it is often that switch the control mode over from one to the other, such as from position to velocity or from velocity to force. In this paper, propose a flow calculation formula for the flow control valve in order to have an LPV system representation. Then, design gain scheduled controllers for the velocity and the force individually. During a switching mode, a control is generated by adding two controller outputs with appropriate ratios. Usefulness of this approach is shown by the experiment results, which are obtained from Injection molding machine application.

Keywords: Injection molding, Electro-hydraulic systems, LPV System, H-infinity, Switching algorithms

1. INTRODUCTION

Hydraulic control system is used in various industrial applications for the power in size, high durability. In most cases, the electro-hydraulic servosystem is applied to obtain the high reproducibility and the fine dynamic performance of the position, velocity or force control in the hydraulic system. The hydraulic plant has many nonlinear factors and components. For example, asymmetric cylinder, mechanical friction, deadband, complex flowpassage relates to hydraulic dynamics, the effective control flow which depends on load condition and hysteresis. In addition to the above, plant parameters are not constant, such as the bulk modulus depends on the containing air quantity or viscosity varies according to the temperature. Also, it is often in indistrial hydraulic applications that a control mode swicthes over from one to the other sequentially, such as from the position to pressure or from the velocity to force. These factors make a controller design complicated. However, because of the electro-hydraulic servosystem capability, a number of studies have been done in designing the controller. Approaches reported recently are adaptive control(Bobrow and Lum, 1996),(Plummer and Vaughan, 1996)and sliding mode control(Ha *et al.*, 1995). There, controller structure becomes complicated and it is not easy to realize the smooth operation and the fast response. The robust control design by H_∞ framework (Tunay *et al.*, 2001) is also applied. The controller design approach, which is based on the linear model, make the closed-loop system stable locally. When the load condition changes significantly, there is a limitation in appling this approch. Here, we examine gain scheduling control of the electro-hydraulic servosystem for the velocity and force control in order to guarantee stability and performance under the significant plant parameter variation.

It is important that we take a varying load condition and a complex flow characteristic of the

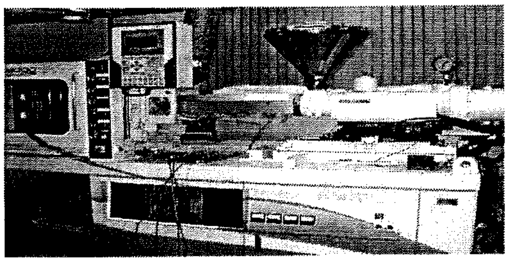

Fig. 1. Injection Molding Machine appearance

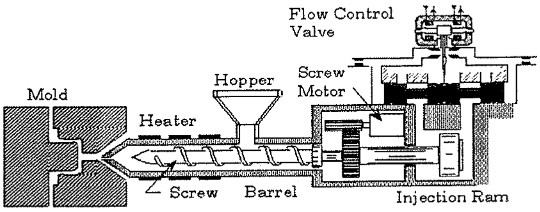

Fig. 2. Test equipment

control valve around the null in consideration to construct a plant model. One of the reasons to make the situation difficult is the discontinuity of the control flow calculation formula around the null. In this paper, we propose a formula, which interpolates the turbulent and the laminar flow in the flow control valve so that the linear model becomes continuous at the boundary for the precise force control. Then, we compose the linear plant model as an LPV (linear parameter varying) system with a scheduling parameter which depends on load force. Based on LPV plant models, we design gain scheduling controllers for the velocity and force. One of the contributions of this paper is that present the way to design the gain scheduled controller of the electro-hydraulic servosystem. The other contribution is that we study switching behavior from velocity to force control mode with gain scheduled controllers applying to the Injection molding machine. Here, we attempt to add up the outputs from the velocity and the force controller according to the weighting coefficient which is determined by the ratio of actual force and the force reference value.

2. INJECTION MOLDING PROCESS

In the injection molding process, there is an injection velocity control and a holding pressure control mode. The velocity profile is generated with respect to the mold shape so that the velocity between melt plastic and mold surface becomes constant. The force control of the holding pressure mode makes the plastic stress uniform in order to minimize the deformation of the product. The electro-hydraulic servosystem is adapted to both of the velocity and force control for the high power, fast response and fine reproducibity. The

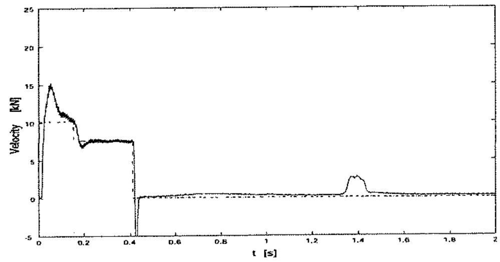

Fig. 3. Velocity behavior (Case 1)

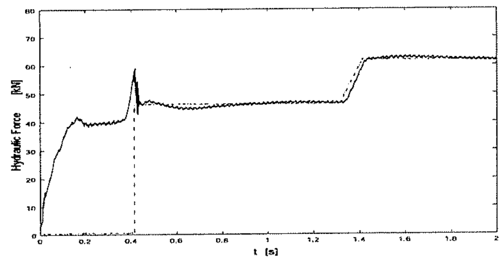

Fig. 4. Force behavior (Case 1)

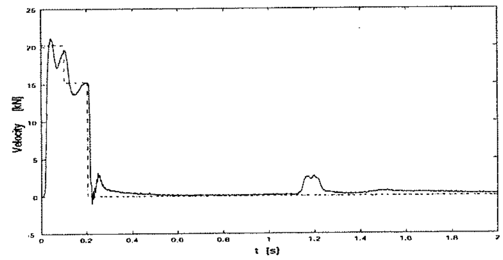

Fig. 5. Velocity behavior (Case 2)

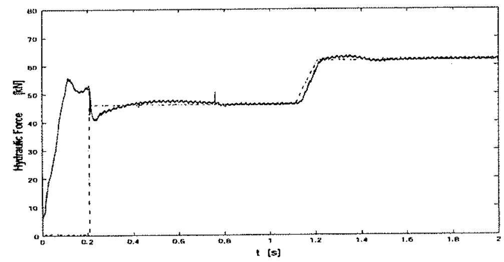

Fig. 6. Force behavior (Case 2)

appearance of the Injection molding machine is shown in Figure 1 and the structure scheme is shown in Figure 2. Main components are a mold, heaters, a screw, a hydraulic injection cylinder, a flow control valve and transducers for the velocity and force. The purpose to design the gain scheduled controller here, is to have stable and good performance under significant palnt parameter variation, caused by the load force change. Also, the switching behavior from velocity to force is important. To make it smooth, we generate a control by adding up two controller outputs, during the transition from velocity to force control. Figures 3,4 and figures 5,6 show the typical behaviors, which we see often, when switch the controlled variable from velocity to force without any measures to make the transition smooth. In figures 3,4, switching occurs at 7.5 cm/s velocity. Figure 3 shows a rapid velocity change and an revrse direction velocity at 0.4 sec. This means that the injection ram moves to backward. This phenomenon should be avoided in the process. In Figure 4, there is a pressure peak at the just before switching happens. In figures 5, 6, switching takes place at 15 cm/s velocity. In this case, the reverse velocity behavior and pressure peak are small, but still exist.

3. FLOW CALCULATION FORMULAS

In the flow control valve, which has a sleeve and spool, there are two typical flow conditions, called the turbulent flow at the metering orifice opening and laminar flow in the clearance between sleeve and spool. Now, think about the flow at the metering orifice A out of four orifices in Figure 7, as an example. There are commonly used flow calculation formulas (1), (3), with lap l_a and clearance c_r, for each conditions. However, these two equations are not continuous at the boundary. Hence, adopt equation (2) in order to interpolate flow and derivation of the turbulent and the laminar conditions.

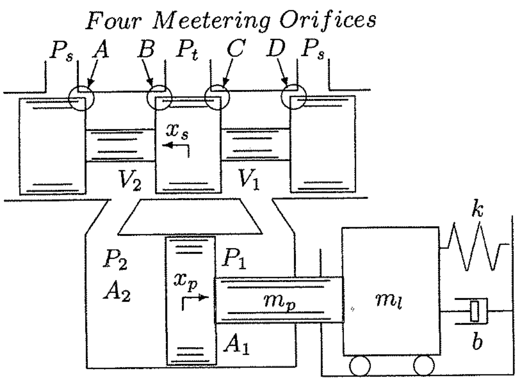

Fig. 7. Configuration of the plant

Then, continuous flow in whole operating range, is given as follows

$$q_{2int} = K_t\sqrt{(x_s-l_a)^2+c_r^2}\sqrt{P_s-P_2} \quad \dots\ l_a \le x_s \quad (1)$$

$$q_{2ini} = \left(\frac{3}{K_l}\right)^3\left(\frac{K_t c_r}{4}\right)^4\frac{(x_s-l_a)^3}{P_s-P_2} + K_t c_r\sqrt{P_s-P_2} \quad \dots\ l_a + x_{sa} \le x_s < l_a \quad (2)$$

$$q_{2inl} = K_l\frac{P_s-P_2}{-(x_s-l_a)} \quad \dots\ x_s < l_a + x_{sa} \quad (3)$$

here, $x_{sa} = -4C_r\sqrt{Ps-P_2}/(3K_tK_l)$

According to these equations (1),(2) and (3), the flow from control valve highly depends on load pressure (or force) and is nonlinear. Because of such a characteristics, a fixed controller at the one operating condition can not satisfy the performances in the over all operating range. $P_2(t)$, $P_1(t)$ are bore and rod pressure, A_2, A_1 are effective piston areas of the cylinder. m_p and m_l are masses, $x_p(t)$ is piston displacement. K_t, K_l are coefficients of the turburant and the laminar flow. Linearization of the equations (1),(2) and (3) at the arbitrary operating point (x_{s0}, P_{20}). We have

$$\delta q_{2int} = K_t\left(\frac{x_{s0}-l_a}{\sqrt{(x_{s0}-l_a)^2+c_r^2}}\sqrt{P_s-P_{20}}\,\delta x_s - \frac{\sqrt{(x_{s0}-l_a)^2+c_r^2}}{2\sqrt{(P_s-P_{20})}}\delta P_2\right) \quad (4)$$

$$\delta q_{2ini} = \left(\frac{3}{K_l}\right)^3\left(\frac{K_t c_r}{4}\right)^4\frac{3(x_{s0}-l_a)^2}{P_s-P_{20}}\delta x_s + \left(\left(\frac{3}{K_l}\right)^3 \times \left(\frac{K_t c_r}{4}\right)^4\frac{(x_{s0}-l_a)^3}{P_s-P_{20}} - \frac{K_t c_r}{2\sqrt{P_s-P_{20}}}\right)\delta P_2 \quad (5)$$

$$\delta q_{2inl} = K_l\left(\frac{P_s-P_{20}}{-(x_{s0}-l_a)}\delta x_s - \frac{1}{-(x_{s0}-l_a)}\delta P_2\right) \quad (6)$$

For the simple presentation such as (7), choose the corresponding equation from (4), (5) or (6) according to spool displacement $x_s(t)$.

$$\delta q_{2in} = K_{2inxs}\delta x_s + K_{2inp2}\delta P_2 \quad (7)$$

The same way as above, derivative equations of the metering orifice B, C and D are described below, respectively.

$$\delta q_{2out} = K_{2outxs}\delta x_s + K_{2outp2}\delta P_2 \quad (8)$$

$$\delta q_{1out} = K_{1outxs}\delta x_s + K_{1outp1}\delta P_1 \quad (9)$$

$$\delta q_{1in} = K_{1inxs}\delta x_s + K_{1inp1}\delta P_1 \quad (10)$$

4. MODELING

4.1 *Linear System*

Figure 8 presents the block diagram of the linear system from the input δv_{sig}, which is applied to the flow control valve, to the controlled variable hydraulic force V_{F_h} and piston velocity $V_{x_{pv}}$. v_{sig} is the control. r_{sig} is the rated signal and r_{str} is the rated spool displacement. w_v and ζ_v represents the control valve dynamics as the second order transfer function. The pressure in the actuator chamber is calculated from the hydraulic compressibility, called as bulk modulus β, and the effective volume change caused by flow in and out from the chamber, plus piston displacement. A mass, an equivalent viscous resistance and a spring compose the load. In the actual injection process, the viscous resistance and spring rate change under the various operating conditions. Here, suppose that they are constant in the velocity and force control mode. Here, take state vector δx as

$$\delta x = (\delta x_{sv},\ \delta x_s,\ \delta P_2,\ \delta P_1,\ \delta x_{pv},\ \delta x_p)^T \quad (11)$$

From the linear block diagram in Figure 8, the linear state space equation is represented in equation (12) and (13).

$$\delta\dot{x} = \begin{pmatrix} -2\zeta_v\omega_v & -\omega_v^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_p(3,2) & a_p(3,3) & 0 & a_p(3,5) & 0 \\ 0 & a_p(4,2) & 0 & a_p(4,4) & a_p(4,5) & 0 \\ 0 & 0 & \frac{10^4 A_2}{mp+m_l} & -\frac{10^4 A_1}{mp+m_l} & \frac{10^2 b}{mp+m_l} & \frac{10^2 k}{mp+m_l} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \delta x + \begin{pmatrix} \frac{r_{str}}{r_{sig}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \delta v_{sig} \quad (12)$$

$$\begin{pmatrix} V_{Fh} \\ V_{x_{pv}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{A_2}{457.78} & -\frac{A_1}{457.78} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \end{pmatrix} \delta x \quad (13)$$

Some of the matrix elements, such as $a_p(3,2)$, are not constant. Here,

$$a_p(3,2) = \beta\frac{K_{2inxs} - K_{2outxs}}{V_2 + A_2(L_n + x_{p0})}$$

$$a_p(3,3) = \beta\frac{K_{2inP_2} - K_{2outP_2}}{V_2 + A_2(L_n + x_{p0})}$$

$$a_p(3,5) = -\beta\frac{A_2}{V_2 + A_2(L_n + x_{p0})}$$

$$a_p(4,2) = \beta\frac{K_{1inxs} - K_{1outxs}}{V_1 + A_1(L_n - x_{p0})}$$

$$a_p(4,4) = \beta\frac{K_{1inP_1} - K_{1outP_1}}{V_1 + A_1(L_n - x_{p0})}$$

$$a_p(4,5) = \beta\frac{A_1}{V_1 + A_1(L_n - x_{p0})}$$

L_n is a half with total piston stroke.

4.2 *LPV System*

As mentioned above, some of the elements in the state space equation vary according to the operating conditions. To have linear state space equation, we adopt a LPV system presentation, which has the parameter that is the function of the load force. Equation 12 has the form as

$$\delta\dot{x} = \frac{\partial}{\partial x} f(x_0)\delta x + B_p \delta v_{sig} \quad (14)$$

$$\delta y = C_p \delta x \quad (15)$$

Two matrices B_p and C_p are constant and all elements in these matrices are decided based on the mechanical specifications. x_0 is a state at the equilibrium point. But, it is difficult to have the solution of x_0 from the implicit function of $f(x_0) + B_p v_{sig0} = 0$. Now, suppose that x_0 is given and then y_0 is calculated by $y_0 = C_p x_0$. Addition to this, suppose that the scheduling parameter θ is a smooth function of the y_0, such as $\theta = \varphi(y_0)$. The linear state space equation, which depends on the scheduling parameter θ, represents the plant behavior in the neighborhood of the equilibrium x_0 (Uchida, 1995), (Rugh and Shamma, 2000). By the way, $a_p(3,2)$, $a_p(4,4)$ and so on, are decided when x_{s0}, P_{20}, P_{10} and x_{p0} are fixed. But, as explained already, it is hard to decide these values from the related equations.

Here, we use a value of the variable which is obtained from the following simulation. Apply PI controller to close the velocity loop in the nonlinear plant model. Then, apply relatively slow enough ramp velocity reference signal ($\delta v_{sig} = 0$) so that we are able to assume all state variables are close enough to the equilibrium states. Now, we have a set of the equilibrium points for the specified operating range in both velocity control. Using these values, whose could be considered as the equilibrium set, figure out the elements of equation (12), such as $a_p(3,2)$ so on, with respect to corresponding scheduling parameter that is described as equation (19) or (20). In the case of force plant model, the way to have PLV representation is same as mentioned above. Then, we represent these elelments by the polynomial approximation, as follows

$$a_p(\theta) = a_{p0} + a_{p1}\theta + a_{p2}\theta^2 + a_{p3}\theta^3 \quad (16)$$

The state space equations (12), (13) are rewrited as equations (17), (18) with the scheduling parameter.

$$\delta\dot{x}_p = A_p(\theta)\delta x_p + B_p \delta v_{sig} \quad (17)$$

$$\delta y_p = C_p \delta x_p \quad (18)$$

Considering the flow characteristic which depends on $\sqrt{\Delta P}$, define the scheduling parameter θ_v for the velocity and θ_n for the force control as

$$\theta_v = \frac{\dfrac{1}{\sqrt{1 - \frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1 - \frac{F_{v \cdot rated}}{F_{max}}}} - 1}, \quad [0 \le \theta_v \le 1] \quad (19)$$

$$\theta_n = \frac{\dfrac{1}{\sqrt{1 - \frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1 - \frac{F_{n \cdot rated}}{F_{max}}}} - 1}, \quad [0 \le \theta_n \le 1] \quad (20)$$

F_{max} is maximum force, $F_{v \cdot rated}$ and $F_{n \cdot rated}$ are the rated force in the velocity and force control mode, F_x is actual force which is measured as V_{F_h}.

5. GAIN SCHEDULED CONTROLLERS

Designing gain scheduled controllers for the volocity and force, we take following issues in consideration. In the velocity loop, the rise time is within 15ms to the step reference and minimizes steady state error. In the force controller design, the force follows to the ramp reference signal, which reaches to the rated force with 15 ms and zero steady state error. In order to construct the generalized plant for the H_∞ controller design framework, two weighting functions, $W_s(s)$ for the sensitivity

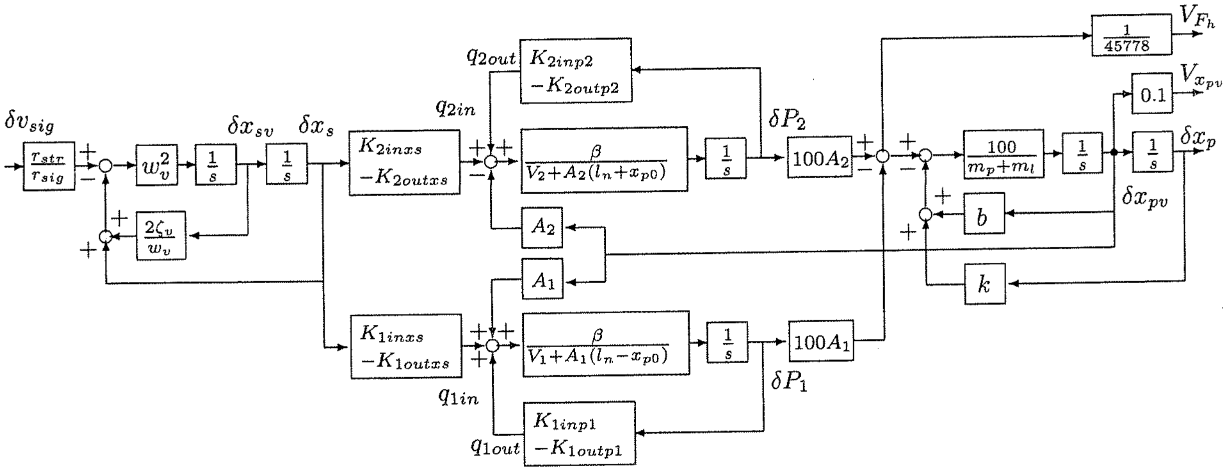

Fig. 8. Block diagram of the linearlized plant

function and $W_a(s)$ for the additive uncertainty at the input of the plant, are specified after the several try and error. For the velocity controller, we use

$$W_{sv}(s) = \frac{0.0025s + 68.75}{s + 0.005}, \quad W_{av}(s) = \frac{5(s+1)}{s+375}$$

and at the force controller design

$$W_{sn}(s) = \frac{0.4s + 5}{s + 0.01}, \quad W_{an}(s) = \frac{25(s+1)}{s+75}$$

Beside these weighting functions, $(0.1s + 0.015)/s$ is added in series to the plant in the velocity control. Also, $(50s + 1)/(s + 0.01)$ is in series to force plant to improve the response. When solve H_∞ controller design problem with LMI formulation, the two positive definite matrices, in many cases described as $\mathcal{X}$ and $\mathcal{Y}$, also let the function of the scheduling parameter, such as

$$\mathcal{X}_v(\theta_v) = \mathcal{X}_{v0} + \mathcal{X}_{v1}\theta_v + \mathcal{X}_{v2}{\theta_v}^2 + \mathcal{X}_{v3}{\theta_v}^3$$

so that minimize the conservatives of the controller. As the results, the generalized plant becomes function of the continuous scheduling parameter and has to solve infinite number of LMIs. In order to reduce this problem to finite number of constraints, a technique that proposed by Azuma (Azuma *et al.*, 2000) to construct a convex hull is introduced. For the more details of the gain scheduled controller for the velocity and force control, see (Sugiyama *et al.*, 2000), (Sugiyama and Uchida, 2001) and (T. Sugiyama and K. Uchida, 2002).

6. SWITCHING SCHEME AND TEST RESULTS

In this experiment, the rated velocity is 200 mm/s, the rated force is $F_{n\cdot rated}$ = 160 kN and the maximum force F_{max} = 183 kN are defined by the specific product and mold capability. The actual load force in the velocity control mode becomes 40 to 50 kN , which is used in calculation of the scheduling parameter θ_v. In the force control mode, the controlled force comes up to 60 kN in the process. The way to add up the outputs form the velocity and force controller is described in Figure 9.

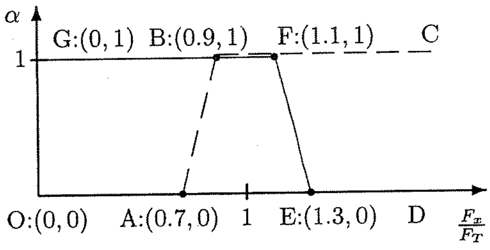

Fig. 9. Explanation of the switching method

Summing up control depends on the force level and whether it overs a set point or not. If the force F_x over the set force level F_r when the switching occurs. In this case, follows the line "D-E-F-G" in Figure 9. Moreover, set the velocity reference as zero. And, the force crosses the point "F", comes from the direction of "E", switch to the force control completely. In the other case, it means that the force level is below to a set point, follows the line "O-A-B-C" and keep to use the velocity reference as it is. In the range "A" to "B" or "E" to "F", we use the add up control. Let's say, u_v is the velocity controller output and u_f is the one of the force controller. The switching control u_t is calculated as

$$u_t = (1 - \alpha)u_v + \alpha u_f \tag{21}$$

In Figure 10,11 and 12,13 show the transitional behaviors that occurs in 7.5 and 15 cm/s velocity. At the beginning of the velocity control, the big overshoot or fluctuation is observed. This is caused by dead time that we do not consider in gain scheduled controller design, so far. But, there is not significant influence on switching operation. Looking at the switchig behaviors, obtain

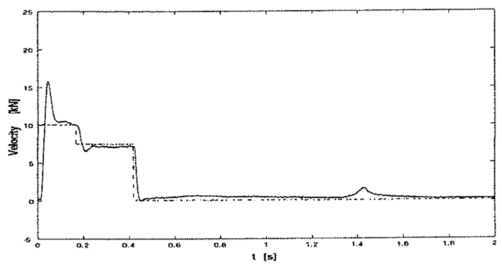

Fig. 10. Velocity behavior with control (Case 3)

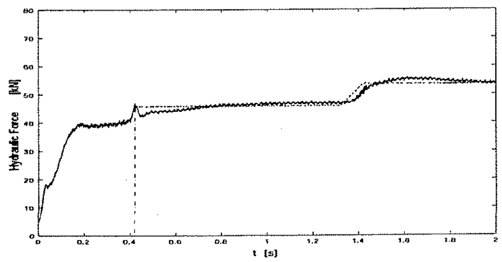

Fig. 11. Force behavior with control (Case 3)

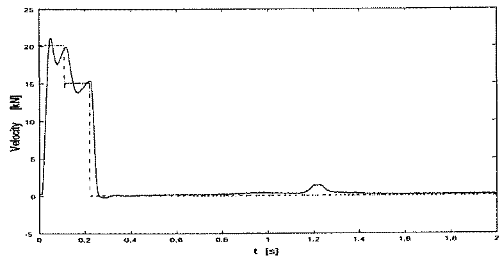

Fig. 12. Velocity behavior with control (Case 4)

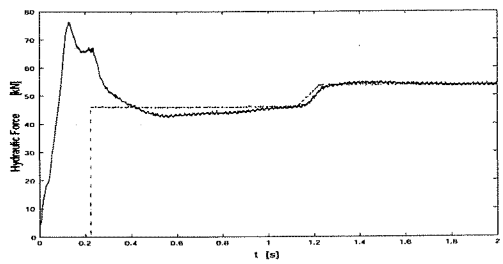

Fig. 13. Force behavior with control (Case 4)

reasonable results by the switching control principle, mentioned in this paper. It is obvious that the velocity comes down near to zero level very smoothly in the both of cases. Force is changing quite smooth, too. The smooth transition is achieved.

7. CONCLUSION

In this paper, we propose the flow calculation formula to have continues flow between the turbulent and laminar flow so that compose the linear plant model with a scheduling parameter. Then, we obtained the values of the system variables by the simulation in order to design the gain scheduled controller for the velocity and force control based on LPV system. The usefulness of the controller design method, described here, is confirmed to the electro-hydraulic servosystem. Also, the way described here to switch from velocity to force makes the transition behavior smooth successfully.

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論文

電気油圧サーボ系における速度および推力のゲインスケジューリング制御

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Gain Scheduled Velocity and Force Controllers for Electro-hydraulic Servosystem

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The load variation of the electro-hydraulic servosystem causes degradation of the control characteristic. This is because the flow characteristic of the flow control valve depends on the load condition. Here, we propose the flow calculation formula to have continues flow between turbulent and laminar flow so that compose the linear plant model with a scheduling parameter. Then, design gain scheduled controllers for the velocity and hydraulic force control.

キーワード：射出成形, 電気油圧サーボシステム, LPV システム, H_∞ 制御, ゲインスケジューリング
Keywords: Injection molding, Electro-hydraulic servo systems, LPV system, H_∞ control, Gain scheduling

1. はじめに

油圧駆動システムはコンパクトで高出力、高い耐久性を有し多くの産業機械に用いられている。油圧装置制御手法の一つである電気油圧サーボ系は、高い再現性と滑らかで良好な動特性が求められる位置や速度、あるいは推力の制御に適用される。

電気油圧サーボ系の特性は使用圧力、油圧アクチュエータや流量制御弁など主要機器の選定に大きく依存する。今日の社会的省資源要求、省エネルギー化を踏まえた機器の選定を行なう場合、従来の制御手法を踏襲すれば制御特性の低下が避けられない。一方、産業機械の高機能化、高性能化が常に求められる。この問題に対し、機械系と制御系の同時設計問題を扱う研究が進められている。しかしながら多くの場合、与えられた制御対象に対し、要求される制御仕様が達成される手法を採用するという立場がとられる。

油圧装置には、多くの非線形要素やモデル化されない高周波振動成分を有する機器が内在する。代表的なものに、アクチュエータ形状の非対称性、摺動摩擦、ヒステリシス、不感帯、バックラッシュ、流路形状、流体の動特性や作動油粘性、体積弾性係数、流量制御弁の負荷流量特性などが挙げられる。また、作動油温度上昇に伴う油圧機器の高周波発振現象がしばしば発生し問題となる。作動油の体積弾性係数や粘度は空気の含有量や流体温度に大きく依存するので、不確定なパラメータとして扱う必要がある[1]。

このように電気油圧サーボ系には、取り扱いが難しい要素が多数含まれるが前述のような利点から、これまでに多くの研究が行なわれている。制御対象のパラメータ変動に対し適応制御[2]の有効性が示され、非線形制御対象へはスライディングモード制御[3][4]の検討が行なわれている。これら手法によれば、PID 制御など従来手法よりもパラメータや負荷変動の影響を低減できるが、制御器の構造が複雑になること、力や速度制御における電気油圧サーボ系の持つ滑らかで高い追従性の実現に至っていないことなどの問題がある。一方、電気油圧サーボ系の制御器設計に用いられる手法の一つに、H_∞ の枠組みによるロバスト制御[5][6]がある。しかし、動作点近傍の線形モデルに基づく制御器設計によれば、局所的な安定性と性能が保証されるが、制御対象のパラメータが比較的大きく変動する場合には適用の限界がある。本論文では、大きなパラメータ変動に対して安定性と性能を保証することをねらい、LPV(Linear Parameter-varying) モデルに基づくゲインスケジューリング制御を検討する。

制御器設計には制御対象のモデルが必要であるが、電気油圧サーボ系に対する制御対象のパラメータ変動と流量制御弁における絞り部のクリアランスと重合度を考慮した流量計算に基づくモデル作成の研究は少ない。また、推力や圧力の制御では制御弁が主に中立点付近で動作するため、これらを考慮したモデルを構築することが重要である。これまでに、絞り部開口面積に比例する流れとクリアランス部のすき間流れの流量計算式は広く知られている。しかし、二つの流量計算式は境界で不連続なため、制御弁動作領域

全体に渡る連続なモデルを記述することを難しくしている。そこで本論文では、境界領域で流量計算値と微小変化量が連続となる補間式を提案する。この補間式によるモデルと推力に依存するスケジューリングパラメータを導入して、制御対象を LPV システムとして記述する。この LPV モデルに基づき、電気油圧サーボ系におけるシリンダ速度と推力制御のゲインスケジュールド制御器を設計する。

本論文では、はじめに射出成形機に用いられる電気油圧サーボ系の概要を説明した後、中立点付近における制御流量の補間式を述べる。この補間式を用いて線形モデルを構成する手順を説明し、次に推力に依存するスケジューリングパラメータを用いて制御対象のモデル集合を LPV システムとして記述する。制御器設計は、H_∞ 制御問題の枠組みを用いるので、スケジューリングパラメータに依存する一般化プラントを構成する。一般化プラントに対する LMI 制約問題の安定化解を求めることにより、制御器パラメータを算出する。本手法により、射出成形機における射出シリンダーの速度と推力のゲインスケジュールド制御器を設計し、制御器を非線形制御対象モデルへ適用したシミュレーション結果と実機における制御結果を示す。

2. 射出成形機に適用される電気油圧サーボ制御

射出成形機の主要構成機器は、図 1 に示すように金型と金型開閉機構、バレル、射出シリンダ、流量制御弁、油圧源、速度と圧力検出器、樹脂材料供給器と制御機器がある。射出成形機では、溶融樹脂を金型内へ射出する行程で油圧シリンダの速度制御が用いられ、金型内へ樹脂が充填完了する直前から行なわれる保圧行程には推力制御が用いられる。樹脂の剪断発熱とヒータで加熱溶融した樹脂は、金型表面接触部の速度が一定になるよう金型形状に従った速度プロファイルで射出される。射出成形の特徴は、金型形状をプラスチックで成形することばかりではなく、金型表面に設けられた模様や情報を写しとることである。成形の良否は、溶融樹脂の金型表面接触から接触部表面の冷却、凝固の過程で決まる。溶融樹脂流動速度の変動は、転写性と呼ばれる金型表面形状を写しとる特性を劣化させる。金型内に樹脂がほぼ充填されると、保圧制御モードに切り替わる。これは、油圧シリンダの推力、あるいは圧力の制御を行なうことである。成形品の内部応力を均一にすることで、樹脂冷却と成形後の経時変化を低減するものである。

3. 流量計算式

図 2 に示すようなスリーブとスプールから構成される流量調整部を持つ制御弁には、二つの代表的な流れの状態がある。一つはスリーブとスプールの位置関係により構成される開口部の流れで、他の一つは開口部が閉ざされた状態におけるスリーブとスプールのすき間を流れである。まず図 2 に示す 4 つの流量調整部のうち A 部についての流量計算式を考える。開口部での流量とすき間部の流量計算には、クリアランス (c_r) と重合度 (l_a) を考慮して式 (1) と (3) が

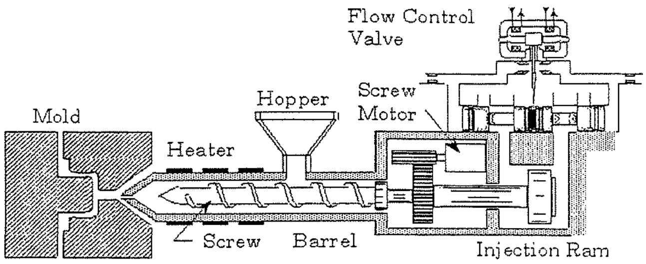

図 1　射出成形機の構成

Fig. 1.　Injection molding machine structure

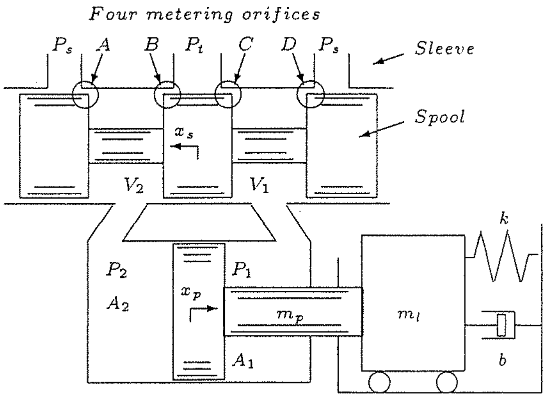

図 2　制御対象の構成

Fig. 2.　Plant structure

一般に用いられる。

$$q_{2int} = K_t\sqrt{(x_s - l_a)^2 + c_r^2}\sqrt{P_s - P_2} \ \dots\ l_a \le x_s \cdots (1)$$

$$q_{2ini} = \left(\frac{3}{K_l}\right)^3 \left(\frac{K_t c_r}{4}\right)^4 \frac{(x_s - l_a)^3}{P_s - P_2} + K_t c_r\sqrt{P_s - P_2} \ \dots\ l_a + x_{sa} \le x_s < l_a \cdots (2)$$

$$q_{2inl} = K_l \frac{P_s - P_2}{-(x_s - l_a)} \ \dots\ x_s < l_a + x_{sa} \cdots (3)$$

ここで、$x_{sa} = -4C_r\sqrt{Ps - P_2}/(3K_tK_l)$ 。$P_2(t), P_1(t)$ はシリンダヘッド側とロッド側圧力、A_2, A_1 はそれぞれヘッド側とロッド側の有効面積である。m_p と m_l はピストン質量と負荷質量、$x_p(t)$ はピストン変位である。K_t、K_l は、乱流と層流状態の流量係数である。しかしながら、これら二つの式により算出される流量は、境界で連続とならない。そこで、これらの流量を連続、かつスプール変位と負荷圧力変化に対する流量変化率も連続となるような補間式 (2) を用いることとする。これにより、推力制御器設計において重要な電気油圧サーボ系油圧駆動部の剛性をより正しく表現することができる。式 (1),(2) と (3) からは、流量制御弁の出力流量が負荷圧力に依存する非線形な特性となることがわかる。この特性により、一つの動作点に対する制御器パラメータが固定される制御手法では、制御対象の全動作範囲に渡る良好な制御性能を満たすことが難し

くなっている。

式 (1),(2) と (3) を動作点 (x_{s0}, P_{20}) で線形化すれば

$$\delta q_{2int} = K_t \left(\frac{x_{s0} - l_a}{\sqrt{(x_{s0} - l_a)^2 + c_r^2}} \sqrt{P_s - P_{20}} \delta x_s - \frac{\sqrt{(x_{s0} - l_a)^2 + c_r^2}}{2\sqrt{(P_s - P_{20})}} \delta P_2 \right) \quad \cdots (4)$$

$$\delta q_{2ini} = \left(\frac{3}{K_l}\right)^3 \left(\frac{K_t c_r}{4}\right)^4 \frac{3(x_{s0} - l_a)^2}{P_s - P_{20}} \delta x_s + \left(\left(\frac{3}{K_l}\right)^3 \times \left(\frac{K_t c_r}{4}\right)^4 \frac{(x_{s0} - l_a)^3}{P_s - P_{20}} - \frac{K_t c_r}{2\sqrt{P_s - P_{20}}} \right) \delta P_2 \quad \cdots (5)$$

$$\delta q_{2inl} = K_l \left(\frac{P_s - P_{20}}{-(x_{s0} - l_a)} \delta x_s - \frac{1}{-(x_{s0} - l_a)} \delta P_2 \right) \quad \cdots (6)$$

を得る。スプール変位 $x_s(t)$ に従い式 (4), (5) または (6) から対応する式を選ぶものし、これらを式 (7) のように簡潔に表す。

$$\delta q_{2in} = K_{2inxs} \delta x_s + K_{2inp2} \delta P_2 \quad \cdots\cdots\cdots\cdots (7)$$

