

The Role of the Wage-Unit in the General Theory*

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Abstract

What did Keynes really want to say in the *General Theory*? The answer of this paper is that he wanted to establish the proposition that a cut in money wages is not a remedy for unemployment. For that purpose economic variables such as income and consumption were measured in terms of the wage-unit. Then, what justifies the use of such an unusual unit? This paper finds that it originally belonged to the supply side represented by the aggregate supply function. It was transposed to the demand side. And production functions hidden in the *GT* are specified for the first time.

Key words: Involuntary Unemployment, Cut in Money Wages, Kahn's Employment Multiplier, Keynes's Investment Multiplier, Production Functions

JEL classification: B22, E12, E24

1 Introduction

Even today anyone who reads Keynes's [10] *General Theory* (hereafter *GT*) will have difficulty as early as in Chapter 3 and ask a question: What did Keynes really want to say by writing the book? Needless to say, this question moved lots of people because of the revolutionary aspects of the *GT*. But, because of the kaleidoscopic nature, it has long been disturbing them, too. For example, Mankiw [17, p.561] said, "*The General Theory* is an obscure book: I am not sure that even Keynes himself knew completely what he really meant." It would seem that a New Keynesian also had difficulty in answering the question.

Nevertheless, it can be said that the most influential answer is still that by Klein [15] and Samuelson [27] who both regarded the principle of effective demand, not the liquidity preference theory, as the essence of the *GT*. According to Samuelson [27, p.278], "To put the matter most simply: investment calls the tune." Recently Krugman also strongly defended Samuelson's interpretation in the introduction to the 2007 edition of the *GT* as follows:

... it is hard to see any significant difference between Samuelson's formulation and Keynes's own equation for equilibrium, right there in Chapter 3: $\phi(N) - \chi(N) = D_2$ [29]. Represented graphically, Keynes's version looks a lot like Samuelson's diagram; quantities are measured in wage units

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rather than constant dollars, and the nifty 45-degree feature is absent, but the logic is exactly the same. (Krugman [16, p. xxxv])⁽¹⁾

The logic is this: Investment demand D_2 that is assumed to be given determines the volume of employment N , which can be below the level of full employment. Actually Krugman's explanation is so customary that there seems to be no need to examine it further. I also think of the principle of effective demand as the theoretical core of the *GT*.

But the concept of the principle of effective demand had already been developed by Kahn [7, 8]. Furthermore, as will be seen in detail, the *GT* took over what Kahn set up to construct a new theory for a reduction in unemployment to a great extent. In this sense the *GT* is an extension of Kahn.

There is, however, a big difference between the two. Kahn regarded the rigidity of money wages as essential for his analysis. Keynes by contraries insisted that the volume of employment in equilibrium was not affected whatever value money wages may take. In the *GT*, Keynes defined voluntary unemployment as follows:

[The classical school argues] that if labour as a whole would agree to a reduction of money-wages more employment would be forthcoming. If this is the case, such unemployment ... ought to be included under the ... category of "voluntary" unemployment" (p. 8)⁽²⁾

That is, if a wage cut leads to a reduction in unemployment, such unemployment is voluntary one. And Keynes asserted that there is a different kind of unemployment which cannot be reduced by a reduction in money wages and called it involuntary unemployment. The latter is of course the unemployment studied thoroughly in the *GT*. Then, Keynes provocatively argued that *a cut in money wages is not a remedy for unemployment*. This proposition, which will be called the *GT* proposition, is what Keynes *really* wanted to show in the *GT* on the basis of the principle of effective demand.

In this relation it should be recalled that main economic variables in the *GT* such as income and investment were measured in terms of the wage-unit, i.e., the money wage of a unit of labor. Why was it adopted as the basic unit? This question was ignored early on. For example, Hicks [5, p. 6] declared, "The concept of the 'wage-unit', which Keynes introduced to buttress his employment approach, has to be abandoned." Hansen [1, pp. 42-43] said, "Thus for Keynes it really made no great difference whether he corrected the nominal monetary magnitudes by a price index or by a wage-rate index." Samuelson and Krugman also hold the same view as seen from the above quotation. Indeed the principle of effective demand obtains irrespective of units. But, as far as the *GT* is concerned, the wage-unit cannot be abandoned and it actually makes a great difference whether the nominal monetary magnitudes are corrected by the price level or by the wage-unit because it is the wage-unit that can be used to show the *GT* proposition, the final goal of the *GT*. That is why Keynes was particular about such an unusual unit.

Then, what justifies the use of the wage-unit? To put it in other words, where on earth did it come from? The answer of this paper is that it had been taken from the supply side to the demand side represented by the effective demand. It originally belonged to the supply side represented by the aggregate supply function. Therefore the supply side also matters to understand what Keynes really wanted to say. The aggregate supply function has already been studied, but, as far as I know, the crucial role which Keynes assigned to the wage-unit for the proof of the *GT* proposition has not been recognized. Also the examination of the aggregate supply function reveals that there are a kind of the

Cobb-Douglas production functions hidden in the *GT*.

This paper is organized as follows. Section 2 discusses the similarity and difference between Kahn [7, 8] and the *GT*. Section 3 makes clear how the wage-unit is treated in the *GT*. This provides preliminaries to verifying the *GT* proposition. It is shown in Section 4 that the *GT* was so structured as to support the proposition but it could not be proved successfully. The supply side is studied in detail in Sections 5 and 6 to specify production functions. Section 7 concludes this paper.

2 The Theoretical Relationship between Kahn and Keynes

As is well known, the *GT* was written under the “supervision” of Richard Kahn.⁽³⁾ Though new analytical tools were augmented by Keynes, the theoretical core of the *GT* already lay in Kahn [7, 8]. To see this, let us begin by summarizing the arguments of Kahn.

The economy analyzed there has two production sectors producing consumption goods and investment goods, respectively. In order to reduce unemployment it is necessary to raise the demand for output which is made up of consumption demand and investment demand. Therefore, either of the two demands must be increased for the reduction in unemployment. Kahn [8] started with the following:⁽⁴⁾

The main object, I take it, of economic policy at a time like the present is to raise the level of output and of employment. Judged by this criterion success can be attained only by raising the demand for output as a whole. Now all methods of raising the demand for output can be classified under two heads: those that involve an increase of investment, ... and those that involve a reduction of saving, increased consumption of consumable goods.

Note that the quotations are from Kahn, not from the *GT*. The remedy for unemployment which may be thought to originate from the *GT* had been clearly stated before Keynes said, “I should readily concede that the wisest course is to advance on both fronts at once. Whilst aiming at a socially controlled rate of investment ... , I should support at the same time all sorts of policies for increasing the propensity to consume.” (p. 325)

The increase in employment which is required for an increase in output of investment goods is called the “primary” employment. Similarly, the increase in employment which is required for an increase in output of consumption goods is termed the “secondary” employment. Kahn treated investment demand as an *independent* variable and showed that the primary employment triggered by an increase in investment demand generates the secondary employment with the result that employment as a whole increases by more than the primary employment or by the “employment multiplier” times as much as the primary employment. Furthermore, it is important to notice that government expenditure, say, on roads is included in the investment demand. Thus Kahn proposed an increase in investment such as the building of roads for the reduction in unemployment. Of course the *GT* adopted this proposal, too.⁽⁵⁾

As is seen from the above, consumption demand is considered to be a *dependent* variable. And the increase in consumption demand resulting from that in investment demand leads to a rise in the price of and an increase in output of consumption goods along the *supply curve* of consumption goods in general.⁽⁶⁾ The supply curve has familiar properties in microeconomics, so that the price and output of

consumption goods are determined by the conditions of supply and demand just like the price and output of any single good.⁽⁷⁾

A crucial point to be noticed here is that Kahn assumed money wages to be fixed. The assumption of fixed money wages means that the supply curve does not shift as a consequence of the reduction in unemployment and that there is a unique relation between the price and output of consumption goods. In fact Kahn [7, p. 175] stated, "Even if this assumption is not entirely reasonable, it is clear that it is *essential* if the analysis is to proceed at all." (My italics.)⁽⁸⁾

Then, take a look at the following:

... we have here established the law that increased *employment* for investment must necessarily stimulate the industries producing for consumption and thus lead to a total increase of *employment* which is a multiple of the primary employment required by the investment itself. (p. 118, My italics.)

It is clear-cut, isn't it? Note that the quotation this time does not come from Kahn but from the *GT*. Someone may be surprised at the similarities between Kahn and Keynes. They are entirely in agreement with the causality running from investment demand to *employment*. Although the *GT* has been thought to be an *income* determination theory, it is much more correct to say that it is an *employment* determination theory as the title of it exactly implies.

