Doctoral thesis

# Effective theories of type IIB string compactified on Calabi-Yau manifolds and their applications

# カラビ・ヤウ多様体上にコンパクト化された IIB 型超弦理論の有効理論とその応用

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# Overview

Superstring theory is a ten-dimensional supersymmetic theory and a candidate for the unified theory including both gauge and gravity interactions. The supersymmetry is the symmetry under the exchange of bosons and fermions, and it is considered that the ten-dimensional spacetime consists of the four-dimensional Minkowski spacetime of our universe and a six-dimensional compactified space which has not been detected so far.

Since the string theory describes quite high energy physics compared to the scale of collider experiments, it is important to reveal the nature of the string theory through the low-energy effective theory. In particular, it is important to derive some experimental predictions and indications from the string models constructed for realizing the standard model of elementary particles at a low energy.

So far, there are several studies to construct the standard-like models based on the heterotic string theory on orbifold [1, 2, 3], where the matter contents of the standard model are involved in the adjoint representation of $E_8$ or $SO(32)$. The discovery of D-branes [4] also leads to developments of the construction of phenomenological models in the type II superstring theory, since the gauge fields as well as the matter fields are induced by the open strings whose endpoints are attached to them. The three generation models are then constructed by the intersecting D-branes in type IIA superstring theory [5, 6] and its T-dual magnetized D-brane models in type IIB superstring theory. It is known that the Calabi-Yau manifold is a promising candidate for the extra-dimensional space of superstring theory to retain supersymmetry in the four-dimensional effective theory. In this thesis, we examine the influence of the geometrical structure of Calabi-Yau space on the effective theory of type IIB string from phenomenological and cosmological aspects.

From the phenomenological point of view, observed masses of quarks and leptons are a clue to construct a realistic string model. They can be determined dynamically in the string model, while they are input parameters to realize the experimental results in the standard model. Then, we can check the consistency of some string models by comparing the theoretical and experimental values of quark and lepton masses. In particular, the geometrical structure of extra-dimension, in general, affects the Yukawa coupling constants in four-dimensional effective theory. They are given by the overlap integral of matter wavefunctions in extra-dimension as explained in Ch. 3. It is then expected to obtain a guiding principle to confirm the extra-dimensional geometry by exploring string models compactified on various Calabi-Yau manifolds and probing them by phenomenological observations.

We also focus on the cosmological aspects with an emphasis on moduli fields. Moduli fields appear through the compactification in the string theory. In particular, the closed string moduli fields include Kähler moduli fields, complex structure moduli fields, and dilaton field, which determine the size and shape of extra-dimensional space, and the string coupling constant, respectively. The dynamical procedure for determining the vacuum expectation values of moduli fields is called the moduli stabilization. It is known that complex structure moduli fields and dilaton field can be stabilized at the perturbative level, where the potential is generated by the three-form fluxes in type IIB string theory. On the other hand, Kähler moduli fields can be stabilized only by the non-perturbative or loop effects in this framework, and their masses tend to be lighter than those of complex structure moduli and dilaton. In Ch. 6, we identify one of the moduli as inflaton and discuss moduli inflation models, in which an accelerated expansion of the early universe is achieved.

By stabilizing the fields except the inflaton field at a scale sufficiently higher than the inflation scale, the instability of their potential can be avoided during the inflation. In this regard, the models of the Kähler moduli playing the role of inflaton are usually discussed rather than those of the complex structure moduli. In this thesis, we propose a new class of inflation models where one of complex structure moduli is identified as the inflaton and it keeps being lighter than the Kähler moduli.

### About this thesis

In this thesis, we construct several string models with Calabi-Yau spaces as internal space in view of phenomenological and cosmological aspects. A great deal of string models on highly symmetric internal space, such as torus and sphere, are constructed because of analytical simplicity. The motivation of this thesis is exploring the string models with the internal spaces involving a more complicated metric and a less geometrical symmetry, in order to take a step to obtain a guideline to discriminate favored internal space experimentally or observationally.

The outline of this thesis is as follows. The topics of the effective theories for the type IIB string theory compactified on Calabi-Yau manifolds are discussed in various views of phenomenological aspect in Part I and cosmological aspect in Part II, respectively. Before entering into Part I, we first introduce the basic complex geometry and explain some spaces we will use later, i.e., complex torus and $T^{1,1}$ geometry in Ch. 1. In Ch. 2, we review the Klebanov-Witten model and kappa-symmetry as preparation for Part I.

In Part I, we focus on the phenomenological aspects along the line of Ref. [7]. In Ch. 3, we review the derivation of matter wavefunctions on magnetized branes and their properties on the background of torus or complex projective space. In Ch. 4, we construct the supersymmetric brane configuration taking into account the magnetic fluxes, and derive the matter wavefunctions explicitly on the branes wrapping the cycle of $AdS_5 \times T^{1,1}$ space. We also compare the resultant overlap integral among the matter fields with the observed values of the masses of quarks and leptons. The hierarchical structure of overlap integrals is realized by the localized wavefunctions of matter fields.

In Part II, we focus on the cosmological aspects along the line of Ref. [8]. In Ch. 5, we review some known inflation mechanisms such as natural inflation and axion monodromy inflation, and correspondingly the recent observational results by the Planck satellite are summarized. In Ch. 6, we propose a new class of inflation models associated with the complex structure moduli. The new potential is obtained by geometrical effects in the type IIB string, which is corresponding to worldsheet instanton effects in type IIA string. Finally in Ch. 7, we summarize this thesis.

# Chapter 1

# Geometrical preparation

## 1.1 Introduction to complex geometry

In this chapter, we introduce some basic properties of the complex geometry and algebraic geometry that are useful to explore various spaces arising in string theory.

As an example of the complex manifolds, we show the definition of the complex torus (elliptic function) first. The complex torus is a typical example for the modular form theory, which includes the periodic complex-valued functions invariant under the certain group action, i.e., modular group.

**Definition 1.1.** For two complex numbers $\omega_1$ and $\omega_2$ satisfying the relation $\tau = \omega_2/\omega_1 \notin \mathbb{R}$, *the lattice* is defined by

$$\Lambda(\omega_1, \omega_2) = \{m\omega_1 + n\omega_2 | m, n \in \mathbb{Z}\}. \tag{1.1}$$

We set $\lambda\Lambda = \Lambda(\lambda\omega_1, \lambda\omega_2)$ where $\lambda \in \mathbb{C}\backslash\{0\}$.

**Proposition 1.1.** The two lattices $\Lambda(\omega_1, \omega_2)$ and $\Lambda(\omega_1', \omega_2')$ are the same iff there exists $M \in GL(2, \mathbb{Z})$ such that

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL(2, \mathbb{Z}); \begin{pmatrix} \omega_1' \\ \omega_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}. \tag{1.2}$$

**Proposition 1.2.** For the complex numbers on upper half-plane $\tau, \tau' \in \mathbb{H} \equiv \{z \in \mathbb{C} \mid \operatorname{Im} z > 0\}$, the following condition is satisfied,

$$\Lambda(1, \tau') = \lambda\Lambda(1, \tau) \Leftrightarrow^{\exists} M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}); \tau' = \frac{a\tau + b}{c\tau + d}. \tag{1.3}$$

$SL(2, Z)$ is called the *modular group*, where the set of matrices is defined as

$$SL(2, \mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}. \tag{1.4}$$

Modular group is generated by the two transformations, $\tau \mapsto \tau + 1$ and $\tau \mapsto -1/\tau$, and the related generators are given by

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{1.5}$$

respectively.

**Definition 1.2.** The complex torus is defined by the following,

$$T(\omega_1, \omega_2) = \frac{\mathbb{C}}{\Lambda(\omega_1, \omega_2)}. \tag{1.6}$$

Next, as another basic example for the complex manifolds, we will move on to the projective spaces.