絞り部 B、C と D についても同様に、それぞれ線形化式 (8)、(9) と (10) を得る。

$$\delta q_{2out} = K_{2outxs} \delta x_s + K_{2outp2} \delta P_2 \cdots\cdots\cdots\cdots (8)$$

$$\delta q_{1out} = K_{1outxs} \delta x_s + K_{1outp1} \delta P_1 \cdots\cdots\cdots\cdots (9)$$

$$\delta q_{1in} = K_{1inxs} \delta x_s + K_{1inp1} \delta P_1 \cdots\cdots\cdots\cdots (10)$$

4. モデリング

〈4・1〉 線形システム 図 3 に流量制御弁への入力 δv_{sig} から観測変数である油圧推力 δV_{F_h} とピストン速度 $\delta V_{x_{pv}}$ までの線形系のブロック図を示す。v_{sig} は操作量で、r_{sig} と r_{str} は流量制御弁の定格入力信号と定格スプール変位である。w_v と ζ_v は、流量制御弁の固有周波数と減衰係数である。圧力変化は、体積弾性係数 β と呼ばれる流体の圧縮性と圧力容器内における流体の有効体積変化により算出する。有効体積変化は、流量制御弁を介して流入・流出する流量の時間積分とピストン変位に伴う体積変化量の総和である。負荷系は質量、等価バネと等価粘性抵抗で構成されている。バネ定数と粘性係数は、材料の乾燥状態や溶融した時の温度条件などで変動するが、ここでは事前に速度制御と保圧制御工程それぞれに代表的な値が実験的に与えられているものとする。線形系のブロック線図 3 から、本制御対象の状態量を次式 (11) のように決める。

$$\delta x = (\delta x_{sv},\ \delta x_s,\ \delta P_2,\ \delta P_1,\ \delta x_{pv},\ \delta x_p)^T \cdots (11)$$

この時、線形システムとしての状態空間表現は、状態方程式 (12) と出力方程式 (13) のよう表すことができる。

$$\delta \dot{x} = \begin{pmatrix} -2\zeta_v \omega_v & -\omega_v^2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_p(3,2) & a_p(3,3) & 0 & a_p(3,5) & 0 \\ 0 & a_p(4,2) & 0 & a_p(4,4) & a_p(4,5) & 0 \\ 0 & 0 & \frac{10^4 A_2}{mp+m_l} & -\frac{10^4 A_1}{mp+m_l} & \frac{10^2 b}{mp+m_l} & \frac{10^2 k}{mp+m_l} \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \delta x + \begin{pmatrix} \frac{r_{str}}{r_{sig}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \delta v_{sig} \cdots (12)$$

$$\begin{pmatrix} \delta V_{Fh} \\ \delta V_{x_{pv}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{A_2}{457.78} & -\frac{A_1}{457.78} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 & 0 \end{pmatrix} \delta x \cdots\cdots (13)$$

ここで

$$\begin{cases} a_p(3,2) = \beta \dfrac{K_{2inxs} - K_{2outxs}}{V_2 + A_2(L_n + x_{p0})} \\ a_p(3,3) = \beta \dfrac{K_{2inP_2} - K_{2outP_2}}{V_2 + A_2(L_n + x_{p0})} \\ a_p(3,5) = -\beta \dfrac{A_2}{V_2 + A_2(L_n + x_{p0})} \\ a_p(4,2) = \beta \dfrac{K_{1inxs} - K_{1outxs}}{V_1 + A_1(L_n - x_{p0})} \\ a_p(4,4) = \beta \dfrac{K_{1inP_1} - K_{1outP_1}}{V_1 + A_1(L_n - x_{p0})} \\ a_p(4,5) = \beta \dfrac{A_1}{V_1 + A_1(L_n - x_{p0})} \end{cases} \cdots\cdots\cdots\cdots (14)$$

また L_n は、ピストン全変位量の 1/2 を表している。

〈4・2〉 LPV システム 式 (12) のように状態空間表現における幾つかの要素は、負荷力変動など動作条件に従い異なる値を持つ。そこで、負荷力と等価な油圧推力の関数として定まるスケジューリングパラメータを導入し、変動する要素をスケジューリングパラメータで表すことにする。このような制御対象表現からは、スケジューリングパラメータを固定するごとに対応する線形状態方程式が決められ、LPV (Linear Parameter-varying) システムと呼ばれる。さて式 (12),(13) は、次式 (15),(16) のような形式となっている。

$$\delta \dot{x} = \frac{\partial}{\partial x} f(x_0) \delta x + B_p \delta v_{sig} \cdots\cdots\cdots\cdots (15)$$

$$\delta y = C_p \delta x \quad \cdots\cdots\cdots\cdots (16)$$

本制御対象では、B_p と C_p は定数行列で、全ての要素は機械的仕様から定めることができる。x_0 は平衡点における状態である。しかし、$f(x_0) + B_p v_{sig0} = 0$ は、状態に関して陰関数のため関係式から x_0 を直接求めることが難しい。いま、x_0 が与えられて y_0 が $y_0 = C_p x_0$ のように計算できるものと仮定する。さらに、スケジューリングパラメータ θ が y_0 の滑らかな関数 $\theta = \varphi(y_0)$ で与えられるものと仮定する。この時、スケジューリングパラメータ θ に依存する線形状態方程式を導くことができれば、その状態方程式は平衡点 x_0 の近傍で制御対象の振舞いを表す (7) (8)。さて、$a_p(3,2)$, $a_p(4,4)$ などは、x_{s0}, P_{20}, P_{10} と x_{p0} が与えられらば式 (14) から決められる。しかし、前述のように全動作領域に渡る平衡状態を直接求めることは難しい。

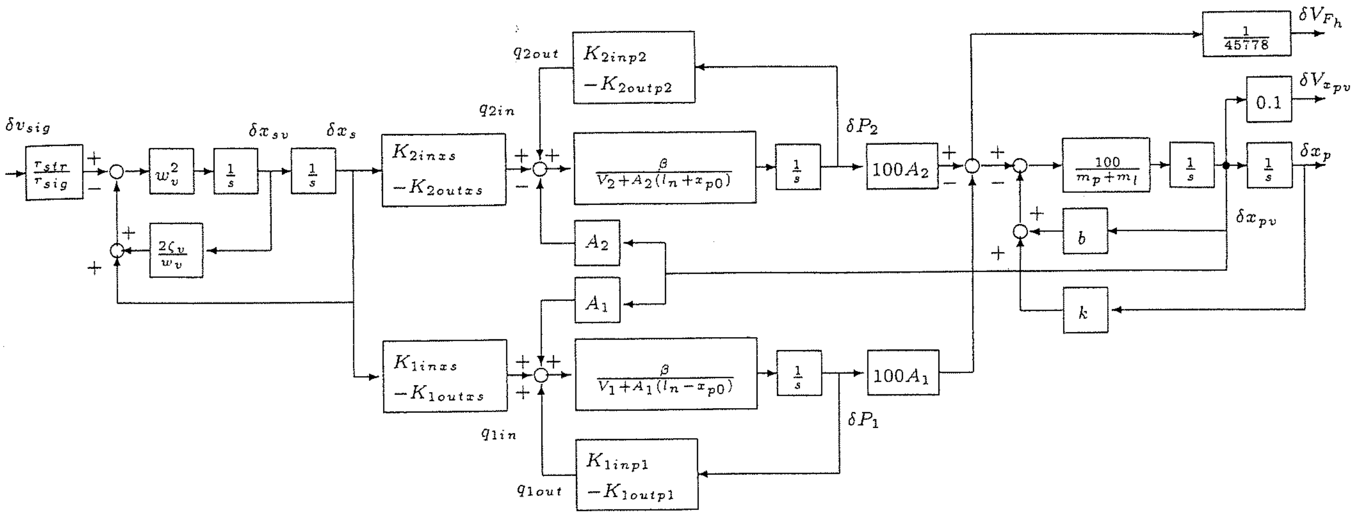

図 3　制御対象のブロック線図

Fig. 3.　Block diagram of the linearized plant

そこで、シミュレーション結果から得られる状態変数の値を用いることとする。式 (1),(2) や (3) を用いて構成する非線形制御対象モデルに対し、従来より用いられる PI 制御を適用し速度と推力制御閉ループ系を構成する。閉ループ系の状態変数が十分ゆっくりと変化し、平衡状態に近いと見なすことができるような、ランプ目標信号を与えて状態量を記録する。ランプ信号の大きさは、所望する全動作範囲に渡るよう調整する。このようにして、速度と推力閉ループ系における動作領域における平衡点の集合を得る。

スケジューリングパラメータ θ に依存する LPV システム表現とするには、まず得られた平衡点の値を式 (14) へ代入し、各要素の値を求める。同時に、式 (21)、または式 (22) から速度と推力制御対象それぞれのスケジューリングパラメータ θ の値を算出する。すべての平衡点の組に対してこれらが求まれば、式 (14) の各要素を θ の関数として多項式近似できる。ただし、式 (12)、(13) に従う制御対象モデルでは、行列要素の値がばらつき数値計算の上からは悪条件となり、後述の LMI 制約問題が解けない。このため状態空間表現から伝達関数表現へ変換した後、周波数軸を $1/(2{\cdot}\pi{\cdot}50)$ でスケーリングしてモデルパラメータのばらつきを低減している。速度制御対象を例にとれば、LPV モデルを

$$G_{pv}(s) = \frac{b_{v2}(\theta){\cdot}s^2 + b_{v1}(\theta){\cdot}s}{s^6 + a_{v5}(\theta){\cdot}s^5 + \ldots + a_{v0}(\theta)} \cdots\cdots \quad (17)$$

のように表すことになる。ここで、分子と分母多項式の係数は θ の 3 次多項式で、例えば $a_{v0}(\theta)$ は

$$a_{v0}(\theta) = a_{v0\cdot0} + a_{v0\cdot1}\theta + a_{v0\cdot2}\theta^2 + a_{v0\cdot3}\theta^3 \ \cdot \ (18)$$

のようである。また、図 4 に a_{v0} の多項式近似の様子を示す。図中、丸印は式 (14) による計算値、実線は θ の多項式で近似したものである。式 (18) を正準形式で状態空間表現へ再度書き直せば、スケジューリングパラメータ θ 依存

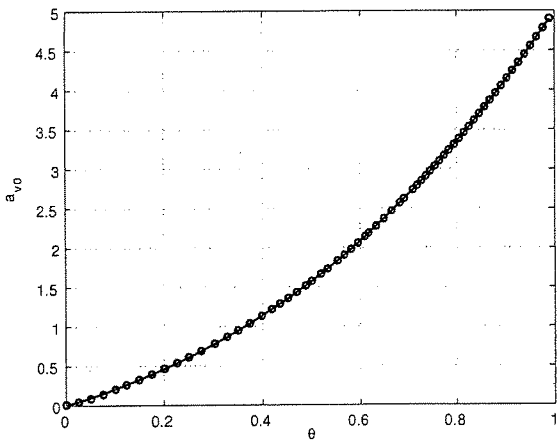

図 4　$a_{v0}(\theta)$ の多項式近似

Fig. 4.　Polynomial approximation of $a_{v0}(\theta)$

の三次行列多項式を含む式 (19) と (20) のようになる。

$$\delta \dot{x}_p = A_p(\theta)\delta x_p + B_p \delta v_{sig} \cdots\cdots\cdots\cdots\cdots\cdots (19)$$

$$\delta y_p = C_p \delta x_p \qquad \cdots\cdots\cdots\cdots\cdots\cdots (20)$$

いま、流量特性が $\sqrt{\triangle P}$ に依存することを考慮して、速度制御のためのスケジューリングパラメータ θ_v と推力制御のための θ_n を下の式 (21) と (22) のように決める。

$$\theta_v = \frac{\dfrac{1}{\sqrt{1-\frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1-\frac{F_{v\cdot rated}}{F_{max}}}} - 1}, \ [0 \le \theta_v \le 1] \cdots\cdots\cdots \ (21)$$

$$\theta_n = \frac{\dfrac{1}{\sqrt{1-\frac{F_x}{F_{max}}}} - 1}{\dfrac{1}{\sqrt{1-\frac{F_{n\cdot rated}}{F_{max}}}} - 1}, \ [0 \le \theta_n \le 1] \cdots\cdots\cdots \ (22)$$

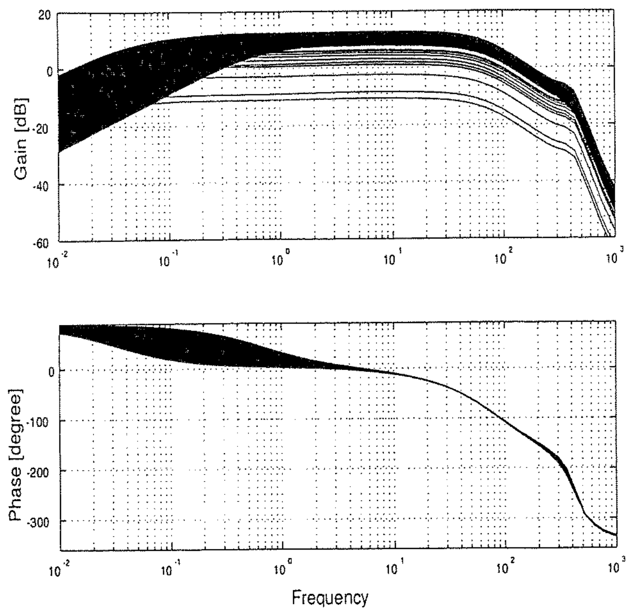

図 5　速度制御における制御対象の変動

Fig. 5.　Plant varying range in the velocity control

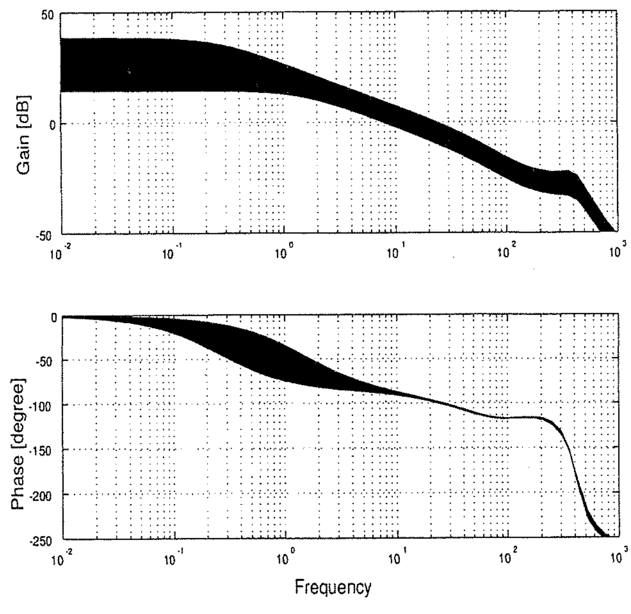

図 6　推力制御における制御対象の変動

Fig. 6.　Plant varying range in the force control

F_{max} は最大推力 (160kN) を表し，$F_{v\text{-}rated}$ と $F_{n\text{-}rated}$ は、それぞれ速度と推力制御時の定格推力である。定格速度 (20cm/s) 時における推力は、40～50kN まで上昇する。一方、保圧制御時に推力は、60kN に達する。また、F_x は V_{F_h} として観測される実推力である。図 5 にスケジューリングパラメータ θ_v に依存する、速度制御対象ボード線図の集合を示す。図 6 は推力制御対象ボード線図の集合を表している。

5. ゲインスケジュールルド制御器

ゲインスケジュールルド制御器設計に際しては、次のような要求仕様を考慮する。速度制御系では、ステップ応答の立上がり時間が 15 ms 以下で定常誤差をできるだけ小さくする。推力制御系では、15ms で定格推力まで到達するランプ信号へ追従し、かつ定常値に対する誤差が無いことする。モデル化の過程で制御対象のパラメータ誤差が存在するので、ロバストな制御器設計のため H_∞ 制御器設計の枠組みを採用する。一般化プラント構成のため感度関数に対する周波数重み関数を $W_s(s)$ として閉ループ系の周波数帯域を指定する。$W_a(s)$ は準相補感度関数に対する周波数重み関数で、制御対象入力端の加法的誤差を考慮する。このような $W_a(s)$ を採用する理由は、一般化プラントも θ の多項式表現となるが、その次数を極力低く抑えるためである。速度制御器設計では、

$$W_{sv}(s) = \frac{0.0025s + 68.75}{s + 0.005}, \quad W_{av}(s) = \frac{5(s+1)}{s+375}$$

推力制御器設計には、

$$W_{sn}(s) = \frac{0.4s+5}{s+0.01}, \quad W_{an}(s) = \frac{25(s+1)}{s+75}$$

を何度かの試行錯誤の後採用する。また、これら周波数重み関数とは別に、閉ループ系の応答特性を改善するため $(0.1s + 0.015)/s$ を速度制御対象へ直列接続し、同様に $(50s+1)/(s+0.01)$ を推力制御対象へ接続して、一般化プラントを構成する。H_∞ 制御問題を解くにあたり、問題を LMI 制約として定式化する。外乱から被制御量までの $\mathcal{L}_2$ ゲインを最小にする、次のような連続正定対象行列 $\mathcal{X}_v(\theta_v)$ と $\mathcal{Y}_v(\theta_v)$ を求める。

$$\mathcal{X}_v(\theta_v) = \mathcal{X}_{v0} + \mathcal{X}_{v1}\theta_v + \mathcal{X}_{v2}{\theta_v}^2 + \mathcal{X}_{v3}{\theta_v}^3$$

$$\mathcal{Y}_v(\theta_v) = \mathcal{Y}_{v0} + \mathcal{Y}_{v1}\theta_v + \mathcal{Y}_{v2}{\theta_v}^2 + \mathcal{Y}_{v3}{\theta_v}^3$$

$\mathcal{X}_v(\theta_v)$ と $\mathcal{Y}_v(\theta_v)$ も θ 依存の三次行列多項式としたのは、制御器の保守性を低減するためである。また、スケジューリングパラメータ θ の最大変化率を設計時点で考慮している。これにより、実装時に変化率情報を必要としない簡潔な線形制御器となる。詳細は文献 (9) (10) を参照。本論文で採用するスケジューリングパラメータ θ は、動作領域内で連続値を取るため無限個の LMI 制約となる問題がある。この問題に対しては、東らが提案する凸多面体 (11) を構成し有限個の LMI 制約へ帰着している。図 7 と 8 は、速度と推力制御に設計されたゲインスケジュールルド制御器のゲイン特性をスケジューリングパラメータ変動領域全体に渡り計算したものである。

6. シミュレーションと実験結果

式 (1),(2) と (3) に従う非線形な制御対象モデルに対し、ゲインスケージュールルド制御器を用いて閉ループ系を構成する。ステップ目標指令を 0 から 10 (cm/s) と 0 から 20

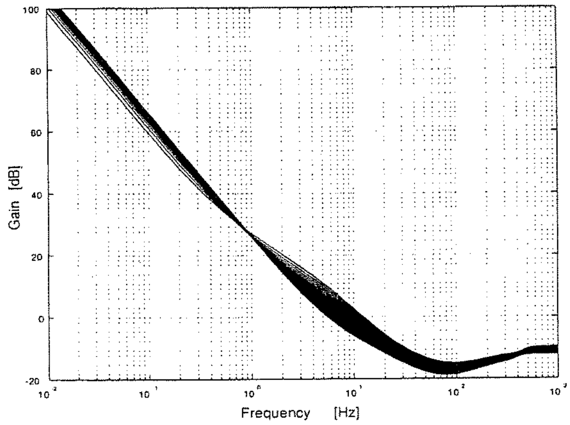

図 7 速度制御用制御器のゲイン特性

Fig. 7. Gain plot (velocity gain scheduling controller)

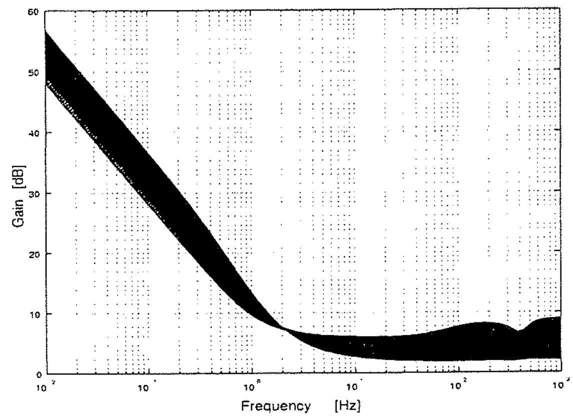

図 8 推力制御用制御器のゲイン特性

Fig. 8. Gain plot (force gain scheduling controller)

(cm/s) とした場合のシミュレーション結果を図 9 に示す。これら二種の目標指令に対し、速度応答は設計仕様を良く満たしている。ただし、非対称シリンダを用いているため、立上がり時間に対して立下り時にはより時間がかかることがわかる。図 10 には、推力制御器を用いたシミュレーション結果を示す。推力が増加、あるいは減少する両方のランプ目標指令へ良好に追従し、定常偏差も零へ収束して設計要求を満足していることが確かめられる。

実機試験は作動油温度範囲 20～50°C で行ない、本研究で用いる実験装置の主な諸元を表 1 に示す。速度制御系の実機試験においては、目標値の 65 % に相当する大きなオーバーシュートが発生する。また、9 から 10 ms のむだ時間が観測される。図 11 は速度目標が 0 から 10 cm/s へ立ち上がる様子を示す。図 12 は 0 から 20 cm/s への応答であるが、操作量を適当な大きさで飽和させることで大きなオーバーシュートの発生を避けている。このため、立ち上がり時間が長くなり要求仕様を満足していない。なお、操作量飽和要素による安定性への影響は、シミュレーションと実験で確認するものとした。

オーバーシュートの原因の一つとして制御対象のむだ時間が挙げられる。電気油圧制御系には多くのむだ時間要素

表 1 実験機パラメータ

Table 1. Experiment machine parameters

Supply pressure	Ps	13.7	[MPa]
Hydraulic fluid			
Bulk modulus	β	686.5	[MPa]
Specific gravity (15/4°C)		0.866	
Kinematic viscosity (40°C)		30.1	[cSt]
Cylinder			
Head side area	A_2	132.7	[cm^2]
Rod side area	A_1	54.2	[cm^2]
Stroke	L_n	12	[cm]
Trapped fluid volume			
Head side	V_2	450	[cm^3]
Rod side	V_1	350	[cm^3]
Flow control valve (Model:J790-004 Moog Japan Ltd.)			
Rated flow (at 6.86MPa drop)		1650	[l/min]
Rated signal	r_{sig}	±10	[V]
Rated stroke	r_{str}	0.32	[cm]
Frequency response (at 14MPa with ±25% of rated signal input)			
90 deg phase lag	w_v	534	[rad/s]
Damping ratio	ζ_v	0.94	
Lap	l_a	5 %以下	
Clearance	c_r	0.0075	[mm]
Mass (m=$m_p + m_l$)	m	14	[kg]
Coefficient of viscous damping	b	198.4	[N/(cm/s)]
Spring stiffness			
Velocity mode	k_v	30538	[N/cm]
Force mode	k_f	93163	[N/cm]

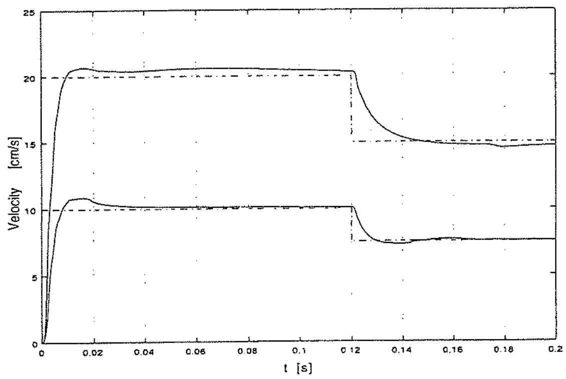

図 9 速度制御シミュレーション結果

Fig. 9. Velocity control simulation results

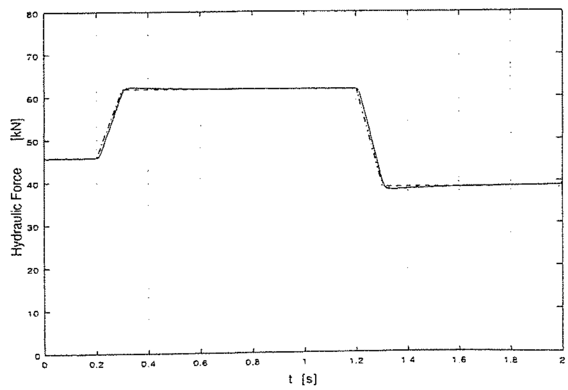

図 10 推力制御シミュレーション結果

Fig. 10. Force control simulation result

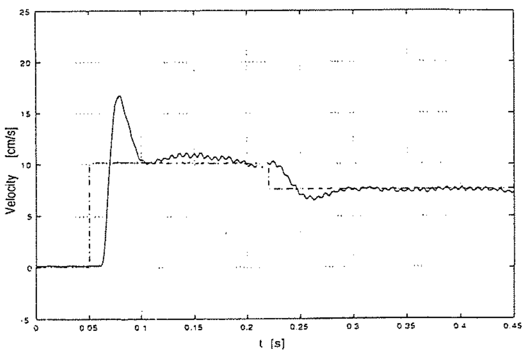

図 11　速度制御結果 (0 ～ 10 cm/s)
Fig. 11.　Velocity test result(0 to 10 cm/s)

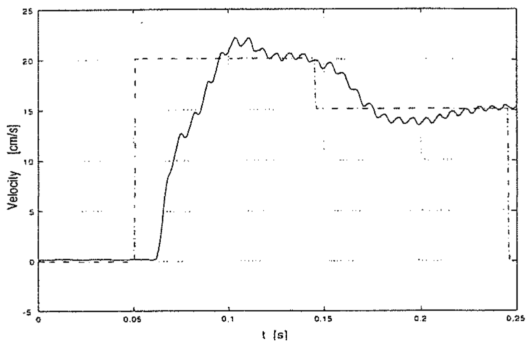

図 12　速度制御結果 (0 ～ 20 cm/s)
Fig. 12.　Velocity test result(0 to 20 cm/s)

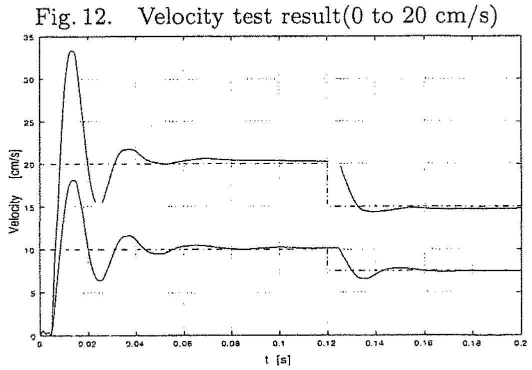

図 13　むだ時間を考慮した場合の速度制御シミュレーション
Fig. 13.　Simulation results of the veloacity control with dead time

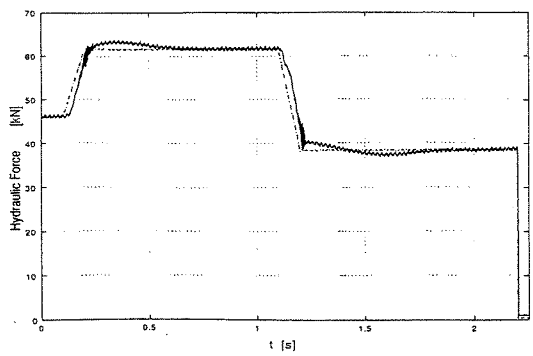

図 14　推力制御結果
Fig. 14.　Test result of the force control

が存在するが、これらを個々に扱うことは困難である。ここでは、流量制御弁のスプール変位にむだ時間が存在するものとしてシミュレーションを行い、実応答に現れるオーバーシュートを再現するむだ時間 3.5 ms を求めた。この時の速度制御シミュレーション結果を図 13 に示す。むだ時間を考慮することで、実験結果と同様の挙動を再現することから、むだ時間がオーバーシュートの主な原因と考えられる。一方、推力制御系では、シミュレーションに近い応答が得られ、これを図 14 に示す。ここでは、速度制御系に現れたオーバーシュートは見られない。これは推力制御対象のむだ時間が速度制御対象のそれよりも十分小さいことと、推力制御対象が持つ応答特性が閉ループ系に要求したものに対し余裕があるためと推察される。

7. まとめ

電気油圧サーボ系に用いられる、流量制御弁の開口部流れとすき間流れを連続に補間する流量計算式を提案し線形モデルの構成を示した。制御対象のパラメータが負荷力に依存し、比較的大きく変動するため、スケジューリングパラメータを導入して制御対象を LPV モデルとして記述した。制御対象モデルのパラメータは実負荷力を想定した非線形制御対象を用いたシミュレーションから求め、速度と推力のゲインスケジュールド制御器を設計した。設計した制御器の有効性をシミュレーションで確認したが、実機試験では速度制御対象に比較的大きなむだ時間が存在することから想定されないオーバーシュートが現れた。一方、推力の制御ではシミュレーションと実機試験共に想定した応答を得ることができ、本論文で述べた制御器設計手法が妥当なものであることが確認された。また、速度制御に対する設計モデルとしては、LPV とむだ時間を考慮したものが妥当であることが判明した。さらに多くのアプリケーションへ適用するために、むだ時間を考慮した設計を行なうことが今後の課題である。

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INFINITE-DIMENSIONAL LMI APPROACH TO ANALYSIS AND SYNTHESIS FOR LINEAR TIME-DELAY SYSTEMS

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This paper considers an analysis and synthesis problem of controllers for linear time-delay systems in the form of delay-dependent memory state feedback, and develops an Linear Matrix Inequality (LMI) approach. First, we present an existence condition and an explicit formula of controllers, which guarantee a prescribed level of L^2 gain of closed loop systems, in terms of infinite-dimensional LMIs. This result is rather general in the sense that it covers, as special cases, some known results for the cases of delay-independent/dependent and memoryless/memory controllers, while the infinity dimensionality of the LMIs makes the result difficult to apply. Second, we introduce a technique to reduce the infinite-dimensional LMIs to a finite number of LMIs, and present a feasible algorithm for synthesis of controllers based on the finite-dimensional LMIs.

1. INTRODUCTION

The fact that the state space of time-delay systems is infinite-dimensional leads generally to infinite-dimensional characterizations for analysis and synthesis in time-delay systems. For example, it is well known that the optimal LQ control for time-delay systems is given in the memory, i. e. infinite-dimensional, state feedback form whose feedback gains are characterized by the infinite-dimensional Riccati equations; as for state feedback control synthesis, we could say that the memory state feedback form is general and natural for time-delay systems, and can expect that memory state feedback controllers achieve better performance than memoryless state feedback controllers [2, 11, 16, 17]. Of course, the infinite-dimensional characterizations give us contrary hard problems in computations and implementations [6]. Our concern is to find a feasible approach to such infinite-dimensional tasks in analysis and synthesis for linear time-delay systems.

Recently the Linear Matrix Inequality (LMI) approach [4] has been developed in analysis and synthesis problems for linear time-delay systems and its advantages in numerical computations are presented [6, 8, 9, 11, 14, 15]; however, the approach is mostly developed under some finite-dimensional assumptions assured by a special form of Lyapunov functional in analysis and/or a memoryless controller form in synthesis. One exception which does not require such finite-dimensional assumptions is a series of the works by Gu [8, 9]; he proposes a discretization technique which can characterize a general Lyapunov functional with a finite number of LMIs. As more recent references on LMI for time-delay systems, which we learned after submitting this paper, [5] (and references inside) and [7] should be mentioned; a synthesis problem of state feedback with delay is discussed in [5], and a memoryless state feedback is designed for a system with distributed time-delays in [7].

In this paper, focusing on input-output L^2 gain performance, we consider an analysis and synthesis problem of memory state feedback controllers for linear systems with time-delay via an LMI approach which is an extension of the LMI approach developed in [2] where stability and stabilizability is focused. First we show a result of L^2 gain analysis of linear systems with time-delay and make a comparison with some previous works in some special cases. Next we discuss a controller synthesis problem based on this result of L^2 gain analysis. We also consider a synthesis problem of controllers with constrained feedback gains. We derive the results of L^2 gain analysis and controller synthesis in the form of infinite-dimensional LMIs, and present a procedure to reduce the infinite-dimensional LMIs to a finite number of LMIs. Finally we show a numerical example.

2. SYSTEM DESCRIPTION

Consider the following linear time-delay system defined on the time interval $[0,\infty)$,

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-h) + Bu(t) + Dw(t), \\ z(t) &= Cx(t), \\ x(\beta) &= 0, \ -h \le \beta \le 0, \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^{m_u}$ is the input, $w(t) \in R^{m_w}$ is the disturbance, and $z(t) \in R^l$ is the output. $A_0 \in R^{n\times n}$, $A_1 \in R^{n\times n}$, $B \in R^{n\times m_u}$, $C \in R^{l\times n}$ and $D \in R^{n\times m_w}$ are constant matrices. The parameter h denotes the time delay and $h > 0$.

The input $u(t)$ is given by the following state feedback controller,

$$u(t) = K_0x(t) + \int_{-h}^{0} K_{01}(\beta)\, x(t+\beta)\, \mathrm{d}\beta, \tag{2.2}$$

where $K_0 \in R^{m_u\times n}$ is a constant matrix and $K_{01}(\beta) \in L^2([-h,0]; R^{m_u\times n})$ is a square integrable matrix function.

In this paper, we use a notation,

$$L(\alpha,\beta) = \begin{bmatrix} P_0 & P_1(\beta) \\ P_1'(\alpha) & P_2(\alpha,\beta) \end{bmatrix} > (<)\ 0, \quad {}^\forall\alpha \in [-h,0],\ {}^\forall\beta \in [-h,0],$$

which means that P_0 and $P_2(\alpha,\beta)$ are symmetric, that is $P_0' = P_0$ and $P_2'(\alpha,\beta) =$

$P_2(\beta,\alpha)$, and the symmetrized matrix,

$$\frac{1}{2}(L(\alpha,\beta)+L'(\alpha,\beta))=\begin{bmatrix} P_0 & \frac{1}{2}(P_1(\alpha)+P_1(\beta)) \\ \frac{1}{2}(P_1'(\alpha)+P_1'(\beta)) & \frac{1}{2}(P_2(\alpha,\beta)+P_2(\beta,\alpha)) \end{bmatrix},$$

is positive definite (negative definite) for each $(\alpha,\beta)\in[-h,0]\times[-h,0]$, where "$'$" denotes transposition of vector and matrix. The notation, $L(\alpha,\beta)\geq(\leq)\ 0$, is similarly defined. Note that, if a matrix function $L(\alpha,\beta)>0$ is continuous in (α,β), there exists a positive number λ such that $L(\alpha,\beta)\geq\lambda I$ for all $(\alpha,\beta)\in[-h,0]\times[-h,0]$, where I denotes identity matrix.

3. L^2 GAIN ANALYSIS

3.1. General result

From (2.1),(2.2),the closed loop system can be written in the following form,

$$\begin{aligned} \dot{x}(t) &= \widetilde{A}_0x(t)+\widetilde{A}_1x(t-h)+\int_{-h}^{0}\widetilde{A}_{01}(\beta)\,x(t+\beta)\,\mathrm{d}\beta+Dw(t), \\ z(t) &= Cx(t), \end{aligned} \tag{3.1}$$

where

$$\widetilde{A}_0=A_0+BK_0,\quad \widetilde{A}_1=A_1,$$
$$\widetilde{A}_{01}(\beta)=BK_{01}(\beta).$$

First we analyze L^2 gain for the closed loop system (3.1). The L^2 gain of the system (3.1) is defined as follows,

$$G=\sup_{w\in L^2,w\neq 0}\frac{||z||_{L^2}}{||w||_{L^2}},$$

where $||\cdot||_{L^2}$ denotes L^2 norm.

Now we introduce the following functional,

$$\begin{aligned} V(x_t) &= x'(t)\,Px(t)+\int_{-h}^{0}x'(t+\beta)\,Qx(t+\beta)\,\mathrm{d}\beta \\ &\quad +x'(t)\int_{-h}^{0}R(\beta)\,x(t+\beta)\,\mathrm{d}\beta+\int_{-h}^{0}x'(t+\alpha)R'(\alpha)\,\mathrm{d}\alpha x(t) \\ &\quad +\int_{-h}^{0}\int_{-h}^{0}x'(t+\alpha)S(\alpha,\beta)\,x(t+\beta)\,\mathrm{d}\alpha\mathrm{d}\beta, \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} &x_t=\{x(t+\beta)\,|\,-h\leq\beta\leq 0\}, \\ &P,Q\in R^{n\times n}, \\ &R(\beta)\in L^2([-h,0];R^{n\times n}), \\ &S(\alpha,\beta)\in L^2([-h,0]\times[-h,0];R^{n\times n}). \end{aligned}$$

By using this functional, we have a result for L^2 gain analysis of the time-delay system (3.1).