Such an employment determination theory was formulated⁽⁹⁾ in Chapter 3 by the equilibrium condition in the goods market

$$\phi(N) = \chi(N) + D_2. \quad (1)$$

Here N is the volume of employment. $\phi(N)$ on the left-hand supply side is called the aggregate supply function, while $\chi(N)$ and D_2 on the right-hand demand side are respectively the "propensity to consume" (or the consumption function) and investment demand. Now "the essence of the General Theory of Employment" is made quite clear as follows:

Hence the volume of employment in equilibrium depends on (i) the aggregate supply function, ϕ , (ii) the propensity to consume, χ , and (iii) the volume of investment, D_2 . This is the essence of the General Theory of Employment. (p. 29)

As is obvious from the above statement,⁽¹⁰⁾ the volume of investment or investment demand D_2 is *not* a function of N in "the essence of the General Theory of Employment." In this sense it is taken as an independent variable. On the other hand, consumption demand $\chi(N)$ ($= D_1$) is a dependent variable. Thus, there is no ambiguity in the determination of employment: Investment demand as an independent variable determines the volume of employment. It is obvious now that most of Kahn's arguments are maintained by Keynes.

There is, however, a crucial difference between Kahn and Keynes. As said earlier, Kahn assumed money wages to be fixed and considered it to be essential for his analysis. On the contrary, Keynes sought to dispense with the assumption of fixed money wages, and said, "The essential character of the argument is precisely the same whether or not money-wages, etc., are liable to change." (p. 27) That is, he attempted to show that "the essence of the General Theory of Employment" holds without such an assumption.

Why did Keynes intend to do so? The immediate reason, in my view, was that Pigou [23], based on a much more rigorous theory, denied that the secondary employment was induced by the primary employment due to public works and claimed that a reduction in money wages caused that in real wages, which in turn leads to an increase in the demand for labor. If Pigou [23] was right, the way to cure unemployment was a cut in money wages, *not* public works. Pigou [23] did not mention Kahn [7], but the former virtually negated the latter and also Keynes who had asked Kahn to write a paper for the effectiveness of public works.⁽¹¹⁾

Then, what Keynes had to do in the first place in defense of Kahn [7] was to show that a reduction in money wages causes neither a fall in real wages nor a decrease in unemployment. Keynes stressed this in Chapter 2 as follows:

In assuming that the wage bargain determines the real wage the classical school have slipt in an illicit assumption. ... There may exist no expedient by which labour as a whole can reduce its *real* wage to a given figure by making revised *money* bargains with the entrepreneurs. This will be our contention. (p. 13)

And, after summarizing “the General Theory of Employment” in Chapter 18, he restated the problem in terms of effective demand in Chapter 19:

... the precise question at issue is whether the reduction in money-wages will or will not be accompanied by the same aggregate effective demand as before measured in money, or, at any rate, by an aggregate effective demand which is not reduced in full proportion to the reduction in money-wages (*i.e.* which is somewhat greater measured in wage-units). (pp. 259–260)

Indeed there is a conventional view that what is the most important in the the *GT* is the invention of the principle of effective demand and/or the liquidity preference theory because they determine income the level of which may be below full employment. I do not intend to deny it completely. But it must be more important for Keynes, *in the context of the academic dispute over unemployment*, to show the proposition that a cut in money wages is not a remedy for unemployment since it does not lead to a fall in real wages. In fact, later Kahn [9, p.126] confirmed that this problem was vital to Keynes:

For the present reader it is ridiculous to write for lower, and falling money-wages. But I am lecturing Keynes and, until shortly before the outbreak of the Second World War, the questions were why wages did not fall and whether employment would not be increased if they were cut. This was the great issue dividing Keynes from Pigou, Robertson and many other of the leading economists of the day.

Therefore, while the principle of effective demand and/or the liquidity preference theory are important as analytical tools, it is, I think, the final goal of the *GT* to prove the proposition on money wages.⁽¹²⁾

Then, did Keynes succeed in establishing the proposition by taking advantage of his new tools? The next section provides preliminaries to answering this question.

3 The Essence of the General Theory of Employment

Generally speaking, if a theory is expressed by any sort of mathematical model, there should be a set of independent (or exogenous) variables and dependent (or endogenous) ones. Analysis on the basis of a theory is to find the relationship between independent variables and dependent ones. Particularly a theory assumes that independent variables determine dependent ones. Thus, the distinction between independent variables and dependent ones is crucial to a theory. If it is ambiguous, the theory makes little sense.

According to (1), such a distinction is quite clear in the *GT*. That is to say, given the aggregate supply function ϕ and the propensity to consume χ , investment demand D_2 is an independent variable, while the volume of employment N is a dependent variable. No one makes a mistake. To repeat, this is “the essence of the General Theory of Employment.” But there are no money wages in it. It is not possible to analyze the effect of money wages on employment by using (1).

In Chapter 18, however, money wages are taken into consideration in the argument of the distinction as follows:

Our independent variables are, in the first instance, the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest

Our dependent variables are the volume of employment and the national income (or national dividend) measured in wage-units. (p. 245)

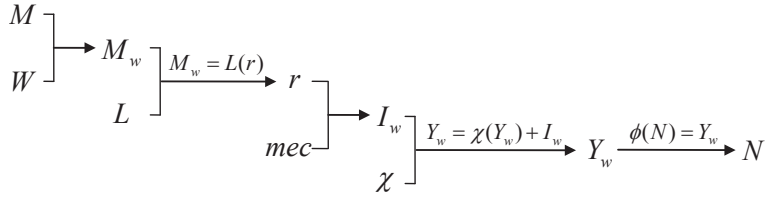
Let us pay attention to the last term “the national income (or national dividend) measured in wage-units.” As can be seen from the title of Chapter 4 “The Choice of Units,” Keynes was very nervous about the units in which economic variables are measured. After some consideration, he concluded as follows:

In dealing the theory of employment I propose ... to make use of only two fundamental units of quantity, namely, quantities of money-value and quantities of employment. The first of these is strictly homogeneous, and the second can be made so. ... We shall call the unit in which the quantity of employment is measured the labour-unit; and the money-wage of a labour-unit we shall call the wage-unit. [Footnote 1] If X stands for any quantity measured in terms of money, it will be convenient to write X_w for the same quantity measured in terms of the wage-unit. (p. 41)

The labour-unit is defined as “an hour’s employment of ordinary labour” and the volume of employment N is now measured in labour-units. The wage-unit, i.e., the money rate of wage of a labour-unit, is denoted by W , so that the above relation between X and X_w can be written as $X_w = X/W$. Thus, “national income (or national dividend)” can be measured either in terms of money as Y or in terms of the wage-unit as Y_w . But it is the latter that was really chosen.

Now look at the equilibrium condition (1) again from this point of view. Then, it would be right to regard it as expressed in terms of the wage-unit. By definition “income is equal to the value of current output.”⁽¹³⁾ It means $\phi(N) = Y_w$. Furthermore, in the *GT*, consumption demand and investment demand both in terms of money are denoted respectively by C and I . Taking these into account, (1)

Figure 1. The Essence of the General Theory of Employment.



can be rewritten as

$$\phi(N) = Y_w = C_w + I_w, \tag{2}$$

where $Y_w = Y/W$, $C_w = C/W (= D_1)$, and $I_w = I/W (= D_2)$. That is, $\phi(N)$ is the aggregate supply function, Y_w income, C_w consumption demand, I_w investment demand, all measured in terms of the wage-unit. It is (2) that Keynes used to examine the role of money wages in the determination of employment.

Remember that in (1) consumption demand D_1 was a function of N . However, in Chapter 8 Keynes modified it by assuming that consumption demand is a function of income measured in terms of the wage-unit, i.e., $C_w = \chi(Y_w)$.⁽¹⁴⁾ On the basis of this consumption function, he ultimately specified the independent variables for his analysis as follows:⁽¹⁵⁾

... the rate of interest [r] depends partly on the state of liquidity-preference (*i.e.* on the liquidity function [L]) and partly on the quantity of money measured in terms of wage-units [M_w]. Thus we can sometimes regard our ultimate independent variables as consisting of (1) the three fundamental psychological factors, namely, the psychological propensity to consume [χ], the psychological attitude to liquidity [L] and the psychological expectation of future yield from capital-assets [mec], (2) the wage-unit [W] as determined by the bargains reached between employers and employed, and (3) the quantity of money [M] as determined by the action of the central bank; so that, if we take as given the factors specified above, these variables determine the national income (or dividend) [Y_w ⁽¹⁶⁾] and the quantity of employment [N]. (pp. 246–247)⁽¹⁷⁾

Here mec stands for the marginal efficiency of capital.