**Definition 1.3** The $n$ dimensional *projective space* is defined by

$$C\mathbb{P}^n = \frac{\mathbb{C}^{n+1}\backslash\{0\}}{\mathbb{C}^*}. \tag{1.7}$$

The equivalence relation $(z_0, z_1, \ldots, z_n) \sim (\lambda z_0, \lambda z_1, \ldots \lambda z_n)$ among the *homogeneous coordinates* is introduced by the $\mathbb{C}^*$ action, where $\lambda \in \mathbb{C}^*$. The *inhomogeneous coordinates* are defined by $t_\mu^{(i)} = z_\mu / z_i \, (\mu \neq i)$ in the patch $U_i$ satisfying $z_i \neq 0$. The coordinate transformation on $U_i \cap U_j$ is $t_\mu^{(i)} = (z_j/z_i) t_\mu^{(j)}$. The complex projective space is a Kähler manifold, and its Kähler metric is called the *Fubini-Study metric*,

$$g_{i\bar{j}} = \frac{(|\mathbf{t}|^2 + 1)\delta_{i\bar{j}} - t_i \bar{t}_{\bar{j}}}{(|\mathbf{t}|^2 + 1)^2}, \tag{1.8}$$

where $|\mathbf{t}|^2 = \sum t_i \bar{t}_{\bar{i}}$. Note that the index $(i)$ for patches is dropped for simplicity. The line element of space can be expressed as

$$ds^2_{\mathrm{CP^n}} = \frac{(|\mathbf{t}|^2 + 1)|d\mathbf{t}|^2 - (\mathbf{t} \cdot d\bar{\mathbf{t}})(\bar{\mathbf{t}} \cdot d\mathbf{t})}{(|\mathbf{t}|^2 + 1)^2}. \tag{1.9}$$

**Example 1.3.1** In the case of $n = 1$, $C\mathbb{P}^1$ is called the *projective line*. It is equivalent to Riemann sphere $\mathbb{C} \cup \{\infty\}$, that is, the space given by adding the point at infinity to the complex plane. In the patches $U_1 = \{z_0 \neq 0\}$ and $U_2 = \{z_1 \neq 0\}$, inhomogeneous coordinates can be denoted as $u^{(1)} = z_1/z_0$ and $u^{(2)} = z_0/z_1$ respectively. The $C\mathbb{P}^1$ metric can be rewritten to the $S^2$ metric,

$$ds^2_{CP^1} = \frac{du d\bar{u}}{(u\bar{u} + 1)^2} = \frac{1}{4}(d\phi^2 + \sin^2 \phi d\theta^2), \tag{1.10}$$

by transforming to the spherical coordinates, namely, $u = re^{i\theta}$ and $r = \tan^{-1}(\phi/2)$.

**Example 1.3.2.** $C\mathbb{P}^2$ identifies the coordinates under the action $(x_1, x_2, x_3) \sim \lambda(x_1, x_2, x_3)$ with the complex parameter $\lambda \in \mathbb{C}^*$.

The above projective space can be generalized to the *weighted projective space* $C\mathbb{P}^n_{a_1, \cdots, a_n}$ by taking the equivalence relation with weights, like $(z_1, z_2, \ldots, z_n) \sim (\lambda^{a_1} z_1, \lambda^{a_2} z_2, \ldots, \lambda^{a_n} z_n)$ where $\lambda \in \mathbb{C}^*$ and $a_1, \ldots, a_n \in \mathbb{N}$.

## 1.2 Toric geometry

As a generalization of the projective space, the $n$-dimensional *toric variety* $X$ is defined by

$$X = \frac{\mathbb{C}^r \backslash Z(\Sigma)}{G}, \tag{1.11}$$

where $G$ is $(\mathbb{C}^*)^{r-n}$ times a finite abelian group that we consider only a case for trivial group in this section, and $Z(\Sigma)$ is a subset fixed by $(\mathbb{C}^*)^{r-n}$ action, that is defined by *fan* $\Sigma$ below. The toric variety is constructed by the fan $\Sigma$, which is the collection of cones. Let $N$ be an $r$-dimensional lattice $N \simeq \mathbb{Z}^r$, which is a subset of vector spaces $N_{\mathbb{R}} = N \otimes \mathbb{R}$.

**Definition 2.1.** A strongly convex rational polyhedral cone $\sigma \in N_{\mathbb{R}}$ is a set

$$\sigma = \{a_1 v_1 + a_2 v_2 + \cdots + a_k v_k \mid a_i \geq 0\}, \tag{1.12}$$

generated by a finite number of vectors $v_1, \ldots, v_k$ in $N$ such that $\sigma \cap (-\sigma) = \{0\}$.

**Definition 2.2.** A collection $\Sigma$ of strongly convex rational polyhedral cones in $N_{\mathbb{R}}$ is called a *fan* if

i) each face of a cone in $\Sigma$ is also a cone in $\Sigma$.

ii)the intersection of two cones in $\Sigma$ is a face of each.

We denote one-dimensional cones as $\Sigma(1)$ and let $\rho_i$, $(i = 1, \ldots r)$ be the primitive generator in $\Sigma(1)$. $Z(\Sigma) \in \mathbb{C}^r$ is $r$-dimensional subvariety of $X$ given by

$$Z(\Sigma) = \bigcup_I \{(z_1, \ldots, z_r) | z_i = 0 \ \ \forall i \in I\}, \tag{1.13}$$

where $z_i$ is the homogeneous coordinate for each $\rho_i$. The union is taken over all sets that do not belong to a cone in $\Sigma$.

The denominator of Eq. (1.11) corresponds to the equivalence relations $(z_1, \ldots, z_r) \sim (\lambda^{Q_a^1} z_1, \ldots, \lambda^{Q_a^r} z_r)$ where $Q_a^i$ are integers given by $\sum_{i=1}^r Q_a^i \rho_i = 0$ and their greatest common divisor is fixed to 1.

**Example 2.1.** $C\mathbb{P}^2$ is constructed from the fan $\Sigma$, which is spanned by the three two-dimensional sets $\{\rho_1, \rho_2, \rho_3\} = \{(1,0), (0,1), (-1,-1)\}$. In this case, the fan $\Sigma$ consists of seven cones, they are a trivial cone $\{0\}$, three one-dimensional cones spanned by $\{(-1,-1)\}$, $\{(1,0)\}$, $\{(0,1)\}$, and three two-dimensional cones spanned by $\{(1,0),(0,1)\}$, $\{(-1,-1),(0,1)\}$, $\{(-1,-1),(1,0)\}$. Since only the three-dimensional set $\{(1,0),(0,1),(-1,-1)\}$ does not span any cones, $Z(\Sigma) = \{(0,0,0)\}$ is obtained. The equivalent relation is given by the weights $(Q^1, Q^2, Q^3) = (1,1,1)$. Thus, $C\mathbb{P}^2 = (\mathbb{C}^3 \backslash \{0,0,0\})/((z_1, z_2, z_3) \sim (\lambda z_1, \lambda z_2, \lambda z_3))$ is given by Eq. (1.11), that is in accord with Eq. (1.7).

## 1.3 The classes of Calabi-Yau space

At the last of this chapter, we focus on some classes of Calabi-Yau space applicable to the physical models, especially on two types of Calabi-Yau space. For one class of Calabi-Yau, the metric of Calabi-Yau space is known, whereas the other type of Calabi-Yau is defined as the hypersurface on the zero locus of certain algebraic equations.

In the string theory, extra-dimensional space is expected to be Ricci-flat Kähler, i.e., Calabi-Yau manifold, though there are no known compact six-dimensional Calabi-Yau metrics. On the other hand, some Calabi-Yau metrics are known if they are not compact spaces, which means that these spaces have infinite volumes. We show an example of the Calabi-Yau space which contains the *Sasaki-Einstein manifold.* The Sasaki-Einstein manifold is the *contact manifold* as the odd real dimensional extension for the Kähler manifold that has an even real dimension. The definition is as follows.

A compact Riemann manifold $(M, g)$ is *Sasaki* iff its metric cone $(C(M), \tilde{g}) = (\mathbb{R}_+ \times M, dr^2 + r^2 g)$ is the Kähler manifold. Note that the origin $r = 0$ is the singular removed in the above definition. A compact Riemann manifold $(M, g)$ is *Sasaki-Einstein* iff its metric cone $(C(M), \tilde{g})$ is Calabi-Yau, i.e., Ricci-flat Kähler manifold. Ricci-flat corresponds to vanishing Ricci curvature.