Theorem 3.1. If there exist constant matrices P, Q and continuously differentiable matrix functions $R(\beta)$, $S(\alpha,\beta)$ which satisfy the following inequalities,

$$L_1(\alpha,\beta) = \left[\begin{array}{cc} \left(\begin{array}{c}\widetilde{A}_0'P + P\widetilde{A}_0 + Q \\ +R'(0) + R(0) + C'C\end{array}\right) & P\widetilde{A}_1 - R(-h) \\ \widetilde{A}_1'P - R'(-h) & -Q \\ \left(\begin{array}{c}\widetilde{A}_{01}'(\alpha)P + R'(\alpha)\widetilde{A}_0 \\ -\frac{\partial}{\partial\alpha}R'(\alpha) + S(\alpha,0)\end{array}\right) & R'(\alpha)\widetilde{A}_1 - S(\alpha,-h) \\ D'P & 0 \end{array}\right.$$

$$\left.\begin{array}{cc} \left(\begin{array}{c}P\widetilde{A}_{01}(\beta) + \widetilde{A}_0'R(\beta) \\ -\frac{\partial}{\partial\beta}R(\beta) + S(0,\beta)\end{array}\right) & PD \\ \widetilde{A}_1'R(\beta) - S(-h,\beta) & 0 \\ \left(\begin{array}{c}R'(\alpha)\widetilde{A}_{01}(\beta) + \widetilde{A}_{01}'(\alpha)R(\beta) \\ -(\frac{\partial}{\partial\beta} + \frac{\partial}{\partial\alpha})S(\alpha,\beta)\end{array}\right) & R'(\alpha)D \\ D'R(\beta) & -\gamma^2 I \end{array}\right] < 0, \qquad (3.3)$$

$$L_2(\alpha,\beta) = \begin{bmatrix} P & R(\beta) \\ R'(\alpha) & S(\alpha,\beta) \end{bmatrix} > 0, \qquad (3.4)$$

$$Q > 0, \qquad (3.5)$$

$$\forall\alpha \in [-h,\ 0],\ \forall\beta \in [-h,\ 0],$$

then the time-delay system (3.1) is internally, asymptotically, stable and the L^2 gain of (3.1) is less than γ.

Proof. (Stability) We shall show that the functional (3.2) is a Lyapnov functional for the system(3.1), that is $V(x_t) > 0$ and $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) > 0$ for $x(t) \neq 0$. $V(x_t) > 0$ follows from (3.4),(3.5) and the expression,

$$V(x_t) = \int_{-h}^{0}\int_{-h}^{0} \begin{bmatrix} h^{-1}x(t) \\ x(t+\alpha) \end{bmatrix}' L_2(\alpha,\beta) \begin{bmatrix} h^{-1}x(t) \\ x(t+\beta) \end{bmatrix} \mathrm{d}\alpha\mathrm{d}\beta + \int_{-h}^{0} x'(t+\beta)\,Qx(t+\beta)\,\mathrm{d}\beta.$$

Differentiating both sides of (3.2) with respect to t along the trajectory of the system (3.1) with $w(t) \equiv 0$ and rearranging terms, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) = \int_{-h}^{0}\int_{-h}^{0} \begin{bmatrix} h^{-1}x(t) \\ h^{-1}x(t-h) \\ x(t+\alpha) \end{bmatrix}' L_0(\alpha,\beta) \begin{bmatrix} h^{-1}x(t) \\ h^{-1}x(t-h) \\ x(t+\beta) \end{bmatrix} \mathrm{d}\alpha\mathrm{d}\beta$$

where

$$L_0(\alpha,\beta) =$$
$$\begin{bmatrix} \begin{pmatrix} \tilde{A_0}'P + P\tilde{A_0} \\ +Q \\ +R(0)' + R(0) \end{pmatrix} & P\tilde{A_1} - R(-h) & \begin{pmatrix} P\tilde{A_{01}}(\beta) + \tilde{A_0}'R(\beta) \\ -\frac{\partial}{\partial\beta}R(\beta) + S(0,\beta) \end{pmatrix} \\ \tilde{A_1}'P - R'(-h) & -Q & \tilde{A_1}'R(\beta) - S(-h,\beta) \\ \begin{pmatrix} \tilde{A_{01}}'(\alpha)P \\ +R'(\alpha)\tilde{A_0} \\ -\frac{\partial}{\partial\alpha}R'(\alpha) \\ +S(\alpha,0) \end{pmatrix} & R'(\alpha)\tilde{A_1} - S(\alpha,-h) & \begin{pmatrix} R'(\alpha)\tilde{A_{01}}(\beta) \\ +\tilde{A_{01}}'(\alpha)R(\beta) \\ -(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta})S(\alpha,\beta) \end{pmatrix} \end{bmatrix}.$$

Using Schur Complement, we can show that the inequality (3.3), that is $L_1(\alpha,\beta) < 0$ is equivalent to the following inequality,

$$L_0(\alpha,\beta) + \begin{bmatrix} C' & PD \\ 0 & 0 \\ 0 & R'(\alpha)D + R'(\beta)D \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & \gamma^2 I \end{bmatrix}^{-1} \begin{bmatrix} C' & PD \\ 0 & 0 \\ 0 & R'(\alpha)D + R'(\beta)D \end{bmatrix}' < 0.$$

Thus, from (3.3) we have $L_0(\alpha,\beta) < 0$, and from the above expression of $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t)$ we can see $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) < 0$ for $x(t) \neq 0$. Then, the internal, asymptotic, stability of the system (3.1) follows from the well-known stability result [10].

(L^2 gain) First note that the internal, asymptotic, stability of the system (3.1) implies $z \in L^2([-h,0];R^l)$ and, in particular, $x(\infty) = 0$ for any $w \in L^2([-h,0];R^{m_w})$. Hence, from $x(\beta) = 0, -h \le \beta \le 0$, we have the identity,

$$\|z\|_{L^2}^2 - \gamma^2\|w\|_{L^2}^2 = \int_0^\infty \left[z'(t)z(t) - \gamma^2 w'(t)\,w(t) + \frac{\mathrm{d}}{\mathrm{d}t}V(x_t) \right] \mathrm{d}t,$$

for $w \in L^2([-h,0];R^l)$. Calculating $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t)$ with (3.2) along the trajectory of the system (3.1) and substituting it into the above identity, we obtain

$$\|z\|_{L^2}^2 - \gamma^2\|w\|_{L^2}^2 = \int_0^\infty \left(\int_{-h}^0 \int_{-h}^0 \begin{bmatrix} h^{-1}x(t) \\ h^{-1}x(t-h) \\ x(t+\alpha) \\ h^{-1}w(t) \end{bmatrix}' L_1(\alpha,\beta) \begin{bmatrix} h^{-1}x(t) \\ h^{-1}x(t-h) \\ x(t+\alpha) \\ h^{-1}w(t) \end{bmatrix} \mathrm{d}\alpha \mathrm{d}\beta \right) \mathrm{d}t.$$

Then $G < \gamma$ follows from (3.3). □

3.2. Results in special cases

It is known that the existence of Lyapunov functional of the form (3.2) is a necessary and sufficient condition for internal stability of linear time-delay systems. From this fact and the analogy of L^2 gain analysis in linear systems with no delay, we suspect that the functional (3.2) might lead to a necessary and sufficient condition, and the LMI conditions in Theorem 3.1 might be rather less-conservative. Instead of pursuing this issue, here, we observe that, for particular choices of structure of the solution $(P, Q, R(\beta), S(\alpha,\beta))$, the LMI conditions (3.3), (3.4), (3.5) in Theorem 3.1 is reduced to the well known condition of delay-independent types [11] or delay-dependent types [14]. To simplify the discussion, we focus on the case of the following system,

$$\begin{aligned}\dot{x}(t) &= \widetilde{A}_0 x(t) + \widetilde{A}_1 x(t-h) + Dw(t),\\ z(t) &= Cx(t).\end{aligned}$$

First note that the positive definiteness of inequalities (3.4) and (3.5) in Theorem 3.1, which are required for (3.2) to be a Lyapunov functional of this system, can be relaxed to positive semidefiniteness except $P > 0$. In view of this, let $R(\beta) \equiv 0$ and $S(\alpha,\beta) \equiv 0$ in the inequality (3.3), we can rewrite (3.3) as

$$\begin{bmatrix} \widetilde{A}_0' P + P\widetilde{A}_0 + Q + C'C & P\widetilde{A}_1 & PD \\ \widetilde{A}_1' P & -Q & 0 \\ D'P & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{3.6}$$

and obtain the next result from Theorem 3.1.

Corollary 3.2. If there exists positive definite P and Q which satisfy the LMI condition (3.6), then the time-delay system is internally, asymptotically, stable and the L^2 gain is less than γ.

The LMI condition (3.6) is equivalent to the Riccati inequality condition derived by Lee et al in [14].

Next let $R(\beta) = PU(\beta)$ and $S(\alpha,\beta) = U'(\alpha)PU(\beta)$, where $U(\beta)$ is a matrix function defined by the following functional differential equation,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\beta}U(\beta) &= (\widetilde{A}_0 + U(0))U(\beta),\\ U(-h) &= \widetilde{A}_1,\ -h \le \beta \le 0.\end{aligned} \tag{3.7}$$

We have a sufficient condition for the inequality (3.3), which is given by

$$\begin{bmatrix} M + C'C & M & PD \\ M & M & PD \\ D'P & D'P & -\gamma^2 I \end{bmatrix} < 0, \tag{3.8}$$

where $M = (\widetilde{A}_0 + U(0))'P + P(\widetilde{A}_0 + U(0))$. Thus we can obtain the next result from Theorem 3.1.

Corollary 3.3. If there exist a positive definite matrix P and a matrix function $U(\beta)$ which is the solution to the equation (3.7) and satisfy the LMI condition (3.8), then the time-delay system is internally, asymptotically, stable and the L^2 gain is less than γ.

Corollary 3.3 is the result derived by He et al in [11] where the LMI condition (3.8) is expressed in the equivalent Riccati inequality form.

The LMI condition (3.6) is independent of the time-delay h and is finite-dimensional. On the other hand, the LMI condition (3.8), which seems the finite-dimensional one at first sight, is infinite-dimensional in actual, since it requires to solve the infinite-dimensional equation (3.7) that depends on the time-delay h.

As shown in Theorem 3.1 and observed above, the Lyapunov functional (3.2) leads generally to infinite-dimensional and delay-dependent conditions or finite-dimensional and delay-independent conditions. In some special cases, however, our approach with a generalization of the functional (3.2) leads us to finite-dimensional and delay-dependent conditions. To illustrate this fact, consider the system with only distributed delay,

$$\begin{aligned} \dot{x}(t) &= \widetilde{A}_0 x(t) + \int_{-h}^{0} \widetilde{A}_{01}(\beta)\, x(t+\beta)\, \mathrm{d}\beta, \\ z(t) &= Cx(t), \end{aligned} \tag{3.9}$$

and consider the following functional,

$$V(x_t) = x'(t)\, Px(t) + \int_{-h}^{0} x'(t+\beta)\, Q(\beta)\, x(t+\beta)\, \mathrm{d}\beta. \tag{3.10}$$

Note that $Q(\beta)$ is here allowed to depend on β. Then calculating the time derivative of (3.10) and rearranging terms as in the proof of Theorem 3.1, we have a sufficient condition for $\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) + z'(t)z(t) - \gamma^2 w'(t)\, w(t) < 0$, which is given as $Q(-h) \geq 0$ and

$$\begin{bmatrix} \widetilde{A}_0' P + P\widetilde{A}_0 + Q(0) & P\widetilde{A}_{01}(\beta) & PD \\ \widetilde{A}_{01}'(\beta) P & -h^{-1}\frac{\mathrm{d}}{\mathrm{d}\beta}Q(\beta) & 0 \\ D'P & 0 & -\gamma^2 I \end{bmatrix} < 0,$$

$$\forall \beta \in [-h,\ 0].$$

This LMI condition is the infinite-dimensional one. However, in the special case of $\widetilde{A}_{01}(\beta) = \widetilde{A}_{01}$, setting $Q(\beta) = (\beta + h)I$ yields the following finite-dimensional LMI condition of delay-dependence,

$$\begin{bmatrix} \widetilde{A}_0' P + P\widetilde{A}_0 + hI & P\widetilde{A}_{01} & PD \\ \widetilde{A}_{01}' P & -h^{-1} I & 0 \\ D'P & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{3.11}$$

Thus we obtain the next result.

Corollary 3.4. If there exists the positive definite matrix P which satisfies the LMI condition (3.11), then the time-delay system (3.9) with $\widetilde{A}_{01}(\beta) = \widetilde{A}_{01}$ is internally, asymptotically, stable and the L^2 gain is less than γ.

In [15], Li and DeSouza derived a finite-dimensional and delay-dependent LMI condition for robust stability and stabilization based on a Lyapunov functional. We can see that their LMI has a similar structure to (3.11), and expect that our framework described by (3.9) and (3.10) presents an essential point of their procedure consisting of a sophisticated system transformation and a special Lyapunov functional.

4. CONTROLLER SYNTHESIS

4.1. Synthesis of controller gain

Now we consider the synthesis of controllers which attain a prescribed level of L^2 gain of the closed loop system (3.1). The problem is to find a gain $(K_0, K_{01}(\beta))$ of the controller (2.2) based on the analysis result of Theorem 3.1.

Theorem 4.1. If there exist constant matrices W, X, Z_0 and continuously differentiable matrix function $Z_{01}(\beta)$ and $Y(\alpha, \beta)$ which satisfy the following inequalities,

$$L_3(\alpha,\beta) = \begin{bmatrix} \begin{pmatrix} WA_0' + WA_0 \\ +X + 2W \\ +BZ_0 + Z_0'B' \end{pmatrix} & A_1W - W & \begin{pmatrix} BZ_{01}(\beta) \\ +WA_0' \\ +Z_0'B' \\ +Y(0,\beta) \end{pmatrix} & WC' & D \\ WA_1' - W & -X & \begin{pmatrix} WA_1' \\ -Y(-h,\beta) \end{pmatrix} & 0 & 0 \\ \begin{pmatrix} Z_{01}'(\alpha)B' \\ +A_0W + BZ_0 \\ +Y(\alpha,0) \end{pmatrix} & \begin{pmatrix} A_1W \\ -Y(\alpha,-h) \end{pmatrix} & \begin{pmatrix} BZ_{01}(\beta) \\ +Z_{01}'(\alpha)B' \\ -(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) \\ Y(\alpha,\beta) \end{pmatrix} & 0 & D \\ CW & 0 & 0 & -I & 0 \\ D' & 0 & D' & 0 & -\gamma^2 I \end{bmatrix} < 0, \quad (4.1)$$

$$L_4(\alpha,\beta) = \begin{bmatrix} W & W \\ W & Y(\alpha,\beta) \end{bmatrix} > 0, \quad (4.2)$$

$$X > 0, \quad (4.3)$$

$$\forall \alpha \in [-h,\ 0],\ \forall \beta \in [-h,\ 0],$$

then the time-delay system (2.1) with the state feedback controller (2.2)

$$K_0 = Z_0 W^{-1}, \quad K_{01}(\beta) = Z_{01}(\beta) W^{-1}, \quad (4.4)$$

is internally, asymptotically, stable and the L^2 gain is less than γ.

Proof. Assume that the conditions of Theorem 4.1 are satisfied and consider the closed loop system (3.1) with the feedback gain given by (4.4). Then, using the inequalities (4.1), (4.2) and (4.3) together with Schur Complement, we can show that the inequalities (3.3), (3.4) and (3.5) admit the following solutions,

$$P = W^{-1}, \quad R(\beta) = W^{-1}, \quad S(\alpha,\beta) = W^{-1}Y(\alpha,\beta)\,W^{-1}, \quad Q = W^{-1}XW^{-1},$$

that is, the conditions of Theorem 3.1 are satisfied. Thus, Theorem 4.1 follows from Theorem 3.1. □

4.2. Constraint on controller gain

To simplify the discussion, we assume that the controlled output $z(t)$ in (2.1) does not directly depend on the control input $u(t)$. This may lead to large control inputs which are synthesized by Theorem 4.1. One conventional way to make such a possibility small is to impose some constraints on the feedback gain.

Now we constrain the feedback gain as follows,

$$K_0'K_0 < \gamma_1 I,\ K_{01}'K_{01}(\beta) < \gamma_2 I,\ {}^{\forall}\beta \in [-h,\ 0], \tag{4.5}$$

where γ_1 and γ_2 are given in advance, and consider the same synthesis problem as in Section 4.1. Based on Theorem 4.1, we have the following theorem.

Theorem 4.2. For given positive numbers p_1, p_2 and q, if there exist W, X, Z_0 and continuously differentiable matrix function $Z_{01}(\beta)$ and $Y(\alpha,\beta)$ which satisfy the following inequalities,

$$L_3(\alpha,\beta) < 0,\ L_4(\alpha,\beta) > 0,\ X > 0, \tag{4.6}$$

$$\begin{bmatrix} p_1 I & Z_0' \\ Z_0 & I \end{bmatrix} > 0, \tag{4.7}$$

$$\begin{bmatrix} p_2 I & Z_{01}'(\beta) \\ Z_{01}(\beta) & I \end{bmatrix} > 0, \tag{4.8}$$

$$\begin{bmatrix} qI & I \\ I & W \end{bmatrix} > 0, \tag{4.9}$$

$${}^{\forall}\alpha \in [-h,\ 0],\ {}^{\forall}\beta \in [-h,\ 0],$$

where $L_3(\alpha,\beta)$ and $L_4(\alpha,\beta)$ are given as (4.1) and (4.2) respectively, then the time-delay system (2.1) with the state feedback controller (2.2)

$$K_0 = Z_0W^{-1},\ \ K_{01}(\beta) = Z_{01}(\beta)\,W^{-1}, \tag{4.10}$$

is internally, asymptotically, stable and the L^2 gain is less than γ. Here K_0 and $K_{01}(\beta)$ are constrained as follows,

$$K_0'K_0 < p_1q^2 I,\ \ K_{01}'(\beta)K_{01}(\beta) < p_2q^2 I.$$

Proof. (4.7), (4.8) and (4.9) are equivalent to the following conditions respectively,

$$Z_0' Z_0 < p_1 I, \quad Z_{01}'(\beta)\, Z_{01}(\beta) < p_2 I, \quad W^{-1} < qI.$$

By using the above conditions, we have the following results,

$$\begin{aligned} K_0' K_0 &= W^{-1} Z_0' Z_0 W^{-1} \\ &< p_1 W^{-1} W^{-1} \\ &< p_1 q^2 I \\ K_{01}'(\beta) K_{01}(\beta) &= W^{-1} Z_{01}'(\beta)\, Z_{01}(\beta)\, W^{-1} \\ &< p_2 W^{-1} W^{-1} \\ &< p_2 q^2 I. \end{aligned}$$

□

Thus by using this theorem and choosing p_1, p_2 and q appropriately, we can obtain the controllers with feedback gains satisfying (4.5) and assuring $G < \gamma$. Next we show an algorithm to choose p_1, p_2 and q.

Algorithm:

Step 1: Let p_{10}, p_{20} and q_0 be initial values of p_1, p_2 and q respectively.

Step 2: Solve inequalities in Theorem 4.2 and the following inequalities,

$$p_1 < p_{10}, \quad p_2 < p_{20}, \quad q < q_0.$$

— If *Step 2* has no solution, the algorithm has no solution for the initial values p_{10}, p_{20}, q_0.

Step 3: Check the next conditions for p_1, p_2 and q of *Step 2*.

$$p_1 q^2 < \gamma_1, \quad p_2 q^2 < \gamma_2. \tag{4.11}$$

— If (4.11) is satisfied, the algorithm is finished. The controller designed in *Step 2* satisfies (4.5).

— If (4.11) is not satisfied, go back to *Step 1*.

When we come back from *Step 3* to *Step 1*, p_{10}, p_{20} and q_0 are generally modified into smaller ones, so that p_1, p_2 and q can be chosen smaller in *Step 2* and satisfy (4.11) in *Step 3*. Note that it is generally more difficult to solve the inequalities of Theorem 4.2 for smaller p_1, p_2 and q. The solvability condition of the inequalities of Theorem 4.2, which might be characterized by open loop properties, e.g. stabilizability, of the system (3.1), is our future task of interest.

To illustrate this algorithm, a design example is presented in Section 6. It is a matter of course that smaller gains $(K_0, K_{01}(\beta))$, which are realized by taking p_1, p_2 and q smaller, do not necessarily guarantee smaller control inputs. One possible way to handle constraints on control inputs such as $u(t)'u(t) \leq \mu$ is to introduce a step of state-reachable set analysis, which is characterized with infinite-dimensional LMIs [13]. As for the existing results on constrained control input, see Chapter 14 of [6] and references inside.

5. REDUCTION TO A FINITE NUMBER OF LMI CONDITIONS

Inequalities in Theorem 4.1 depend on parameters α and β. It seems difficult to solve these infinite-dimensional (parameter-dependent) inequalities directly. In our approach, we reduce these infinite-dimensional inequalities to a finite number of LMIs by using the technique in [3, 2], and obtain the solution of the infinite-dimensional inequalities by computing the finite number of LMIs.

Here we restrict solutions in Theorem 4.1 to the following forms,

$$\begin{aligned} Y(\alpha,\beta) &= Y_0 + g_1(\alpha,\beta)\,Y_1 + g_2(\alpha,\beta)\,Y_2 + \cdots + g_{l_Y}(\alpha,\beta)\,Y_{l_Y}, \\ Z_{01}(\beta) &= Z_0^{01} + h_1(\beta)Z_1^{01} + h_2(\beta)Z_2^{01} + \cdots + h_{l_z}(\beta)\,Z_{l_z}^{01}, \end{aligned} \tag{5.1}$$

where $g_i : R^2 \to R$ is a continuous differentiable function of α and β such that

$$g_i(\alpha,\beta) = g_i(\beta,\alpha),$$

$h_i : R \to R$ is a continuous differentiable function of β, and the unknown matrices satisfy

$$Y_i \in R^{n\times n},\ Y_i' = Y_i\ (i = 0,1,\cdots,l_Y),$$
$$Z_i^{01} \in R^{m_u\times n}\ (i = 0,1,\cdots,l_Z).$$

Note that (5.1) satisfies matrix inequalities (4.1), (4.2). Then inequalities in Theorem 4.1 can be written in the form of the following parameter dependent LMI condition,

$$F_0(M) + f_1(\theta)\,F_1(M) + \cdots + f_r(\theta)\,F_r(M) < 0, \tag{5.2}$$

where

$$\theta \in \Theta = \{[\alpha\ \beta]' \,|\, \alpha \in [-h,\ 0], \beta \in [-h,\ 0]\},$$

and $f_i : R^2 \to R$ is a continuous function of α and β, and a symmetric matrix function F_i depends affinely on the unknown matrix $M = [Y_0,\cdots,Y_{l_Y}, Z_0^{01},\cdots,Z_{l_Z}^{01}]$. The parameter dependent LMI condition (5.2) can be reduced to a finite number of LMI conditions as follows.

Theorem 5.1. [3] Let $\{p_1, p_2, \cdots, p_q\}$ be vertices of a convex polyhedron which includes the curved surface T,

$$T = \{[f_1(\theta)\ f_2(\theta)\ \cdots\ f_r(\theta)]'\ \mid \theta \in \Theta\}. \tag{5.3}$$

Assume that there exists M which satisfies the following LMI condition for all $p_i (i = 1, 2, \cdots, q)$,

$$F_0(M) + p_{i1}F_1(M) + \cdots + p_{ir}F_r(M) < 0, \tag{5.4}$$

where p_{ij} is the jth element of p_i. Then M satisfies (5.2) for all $\theta \in \Theta$.

A general technique to construct a convex polyhedron which includes the curved surface T is proposed in [3].

In the special case that $r = 2s$,

$$f_i(\alpha, \beta) = \begin{cases} f_i(\alpha), & i = 1, 2, \cdots, s, \\ f_i(\beta), & i = s+1, s+2, \cdots, 2s, \end{cases}$$

and $f_i(\alpha)$ and $f_i(\beta)$ are polynomial functions of α and β, respectively, we can use a simple technique to construct such a convex polyhedron, which is given by

Theorem 5.2. [12] Let $p^{ij} \in R^{2s}$ be defined such that

$$p^{ij} = \begin{bmatrix} p^i \\ p^j \end{bmatrix}, \quad i, j = 0, 1, \cdots, s,$$

where

$$\begin{aligned} p^0 &= [h_1 \; h_1^2 \; \cdots h_1^s]' \in R^s, \\ p^1 &= [h_2 \; h_1^2 \; \cdots h_1^s]' \in R^s, \\ &\vdots \\ p^s &= [h_2 \; h_2^2 \; \cdots h_2^s]' \in R^s. \end{aligned}$$

Then the convex polyhedron whose vertices are given by $p^{ij}, i, j = 0, 1, \cdots, s$ includes the curved surface $T = \{[\alpha \; \alpha^2 \; \cdots \; \alpha^s \; \beta \; \beta^2 \; \cdots \; \beta^s]' | \alpha \in [h_1, h_2], \beta \in [h_1, h_2]\}$

Actually taking $h_1 = -h$ and $h_2 = 0$ in Theorem 5.2, we have a desired convex polyhedron. To make the volume of the convex polyhedron smaller for less conservative solutions, we may divide the interval $[-h, 0]$ into sub-intervals $[h_a, h_b]$, $[h_c, h_d]$, $\cdots$, and apply repeatedly Theorem 5.2 in each sub-interval.

6. NUMERICAL EXAMPLE

Consider the next time-delay system,

$$\Sigma_p : \begin{aligned} &\dot{x}(t) = x(t) + 0.3x(t-1) + u(t), \\ &y(t) = x(t), \\ &x(\beta) = 0, \; -h \le \beta \le 0. \end{aligned} \tag{6.1}$$

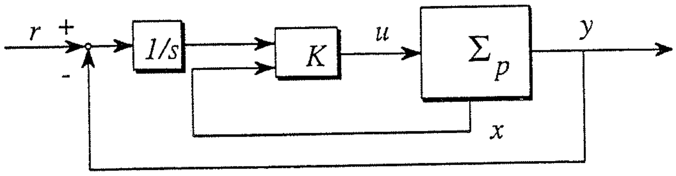


Fig. 1. The closed loop system.

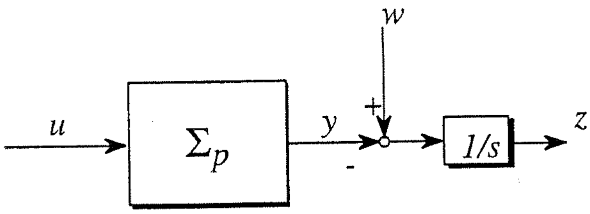


Fig. 2. Generalized plant.

Now we design a state feedback controller K of the form (2.2) such that the error, $r-y$, is asymptotically zero. As shown in Figure 1, an integrator is added in order to assure the asymptotically-zero error for step references. When we use the technique of Section 5, we restrict solutions of Theorem 4.1 and Theorem 4.2 as follows,

$$\begin{aligned} Z_{01}(\beta) &= Z_0 + \beta Z_1 + \beta^2 Z_2, \\ Y(\alpha,\beta) &= Y_0 + (\alpha+\beta)Y_1 + (\alpha^2+\beta^2)\,Y_2. \end{aligned}$$

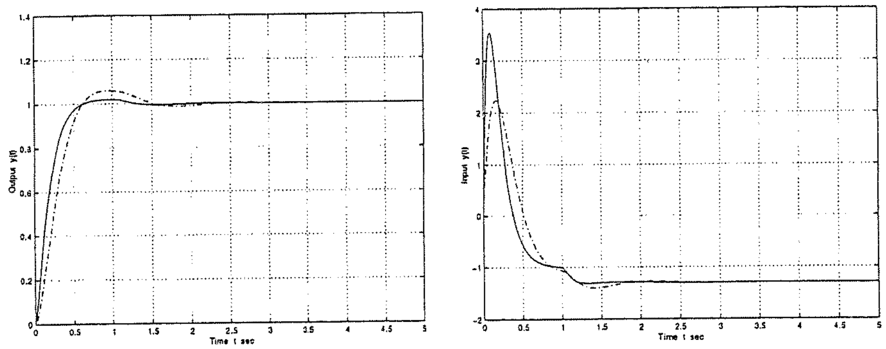
First we apply Theorem 4.1 to Figure 2 and obtain the state feedback controller with the next feedback gains,

$$\begin{aligned} K_0 &= \left[\ 115.48 \quad -24.94\ \right], \\ K_{01}(\beta) &= \left[\ 75.79 \quad -12.45\ \right] + \beta \left[\ -9.31 \quad -3.09\ \right] + \beta^2 \left[\ 15.14 \quad -3.53\ \right]. \end{aligned} \tag{6.2}$$

Second setting $p_1 = 3.49 \times 10^4$, $p_2 = 1.28 \times 10^2$, $q = 2.56$ and using Theorem 4.2, we obtain the state feedback controller (2.2) with the next feedback gains,

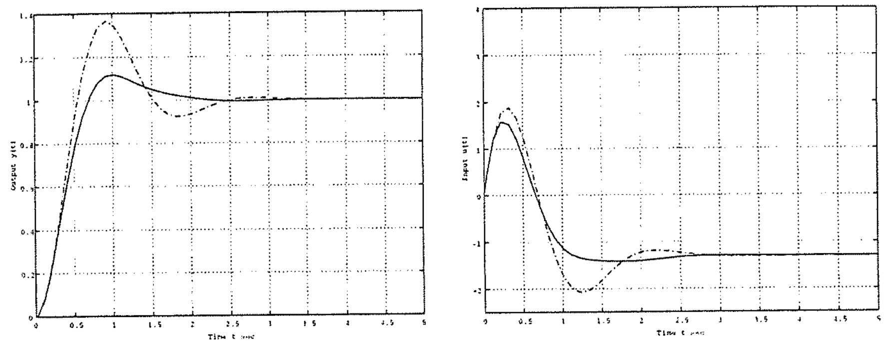
$$\begin{aligned} K_0 &= \left[\ 36.16 \quad -11.74\ \right], \\ K_{01}(\beta) &= \left[\ 23.49 \quad -4.01\ \right] + \beta \left[\ 1.71 \quad -0.53\ \right] + \beta^2 \left[\ -0.07 \quad -0.19\ \right]. \end{aligned} \tag{6.3}$$

The simulation results are shown in Figure 3, where the reference is 1 ($r = 1$). In this figure, the solid line and the dashdot line denote the simulation result of the case (6.2) and (6.3) respectively. The error $r-y$ is asymptotically zero at both cases. Note that the asymptotically-zero error is assured for arbitrary L^2 type references, since both feedback schemes provide finite L^2 gain from reference r to error $r-y$.



(a) Output y (b) Input u

Fig. 3. Simulation result (Memory feedback case).



(a) Output y (b) Input u

Fig. 4. Simulation result (Memoryless feedback case).

We see also that, by using Theorem 4.2, we can make the maximum of the control input small.

Our approach with corresponding specializations, which is equivalent to the approach of [14] for unconstrained gain case, provides memoryless controllers. On this example, the feedback gain for unconstrained gain case is calculated as

$$K_0 = \begin{bmatrix} 14.49 & -5.27 \end{bmatrix}, \tag{6.4}$$

and the feedback gain for the constrained gain case is calculated as

$$K_0 = \begin{bmatrix} 12.94 & -3.20 \end{bmatrix}. \tag{6.5}$$

The simulation results are shown in Figure 4, where the solid line and the dashdot line denote the result of (6.4) and (6.5) respectively. Compared with the results shown in Figure 3, we see worse tracking properties for both cases. We also see that the maximum of the control input given by (6.5) is larger than that of the control input given by (6.4), that is, in the memoryless feedback case, our algorithm cannot succeed in making the maximum control input small.

7. CONCLUSION

In this paper, we considered L^2 gain analysis and control synthesis problems for linear systems with time-delay via an LMI approach. We derived conditions for analysis and synthesis in the form of infinite-dimensional LMIs and showed a technique to reduce the infinite-dimensional LMIs to a finite number of LMIs which provide feasible formulas. We demonstrated the efficacy of our approach by a numerical example.

The LMI approach presented in this paper requires the exact value of the time-delay h. This may make us anxious that the constructed controller is sensitive to any variation of time-delay. However, the closed loop system which is formed by the controller of Theorem 4.1 is robustly stable against sufficiently small variation of time-delay, which is discussed in [1].

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Another Look at Finite Horizon H^∞ Control Problems for Systems with Input Delays

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Abstract. We discuss a finite horizon H^∞ control problem for systems with input delays. Clarifying a relationship between two H^∞ control problems in input delay case and in measurement delay case, we derive a solution in input delay case based on the known result for the H^∞ control problem in measurement delay case, and show that the solution has the same predictor-observer structure as the solution in measurement delay case has. Using this structural information on the solution, we also present a direct proof of the solution to the finite horizon H^∞ control problem for systems with input delays, which is based only on completion of squares.

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Keywords: Time delay systems; Input delay; Finite horizon; H^∞ control; Predictor

1. Introduction

In control system designs, input delay appears rather often and is considered as a small but cumbersome obstacle. For systems with input delays, the H^∞ control problem was actively investigated in parallel with development of H^∞ control theory. The problem was solved by Kojima and Ishijima (1994) in state-space form, the parameterization of all the solutions was given by Tadmor (1995), and a particular (predictor-observer) structure of the solutions has been recently been pointed out by Mirkin (2000). We can find detailed reviews of this area in (Tadmor, 2000, Mirkin, 2000) and the references insides. In this paper, we revisit the H^∞ control problem for systems with input delays in the framework of finite horizon. The first objective is to discuss further the predictor-observer structure of the solution, which is pointed out by Milkin (2000), from a novel viewpoint. Being suggested by the first discussion, secondly, we try to develop an elementary approach to the problem, which is completely different from the abstract approach based on evolution equations taken in (Kojima and Ishijima, 1994, Tadmor, 1995) and requires only completion of squares.

More specifically, the content of this paper is stated and organized as follows. In Section 2, we formulate the H^∞ control problem for systems with input delays together with two related H^∞ control problems. In Section 3, we first clarify a relationship between our problem and an H^∞ control problem for systems with measurement delays. Next, we derive a solution based on the known result for the H^∞ control problem in measurement delay case, and show that the solution has the same predictor-observer structure as the solution in measurement delay case has. In Section 4, using this structural information on the solution, we present a direct proof of the solution to the finite horizon H^∞ control problem for systems with input delays, which is based only on completion of squares.

Notations: $L^2(a,b;R^k)$ is the space of square integrable functions of k-dimension defined on the time interval $[a,b]$. When $a=t_0$ and $b=t_1$, the L^2-norm of f in $L^2(a,b;R^k)$ is denoted as $\|f\|_2^2$. $\|x\|$ denotes the Euclidean norm of x in R^k. For symmetric matrices X and Y, $X \geq Y$ $(X > Y)$ implies that $X-Y$ is positive semidefinite (positive definite). I is the identity matrix of appropriate dimension. $(\,)'$ denotes the transpose of vector or matrix. $\rho(X)$ denotes the spectral radius of matrix X.

2. System Description and Problem Statement

Consider the linear time-varying system with the time-delay $h>0$ in the control input, which is defined on the interval $[t_0,t_1]$ and described by

$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t-h) + D(t)v(t),$$
$$y(t) = C(t)x(t) + w(t), \qquad (1)$$
$$g(t) = \begin{bmatrix} z(t) \\ u(t) \end{bmatrix}, \qquad z(t) = F(t)x(t)$$

where $x(t)$ the n-dimensional internal-variable; $u(t)$ is the r-dimensional control input; $y(t)$ is the m-dimensional measurement output; $g(t)$ is the $(q+r)$-dimensional controlled output; $d(t) = (v(t)', w(t)')'$ is the $(p+m)$-dimensional disturbance; the initial condition $(x(t_0), u_{t_0})$ (where $u_t = \{u(t+\beta), -h \le \beta \le 0\}$) in $R^n \times L^2(-h,0;R^r)$, is given by a constant matrix $N > 0$ and an n-dimensional parameter ξ as

$$x(t_0) = N\xi, \qquad u_{t_0} = 0. \qquad (2)$$

$A(t), B(t), C(t), D(t)$ and $F(t)$ are matrices of appropriate dimensions whose elements are continuous functions of time. For the system (1) with the initial condition (2), the admissible control $u(t) = \Phi_{ID}(t,y)$ is given by a causal function of the measurement data specifically to be the form

$$u(t) = \Phi_{ID}(t, \{y(s), t_0 \le s \le t\}), \qquad t_0 \le t \le t_1 - h. \qquad (3)$$

Problem ID *(H^∞ Control Problem with Input Delay):* Given the system described by (1) and (2) and a constant number $\gamma > 0$, the problem is to find an admissible control (3) which satisfies the inequality

$$\|g\|_2^2 < \gamma^2(\|d\|_2^2 + \xi' N\xi) \qquad (u_{t_1} = 0) \qquad \text{(ID)}$$

for all $d = (v', w')'$ in $L^2(t_0, t_1; R^{p+m})$ and all ξ in R^n such that $(d, \xi) \neq 0$.

Remark : The terminal penalty $\xi' N\xi$ implies that the controls are determined so as to attenuate the effect of the uncertain initial internal-variable $x(t_0)$ which is known to be in $\mathrm{Im}(N)$; if $N = 0$, the initial internal-variable is completely (to be zero), and if N is nonsingular (positive definite), the initial internal-variable is completely unknown. For simplicity, in this paper, we discuss only the nonsingular case. The extension to the singular case can be done by a perturbation technique (Uchida and Fujita, 1992).

The H^∞ control problem for systems with input delays was solved by Kojima and Ishijima (1994), the parameterization of all the solutions was given by Tadmor (1995), and a particular (predictor-observer) structure of the solutions has been recently been pointed out by Mirkin (2000). Problem ID is an extension of the problem discussed in these literatures in the points that the system with a finite horizon is time varying and the criterion includes a terminal penalty, and could be solved by extending the arguments of (Kojima and Ishijima, 1994, Tadmor, 1995). In this paper, instead of pursuing this line, we will develop another approach and provide a new characterization of the solutions, which is inspired by the observation in (Mirkin, 2000).