Figure 1 shows how five independent variables, M , W , L , mec , and χ , determine Y_w and N . The process can easily be understood by dividing it into four steps. First, the quantity of money M is determined by the central bank, while the wage-unit W is determined by the bargains between employers and employed. As a result, the quantity of money measured in terms of wage-units $M_w (= M/W)$ is fixed. Second, following the liquidity preference theory, the rate of interest r is so determined as to equate the demand for money $L(r)$ with M_w .⁽¹⁸⁾ Third, investment demand measured in terms of wage-units I_w is determined at the point where the marginal efficiency of capital mec is equal to the rate of interest r . Fourth, following the principle of effective demand, I_w and the propensity to consume χ determine between them income Y_w measured in terms of wage-units,⁽¹⁹⁾ and at the same time the volume of employment N through the relation $\phi(N) = Y_w$ in (2).⁽²⁰⁾

The process of the determination of N after the third step corresponds to “the essence of the General Theory of Employment” (1). And Figure 1 as a whole represents “the essence of the General Theory of Employment” when money wages are considered explicitly.⁽²¹⁾

4 The *GT* Proposition

Now we are in a position to verify Keynes's proposition that a cut in money wages is not a remedy for unemployment since it does not lead to a fall in real wages. Let us call it the *GT* proposition. Correctly speaking, Keynes tried to address two questions concerning a reduction in money wages in Chapter 19 "Changes in Money-Wages." They were of course important to him. But this paper focuses on the first question of the two since the arguments in the previous sections of this paper are helpful for the most part to consider it. The first question is as follows:

Does a reduction in money-wages have a direct tendency, *cet. par.* to increase employment, "*cet. par.*" being taken to mean that the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest are the same as before for the community as a whole? (p. 260)

And the question was answered as follows:⁽²²⁾

The first question we have already answered in the negative in the preceding chapters. For we have shown that the volume of employment [N] is uniquely correlated with the volume of effective demand measured in wage-units [$Y_w (= D_w)$], and that the effective demand, being the sum of expected consumption [C_w] and the expected investment [I_w], cannot change, if the propensity to consume [χ], the schedule of the marginal efficiency of capital [mec] and the rate of interest [r] are all unchanged. (pp. 260-261)

The *GT* proposition was proved in this way.⁽²³⁾ As far as the above quotations are concerned, it appears that a reduction in money wages does not affect the volume of employment. But did Keynes really succeed in the proof? I do not think so for two reasons.

The first reason is that there is no proof that I_w remains constant in response to a fall in the wage-unit W . Since N is uniquely correlated with Y_w and Y_w in turn is determined uniquely by I_w , I_w must not change when W is reduced. As is seen from Figure 1, I_w remains unchanged if and only if the rate of interest r is constant, given mec . Furthermore, r is constant if and only if the quantity of money M is decreased in the same proportion to W to keep M_w constant, given L . Such a relation between W and M must be assumed implicitly for the *GT* proposition to hold.⁽²⁴⁾ Nonetheless, even if such an assumption is made explicit, it remains an open, and that critical, question how investment demand is determined *in terms of the wage-unit*.

The second reason is that the propensity to consume was modified in a fashion suitable to the *GT* proposition. As mentioned in the last section, the propensity to consume χ was modified in Chapter 8 as follows:

Since we are here concerned in determining what sum will be spent on consumption when employment is at a given level, we should, strictly speaking, consider the function which relates the former quantity (C) to the latter (N). It is more convenient, however, to work in terms of a slightly different function, namely, the function which relates the consumption in terms of wage-units (C_w) to the income in terms of wage-units (Y_w) corresponding to a level of employment N

We will therefore define what we shall call *the propensity to consume* as the functional relationship χ between Y_w , a given level of income in terms of wage-units, and C_w the expenditure on consumption out of that level of income, so that

$$C_w = \chi(Y_w) \text{ or } C = W \cdot \chi(Y_w).$$

(p. 90, My underlines.)

According to Keynes's own belief, "Consumption (C) is obviously much more a function of (in some sense) *real* income than of money-income." (p. 91)⁽²⁵⁾ Nonetheless, he defined the propensity to consume as a function of income measured in terms of wage-units (Y_w) because "real income, in terms of product, may be incapable of precise numerical measurement". (p. 114) "Real income measured in terms of product and income measured in terms of wage-units will, however, increase and decrease together (in the short period when capital equipment is virtually unchanged). ... therefore, ... it is often convenient to regard income in terms of wage-units (Y_w) as an adequate working index of changes in real income." (p. 114) That is how he justified the use of income measured in terms of wage-units (Y_w) in lieu of real income.

It is certain that Keynes regarded real income and income measured in terms of wage-units as "virtually interchangeable" (p. 114) for the sake of convenience. But it is Keynes, too, who said that "the aggregate income measured in terms of wage-unit is, as a rule, the principal variable upon which the consumption-constituent of the aggregate demand function will depend." (p. 96) The propensity to consume modified in terms of wage-units seems to be satisfactory enough to him. In fact, his arguments actually proceed on the basis of the propensity to consume in terms of wage-units, i.e., $C_w = \chi(Y_w)$. For example, the marginal propensity to consume is defined as dC_w/dY_w .

However, as is apparent from (2), the propensity to consume that uniquely determines equilibrium income measured in terms of the wage-unit when investment demand measured in terms of the wage-unit is given is only the propensity to consume modified in terms of wage-units.⁽²⁶⁾ In other words, Keynes could not choose but take it in order that the *GT* proposition may obtain. It was not a happy choice but an inevitable one. As far as I know, no one has noticed the implication of the modification of the propensity to consume *in terms of wage-units*.

For the two reasons above it cannot be said that the *GT* proposition was proved. The final goal of the *GT* was not achieved.

5 Digging up the Production Functions

The previous section focused on the latter half of (2), i.e., $Y_w = C_w + I_w$.⁽²⁷⁾ Further, since the propensity to consume is defined as $C_w = \chi(Y_w)$, it becomes

$$Y_w = \chi(Y_w) + I_w. \tag{3}$$

Thus, given χ , investment demand I_w completely determines the equilibrium value of income Y_w . Apart from the unit used, this is the traditional way to get at the equilibrium in the goods market. And, as Patinkin [21, p. 360] said, this is the reason why it is criticized as "Keynesian models neglect the supply side of the market."

But such a criticism is quite a misunderstanding because (3) is just another expression of "the essence of the General Theory of Employment" (1) which is made up of the propensity to consume,

the volume of investment, and the aggregate supply function. No one doubts that (1) represents the equilibrium between supply and demand in the goods market in a natural sense. When it comes to (3), on the other hand, it is wrongly believed that the equilibrium in the goods market can be found without the supply side, probably because the principle of effective demand is too much emphasized. (3) is equivalent to (1) through the definition $\phi(N) = Y_w$. Income, which is usually thought to represent the demand side, works to connect the two sides as seen from (2).

In order to prove the *GT* proposition, Keynes used (3), not (1). As far as the proof is concerned, it would be reasonable to choose the simpler one. Then don't we need to consider (1) any more? Indeed Keynes put emphasis solely on the aggregate demand function, saying "The aggregate supply function, ... which depends in the main on the physical conditions of supply, involves few considerations which are not already familiar. The form may be unfamiliar but the underlying factors are not new." (p. 89)⁽²⁸⁾ But we do not have to take it seriously because, as will be shown below, the detailed study of the aggregate supply function on the left-hand side of (1) enables us to understand the supply side in Keynes's theory of employment deeply. Most importantly, it reveals that there are production functions of a special type hidden in the *GT*.

To examine Keynes's aggregate supply function it is helpful to begin by citing Pigou's [23] *Theory of Unemployment*. Let $F(x)$ and $F'(x)$ be respectively the production function of the wage-goods industries and its derivative with respect to x , the number of men employed there. Then,

... the general rate of wage is $F'(x)$. There are also engaged in other industries y further wage-earners, the wage payment to whom amounts, of course, to $yF'(x)$. (Pigou [23, pp. 89-90])

Although Keynes criticized the theoretical structure of Pigou [23], it is important to notice that Keynes confirmed that $F(x)$ "represents the physical conditions of production in the wage-goods industries" (p. 279)

On the basis of Pigou's setting above, Keynes explained the relationship between the production function $F(x)$ of the consumption-goods sector and his consumption demand measured in terms of the wage-unit C_w as follows:

In so far as we can identify Professor Pigou's wage-goods with my consumption-goods, and his "other goods" with my investment-goods, it follows that his $\frac{F(x)}{F'(x)}$, being the value of the output of the wage-goods industries in terms of the wage-unit, is the same as my C_w . (p. 273)⁽²⁹⁾

This is the crucial part of the *GT* to know what is the aggregate supply function.