The complex three-dimensional manifold discussed in the physical literature can be given by a real five-dimensional Sasaki-Einstein manifold, e.g., $T^{1,1}$, $Y^{p,q}$ and $L^{a,b,c}$. $T^{1,1}$ metric is given by

$$ds^2_{T^{1,1}} = \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2, \tag{1.14}$$

where $\theta_i$, $\phi_i$ and $\psi$ are the coordinates of $T^{1,1}$ with $0 \leq \theta_i < \pi$, $0 \leq \phi_i < 2\pi$ and $0 \leq \psi < 4\pi$, respectively. The *conifold* is one of the Calabi-Yau spaces defined by the metric cone of $T^{1,1}$, $ds^2 = dr^2 + r^2 ds^2_{T^{1,1}}$.

For the other examples, $Y^{p,q}$ is known as cohomogeneity-one manifold. There exists countably many Sasaki-Einstein matric $Y^{p,q}$, where $(p, q)$ are the disjoint set of natural numbers satisfying the conditions $g.c.d(p, q) = 1$ and $p > q$. In Ref. [9], $Y^{p,q}$ metric is found as

$$\begin{aligned} ds^2_{Y^{p,q}} = {} & \frac{1-cy}{6}(d\theta^2 + \sin^2\theta d\phi^2) + \frac{1}{w(y)v(y)}dy^2 + \frac{v(y)}{9}[d\psi - \cos\theta d\phi]^2 \\ & + w(y)\left[d\alpha + \frac{ac - 2y + y^2 c}{6(a - y^2)}[d\psi - \cos\theta d\phi]\right]^2, \end{aligned} \tag{1.15}$$

where

$$w(y) = \frac{2(a - y^2)}{1 - cy}, \quad v(y) = \frac{a - 3y^2 + 2cy^3}{a - y^2}, \tag{1.16}$$

$$0 \leq \theta \leq \pi, \;\; 0 \leq \phi \leq 2\pi, \;\; y_1 \leq y \leq y_2, \;\; 0 \leq \psi \leq 2\pi, \;\; 0 \leq \alpha \leq 2\pi\ell. \tag{1.17}$$

We can reproduce $T^{1,1}$ metric by taking $c = 0$, and in all other cases we can take $c = 1$ without loss of generality by rescaling $y$. The first line of Eq. (1.15) corresponds to the four-dimensional base space $B_4$ of $Y^{p,q}$, that is topologically $S^2 \times S^2$. We rewrite Eq. (1.15) to $ds^2_{Y^{p,q}} = ds^2(B_4) + w(y)(d\alpha + A)^2$ with local one-form $A$. For the directions $(y, \psi)$ which are fibered over $S^2(\theta,\phi)$, we impose $1-y>0, a-y^2>0$ and $v(y)\geq 0$, and let $y_1$, $y_2$ and $y_3$ ($y_1\leq y_2\leq y_3$) be zero points of $a-3y^2+2y^3=0$. There exists $y$ such that $y_1\leq y\leq y_2$ ($y_1<0$, $y_2>0$) if we set $0<a<1$.

By taking the periods of $\alpha$ as $0\leq\alpha\leq 2\pi\ell$, $\ell^{-1}A$ is a connection on a $U(1)$ bundle over the base space $B_4=S^2\times S^2$. Let $C_1$ and $C_2$ be two $S^2$ cycles on $B_4$, which are characterized by the Chern numbers in $H^2(S^2;\mathbb{Z})=\mathbb{Z}$ denoted as $p$ and $q$. We define the following two periods:

$$P_1=\frac{1}{2\pi}\int_{C_1}dA=p\ell,\quad P_2=\frac{1}{2\pi}\int_{C_2}dA=q\ell. \tag{1.18}$$

After some calculations (for detail, see, Ref. [9]), we have the following relations,

$$P_1=\frac{y_1-y_2}{6y_1y_2},\quad P_2=-\frac{(y_1-y_2)^2}{9y_1y_2}, \tag{1.19}$$

and

$$\frac{P_1}{P_2}=\frac{3}{2(y_2-y_1)}=\frac{p}{q}. \tag{1.20}$$

Note that $y_1$ and $y_2$ are the functions depending on $a$ determined by $p$ and $q$. Thus, the explicit form of $\ell$ is derived by Eqs. (1.18) and (1.19) as follows,

$$\ell=\frac{q}{3q^2-2p^2+p\sqrt{4p^2-3q^2}}. \tag{1.21}$$

Next, we show the case where the Calabi-Yau space is constructed as a hypersurface. The *quintic* is the space described by the zero locus of a degree-five polynomial in $C\mathbb{P}^4$, $\sum_{a_1+\cdots+a_5=5}c^{\mu_1\cdots\mu_5}(x_{\mu_1})^{a_1}\cdots(x_{\mu_5})^{a_5}$. In general, degree-$d$ polynomials in $n$ variables describe a manifold with $\begin{pmatrix} d+n-1 \\ n-1 \end{pmatrix}$ independent parameters out of $c^{\mu_1\cdots\mu_5}$. This number of independent parameters corresponds to the complex structure and 101 parameters in quintic. Then, hodge numbers of quintic are $h^{1,1}=1, h^{2,1}=101$ and Euler number is $\chi=-200$. In particular, $\sum_{\mu=1}^4(x_\mu)^5$ is called *Fermat quintic threefold.*

The conifold is the complex three-dimensional hypersurface in $\mathbb{C}^4$,

$$z_1z_2-z_3z_4=0, \tag{1.22}$$

with conical singularity at the origin, that is the deformation limit of the quintic. Indeed, the conifold is the cone, since any real multiple of $(z_1,z_2,z_3,z_4)$ also satisfies the above relation. After the coordinate transformation $w_1=(z_1+z_2)/2$, $w_2=i(z_1-z_2)/2$, $w_3=i(z_3+z_4)/2$ and $w_4=(z_3-z_4)/2$, the above equation is rewritten in the following form,

$$w_1^2+w_2^2+w_3^2+w_4^2=0. \tag{1.23}$$

The intersection of Eq. (1.23) and $|w_1|^2 + |w_2|^2 + |w_3|^2 + |w_4|^2 = r^2$ is

$$w_{1R}^2 + w_{2R}^2 + w_{3R}^2 + w_{4R}^2 = \frac{r^2}{2}, \tag{1.24}$$

$$w_{1I}^2 + w_{2I}^2 + w_{3I}^2 + w_{4I}^2 = \frac{r^2}{2}, \tag{1.25}$$

$$w_{1R}w_{1I} + w_{2R}w_{2I} + w_{3R}w_{3I} + w_{4R}w_{4I} = 0, \tag{1.26}$$

where $w_{iR} = \mathrm{Re}\, w_i$, $w_{iI} = \mathrm{Im}\, w_i$ and $r$ is a real parameter. Equation (1.24) shows that $w_R$ spans the coordinates of $S^3$ on a fixed $r$, while Eq.(1.25) indicates that $w_I$ parametrizes the coordinates of $S^2$, that is perpendicular to $S^3$ direction. Then, it turns out that $T^{1,1}$ is the $S^2$ fibered over $S^3$.

# Chapter 2

# Type II string theory compactified on Calabi-Yau

## 2.1 Type IIA/IIB supergravity

In this section, we introduce type II string theory, in particular, the low-energy effective action described by the supergravity. The bosonic part of type II string includes the Neveu-Schwarz-Neveu-Schwarz (NS) and Ramond-Ramond (RR) fields. In this theory, the matter contents are dilaton $\phi$, metric tensor $G_{MN}$ and anti-symmetric tensor $B_{MN}$ called B-field in NSNS sector, and $p$-form gauge field $C_p$ in RR sector. In type IIA, $p$ is an even number but in type IIB, $p$ is an odd number. From the calculation of scattering amplitudes between strings in the world-sheet, the equations of motion of the fields in type IIB superstring are derived. Then, the ten-dimensional (10D) low-energy effective action of the type IIB superstring in the string frame is extracted as

$$
\begin{aligned}
S_{IIB} &= S_{NS}^{(B)} + S_{R}^{(B)} + S_{CS}^{(B)}, && (2.1)\\
S_{NS}^{(B)} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}|H_3|^2\right), && (2.2)\\
S_{R}^{(B)} &= -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right), && (2.3)\\
S_{CS}^{(B)} &= -\frac{1}{4\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3, && (2.4)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{F}_3 &= F_3 - C_0 H_3, && (2.5)\\
\tilde{F}_5 &= F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3, && (2.6)
\end{aligned}
$$

with $G = \det G_{MN}$, and $F_3 = dC_2$.