We first consider an auxiliary problem to Problem ID. The system is defined on $[t_0, t_1]$ and described by

$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t-h) + D(t)v(t),$$
$$y(t) = C(t)x(t) + w(t), \qquad (4)$$
$$g(t) = \begin{bmatrix} z(t) \\ u(t-h) \end{bmatrix}, \; z(t) = F(t)x(t)$$

with the initial condition

$$x(t_0) = N\xi, \qquad (5)$$

and the admissible control $u(t) = \Phi_{AID}(t,y)$ is given by a causal function of the measurement data specifically to be the form

$$u(t) = \begin{cases} \Phi_{AID}(t, \{y(s), t_0 \le s \le t\}), & t_0 \le t < t_1 - h \\ \Phi_{AID}(t), & t_0 - h \le t \le t_0. \end{cases} \qquad (6)$$

Problem AID *(Auxiliary H^∞ Control Problem with Input Delay):* Given the system described by (4) and (5) and a constant number $\gamma > 0$, the problem is to find an admissible control (6) which satisfies the inequality

$$\|g\|_2^2 < \gamma^2(\|d\|_2^2 + \xi' N\xi) \qquad \text{(AID)}$$

for all $d = (v', w')'$ in $L^2(t_0, t_1; R^{p+m})$ and all ξ in R^n such that $(d, \xi) \neq 0$.

The difference between Problem ID and Problem AID is found only in the role of $u_{t_0} = \{u(t_0 + \beta), -h \le \beta \le 0\}$, that is, u_{t_0} is fixed (to be zero function) as a part of the initial condition in Problem ID, while u_{t_0} is a part of the control input to be determined in Problem AID. Although Problem AID itself is an H^∞ control problem applicable to some control designs, we will use Problem AID to bridge a gap between Problem ID and another H^∞ control problem introduced in the following.

We consider next an H^∞ problem which does not have input delays but has measurement delays. The system is defined on $[t_0, t_1]$ and described by

$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t) + D(t)v(t),$$
$$y(t) = C(t)x(t) + w(t), \tag{7}$$
$$g(t) = \begin{bmatrix} z(t) \\ u(t) \end{bmatrix}, \qquad z(t) = F(t)x(t)$$

with the initial condition

$$x(t_0) = N\xi, \tag{8}$$

and the admissible control $u(t) = \Phi_{MD}(t, y)$ is given by a causal function of the delayed measurement data specifically to be the form

$$u(t) = \begin{cases} \Phi_{MD}(t, \{y(s), t_0 \le s \le t-h\}), & t_0 + h \le t \le t_1 \\ \Phi_{MD}(t), & t_0 \le t \le t_0 + h. \end{cases} \tag{9}$$

Problem MD *(H^∞ Control Problem with Measurement Delay):* Given the system described by (7) and (8) and a constant number $\gamma > 0$, the problem is to find an admissible control (9) which satisfies the inequality

$$\|g\|_2^2 < \gamma^2(\|d\|_2^2 + \xi' N\xi) \tag{MD}$$

for all $d = (v', w')'$ in $L^2(t_0, t_1; R^{p+m})$ and all ξ in R^n such that $(d, \xi) \ne 0$.

The H^∞ control problem for systems with measurement delays was also solved completely in (Basar and Bernhard, 1991, Nagpal and Ravy, 1997), and, as is expected from existence of information delays in constructing control inputs, the solution has a natural predictor-observer structure.

Our approach to Problem ID, which we will take in the following sections, is summarized as follows. We establish first some relationships between Problem ID and Problem MD via Problem AID, and try to solve Problem ID based on the relationships and the solution of Problem MD, so that the solution of Problem ID has the same predictor-observer structure as the solution of Problem MD has.

3. Structure and Characterization of Solution

To find relations among three Problems ID, AID and MD, we observe the detail of the term of the controlled output in each criterion. In (ID), if we take an admissible control given by (3), we have

$$\|g\|_2^2 = \int_{t_0}^{t_1} \|z(t)\|^2 dt + \int_{t_0+h}^{t_1} \|\Phi_{ID}(t-h, \{y(s), t_0 \le s \le t-h\})\|^2 dt. \tag{10}$$

In (AID), if we take an admissible control given by (6), we have

$$\|g\|_2^2 = \int_{t_0}^{t_1} \|z(t)\|^2 dt + \int_{t_0+h}^{t_1} \|\Phi_{AID}(t-h, \{y(s), t_0 \le s \le t-h\})\|^2 dt + \int_{t_0}^{t_0+h} \|\Phi_{AID}(t-h)\|^2 dt. \tag{11}$$

In (MD), if we take an admissible control given by (9), we have

$$\|g\|_2^2 = \int_{t_0}^{t_1} \|z(t)\|^2 dt + \int_{t_0+h}^{t_1} \|\Phi_{MD}(t, \{y(s), t_0 \le s \le t-h\})\|^2 dt + \int_{t_0}^{t_0+h} \|\Phi_{AID}(t)\|^2 dt. \tag{12}$$

The following result is an immediate conclusion from the descriptions of three Problems (ID), (AID) and (MD) and the expressions of (10), (11) and (12).

Proposition 1: a) If $u(t) = \Phi_{ID}(t, y)$ defined by (3) is a solution to Problem ID, the control $u(t) = \Phi_{ID}(t, y)$ together with $u_{t_0} = 0$ is a solution to Problem AID. Conversely, if a control $u(t) = \Phi_{AID}(t, y)$ defined by (6) is a solution to Problem AID and satisfies $u_{t_0} = 0$, the control $u(t) = \Phi_{AID}(t, y)$ is a solution to Problem ID.

b) If a control $u(t) = \Phi_{AID}(t, y)$ given by (6) is a solution to Problem AID, the delayed control $u(t) = \Phi_{AID}(t-h, y)$ is a solution to Problem MD. Conversely, if $u(t) = \Phi_{MD}(t, y)$ defined by (9) is a solution to Problem MD, the advanced control $u(t) = \Phi_{MD}(t+h, y)$ is a solution to Problem AID.

c) If $u(t) = \Phi_{MD}(t, y)$ defined by (9) is a solution to Problem MD and satisfies $u_{t_0+h} = 0$, the advanced control $u(t) = \Phi_{MD}(t+h, y)$ is a solution to Problem ID. Conversely, if $u(t) = \Phi_{ID}(t, y)$ given by (3) is a solution to Problem ID, the delayed control $u(t) = \Phi_{ID}(t-h, y)$ together with $u_{t_0+h} = 0$ is a solution to Problem MD.

Using the fact c) in Proposition 1 and a solution to Problem MD, we will derive a solution of Problem ID. Now we present the solution to Problem MD, which is a slight modification of the result given by Basar and Bernhard (1991). We need to introduce the following four conditions.

(C1) There exists a solution $M(t)$, $t_0 \le t \le t_1$ to the Riccati differential equation

$$-\frac{d}{dt}M(t) = M(t)A(t) + A(t)'M(t) + F(t)'F(t) - M(t)(B(t)B(t)' - \gamma^{-2}D(t)D(t)')M(t),$$
$$M(t_1) = 0. \quad (13)$$

(C2) There exists a solution $P(t)$, $t_0 \le t \le t_1 - h$ to the Riccati differential equation

$$\frac{d}{dt}P(t) = A(t)P(t) + P(t)A(t)' + D(t)D(t)' - P(t)(C(t)'C(t) - \gamma^{-2}F(t)'F(t))P(t),$$
$$P(t_0) = N. \quad (14)$$

(C3) There exists a solution $Q(t+\beta, t-h)$, $t_0 + h \le t \le t_1$, $-h \le \beta \le 0$ to the Riccati differential equation

$$\frac{\partial}{\partial \beta}Q(t+\beta,t-h) = A(t+\beta)Q(t+\beta,t-h) + Q(t+\beta,t-h)A(t+\beta)' + D(t+\beta)D(t+\beta)' + \gamma^{-2}Q(t+\beta,t-h)F(t+\beta)'F(t+\beta)Q(t+\beta,t-h),$$
$$Q(t-h,t-h) = P(t-h). \quad (15)$$

(C4) $\rho(M(t+\beta)Q(t+\beta,t-h)) < \gamma^2$, $t_0 + h \le t \le t_1$, $-h \le \beta \le 0$.

Proposition 2: Assume that the conditions (C1)-(C4) are satisfied. Then, a solution to Problem MD is given by

$$u(t) = \begin{cases} -B(t)'S(t,t-h)\bar{x}(t,t-h), & t_0 + h \le t \le t_1 \\ 0, & t_0 \le t \le t_0 + h \end{cases} \quad (16)$$

where $S(t,t-h)$ is defined by

$$S(t+\beta,t-h) = M(t+\beta)(I - \gamma^{-2}Q(t+\beta,t-h)M(t+\beta))^{-1}, \quad -h \le \beta \le 0 \quad (17)$$

and $\bar{x}(t,t-h)$ is predicted with the "predictor"

$$\frac{\partial}{\partial \beta}\bar{x}(t+\beta,t-h) = (A(t+\beta) + \gamma^{-2}Q(t+\beta,t-h)F(t+\beta)'F(t+\beta) - B(t+\beta)B(t+\beta)'S(t+\beta,t-h))\bar{x}(t+\beta,t-h), \quad -h \le \beta \le 0 \quad (18)$$

from the estimate $\bar{x}(t-h,t-h) = \hat{x}(t-h)$ which is estimated with the "observer"

$$\frac{d}{dt}\hat{x}(t) = (A(t) + \gamma^{-2}P(t)F(t)'F(t) - B(t)B(t)'S(t))\hat{x}(t) + P(t)C(t)'(y(t) - C(t)\hat{x}(t)),$$
$$\hat{x}(t_0) = 0. \quad (19)$$

The proof can be found in the next section. From b) in Proposition 1 and Proposition 2, a solution to Problem AID is given by

$$u(t) = \begin{cases} -B(t+h)'S(t+h,t)\bar{x}(t+h,t), & t_0 \le t \le t_1 - h \\ 0, & t_0 - h \le t \le t_0 \end{cases} \quad (20)$$

which is the advanced form of the control (16). Moreover, since the solution (16) satisfies $u_{t_0+h} = 0$, it follows from c) in Proposition 1 and Proposition 2 that the advanced version of (16) given as

$$u(t) = -B(t+h)'S(t+h,t)\bar{x}(t+h,t), \quad t_0 \le t \le t_1 - h \quad (21)$$

is a solution to Problem ID. Here note that the solutions (20) and (21) have the same predictor-observer structure. That is, in constructing the controls (20) and (21), the estimate $\hat{x}(t)$ is estimated with the observer (19) based on the data $\{y(s), t_0 \le s \le t\}$, and $\bar{x}(t+h,t)$ is predicted with the predictor (18) from the estimate $\bar{x}(t,t) = \hat{x}(t)$. It is also noted that the conditions (C1)-(C4) form the same sufficient condition for existence of solutions to Problems AID and ID. We can summarize these facts, together with necessity of the conditions (C1)-(C4), in the following form.

Theorem: a) There exists a solution to Problem ID if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (21) is a solution to Problem ID.

b) There exists a solution to Problem AID if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (20) is a solution to Problem AID.

c) There exists a solution to Problem MD if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (16) is a solution to Problem MD.

In the next section, we provide a direct proof of this theorem by using an elementary argument based only on completion of squares (Uchida and Fujita, 1990).

4. Proof of Theorem (Completion of Squares)

We prove only b) in Theorem, because a) and c) follows from b) and Proposition 1. Before starting the proof, we present a preliminary result.

Lemma 1: Let $Q(t+\beta,t-h)$, $t_0+h\le t\le t_1$, $-h\le\beta\le 0$ be a solution to the Riccati differential equation (15) in (C3) with the initial condition given in (C2). The solution satisfies the Riccati differential equations

$$\begin{aligned}
&\frac{d}{dt}Q(t,t-h)=A(t)Q(t,t-h)+Q(t,t-h)A(t)'\\
&\quad +D(t)D(t)'+\gamma^{-2}Q(t,t-h)F(t)'F(t)Q(t,t-h)\\
&\quad -\Psi(t,t-h)P(t-h)C(t-h)'C(t-h)P(t-h)\Psi(t,t-h)',\\
&\frac{\partial}{\partial\beta}Q(t_0+h+\beta,t_0)=A(t_0+h+\beta)Q(t_0+h+\beta,t_0)\\
&\quad +Q(t_0+h+\beta,t_0)A(t_0+h+\beta)'+D(t_0+h+\beta)D(t_0+h+\beta)'\\
&\quad +\gamma^{-2}Q(t_0+h+\beta,t_0)F(t_0+h+\beta)'F(t_0+h+\beta)Q(t_0+h+\beta,t_0),\\
&\quad Q(t_0,t_0)=N.
\end{aligned} \tag{22}$$

where $\Psi(\tau,t-h)$ is the transition matrix associated with $A(\tau)+\gamma^{-2}Q(\tau,t-h)F(\tau)'F(\tau)$.

Lemma 2: The condition formed by (C1), (C2), (C3) and (C4) is equivalent to the condition formed by (C14), (C2) and (C3), where (C14) is defined as follows.

(C14) There exists a solution $S(t+\beta,t-h)$, $t_0+h\le t\le t_1$, $-h\le\beta\le 0$ to the Riccati differential equations

$$\begin{aligned}
&-\frac{d}{dt}S(t,t-h)=S(t,t-h)\Gamma(t,t-h)+\Gamma(t,t-h)'S(t,t-h)\\
&\quad +F(t)'F(t)-S(t,t-h)(B(t)B(t)'-\gamma^{-2}\Psi(t,t-h)P(t-h)\times\\
&\qquad\qquad \times C(t-h)'C(t-h)P(t-h)\Psi(t,t-h)')S(t,t-h),\\
&\quad S(t_1,t_1-h)=0,\\
&\frac{\partial}{\partial\beta}S(t+\beta,t-h)=S(t+\beta,t-h)\Gamma(t+\beta,t-h)\\
&\qquad +\Gamma(t+\beta,t-h)'S(t+\beta,t-h)+F(t+\beta)'F(t+\beta)\\
&\qquad -S(t+\beta,t-h)B(t+\beta)B(t+\beta)'S(t+\beta,t-h),\\
&\Gamma(t+\beta,t-h)=A(t+\beta)+\gamma^{-2}Q(t+\beta,t-h)F(t+\beta)'F(t+\beta).
\end{aligned} \tag{23}$$

Proof of Sufficiency of b) in Theorem: Assume that the conditions (C1), (C2), (C3) and (C4) are satisfied so that (C14) is also satisfied, and consider the functionals

$$\begin{aligned}
&V_1(t+\beta,t-h)=\bar{x}(t+\beta,t-h)'S(t+\beta,t-h)\bar{x}(t+\beta,t-h)\\
&V_2(t+\beta,t-h)=(x(t+\beta)-\bar{x}(t+\beta,t-h))'\gamma^2Q(t+\beta,t-h)^{-1}\times\\
&\qquad\qquad \times(x(t+\beta)-\bar{x}(t+\beta,t-h))
\end{aligned}$$

where $Q(t-h,t-h)=P(t-h)$, and further assume that, for a fixed admissible control $u(t)$, $x(t+\beta)$ is generated by (3) and (4) and $\bar{x}(t+\beta,t-h)$ given by

$$\bar{x}(t+\beta,t-h)=\Psi(t+\beta,t-h)\hat{x}(t-h)+\int_{t-h}^{t+\beta}\Psi(t+\beta,s)B(s)u(s-h)ds, \tag{24}$$

$$\begin{aligned}
&\frac{d}{dt}\hat{x}(t-h)=(A(t-h)+\gamma^{-2}P(t-h)F(t-h)'F(t-h))\hat{x}(t-h)\\
&\quad +B(t-h)u(t-h)+P(t-h)C(t-h)'(y(t-h)-C(t-h)\hat{x}(t-h)),\\
&\quad \hat{x}(t_0)=0.
\end{aligned} \tag{25}$$

Substituting the definitions (24) and (25) together with the formulas (14), (22) and (23) into the following identities

$$\begin{aligned}
&\int_{t_0+h}^{t_1}\{\frac{d}{dt}V_1(t,t-h)+\frac{\partial}{\partial\beta}V_2(t+\beta,t-h)\big|_{\beta=0}\\
&\qquad -\frac{\partial}{\partial\beta}V_2(t+\beta,t-h)\big|_{\beta=-h}+\frac{d}{dt}V_2(t-h,t-h)\}dt\\
&=V_1(t_1,t_1-h)+V_2(t_1,t_1-h)-V_1(t_0+h,t_0)-V_2(t_0+h,t_0)
\end{aligned}$$

in the interval $[t_0+h,t_1]$ and

$$\begin{aligned}
&\int_{-h}^{0}\{\frac{\partial}{\partial\beta}V_1(t_0+h+\beta,t_0)+\frac{\partial}{\partial\beta}V_2(t_0+h+\beta,t_0)\}d\beta\\
&=V_1(t_0+h,t_0)+V_2(t_0+h,t_0)-V_1(t_0,t_0)-V_2(t_0,t_0),
\end{aligned}$$

in the interval $[t_0,t_0+h]$, and rearranging terms, we obtain

$$\begin{aligned}
&\|g\|_2^2-\gamma^2(\|d\|_2^2+\xi'N\xi)=\int_{t_0}^{t_1}\{\|u(t-h)-u_{\min}(t-h)\|^2\\
&\qquad -\gamma^2\|v(t)-v_{\max}(t)\|^2-\gamma^2\|w(t)-w_{\max}(t)\|^2\}dt\\
&\quad -\gamma^2(x(t_1)-\bar{x}(t_1,t_1-h))'Q(t_1,t_1-h)^{-1}(x(t_1)-\bar{x}(t_1,t_1-h))
\end{aligned} \tag{26}$$

where $u_{\min}(t)$, $v_{\max}(t)$ and $w_{\max}(t)$ are defined by

$$u_{\min}(t)=\begin{cases}-B(t+h)'S(t+h,t)\bar{x}(t+h,t), & t_0\le t\le t_1-h\\ -B(t+h)'S(t+h,t_0)\bar{x}(t+h,t_0), & t_0-h\le t\le t_0\end{cases}$$

$$v_{\max}(t)=D(t)'Q(t,t-h)^{-1}(x(t)-\bar{x}(t,t-h))$$

$$w_{\max}(t)=\begin{cases}0, & t_1-h\le t\le t_1\\ -C(t)(x(t)-\hat{x}(t))+\gamma^{-2}C(t)P(t)\times\\ \quad\times\Psi(t+h,t)'S(t+h,t)\bar{x}(t+h,t), & t_0\le t\le t_1-h.\end{cases}$$

From (26), we see that $u(t)=u_{\min}(t)$ assures $\|g\|_2^2-\gamma^2(\|d\|_2^2+\xi'N\xi)\le 0$, and also see that the equality holds only if $(v(t),w(t))=(v_{\max}(t),w_{\max}(t))$ and $x(t_1)=\bar{x}(t_1,t_1-h)$ so that $(d,\xi)=0$. Thus $u(t)=u_{\min}(t)$ is a solution to Problem AID. Furthermore, when $u(t)=u_{\min}(t)$, it follows from (24) and (25) that $\bar{x}(t+h,t)$ is generated also by (18) and (19) and $\bar{x}(t+h,t_0)=0$, $t_0-h\le t\le t_0$. (Note that the above proof together with Proposition 1 proves Proposition 2.)

Proof of Necessity of b) in Theorem: We show that, if there exists a solution to Problem AID, (C2), (C3) and (C14) must be satisfied. Then, necessity of the conditions (C1), (C2), (C3) and (C4) follows from Lemma 2.

Suppose that the condition (C2) does not hold; then, we can find the smallest time $t^* \in [t_0, t_1)$ such that (14) has a solution $P(t),\ t_0 \le t < t^*$ and there exists a nonzero vector α such that $\lim_{T \to t^*} P(t)^{-1}\alpha = 0$. Now using the functional $V_2(t-h, t-h),\ t_0 + h \le t \le T + h$ and a same argument as in the proof of sufficiency, choosing nonzero (d, ξ) such that

$$(v(t), w(t)) = \begin{cases} (0, 0), & T \le t \le t_1 - h \\ (v_{\max}(t), -C(t)(x(t) - \hat{x}(t))), & t_0 \le t < T \end{cases} \qquad (T < t^*)$$

and ξ guarantees $x(T) - \hat{x}(T) = \alpha$, and taking T as $T \to t^*$, we have

$$\|g\|_2^2 - \gamma^2(\|d\|_2^2 + \xi' N \xi) \ge \int_{t_0}^{t^*} \{\|F(t)\hat{x}(t)\|^2 + \|u(t)\|^2\} dt \ge 0$$

for all admissible controls $u(t)$. This inequality contradicts the existence of a solution. Thus, (C2) must hold.

As to the condition (C3), by using the functionals $V_1(t_0 + h + \beta, t_0)$ and $V_1(t + \beta, t - h)$ and modifying slightly the above argument for (C2), we can show the existence of solutions $Q(t_0 + h + \beta, t_0)$ and $Q(t + \beta, t - h)$ to the Riccati equations (15). Thus, (C3) must hold.

Suppose that the condition (C14) does not hold; then, we can find the largest time $t_* \in (t_0 + h, t_1]$ such that the first equation of (23) has a solution $S(t, t-h),\ t_* < t \le t_1$ and there exists a nonzero vector β such that $\lim_{T \to t_*} S(T, T-h)\beta = \infty$. Now using the functionals $V_1(t + \beta, t - h)$ and $V_2(t + \beta, t - h)$, $T \le t \le t_1$, repeating the same argument as in the proof of sufficiency, and choosing nonzero (d, ξ) such that

$$(v(t), w(t)) = \begin{cases} (v_{\max}(t), w_{\max}(t)), & T < t \le t_1 \\ (0, 0), & t_0 + h \le t \le T \end{cases} \qquad (t_* < T)$$

and ξ assure $x(t_1) - \bar{x}(t_1, t_1 - h) = 0$ and $\bar{x}(T, T-h) = \beta$, where $\beta \ne 0$ assures $(d, \xi) \ne 0$, we have

$$\|g\|_2^2 - \gamma^2(\|d\|_2^2 + \xi' N \xi) \ge \int_T^{t_1} \|u(t) - u_{\min}(t)\| dt$$
$$+ (x(T) - \beta)' Q(T, T-h)^{-1}(x(T) - \beta) + \beta' S(T, T-\beta)\beta - \xi' N \xi.$$

Taking T as $T \to t^*$, the right hand side of the above inequality becomes arbitrary large. This contradicts the existence of a solution. Thus, the first equation of (23) has a solution on the whole interval. Using this solution $S(t, t-h)$ as a terminal condition for the second equation of (23) and repeating the same argument, we can show that the second equation of (23) has a solution $S(t + \beta, t - h),\ -h \le \beta \le 0$. Thus, the condition (C14) must hold.

5. Conclusion

We discussed a finite horizon H^∞ control problem for systems with input delays. We derived a solution based on the known result for the H^∞ control problem in measurement delay case, and showed that the solution has the same predictor-observer structure as the solution in measurement delay case has. Using this structural information on the solution, we also presented a direct proof of the solution to the finite horizon H^∞ control problem for systems with input delays, which is based only on completion of squares.

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AN APPROACH TO CONSTRAINED STATE FEEDBACK H^∞ CONTROL SYNTHESIS

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Abstract: In this paper, we propose an approach to H^∞ controller synthesis problem for linear systems with time-delay, when the control input is constrained. This approach is based on a state reachable set analysis. We deal with a case of memory state feedback controllers.

Keywords: Time-delay, H^∞ control, Input constraints, State Reachable Sets, Linear Matrix Inequality(LMI)

1. INTRODUCTION

In this paper, we consider H^∞ control in memory state feedback for the state delay systems, when the control input is constrained. First, we propose an H^∞ controller synthesis which make the closed loop system asymptotically stable and its L^2 gain less than a specified value. Next, extending the analysis method of state reachable sets for systems no delay proposed in (Watanabe and Fujita, 1998),(Boyd and Valakrisham, 1994), we provide a method to evaluate an admissible range of state. Finally, based on the reachable set analysis, we propose an H^∞ controller synthesis method when control constraint is imposed.

For recent and related developments in this area, see (Niculescu and L.Dugard, 1996) and (Tarbourieh, 2000), where the memory feedback case presented in this paper is not discussed.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

System Description

Consider a linear system with delay in state. The system is defined over the interval $[0,\infty)$ and described by

$$\begin{aligned}\dot{x}(t) =& A_0x(t) + A_1x(t-h)\\ &+\int_{-h}^{0} A_{01}(\beta)x(t+\beta)d\beta + Bu(t) + Dw(t) \qquad (1)\\ z(t) =& Cx(t)\end{aligned}$$

Here, $w(t)$ is the disturbance vector; $u(t)$ is the control input vector; $z(t)$ is the controlled output vector; and the state at time t of the system is described by $(x(t), x_t)$, here, $x_t = \{x(t+\beta)| -h \leq \beta \leq 0\} \in L^2([-h,0];R^n)$. The initial condition is $(x(0), x_0) \in R^n \times L^2([-h,0];R^n)$. The number h denotes the length of time delay and $h > 0$. The parameters A_0, A_1, B, D, C are constant matrices and the parameter $A_{01}(\beta)$, is a matrix function whose elements are bounded continuous functions.

In this paper, we consider this constraint condition about disturbance w

$$\begin{aligned} w(t) \in \mathcal{W}, \quad \forall t \in [0,\infty) \qquad (2)\\ \mathcal{W} = \{w | w'W_Dw \leq 1\}.\end{aligned}$$

Here, W_D is the given and positive definite matrix.

We consider the feedback controller for the time-delay system as described by

$$u(t) = K_0 x(t) + \int_{-h}^{0} K_{01}(\beta) x(t+\beta) d\beta. \quad (3)$$

Here, K_0 is a constant matrix and $K_{01}(\beta)$ is a matrix function whose elements are in $L^2[-h,0]$.A closed loop system applied the controller (3) to the system (1) is described as

$$\dot{x}(t) = \tilde{A}_0 x(t) + \tilde{A}_1 x(t-h) + \int_{-h}^{0} \tilde{A}_{01}(\beta) x(t+\beta) d\beta$$

where, $\tilde{A}_0 = A_0 + BK_0$, $\tilde{A}_1 = A_1$, $\tilde{A}_{01}(\beta) = A_{01}(\beta) + BK_{01}$.

Now, we prepare a notation for defining a quadratic form of the state. Denote by $\{P,R,S\}$ a triplet of three matrices P, $R(\beta)$ and $S(\alpha,\beta)$ with the same dimensions such that P is a constant matrix, $R(\beta)$ is a matrix function whose elements are in $L^2[-h,0]$ and $S(\alpha,\beta)$ is a matrix function whose elements are in $L^2([-h,0]\times[-h,0])$. A triplet $\{P,R,S\}$ is called symmetric if $P' = P$ and $S'(\alpha,\beta) = S(\alpha,\beta)$. For a given symmetric triplet $\{P,R,S\}$, a quadratic form associated with this triplet is defined as follows:

$$\begin{aligned}
&(\xi,\zeta)\{P,R,S\}(\xi,\zeta) \\
&:= \xi' P \xi + 2\xi' \int_{-h}^{0} R(\beta)\zeta(\beta) d\beta \quad (4) \\
&+ \int_{-h}^{0}\int_{-h}^{0} \zeta'(\alpha) S(\alpha,\beta) \zeta(\beta) d\alpha d\beta,
\end{aligned}$$

here, (ξ,ζ) satisfies $(\xi,\zeta) \in R^n \times L^2([-h,0];R^n)$. A symmetric triplet $\{P,R,S\}$ is called positive semi-definite if $(\xi,\zeta)'\{P,R,S\}(\xi,\zeta) \geq 0$ for all $(\xi,\ \zeta)$ and, in particular called positive definite if there exists a positive number ϵ such that $(\xi,\zeta)'\{P,R,S\}(\xi,\zeta(\beta)) \geq (\xi,\zeta)'\{\epsilon I\ 0\ 0\}(\xi,\zeta)$ for all ξ, ζ, where I denotes identity matrix. We denote $\{P,R,S\} \geq 0$ (>0) when $\{P,R,S\}$ is positive semi-definite (definite). Negative semi-definiteness and negative definiteness are similarly defined.

We describe $L^2([-h,0];R^n)$ as L^2 for the simplicity.

State Reachable Sets

Here, we assume the case that the disturbance of system (1) is constrained by (2). Now, we make the following definitions.

Definition 1. For $\lambda = (\xi,\zeta) \in R^n \times L^2$, if there exists a disturbance w that satisfies (2) and there exists a time $T < \infty$ that satisfies $\xi = x(T), \zeta = x_T$, then λ is called state reachable from $(x(0),x_0)$.

Definition 2. A reachable set $\mathcal{R}(x(0),x_0)$ from $(x(0),x_0)$ is defines as

$$\begin{aligned}
&\mathcal{R}(x(0),x_0) \\
&= \{(\xi,\zeta) \in R^n \times L^2 \\
&\quad : (\xi,\zeta) \text{ is state reachable from } (x(0),x_0)\}.
\end{aligned}$$

Now, for simplicity we assume about the matrix $\mathcal{W}$ that constrains the disturbance w as

$$W_D = I$$

And we define a set $\mathcal{E}$ as follows.

Definition 3. For any positive definite triplet $\{P,R,S\}$, $\mathcal{E}$ is defined as

$$\begin{aligned}
&\mathcal{E}(P,R,S) = \\
&\{\lambda = (\xi,\zeta) \in R^n \times L^2 | \lambda^T\{P,R,S\}\lambda \leq 1\}.
\end{aligned}$$

Lemma 1. Assume that there exists a positive definite triplet $\{P,R,S\}$ that satisfies LMI condition LMI-1 for any $\lambda = (\xi,\zeta) \in R^n \times L^2$, $\lambda'\{P,R,S\}\lambda \geq 1$ and for any $w \in \mathcal{W}$, where

(LMI − 1)

$$\int_{-h}^{0}\int_{-h}^{0} \begin{bmatrix} \frac{1}{h}\xi \\ \frac{1}{h}\zeta(-h) \\ \zeta(\alpha) \\ \frac{1}{h}w \end{bmatrix}^T \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ \Delta_{12}' & \Delta_{22} & \Delta_{23} & \Delta_{24} \\ \Delta_{13}' & \Delta_{23}' & \Delta_{33} & \Delta_{34} \\ \Delta_{14}' & \Delta_{24}' & \Delta_{34}' & \Delta_{44} \end{bmatrix} (\alpha,\beta) \begin{bmatrix} \frac{1}{h}\xi \\ \frac{1}{h}\zeta(-h) \\ \zeta(\beta) \\ \frac{1}{h}w \end{bmatrix} d\beta d\alpha \leq 0.$$

Where,

$$\begin{aligned}
\Delta_{11} &= A_0{}'P + PA_0 + R(0)' + R(0) \\
\Delta_{12} &= PA_1 - R(-h) \\
\Delta_{13}(\beta) &= PA_{01}(\beta) + A_0{}'R(\beta) \\
&\quad - \frac{\partial}{\partial\beta}R(\beta) + S(0,\beta) \\
\Delta_{14} &= PB, \quad \Delta_{22} = 0 \\
\Delta_{23}(\beta) &= A_1{}'R(\beta) - S(-h,\beta), \quad \Delta_{24} = 0 \\
\Delta_{33}(\alpha,\beta) &= R(\alpha)'A_{01}(\beta) + A_{01}(\alpha)'R(\beta) \\
&\quad - (\frac{\partial}{\partial\beta} + \frac{\partial}{\partial\alpha})S(\alpha,\beta) \\
\Delta_{34}(\alpha) &= R(\alpha)'B, \quad \Delta_{44} = 0
\end{aligned}$$

and, if Δ_{ij} is a function of parameter α or β, Δ_{ij}' is defined as follows:

$$\begin{aligned}
&\Delta_{ij}'(\beta) = \Delta_{ji}(\alpha), \qquad \Delta_{ij}'(\alpha) = \Delta_{ji}(\beta) \\
&\Delta_{ij}'(\alpha,\beta) = \Delta_{ji}(\beta,\alpha)
\end{aligned}$$

Then, the state reachable set $\mathcal{R}(0,0)$ of the time-delay system described by (1) and (2) satisfies

$$\mathcal{R}(0,0) \subset \mathcal{E}(P,R,S)$$

□

Problem Statement

Our objective of this paper is to design a state-feedback controller (3) when the closed loop system is asymptotically stable and the L^2 gain, defined by (5), of the system is less than a scalar γ for any disturbance w which constrained by $\mathcal{W}$, and the input u of the closed loop system is included by $\mathcal{U}$:

$$\mathcal{U} = \{u(t)|u(t)'Uu(t) \leq 1\},$$

where, W_D is the given and positive definite matrix.

Here, an L^2 gain g is defined by

$$g = \sup_{w \in L^2, w \neq 0} \frac{\|z\|_{L_2}}{\|w\|_{L_2}}. \qquad (5)$$

In this case, we call the L^2-gain as "semi-global L^2-gain", because the disturbance is constrained by $\mathcal{W}$.

3. H^∞ CONTROLLER SYNTHESIS

In this section, we propose a synthesis method of state-feedback controller that makes the closed loop system asymptotically stable and semi-global L^2 gain of the closed system less than γ. First, consider the case that we do not limit the size of the input.

Theorem 1. Assume that there exist non-negative scalar p, matrices and matrix functions W,$Y(\alpha,\beta)$, Z_0 and $Z_{01}(\beta)$ that satisfies conditions LMI-2,

(LMI − 2)

$$\begin{bmatrix} W & W \\ W & Y(\alpha,\beta) \end{bmatrix} > 0, \quad X > 0,$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} \\ \Gamma_{12}' & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{13}' & \Gamma_{23}' & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{14}' & \Gamma_{24}' & \Gamma_{34}' & \Gamma_{44} \end{bmatrix} (\alpha,\beta) \leq 0$$

Where,

$$\Gamma_{11} = WA_0' + A_0W + Z_0'B + BZ_0 + 2W + pW$$
$$\Gamma_{12} = A_1W - W$$
$$\Gamma_{13}(\beta) = A_{01}(\beta)W + BZ_{01}(\beta) + WA_0' + Z_0B' + Y(0,\beta) + pW$$
$$\Gamma_{14} = BW, \quad \Gamma_{22} = 0$$
$$\Gamma_{23}(\beta) = WA_1 - Y(-h,\beta)$$
$$\Gamma_{24} = 0$$
$$\Gamma_{33}(\alpha,\beta) = A_{01}(\beta)W + BZ_{01}(\beta) + WA_{01}(\beta)' + Z_{01}(\beta)'B' - (\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta})Y(\alpha,\beta) + pY(\alpha,\beta)$$
$$\Gamma_{34} = BW, \quad \Gamma_{44} = -pI$$

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ \Lambda_{12}' & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} \\ \Lambda_{13}' & \Lambda_{23}' & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} \\ \Lambda_{14}' & \Lambda_{24}' & \Lambda_{34}' & \Lambda_{44} & \Lambda_{45} \\ \Lambda_{15}' & \Lambda_{25}' & \Lambda_{35}' & \Lambda_{45}' & \Lambda_{55} \end{bmatrix} (\alpha,\beta) < 0.$$

Where,

$$\Lambda_{11} = WA_0' + A_0W + X + 2W + BZ_0 + Z_0'B'$$
$$\Lambda_{12} = A_1W - W$$
$$\Lambda_{13}(\beta) = A_{01}(\beta)W + BZ_{01}(\beta) + WA_0' + Z_0'B' + Y(0,\beta)$$
$$\Lambda_{14} = WC', \quad \Lambda_{15} = D, \quad \Lambda_{22} = -X$$
$$\Lambda_{23}(\beta) = WA_1' - Y(-h,\beta), \quad \Lambda_{24} = 0$$
$$\Lambda_{25} = 0$$
$$\Lambda_{33}(\alpha,\beta) = A_{01}(\beta)W + WA_{01}(\alpha)' + BZ_{01}(\beta) + Z_{01}(\alpha)'B' - (\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta})Y(\alpha,\beta)$$
$$\Lambda_{34} = 0, \quad \Lambda_{35} = D, \quad \Lambda_{44} = -I$$
$$\Lambda_{45} = 0, \quad \Lambda_{55} = -\gamma^2 I$$

where, $W = W'$, $Y(\alpha,\beta) = Y'(\beta,\alpha)$. Then, the closed loop system with the feedback controller(3) for the system(1) is asymptotically stable and the trajectory of the closed loop system exists in a range $\mathcal{M}$:

$$\mathcal{M} = \{(x(t),x_t)| \\ (x(t),x_t)'\{W^{-1},W^{-1},W^{-1}YW^{-1}\}(x(t),x_t) \leq 1\},$$

and the semi-global L^2 gain of the closed loop system is less than $\gamma > 0$. Here, the feedback gain K_0 and $K_{01}(\beta)$ are given by

$$K_0 = Z_0W^{-1}, \quad K_{01}(\beta) = Z_{01}(\beta)W^{-1}$$

□

Here, we note that the state reachable set of the closed loop system where a disturbance w is constrained by $\mathcal{W}$ is evaluated by

$$\mathcal{E}(W^{-1},W^{-1},W^{-1}YW^{-1}),$$

so we obtain the next theorem to design a state feedback gain when the input of the closed loop system is in $\mathcal{U}$.