Let us see what the above quotation means more in detail. Denote the output of consumption goods, their price, and the profit of the consumption-goods sector by O_x , p_x , and Π_x , respectively. Then, the production function and the profit can be written respectively as

$$O_x = F(x), \tag{4}$$

and

$$\begin{aligned} \Pi_x &= p_x O_x - Wx \\ &= p_x F(x) - Wx, \end{aligned}$$

where the wage-unit W is given. The first-order condition for profit maximization $d\Pi_x/dx = 0$ becomes

$$\frac{W}{p_x} = F'(x).$$

$F'(x)$ is the marginal productivity of labor and the real wage W/p_x is equal to it because of the profit-maximizing behavior of firms. All this is perfectly elementary.

Now the equilibrium condition in the consumption-goods market is expressed as

$$p_x O_x = C, \tag{5}$$

where C is consumption demand measured in terms of money. The first-order condition above can also be written as

$$p_x = \frac{W}{F'(x)}. \tag{6}$$

Then, substituting (4) and (6) into (5) yields

$$\frac{F(x)}{F'(x)} W = C. \tag{7}$$

Finally dividing both sides of (7) by W gives

$$\frac{F(x)}{F'(x)} = C_w, \tag{8}$$

where $C_w = C/W$. This is what the above quotation means.

Both sides of (8) are measured in terms of the wage-unit. The left-hand side is “the value of the output of the wage-goods industries in terms of the wage-unit.” And it is also the aggregate supply price since the proceeds $p_x O_x$ “will just make it worth the while of the entrepreneurs to give that employment” in the sense that the profit is maximized at the level x of employment. An important point is that the wage-unit W in itself belongs to the left-hand side representing the supply side as in (7). But it is transposed to the right-hand side representing the demand side as in (8). Hence the propensity to consume measured in the wage-unit. The justification of the use of quantities of employment for measuring consumption demand cannot be understood until the aggregate supply price is explicitly introduced into the analysis.⁽³⁰⁾

At any rate, a production function was already used by Pigou [23] for the theory of (un)employment and Keynes took advantage of it in the (unfamiliar) form of the aggregate supply function. The two is theoretically connected through $F(x)$.

However, both Pigou and Keynes talked about the production function or the aggregate supply function of the consumption-goods sector alone.⁽³¹⁾ Is it strange to deal only with the production function of the consumption-goods sector? Can the production function of the investment-goods sector be dispensed with? In fact later Pigou [24] extended his theory by introducing the production function of the investment-goods sector $\psi(y)$. How about Keynes? Admittedly he did not make it explicit in the *GT*. But it must be hidden there since the *GT* is based on the two-sector model consisting of the consumption-goods and investment-goods sectors. The consumption-goods sector has its own production function. Why not the investment-goods sector? Then, let us examine the production function of the investment-goods sector as well as that of the consumption-goods sector in the *GT*.

First denote the production function of the consumption-goods sector and that of the investment-goods sector respectively by

$$O_1 = F(N_1), F'(N_1) > 0, F''(N_1) < 0,$$

and

$$O_2 = G(N_2), G'(N_2) > 0, G''(N_2) < 0.$$

Subscript 1 represents consumption goods, whereas subscript 2 investment goods as in the *GT*. Then, O_1 and O_2 are the output of consumption goods and that of investment goods, while N_1 and N_2 are the volume of employment in the consumption-goods sector and that in the investment-goods sector. O_1 and O_2 cannot be added but the sum of N_1 and N_2 makes sense. Thus, $N = N_1 + N_2$. The function $O_1 = F(N_1)$ is just the same as (4), N_1 corresponding to x . The two conditions $F''(N_1) < 0$ and $G''(N_2) < 0$ mean the decreasing returns in both sectors.

Next, let p_1 and p_2 be the price of consumption goods and that of investment goods, respectively. The first-order conditions for profit maximization imply

$$p_1 = \frac{W}{F'(N_1)}, \tag{9}$$

and

$$p_2 = \frac{W}{G'(N_2)}. \tag{10}$$

(9) is just the same as (6). Then, the aggregate supply function of the consumption-goods sector $\phi_1(N_1)$ and that of the investment-goods sector $\phi_2(N_2)$ are given respectively by

$$\phi_1(N_1) = \frac{F(N_1)}{F'(N_1)}$$

and

$$\phi_2(N_2) = \frac{G(N_2)}{G'(N_2)}.$$

$\phi_1(N_1)$ and $\phi_2(N_2)$ are both measured in terms of the wage-unit and so can be added. Thus,

$$\phi(N) = \phi_1(N_1) + \phi_2(N_2), \tag{11}$$

where $\phi(N)$ is exactly what appears in (2).

Finally, considering the correspondence of $\phi_1(N_1)$ and $\phi_2(N_2)$ with the equilibrium condition in the goods market $Y_w = C_w + I_w$ yields the following two equations:

$$\phi_1(N_1) = C_w$$

and

$$\phi_2(N_2) = I_w. \tag{12}$$

The first equation is just the same as (8), i.e., the equilibrium condition in the consumption-goods market.³²⁾

Then, what shape do the production functions $F(N_1)$ and $G(N_2)$ take? Keynes implicitly gave the conditions necessary to answer this question as follows:

... $\Delta Y_w = \Delta C_w + \Delta I_w$, where ΔC_w and ΔI_w are the increments of consumption and investment; so that we can write $\Delta Y_w = k\Delta I_w$, where $1 - \frac{1}{k}$ is equal to the marginal propensity to consume.

Let us call k the *investment multiplier*. It tells us that, when there is an increment of aggregate investment, income will increase by an amount which is k times the increment of investment. ...

Mr. Kahn's multiplier is a little different from this, being what we call the *employment multiplier* designated by k' , since it measures the ratio of the increment of total employment which is associated with a given increment of primary employment in the investment industries. That is to say, if the increment of investment ΔI_w leads to an increment of primary employment ΔN_2 in the investment industries, the increment of total employment $\Delta N = k'\Delta N_2$.

There is no reason in general to suppose that $k = k'$ But to elucidate the ideas involved, it will be convenient to deal with the simplified case where $k = k'$. (pp.115-116)

According to the above quotations, as for the marginal propensity to consume,

$$dC_w = \left(1 - \frac{1}{k}\right) dY_w,$$

and as for the investment multiplier,

$$dY_w = kdI_w.$$

Then, when there is an increment of aggregate investment I_w ,

$$\begin{aligned} dN_1 &= \frac{1}{\phi'_1(N_1)} dC_w \\ &= \frac{1}{\phi'_1(N_1)} \left(1 - \frac{1}{k}\right) dY_w \\ &= \frac{1}{\phi'_1(N_1)} \left(1 - \frac{1}{k}\right) kdI_w, \end{aligned} \tag{13}$$

and

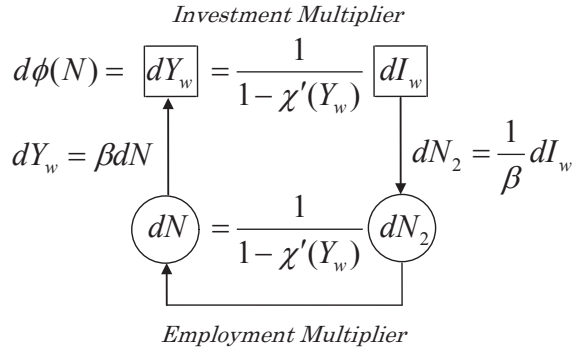
$$dN_2 = \frac{1}{\phi'_2(N_2)} dI_w. \tag{14}$$

Using (13) and (14), the employment multiplier k' can be calculated as follows:³³

$$\begin{aligned} k' &= \frac{dN}{dN_2} \\ &= \frac{dN_1}{dN_2} + 1 \\ &= \frac{\phi'_2(N_2)}{\phi'_1(N_1)} (k - 1) + 1. \end{aligned}$$

Put $k = k'$ "to elucidate the ideas involved."³⁴ Then, the above relation leads to

Figure 2. The Relationship between Keynes's Investment Multiplier and Kahn's Employment Multiplier.



$$\phi'_1(N_1) = \phi'_2(N_2) (= \beta),$$

which in turn means

$$\phi_1(N_1) = \beta N_1 + B_1, \tag{15}$$

and

$$\phi_2(N_2) = \beta N_2 + B_2, \tag{16}$$

where β is a positive parameter, and B_1 and B_2 are both integration constants. Note that (15) and (16) hold for all possible value of N_1 and N_2 . It follows from (11) that

$$\phi(N) = \beta N + B, \tag{17}$$

where $B = B_1 + B_2$. Moreover, Y_w is expressed as

$$Y_w = \beta N + B, \tag{18}$$

due to (2) and (17). Note that Y_w is a function of total employment N alone.