On the other hand, the 10D low-energy effective action of type IIA string in the string frame becomes

$$
\begin{aligned}
S_{IIA} &= S_{NS}^{(A)} + S_{R}^{(A)} + S_{CS}^{(A)}, && (2.7)\\
S_{NS}^{(A)} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x(-G)^{1/2}e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}|H_3|^2\right), && (2.8)\\
S_{R}^{(A)} &= -\frac{1}{4\kappa_{10}^2}\int d^{10}x(-G)^{1/2}\left(|F_2|^2 + |\tilde{F}_4|^2\right), && (2.9)\\
S_{CS}^{(A)} &= -\frac{1}{4\kappa_{10}^2}\int B_2\wedge F_4\wedge F_4, && (2.10)
\end{aligned}
$$

where

$$
\tilde{F}_4 = F_4 - C_1\wedge F_3, \qquad (2.11)
$$

with $F_4 = dC_3$.

Furthermore, we can consider the D-branes which are charged under the RR gauge fields in the type II string theory. The open string modes on D-branes give rise to the non-abelian gauge fields in the low-energy effective theory. In the type II string with D-brane setup, we thus can take into account the gauge and gravitational interactions at the same time. However, as will be mentioned in Sec. 2.3, one has to consider the kappa-symmetry to ensure the stability of D-branes.

## 2.2 Klebanov-Witten model

In this section, we proceed to review the Klebanov-Witten model [10], which is the type IIB supergravity solution. In this model, one begins with placing a stack of $N_c$ D3-branes at the singularity of conifold, whose metric is given as mentioned in Sec. 1.3 by

$$
\begin{aligned}
&ds^2 = dr^2 + r^2 ds^2_{T^{1,1}},\\
&ds^2_{T^{1,1}} = \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2, \qquad (2.12)
\end{aligned}
$$

where $\theta_i$, $\phi_i$ and $\psi$ are the coordinates of $T^{1,1}$ with $0 \le \theta_i < \pi$, $0 \le \phi_i < 2\pi$ and $0 \le \psi < 4\pi$, while $r$ is the coordinate of $AdS_5$, in other words, radial direction of conifold. D3-branes are put near the conical singularity under $g_s N_c \gg 1$, where $g_s$ is the string coupling. Thus, the resulting geometry becomes $AdS_5 \times T^{1,1}$ around the conical singularity supported by the Ramond-Ramond self-dual five-form flux $F^{(5)}$. In terms of AdS/CFT correspondence, its dual theory is the four-dimensional $\mathcal{N} = 1$ superconformal field theory with $SU(N)\times SU(N)$ gauge group.

In the near-horizon limit, type IIB supergravity solution is written as,

$$\begin{aligned} ds_{10}^2 &= h(r)^{-1/2} d^2 x_{1,3} + h(r)^{1/2} (dr^2 + r^2 ds_{T^{1,1}}^2), \\ h(r) &= \frac{L^4}{r^4}, \\ g_s F^{(5)} &= d^4 x \wedge dh^{-1} + \text{Hodge dual}, \\ L^4 &= \frac{27}{4} \pi g_s N_c \alpha'^2, \end{aligned} \tag{2.13}$$

where $\alpha'$ denotes the regge slope, $dx_{1,3}^2$ is the line-element in the four-dimensional Minkowski spacetime $dx_{1,3}^2 = \sum_{\mu,\nu=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu$ with $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, and $h(r)$ is the warp factor given by the backreactions of D3-branes.

From the phenomenological point of view, this model suffers from the problem that the gravitational effect decouples since the conifold is non-compact. In Sec.3 and Sec.4, we focus on the effective description of the gravitational theory on $AdS_5 \times T^{1,1}$ by assuming that conifold geometry is a local description of certain compact Calabi-Yau threefold. As will be discussed in Sec. 4.3.3, the method to construct a global Calabi-Yau is known and one can consider the situation that the local conifold can be glued to the global compact Calabi-Yau manifold in the large radius limit of $r$.

## 2.3 Kappa-symmetry

Kappa-symmetry is a local fermionic symmetry of the worldvolume of D-brane. For the bosonic and fermionic fields $(X^m, \theta)$ $(m = 0, 1, \ldots, p)$ on D$p$-brane, $(p + 1)$-dimensional action is formulated by a $d$-dimensional global symmetry, that is the super-Poincare symmetry. The other symmetries are general coordinate invariance, and kappa-symmetry which eliminates half the degrees of freedom of $\theta$. In other words, the $d$-dimensional supersymmetry on the brane reduces to be half due to the kappa-symmetry. The half supersymmetry are represented linearly on the worldvolume, while the other half one is broken and generates a goldstino, fermion which becomes the physical fermion on worldvolume. After taking a proper gauge, the bosonic degrees of freedom become $(10-p-1)$ for $X^m$ and $(p-1)$ for $A_\mu$, totally 8. For the fermion, 32 degrees of freedom for $\theta$ reduce to 16 by kappa-symmetry, and then divided into 8 by the equation of motion.

The supersymmetry is preserved when the vacuum expectation values of supersymmetric transformations of gravitino, dilatino, and gaugino are vanishing. In particular, the supersymmetric transformation of gravitino in the extra-dimensional spacetime requires the following condition,

$$\nabla_m \epsilon = 0, \tag{2.14}$$

where $m$ denote the indices of extra-dimensional space and $\nabla$ is the covariant derivative of the Killing spinor $\epsilon$. Let us consider the Killing spinor on $AdS_5$ for Klebanov-Witten background (2.13), given by

$$\epsilon = r^{\frac{\Gamma_*}{2}} \left( 1 + \frac{\Gamma_r}{2L^2} x^\alpha \Gamma_{x^\alpha} (1 - \Gamma_*) \right) \eta, \tag{2.15}$$

with

$$\Gamma_* = i\Gamma_{x^0 x^1 x^2 x^3}, \tag{2.16}$$

where $\Gamma_{x^i}$ are the gamma matrices on four-dimensional Minkowski spacetime, $\Gamma_r$ is the gamma matrix that corresponds to radial direction for $AdS_5$, and $\eta$ is a constant spinor. Furthermore, the stable D$p$-branes on the background $AdS_5 \times T^{1,1}$ are properly embedded in a kappa-symmetric way. As stated in Ref. [11], the kappa-symmetric conditions are equivalent to the following condition,

$$\Gamma_\kappa \epsilon = \epsilon, \tag{2.17}$$

where

$$\Gamma_\kappa = \frac{1}{(p+1)!\sqrt{-g}} \epsilon^{\mu_1 \cdots \mu_{p+1}} (\tau_3)^{\frac{p-3}{2}} i\tau_2 \otimes \gamma_{\mu_1 \cdots \mu_{p+1}}, \tag{2.18}$$

with $\tau_{2,3}$ being the Pauli matrices and the gamma matrices $\tilde{\gamma}_{\mu_1 \cdots \mu_{p+1}}$ which are pull-backed into the worldvolume of D$p$-brane. $g$ is the determinant of the metric on this D-brane. This kappa-symmetry condition (2.17) ensures the invariance under the local supersymmetry transformations for dilatino and gravitino. We will show that the string model on the D7-brane considered in Ch. 4 satisfies the above condition (2.17) which is summarized in Ref. [12]. See for the detail of the effective action of the D7-branes in the type IIB superstring on the Calabi-Yau orientifold, e.g., Refs. [13, 14].