Theorem 2. If there exists a non-negative scalar p and matrices or matrix functions $W,Y(\alpha,\beta),Z_0$ and $Z_{01}(\beta)$ that satisfies conditions LMI-3 and LMI-2 in Theorem2,

(LMI − 3)

$$\begin{bmatrix} W & W & Z_0' \\ W & Y & Z_{01}(\beta)' \\ Z_0 & Z_{01}(\alpha) & U^{-1} \end{bmatrix} > 0$$

then, the closed loop is asymptotically stable and its semi global L^2 gain is less than γ, and its input is included in $\mathcal{U}$. Here, the feedback gains are given by

$$K_0 = Z_0W^{-1}, \quad K_{01}(\beta) = Z_{01}(\beta)W^{-1} \qquad (6)$$

Proof.

From (6),

$$u(t)'Uu(t) = (x(t),x_t)'\{W^{-1}Z_0'Z_0W^{-1}, W^{-1}Z_{01}(\beta)'Z_0W^{-1}, W^{-1}Z_{01}(\beta)'Z_{01}(\beta)W^{-1}\}(x(t),x_t).$$

Now, the state reachable set is included in

$$\mathcal{E}(W^{-1},W^{-1},W^{-1}YW^{-1}) = \{\lambda \in R^n \times L^2 | \lambda'\{W^{-1},W^{-1},W^{-1}YW^{-1}\}\lambda \le 1\},$$

so, if the following condition is satisfied, the input of the system is included in $\mathcal{U}$.

$$\begin{aligned}&\{W^{-1},W^{-1},W^{-1}YW^{-1}\} \\ &- \quad \{W^{-1}Z_0'Z_0W^{-1}, \\ &\quad W^{-1}Z_0'Z_{01}(\beta)W^{-1}, \\ &\quad W^{-1}Z_{01}(\beta)'Z_{01}(\beta)W^{-1}\} > 0 \\ &\Leftarrow \begin{bmatrix} W^{-1} & W^{-1} \\ W^{-1} & W^{-1}YW^{-1}\end{bmatrix} \\ &- \begin{bmatrix} W^{-1}Z_0'Z_0W^{-1} & W^{-1}Z_0'Z_{01}(\beta)W^{-1} \\ * & W^{-1}Z_{01}(\beta)'Z_{01}(\beta)W^{-1}\end{bmatrix} > 0 \\ &\iff \\ &\begin{bmatrix} W & W \\ W & Y\end{bmatrix} - \begin{bmatrix} Z_0'Z_0 & Z_0'Z_{01}(\beta) \\ * & Z_{01}(\beta)'Z_{01}(\beta)\end{bmatrix} > 0 \\ &\iff \begin{bmatrix} W & W \\ W & Y\end{bmatrix} - \begin{bmatrix} Z_0' \\ Z_{01}(\beta)'\end{bmatrix}\begin{bmatrix} Z_0' \\ Z_{01}(\beta)'\end{bmatrix}' > 0 \\ &\iff \textbf{(LMI-3)}\end{aligned}$$

Q.E.D.

As a special case, we consider the case when $A_{01}(\beta)$ in (1) and $K_{01}(\beta)$ in (3) are zero. In this case, the system has only the point delay and the controller is memoryless controller. The condition of this system which corresponds to that of Theorem 2 is described in the following theorem.

Theorem 3. If there exists a non-negative scalar p and matrices $W,'(\alpha,\beta), Z_0$ that satisfies the conditions:

$$W > 0,\ X > 0,$$

$$\begin{bmatrix} \begin{pmatrix} WA_0' + A_0W \\ +BZ_0 + Z_-'B' + pW \end{pmatrix} & A_1W & D \\ WA_1' & 0 & 0 \\ D' & 0 & -pW_D \end{bmatrix} \le 0,$$

$$\begin{bmatrix} \begin{pmatrix} WA_0' + A_0 + W \\ +X \\ +BZ_0 + Z_0'B' \end{pmatrix} & WC' & A_1W & D \\ CW & -I & 0 & 0 \\ WA_1' & 0 & -X & 0 \\ D' & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0,$$

$$\begin{bmatrix} W & Z_0' \\ Z_0 & U^{-1}\end{bmatrix} > 0$$

then, the closed loop system is asymptotically stable and its semi-global L^2 gain is less than γ, and its input is included in $\mathcal{U}$. Here, the feedback gain is given by

$$K_0 = Z_0W^{-1}$$

4. ALGORITHM

The condition given by Theorem2 includes bilinear terms, difficult to solve . Here, we propose an algorithm which overcomes such difficulty in solving LMI conditions iteratively.

Algorithm

Step 1 Define the initial values of W and $Y(\alpha,\beta)$ as $\tilde{W} = \epsilon_1 I, \tilde{Y}(\alpha,\beta) = \epsilon_2 I$

Step 2 Solve the following LMIs **(LMI-2')**, **(LMI-3)** and **(LMI-4)** given as

(LMI – 2')

$$\begin{bmatrix} W & W \\ W & Y(\alpha,\beta)\end{bmatrix} > 0, \quad X > 0,$$

$$\begin{bmatrix} \tilde{\Gamma}_{11} & \Gamma_{12} & \tilde{\Gamma}_{13} & \Gamma_{14} \\ \Gamma_{12}' & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \tilde{\Gamma}_{13}' & \Gamma_{23}' & \tilde{\Gamma}_{33} & \Gamma_{34} \\ \Gamma_{14}' & \Gamma_{24}' & \Gamma_{34}' & \Gamma_{44}\end{bmatrix}(\alpha,\beta) \le 0$$

where,

$$\tilde{\Gamma}_{11} = WA_0' + A_0W + Z_0'B + BZ_0 + 2W + p\tilde{W}$$

$$\tilde{\Gamma}_{13}(\beta) = A_{01}(\beta)W + BZ_{01}(\beta) + WA_0' + Z_0B' + Y(0,\beta) + p\tilde{W}$$

$$\tilde{\Gamma}_{33}(\alpha,\beta) = A_{01}(\beta)W + BZ_{01}(\beta) + WA_{01}(\beta)' + Z_{01}(\beta)'B' - (\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta})Y(\alpha,\beta) + p\tilde{Y}(\alpha,\beta)$$

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\ \Lambda_{12}' & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} \\ \Lambda_{13}' & \Lambda_{23}' & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} \\ \Lambda_{14}' & \Lambda_{24}' & \Lambda_{34}' & \Lambda_{44} & \Lambda_{45} \\ \Lambda_{15}' & \Lambda_{25}' & \Lambda_{35}' & \Lambda_{45}' & \Lambda_{55}\end{bmatrix}(\alpha,\beta) < 0$$

(LMI − 3)

$$\begin{bmatrix} W & W & Z_0' \\ W & Y & Z_{01}(\beta)' \\ Z_0 & Z_{01}(\alpha) & U^{-1} \end{bmatrix} > 0$$

(LMI − 4)

$$\begin{bmatrix} W & W \\ W & Y \end{bmatrix} \leq \begin{bmatrix} \tilde{W} & \tilde{W} \\ \tilde{W} & \tilde{Y} \end{bmatrix}$$

Step 3 If solutions $W, Y(\alpha,\beta)$ are obtained, then the W, $Y(\alpha,\beta)$ are solutions to the problem. If the solutions W, $Y(\alpha,\beta)$ is not satisfactory, put $\tilde{W} = W, \tilde{Y}(\alpha,\beta) = Y(\alpha,\beta)$ and go to Step 2.

Remark 1. The conditions in step 2 are linear, but infinite-dimensional ones, and the computational complexity is very high. Here, using a technique which we show in the next section, the conditions are reduced to the finite dimensional conditions, and becomes feasible ones.

Remark 2. A special case of this algorithm is applicable to the conditions in Theorem 3. Here, we note that when we apply the special case of this algorithm to the conditions in Theorem 3, we can show a convegence property of the algorithm.

5. REDUCTION TO FINITE-DIMENSIONAL LMIS

These LMI conditions, proposed in the algorithm, are infinite-dimensional LMI conditions. These infinite-dimensional LMI conditions are still difficult to solve,and some idea is needed to solve them. Now, we propose a method to reduce infinite-dimensional LMI conditions to finite-dimensional LMI conditions(Ikeda *et al.*, 2001). This method doesn't need descritization.

Now, we assume that, in LMI conditions **(LMI-2')**, **(LMI-3)** and **(LMI-4)**, $A_{01}(\beta), Z_{01}(\beta)$ and $Y(\alpha,\beta)$ is described like:

$$A_{01}(\beta) = A_{01}^0 + \beta A_{01}^1 + \beta^2 A_{01}^2 + \cdots + \beta^l A_{01}^l$$
$$Z_{01}(\beta) = Z_{01}^0 + \beta Z_{01}^1 + \beta^2 Z_{01}^2 + \cdots + \beta^l Z_{01}^l$$
$$Y(\alpha,\beta) = Y_0 + (\alpha+\beta)Y_1 + \cdots + (\alpha^l + \beta^l)Y_l,$$

then, LMI conditions in **(LMI-2')**, **(LMI-3)** and **(LMI-4)** are written as:

$$F_0(M) + f_1(\theta)F_1(M) + f_2(\theta)F_2(M) + \cdots + f_r(\theta)F_r(M) \leq 0$$

Here, θ is a vector like:

$$\theta \in \Theta \equiv \{[\alpha\ \beta]' | \alpha, \beta \in [-h, 0]\}$$

and $f_i : R^2 \mapsto R$ is a polynomial function of θ, F_i is an affine matrix function for a symmetric and unknown matrix M. Here, an LMI written like this form is able to be reduced to a finite-dimensional LMI without so-called discritization.

6. NUMERICAL EXAMPLE

In this section, we show a numerical example with a system which has only point delay:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h) + Bu(t) + Dw(t) \quad (7)$$
$$z(t) = Cx(t).$$

and a memoryless controller:

$$u(t) = K_0 x(t). \quad (8)$$

In this case, the system has only point delay, then, the matrix inequality conditions given by Theorem 3 are finite dimensional.

The system parameters of (7) are given as:

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -1.5 \end{bmatrix}, \ A_1 = \begin{bmatrix} -0.2 & 0.0 \\ 0.0 & 0.5 \end{bmatrix},$$
$$B = \begin{bmatrix} 1.0 \\ 0.5 \end{bmatrix}, \ D = \begin{bmatrix} -0.1 \\ -0.2 \end{bmatrix}.$$

The values of W_D, U which determines the range of w, u respectively and the value of the L^2 gain are chosen as:

$$W_D = 1.0 \times 10^1, \ U = 1.0 \times 10^{-2}, \ \gamma = 1.732.$$

With these conditions, we solve the LMI conditions by the special case algorithm introduced the section 4, and we obtain these solutions in the 1st iteration.

$$W = \begin{bmatrix} 0.0536 & 0.0253 \\ 0.0253 & 0.0151 \end{bmatrix}, \ X = \begin{bmatrix} 3.1548 & 1.5652 \\ 1.5652 & 0.7781 \end{bmatrix},$$
$$Z_0 = \begin{bmatrix} -2.2261 \\ -1.1356 \end{bmatrix}, \ K_0 = \begin{bmatrix} -32.2513 \\ -21.0170 \end{bmatrix},$$
$$p = 3.5357 \times 10^{-5}.$$

And in the 20th iteration, we obtain these solutions. Here, we really found the decreasing property of W with iterations.

$$W = \begin{bmatrix} 0.0021 & -0.0007 \\ -0.0007 & 0.0021 \end{bmatrix},$$
$$X = \begin{bmatrix} 0.4035 & 0.1994 \\ 0.1994 & 0.1005 \end{bmatrix},$$
$$Z_0 = \begin{bmatrix} -0.3000 \\ -0.1579 \end{bmatrix}, \ K_0 = \begin{bmatrix} -184.4005 \\ -134.7405 \end{bmatrix},$$
$$p = 0.0306.$$

The responce of the step disturbance ($0.5 \leq t \leq 1.5$) with this feedback gain is described in Fig.1.

We found that $u(t)$ stays in the range $\mathcal{U}$ which is defined by the given matrix U.

7. CONCLUSION

We proposed an H^∞ controller synthesis method for linear time-delay system, when a size of the

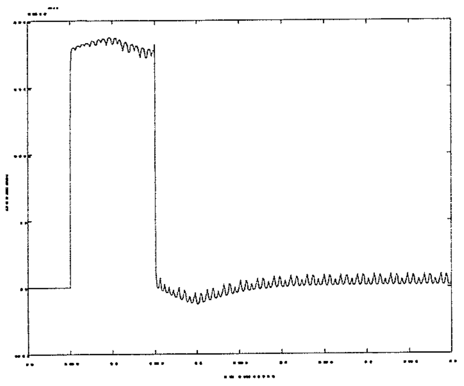

Fig. 1. Simulation result : Input $u(t)$

input is constrained. The controller makes the closed loop is semi-globally asymptotically stable and its L^2 gain less than a specified value.

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論 文

線形むだ時間システムにおける L^2ゲイン解析と状態フィードバック制御 — 無限次元 LMI アプローチ —

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L^2-gain Analysis and State Feedback Control Synthesis for Linear Time-Delay Systems — Infinite-Dimensional LMI Approach —

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In this paper, we propose a new method of L^2-gain analysis and state feedback control synthesis for time-delay systems via infinite-dimensional LMI, which reflects explicitly infinite-dimensionality features of time-delay systems. First, we show an infinite-dimensional LMI condition for L^2-gain analysis. The infinite-dimensional LMI condition depends two parameters, and difficult to solve. Next, we propose a method of solving such infinite-dimensional LMI conditions using a convex polyhedron. The method consists of two steps: the first step is to make a convex polyhedron, and the second step is to reduce an infinite-dimensional LMI condition to a finite-dimensional LMI condition using the convex polyhedron. Finally, we discuss a state-feedback control synthesis based on the infinite-dimensional LMI conditions for L^2-gain analysis and the method of solving infinite-dimensional LMIs, when the feedback gain is constrained.

キーワード：むだ時間システム (time-delay system), L^2ゲイン (L^2-gain), 線形行列不等式 (Linear Matrix Inequality:LMI), 状態フィードバック (state feedback)

1. はじめに

近年，線形制御システムの解析･設計において，有効な数値計算アルゴリズムを有する線形行列不等式 (Linear Matrix Inequalities: LMIs) を用いる手法が注目されている。[5] [10] LMI 手法の要点は，システムの状態の 2 次形式で与えられるリアプノフ関数あるいはストレージ関数 (storage function) の構成を LMI の解法に帰着させることにある。

LMI 手法を線形むだ時間システムの解析・設計に拡張するする試みもすでに数多くなされているが，著者らの知る限り Gu の安定性解析の結果 [11] を除けば，むだ時間がない場合と同様の有限次元 LMI の枠組み内での結果となっている [13]。そこでは，状態の 2 次形式としては，集中定数部分の 2 次形式を中心とする限定されたものを用い，また，状態フィードバック制御則としてはメモリーレスフィードバックに限定することによって LMI 手法の実行が可能になっており，いわば，議論の出発点から有限次元に限定したものである。しかし，むだ時間システムの最大の特徴はその無限次元性にある [6]。著者らはこの無限次元性をできるだけ反映した解析・設計法になることを念頭に置き，無限次元 LMI に基づくむだ時間システムの新しい解析・設計法の検討を始めている [4]。本稿では同じ観点から，L^2ゲイン解析と状態フィードバックコントローラの設計について，無限次元 LMI に基づく新しい解析・設計法を提案する。

従来法と比較した時の提案法の特徴をまとめると，つぎの 2 点になる。1 点目は，状態 2 次形式として集中定数部分と分布定数部分からなるむだ時間システムの無限次元の状態の一般的な 2 次形式を考え，この 2 次形式の構成要件を 2 個のパラメータに依存する無限次元 LMI 条件に帰着させることである。従来法では集中定数部分の 2 次形式に限定している。2 点目は，この無限次元 LMI を，パラメータ空間上に適当な凸多面体を構成することによって実際に解くことが可能な有限個の LMI に帰着させることである。無限次元 LMI 条件は特別な限定をしていないことから，有限次元に限定した従来法より保守性の少ない条件である。実際，2 点目の無限次元 LMI の解法において保守性を少なく

することができれば，従来法より良い結果を得ることが期待できる。なお，2 点目の無限次元の LMI の解を，無限次元 LMI を有限個の LMI に帰着させてから解く方法は，著者らによって提案された，ゲインスケジューリング問題におけるパラメータ依存 LMI の解法のアイデアと同じものである。[3] この手法においては，保守性の少ない解を得るためにはパラメータ空間における凸多面体の構成が重要であり，一般的な構成法が提案されている。[1] 本稿では，対象がむだ時間のシステムの L^2ゲイン解析・設計に対する無限次元 LMI であること，さらに，無限次元 LMI が 2 個のパラメータに依存するという特殊性を考慮して，保守性のより少ない新しい凸多面体の構成法を提案する。

以下，本稿では，まず第 2 章で対象システムを設定し，L^2安定のための無限次元 LMI 条件を導出する。そして，第 3 章では，その無限次元の LMI 条件を有限個の LMI 条件に帰着させる手法および凸多面体の構成法を提案する。さらに，第 4 章では，第 3 章までで述べた L^2ゲイン解析の結果に基づいた状態フィードバックコントローラの設計法，ならびに，その際にゲインの大きさを制限した場合の設計法について提案し，最後に第 5 章でまとめる。

なお，以下において “′” は行列あるいはベクトルの転置を表す。対称行列 P_0および対称行列値関数 $P_2(\alpha,\beta)$ に対して，記法

$$L(\alpha,\beta)=\begin{bmatrix} P_0 & P_1(\beta) \\ P_1(\beta)' & P_2(\alpha,\beta)\end{bmatrix} > (<)0,$$
$$\forall\alpha\in[-h,0],\beta\in[-h,0]$$

は，

$$\frac{1}{2}(L(\alpha,\beta)+L'(\alpha,\beta))=$$
$$\begin{bmatrix} P_0 & \frac{1}{2}(P_1(\alpha)+P_1'(\beta)) \\ \frac{1}{2}(P_1'(\alpha)+P_1(\beta)) & \frac{1}{2}(P_2(\alpha,\beta)+P_2(\beta,\alpha))\end{bmatrix}$$

が各 $(\alpha,\beta)\in[-h,0]\times[-h,0]$ において，正定 (負定) であることを意味する。また，$L^2([-h,0];R^n)$ は区間 $[-h,0]$ で定義される二乗可積分な n 次元実ベクトル値関数の集合を，$C([-h,0];R^n)$，$C^1([-h,0];R^n)$ は区間 $[-h,0]$ で定義される連続および 1 回連続微分可能な n 次元実ベクトル値関数の集合を，それぞれ表すものとする。

2. L^2ゲイン解析

〈2·1〉 対象システム　つぎの状態にむだ時間を含む線形システムを考える。

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)$$
$$+\int_{-h}^{0}A_{01}(\beta)x(t+\beta)d\beta \cdots\cdots\cdots\cdots\cdots (1)$$
$$+Bu(t)+Dw(t)$$
$$z(t)=Cx(t)$$
$$x(0)=\phi(0)\ ,\ x(\beta)=\phi(\beta)\ ,\ -h\le\beta\le 0$$

ここで，$x(t)\in R^n$は状態，$u(t)\in R^m$は制御入力，$w(t)\in R^p$は外部入力，$z(t)\in R^q$は制御量，$(\phi(0),\phi(\beta))\in R^n\times L^2([-h,0];R^n)$ は初期状態，h はシステムのむだ時間を表す。

また，$A_0,A_1\in R^{n\times n}$，$A_{01}(\beta)\in L^2([-h,0];R^{n\times n})$，$B\in R^{n\times m}$，$C\in R^{q\times n}$，$D\in R^{n\times p}$，$h>0$である。

〈2·2〉 内部安定性　むだ時間システム (1) において $u(t)=0$，$w(t)=0$として得られるつぎのむだ時間システムを考える。

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)$$
$$+\int_{-h}^{0}A_{01}(\beta)x(t+\beta)d\beta \cdots\cdots\cdots\cdots\cdots (2)$$

むだ時間システム (2) の内部安定性をつぎのように定義する。

〔定義 1〕 むだ時間システム (2) において，任意の初期状態 $(\phi(0),\phi(\beta))$ に対し解 $x(t)$ が零に漸近するとき，むだ時間システム (2) は内部安定であるという。

〈2·3〉 L^2入出力安定性　つづいて，この節では，むだ時間システム (1) において $u(t)=0$として得られるつぎのむだ時間システムを考える。

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)$$
$$+\int_{-h}^{0}A_{01}(\beta)x(t+\beta)d\beta+Dw(t) \cdots\cdots (3)$$
$$z(t)=Cx(t)$$

むだ時間システム (3) の L^2入出力安定性をつぎのように定義する。

〔定義 2〕 むだ時間システム (3) において，初期状態 $(\phi(0),\phi(\beta))=0$ としたとき，任意の入力 $w(t)\in L^2([0,\infty);R^p)$ に対し出力も常に $z(t)\in L^2([0,\infty);R^q)$ となるとき，むだ時間システム (3) は L^2入出力安定であるという。

さらに，L^2入出力安定であるようなむだ時間システム (3) の L^2ゲイン G をつぎのように定義する。

〔定義 3〕 むだ時間システム (3) が L^2入出力安定であるとき，むだ時間システム (3) の L^2ゲイン G をつぎで定義する。

$$G=\sup_{w\in L^2,w\neq 0}\frac{\|z\|_{L^2}}{\|w\|_{L^2}} \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

ここで $\|\cdot\|_{L^2}$は L_2ノルムを表す。

〈2·4〉 L^2ゲイン解析　むだ時間システム (3) が内部安定かつγ以下の L^2ゲインをもつための条件，すなわち $G\le\gamma$ となるための条件は次の定理により与えられる。

〔定理 1〕 下記の LMI 条件 (LMI−1) を満たす行列および行列値関数 P,Q,$R(\beta)$, $S(\alpha,\beta)$ が存在するならば，むだ時間システム (1) において，$u(t)=0$としたむだ時間シ

ステム (3) は内部安定でかつγ以下のL^2ゲインを有する。
(LMI−1)

$$L_1(\alpha,\beta) = \begin{bmatrix} P & R(\beta) \\ R(\alpha)' & S(\alpha,\beta) \end{bmatrix} > 0 \text{ , (5)}$$

$$Q > 0$$

$$L_2(\alpha,\beta) =$$

$$\begin{bmatrix} \begin{pmatrix} A_0'P + PA_0 + Q \\ +R(0)' + R(0) + C'C \end{pmatrix} & PA_1 - R(-h) & \begin{pmatrix} PA_{01}(\beta) + A_0'R(\beta) \\ -\frac{\partial}{\partial\beta}R(\beta) + S(0,\beta) \end{pmatrix} & PD \\ A_1'P - R(-h)' & -Q & A_1'R(\beta) - S(-h,\beta) & 0 \\ \begin{pmatrix} A_{01}(\alpha)'P + R(\alpha)'A_0 \\ -\frac{\partial}{\partial\alpha}R(\alpha)' + S(\alpha,0) \end{pmatrix} & R(\alpha)'A_1 - S(\alpha,-h) & \begin{pmatrix} R(\alpha)'A_{01}(\beta) \\ +A_{01}(\alpha)'R(\beta) \\ -(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta})S(\alpha,\beta) \end{pmatrix} & R(\alpha)'D \\ D'P & 0 & D'R(\beta) & -\gamma^2 I \end{bmatrix} < 0 \cdot (6)$$

なお，ここで，

$$P = P',\ Q = Q' \in R^{n\times n}$$
$$R(\beta) \in C^1([-h,0];R^{n\times n})$$
$$S(\alpha,\beta) = S(\beta,\alpha)'$$
$$\in C^1([-h,0]\times[-h,0];R^{n\times n})$$
$$\forall\alpha,\beta \in [-h,0]$$

である。
証明は付録 1 に示す。

3. 無限次元 LMI から有限次元 LMI への帰着

前節で得られた条件は無限次元の LMI 条件であり，そのままでは解くことは一般に困難である。そこで，本節ではこれらの無限次元 LMI 条件を解くための手法を提案する。

〈3·1〉 基本的手順　　ここで，(LMI−1) において，システムパラメータ $A_{01}(\beta)$ および解 $R(\beta), S(\alpha,\beta)$ が

$$A_{01}(\beta) = A_{01}^0 + \beta A_{01}^1 + \beta^2 A_{01}^2 + \cdots + \beta^l A_{01}^l$$
$$R(\beta) = R_0 + \beta R_1 + \beta^2 R_2 + \cdots + \beta^l R_l$$
$$S(\alpha,\beta) = S_0 + (\alpha+\beta)S_1 + \cdots + (\alpha^l + \beta^l)Y_l$$

で与えられるなら，いずれの LMI もつぎのようなパラメータ依存 LMI

$$F_0(M) + f_1(\theta)F_1(M) + f_2(\theta)F_2(M) + \cdots$$
$$+ f_l(\theta)F_l(M) < 0 \quad \cdot (7)$$

の形で記述できる。ただし，θはつぎのようなベクトルである。

$$\theta \in \Theta := \{[\alpha \quad \beta]' | \alpha \in [-h,0], \beta \in [-h,0]\}$$

ここで，$f_i : R^2 \mapsto R$はθの多項式関数であり，F_iは対称かつ未知行列Mに対しアフィンな行列値関数である。

このとき，つぎの定理により，(7) の条件は有限個の LMI 条件に帰着させることができる。

〔定理 2〕[1] Hを，$k(\geq l+1)$ 個の頂点 $\{p_1, p_2, \cdots, p_k\}$ $(p_i \in R^r, i = 1,2,\cdots,k)$ を持ち，曲面 $T = \{[f_1(\theta), f_2(\theta), \cdots, f_l(\theta)]' | \theta \in \Theta\}$ を内部に含む R^lにおける凸多面体とする。このとき，Hの任意の頂点に対しつぎの LMI を満たす行列 Mが存在すれば，その行列 M は任意の$\theta \in \Theta$に対し，(7) を満たす。

$$F_0(M) + p_{i1}F_1(M) + p_{i2}F_2(M) + \cdots$$
$$+ p_{il}F_l(M) > 0 \qquad \cdots (8)$$

ここで，p_{ij}は頂点p_iのj番目の要素を表す。

注意 1 この定理は凸多面体 Hの任意の頂点において (8) が成立すればその内部の点においても必ず (8) が成立することを示している。したがって，Tを含む体積の小さいHを構成することにより，L^2ゲインの評価に関する保守性を改善することができる。

文献 (1) では，凸多面体の構成方法として，

$$r_1 = [\min_{\theta\in\Theta} f_1(\theta) \ \min_{\theta\in\Theta} f_2(\theta) \ \cdots \ \min_{\theta\in\Theta} f_l(\theta)]',$$
$$r_2 = [\max_{\theta\in\Theta} f_1(\theta) \ \max_{\theta\in\Theta} f_2(\theta) \ \cdots \ \max_{\theta\in\Theta} f_l(\theta)]',$$

の 2 点を対角に持つ超直方体 (2^l個の頂点を持つ) を構成する方法を提案している。この方法は一般的な関数 f_iおよび パラメータθの個数が任意の一般的な場合に適用できるものであり，本節での問題にもそのまま適用できる。しかしながら，この方法はその一般性のために，保守性の大きい構成法になる可能性ももっている。そこで，以下では，本節の問題におけるパラメータが 2 個しかも f_iは多項式関数であるという特殊性を考慮することでより保守性の少ない凸多面体の構成を提案する。

〈3·2〉 凸多面体の構成　　前節で述べた線形むだ時間システムに関係するパラメータ依存 LMI に対しては，Tは 2 つのパラメータα, βに依存したものになる。これは，定理 2 において，$l = 2s$ で，

$$f_i(\alpha,\beta) = \begin{cases} f_i(\alpha), & i = 1,2,\cdots,s, \\ f_i(\beta), & i = s+1, s+2, \cdots, 2s \end{cases}$$

である場合に相当する。ここで，$f_i(\alpha)$，$f_i(\beta)$ はα，βの多項式である。この場合，凸多面体が囲む曲面 Tは $T = \{[\alpha \quad \alpha^2 \cdots \alpha^r \quad \beta \quad \beta^2 \cdots \quad \beta^r] | \alpha \in [-h,0], \beta \in [-h,0]\}$ となり，これを内部に含む R^{2r}における凸多面体の構成について考える。以下では区間 $[-h,0]$ を一般化した区間 $[h_1,h_2]$ に対する結果を述べる。

〔定理 3〕 $T = \{[\alpha \quad \alpha^2 \cdots \alpha^s \quad \beta \quad \beta^2 \cdots \quad \beta^s] | \alpha \in [h_1,h_2], \beta \in [h_1,h_2]\}$ とする。このとき，つぎで与えられる頂点 p^{ij} $(0 \leq i \leq s,\ 0 \leq j \leq s)$ を持つ R^lの凸多面体 H は Tを内部に含む。

ここで，p^{ij}は

$$p^{ij} = [p^i \ p^j], \quad i,j = 0,1,\cdots,s$$

であり，p^iは

$$
\begin{aligned}
p^0 &= [h_1\ h_1^2\ \cdots\ h_1^s]' \in R^s \\
p^1 &= [h_2\ h_1^2\ \cdots\ h_1^s]' \in R^s \\
&\ \vdots \\
p^s &= [h_2\ h_2^2\ \cdots\ h_2^s]' \in R^s
\end{aligned}
$$

である。
証明は付録2に示す。

実際には，定理3において，h_1，h_2をそれぞれ，$h_1 = -h$，$h_2 = 0$とすることで，目的とする凸多面体を構成することができる。

なお，さらに保守性を少なくするためにさらに体積の小さい凸多面体を構成するためにはパラメータの範囲$[-h,\ 0]$を$[-h,\ h_a]$，$[h_a,\ h_b]$，$\cdots$のようにいくつかの区間に分割し，それらの区間ごとに定理3で示した凸多面体を構成することが可能である。

〈3·3〉 数 値 例　提案手法の有効性を検証するため，数値例を示す。対象システム(1)のL^2ゲインは定理1中のLMI条件(LMI-1)によって与えられる。そこで，定理1によって得られる無限次元のLMI条件を，有限個のLMI条件に帰着させる時に，文献(1)の方法と，定理3による方法を適用し，それぞれの方法により得られたL^2ゲインの上限の値と，条件を解くのに要した計算機のCPU時間を比較する。なお，演算に当たってはLMI Control Toolbox for Use with MATLAB[12]を使用した。

システムパラメータをつぎのようにする。

$$
A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -1.5 \end{bmatrix},\ A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}
$$

$$
A_{01}(\beta) = \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix},\ C = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}
$$

$$
D = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},\ h = 2.0.
$$

ここで，(LMI-1)において，$R(\beta), S(\alpha,\beta)$は，

$$
\left.\begin{aligned}
R(\beta) &= R_0 + \sum_{i=1}^{i} \beta^i R_i \\
S(\alpha,\beta) &= S_0 + \sum_{i=1}^{i} (\alpha^i + \beta^i) S_i
\end{aligned}\right\} (i = 2 \sim 4)
$$

とした。

結果は表1,2のようになった。

これらの結果から，パラメータ$(R(\beta), S(\alpha,\beta))$の次数にかかわらず，提案した凸多面体の構成法を利用することにより，L^2ゲインの評価に関する保守性が改善されていることがわかる。また，計算に要するCPU時間についても同様に改善されている。

4. ゲインの大きさを制限した制御則の構成法

〈4·1〉 状態フィードバック制御問題　ここでは，むだ時間システム(1)に対し，

$$
u(t) = K_0 x(t) + \int_{-h}^{0} K_{01}(\beta) x(t+\beta) d\beta \cdots\cdots (9)
$$

$$
K_0 \in R^{m\times n}\ ,\ K_{01}(\beta) \in L^2([-h, 0]; R^{m\times n})
$$

のような状態の履歴を用いるメモリーフィードバック則を適用した閉ループシステムを考える。その閉ループ系のL^2ゲインの大きさがγ以下になるようなコントローラ(9)の設計法について述べる。定理2の結果を基にこの条件を満たすコントローラのゲイン$K_0, K_{01}(\beta)$を求めるための条件は次の定理で与えられる。

〔定理4〕 次のLMI条件を満たす行列および行列値関数$W, X, Z_0, Z_{01}(\beta), Y(\alpha,\beta)$が存在するとき，

$$
K_0 = Z_0 W^{-1},\quad K_{01}(\beta) = Z_{01}(\beta) W^{-1}
$$

で表されるゲインを用いたコントローラを適用した閉ループシステムは内部安定となり，そのL^2ゲインの値はγ以下となる。

(LMI−2)

$$
L_3(\alpha,\beta) = \begin{bmatrix} W & W \\ W & Y(\alpha,\beta) \end{bmatrix} > 0, \cdots\cdots (10)
$$

$$
X > 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (11)
$$

$$
L_4(\alpha,\beta) =
$$

$$
\begin{bmatrix}
\begin{pmatrix} WA_0' + WA_0 \\ +X + 2W \\ +BZ_0 + Z_0'B' \end{pmatrix} & A_1W - W & \begin{pmatrix} BZ_{01}(\beta) \\ +WA_0' \\ +Z_0'B' \\ +Y(0,\beta) \end{pmatrix} & WC' & D \\
WA_1' - W & -X & \begin{pmatrix} WA_1' \\ -Y(-h,\beta) \end{pmatrix} & 0 & 0 \\
\begin{pmatrix} Z_{01}(\alpha)'B' \\ +A_0W + BZ_0 \\ +Y(\alpha,0) \end{pmatrix} & \begin{pmatrix} A_1W \\ -Y(\alpha,-h) \end{pmatrix} & \begin{pmatrix} BZ_{01}(\beta) \\ +Z_{01}(\alpha)'B' \\ -(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) \\ Y(\alpha,\beta) \end{pmatrix} & 0 & D \\
CW' & 0 & 0 & -I & 0 \\
D' & 0 & D' & 0 & -\gamma^2 I
\end{bmatrix} < 0 \cdots (12)
$$

表1　L^2ゲインの比較

Table 1.　Comparison of L^2 gains

$S(\alpha,\beta), R(\alpha)$の次数	2次	3次	4次
従来の手法	0.9050	0.9047	1.0620
提案の手法	0.9046	0.9033	0.90268

表2　CPU時間の比較(単位:秒)

Table 2.　Comparison of CPU time (sec)

$S(\alpha,\beta), R(\alpha)$の次数	2次	3次	4次
従来の手法	159.89	1861.0	20409
提案の手法	81.12	368.66	1208.5

なお，ここで，

$$W = W',\ X = X' \in R^{n\times n},$$
$$Y(\alpha,\beta) = Y(\beta,\alpha)' \in C^1([-h,0]\times[-h,0];R^{n\times n})$$
$$Z_0 \in R^{m\times n}$$
$$Z_{01}(\beta) \in C([-h,0];R^{m\times n})$$
$$\forall\alpha \in [-h,0],\quad \forall\beta\in[-h,0]$$

である。

この定理は，定理2において，

$$P^{-1} = R(\beta)^{-1} = W,\ P^{-1}QP^{-1} = X$$
$$S(\alpha,\beta) = W^{-1}Y(\alpha,\beta)W^{-1}$$
$$K_0P^{-1} = Z_0,\ K_{01}(\beta)P^{-1} = Z_{01}(\beta) \quad \cdots\cdot (13)$$

とすることにより直ちに証明できる。

〈4・2〉 ゲインの大きさを制限した制御則の構成法　前節で述べたコントローラの設計法では，制御量zが制御入力uを含まないため，定理4をそのまま用いると制御入力が大きくなってしまうことがある。これを避けるための一つの方法として，フィードバックゲインの大きさを制限することを考える。

いま，次のようにフィードバックゲインの大きさを制限しよう。

$$K_0'K_0 < \gamma_1 I,\quad K_{01}(\beta)'K_{01}(\beta) < \gamma_2 I \quad \cdots\cdots (14)$$
$$\forall\beta\in[-h,0]$$

ここで，γ_1,γ_2はあらかじめ与えられているとする。このとき，次の定理を得る。

〔定理5〕　与えられた正の定数p_1,p_2,qに対し，定理4のLMI条件(LMI−2)と下記のLMI条件(LMI−3)とを満たす行列および行列値関数W, X, $Y(\alpha,\beta)$, Z_0, $Z_{01}(\beta)$が存在するならば，システム(1)に対し状態フィードバック制御則

$$K_0 = Z_0W^{-1},\ K_{01}(\beta) = Z_{01}(\beta)W^{-1} \quad (15)$$

を用いて構成した閉ループシステムは内部安定でそのL^2ゲインの値はγ以下となり，フィードバックゲインは以下の不等式を満たす。

$$K_0'K_0 < p_1q^2I,\ K_{01}(\beta)'K_{01}(\beta) < p_2q^2I \quad (16)$$

(LMI−3)

$$\begin{bmatrix} p_1I & Z_0' \\ Z_0 & I\end{bmatrix} > 0 \quad \cdots\cdots (17)$$
$$\begin{bmatrix} p_2I & Z_{01}'(\beta) \\ Z_{01}(\beta) & I\end{bmatrix} > 0 \quad \cdots\cdots (18)$$
$$\begin{bmatrix} qI & I \\ I & W\end{bmatrix} > 0 \quad \cdots\cdots (19)$$