As summarized earlier, Kahn showed that the primary employment resulting from an increase in investment demand eventually leads to the increase in employment as a whole by the employment multiplier times as much as the primary employment. In order to consider it within the framework of the *GT*, denote the increase in investment demand by dI_w , the resulting primary employment by dN_2 , and the increase in employment as a whole by dN . Then, using (3), (12), (16), and (18), the relationship between dN_2 and dN can be written as

$$dN = \frac{1}{1 - \chi'(Y_w)} dN_2,$$

where Y_w is given by (18). The employment multiplier is $\frac{1}{1 - \chi'(Y_w)}$. Figure 2 shows the relationship between Kahn's employment multiplier and Keynes's investment multiplier. A starting point is always dI_w . The investment multiplier describes the effect of dI_w on dY_w , while the employment multiplier that of dN_2 resulting dI_w on dN . As is seen from the figure, both multiplier coincides. That is, I think, why Keynes was able to say, "It follows that we shall measure changes in current output by reference to the number of hours of labour paid for (whether to satisfy consumers or to produce fresh

capital equipment)" (p. 44)

Now remember $\phi_1(N_1) = F(N_1)/F'(N_1)$. Then, (15) becomes

$$\frac{F'(N_1)}{F(N_1)} = \frac{1}{\beta N_1 + B_1}.$$

Integrating both sides yields

$$F(N_1) = a_1 (\beta N_1 + B_1)^{1/\beta}, \quad (19)$$

where a_1 is a positive constant. Further, the condition $F''(N_1) < 0$ implies that $\beta > 1$. Similarly using (16), the production function of the investment industries can be specified as

$$G(N_2) = a_2 (\beta N_2 + B_2)^{1/\beta}, \quad (20)$$

where a_2 is a positive constant.⁽³⁵⁾

The production functions have been specified to a considerable extent. But, are they consistent with the GT which includes various statements? The next task to do is to answer this question.

6 Further Considerations of the Production Functions

It is obvious that Keynes always assumed the decreasing returns as the volume of employment increases.⁽³⁶⁾ Such an assumption was already taken into account by the two conditions $F''(N_1) < 0$ and $G''(N_2) < 0$. The following statement also reflects it.

Quantity of investment is measured, above, by the number of men employed in producing it. Thus if there are diminishing returns per unit of employment as employment increases, what is double the quantity of investment on the above scale will be less than double on a physical scale (if such a scale is available). (Footnote 1, p. 126)

Now "such a scale is available" since the production function of the investment industries has been derived. It follows that the above statement claims that $G(2N_2) < 2G(N_2)$ obtains. That is,

$$a_2 (\beta 2N_2 + B_2)^{1/\beta} < 2a_2 (\beta N_2 + B_2)^{1/\beta}.$$

Arranging it yields

$$2 - \frac{B_2}{\beta N_2 + B_2} < 2^{1/\beta}. \quad (21)$$

In order for the above inequality to hold for all possible values of N_2 such that $\beta N_2 + B_2 > 0$, $B_2 \geq 0$.⁽³⁷⁾ The above quotations are concerned with the decreasing returns in the investment-goods sector and it also applies to the consumption-goods sector. Thus, $B_1 \geq 0$. Therefore,

$$B \geq 0. \quad (22)$$

Next, let us check out if the following two statements are satisfied.⁽³⁸⁾

1. Income measured in terms of wage-units will increase more than in proportion to the amount of

employment.

2. The increase of employment will tend to increase the proportion of aggregate income which accrues to the entrepreneurs.

Statement 1 claims that the ratio Y_w/N rises as N increases. On the other hand, the “proportion of aggregate income which accrues to the entrepreneurs” in Statement 2 is expressed as follows:

$$\frac{Y - WN}{Y} = 1 - \frac{N}{Y_w}.$$

Thus, Statement 2 claims that $1 - N/Y_w$ rises as N increases. In sum, both statements are mathematically equivalent.

Y_w in equilibrium has already been obtained as (18). Then, Statements 1 and 2 hold if and only if

$$B < 0. \tag{23}$$

But (23) contradicts (22)! It is found that there is no value of the sum B of integration constants which satisfies the above statements at once. Here the arguments met a dead end.

A way out is to leave the GT in this respect and to look at Keynes [12] who changed his previous conviction expressed in Statement 2.^{(39), (40)} Keynes [12, pp. 48-49] admitted “the stability of the proportion of the national dividend accruing to labour” as “one of the most surprising, yet best established, facts in the whole range of economic statistics” and even “a bit of a miracle.” The “proportion of the national dividend accruing to labour” is given by

$$\frac{WN}{Y} = \frac{N}{Y_w}.$$

Thus, the 1939 statement claims that N/Y_w is constant, irrespective of the level of employment N . As is apparent, this holds if and only if

$$B = 0. \tag{24}$$

(22) is still valid, so that (24) implies both $B_1 = 0$ and $B_2 = 0$.⁽⁴¹⁾

In this case, the aggregate supply functions (15) - (17) and income measured in terms of wage-units (18) become respectively

$$\phi_1(N_1) = \beta N_1, \tag{25}$$

$$\phi_2(N_2) = \beta N_2, \tag{26}$$

$$\phi(N) = \beta N, \tag{27}$$

and

$$Y_w = \beta N, \tag{28}$$

where $\beta > 1$ as before.⁽⁴²⁾ $1/\beta$ represents “the proportion of the national dividend accruing to labour” (or labor share) because $N/Y_w = WN/Y = 1/\beta$. Moreover, putting $A_1 = a_1(\beta)^{1/\beta}$, $A_2 = a_2(\beta)^{1/\beta}$, and $1 - \alpha = 1/\beta$ for convenience sake, the production functions (19) and (20) are simplified further to

$$F(N_1) = A_1 N_1^{1-\alpha}, \quad (29)$$

and

$$G(N_2) = A_2 N_2^{1-\alpha}, \quad (30)$$

where $A_1 > 0$, $A_2 > 0$, and $0 < \alpha < 1$.⁽⁴³⁾ Things become very simple.⁽⁴⁴⁾

Needless to say, the *GT* deals with the short run in which capital stock is regarded as given. Technology is given, too. In fact, there are no variables representing capital stock or technology in (29) and (30). But it can be said that both of them are embedded in A_1 in (29) and A_2 in (30). So it is interesting to ask what functions A_1 and A_2 are. Particularly such a question must be addressed if the long run is to be studied.

So, for the present, let A_1 and A_2 be functions of capital stock K_1 and K_2 in each sector, i.e., $A_1 = A_1(K_1)$ and $A_2 = A_2(K_2)$. Then, the rate of profit in the consumption-goods sector can be written as

$$\begin{aligned} \frac{\Pi_1}{p_2 K_1} &= \frac{p_1 A_1(K_1) N_1^{1-\alpha} - W N_1}{p_2 K_1} \\ &= \frac{p_1 \left[A_1(K_1) N_1^{1-\alpha} - \frac{W}{p_1} N_1 \right]}{p_2 K_1} \\ &= \frac{p_1 \left[A_1(K_1) N_1^{1-\alpha} - A_1(K_1) (1-\alpha) N_1^{-\alpha} N_1 \right]}{p_2 K_1} \\ &= \frac{p_1 \alpha A_1(K_1) N_1^{1-\alpha}}{p_2 K_1}. \end{aligned}$$

And the rate of profit in the investment-goods sector is given by

$$\begin{aligned} \frac{\Pi_2}{p_2 K_2} &= \frac{p_2 A_2(K_2) N_2^{1-\alpha} - W N_2}{p_2 K_2} \\ &= \frac{A_2(K_2) N_2^{1-\alpha} - \frac{W}{p_2} N_2}{K_2} \\ &= \frac{A_2(K_2) N_2^{1-\alpha} - A_2(K_2) (1-\alpha) N_2^{-\alpha} N_2}{K_2} \\ &= \frac{\alpha A_2(K_2) N_2^{1-\alpha}}{K_2}. \end{aligned}$$

It would be natural to assume that *in the long run* the two rates of profit coincide under the condition that the two prices p_1 and p_2 are equal.^{(45), (46)} Then,

$$\frac{\alpha A_1(K_1) N_1^{1-\alpha}}{K_1} = \frac{\alpha A_2(K_2) N_2^{1-\alpha}}{K_2}.$$

The equation above is rewritten as

$$\frac{A_1(K_1)}{A_2(K_2)} = \frac{K_1}{K_2} \left(\frac{N_2}{N_1} \right)^{1-\alpha}.$$

The above relation must hold for all possible values of N_1 and N_2 . So put $N_1 = K_1$ and $N_2 = K_2$. Then,

$$\frac{A_1(K_1)}{A_2(K_2)} = \frac{A^{1-\alpha} K_1^\alpha}{A^{1-\alpha} K_2^\alpha},$$

where A is an arbitrary positive constant that reminds you of the level of technology.