# Part I

# Phenomenological aspects

# Chapter 3

# Wavefunction localization

## 3.1 Introduction

String phenomenology and cosmology are the important subjects to test the string theory in the low-energy scale by employing the data of experiments and observations. In the perturbative string theory, the fundamental parameters are the string coupling constant and string length. Thus, the masses of and mixing angles between elementary particles are believed to be dynamically generated, although in the standard model, they are just the parameters so that the experimental values are correctly reproduced at the electroweak scale. From these aspects, the experimental values of masses and mixing angles of quarks and leptons are expected to be a guiding principle to explore the realistic string model. String cosmology is also an important field to test the cosmological observables predicted in the string theory. In Part I, we study the reproducibility of the observed values of the masses of quarks and leptons on the basis of several string models by focusing on the wavefunction localization of matter fields.

In the higher-dimensional theory and string theory, the 4D coupling constants such as Yukawa couplings are obtained by compactifying the theory on certain internal manifold. In particular, the Yukawa couplings are derived from the overlap integral of matter wavefunctions in the extra-dimensional space. Thus, the structure of Yukawa couplings is sensitive to the geometrical structures of extra-dimensional space behind the 4D spacetime. In this chapter, we take into account the flat and curved backgrounds and show the properties of matter wavefunctions. We expect that string model building followed by such an approach could give us a hint for the structure of internal geometry such as the unknown Calabi-Yau manifold.

In the type II string theory, the gauge fields are originating from the D-branes, that is the solitonic objects. The action for $N$-stacks of D-branes is given by the $SU(N)$ supersymmetric Yang-Mills (SYM) theory. It allows us to consider our matter fields in the type II setup. There are several studies in which toroidal backgrounds are considered as the simplest background. In type IIB string theory on toroidal background, the authors of Ref. [15] suggested that the spectrum on D-branes with worldvolume fluxes, i.e., magnetized D-branes is chiral due to the fluxes as reviewed in the next section. Furthermore, the multiple magnetic fluxes induce the multiple degenerated zero-modes. It is thus possible to identify these degeneracies of zero-modes as the generations of quarks and leptons. Since the matter wavefunction on the toroidal background is quasi-localized in the extra-dimensional space, the Yukawa couplings between quarks/leptons and Higgs fields have a hierarchical structure. From the fact that the obtained Yukawa couplings enjoy the discrete flavor symmetries [16], one can realize the semi-realistic spectrum of quarks and leptons [16, 17] on the basis of the 10D SYM theory on three-factorized tori.

In addition, there are several approaches to study the matter spectrum on the curved backgrounds such as $\mathbb{P}^1$, $\mathbb{P}^1 \times \mathbb{P}^1$ and $\mathbb{P}^2$ [18], in which the matter wavefunctions on local magnetized D7-branes are derived. They discuss the local models embedded in a certain global Calabi-Yau threefold. Since, we do not know the explicit metric of any Calabi-Yau threefold, it is hard to obtain the analytical matter wavefunctions on this background. In the next chapter, we also discuss the curved background, i.e., conifold in a local description and we show an idea how to embed the local model in the global Calabi-Yau manifold.

In Part I, after briefly reviewing the matter wavefunction on tori and projective space, we consider the Klebanov-Witten model in the next chapter. The geometry of Klebanov-Witten model is described by the $AdS_5 \times T^{1,1}$ which can be induced by the stack of a large number of D3-branes placed at the tip of conifold [10]. We can discuss the local model in Calabi-Yau background, i.e., the conifold. In addition to the Klebanov-Witten model, the conifold geometry is discussed in other string models. As an example, a large number of D3-branes placed at the same point in the internal space give the backreaction to the geometry and consequently the background close to the D3-branes is described by the conifold [19]. In the landscape of string theory, the existence of conifold singularities is assisted by a statistical analysis [20].

To discuss the local model in the Klebanov-Witten background, we consider the probe D7-branes wrapping the internal cycles in the conifold. On this curved background, we study the matter wavefunction and its phenomenological consequences. The D-branes should wrap the internal cycles in a supersymmetric and kappa-symmetric way [21]. Since the kappa-symmetry is closely related to the local supersymmetry on the internal cycles wrapped by D-branes, the stability of D-branes is then guaranteed in this setup. On the $AdS_5 \times T^{1,1}$ background, the embedding of D-branes is developed in Ref. [12], in which they concentrate on not only the spacetime filling D-brane, but also the not realistic D-branes. The several D-brane configurations are paid attention to the physics of AdS/CFT correspondence. (For detail, see, e.g., Refs. [12, 22].) Note that as discussed in Ref. [23], the semi-realistic spectrum of D3-branes is realized at the orbifold singularity at the bottom of a warped throat such as the conifold. These D-brane configurations are also interesting from the cosmological point of view. On these warped these backgrounds, the possibility has been pointed out of brane inflation on the warped deformed conifold [24] as well as the natural inflation on the warped resolved conifold [25].

In the next chapter, we focus on the matter wavefunction living on the spacetime filling D7-branes wrap the supersymmetric and kappa-symmetric four-cycles on $AdS_5 \times T^{1,1}$. Furthermore, we carefully study the matter wavefunction with and without the magnetic fluxes. It turns out that the analytical solutions of both the Dirac and Laplace equations are quasi-localized toward the tip of conifold and the localized chiral zero-modes are degenerate due to the magnetic fluxes. The quasi-localized wavefunctions yield the hierarchical structures of Yukawa coupling in the same way on the other background.

The remaining of this chapter is organized as follows. In Sec. 3.2, we first review the wavefunctions of matter field on toroidal background. Along the line of Ref. [18], in Sec. 3.3, we extend this analysis of toroidal background to the other curved background such as $\mathbb{P}^1 \times \mathbb{P}^1$ and $\mathbb{P}^2$ cycles with the magnetic fluxes. Then, on this curved background, it is shown that the twisting of magnetic fluxes is important to solve the equations of motion of matter fields.

## 3.2 Wavefunctions for magnetized toroidal compactification

First, in this section, we introduce how to obtain the 4D Yukawa coupling constants for toroidal compactification.

The extra-dimensional space of the string theory is often considered to be Ricci-flat Kähler space from the view point of supersymmetry, when we construct the supersymmetric model. The supersymmetry is motivated by the stability of system, though few applicable Ricci-flat Kähler metric is known. Moreover, even if we know complicated Calabi-Yau metric, it is too hard to solve equations of motion analytically to obtain some explicit observables. One way to get over such an obstacle is to consider the model including highly symmetric extra-dimensional space globally or locally, like complex torus, sphere and so on.

One of the complex three-dimensional compactified spaces of the simplest metric is the factorizable three tori, $T^2 \times T^2 \times T^2$, which does not give rise to any curvature terms in the field equations of motion. As the first step to obtain the Yukawa coupling constants for the factorizable three tori, we briefly review the six-dimensional string model compactified on the 2-tori, $T^2$.

The Lagrangian density of the six-dimensional $SU(N)$ SYM in $R^{1,3} \times T^2$ is written as

$$\mathcal{L}_{6D} = -\frac{1}{4g^2}\mathrm{Tr}\{F^{MN}F_{MN}\} + \frac{i}{2g^2}\mathrm{Tr}\{\bar{\lambda}\Gamma^M D_M \lambda\}, \quad (M, N = 0, \ldots, 5), \tag{3.1}$$

where $g$ is the gauge coupling constant, $\lambda$ is the Majorana-Weyl spinor and its covariant derivative is $D_M\lambda = \partial_M \lambda - i[A_M, \lambda]$. The field strength is defined by $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$. Trace is denoted for the adjoint representation of $U(N)$ gauge group. We denote the $R^{1,3} \times T^2$ coordinates by $(x_\mu, y_m)$ where $\mu = 0, \ldots, 3, m = 4, 5$.

We can decompose the spinor field and gauge field as

$$\lambda(x, y) = \sum_n \chi_n(x) \otimes \psi_n(y_4, y_5), \quad A_m(x, y) = \sum_n \varphi_{n,m}(x) \otimes \phi_{n,m}(y_4, y_5), \tag{3.2}$$

on $R^{1,3} \times T^2$. In this thesis, we focus only on zero-modes corresponding to $n = 0$ and drop the index 0 hereafter.