証明は付録3に示す。

与えられたγ_1,γ_2に対して，(14)を満たし，システムのL^2ゲインGをある値γ以下となるようなフィードバックゲインを求めるアルゴリズムはつぎのように与えられる。

アルゴリズム

(Step 1) p_1,p_2,qの初期値を定め，p_{10},p_{20},q_0とする。

(Step 2) (LMI−1),(LMI−2)と以下のLMIを解く。

$$p_1 < p_{10},\ p_2 < p_{20},\ q < q_0$$

(Step 3) Step 2により求めた解p_1,p_2,qについて，

$$\gamma_1 > p_1q^2,\ \gamma_2 > p_2q^2$$

をみたすかどうか調べる。

(Step 4) ・Step 3が満たされるとき，求められたゲインは(14)を満たし，アルゴリズムは終了する。・Step 3が満たされないとき，$p_{10}=p_1,p_{20}=p_2,q_0=q$としてStep 2に戻る。

〈4・3〉 数値例

数値例1　本節では，前節までで示したコントローラの設計について，定理4による方法と，定理5で提案した方法で設計を行ない，その比較を行う。なお，これらの方法で得られる無限次元のLMI条件は第3章の方法により有限個のLMI条件に帰着させて解いている。

対象とするシステムは，

$$\dot{x}(t) = x(t) + 0.3x(t-1) + u(t)$$
$$y(t) = x(t) \quad \cdots\cdots (20)$$
$$x(\beta) = 0,\ -h \leq \beta \leq 0$$

また，$Z_{01}(\beta), Y(\alpha,\beta)$の形は，

$$Z_{01}(\beta) = Z_{01}^0 + \beta Z_{01}^1 + \beta^2 Z_{01}^2$$
$$Y(\alpha,\beta) = Y_0 + (\alpha+\beta)Y_1 + (\alpha^2+\beta^2)Y_2$$

で表されるとする。

ここでは，このシステムに対して目標信号rへのトラッキング問題を考え，図1に示すように積分器を含む形でフィードバックコントローラ(9)の設計を行なう。そのために，図2のような積分器を重み関数とする一般化プラントを導入し，L^2ゲインを低減化する設計を行なった。

さらに，ゲインを制限する値として，p_1,p_2,qを，

$$p_1 = 3.49\times10^4, p_2 = 1.28\times10^2, q = 2.56$$

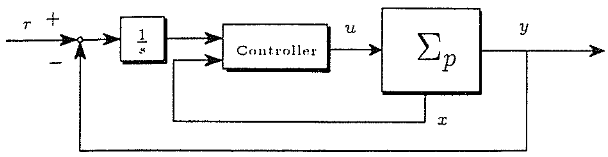


図1　閉ループシステム

Fig. 1.　The closed loop system

とした。なお，$\gamma_1 = 2.5 \times 10^5$，$\gamma_2 = 8.5 \times 10^2$ とした。

このシステムについて定理 4，定理 5 に基づき導出した無限次元 LMI を第 3 章の方法に基づき有限個に帰着させ解いた結果，得られたコントローラは次のようになった。

定理 4 によるコントローラ

$$K_0 = [115.48 \quad -24.94]$$

$$\begin{aligned} K_{01}(\beta) = &[75.79 \quad -12.45] \\ &+ \beta[-9.34 \quad -3.09] \\ &+ \beta^2[15.14 \quad -3.53] \end{aligned}$$

定理 5 によるコントローラ

$$K_0 = [36.16 \quad -11.74]$$

$$\begin{aligned} K_{01}(\beta) = &[23.49 \quad -4.01] \\ &+ \beta[1.71 \quad -0.53] \\ &+ \beta^2[-0.07 \quad -0.19] \end{aligned}$$

ここで，$r = 1$ としたときの結果を図 3 に示す。実線は定理 4 によるコントローラ，一点鎖線は定理 5 によるコントローラを用いたものである。

数値例 2　前節までに示したコントローラの設計法は，形式的に $C = 0, D = 0$ とおくことによってそのままレギュレータ (安定化コントローラ) の設計法となる。本節ではレギュレータの設計例を示す。システムパラメータを

$$A_0 = \begin{bmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix},\quad A_1 = \begin{bmatrix} -1.0 & -1.0 \\ 0.0 & -1.0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix},\quad h = 0.5$$

とし，ゲインを制限する値として，$\gamma_1 = 4.0 \times 10^3$，$\gamma_2 = 3.0 \times 10^2$ と仮定した。$Z_0(\beta), Y(\alpha, \beta)$ を数値例 1 と同じ形式として求めたコントローラはつぎのようになった。

定理 4 によるコントローラ

$$K_0 = [\ -63.6094 \quad -29.0628\ ]$$

$$\begin{aligned} K_{01}(\beta) = &[\ -20.8643 \quad -8.9807\ ] \\ &+\beta[\ -18.3488 \quad -10.1254\ ] \\ &+\beta^2[\ -4.9496 \quad -1.5358\ ] \end{aligned}$$

定理 5 によるコントローラ

$$K_0 = [\ -16.2512 \quad -9.8343\ ]$$

$$\begin{aligned} K_{01}(\beta) = &[\ -9.3076 \quad -4.3236\ ] \\ &+\beta[\ -2.0446 \quad -2.1647\ ] \\ &+\beta^2[\ 8.0998 \quad 3.9455\ ] \end{aligned}$$

一方，求められたこれらのゲインを用いた閉ループ系の応答は図 4，5 のようになった。

これらの結果から，提案した定理およびアルゴリズムを用いることにより，むだ時間を含むシステムに対し，制御入力の値が制限されたフィードバック制御則の構成がある程度まで可能であることがわかる。

5. おわりに

本稿では線形むだ時間システムにおける L^2 ゲイン解析および状態フィードバックコントローラ設計問題に対する新しい LMI 手法を提案し，数値例によりその有効性を示した。また，入力が大きくなってしまうことを防ぐため，ゲインの大きさを制限する LMI 条件を付加する方法についても検討した。ゲインの大きさを小さくすることにより入力を小さくする方法には一定の限界がある。制御入力の大きさを制限した制御則の構成の本格的な議論には状態可到達集合の評価が必要であり，今後の課題である。

（平成 12 年 12 月 25 日受付，同 13 年 7 月 4 日再受付）

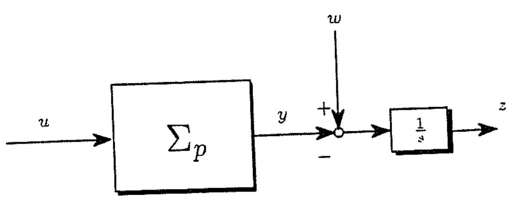
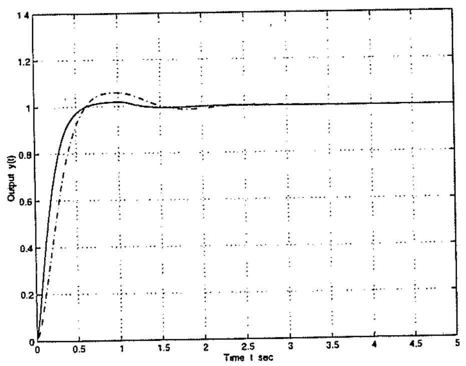
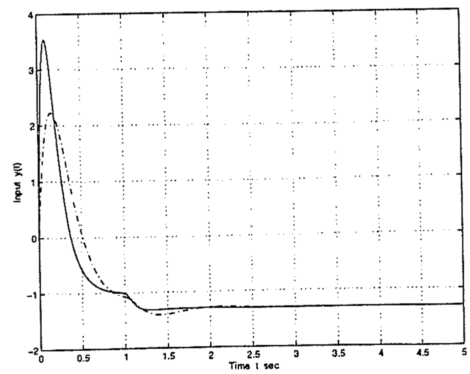


図 **2**　一般化プラント

Fig. 2.　Generalized plant



(a) Output y



(b) Input u

図 **3**　シミュレーション結果 (数値例 1)

Fig. 3.　Simulation result

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付　録

1. 定理 1 の証明

LMI−1 が満たされていると仮定する。

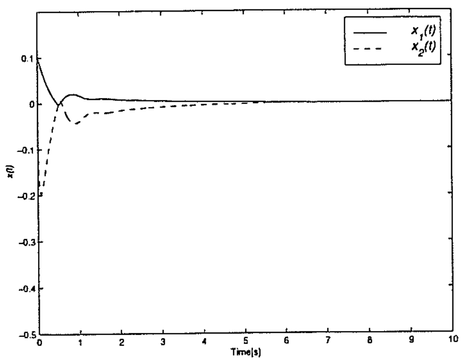
・内部安定性

次の関数を考える

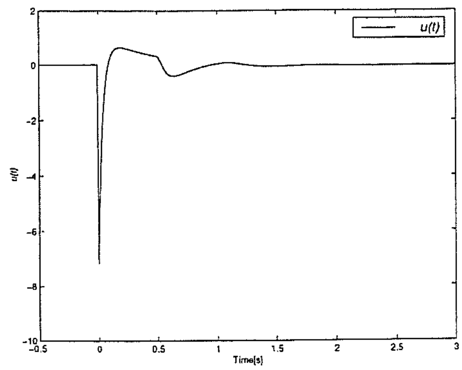
$$
\begin{aligned}
V(x(t),x_t) &= x(t)'Px(t) \\
&+ \int_{-h}^{0} x(t+\beta)'Qx(t+\beta)d\beta \\
&+ x(t)'\int_{-h}^{0} R(\beta)x(t+\beta)d\beta \\
&+ \int_{-h}^{0} x(t+\alpha)'R(\alpha)'d\alpha x(t) \\
&+ \int_{-h}^{0}\int_{-h}^{0} x(t+\alpha)'S(\alpha,\beta)x(t+\beta)d\alpha d\beta
\end{aligned}
\qquad \text{(付 1)}
$$

ここで，

$$x_t = \{x(t+\beta) \mid -h \le \beta \le 0\},$$
$$P, Q \in R^{n\times n},$$
$$R(\beta) \in C^1([-h,0];R^{n\times n}),$$



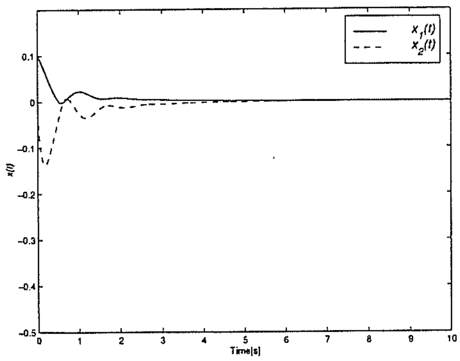
(a)　状態 $x(t)$



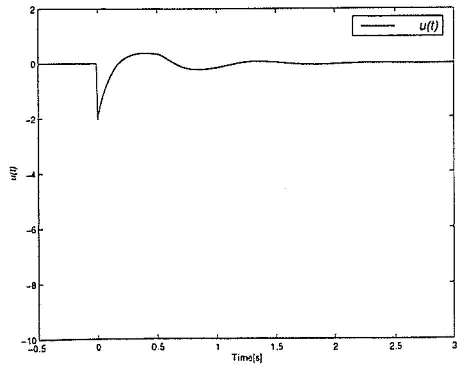
(b)　入力 $u(t)$

図 **4**　定理 4 を用いた結果結果 (数値例 2)

Fig. 4.　Simulation result (Theorem 4)



(a)　状態 $x(t)$



(b)　入力 $u(t)$

図 **5**　定理 5 を用いた結果 (数値例 2)

Fig. 5.　Simulation result (Theorem 5)

$$S(\alpha,\beta) \in C^1([-h,0]\times[-h,0];R^{n\times n})$$

である。

(1)のリアプノフ関数として(付1)を考え，$V(x(t),x_t)>0$，$\frac{d}{dt}V(x(t),x_t)<0$ $((x(t),x_t)\neq 0)$ を示すことにより，内部安定性を証明する。いま，(5)，(6)と，次式より，

$$V(x(t),x_t)=$$
$$\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix} h^{-1}x(t)\\ x(t+\alpha)\end{bmatrix}' L_1(\alpha,\beta)\begin{bmatrix} h^{-1}x(t)\\ x(t+\alpha)\end{bmatrix} d\alpha d\beta$$
$$+\int_{-h}^{0} x(t+\beta)'Qx(t+\beta)d\beta.$$

$V(x(t),x_t)>0$がいえる。

システム(1)の軌跡に沿って(付1)の両辺をtで微分すると，

$$\frac{d}{dt}V(x(t),x_t)=\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix} h^{-1}x(t)\\ h^{-1}x(t-h)\\ x(t+\alpha)\end{bmatrix}' L_0(\alpha,\beta)\begin{bmatrix} h^{-1}x(t)\\ h^{-1}x(t-h)\\ x(t+\alpha)\end{bmatrix} d\alpha d\beta$$

となる。ここで，L_0は次で表される。

$$L_0(\alpha,\beta)=$$
$$\left[\begin{array}{ccc} \begin{pmatrix} A_0{}'P+PA_0\\ +Q\\ +R(0)'+R(0)\end{pmatrix} & PA_1-R(-h) & \begin{pmatrix} PA_{01}(\beta)+A_0{}'R(\beta)\\ -\frac{\partial}{\partial\beta}R(\beta)+S(0,\beta)\end{pmatrix}\\ A_1'P-R(-h)' & -Q & A_1{}'R(\beta)-S(-h,\beta)\\ \begin{pmatrix} A_{01}(\alpha)'P\\ +R(\alpha)'A_0\\ -\frac{\partial}{\partial\alpha}R(\alpha)'\\ +S(\alpha,0)\end{pmatrix} & \begin{pmatrix} R(\alpha)'A_1\\ -S(\alpha,-h)\end{pmatrix} & \begin{pmatrix} R(\alpha)'A_{01}(\beta)\\ +A_{01}(\alpha)'R(\beta)\\ -(\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\beta})S(\alpha,\beta)\end{pmatrix}\end{array}\right].$$

ここでSchur Complementを用いることで(6)，すなわち$L_2(\alpha,\beta)<0$が次の条件と等価であることがわかる。

$$L_0(\alpha,\beta)+\begin{bmatrix} C' & PD\\ 0 & 0\\ 0 & R(\alpha)'D+R(\beta)'D\end{bmatrix}\begin{bmatrix} I & 0\\ 0 & \gamma^2 I\end{bmatrix}^{-1}\begin{bmatrix} C' & PD\\ 0 & 0\\ 0 & R(\alpha)'D+R(\beta)'D\end{bmatrix}'<0.$$

以上より，システム(1)は内部安定であることがわかる。

・L^2ゲインがγ以下

最初に，$w(t)=0$としたシステム(1)が内部安定でL^2入出力安定であるとき，$w\in L^2([0\ \infty];R^p)$に対して$x(\infty)=0$であることに注意する。このことと，$x(\beta)=0, -h\le\beta\le 0$より，すべての$w\in L^2([0,\ \infty];R^p)$に対して，

$$\|z\|_{L^2}^2-\gamma^2\|w\|_{L^2}^2 =\int_0^{\infty}[z(t)'z(t)-\gamma^2 w(t)'w(t)+\frac{d}{dt}V(x(t),x_t)]dt$$

であるということがわかる。ここで$\frac{d}{dt}V(x(t),x_t)$を求め，上式に代入することで，次式が得られる。

$$\|z\|_{L^2}^2-\gamma^2\|w\|_{L^2}^2 =\int_0^{\infty}(\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix} h^{-1}x(t)\\ h^{-1}x(t-h)\\ x(t+\alpha)\\ h^{-1}w(t)\end{bmatrix}' L_2(\alpha,\beta)\begin{bmatrix} h^{-1}x(t)\\ h^{-1}x(t-h)\\ x(t+\alpha)\\ h^{-1}w(t)\end{bmatrix} d\alpha d\beta)dt.$$

このとき，(6)より，$G\le\gamma$となる。 □

2. 定理3の証明

定理3においてはα,βの区間としては$\alpha\in[h_1,h_2],\beta\in[h_1,h_2]$としているが，ここでは，記述を簡単にするため$\alpha\in[0,1],\beta\in[0,1]$とした場合を考えるが，一般性が失われることはない。

$e_1,e_2,\cdots,e_{2s}$はR^{2s}の自然基底とする，すなわち，

$$\begin{aligned} e_1 &= [1\,0\,0\,\cdots 0\ \ 0\,0\,0\cdots 0],\\ e_2 &= [0\,1\,0\,\cdots 0\ \ 0\,0\,0\cdots 0],\\ &\vdots\\ e_s &= [0\,0\,0\,\cdots 1\ \ 0\,0\,0\cdots 0],\\ e_{s+1} &= [0\,0\,0\,\cdots 0\ \ 1\,0\,0\cdots 0],\\ e_{s+2} &= [0\,0\,0\,\cdots 0\ \ 0\,1\,0\cdots 0],\\ &\vdots\\ e_{2s} &= [0\,0\,0\,\cdots 0\ \ 0\,0\,0\cdots 1], \end{aligned}$$

である。なお，ここで，

$$\begin{aligned} e_{s+1} &= p^{01}-p^{00}\\ &= p^{11}-p^{10}=p^{21}-p^{20}=\cdots,\\ e_{s+2} &= p^{02}-p^{01}\\ &= p^{12}-p^{11}=p^{22}-p^{21}=\cdots,\quad \cdots\cdots \quad (付2)\\ &\vdots\\ e_{2s} &= p^{0s}-p^{0(s-1)}\\ &= p^{1s}-p^{1(s-1)}=p^{2s}-p^{2(s-1)}=\cdots \end{aligned}$$

であることに注意する。

いま，βを$0\le\beta\le 1$の任意の値に固定した時，p^{i0}とp^{is}間の点$p^{i\prime}(i=0,1,\cdots,s)$は，

$$p^{i'} = p^{i0} + \beta e_{s+1} + \beta^2 e_{s+2} + \cdots + \beta^s e_{2s} \quad \cdot \text{(付 3-a)}$$
$$= p^{i0} + \beta(p^{i1} - p^{i0}) + \beta^2(p^{i2} - p^{i1}) + \cdots$$
$$+\beta^s(p^{is} - p^{i(s-1)}) \quad \cdots\cdots\cdots\cdots\cdots \text{(付 3-b)}$$

と表される。すると，T上の点Pはつぎのように書ける。

$$\begin{aligned} P &= \alpha e_1 + \alpha^2 e_2 + \cdots + \alpha^s e_s \\ &\quad +\beta e_{s+1} + \beta^2 e_{s+2} + \cdots + \beta^s e_{2s} \\ &= p^{0'} - p^{0'} + \alpha e_1 + \alpha^2 e_2 + \cdots + \alpha^s e_s \\ &\quad +\beta e_{s+1} + \beta^2 e_{s+2} + \cdots + \beta^s e_{2s} \end{aligned}$$

(付 2) より，

$$\begin{aligned} P &= p^{0'} - p^{00} - \beta e_{s+1} - \beta^2 e_{s+2} - \cdots - \beta^s e_{2s} \\ &\quad +\alpha e_1 + \alpha^2 e_2 + \cdots + \alpha^s e_s \\ &\quad +\beta e_{s+1} + \beta^2 e_{s+2} + \cdots + \beta^s e_{2s} \\ &= p^{0'} + \alpha e_1 + \alpha^2 e_2 + \cdots + \alpha^s e_s \end{aligned}$$

(付 2)，(付 3-a)，(付 3-b) より，

$$\begin{aligned} P &= p^{0'} + \alpha(p^{1'} - p^{0'}) + \alpha^2(p^{2'} + p^{1'}) + \cdots \\ &\quad +\alpha^s(p^{s'} + p^{(s-1)'}) \\ &= (1-\alpha)p^{0'} + (\alpha - \alpha^2)p^{1'} + \cdots \\ &\quad +(\alpha^{(s-1)} - \alpha^s)p^{(s-1)'} + \alpha^s p^{s'} \end{aligned}$$

さらに (付 3-a)，(付 3-b) より，

$$\begin{aligned} P &= (1-\alpha)p^{00} + (1-\alpha)\beta(p^{01} - p^{00}) + \cdots \\ &\quad +(1-\alpha)\beta^s(p^{0s} - p^{0(s-1)}) + (\alpha - \alpha^2)P^{10} \\ &\quad +(\alpha - \alpha^2)\beta(p^{11} - p^{10}) + \cdots \\ &\quad +(\alpha - \alpha^2)\beta^s(p^{1s} - p^{1(s-1)}) \\ &\quad \vdots \\ &\quad +(\alpha^{s-1} - \alpha^s)P^{(s-1)0} \\ &\quad +(\alpha^{(s-1)} - \alpha^s)\beta(p^{(s-1)1} - p^{(s-1)0}) \\ &\quad +\cdots + (\alpha^{s-1} - \alpha^s)\beta^s(p^{(s-1)s} - p^{(s-1)(s-1)}) \\ &\quad +\alpha^s p^{s'} \\ &= (1-\alpha)(1-\beta)p^{00} + (1-\alpha)(\beta - \beta^2)p^{01} + \cdots \\ &\quad +(1-\alpha)(\beta^{s-1} - \beta^s)p^{0(s-1)} + (1-\alpha)\beta^s p^{0s} \\ &\quad +(\alpha - \alpha^2)(1-\beta)p^{10} \\ &\quad +(\alpha - \alpha^2)(\beta - \beta^2)p^{11} + \cdots \\ &\quad +(\alpha - \alpha^2)(\beta^{s-1} - \beta^s)p^{1(s-1)} + (\alpha - \alpha^2)\beta^s p1s \\ &\quad \vdots \\ &\quad +(\alpha^{s-1} - \alpha^s)(1-\beta)p^{(s-1)0} \\ &\quad +(\alpha^{s-1} - \alpha^s)(\beta - \beta^2)p^{(s-1)1} + \cdots \\ &\quad +(\alpha^{s-1} - \alpha^s)(\beta^{s-1} - \beta^s)p^{(s-1)(s-1)} \\ &\quad +(\alpha^{s-1} - \alpha^s)\beta^s p^{(s-1)s} \\ &\quad +\alpha^s p^{s'} \end{aligned}$$

このとき，$\alpha \in [0,1], \beta \in [0,1]$ より，すべての p^{ij}の係数は 0 以上である。さらに各係数の和は 1 である。これらより，p_{ij}，$(0 \le i \le s,\quad 0 \le j \le s)$ を頂点とする凸多面体 Hは Tを内部に含んでいる。 □

3. 定理 5 の証明

内部安定性と L^2ゲインについては定理 4 より明らかである。そこでゲイン (15) が (16) を満たすことを示す。(17),(18),(19) はそれぞれ以下と等価である。

$$Z_0' Z_0 < p_1 I$$
$$Z_{01}(\beta)' Z_{01}(\beta) < p_2 I$$
$$W^{-1} < qI$$

したがって，

$$\begin{aligned} K_0' K_0 &= W^{-1} Z_0' Z_0 W^{-1} \\ &< p_1 W^{-1} W^{-1} \\ &< p_1 q^2 I \\ K_{01}(\beta)' K_{01}(\beta) &= W^{-1} Z_{01}(\beta)' Z_{01}(\beta) W^{-1} \\ &< p_2 W^{-1} W^{-1} \\ &< p_2 q^2 I \end{aligned}$$

となり，ゲインが制限される。 □

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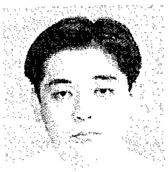

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Memory state feedback control synthesis for linear systems with time delay via a finite number of linear matrix inequalities

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Abstract

In this paper, we consider a synthesis problem of delay-dependent memory state feedback control which stabilizes linear time-delay systems. First we derive conditions for stability analysis and controller synthesis in the form of infinite-dimensional (parameter-dependent) linear matrix inequalities (LMIs), while infinite dimensionality of the LMIs may lead to less conservative results, but makes the conditions difficult to use. Second we show a technique to reduce the infinite-dimensional LMIs to a finite number of LMIs. A numerical example is given to demonstrate out approach. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Time-delay systems; State feedback control; Linear matrix inequalities

1. Introduction

In state feedback control synthesis problems for linear time-delay systems, two typical classes of state feedback controllers would be delay-independent memoryless state feedback controllers

$$u(t) = Kx(t)$$

and delay-dependent memory state feedback controllers

$$u(t) = K_0x(t) + \int_{-h}^{0} K_{01}(\beta)x(t+\beta)\,\mathrm{d}\beta.$$

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It is well known that the linear-quadratic optimal control has a memory state feedback form. From the view point of the infinite-dimensional state space of time-delay systems, memory controllers are natural state feedback controllers, and we can expect that memory controllers achieve better performances than memoryless controllers. An inevitable issue of memory controller syntheses is difficulty arisen from their infinite dimensionality in computations and implementations. In this paper, we discuss a synthesis problem of memory controllers which stabilize a linear time-delay system, and propose a synthesis procedure based on a finite number of linear matrix inequalities (LMIs).

Syntheses of memory controllers for linear time-delay systems have been discussed from the various viewpoints (see, e.g. Refs. [9,14,15]). We discuss here a synthesis problem from the viewpoint of stabilization. Recently the LMI approach [3,5] has been developed in analysis and synthesis problems for linear time-delay systems and its advantages in numerical computations are presented in Refs. [4,13,16]; however, the approach has mostly been limited to the case of utilizing finite-dimensional LMIs and synthesizing memoryless controllers, and one exception is a discretization technique by Gu [6,7], for stability analysis, which can characterize infinite-dimensional integral kernels of a general Lyapunov functional with a finite number of LMIs, but is difficult to apply to memory controller syntheses. For more recent progresses of stability analysis and synthesis for linear time-delay systems with and without LMIs, we can see Refs. [11,12].

In this paper, we propose a new LMI approach to a memory state feedback control synthesis for linear systems with delay in the state. First we consider stability analysis and state feedback controller synthesis and derive conditions in the form of infinite-dimensional (parameter-dependent) LMIs. Second we reduce the infinite-dimensional LMIs to a finite number of LMIs by applying the technique proposed in results [1,2]. Finally we demonstrate its efficacy by a numerical case study.

2. System description and notation

Consider the linear time-delay system,

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-h) + Bu(t), \\ x(\beta) &= \phi(\beta), \quad -h \leqslant \beta \leqslant 0, \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the input, $\phi(\beta) \in R^n$ is a continuous initial function, and $A_0 \in R^{n\times n}$, $A_1 \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant matrices. The parameter h denotes the time delay of this system and $h > 0$. For the linear time-delay system (1), we consider the following state feedback controller,

$$u(t) = K_0x(t) + \int_{-h}^{0} K_{01}(\beta)x(t+\beta)\,\mathrm{d}\beta, \tag{2}$$

where $K_0 \in R^{m\times n}$ is a constant matrix and $K_{01}(\beta) \in L^2([-h,0];R^{m\times n})$ is a matrix function.

The purpose of this paper is to design a delay-dependent state feedback controller (2) which stabilizes the time-delay system (1).

In this paper, we use the following notation

$$\begin{bmatrix} P_0 & P_1(\beta) \\ P_1'(\alpha) & P_2(\alpha,\beta) \end{bmatrix} > 0, \quad \forall\alpha \in [-h,0], \ \forall\beta \in [-h,0], \tag{3}$$

in the sense that

$$\begin{bmatrix} P_0 & \frac{1}{2}(P_1(\alpha)+P_1(\beta)) \\ \frac{1}{2}(P_1'(\alpha)+P_1'(\beta)) & \frac{1}{2}(P_2(\alpha,\beta)+P_2(\beta,\alpha)) \end{bmatrix} > 0, \tag{4}$$

$$P_0' = P_0, \tag{5}$$

$$P_2'(\alpha,\beta) = P_2(\beta,\alpha), \quad \forall\alpha\in[-h,0], \ \forall\beta\in[-h,0], \tag{6}$$

where ">" denotes positive definiteness of matrix and "′" denotes transposition of vector and matrix.

3. State feedback control synthesis

3.1. Stability analysis

We consider a linear time-delay system as follows,

$$\dot{x}(t) = \tilde{A}_0 x(t) + \tilde{A}_1 x(t-h) + \int_{-h}^{0} \tilde{A}_{01}(\beta)x(t+\beta)\,\mathrm{d}\beta, \tag{7}$$

$$x(\beta) = \phi(\beta), \quad -h \leqslant \beta \leqslant 0,$$

and derive a asymptotical stability condition of the above systems (7).

Now we define a functional V as follows,

$$V(x_t) = x'(t)Px(t) + \int_{-h}^{0} x'(t+\beta)Qx(t+\beta)\,\mathrm{d}\beta + x'(t)\int_{-h}^{0} R(\beta)x(t+\beta)\,\mathrm{d}\beta + \int_{-h}^{0} x'(t+\alpha)R'(\alpha)\,\mathrm{d}\alpha\, x(t) + \int_{-h}^{0}\int_{-h}^{0} x'(t+\alpha)S(\alpha,\beta)x(t+\beta)\,\mathrm{d}\alpha\,\mathrm{d}\beta, \tag{8}$$

where

$$x_t = \{x(t+\beta) \mid -h \leqslant \beta \leqslant 0\}, \quad P, Q \in R^{n\times n}, \quad R(\beta) \in L^2([-h,0]; R^{n\times n}),$$
$$S(\alpha,\beta) \in L^2([-h,0]\times[-h,0]; R^{n\times n}).$$

The following theorem provides a sufficient condition for the linear time-delay system (7) to be asymptotically stable.

Theorem 3.1 [8]. *The linear time-delay system* (7) *is asymptotically* (*exponentially*) *stable if there exists a Lyapunov functional* (8) *such that*

$$V(x_t) \geqslant \epsilon_1 |x(t)|^2 \quad \text{for } \epsilon_1 > 0 \tag{9}$$

and its derivative along the solution of (7) satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t} V(x_t) \leqslant -\epsilon_2 |x(t)|^2 \quad \text{for } \epsilon_2 > 0. \tag{10}$$

By using the functional (8), we have the next theorem for the asymptotical stability analysis.

Theorem 3.2. *If there exist P, Q and continuously differentiable matrix functions $R(\beta)$, $S(\alpha,\beta)$ which satisfy the following conditions, LMI-1:*

$$L_1(\alpha,\beta)=\begin{bmatrix} \begin{pmatrix} \tilde{A}_0'P+P\tilde{A}_0 \\ +Q+R'(0)+R(0) \end{pmatrix} & P\tilde{A}_1-R(-h) & \begin{pmatrix} P\tilde{A}_{01}(\beta)+\tilde{A}_0'R(\beta) \\ -\frac{\partial}{\partial\beta}R(\beta)+S(0,\beta) \end{pmatrix} \\ \tilde{A}_1'P-R'(-h) & -Q & \tilde{A}_1'R(\beta)-S(-h,\beta) \\ \begin{pmatrix} \tilde{A}_{01}'(\alpha)P+R'(\alpha)\tilde{A}_0 \\ -\frac{\partial}{\partial\alpha}R'(\alpha)+S(\alpha,0) \end{pmatrix} & R(\alpha)'\tilde{A}_1-S(\alpha,-h) & \begin{pmatrix} R'(\alpha)\tilde{A}_{01}(\beta)+\tilde{A}_{01}'(\alpha)R(\beta) \\ -(\frac{\partial}{\partial\beta}+\frac{\partial}{\partial\alpha})S(\alpha,\beta) \end{pmatrix} \end{bmatrix}$$

$$<0, \qquad (11)$$

$$L_2(\alpha,\beta)=\begin{bmatrix} P & R(\beta) \\ R'(\alpha) & S(\alpha,\beta) \end{bmatrix}>0, \qquad (12)$$

$$Q>0, \quad \forall\alpha\in[-h,0], \quad \forall\beta\in[-h,0], \qquad (13)$$

then the linear time-delay system (7) is asymptotically stable.

Proof. To show that the linear time-delay system (7) is asymptotically stable, we prove that the functional (8) is a Lyapunov functional, that is, the conditions (9) and (10) of Theorem 3.1 are satisfied.

From Eq. (12), there exist continuous functions $\lambda_1(\alpha,\beta)$ and $\lambda_2(\alpha,\beta)$ which satisfy

$$\lambda_1(\alpha,\beta)I \geqslant \begin{bmatrix} P & \frac{1}{2}(R(\alpha)+R(\beta)) \\ \frac{1}{2}(R'(\alpha)+R'(\beta)) & \frac{1}{2}(S(\alpha,\beta)+S(\beta,\alpha)) \end{bmatrix} \geqslant \lambda_2(\alpha,\beta)I, \quad \lambda_1(\alpha,\beta)\geqslant\lambda_2(\alpha,\beta)>0,$$
$$\forall\alpha\in[-h,0], \quad \forall\beta\in[-h,0],$$

where

$$\lambda_1,\lambda_2:[-h,0]\times[-h,0]\rightarrow R$$

and I denotes a unit matrix. Thus the next inequality is satisfied.

$$\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x'(t) & x'(t+\alpha)\end{bmatrix}\begin{bmatrix} P & \frac{1}{2}(R(\alpha)+R(\beta)) \\ \frac{1}{2}(R'(\alpha)+R'(\beta)) & \frac{1}{2}(S(\alpha,\beta)+S(\beta,\alpha)) \end{bmatrix}\begin{bmatrix}\frac{1}{h}x(t) \\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta$$
$$=\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x'(t) & x'(t+\alpha)\end{bmatrix}\left\{\frac{L_2(\alpha,\beta)+L_2'(\alpha,\beta)}{2}\right\}\begin{bmatrix}\frac{1}{h}x(t) \\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta>0. \qquad (14)$$

Since the functional (8) can be written as follows,

$$V(x_t) = \int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x'(t) & x'(t+\alpha)\end{bmatrix}L_2(\alpha,\beta)\begin{bmatrix}\frac{1}{h}x(t)\\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta + \int_{-h}^{0}x'(t+\beta)Qx(t+\beta)\mathrm{d}\beta$$
$$= \int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x'(t) & x'(t+\alpha)\end{bmatrix}\left\{\frac{L_2(\alpha,\beta)+L_2'(\alpha,\beta)}{2}\right\}\begin{bmatrix}\frac{1}{h}x(t)\\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta$$
$$+ \int_{-h}^{0}x'(t+\beta)Qx(t+\beta)\mathrm{d}\beta,$$

the inequalities (14) and (13) show that the functional $V(x_t)$ satisfies the condition (9).

From Eq. (11), there exist continuous functions $\lambda_3(\alpha,\beta)$ and $\lambda_4(\alpha,\beta)$ which satisfy

$$\lambda_3(\alpha,\beta)I \leqslant \left\{\frac{L_1(\alpha,\beta)+L_1'(\alpha,\beta)}{2}\right\} \leqslant \lambda_4(\alpha,\beta)I, \quad \lambda_3(\alpha,\beta)\leqslant\lambda_4(\alpha,\beta)<0, \quad \forall\alpha\in[-h,0],$$
$$\forall\beta\in[-h,0],$$

where

$$\lambda_3,\lambda_4 : [-h,0]\times[-h,0]\to R.$$

Thus the next inequality is satisfied.

$$\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x(t)\\ \frac{1}{h}x(t-h)\\ x(t+\alpha)\end{bmatrix}'\left\{\frac{L_1(\alpha,\beta)+L_1'(\alpha,\beta)}{2}\right\}\begin{bmatrix}\frac{1}{h}x(t)\\ \frac{1}{h}x(t-h)\\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta<0. \tag{15}$$

The time derivative of Eq. (8) along the trajectory of Eq. (7) is given as follows,

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x_t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)'Px(t) + x(t)'P\frac{\mathrm{d}}{\mathrm{d}t}x(t) + \int_{-h}^{0}\frac{\partial}{\partial t}x(t+\beta)'Qx(t+\beta)\,\mathrm{d}\beta$$
$$+\int_{-h}^{0}x(t+\beta)'Q\frac{\partial}{\partial t}x(t+\beta)\,\mathrm{d}\beta + \frac{\mathrm{d}}{\mathrm{d}t}x(t)'\int_{-h}^{0}R(\beta)x(t+\beta)\,\mathrm{d}\beta + x(t)'$$
$$\times\int_{-h}^{0}R(\beta)\frac{\partial}{\partial t}x(t+\beta)\,\mathrm{d}\beta + \int_{-h}^{0}x(t+\alpha)'R(\alpha)'\,\mathrm{d}\alpha\cdot\frac{\mathrm{d}}{\mathrm{d}t}x(t) + \int_{-h}^{0}\frac{\partial}{\partial t}x(t+\alpha)'R(\alpha)'\,\mathrm{d}\alpha\cdot x(t)$$
$$+\int_{-h}^{0}\int_{-h}^{0}\frac{\partial}{\partial t}x(t+\alpha)'S(\alpha,\beta)x(t+\beta)\,\mathrm{d}\alpha\,\mathrm{d}\beta + \int_{-h}^{0}\int_{-h}^{0}x(t+\alpha)'S(\alpha,\beta)\frac{\partial}{\partial t}x(t+\beta)\,\mathrm{d}\alpha\,\mathrm{d}\beta$$
$$= \int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}x(t)\\ \frac{1}{h}x(t-h)\\ x(t+\alpha)\end{bmatrix}'L_1(\alpha,\beta)\begin{bmatrix}\frac{1}{h}x(t)\\ \frac{1}{h}x(t-h)\\ x(t+\beta)\end{bmatrix}\mathrm{d}\alpha\,\mathrm{d}\beta, \tag{16}$$

where we used the following identities,

$$\frac{\partial}{\partial t}x(t+\beta)=\frac{\partial}{\partial\beta}x(t+\beta), \quad \frac{\partial}{\partial t}x(t+\alpha)=\frac{\partial}{\partial\alpha}x(t+\alpha).$$

From the inequality (15) and the presentation (16), we can show that the time derivative of Eq. (8) along the solution of Eq. (7), i.e. $(\mathrm{d}/\mathrm{d}t)V(x_t)$ satisfies the condition (10). □

Remark 3.3. It is known (see, e.g. Ref. [11]) that the existence of Lyapunov functionals of the form (8) with $Q = 0$ is necessary and sufficient for linear time-delay systems (7) to be stable. In this sense, the LMI condition (LMI-1) given in Theorem 3.1 is a rather general one including some known conditions as special cases; for example, if we set $R(\beta) \equiv 0$ and $S(\alpha,\beta) \equiv 0$ in LMI-1, LMI-1 is reduced to the well-known LMI condition for stability (see, e.g. Refs. [4,12]).