It follows that (29) and (30) can take the following forms:

$$F(N_1, K_1) = K_1^\alpha (AN_1)^{1-\alpha},$$

and

$$G(G_2, K_2) = K_2^\alpha (AN_2)^{1-\alpha}.$$

These belong to the Cobb-Douglas production function with the Harrod neutral technological progress, the most trusted production function in economics.⁽⁴⁷⁾ Here it should be added at once that these production functions are not what Keynes intended because he believed the definition of the physical unit of capital “to be both insoluble and unnecessary.” (p. 138) But it should also be emphasized that the aggregate supply functions $\phi_1(N_1)$, $\phi_2(N_2)$, and $\phi(N)$, and income Y_w measured in terms of wage-units *always* take the forms of (25) – (28), whether capital stock and/or technology are made explicit or not.⁽⁴⁸⁾

7 Conclusion

All in all, this paper has done two things. First, it showed that the final goal of the *GT* is to prove the proposition that a cut in money wages is not a remedy for unemployment since it does not lead to a fall in real wages (the *GT* proposition). The traditional view is that the essence of the *GT* lies in the principle of effective demand and/or the liquidity preference theory because they make it possible to explain why involuntary unemployment exists. But if a wage cut leads to a reduction in unemployment, such unemployment is *not* involuntary unemployment. And involuntary unemployment is the kind of unemployment the classical school cannot explain. Therefore, the *GT* proposition is, I believe, what Keynes really wanted to say.

It is the wage-unit that was adopted as the basic unit in order to verify the *GT* proposition. The way is very simple. Income, consumption demand, and investment demand are all measured in terms of the wage-unit. Then, if these variables do not change in response to a fall in money wages, the proposition obtains. However, Keynes did not succeed in the proof because the constancy of investment demand to a change in money wages was not well founded and the propensity to consume was modified *ad hoc*.

Second, this paper proceeded to the supply side since the wage-unit used for the *GT* proposition originally belonged to the supply side, and specified production functions which have long been hiding themselves in the *GT*. Keynes mentioned a kind of production functions here and there in the *GT*. It is obvious that he was thinking on the basis of production functions. Then, what shape do they take? This question was answered for the first time by paying attention to the relationship between two

multipliers. That is, when Kahn's employment multiplier and Keynes's investment multiplier coincide, such production functions proved to be of the familiar Cobb-Douglas type. Needless to say, the Cobb-Douglas production function had already been discovered in 1928 by Cobb and Douglas from an empirical viewpoint. Then, it is an amazing fact that Keynes was taking quite a different route to similar production functions, isn't it? In any case, this result renders things very simple.

Although the *General Theory* seems to be an "obscure" book, this paper showed that it is so robust as to be analyzed rigorously from a mathematical point of view. In such a sense it remains the foundation of macroeconomics.

Appendix: Applications of Results (25) – (30) to Formulas in Chapter 20

Here are the applications of results (25) – (30) to various formulas in Chapter 20 which are calculated in general form. Things certainly become simple.

(A1.1) The employment function for a given industry (p. 280):

$$\begin{aligned} N_r &= F_r(D_{wr}) \\ &= (1 - \alpha)D_{wr}, \end{aligned}$$

where it is implicitly assumed that $D_{w1} = C_w$ and $D_{w2} = I_w$.

(A1.2) The employment function for industry as a whole (p. 282):

$$\begin{aligned} N &= F(D_w) \\ &= (1 - \alpha)D_w, \end{aligned}$$

since $N = N_1 + N_2$. To derive $N = F(D_w)$, Keynes assumed that "corresponding to a given level of aggregate effective demand there is a unique distribution of it between different industries." (p. 282) However such a strong assumption is not necessary.

(A2.1) The elasticity of employment with respect to effective demand in terms of wage-units for a given industry (p. 282):

$$\begin{aligned} e_{er} &= \frac{dN_r}{dD_{wr}} \frac{D_{wr}}{N_r} \\ &= 1. \end{aligned}$$

(A2.2) The elasticity of employment with respect to effective demand in terms of wage-units for industry as a whole (p. 282):

$$\begin{aligned} e_e &= \frac{dN}{dD_w} \frac{D_w}{N} \\ &= 1. \end{aligned}$$

(A3) The elasticity of output with respect to effective demand in terms of wage-units for a given industry (p. 283):

$$\begin{aligned} e_{Or} &= \frac{dO_r}{dD_{wr}} \frac{D_{wr}}{O_r} \\ &= 1 - \alpha, \end{aligned}$$

since $O_r = A_r[(1 - \alpha)D_{wr}]^{1-\alpha}$ in equilibrium. Keynes said, "Ordinarily, of course, e_{O_r} will have a value intermediate between zero and unity." (p. 284) He was quite right.

(A4) The relationship between an increase in effective demand in terms of wage-units and the corresponding increase in the expected profit P_{wr} (p. 283):

$$\begin{aligned} \Delta D_{wr} &= \frac{1}{1 - e_{O_r}} \Delta P_{wr} \\ &= \frac{1}{\alpha} \Delta P_{wr}, \end{aligned}$$

due to (A3). There is another way to obtain the above relation. In equilibrium,

$$\begin{aligned} P_{wr} &= p_{wr}O_r - N_r \\ &= \beta N_r - N_r \\ &= (\beta - 1)N_r. \end{aligned}$$

Then,

$$\begin{aligned} \frac{dP_r}{dD_w} &= \frac{dP_r}{dN_r} \frac{dN_r}{dD_{wr}} \\ &= (\beta - 1)(1 - \alpha) \\ &= \alpha, \end{aligned}$$

because of (A1.1).

(A5) The relationship between the two elasticities and the production function (p. 283):

$$\frac{1 - e_{O_r}}{e_{er}} = - \frac{N_r \phi''(N_r)}{p_{wr} \{\phi'(N_r)\}^2}.$$

As for the left-hand side of the above relation,

$$\begin{aligned} \frac{1 - e_{O_r}}{e_{er}} &= \frac{1 - (1 - \alpha)}{1} \\ &= \alpha, \end{aligned}$$

due to (A1.1) and (A2.1). As for the right-hand side, it should be noticed that $\phi(N_r)$ is not the aggregate supply function but the production function, i.e., $\phi(N_1) = F(N_1) = A_1 N_1^{1-\alpha}$ and $\phi(N_2) = G(N_2) = A_2 N_2^{1-\alpha}$. Thus,

$$\begin{aligned} \phi'(N_r) &= (1 - \alpha)A_r N_r^{-\alpha}, \\ \phi''(N_r) &= -\alpha(1 - \alpha)A_r N_r^{-\alpha-1}, \end{aligned}$$

and also

$$p_{wr} = \frac{1}{\phi'(N_r)}.$$

Therefore,

$$\begin{aligned} -\frac{N_r \phi''(N_r)}{p_{wr} \{\phi'(N_r)\}^2} &= -\frac{N_r \phi''(N_r)}{\phi'(N_r)} \\ &= \alpha. \end{aligned}$$

(A6) The sum of the elasticities of price and of output in response to changes in effective demand measured in terms of wage-units (p. 284):

$$e'_{pr} + e_{Or} = 1.$$

e_{Or} has already been obtained in (A3). On the other hand,

$$\begin{aligned} p_{wr} &= \frac{1}{\phi'(N_r)} \\ &= \frac{1}{(1-\alpha)A_r N_r^{-\alpha}} \\ &= \frac{1}{(1-\alpha)A_r [(1-\alpha)D_{wr}]^{-\alpha}}. \end{aligned}$$

Therefore,

$$\begin{aligned} e'_{pr} &= \frac{dp_{wr}}{dD_{wr}} \frac{D_{wr}}{p_{wr}} \\ &= \alpha. \end{aligned}$$

As a result,

$$\begin{aligned} e'_{pr} + e_{Or} &= \alpha + (1-\alpha) \\ &= 1. \end{aligned}$$