Let us first consider Abelian case. After coordinate transformation, torus coordinate can be summarized as the complex coordinate $(z, \bar{z}) = (y_4 + \tau y_5, y_4 + \bar{\tau} y_5)$ with complex structure modulus of torus $\tau \in \mathbb{C}$. We take the proper gauge so that gauge field and its field strength are expressed as

$$F = \frac{2\pi M}{\mathrm{Im}\tau}, \quad A_z = \frac{\pi M}{\mathrm{Im}\tau}\mathrm{Im}(\bar{z} + \bar{\zeta}), \quad A_{\bar{z}} = -\frac{\pi M}{\mathrm{Im}\tau}\mathrm{Im}(z + \zeta), \tag{3.3}$$

where $\zeta$ is the wilson line. Note that the boundary conditions are expressed as $A(z+1) = A(z) + d\xi_1,\ A(z+\tau) = A(z) + d\xi_2,\ \psi(z+1) = e^{i\xi_1}\psi(z)$ and $\psi(z+\tau) = e^{i\xi_2}\psi(z)$, where $\xi_1 = (\pi M \mathrm{Im}\,(z+\zeta))/(\mathrm{Im}\,\tau)$ and $\xi_2 = (\pi M \mathrm{Im}\,\bar{\tau}(z+\zeta))/(\mathrm{Im}\,\tau)$.

By solving zero-mode Dirac equation on the flux background

$$\Gamma^m(\partial_m\psi - i[A_m,\psi]) = 0, \tag{3.4}$$

the following solution can be obtained

$$\psi_+^j = \mathcal{C}_j e^{i\pi qM(z+\zeta)\frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\tau}} \cdot \vartheta\left[\begin{array}{c}\frac{j}{qM}\\ 0\end{array}\right](qM(z+\zeta), qM\tau), \tag{3.5}$$

where $\vartheta$ is the Jacobi theta-function,

$$\vartheta\left[\begin{array}{c}a\\ b\end{array}\right](\nu,\tau) = \sum_{l\in\mathbb{Z}} e^{\pi i(a+l)^2\tau + 2\pi i(a+l)(\nu+b)}. \tag{3.6}$$

From the structure of Jacobi theta-function, the matter wavefunction is quasi localized due to the magnetic flux. Since the Yukawa coupling constants are given by the overlap integral of matter wavefunction, this localized structure of wavefunction is useful to obtain a hierarchy among the elementary particles.

The extension to 10D model compactified on $T^2\times T^2\times T^2$ is straightforward. In general, as a low energy effective theory of D9-branes, 10D $U(N)$ SYM Lagrangian is written as

$$\mathcal{L}_{10D} = -\frac{1}{4g^2}\mathrm{Tr}\{F^{MN}F_{MN}\} + \frac{i}{2g^2}\mathrm{Tr}\{\bar{\lambda}\Gamma^M D_M\lambda\}, \quad (M,N=0,\ldots,9). \tag{3.7}$$

For the compactification on the three factorizable tori, the mode expansion of the spinor field and gauge field can be given as follows,

$$\lambda(x,y) = \sum_n \chi_n(x)\otimes\psi_n^{(1)}(y_4,y_5)\otimes\psi_n^{(2)}(y_6,y_7)\otimes\psi_n^{(3)}(y_8,y_9), \tag{3.8}$$

$$A_m(x,y) = \sum_n \varphi_{n,m}(x)\otimes\phi_{n,m}^{(1)}(y_4,y_5)\otimes\phi_{n,m}^{(2)}(y_6,y_7)\otimes\phi_{n,m}^{(3)}(y_8,y_9), \tag{3.9}$$

where the superscripts of $\psi$ and $\phi$ indicate the number of tori.

The 10D gauge interaction is contained in the second term of Eq. (3.7) with the covariant derivative such as

$$\int d^{10}x\{\bar{\lambda}\Gamma^0\Gamma^M[A_M,\lambda]\}, \tag{3.10}$$

which leads to the 4D Yukawa coupling for the internal vector $A_m$, via the triple overlap integral of the extra dimensional factors shown in Eqs. (3.8) and (3.9).

## 3.3 Some extensions to curved spaces

In this section, we review the development of obtaining zero-mode solution on some curved space with an emphasis on the spacetime filling D7-branes wrapping $\mathbb{P}^1 \times \mathbb{P}^1$ or $\mathbb{P}^2$ local cycle of Calabi-Yau space [18]. We first describe the dimensional reduction of SYM action from 10D to 8D. By decomposing the fields living on spacetime filling D7-branes, we perform the mode expansions of scalars, spinors and vectors of the internal space, step by step.

The 10D fields can be decomposed into 8D ones by integrating out the functions depending on the transverse coordinate to the D7-branes. The 10D gauge boson $A_M$ contained in Eq. (3.7) is decomposed into a complex scalar $\phi$, a 4D spacetime vector $A_\mu$ ($\mu = 0, 1, 2, 3$) and an extra-dimensional vector $A_m$ ($m = 4, 5, 6, 7$). These fields are expanded as

$$\begin{aligned} \phi(x, x') &= \sum_{i=-\infty}^{\infty} \phi^{(i)}(x)\Phi^{(i)}(x'), \\ A_\mu(x, x') &= \sum_{i=-\infty}^{\infty} A_\mu^{(i)}(x)A^{(i)}(x'), \\ A_m(x, x') &= \sum_{i=-\infty}^{\infty} \phi_m^{(i)}(x)A_m^{(i)}(x'), \end{aligned} \tag{3.11}$$

where $x$ is the 4D Minkowski spacetime coordinate and $x'$ is the extra-dimensional coordinate on the D7-brane worldvolume, respectively. The index $i$ represents the number of Kaluza-Klein modes.

Then, as a preliminary for the mode expansion of the spinor field, let us set the notation of 10D gamma matrices as shown

$$\Gamma^\mu = \gamma^\mu \otimes \mathbb{1} \otimes \mathbb{1}, \quad \Gamma^m = \gamma^5 \otimes \tilde{\gamma}^{m-3} \otimes \mathbb{1}, \quad \Gamma^p = \gamma^5 \otimes \tilde{\tilde{\gamma}}^5 \otimes \tau^p, \tag{3.12}$$

where $p = 8, 9$. $\gamma^\mu$ and $\tilde{\gamma}^m$ denote the 4D Minkowski gamma matrices and the Euclidean gamma matrices for the internal coordinates of D7-brane respectively,

$$\gamma^0 = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}, \tag{3.13}$$

$$\tilde{\gamma}^1 = \begin{pmatrix} 0 & -i\mathbb{1} \\ i\mathbb{1} & 0 \end{pmatrix}, \quad \tilde{\gamma}^2 = \begin{pmatrix} 0 & \sigma_z \\ \sigma_z & 0 \end{pmatrix}, \quad \tilde{\gamma}^3 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad \tilde{\gamma}^4 = \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix}, \tag{3.14}$$

and then $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\tilde{\gamma}^5 = \tilde{\gamma}^1\tilde{\gamma}^2\tilde{\gamma}^3\tilde{\gamma}^4$. The transverse directions are described by the Pauli matrices, $\tau^8 = \sigma_x$ and $\tau^9 = \sigma_y$.

In this notation, we can decompose the 10D Majorana-Weyl spinor $\lambda$ as

$$\lambda = (\lambda_1 + \lambda_4) \oplus (\lambda_2 + \lambda_3), \tag{3.15}$$

with

$$\begin{aligned}
\lambda_1 &= \xi_1^+(x)\psi_1^+(x')\theta_1^+(u),\\
\lambda_2 &= \xi_2^+(x)\psi_2^-(x')\theta_2^-(u),\\
\lambda_3 &= \xi_3^-(x)\psi_3^-(x')\theta_3^+(u),\\
\lambda_4 &= \xi_4^-(x)\psi_4^+(x')\theta_4^-(u),
\end{aligned} \tag{3.16}$$

where $u$ is the transverse coordinate, and the signs in the subscript of each factor denote the chirality of spinor component. In this and next sections, we assume that the third factors in Eq. (3.16) which depend on the transverse coordinate $u$ are treated as constants as well as in the bosonic case.