3.2. Controller synthesis

From Eqs. (1) and (2), the closed loop system is given as follows,

$$\dot{x}(t) = \tilde{A}_0 x(t) + \tilde{A}_1 x(t-h) + \int_{-h}^{0} \tilde{A}_{01}(\beta) x(t+\beta)\,\mathrm{d}\beta, \tag{17}$$

$$\begin{aligned} \tilde{A}_0 &= A_0 + BK_0, \\ \tilde{A}_1 &= A_1, \\ \tilde{A}_{01} &= BK_{01}(\beta). \end{aligned}$$

Applying Theorem 3.2 (for stability analysis) to this closed loop system with an unknown feedback gain $(K_0, K_{01}(\beta))$, we have the next theorem for state feedback controller synthesis.

Theorem 3.4. *If there exist W, X, Z_0 and continuously differentiable matrix function $Z_{01}(\beta)$ and $Y(\alpha,\beta)$ which satisfy the following conditions, LMI-2:*

$$L_3(\alpha,\beta) = \begin{bmatrix} \begin{pmatrix} WA_0' + A_0 W + X \\ +2W + BZ_0 + Z_0'B' \end{pmatrix} & A_1 W - W & \begin{pmatrix} BZ_{01}(\beta) + WA_0' \\ +Z_0'B' + Y(0,\beta) \end{pmatrix} \\ WA_1' - W & -X & WA_1 - Y(-h,\beta) \\ \begin{pmatrix} Z_{01}'(\alpha)B' + A_0 W \\ +BZ_0 + Y(\alpha,0) \end{pmatrix} & A_1' W - Y(\alpha,-h) & \begin{pmatrix} BZ_{01}(\beta) + Z_{01}'(\alpha)B' \\ -(\frac{\partial}{\partial\alpha} + \frac{\partial}{\partial\beta}) Y(\alpha,\beta) \end{pmatrix} \end{bmatrix} < 0, \tag{18}$$

$$L_4(\alpha,\beta) = \begin{bmatrix} W & W \\ W & Y(\alpha,\beta) \end{bmatrix} > 0, \tag{19}$$

$$X > 0, \quad \forall\alpha \in [-h,0], \quad \forall\beta \in [-h,0], \tag{20}$$

where $W, X \in R^{n\times n}$, $Y(\alpha,\beta) \in R^{n\times n}$, $Z_0 \in R^{m\times n}$ and $Z_{01}(\beta) \in R^{m\times n}$, then the time-delay system (1) with the state feedback controller (2) given by,

$$K_0 = Z_0 W^{-1}, \tag{21}$$

$$K_{01}(\beta) = Z_{01}(\beta) W^{-1}, \tag{22}$$

is asymptotically stable.

Proof. By setting

$$P^{-1} = R^{-1}(\beta) = W, \tag{23}$$

$$P^{-1}QP^{-1} = X, \tag{24}$$

$$P^{-1}S(\alpha,\beta)P^{-1} = Y(\alpha,\beta), \tag{25}$$

$$K_0P^{-1} = Z_0, \tag{26}$$

$$K_{01}(\beta)P^{-1} = Z_{01}(\beta), \tag{27}$$

we can see that LMI-1 and LMI-2 are equivalent. Hence, from Theorem 3.2, the time-delay system (1) with the state feedback controller (2) given by Eqs. (21) and (22) is asymptotically stable. □

Remark 3.5. Note that, in the above synthesis, we restrict the solution $(P, Q, R(\beta), S(\alpha,\beta))$ of LMI-1 to the special form satisfying $P = R(\beta)$. The more general synthesis is possible, while the more complicated LMIs will be solved. To the contrary, if we take $R(\beta) \equiv 0$ and $S(\alpha,\beta) \equiv 0$ in LMI-1, LMI-2 is reduced to the well-known LMI condition for memoryless controller synthesis [4].

4. Reduction to a finite number of LMI conditions

Inequalities in LMI-2 depend on parameters α and β. It seems difficult to solve these infinite-dimensional (parameter-dependent) inequalities directly. In our approach we reduce these infinite-dimensional LMIs by using the technique in Refs. [1,2], and construct a solution to the infinite-dimensional LMIs by using a solution to the finite number of LMIs.

We restrict solutions in LMI-2 to the following forms,

$$Y(\alpha,\beta) = Y_0 + g_1(\alpha,\beta)Y_1 + g_2(\alpha,\beta)Y_2 + \cdots + g_{l_Y}(\alpha,\beta)Y_{l_Y}, \tag{28}$$

$$Z_{01}(\beta) = Z_0^{01} + h_1(\beta)Z_1^{01} + h_2(\beta)Z_2^{01} + \cdots + h_{l_Z}(\beta)Z_{l_Z}^{01}, \tag{29}$$

where $g_i : R^2 \to R$ is a continuous differentiable function of α and β such that

$$g_i(\alpha,\beta) = g_i(\beta,\alpha),$$

$h_i : R \to R$ is a continuous differentiable function of β, and the unknown matrices Y_i and Z_i^{01} are assumed as

$$Y_i \in R^{n\times n}, Y_i' = Y_i (i = 0, 1, \ldots, l_Y),$$
$$Z_i^{01} \in R^{m\times n} (i = 0, 1, \ldots, l_Z).$$

Note that Eq. (28) satisfies the condition (6). Then inequalities in LMI-2 can be written in the form of the following parameter-dependent LMI condition,

$$F_0(M) + f_1(\theta)F_1(M) + \cdots + f_r(\theta)F_r(M) < 0, \tag{30}$$

where

$$\theta \in \Theta = \{[\alpha\beta]' | \alpha \in [-h, 0], \beta \in [-h, 0]\} \tag{31}$$

and $f_i : R^2 \to R$ is a continuous function of α and β, and a symmetric matrix function F_i depends affinely on the unknown matrix M which consists of Y_i, $i = 0, 1, \ldots, l_Y$ and Z_i^{01}, $i = 0, 1, \ldots, l_Z$. The parameter-dependent LMI condition (30) can be reduced to a finite number of LMI conditions as follows.

Theorem 4.1 [1,2]. *Let $\{p_1, p_2, \ldots, p_q\}$ be vertices of a convex polyhedron which includes the curved surface T,*

$$T = \{[f_1(\theta)f_2(\theta)\cdots f_r(\theta)]'|\theta \in \Theta\}. \tag{32}$$

Assume that there exists M which satisfies the following LMI condition for all p_i $(i = 1, 2, \ldots, q)$,

$$F_0(M) + p_{i1}F_1(M) + \cdots + p_{ir}F_r(M) < 0, \tag{33}$$

where p_{ij} is the jth element of p_i. Then M satisfies Eq. (30) for all $\theta \in \Theta$.

The techniques to construct the convex polyhedron which includes the curved surface T are proposed in Refs. [1,2,10]. Thus we can reduce the infinite-dimensional LMIs to a finite number of LMIs without discretization of the infinite-dimensional LMIs.

Remark 4.2. It is a matter of course that the above procedure can be used to solve LMI-1. If the system parameter $\tilde{A}_{01}$ is given in the following form,

$$\tilde{A}_{01}(\beta) = \tilde{A}_0^{01} + a_1(\beta)\tilde{A}_1^{01} + a_2(\beta)\tilde{A}_2^{01} + \cdots + a_{l_A}(\beta)\tilde{A}_{l_A}^{01},$$

where $a_i : R \to R$ is a continuous function of β and

$$\tilde{A}_i^{01} \in R^{n\times n}(i = 0, 1, \ldots, l_A)$$

and restrict solutions in the following forms,

$$R(\beta) = R_0 + r_1(\beta)R_1 + r_2(\beta)R_2 + \cdots + r_{l_R}(\beta)R_{l_R},$$
$$S(\alpha, \beta) = S_0 + s_1(\alpha, \beta)S_1 + s_2(\alpha, \beta)S_2 + \cdots + s_{l_S}(\alpha, \beta)S_{l_S},$$

where $r_i : R \to R$ is a continuous differentiable function of β, $s_i : R^2 \to R$ is a continuous differentiable function of α and β such that

$$s_i(\alpha, \beta) = s_i(\beta, \alpha)$$

and the unknown matrices R_i and S_i are assumed as

$$R_i \in R^{n\times n}(i = 0, 1, \ldots, l_R),$$
$$S_i \in R^{n\times n}, S_i' = S_i, (i = 0, 1, \ldots, l_S)$$

then we obtain the condition again in the form of the parameter dependent LMI condition (30).

5. Illustrative example

Consider the following system with delay in the state,

$$\dot{x}(t) = A_0x(t) + A_1x(t - h) + Bu(t), \tag{34}$$

where

$$A_0 = \begin{bmatrix} 0.0 & 1.0 \\ 0.0 & 0.0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.3 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \quad h = 0.5$$

and the initial function $\phi(\beta)$ is given as follows

$$\phi(\beta) = \begin{bmatrix} 0.1 \\ 0.0 \end{bmatrix}, \quad -h \leqslant \beta \leqslant 0.$$

Note that the open loop system ($u(t) \equiv 0$) is unstable (see Fig. 1).

For the system (34), we construct the memory state feedback controller (2) by using the formula of Theorem 3.4 and the technique presented in Section 4. In this example, we restrict solutions $Y(\alpha,\beta)$, $Z_{01}(\beta)$ in Theorem 3.4 as follows,

$$Y(\alpha,\beta) = Y_0 + (\alpha+\beta)Y_1 + (\alpha^2+\beta^2)Y_2, \tag{35}$$

$$Z_{01}(\beta) = Z_0^{01} + \beta Z_1^{01} + \beta^2 Z_2^{01}. \tag{36}$$

Finally we obtain the following controller,

$$u(t) = K_0 x(t) + \int_{-h}^{0} K_{01}(\beta) x(t+\beta)\,\mathrm{d}\beta, \tag{37}$$

$$K_0 = [-3.74 \times 10^2 \;\; -5.31 \times 10^1],$$
$$K_{01}(\beta) = [-1.70 \times 10^2 \;\; -1.88 \times 10^1] + \beta[-1.29 \times 10^2 \;\; -2.00 \times 10^1] + \beta^2[2.79 \times 10^2 \;\; -3.45 \times 10^1].$$

The simulation result is shown in Figs. 2 and 3. We can see that the state feedback controller (37) stabilizes the time-delay system (34).

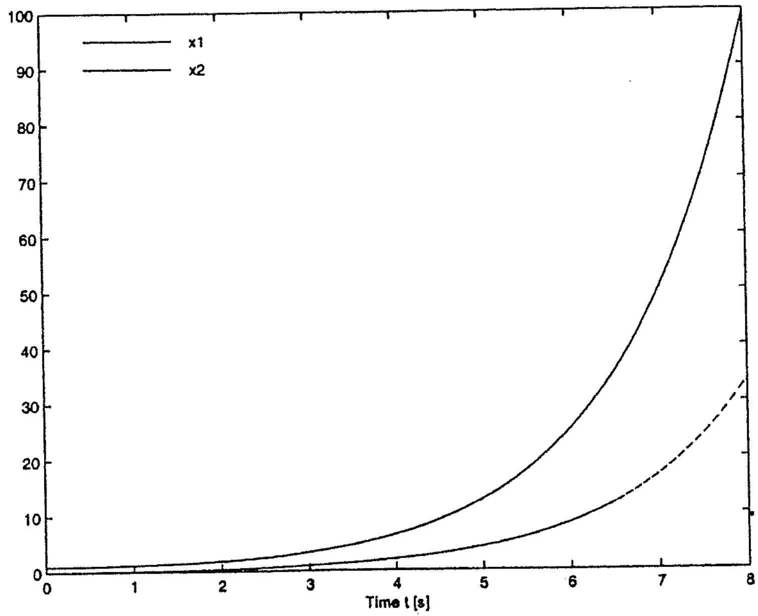

Fig. 1. The state $x_1(t)$ and $x_2(t)$ of the open loop system.

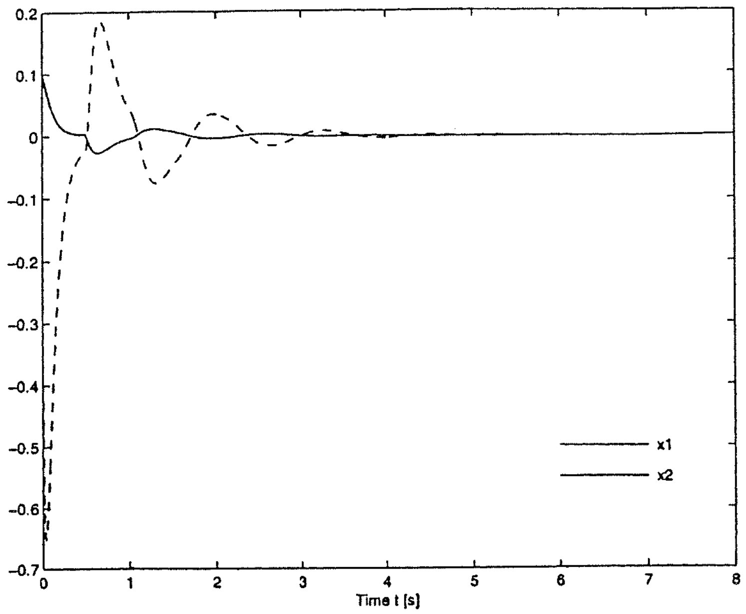

Fig. 2. The state $x_1(t)$ and $x_2(t)$ of the closed loop system.

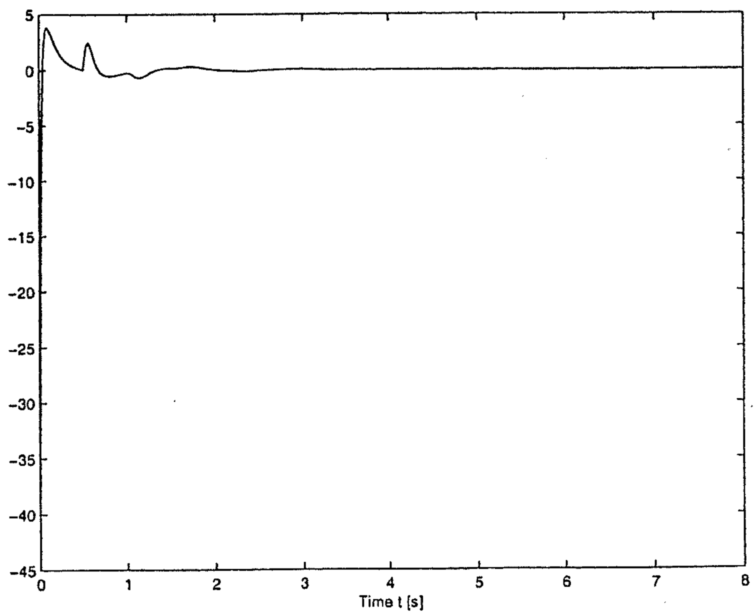

Fig. 3. The input $u(t)$.

6. Conclusion

In this paper, a memory state feedback controller synthesis problem for a system with delay in the state has been discussed and the controllers have been characterized by infinite-dimensional (parameter-dependent) LMI conditions. We have reduced the infinite-dimensional LMI conditions to a finite number of parameter independent LMI conditions without discretization of infinite-dimensional LMIs. A numerical example has been given to illustrate the result.

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H^∞ Control for Linear Systems with Multiple Time-delays and Finite Dimensional Characterizations

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Abstract

This article considers finite-dimensional characterizations of the solutions to H^∞ control for linear systems with multiple time-delays and show that if the controlled output is chosen such that it satisfies the "prediction condition" there exists an H^∞ control problem for finite-dimensional linear systems, which is equivalent to an H^∞ control problem for the linear systems with multiple time-delays.

1 Introduction

Recently control problems of linear systems with time-delays attract attentions together with progress of researches about networking, communication, networked control and so on[1][10][11]. The main difficulty of control problems for linear systems with time-delays is the infinite-dimensional characteristic, that makes controller synthesis problems hard. To overcome this difficulty, many techniques have been proposed.

One of these is the spectrum decomposition and prediction approach which guarantees finite-dimensional characterizations of the solutions to control problems. In [13, 14] and [8], LQ control problems and H^∞ control problems are discussed using such approach. On the other hand, the memoryless feedback control synthesis via linear matrix inequalities(LMI) may be one of such approaches [5][9][12]. We have recently proposed the infinite-dimensional LMI characterization of the solutions[3][4] and a finite-dimensional LMI algorithm[2]. Other finite-dimensional LMI algorithm is discretization technique of Lyapunov functional[6][7].

In this article, we discuss finite-dimensional characterizations of the solutions to H^∞ control for linear systems with multiple time-delays and show that if the controlled output is chosen such that it satisfies the "prediction condition" there exists an H^∞ control problem for finite-dimensional linear systems which is equivalent to an H^∞ control problem for the linear systems with multiple time-delays.

2 System Description and Problem Statement

In this paper, we deal with the following linear systems with multiple time-delays in the control input and the controlled output.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_1 w(t) + B_{20}u(t), \\ &\quad + \sum_{i=1}^{n_h} B_{2i}u(t-h_i), \\ z(t) &= C_{10}x(t) + D_1 u(t) \\ &\quad + \sum_{i=1}^{n_h}\int_{-h_i}^{0} C_{1i}(\beta)u(t+\beta)d\beta, \\ y(t) &= C_2 x(t) + D_2 w(t),\end{aligned} \tag{1}$$

where $x(t)$ is the internal variable, $w(t)$ is the disturbance, $z(t)$ is the controlled output and $y(t)$ is the measurement output. $h_i (i = 1, \cdots, n_h)$ is time-delay of this system and this system has n_h

time-delays. The initial conditions of this system are given as

$$x(0) = 0,$$
$$u(\beta) = 0, \ -h_{max} \le \beta \le 0,$$

where $h_{max} = max\{h_1, \cdots, h_{n_h}\}$. The parameter $C_{1i}(\beta)(i = 1, \cdots, n_h)$ is a matrix function whose elements are bounded continuous functions and another matrices, which have appropriate dimension, are constant.

H^∞ control problem discussed in this article is to find an admissible control $u(t)$ which guarantees that

1. the closed loop system is asymptotically stable,
2. the L^2 gain of the closed loop system is less than 1, where the L^2 gain is defined as follows.

$$G_{L2} = \sup_{w \in L^2, w \neq 0} \frac{\| z \|_{L^2}}{\| w \|_{L^2}},$$

where $\| f \|_{L^2}$ is defined as

$$\| f \|_{L^2} = \left(\int_0^\infty \|f(t)\|^2 dt \right)^{\frac{1}{2}}, \ f \in L^2.$$

3 Prediction condition and finite-dimensional characterization

First we introduce a prediction condition and show that the linear system with multiple time-delays (1) can be described as finite-dimensional linear system. For the controlled output, we consider the following prediction condition,

$$C_{1i}(\beta) = C_{10} e^{-A(\beta + h_i)} B_{2i}, \ -h_i \le \beta \le 0, \quad i = 1, 2, \cdots, n_h. \tag{2}$$

Using this prediction condition (2), we have the following result.

Lemma 3.1. Suppose thet the condition (2) is satisfied. Then the system (1) is equivalent to the following finite-dimensional system

$$\begin{aligned} \dot{p}(t) &= Ap(t) + B_1 w(t) + B_h u(t), \\ z(t) &= C_{10} p(t) + D_1 u(t), \\ q(t) &= C_2 p(t) + D_2 w(t), \end{aligned} \tag{3}$$

where $p(t)$ is the internal variable with $p(0) = 0$, $q(t)$ is the measurement output and

$$B_h = B_{20} + \sum_{i=1}^{n_h} e^{-Ah_i} B_{2i}.$$

Proof: Defining

$$x(t) = p(t) - \sum_{i=1}^{n_h} \int_{-h_i}^0 e^{-A(\beta + h_i)} B_{2i} u(t + \beta) d\beta, \tag{4}$$

$$y(t) = q(t) - C_2 \sum_{i=1}^{n_h} \int_{-h_i}^0 e^{-A(\beta + h_i)} B_{2i} u(t + \beta) d\beta, \tag{5}$$

and using (2), we can show from (3) that $x(t)$ and $y(t)$ defined by (4) and (5) obey the linear system with multiple time-delays (1).

Conversely defining

$$p(t) = x(t) + \sum_{i=1}^{n_h} \int_{-h_i}^0 e^{-A(\beta + h_i)} B_{2i} u(t + \beta) d\beta, \tag{6}$$

$$q(t) = y(t) + C_2 \sum_{i=1}^{n_h} \int_{-h_i}^0 e^{-A(\beta + h_i)} B_{2i} u(t + \beta) d\beta, \tag{7}$$

and using (2), e can show from (1) that $p(t)$ and $q(t)$ defined by (6) and (7) obey the finite-dimensional linear system (3). □

From lemma 3.1, the linear systems with multiple time-delays (1) can be converted to the finite-dimensional linear system (3). Based on this result, we have the next theorem for the output feedback H^∞ control problem defined in section 2.

Theorem 3.2. Suppose thet the condition (2) is satisfied. Then the output feedback H^∞ control problem for the linear systems with multiple time-delays described in section 2 is equivalent to the output feedback H^∞ control problem for the finite-dimensional linear system (3).

Proof: Let

$$u(t) = \Gamma(t, q(\cdot), u),$$

be a solution to the H^∞ control problem for the finite-dimensional linear system (3).

From lemma 1, defining

$$x(t) = p(t) - \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(t+\beta) d\beta,$$

$$y(t) = q(t) - C_2 \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(t+\beta) d\beta,$$

the finite-dimensional linear system (3) is equivalent to the linear systems with multiple time-delays (1). Thus

$$u(t) = \Gamma(t, y(\cdot) + C_2 \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(\cdot+\beta) d\beta, u),$$

is a solution to the H^∞ control problem described in section 2.

Conversely, let

$$u(t) = \Delta(t, y(\cdot), u),$$

be a solution to the H^∞ control problem for the linear systems with multiple time-delays (1). From lemma 3.1, defining

$$p(t) = x(t) + \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(t+\beta) d\beta,$$

$$q(t) = y(t) + C_2 \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(t+\beta) d\beta,$$

the linear systems with multiple time-delays (1) is equivalent to the finite-dimensional linear system (3). Thus

$$u(t) = \Delta(t, q(\cdot) - C_2 \sum_{i=1}^{n_h} \int_{-h_i}^{0} e^{-A(\beta+h_i)} B_{2i} u(\cdot+\beta) d\beta, u)$$

is a solution to the H^∞ control problem for the finite-dimensional linear system(3).

□

Controlled output are chosen correspondingly to purposes of control design. Finally we give some brief comments.

Remark 3.3. If it is required to control the predictive value of the internal variable, the prediction condition will be satisfied automatically. In the previous work[8], the authors proved the same equivalence as in Theorem 3 under the assumption that the condition

$$\begin{aligned} &C_{11}(\beta) = 0, \ -h \le \beta \le 0, \\ &C_{10} A^i B_{21} = 0, \ i = 0, 1, 2, \cdots \end{aligned} \tag{8}$$

and showed that the framework of H^∞ control problems for input delayed systems satisfying (8) includes for the robust stabilization problems against additive or multiplicative perturbations(including uncertain delay case). This result is single time-delay case and can be easily extended to multiple time-delay case as follows.

$$\begin{aligned} &C_{11}(\beta) = 0, \ -h_{max} \le \beta \le 0, \\ &C_{10} A^j B_{2i} = 0, \\ &\quad i = 1, 2, \cdots, n_h, \ j = 0, 1, 2, \cdots \end{aligned} \tag{9}$$

where

$$h_{max} = max\{h_1, \cdots, h_{n_h}\}$$

It is easy to verify that the condition (9) is a sufficient condition for the prediction condition (2) and therefore see that such robust stabilization problems discussed in [8] can be handled also in the framework of this article. To clarity more general meanings of the prediction condition from the view point of control design is still open interesting problem.

4 conclusion

In this article, we discussed finite-dimensional characterizations of the solutions to H^∞ control for linear systems with multiple time-delays. We showed that a linear systems with multiple time-delays is equivalent to a finite-dimensional linear system if the controlled output is chosen such that it satisfies the "prediction condition". And we showed that there exists an H^∞ control problem for finite-dimensional linear system which is equivalent to an H^∞ control problem for the linear system with multiple time-delays.

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On Input Delay and Measurement Delay in H^∞ Control Problems

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Abstract. Focusing on information delays in input and output, we discuss finite horizon H^∞ control problems for systems with delays in control input and in measurement output. Clarifying a relationship between two H^∞ control problems in input delay case and in measurement delay case, we derive a solution in input delay case based on the known result for the H^∞ control problem in measurement delay case, and show that the solution has the same predictor-observer structure as the solution in measurement delay case has.

1. Introduction

In control system designs, information delay appears rather often and is considered as a small but cumbersome obstacle. This paper is concerned itself with general state space solutions to H^∞ control problems for linear systems with delays in control input and in measurement output. For systems with delay, the H^∞ control problem was actively investigated in parallel with development of H^∞ control theory. On the other hand, the fact that the state space of systems with delay is infinite-dimensional leads generally to infinite-dimensional characterizations for analysis and synthesis in systems with delay. Actually, the standard H^∞ control problem for linear systems with delay was solved, as a special case of H^∞ control problems for distributed parameter (infinite-dimensional) systems, in the state space form based on two Riccati operator (infinite-dimensional) equations (see, e.g. van Keulen, 1994). Later, more explicit and feasible solutions, which are based on two algebraic Riccati equations and a transcendential equation (Kojima and Ishijima, 1994) or differential equation (Tadmor, 1995), were presented by focusing the cases with delays in control input. For the case with delays in measurement output, the explicit solutions were given in Basar and Bernhard (1991) and Nagpal and Ravy (1997). A particular (predictor-observer) structure of the solutions for the input delay case has been recently been pointed out by Mirkin (2000). We can find detailed reviews of this area in (Tadmor, 2000, Mirkin, 2000) and the references insides. In this paper, we revisit the H^∞ control problem for systems with input delays in the framework of finite horizon. The objective is to discuss further the predictor-observer structure of the solution, which is pointed out by Mirkin (2000), from a novel viewpoint.

Being suggested by the existing solution for the case with measurement delay, we also try to develop an elementary approach to the problem, which is completely different from the abstract approach based on evolution equations taken in (Kojima and Ishijima, 1994, Tadmor, 1995).

More specifically, the content of this paper is stated and organized as follows. In Section 2, we formulate the H^∞ control problem for systems with input delays together with two related H^∞ control problems. In Section 3, we first clarify a relationship between our problem and an H^∞ control problem for systems with measurement delays. Next, we derive a solution based on the known result for the H^∞ control problem in measurement delay case, and show that the solution has the same predictor-observer structure as the solution in measurement delay case has.

Notations: $L^2(a,b;R^k)$ is the space of square integrable functions of k-dimension defined on the time interval $[a,b]$. When $a=t_0$ and $b=t_1$, the L^2-norm of f in $L^2(a,b;R^k)$ is denoted as $\|f\|_2^2$. $\|x\|$ denotes the Euclidean norm of x in R^k. For symmetric matrices X and Y, $X \geq Y$ $(X > Y)$ implies that $X-Y$ is positive semidefinite (positive definite). I is the identity matrix of appropriate dimension. $(\,)'$ denotes the transpose of vector or matrix. $\rho(X)$ denotes the spectral radius of matrix X.

2. Finite Horizon H^∞ Control Problem

Consider the linear time-varying system with the time-delay $h>0$ in the control input, which is defined on the time interval $[t_0,t_1]$ and described by

$$\begin{aligned} \frac{d}{dt}x(t) &= A(t)x(t)+B(t)u(t-h)+D(t)v(t), \\ y(t) &= C(t)x(t)+w(t), \\ g(t) &= \begin{bmatrix} z(t) \\ u(t) \end{bmatrix}, \quad z(t)=F(t)x(t) \end{aligned} \tag{1}$$

where $x(t)$ the n-dimensional internal-variable; $u(t)$ is the r-dimensional control input; $y(t)$ is the m-dimensional measurement output; $g(t)$ is the $(q+r)$-dimensional controlled output; $d(t)=(v(t)',w(t)')'$ is the $(p+m)$-dimensional disturbance; the initial condition $(x(t_0),u_{t_0})$ in $R^n \times L^2(-h,0;R^r)$, where $u_t=\{u(t+\beta), -h\leq \beta \leq 0\}$, is given by a constant matrix $N>0$ and an n-dimensional parameter ξ as

$$x(t_0)=N\xi, \qquad u_{t_0}=0. \tag{2}$$

$A(t), B(t), C(t), D(t)$ and $F(t)$ are matrices of appropriate dimensions whose elements are continuous functions of time. For the system (1) with the initial condition (2), the admissible control $u(t)=\Phi_{ID}(t,y)$ is given by a causal function of the measurement data specifically to be the form

$$u(t)=\Phi_{ID}(t,\{y(s),t_0\le s\le t\}), \qquad t_0\le t\le t_1-h. \tag{3}$$

Problem ID *(H^∞ Control Problem with Input Delay):* Given the system described by (1) and (2) and a constant number $\gamma>0$, the problem is to find an admissible control (3) which satisfies the inequality

$$\|g\|_2^2<\gamma^2(\|d\|_2^2+\xi' N\xi) \qquad (u_{t_1}=0) \tag{ID}$$

for all $d=(v',w')'$ in $L^2(t_0,t_1;R^{p+m})$ and all ξ in R^n such that $(d,\xi)\ne 0$.

The H^∞ control problem for systems with input delays was solved by Kojima and Ishijima (1994), the parameterization of all the solutions was given by Tadmor (1995), and a particular (predictor-observer) structure of the solutions has been recently been pointed out by Mirkin (2000). Problem ID is an extension of the problem discussed in these literatures in the points that the system with a finite horizon is time varying and the criterion includes a terminal penalty, and could be solved by extending the arguments of (Kojima and Ishijima, 1994, Tadmor, 1995). In this paper, instead of pursuing this line, we will develop another approach and provide a new characterization of the solutions, which is inspired by the observation in (Mirkin, 2000).

We first consider an auxiliary problem to Problem ID. The system is defined on $[t_0,t_1]$ and described by

$$\begin{aligned}\frac{d}{dt}x(t)&=A(t)x(t)+B(t)u(t-h)+D(t)v(t),\\ y(t)&=C(t)x(t)+w(t),\\ g(t)&=\begin{bmatrix} z(t)\\ u(t-h)\end{bmatrix}, \qquad z(t)=F(t)x(t)\end{aligned} \tag{4}$$

with the initial condition

$$x(t_0)=N\xi, \tag{5}$$

and the admissible control $u(t)=\Phi_{AID}(t,y)$ is given by a causal function of the measurement data specifically to be the form

$$u(t)=\begin{cases}\Phi_{AID}(t,\{y(s),t_0\le s\le t\}), & t_0\le t\le t_1-h\\ \Phi_{AID}(t), & t_0-h\le t\le t_0.\end{cases} \tag{6}$$

***Problem AID** (Auxiliary H^∞ Control Problem with Input Delay):* Given the system described by (4) and (5) and a constant number $\gamma > 0$, the problem is to find an admissible control (6) which satisfies the inequality

$$\|g\|_2^2 < \gamma^2(\|d\|_2^2 + \xi' N\xi) \tag{AID}$$

for all $d = (v', w')'$ in $L^2(t_0, t_1; R^{p+m})$ and all ξ in R^n such that $(d, \xi) \neq 0$.

The difference between Problem ID and Problem AID is found only in the role of $u_{t_0} = \{u(t_0 + \beta), -h \leq \beta \leq 0\}$, that is, u_{t_0} is fixed (to be zero function) as a part of the initial condition in Problem ID, while u_{t_0} is a part of the control input to be determined in Problem AID. Although Problem AID itself is an H^∞ control problem applicable to some control designs, we will use Problem AID to bridge a gap between Problem ID and another H^∞ control problem introduced in the following.

We consider next an H^∞ problem which does not have input delays but has measurement delays. The system is defined on $[t_0, t_1]$ and described by

$$\begin{aligned} \frac{d}{dt}x(t) &= A(t)x(t) + B(t)u(t) + D(t)v(t), \\ y(t) &= C(t)x(t) + w(t), \\ g(t) &= \begin{bmatrix} z(t) \\ u(t) \end{bmatrix}, \qquad z(t) = F(t)x(t) \end{aligned} \tag{7}$$

with the initial condition

$$x(t_0) = N\xi, \tag{8}$$

and the admissible control $u(t) = \Phi_{MD}(t, y)$ is given by a causal function of the delayed measurement data specifically to be the form

$$u(t) = \begin{cases} \Phi_{MD}(t, \{y(s), t_0 \leq s \leq t - h\}), & t_0 + h \leq t \leq t_1 \\ \Phi_{MD}(t), & t_0 \leq t \leq t_0 + h. \end{cases} \tag{9}$$

***Problem MD** (H^∞ Control Problem with Measurement Delay):* Given the system described by (7) and (8) and a constant number $\gamma > 0$, the problem is to find an admissible control (9) which satisfies the inequality

$$\|g\|_2^2 < \gamma^2(\|d\|_2^2 + \xi' N\xi) \tag{MD}$$

for all $d=(v',w')'$ in $L^2(t_0,t_1;R^{p+m})$ and all ξ in R^n such that $(d,\xi)\neq 0$.

The H^∞ control problem for systems with measurement delays was also solved completely in (Basar and Bernhard, 1991, Nagpal and Ravy, 1997), and, as is expected from existence of information delays in constructing control inputs, the solution has a natural predictor-observer structure.

Our approach to Problem ID, which we will take in the following sections, is summarized as follows. We establish first some relationships between Problem ID and Problem MD via Problem AID, and try to solve Problem ID based on the relationships and the solution of Problem MD, so that the solution of Problem ID has the same predictor-observer structure as the solution of Problem MD has.

3. Result

To find relations among three Problems ID, AID and MD, we only observe the detail of the term of the controlled output in the left hand sides of the criteria (ID), (AID) and (MD). The following result is an immediate conclusion from the descriptions of three Problems (ID), (AID) and (MD) and the above observation.

Proposition 1: a) If $u(t)=\Phi_{ID}(t,y)$ defined by (3) is a solution to Problem ID, the control $u(t)=\Phi_{ID}(t,y)$ together with $u_{t_0}=0$ is a solution to Problem AID. Conversely, if a control $u(t)=\Phi_{AID}(t,y)$ defined by (6) is a solution to Problem AID and satisfies $u_{t_0}=0$, the control $u(t)=\Phi_{AID}(t,y)$ is a solution to Problem ID.

b) If a control $u(t)=\Phi_{AID}(t,y)$ given by (6) is a solution to Problem AID, the delayed control $u(t)=\Phi_{AID}(t-h,y)$ is a solution to Problem MD. Conversely, if $u(t)=\Phi_{MD}(t,y)$ defined by (9) is a solution to Problem MD, the advanced control $u(t)=\Phi_{MD}(t+h,y)$ is a solution to Problem AID.

c) If $u(t)=\Phi_{MD}(t,y)$ defined by (9) is a solution to Problem MD and satisfies $u_{t_0+h}=0$, the advanced control $u(t)=\Phi_{MD}(t+h,y)$ is a solution to Problem ID. Conversely, if $u(t)=\Phi_{ID}(t,y)$ given by (3) is a solution to Problem ID, the delayed control $u(t)=\Phi_{ID}(t-h,y)$ together with $u_{t_0+h}=0$ is a solution to Problem MD.

Using the fact c) in Proposition 1 and a solution to Problem MD, we will derive a solution of Problem ID. Now we present the solution to Problem MD, which is a slight modification of the result given by Basar and Bernhard (1991). We need to introduce the following four conditions.