(A7) The relationship between the elasticities of output and of money wages in response to changes in effective demand in terms of money (pp. 285–286):

Although this relationship was mentioned, it was not written explicitly. On p. 285 Keynes derived the elasticity of money-prices in response to changes in effective demand measured in terms of money as $e_p = 1 - e_O(1 - e_w)$, where $e_w \left(= \frac{DdW}{WdD} \right)$ is the elasticity of money wages in response to changes in effective demand in terms of money. But neither p nor O was not defined. Rather, the relation should be written for a given industry as $e_{pr} = 1 - e_{Or}(1 - e_w)$, where $e_{pr} = \frac{D_r dp_r}{p_r dD_r}$, $e_{Or} = \frac{D_{wr} dO_r}{O_r dD_{wr}}$ as before, and $e_w = \frac{D_r dw}{w dD_r}$. Thus, the relationship (A7) can be calculated as follows:

$$\begin{aligned} \frac{dO_r}{dD_r} \frac{D_r}{O_r} &= 1 - \frac{dp_r}{dD_r} \frac{D_r}{p_r} \\ &= e_{Or} (1 - e_w) \\ &= (1-\alpha)(1 - e_w). \end{aligned}$$

Note that if $e_w = 0$, or money wages are fixed, (A7) reduces to (A3). It is easy to rewrite (A7) in terms of the volume of employment as follows:

$$\begin{aligned} \frac{dN_r}{dD_r} \frac{D_r}{N_r} &= \left(\frac{dO_r}{dN_r} \frac{N_r}{O_r} \right)^{-1} \left(1 - \frac{dp_r}{dD_r} \frac{D_r}{p_r} \right) \\ &= (1 - \alpha)^{-1} (1 - \alpha)(1 - e_w) \\ &= 1 - e_w. \end{aligned}$$

As to the equation $e_p = 1 - e_o(1 - e_w)$, Keynes said, "... if $e_w = 1$, output will be unaltered and prices will rise in the same proportion as effective demand in terms of money." (p.286) It also applies to (A7). In other words, e_w must be less than one to increase output and employment by increasing *money* expenditure. This can also be understood at once, for example, by paying attention to the relation between W and C in (7). It follows that the stickiness of money wages matters when the effect of an increase in money expenditure is examined. This conclusion is virtually the same as that of Modigliani [19] who stressed the role of rigid money wages in the *GT* long ago.

Notes

- (1) The number [29] in the above citation refers to page 29 of the *GT*.
- (2) The page number not designated is that of the *GT* in what follows.
- (3) See Keynes [14, p. 422].
- (4) The paper was presented in 1932 at the annual meeting of the American Statistical Association.
- (5) See, e.g., pp. 116-117.
- (6) For example, Kahn [7, p. 182] said, "It should now be clear that the whole question ultimately turns on the nature of the supply curve of consumption-goods."
- (7) For a similar statement by Keynes, see p. 276.
- (8) Immediately after the quotation, he added the note: "But it is suggested, though with some hesitation, that over a limited, and not so very limited, range the assumption is not appreciably wide of reality."
- (9) The chapter number is always that of the *GT* in what follows.
- (10) See also pp. 89-90.
- (11) Pigou's belief that the rigidity of money wages is an actual cause of unemployment and that the "plasticity" or flexibility of the former becomes a remedy for the latter went back to Pigou [22]. On the contrary, Keynes had a strong hatred for a wage reduction. See pp. 267-269 and 340. It should be added, however, he recognized the actual stickiness of money wages. See p. 232.
- (12) For another difference between Kahn and Keynes, see note 21 below.
- (13) See p. 63. See also pp. 20, 209.
- (14) The rationale of this consumption function will be explained in the next section.
- (15) Brackets and symbols therein are added by me for exposition.
- (16) Correctly speaking, Y_w is not national income (or dividend) but (gross) income. The difference between (gross) income and national income is capital depreciation or the supplementary cost in Keynes's terms. It is true that Keynes paid enough attention to the actual importance of capital depreciation in consumption demand. For example, see the tables on pp. 102-103. Nonetheless, it is sometimes more appropriate to deem that the *GT* assumes no capital depreciation. For example, see footnote 2 on p. 126 which mentioned the marginal and average propensity to consume.
- (17) Pigou [26, p. 65] thought that this passage contains the kernel of Keynes's contribution to economic thinking. Also it corresponds to what Hicks [3, p. 152] called "Mr. Keynes' *special theory*."
- (18) It goes without saying that traditionally the liquidity preference theory has been formulated by such an equation as $M = L_1(Y) + L_2(r)$. Indeed the equation and the justification of it can be found on p. 171 of Chapter 15 and it is what Hicks [3] adopted to make the *GT* "appreciably more orthodox." But, as Hicks admitted, *quite a large part of the argument of the GT runs without the term* $L_1(Y)$. For example, the diagram on p. 180 corresponds exactly to this case. See also pp. 183-185.

- (19) Hence investment demand is not a function of income. Investment demand and income do not interdepend as in the *IS-LM* model. Investment demand determines income, “not the other way round.” For further evidence, see Keynes [10, pp. 9, 110, 375], Keynes [11, pp. 221, 223], and Keynes [13, p. xxxiii].
- (20) The inverse function of $\phi(N) = Y_w$, i.e., $N = \phi^{-1}(Y_w)$, is what Keynes called the employment function.
- (21) In Figure 1 the quantity of money is supposed to finally determine income. It is the causality Keynes believed in. However, Kahn took the reverse causality as true. See Kahn [8, pp. 169–170].
- (22) Brackets and symbols therein are added again by me for exposition.
- (23) The second question is as follows: “(2) does a reduction in money-wages have a certain or probable tendency to affect employment in a particular direction through its certain or probable repercussions on these three factors [i.e., the propensity to consume, the schedule of the marginal efficiency of capital and the rate of interest]?” (p. 260) Keynes answered the second question almost negatively. It is interesting to know that Tobin [29] does not agree with Keynes on the effect of a change in money wage rates on the aggregate employment and output. See also Solow [28] for the consideration of the elasticity of labor demand with respect to the nominal wage rate.
- (24) It is Keynes that admitted, though with reservations, “We can, therefore, theoretically at least, produce precisely the same effects on the rate of interest by reducing wages, whilst leaving the quantity of money unchanged, that we can produce by increasing the quantity of money whilst leaving the level of wages unchanged.” (p. 266) In the discussion of the wage-theorem, Hicks [6, pp. 59–60] correctly pointed out this implicit assumption in the *GT*, though he mentioned a “rise” in money wages and the corresponding “increase” in the money supply. For the relationship between money wages and money expenditure, see (A7) in the appendix.
- (25) For a similar statement, see Harrod [2, p. 167].
- (26) A special case in which Y_w is uniquely determined is $C_w = cY_w$, c being a positive parameter less than unity. It can be rewritten in terms of money as $C = WcY_w$ or $C = cY$. But $C = cY + b$ with b as a non-zero constant, the most familiar consumption function, does not belong to the category represented by $C_w = \chi(Y_w)$ since Y_w in equilibrium varies according to the value of W .
- (27) Incidentally neither $Y_w = C_w + I_w$ nor $Y = C + I$ can be found as mathematical expressions in the *GT*.
- (28) It is interesting to point out that Hicks [4, p. 78], which was written under the strong influence of the *GT*, expressed a similar feeling on the theory of the supply side or the firm.
- (29) See also Pigou [26, p. 21].
- (30) Employment is a factor of *production*, and no wonder.
- (31) This applies to Kahn, too.
- (32) But (12) is not the equilibrium condition in the investment-goods market. It should be written, if necessary, as $\phi_2(N_2) = S_w$. Here S is saving defined as $S = Y - C$ as usual and $S_w = S/W$. I_w is not the purchasing power. Only current saving can buy investment goods currently produced. In the *GT*, however, it does not seem that current saving always goes to the purchase of investment goods, as Keynes said, “Saving, in fact, is a mere residual.” (p. 64)
- (33) The *GT* says that “in the more general case it [i.e., the employment multiplier] is also a function of the physical conditions of production in the investment and consumption industries respectively.” (footnote 1 on p. 117) This is one of the strongest pieces of evidence that the *GT* assumes a two-sector economy.
- (34) As will be seen, the condition $k = k'$ does not mean that $k(= k')$ always takes the same value.
- (35) Although within the framework of a one-sector model, the aggregate supply function was rigorously analyzed by Wells [31], Marty [18], and Veendorp and Werkema [30]. They all assumed an aggregate production function in which output becomes zero when no one works. In the case of a two-sector model considered in this paper, such an assumption means $B_1 = B_2 = 0$, and thus $B = 0$. The implication of integration constants will be further discussed in the next section.
- (36) See pp. 17, 114, 122, 268, and 328.
- (37) If “double” is replaced by “ $\lambda (> 1)$ times,” $G(\lambda N_2) < \lambda G(N_2)$, and so

$$\lambda - (\lambda - 1) \frac{B_2}{\beta N_2 + B_2} < \lambda^\beta .$$

The condition $B_2 \geq 0$ holds in this case, too.