There are some developments of solving Laplace and Dirac equations in the local $\mathbb{P}^1 \times \mathbb{P}^1$ and $\mathbb{P}^2$ cycles with the magnetic fluxes. Since the explicit metrics of them are known as Fubini-Study metric mentioned in Ch. 1, we can discuss the matter wavefunctions analytically. In the first case for $\mathbb{P}^1 \times \mathbb{P}^1$, the holomorphic coordinates are $(z, w)$ and line element is written by

$$ds^2 = \frac{4R_1^2 dz d\bar{z}}{(1+z\bar{z})^2} + \frac{4R_2^2 dw d\bar{w}}{(1+w\bar{w})^2}, \tag{3.17}$$

with the Abelian magnetic fluxes $M$ on the first $\mathbb{P}^1$ and $N$ on the second $\mathbb{P}^1$,

$$F = -\frac{iM}{(1+z\bar{z})} dz \wedge d\bar{z} - \frac{iN}{(1+w\bar{w})} dw \wedge d\bar{w}, \tag{3.18}$$

$$A = \frac{iM}{2(1+z\bar{z})}(\bar{z}dz - zd\bar{z}) + \frac{iN}{2(1+w\bar{w})}(\bar{w}dw - wd\bar{w}), \tag{3.19}$$

where $R_1$ and $R_2$ are the radii of spheres. In the second case for $\mathbb{P}^2$ with the holomorphic coordinate $(z_1, z_2)$, the metric is expressed as

$$g_{i\bar{j}} = \frac{1}{2}\left(\frac{\delta_{i\bar{j}}}{(1+\rho^2)} - \frac{z_{\bar{i}} z_j}{(1+\rho^2)^2}\right), \tag{3.20}$$

and the magnetic fluxes are introduced as follows,

$$F = iM g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}, \tag{3.21}$$

$$A = \frac{M}{4i(1+\rho^2)}[(\bar{z}_1 dz_1 + \bar{z}_2 dz_2) - (z_1 d\bar{z}_1 + z_2 d\bar{z}_2)], \tag{3.22}$$

where $\rho^2 = \sum_i z_i \bar{z}_i$. In these cases, holomorphic wavefunctions for scalar, fermion and vector modes are obtained as the specific solutions, the rough forms of which are $\Psi = f(z, w)(1 + z\bar{z})^{h(M)}(1 + w\bar{w})^{h(N)}$ for $\mathbb{P}^1 \times \mathbb{P}^1$ case and $\Psi = f(z_1, z_2)(1 + \rho^2)^{h(M)}$ for $\mathbb{P}^2$ case. The functions $h(M)$ are depending on the configuration of magnetic fluxes and they are different for each of the modes, and $f$ are arbitrary functions constrained by normalization condition. For detail, see, Ref. [18]. For both $\mathbb{P}^1 \times \mathbb{P}^1$ and $\mathbb{P}^2$ cases, the number of generation is determined by the magnetic fluxes similar to the $T^2$ case described in the previous section.

4D Yukawa coupling constants are derived by the overlap integrals of Eqs. (3.11) and (3.16), as mentioned in Eq. (3.10). Note that fermions couple to not only internal vector $A_m$ but also transverse vector $A_p$, through the gauge interactions. For more details, see, Ref. [18].

Next, we consider the decomposition of the $U(N)$ gauge group by the magnetic fluxes. When there are magnetic fluxes along $U(1)$ of $U(N)$, i.e., constant field strength exists in the diagonal component of $U(N)$ matrices, the original $U(N)$ is decomposed into $U(N_1)\times U(N_2)\times\cdots\times U(N_n)$ with $N = N_1 + N_2 + \cdots + N_n$. Accordingly, the bifundamental representation of these product gauge groups exists in the off-diagonal components of SYM gauge fields, $\Phi_{\mathfrak{ab}}$ where $\mathfrak{a}, \mathfrak{b} = 1, \cdots, N$ stand for the index of adjoint representation of $U(N)$.

As an example, we consider the gauge symmetry breaking $U(N_1 + N_2) \to U(N_1) \times U(N_2)$. This breaking can be achieved by the $U(1)$ magnetic flux $F_{\mathfrak{ab}} \propto \mathrm{diag}(m_1, \ldots, m_N)$ with $m_1 = \cdots = m_{N_1} \equiv M_{(N_1)}$ and $m_{N_1+1} = \cdots = m_N \equiv M_{(N_2)}$, which is inserted along the $U(1)$ subgroup of $U(N_1 + N_2)$. Then, the bifundamental fields $(\mathbf{N}_1, \bar{\mathbf{N}}_2), (\bar{\mathbf{N}}_1, \mathbf{N}_2)$ are the candidates of standard model fields. Since the bifundamental field $(\mathbf{N}_1, \bar{\mathbf{N}}_2)$ ($(\bar{\mathbf{N}}_1, \mathbf{N}_2)$) feels $M_{(N_1)} - M_{(N_2)}$ ($M_{(N_2)} - M_{(N_1)}$) units of flux as also seen in the Laplace and Dirac equations, the multiple chiral zero-modes appear in the low-energy effective theory in a similar way to the magnetized tori [15]. The degeneracies of zero-modes are determined by the number of fluxes themselves. It is possible to identify these degeneracies as the generation of matter fields.

Following the procedure in Ref. [18], we solve the equations of motion for scalars and fermions on another background, i.e., D7-branes wrapping the four-cycle in the conifold in the next chapter. Before going to their details, let us discuss a shift of the number of magnetic fluxes, called as *twisting*, by which we can take into account the curvature in transverse directions to D7-branes. After the dimensional reduction of the 10D SYM action to 8D one so as to preserve the supersymmetry, the matter contents are an 8D gauge boson, a gaugino, a complex scalar and its superpartner fermion. First, we focus on two types of scalar models. One is the scalar models for the extra-dimensional space whose values are taken in the tangent bundle of the D7-brane. The origin of these scalar modes is the 4D gauge fields, i.e., vector degree of freedom in Minkowski spacetime of the D7-branes. The other is originating from the transverse scalar modes i.e., position moduli whose values are taken in the normal bundle of the D7-brane. These scalar modes obey the following Laplace equation,

$$-\tilde{D}_m \tilde{D}^m \Phi = m^2 \Phi, \tag{3.23}$$

with $\tilde{D}_m$ being the covariant derivative, $\tilde{D}_m \Phi = \nabla_m \Phi - i[A_m, \Phi]$.

In a flat space, the bosons have the representation of $SO(3,1) \times SO(4) \times U(1)_R$, whereas fermions have $\pm 1/2$ charge under $U(1)_R$. To ensure the existence of four scalar supercharges in 4D Minkowski spacetime, we have to consider the twisted SYM theory on the D7-branes by embedding the new central $U(1)$ charge in $SO(4)$ as pointed out in Ref. [26]. Indeed, a nontrivial curvature of the normal bundle changes the structure of covariant derivatives in Eq. (3.23). As a consequence, the equations of motion are modified. Both the nontrivial curvature effect and the magnetic flux are proportional to the Kähler form. Thus, the number of magnetic fluxes in the Laplace equation (3.23) is shifted. In this thesis, we call such effects twisting. Although we discuss the twisting in a flat space so far, the structure of twisting depends on the isometry of the internal four-cycles wrapped by the D7-branes. As discussed in the next chapter, we will show the D7-branes wrapping the four-cycle in the conifold in Eq. (4.3). When the conifold is embedded into a global Calabi-Yau space, these twisting structure is further modified as taken into account in Sec. 4.3.3.