(C1) There exists a solution $M(t),\ t_0 \le t \le t_1$ to the Riccati differential equation

$$-\frac{d}{dt}M(t) = M(t)A(t) + A(t)'M(t) + F(t)'F(t) - M(t)(B(t)B(t)' - \gamma^{-2}D(t)D(t)')M(t), \qquad M(t_1) = 0. \tag{10}$$

(C2) There exists a solution $P(t),\ t_0 \le t \le t_1 - h$ to the Riccati differential equation

$$\frac{d}{dt}P(t) = A(t)P(t) + P(t)A(t)' + D(t)D(t)' - P(t)(C(t)'C(t) - \gamma^{-2}F(t)'F(t))P(t), \qquad P(t_0) = N. \tag{11}$$

(C3) There exists a solution $Q(t+\beta),\ -h \le \beta \le 0$ to the Riccati differential equation for each t in $[t_0 + h, t_1]$

$$\frac{\partial}{\partial\beta}Q(t+\beta) = A(t+\beta)Q(t+\beta) + Q(t+\beta)A(t+\beta)' + D(t+\beta)D(t+\beta)' + \gamma^{-2}Q(t+\beta)F(t+\beta)'F(t+\beta)Q(t+\beta), \qquad Q(t-h) = P(t-h). \tag{12}$$

(C4) $\rho(M(t+\beta)Q(t+\beta)) < \gamma^2, \qquad t_0 + h \le t \le t_1, \quad -h \le \beta \le 0.$

Proposition 2: Assume that the conditions (C1)-(C4) are satisfied. Then, a solution to Problem MD is given by

$$u(t) = \begin{cases} -B(t)'S(t)\bar{x}(t), & t_0 + h \le t \le t_1 \\ 0, & t_0 \le t \le t_0 + h \end{cases} \tag{13}$$

where $S(t)$ is defined by

$$S(t+\beta) = M(t+\beta)(I - \gamma^{-2}Q(t+\beta)M(t+\beta))^{-1}, \qquad -h \le \beta \le 0, \tag{14}$$

and $\bar{x}(t)$ is predicted with the "predictor"

$$\frac{\partial}{\partial\beta}\bar{x}(t+\beta) = (A(t+\beta) + \gamma^{-2}Q(t+\beta)F(t+\beta)'F(t+\beta) - B(t+\beta)B(t+\beta)'S(t,\beta))\bar{x}(t+\beta), \qquad -h \le \beta \le 0 \tag{15}$$

from the estimate $\bar{x}(t-h) = \hat{x}(t-h)$ which is estimated with the "observer"

$$\frac{d}{dt}\hat{x}(t) = (A(t) + \gamma^{-2}P(t)F(t)'F(t) - B(t)B(t)'S(t))\hat{x}(t) + P(t)C(t)'(y(t) - C(t)\hat{x}(t)), \qquad \hat{x}(t_0) = 0. \tag{16}$$

From b) in Proposition 1 and Proposition 2, a solution to Problem AID is given by

$$u(t)=\begin{cases}-B(t+h)'S(t+h)\bar{x}(t+h), & t_0\le t\le t_1-h\\ 0, & t_0-h\le t\le t_0\end{cases} \tag{17}$$

which is the advanced form of the control (13). Moreover, since the solution (13) satisfies $u_{t_0+h}=0$, it follows from c) in Proposition 1 and Proposition 2 that the advanced version of (13) given as

$$u(t)=-B(t+h)'S(t+h)\bar{x}(t+h), \qquad t_0\le t\le t_1-h \tag{18}$$

is a solution to Problem ID. Here note that the solutions (17) and (18) have the same predictor-observer structure. That is, in constructing the controls (17) and (18), the estimate $\hat{x}(t)$ is estimated with the observer (16) based on the data $\{y(s), t_0\le s\le t\}$, and $\bar{x}(t+h)$ is predicted with the predictor (15) from the estimate $\bar{x}(t)=\hat{x}(t)$. It is also noted that the conditions (C1)-(C4) form the same sufficient condition for existence of solutions to Problems AID and ID. We can summarize these facts, together with necessity of the conditions (C1)-(C4), in the following form. The proof can be given by using an elementary argument based only on completion of squares (Uchida and Fujita, 1990), and the detail is found in (Uchida et al., 2001).

Theorem: a) There exists a solution to Problem ID if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (18) is a solution to Problem ID.

b) There exists a solution to Problem AID if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (17) is a solution to Problem AID.

c) There exists a solution to Problem MD if and only if the conditions (C1)-(C4) are satisfied. If the conditions (C1)-(C4) are satisfied, the control (13) is a solution to Problem MD.

5. Conclusion

We discussed a finite horizon H^∞ control problem for systems with input delays. We derived a solution based on the known result for the H^∞ control problem in measurement delay case, and showed that the solution has the same predictor-observer structure as the solution in measurement delay case has.

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論　文

線形むだ時間システムの可到達集合解析と拘束つき H^∞ コントローラ設計への応用

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Reachable Set Analysis of Linear Time-Delay Systems and Application to Constrained H^∞ Controller Synthesis

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In this paper, a new notion of reachable set for linear time-delay systems is introduced and a method to evaluate the reachable set is proposed. The evaluation method is given in linear matrix inequality form. An important application of reachable set analisys is constrained controller synthesis. We also propose synthesis method of constrained H^∞ controllers for time-delay systems.

キーワード：むだ時間システム，状態可到達集合，入力拘束
Keywords: time-delay systems, state reachable set, input constraint

1. はじめに

物質の輸送遅れや情報の遅れを伴う制御対象に対する設計問題は，むだ時間システムの制御問題として定式化される。むだ時間システムの制御問題には例えば，状態空間の無限次元性のような，集中定数システムの場合にはない特徴があり，これを克服するために独特の制御理論が既に展開されているが [3]，未解決の課題も多く残されている。本論文で議論する線形むだ時間システムの可到達集合解析もそのような課題の一つである。

可到達集合という概念は古くは制御によって到達できる状態の領域を意味し，主に可制御性の評価に用いられてきた。近年，ロバスト性などの制御性能を評価するために，外乱による状態の変動領域を意味する閉ループシステムの可到達集合とその解析が注目されている [1][2]。本論文の可到達集合解析も後者の意味であるが，これまでのところむだ時間システムに対する研究は報告されていない。本論文では，状態の無限次元性に注目して線形むだ時間システムに対する可到達集合を定義し，無限次元線形行列不等式アプローチ [5] に基づく解析法を提案する。

本論文のもう一つの目的は，本論文で提案する可到達集合解析法を応用して，線形むだ時間システムに対して制御入力の大きさに制限がある場合の H^∞ コントローラの設計法を提案することである。むだ時間システムに対する拘束つき制御設計問題を扱った従来の研究としては文献 (6) や文献 (7) があるが，本論文の問題とは異なり外乱は考慮せず，初期状態の存在領域に依存した設計法が議論されている。また文献 (5) では，制御入力の大きさではなく制御ゲインの大きさに制限がある場合の制御設計法が提案されている。本論文の構成は以下の通りである。2 節では線形むだ時間システムを導入し，3 節では可到達集合を定義し，無限次元双線形行列不等式の形式で可到達集合を評価するための条件を導く。また，この条件を有限次元線形行列不等式を用いて解くアルゴリズムを提案する。4 節ではその応用として，入力の大きさに制限がある場合の H^∞ コントローラの設計法を提案する。5 節はまとめである。

なお，以下において，“ $'$ ” は行列あるいはベクトルの転置を表わす。対称行列 M_0 および対称行列値関数 $M_2(\alpha,\beta)$ に対して，記法

$$L(\alpha,\beta)=\begin{bmatrix} M_0 & M_1(\beta) \\ M_1(\alpha)' & M_2(\alpha,\beta)\end{bmatrix} > (<) 0$$
$$\forall \alpha,\beta\in[-h,0]$$

は，

$$\frac{1}{2}(L(\alpha,\beta)+L(\beta,\alpha)')=$$
$$\begin{bmatrix} M_0 & \frac{1}{2}(M_1(\beta)+M_1(\alpha)') \\ \frac{1}{2}(M_1(\alpha)+M_1(\beta)') & \frac{1}{2}(M_2(\alpha,\beta)+M_2(\beta,\alpha)')\end{bmatrix}$$

が各 $(\alpha,\beta)\in[-h,0]\times[-h,0]$ において，正定 (負定) で

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あることを意味する。また，R^n は n 次元実ベクトル空間を，$R^{n\times m}$ は $n\times m$ の実行列の集合を，$L^2([-h,0];R^n)$ は区間 $[-h,0]$ で定義される二乗可積分な n 次元実ベクトル値関数の集合を，$C([-h,0];R^n)$，$C^1([-h,0];R^n)$ は区間 $[-h,0]$ で定義される連続および 1 回連続微分可能な n 次元実ベクトル値関数の集合を，それぞれ表すものとする。

2. 線形むだ時間システム

つぎの状態にむだ時間を含む線形システムを考える。

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)+\int_{-h}^{0}A_{01}(\beta)x(t+\beta)d\beta +Dw(t) \cdots\cdots (1)$$

ここで，$x(t)\in R^n$，$w(t)\in R^m$ は外乱，$A_0,A_1\in R^{n\times n}$，$D\in R^{n\times m}$，$A_{01}(\beta)\in L^2([-h,0];R^{n\times n})$，$h>0$ はむだ時間である。初期状態を $(x(0),x_0)\in R^n\times L^2([-h,0];R^n)$，時刻 t におけるシステムの状態は $(x(t),x_t)$ である。ただし $x_t=\{x(t+\beta)|-h\le\beta\le 0\}$ である。

ここで，簡単のため，次の記法を導入する。

定数行列 $P\in R^{n\times n}$，行列値関数 $R(\beta)\in L^2([-h,0];R^{n\times n})$ および $S(\alpha,\beta)\in L^2([-h,0]\times[-h,0];R^{n\times n})$ の組を $\{P,R,S\}$ と表す。とくに，$P'=P$ および $S(\alpha,\beta)'=S(\beta,\alpha)$ であるとき，$\{P,R,S\}$ は対称であるという。対称な $\{P,R,S\}$ に対して，2 次形式を次のように定義する。

$$\begin{aligned}(\xi,\zeta)'\{P,R,S\}(\xi,\zeta) &:=\xi'P\xi+2\xi'\int_{-h}^{0}R(\beta)\zeta(\beta)d\beta\\ &+\int_{-h}^{0}\int_{-h}^{0}\zeta(\alpha)'S(\alpha,\beta)\zeta(\beta)d\alpha d\beta\end{aligned}$$

ここで，$(\xi,\zeta)\in R^n\times L^2([-h,0];R^n)$ であるすべての (ξ,ζ) に対して

$$(\xi,\zeta)'\{P,R,S\}(\xi,\zeta)\ge 0$$

であるとき，$\{P,R,S\}$ は半正定であるといい，$\{P,R,S\}\ge 0$ と表す。特に，

$$(\xi,\zeta)'\{P,R,S\}(\xi,\zeta)\ge(\xi,\zeta)'\{\epsilon I,0,0\}(\xi,\zeta)$$

となるような正数 ϵ が存在するとき，$\{P,R,S\}$ は正定であるといい，$\{P,R,S\}>0$ と表す。なお，負定，半負定についても同様に定義する。

3. 可到達集合解析

〈3·1〉 可到達集合解析と条件　いま，システム (1) の外乱 w は，ある正定行列 Δ に対する集合 $W=\{w\in R^m|w'\Delta w\le 1\}$ によって次のように拘束されているものとする。

$$w(t)\in W,\quad \forall t\in[0,\infty)\cdots\cdots (2)$$

〔定義 1〕 $(\xi,\zeta)\in R^n\times L^2([-h,0];R^n)$ に対し，(2) 式を満たす外乱 w および時刻 $T\in[0,\infty)$ が存在し，システム(1) の軌道が $\xi=x(T),\zeta=x_T$ を満たすとき，(ξ,ζ) は初期状態 $(x(0),x_0)$ から状態可到達という。また，$\xi=x(T)$ を満たすとき，R^n-可到達という。状態可到達集合 $\mathcal{R}(x(0),x_0)$ および R^n-可到達集合 $\mathcal{R}_{R^n}(x(0),x_0)$ は，それぞれ次のように定義される。

$$\begin{aligned}\mathcal{R}(x(0),x_0)&=\{(\xi,\zeta)\in R^n\times L^2([-h,0];R^n)\\ &\qquad |(\xi,\zeta)\text{ は }(x(0),x_0)\text{ から状態可到達 }\}\\ \mathcal{R}_{R^n}(x(0),x_0)&=\{\xi\in R^n|\xi\text{は }x(0)\text{ から }R^n\text{-可到達 }\}\end{aligned}$$

いま，正定な $\{P,R,S\}$ に対し，集合 $\mathcal{E}(P,R,S)$ を次のように定義する。

$$\mathcal{E}(P,R,S)=\{\lambda=(\xi,\zeta)\in R^n\times L^2|\lambda'\{P,R,S\}\lambda\le 1\}$$

このとき，次の補題を準備する。

〔補題 1〕 $\{P,R,S\}$ が存在し，$\lambda'\{P,R,S\}\lambda\ge 1$ を満たす任意の $\lambda=(\xi,\zeta)\in R^n\times L^2([-h,0];R^n)$，および任意の $w\in W$ に対して，次の条件を満たしているものとする。

$$\int_{-h}^{0}\int_{-h}^{0}\begin{bmatrix}\frac{1}{h}\xi\\ \frac{1}{h}\zeta(-h)\\ \zeta(\alpha)\\ \frac{1}{h}w\end{bmatrix}' L_0(P,R,S;\alpha,\beta)\begin{bmatrix}\frac{1}{h}\xi\\ \frac{1}{h}\zeta(-h)\\ \zeta(\beta)\\ \frac{1}{h}w\end{bmatrix}d\beta d\alpha\le 0 \cdots\cdots (3)$$

ここで，

$$L_0(P,R,S;\alpha,\beta)=\left[\begin{array}{c:c}A_0'P+PA_0+R(0)'+R(0) & PA_1-R(-h)\\ A_1'P-R(-h)' & 0\\ \left(\begin{array}{c}A_{01}(\alpha)'P+R(\alpha)A_0\\ -\frac{\partial}{\partial\alpha}R(\alpha)'+S(\alpha,0)\end{array}\right) & R(\alpha)'A_1-S(\alpha,-h)\\ B'P & 0\end{array}\right.$$

$$
\begin{array}{c}
\begin{pmatrix} PA_{01}(\beta)+A_0{}'R(\beta) \\ -\frac{\partial}{\partial\beta}R(\beta)+S(0,\beta) \end{pmatrix} \\
A_1{}'R(\beta)-S(-h,\beta) \\
\begin{pmatrix} R(\alpha)'A_{01}(\beta)+A_{01}(\alpha)'R(\beta)- \\ (\frac{\partial}{\partial\beta}+\frac{\partial}{\partial\alpha})S(\alpha,\beta) \end{pmatrix} \\
D'R(\beta)
\end{array}
\quad
\left.\begin{array}{c} PD \\ 0 \\ R(\alpha)'D \\ 0 \end{array}\right]
$$

である。このとき，システム (1) の状態可到達集合 $\mathcal{R}(0,0)$ は次のように評価される。

$$\mathcal{R}(0,0)\subset\mathcal{E}(P,R,S)$$

□

証明は付録 1 に示す。

この補題から，次のように双線形行列不等式 (BMI) 条件に基づいた状態可到達集合の評価を得ることができる。

〔定理 1〕 $\{P,R,S\}$ および非負の定数 p が存在し，次の条件を満たしているものとする。

$$\begin{bmatrix} P & R(\beta) \\ R(\alpha)' & S(\alpha,\beta) \end{bmatrix}>0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

$$L_0(P,R,S;\alpha,\beta)+pL_1(P,R,S,\Delta;\alpha,\beta)\le 0$$
$$\forall(\alpha,\beta)\in[-h,0] \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

ここで，

$$L_1(P,R,S,\Delta;\alpha,\beta)$$
$$=\begin{bmatrix} P & 0 & R(\beta) & 0 \\ 0 & 0 & 0 & 0 \\ R(\alpha)' & 0 & S(\alpha,\beta) & 0 \\ 0 & 0 & 0 & -\Delta \end{bmatrix}$$
$$P'=P,\ S(\alpha,\beta)'=S(\beta,\alpha)$$
$$R(\beta)\in C^1([-h,0];R^{n\times n})$$
$$S(\alpha,\beta)\in C^1([-h,0]\times[-h,0];R^{n\times n})$$

である。このとき，システム (1) の状態可到達集合 $\mathcal{R}(0,0)$ は次のように評価される。

$$\mathcal{R}(0,0)\subset\mathcal{E}(P,R,S)$$

□

証明は付録 2 に示す。

R^n-可到達集合に関しては，$R(\beta)\equiv 0, S(\alpha,\beta)\equiv 0$ として，補題 1 と定理 1 の議論を繰り返すことにより，次のように評価できる。いま，正定行列 P に対して，集合 $\mathcal{E}_{R^n}(P)$ を

$$\mathcal{E}_{R^n}(P)=\{\xi\in R^n|\xi'P\xi\le 1\}$$

で定義する。

〔定理 2〕 定数行列 P および非負の実数 p が存在し，次の条件を満たしているものとする。

$$P>0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (6)$$
$$L_0(P,0,0;\alpha,\beta)+pL_1(P,0,0,\Delta;\alpha,\beta)\le 0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

このとき，システム (1) の R^n-可到達集合 $\mathcal{R}_{R^n}(0,0)$ は次のように評価される。

$$\mathcal{R}_{R^n}(0,0)\subset\mathcal{E}_{R^n}(P)$$

□

定理 1 および定理 2 の評価は集合の包含関係で与えられている。ここでは，これらの包含関係から直ちに導かれるひとつの評価式を示す。いま，定理 1 の条件を満たす組 $\{P,R,S\}$ に対しては次のような固有値の最小値が計算できることに注意する。

$$0<\mu_{\min}I\le\begin{bmatrix} P & R(\beta) \\ R(\alpha)' & S(\alpha,\beta) \end{bmatrix},\forall\alpha,\beta\in[-h,0]$$

このとき，定理 1 から，状態 (0,0) から出発するシステム(1) の軌道は

$$x(t)'x(t)+\int_{-h}^{0}\int_{-h}^{0}x_t(\alpha)'x_t(\beta)d\alpha d\beta\le\mu_{\min}^{-1}$$

を満たす。一方，定理 2 からは，P の最小固有値

$$0<\nu_{\min}I\le P$$

を計算すれば，状態 (0,0) から出発するシステム(1) の軌道は

$$x(t)'x(t)\le\nu_{\min}^{-1}$$

を満たすことがわかる。

〈3・2〉 LMI 条件による解法 前節の議論から行列不等式を解くことによって可到達集合の評価が可能となったが，行列不等式(5) あるいは(7) は双線形であり，解を求めることは容易ではない。本節では，線形行列不等式 (LMI) の反復計算によって BMI を解くアルゴリズムを提案する。まず，定理 1 の条件の解法について考えよう。

アルゴリズム I

(ステップ 1) 初期値として非負の定数 $\bar{p}$ を選ぶ

(ステップ 2) $\bar{p}$ に対して，次の LMI 条件を満たす組 $\{P,R,S\}$ および定数 p を求める。

$$0<\begin{bmatrix} P & R(\beta) \\ R(\alpha)' & S(\alpha,\beta) \end{bmatrix}$$
$$L_0(P,R,S;\alpha,\beta)+\bar{p}L_1(P,R,S,0;\alpha,\beta)$$
$$+pL_1(0,0,0,\Delta;\alpha,\beta)\le 0$$
$$0\le p\le\bar{p}$$

(ステップ3) p を初期値として (ステップ 2) を繰り返す。適当な終了条件により反復を終了する。

このアルゴリズムは，(ステップ 2) の LMI 条件を解くことができれば，反復を途中で止めてもその解がもとの BMI 条件(5) の解となっている。

$R(\beta) \equiv 0$ および $S(\alpha,\beta) \equiv 0$ とおけば，アルゴリズム I はそのまま定理 2 の条件の解法となるが，定理 2 の条件に対しては，集中系に対して提案されたアルゴリズム[(2)]を一般化した別のアルゴリズムを考えることができる。まず，条件(7) は，$Q = P^{-1}$ と置き換えることによって，Q と p に関する BMI 条件，

$$\begin{bmatrix} QA_0' + A_0Q + pQ & A_1 & A_{01}(\beta) & D \\ A_1' & 0 & 0 & 0 \\ A_{01}(\alpha)' & 0 & 0 & 0 \\ D' & 0 & 0 & -p\Delta \end{bmatrix} \leq 0 \qquad \cdots\cdots (8)$$

となることに注意しよう。

アルゴリズム **II**(R^n-可到達集合の場合)

(ステップ1) 初期値として正定行列 $\bar{Q}$ を選ぶ。

(ステップ2) $\bar{Q}$ に対して，次の LMI 条件を満たす正定行列 Q および定数 p を求める。

$$0 < Q \leq \bar{Q}$$

$$\begin{bmatrix} QA_0' + A_0Q + p\bar{Q} & A_1 & A_{01}(\beta) & D \\ A_1' & 0 & 0 & 0 \\ A_{01}(\alpha)' & 0 & 0 & 0 \\ D' & 0 & 0 & -p\Delta \end{bmatrix} \leq 0$$

$$0 \leq p$$

(ステップ3) Q を初期値として (ステップ 2) を繰り返す。適当な終了条件により反復を終了する。

このアルゴリズムによって得られた Q に対して，$P = Q^{-1}$ とおけば，定理 2 の評価が使えることになる。アルゴリズム I と同様に，このアルゴリズムも途中で反復を途中で止めても，解が得られることが保証される。加えて，このアルゴリズムは，(ステップ 2) の条件 $Q \leq \bar{Q}$ より，

$$\mathcal{E}_{R^n}(Q^{-1}) \subset \mathcal{E}_{R^n}(\bar{Q}^{-1})$$

となるため，反復を繰り返すことによって評価の改善が期待できる。また，アルゴリズムの収束も保証できる。

〈3·3〉 有限次元 LMI 条件　アルゴリズム I あるいは II によって，可到達集合解析は LMI 条件を解くことに帰着された。しかしながらこれらの LMI 条件は一般にパラメータ α, β に依存する無限次元条件であり，すべてのパラメータ値に対して解く必要がある。文献 (4) (5) では，$A_{01}(\beta)$ の各要素が β の多項式，すなわち

$$A_{01}(\beta) = A_{01}^0 + \beta A_{01}^1 + \cdots + \beta^l A_{01}^l$$

であるとき，多項式の解

$$R(\beta) = R^0 + \beta R^1 + \cdots + \beta^k R^k$$
$$S(\alpha,\beta) = S^0 + (\alpha+\beta)S^1 + \cdots + (\alpha^r + \beta^r)S^r$$

を仮定し，すべてのパラメータ値 (α,β) に対する LMI 条件を，パラメータ (α,β) の超曲面を囲む凸多面体の頂点における有限個の LMI 条件に変換する手法を提案している。本論文でも，以下この手法を採用する。

〈3·4〉 数 値 例　対象とするシステムを，

$$\dot{x}(t) = A_0x(t) + A_1x(t-h) + Dw(t)$$

とする。システムのパラメータは，

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix},\ A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix},$$
$$D = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix},\ h = 0.5,$$

で与えられている。また，外乱の拘束条件を決定する Δ は，$\Delta = 1.0$ とおいた。これを，先に示したアルゴリズム I および II を用いて解く。

アルゴリズム I による結果　アルゴリズム I においては $R(\beta)$，$S(\alpha,\beta)$ は

$$R(\beta) = R_0 + \beta R_1 + \beta^2 R_2,$$
$$S(\alpha,\beta) = S_0 + (\alpha+\beta)S_1 + (\alpha^2+\beta^2)S_2$$

とおく。これにより，先に述べた条件式の有限次元化の手法を用いることができる。

20 回反復計算を行った結果，

$$P = 10^{-6}\begin{bmatrix} 0.1962 & -0.1583 \\ -0.1583 & 0.1573 \end{bmatrix},$$

$$R(\beta) = 10^{-6}\left\{ \begin{bmatrix} -0.0437 & 0.1740 \\ 0.0012 & -0.1728 \end{bmatrix} + \beta \begin{bmatrix} -0.0274 & 0.0705 \\ 0.00074 & -0.06983 \end{bmatrix} + \beta^2 \begin{bmatrix} -0.0287 & 0.0730 \\ 0.00051 & -0.07252 \end{bmatrix} \right\},$$

$$S(\alpha,\beta) = 10^{-7}\left\{ \begin{bmatrix} 0.4890 & -0.0140 \\ -0.0140 & 1.8990 \end{bmatrix} + (\alpha+\beta) \begin{bmatrix} 0.2588 & -0.0073 \\ -0.0073 & 0.6898 \end{bmatrix} + (\alpha^2+\beta^2) \begin{bmatrix} 0.2494 & -0.0047 \\ -0.0047 & 0.6779 \end{bmatrix} \right\},$$

$$p = 0.1216$$

となる。これにより状態可到達集合の上限 $\mathcal{E}(P,R,S)$ が求められた。ここで，定理 1 の結果より可到達集合の評価は $\{P,R,S\}$ の最小固有値の下限 μ_{min} により評価される。いま，この μ_{min} は，1.263×10^{-11} と求まる。

アルゴリズムIIによる結果　つづいて，R^n-可到達集合をアルゴリズムIIを用いて評価する。アルゴリズム中における $\bar{Q}$ を $\bar{Q}=10^4\times I$ として，反復計算を行った。なお，反復は Q が収束したと認められる11回行った。その結果 Q と p は，

$$Q=10^3\times\begin{bmatrix}0.6016 & 0.7920\\ 0.7920 & 2.8987\end{bmatrix}$$

$$p=0.2596$$

を得た。このとき，解 Q の最小固有値は，0.3550×10^3 となる。なお，一回目の計算の結果として得た Q は

$$Q=10^3\times\begin{bmatrix}5.3808 & 1.3470\\ 1.3470 & 8.8143\end{bmatrix}$$

で，その最小固有値は 4.9154×10^3 であった。以上より，反復を行うことで行列 Q，およびその最小固有値が小さくなっていることが確認でき，先に述べたように R^n-可到達集合の評価は $P(=Q^{-1})$ の最小固有値の逆数で評価されるので，R^n-可到達集合の評価が反復により改善されていることがわかる。

4. 入力の大きさに制限がある場合の H^∞ コントローラの設計

〈4・1〉 制御問題の記述　本節では，前節までに展開した可到達集合解析を応用して，入力の大きさに制限がある場合の H^∞ コントローラの設計法について検討する。本節において対象とするシステムは，制御入力と制御量を持った状態に点むだ時間のみを含む次の線形システムである。

$$\left.\begin{array}{l}\dot{x}(t)=A_0x(t)+A_1x(t-h)+Bu(t)+Dw(t)\\ z(t)=Cx(t)\end{array}\right\}\cdots(9)$$

ここで，$x(t)\in R^n$，$u(t)\in R^m$ は制御入力，$w(t)\in R^p$ は外乱，$z(t)\in R^q$ は制御量であり，$A_0,A_1\in R^{n\times n}$，$B\in R^{n\times m}, C\in R^{n\times q}$，$D\in R^{n\times p}$，$h>0$ である。初期状態は $(0,0)$ とし，外乱は(2)式を満たすものに限定する。

コントローラとしては，次のようなメモリーレス状態フィードバックを考える。

$$u(t)=K_0x(t)\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

ここで，$K_0\in R^{n\times n}$ である。制御入力の大きさの制限を，ある正定行列 Γ に対する集合 $U=\{u\in R^m|u'\Gamma u\le 1\}$ を用いて表わす。すなわち，

$$u(t)\in U,\quad \forall t\in[0,\infty)\quad\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

条件(2)を満たす外乱を $w\in L^2([0,\infty);R^p)$ に制限し，このような外乱のクラスを $\mathcal{W}$ と表わす。外乱から制御量までの L^2 ゲイン g を次のように定義する。

$$g=\sup_{w\in\mathcal{W},\ w\neq0}\frac{\|z\|_{L^2}}{\|w\|_{L^2}}$$

ここで，$\|\cdot\|_{L^2}$ は L^2 ノルムを表わす。w はその値が条件(2)により制限されていることから L^2 ゲイン g は準大域的なゲインである。

問題は大きさの制限(11)を受ける状態フィードバック制御(10)を設計し，線形システム(9)に適用して構成される閉ループシステムを内部安定(漸近安定)化し，L^2 ゲインを指定された値 γ 以下に抑制することである。

〈4・2〉 コントローラの構成条件　制御入力の大きさに制限がない場合には，文献(6)で示された有界実補題をLMIの形式に直すことにより次の結果を得る。すなわち，

$$\begin{bmatrix}\begin{pmatrix}Q(A_0+BK_0)'\\+(A_0+BK_0)Q+X\end{pmatrix} & QC' & A_1Q & D\\ CQ & -I & 0 & 0\\ QA_1' & 0 & -X & 0\\ D' & 0 & 0 & -\gamma^2I\end{bmatrix}<0$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(12)$$

を満たす定数行列 $Q>0$，$X>0$，K_0 が存在すれば，制御則(10)を線形システム(9)に適用して得られる閉ループシステムは内部安定であり，$g\le\gamma$ を満足する。一方，この閉ループシステムに対して定理2を適用すると，A_0 を A_0+BK で置き換えた行列不等式(8)(ただし $A_{01}(\alpha)\equiv0, A_{01}(\beta)\equiv0$)を満たす行列定数 $Q>0$，K_0，および非負の定数 p が存在すれば，閉ループシステムの軌道は

$$x(t)'Q^{-1}x(t)\le1,\quad \forall t\in[0,\infty)\cdots\cdots\cdots\cdots\cdots(13)$$

を満たすことがわかる。制御入力(10)に対する大きさの制限(11)は，

$$x(t)'K_0'\Gamma K_0x(t)\le1,\quad \forall t\in[0,\infty)\cdots\cdots\cdots\cdots(14)$$

と表わすことができるから，結局 R^n-可到達の評価(13)を前提とすれば，

$$K_0'\Gamma K_0\le Q^{-1}\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

が成立するとき，制御入力の大きさの制限(11)は満たされる。行列不等式(12)，A_0 を A_0+BK に置き換えた行列不等式(8)および行列不等式(15)において，新しい変数 $Z_0(=K_0Q)$ を導入してこれらの行列不等式を書き直し，本節の議論をまとめると次の結果を得る。

〔定理3〕 定数行列 Q，X，Z_0 および非負の定数 p が存在し，次の条件を満たしているものとする。

$$Q > 0, \quad X > 0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (16)$$

$$\begin{bmatrix} \begin{pmatrix} QA_0' + A_0Q \\ +Z_0'B' + BZ_0 \\ +X \end{pmatrix} & QC' & A_1Q & D \\ CQ & -I & 0 & 0 \\ QA_1' & 0 & -X & 0 \\ D' & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (17)$$

$$\begin{bmatrix} \begin{pmatrix} QA_0' + A_0Q + pQ \\ +Z_0'B' + BZ_0 \end{pmatrix} & A_1 & D \\ A_1' & 0 & 0 \\ D' & 0 & -p\Delta \end{bmatrix} \leq 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots (18)$$

$$\begin{bmatrix} Q & Z_0' \\ Z_0 & \Gamma^{-1} \end{bmatrix} > 0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (19)$$

このとき，$K_0 = Z_0Q^{-1}$ をフィードバックゲインとする制御則(10) と線形システム(9) から構成される閉ループシステムは内部安定であり，$g \leq \gamma$ を満たす。また，制御入力の大きさは制限(11) を満足する。 □

条件(18) は双線形であるため，この行列不等式を解くためには〈3・2〉節で述べたアルゴリズム II を用いることになる。条件(19) は Schur の補題 (1) を用いて不等式(15) を書き直したものである。

〈4・3〉 数 値 例 　次のシステムを考える。

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-h) + Bu(t) + Dw(t) \\ z(t) &= Cx(t). \end{aligned}$$

ここで，各行列パラメータは次のように設定する。

$$A_0 = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0 \\ -1.0 \end{bmatrix}, \quad C = \begin{bmatrix} 0.0 & 1.0 \end{bmatrix}, \quad D = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix},$$

$$h = 0.5.$$

また，システムの外乱は，(2) 式のように拘束されている。ここで，$\Delta = 1.0 \times 10^{-4}$ とおく。また，入力の拘束を決定する Γ を $\Gamma = 1.0 \times 10^{-5}I$ とし，システムの L^2 ゲインは $\gamma = 1.0$ とする。反復アルゴリズムを用いて 15 回の反復を行う。解 Q は，1 回目の反復において，$Q = 10^3 \times \begin{bmatrix} 3.7905 & 0.001502 \\ 0.001502 & 0.04985 \end{bmatrix}$ のように，また，15 回目の反復において $Q = 10^2 \times \begin{bmatrix} 5.2576 & 0.09204 \\ 0.09204 & 0.1243 \end{bmatrix}$ のように得られる。これより，Q の値は反復により小さくなっていることがわかる。また，この Q より，求めたいコントローラのゲインは，

$$K = \begin{bmatrix} 0.3552 & 83.84 \end{bmatrix}$$

となる。このフィードバック系に対し，(2) の拘束の限界値をとるステップ外乱 ($w = 100.0, \ 0.5 \leq t \leq 1.5$) を加えたときの入力の様子を Fig.1 に，状態の様子を Fig.2 に示す。このとき，入力はあらかじめ設定した入力拘束を満たしていることがわかる。

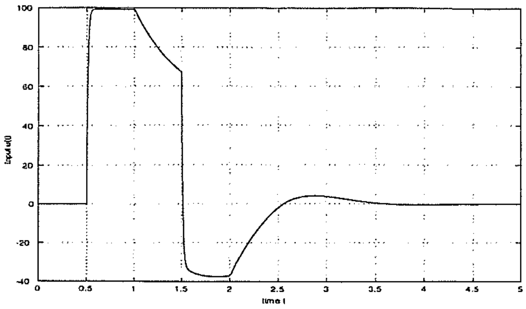

図 1　システムの入力の様子

Fig. 1.　Simulation result of $u(t)$

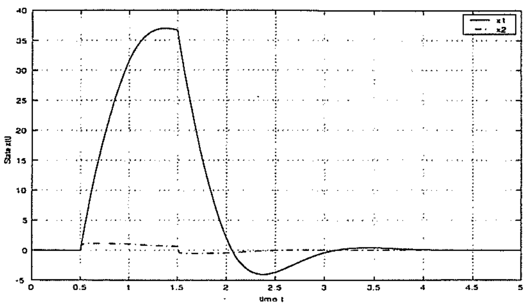

図 2　システムの状態の様子

Fig. 2.　Simulation result of $x(t)$

5. おわりに

本論文では，状態にむだ時間を含む線形システムに対して，可到達集合の概念を導入し，可到達集合を評価する方法を提案した。さらに，その応用として，入力に拘束のある場合の H^∞ コントローラの設計法を示し，数値例を用いて検証を行った。

可到達集合解析の応用として導出した拘束つき H^∞ コントローラの構成条件により，Γ の設定により拘束の大きさに合わせたコントローラの構成が可能となった。しかしながら，この条件は複数の LMI を連立させて解く必要があるため，一般的にはかなり保守的な結果となる。また，Γ, Δ の相対的な大きさによって複数の LMI 条件の可解性が左右される問題の解明も必要である。複数の LMI 条件による保守性を低減化することが今後の課題である。

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付　録

1. 補題1の証明

記述を簡単にするため, $V(\lambda) := \lambda\{P,R,S\}\lambda$ とおく。いま, 条件(2)を満たすある外乱 w に対して, 初期状態 $(0,0)$ から出発するシステム(1)の軌道が

$$V(x(t),x_t) \le 1, \quad 0 \le t \le t_1$$
$$V(x(t),x_t) > 1, \quad t_1 < t \le t_2$$

を満足すると仮定する。この不等式は平均値の定理を用いれば, ある時刻 $t^* \in (t_1,t_2)$ で,

$$\frac{d}{dt}V(x(t^*),x_{t^*}) > 0$$

を意味する。一方, $V(x(t^*),x_{t^*}) > 1$ であるから, 条件(3)より $\frac{d}{dt}V(x(t^*),x_{t^*}) \le 0$ でなければならず, 矛盾が生じる。したがって, 条件(2)が満たすすべての外乱 w に対して, 初期状態 $(0,0)$ から出発するシステム(1)の軌道は必ず集合 $\mathcal{E}(P,R,S)$ に含まれる。 □

2. 定理1の証明

まず, 条件(4)は $\{P,R,S\}$ が正定となるための十分条件である。補題の条件は, $\lambda\{P,R,S\}\lambda \ge 1$, $w'\Delta w \le 1$ に対して,

$$I := \int_{-h}^{0}\int_{-h}^{0} \begin{bmatrix} \frac{1}{h}\xi \\ \frac{1}{h}\zeta(-h) \\ \frac{1}{h}\zeta(\alpha) \\ \frac{1}{h}w \end{bmatrix}' L_0(P,R,S;\alpha,\beta) \begin{bmatrix} \frac{1}{h}\xi \\ \frac{1}{h}\zeta(-h) \\ \frac{1}{h}\zeta(\alpha) \\ \frac{1}{h}w \end{bmatrix} d\alpha d\beta \le 0$$

が成立することである。$\mathcal{S}$-procedure[1] を用いると, この条件の成立は, ある非負の実数 p が存在して,

$$I + p(\lambda'\{P,R,S\}\lambda - 1) + p(1 - w'\Delta w) \le 0 \qquad \cdots\cdots (付1)$$

が成立することにより保証される。定理1の条件(5)は(付1)が成立するための十分条件である。したがって, 補題よりこの定理の結論を得ることができる。 □

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