- (38) Statement 1 comes from p. 114, while Statement 2 from p. 121.
- (39) Keynes provided a numerical example for Statement 2 in footnote 1 on p. 17. However, a counterexample is easy to give.
- (40) Professor Tadasu Matsuo kindly showed me another solution to this problem. It is to assume that $B_1 < 0$ and $B_2 < 0$, which implies that (23) still holds. It should be noted in this case, as he correctly pointed out, that the volume of employment in each sector must be greater than some positive value. In the investment-goods sector, for example, let N_2^* be a value such that $G'(N_n^*) = G(N_n^*)/N_2^*$. Then, the condition $N_2 > N_2^*$ must be satisfied, in which (21) obtains and also the firms in the sector can earn positive profits. What an ingenious idea! His solution is quite right theoretically, and I appreciate his insight. As is apparent now, it is Statement 2 that matters for this problem.
- (41) There are two statements concerning real income on p. 114. (Statement (i) below is also mentioned on pp. 91-92.)
 - (i) Income measured in terms of wage-units will increase in a greater proportion than real income.
 - (ii) The amount of employment will increase more than in proportion to real income.

A problem is that real income is not defined in the *GT*. But it would be reasonable to define it as Y/p_1 . Because of (9),

$$\frac{Y}{p_1} = \frac{Y_w}{F'(N_1)}.$$

Therefore, the ratio of Y_w to Y/p_1 is written as

$$\frac{Y_w}{Y/p_1} = \frac{1}{F'(N_1)}.$$

Since $F''(N_1) < 0$, the ratio rises as N_1 increases. It follows that Statement (i) holds in general.

The ratio of N to Y/p_1 is written as

$$\frac{N}{Y/p_1} = \frac{1}{F'(N_1)} \frac{N}{Y_w}.$$

If N/Y_w is always constant, Statement (ii) also obtains.

- (42) In footnote 2 on p. 55 or a numerical example on pp. 125-127, these aggregate supply functions seem to be used with β set at unity.
- (43) Pigou [24, pp. 152-153] constructed a two-sector model under the assumption that the proportionate share of income accruing to labor has the same value in *both* sectors. He did not specify production functions, but, as is well-known to economists, such an assumption leads directly to the production function like (29) and (30). The fact that the proportions of income accruing to wage-earners and non-wage-earners respectively remained stable over long periods was accepted by Pigou [25, pp. 95-96].
- (44) In the appendix the various formulas in Chapter 20 are simplified by using results (25) - (30).
- (45) The equality between the two prices is just a simplification. The constancy of the ratio p_1/p_2 is required.
- (46) According to the *GT*, the price of investment goods is so determined as to bring about "the equality between the stock of capital-goods offered and the stock demanded" with the result that the marginal efficiency of capital is equal to the rate of interest. See pp. 186 (footnote 1) and 248. The condition $\Pi_1/(p_2K_1) = \Pi_2/(p_2K_2)$ corresponds to such a situation. That is, Keynes's investment theory is a long-run one.
- (47) Simple calculations show that the equality between the two rates of profit in the long run are not necessarily warranted by these production functions alone. They always take the same value in the steady state where the capital-labor ratios coincide in both sectors.
- (48) Okishio [20] gave a noteworthy analysis of Chapter 10 of the *GT*. In relation to this section, two points should be mentioned. First, he examined Statements 1 and 2 in the text and Statements (i) and (ii) in note 41 of this paper. Second, he obtained the following relationship among consumption demand, investment demand, the investment multiplier, and the employment multiplier: $dC_w/dN_1 \gtrsim dI_w/dN_2 \Leftrightarrow dY_w/dI_w \gtrsim dN/dN_2$.

In particular, note that the relation includes the case of $k = k'$ discussed in the text. In writing this section I learned much from him. However, the differences should also be pointed out. First, his examination of the four statements was based on a one-good model. Second, production functions of the Cobb-Douglas type were not derived, but used to confirm the mathematical relation.

References

- [1] Hansen, Alvin H., 1953, *A Guide to Keynes*, New York: McGraw-Hill.
- [2] Harrod, Roy F., 1969, *Money*, London: Macmillan.
- [3] Hicks, John R., 1937, "Mr. Keynes and the "Classics"; A Suggested Interpretation," *Econometrica*, 5, 147-159.
- [4] Hicks, John R., 1939, *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*, Oxford: Clarendon Press. 2nd Edition, 1946.
- [5] Hicks, John R., 1950, *A Contribution to the Theory of the Trade Cycle*, Oxford: Clarendon Press.
- [6] Hicks, John R., 1974, *The Crisis in Keynesian Economics*, Oxford: Basil Blackwell.
- [7] Kahn, Richard F., 1931, "The Relation of Home Investment to Unemployment," *Economic Journal*, 41, 173-198.
- [8] Kahn, Richard F., 1933, "Public Works and Inflation," *Journal of the American Statistical Association*, 27, Supplement, 168-173.
- [9] Kahn, Richard F., 1984, *The Making of Keynes' General Theory*, Cambridge: Cambridge University Press.
- [10] Keynes, John M., 1936, *The General Theory of Employment, Interest and Money*, London: Macmillan.
- [11] Keynes, John M., 1937, "The General Theory of Employment," *Quarterly Journal of Economics*, 51, 209-223.
- [12] Keynes, John M., 1939, "Relative Movements of Real Wages and Output," *Economic Journal*, 49, 34-51.
- [13] Keynes, John M., 1973, *The Collected Writings of John Maynard Keynes, Vol. VII, The General Theory of Employment, Interest and Money*, London: Macmillan.
- [14] Keynes, John M., 1973, *The Collected Writings of John Maynard Keynes, Vol. XIV, The General Theory and After, Part II, Defence and Development*, edited by Donald Moggridge. London: Macmillan.
- [15] Klein, Lawrence R., 1947, *The Keynesian Revolution*, London: Macmillan.
- [16] Krugman, Paul, 2007, "Introduction to New Edition," in the 2007 Edition of Keynes [10], xxv-xxxviii.
- [17] Mankiw, N. Gregory, 1992, "The Reincarnation of Keynesian Economics," *European Economic Review*, 36, 559-565.
- [18] Marty, Alvin L., 1961, "A Geometrical Exposition of the Keynesian Supply Function," *Economic Journal*, 71, 560-565.
- [19] Modigliani, Franco, 1944, "Liquidity Preference and the Theory of Interest and Money," *Econometrica*, 12, 43-88.
- [20] Okishio, Nobuo, 1983, "A Commentary on the Chap. 10 of The General Theory," *National Economic Review*, 148, 15-32 (in Japanese).
- [21] Patinkin, Don, 1949, "Involuntary Unemployment and the Keynesian Supply Function," *Economic Journal*, 59, 360-383.
- [22] Pigou, Arthur C., 1913, *Unemployment*, London: Williams and Norgate.
- [23] Pigou, Arthur C., 1933, *The Theory of Unemployment*, London: Macmillan.
- [24] Pigou, Arthur C., 1941, *Employment and Equilibrium: A Theoretical Discussion*, London: Macmillan. Second Edition, 1949.
- [25] Pigou, Arthur C., 1949, *The Veil of Money*, London: Macmillan.
- [26] Pigou, Arthur C., 1950, *Keynes's 'General Theory': A Retrospective View*, London: Macmillan.
- [27] Samuelson, Paul A., 1948, *Economics: An Introductory Analysis*, New York: McGraw-Hill.
- [28] Solow, Robert M., 1980, "On Theories of Unemployment," *American Economic Review*, 70, 1-11.
- [29] Tobin, James, 1947, "Money Wage Rates and Employment," in Seymour E. Harris, ed., *The New Economics: Keynes' Influence on Theory and Public Policy*, New York: Alfred A. Knopf, 572-587.

- [30] Veendorp, E. C. H., and H. G. Werkema, 1961, "Mr. Wells' Aggregate Supply Function-A Comment," *Economic Journal*, 71, 634-636.
- [31] Wells, Paul, 1960, "Keynes' Aggregate Supply Function: A Suggested Interpretation," *Economic Journal*, 70, 536-542.