We next study the vector modes in an extra-dimensional space which are the scalar degrees of freedom in 4D Minkowski spacetime. These modes are called the internal vector modes, i.e., Wilson-line moduli. From their equations of motion,

$$\tilde{D}_m \tilde{D}^m \Phi_n + 2iF^m_{\ n}\Phi_m - [\nabla^m, \nabla_n]\Phi_m = -m^2 \Phi_n, \tag{3.24}$$

it turns out that they do not feel the twisting, since they take the value in the tangent bundle.

Finally, let us take a look at the fermionic modes which obey the following Dirac equation,

$$\Gamma^m \tilde{D}_m \Psi = 0, \tag{3.25}$$

where the covariant derivative $\tilde{D}_m$ involves both the gauge connection and spin connection terms. As can be seen in the next chapter, they also receive the twisting in a similar way to the scalar modes.

In the next chapter, we employ the setup of the dimensional reduction from 10D SYM action (3.7) to 8D one on the D7-brane as we introduced early in this subsection, though those branes are embedded in the $AdS_5 \times T^{1,1}$ local spacetime around the tip of conifold. By adopting the effect of the twisting, we count the numbers of zero-modes which localize toward the tip in a similar way to the other extra-dimensional space model, and then study their wavefunctions in the conifold region.

# Chapter 4

# Wavefunctions on magnetized branes in the conifold

## 4.1 Supersymmetric brane probes on the conifold

We have seen that the hierarchical structure of 4D Yukawa coupling constants is caused by the quasi-localization of matter wavefunction living on D-brane. In this chapter, we study the matter field wavefunction on the spacetime filling D7-brane wrapping the non-trivial four-cycle in the conifold*.

In this section, we add a D7-brane to the Klebanov-Witten background as mentioned in Sec. 2.2, assuming that the backreaction to the background spacetime is negligible.† We will discuss the property of quasi-localization of matter wavefunction on this background yielded by certain magnetic fluxes in the next section. While the original Klebanov-Witten model is constructed on the non-compact Calabi-Yau space, i.e., the conifold, we assume that the conifold is locally described in a certain limit of some global Calabi-Yau manifold throughout this chapter. The supersymmetric embedding of D7-brane is guaranteed from the analysis of kappa-symmetry which involves at least $\mathcal{N} = 1$ supersymmetry.

In general, BPS configurations of the probe brane must satisfy the following condition for a worldvolume kappa-symmetry,

$$\Gamma_\kappa \epsilon = \epsilon, \tag{4.1}$$

where $\Gamma_\kappa$ denotes the kappa-symmetry matrix,

$$\Gamma_\kappa = -\frac{i}{8!\sqrt{-g}} \epsilon^{\mu_1 \cdots \mu_8} \gamma_{\mu_1 \cdots \mu_8}, \tag{4.2}$$

in the case of D7-brane, $\epsilon$ refers to a Killing spinor for Klebanov-Witten background (2.13) and $\gamma_{\mu_1 \cdots \mu_8}$ is the antisymmetrized product of the gamma matrices pull-backed into the worldvolume

*We can also consider the spacetime filling D5-brane embedded in the conifold, where the induced metric on the D5-brane is described by the projective space. In this case, wavefunction as well as the Yukawa couplings are obtained in Ref. [18].

†See for the backreaction of D7-brane, e.g., Ref. [27].

of D7-brane. Their worldvolume coordinates are denoted by $(x^0, x^1, x^2, x^3, \theta_1, \phi_1, \theta_2, \phi_2)$, where $\theta_i$, $\phi_i$ with $i = 1, 2$ are the coordinates of $T^{1,1}$ given in Eq. (2.12). The generalization of kappa-symmetry matrix in arbitrary dimensions is summarized in Sec. 2.3. As discussed in Ref. [12], Eq. (4.1) is solved in such a way that the transverse coordinates of the D7-brane $(\psi, r)$ change depending on their worldvolume coordinates $\theta_i$ and $\phi_i$ with $i = 1, 2$ as

$$\begin{aligned} r^3 &= \frac{c^2}{\left(\sin\frac{\theta_1}{2}\right)^{n_1+1}\left(\cos\frac{\theta_1}{2}\right)^{1-n_1}\left(\sin\frac{\theta_2}{2}\right)^{n_2+1}\left(\cos\frac{\theta_2}{2}\right)^{1-n_2}}, \\ \psi &= n_1\phi_1 + n_2\phi_2 + \text{const.}, \end{aligned} \tag{4.3}$$

where the integers $(n_1,\ n_2)$ and a constant $c$ determine the allowed region of D7-brane in the direction of $AdS_5$ [12]. From Eq. (4.3), $r$ depends on the sinusoidal functions of the angles $\theta_i$, which implies that there is a non-zero minimal value of $r$ under $|n_i| \leq 1$ with $i = 1, 2$. On the other hand, when $\theta_i$ becomes 0 and/or $\pi$, the radial direction $r$ diverges and the four-cycle wrapped by corresponding D7-brane extends over the outside of conifold.

In general, the holomorphic coordinates $z_a$ for $a = 1, 2, 3, 4$ appearing in the defining equation of conifold, $z_1z_2 - z_3z_4 = 0$, can be written in terms of the worldvolume coordinates as

$$\begin{aligned} z_1 &= r^{3/2}e^{\frac{i}{2}(\psi-\phi_1-\phi_2)}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}, \qquad & z_2 &= r^{3/2}e^{\frac{i}{2}(\psi+\phi_1+\phi_2)}\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}, \\ z_3 &= r^{3/2}e^{\frac{i}{2}(\psi+\phi_1-\phi_2)}\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}, \qquad & z_4 &= r^{3/2}e^{\frac{i}{2}(\psi-\phi_1+\phi_2)}\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}. \end{aligned} \tag{4.4}$$

Then the induced metric of the D7-brane

$$\begin{aligned} ds^2_{D7} = \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) &+ \frac{1}{9}(C_1(\theta_1)d\theta_1 + C_2(\theta_2)d\theta_2)^2 \\ &+ \frac{1}{9}(C_1(\theta_1)\sin\theta_1 d\phi_1 + C_2(\theta_2)\sin\theta_2 d\phi_2)^2, \end{aligned} \tag{4.5}$$

with

$$C_i(\theta_i) \equiv \frac{n_i + \cos\theta_i}{\sin\theta_i} \quad (i = 1, 2), \tag{4.6}$$

can be expressed in terms of the holomorphic coordinates (4.4), that will be explicitly shown later in the next section by setting certain values for $n_1$ and $n_2$.

## 4.2 Wavefunctions on the D7-branes with magnetic fluxes

Hereafter, we restrict ourselves to the case $(n_1, n_2) = (1, 1)$, for simplicity. One can extend our approach to general integers $n_1$ and $n_2$ straightforwardly while some calculations become complicated. After setting $n_1 = n_2 = 1$, the transverse position of the D7-brane denoted by coordinates $(r, \psi)$ can be expressed by its internal coordinates $(\theta_i, \phi_i)$ as

$$\begin{aligned} r^3 &= \left( \frac{c}{\sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}} \right)^2 , \\ \psi &= \phi_1 + \phi_2, \end{aligned} \tag{4.7}$$

in other words, it corresponds to the case of D7-brane embedding on the $z_1 = c$ plane with the holomorphic coordinates shown in Eq. (4.4). In this case, holomorphic coordinates of D7-brane $(z_3, z_4)$ are written as

$$z_3 = c \cot\frac{\theta_1}{2} e^{i\phi_1}, \quad z_4 = c \cot\frac{\theta_2}{2} e^{i\phi_2}. \tag{4.8}$$

By substituting Eq. (4.7) into Eq. (4.5), the induced metric of D7-brane can be rewritten by holomorphic coordinates as

$$ds^2_{D7} = \frac{4(|z_3|^2 + \frac{3}{2}c^2)}{9(|z_3|^2 + c^2)^2} |dz_3|^2 + \frac{4(|z_4|^2 + \frac{3}{2}c^2)}{9(|z_4|^2 + c^2)^2} |dz_4|^2 + \frac{4(\bar{z}_3 z_4 dz_3 d\bar{z}_4 + z_3 \bar{z}_4 d\bar{z}_3 dz_4)}{9(|z_3|^2 + c^2)(|z_4|^2 + c^2)}. \tag{4.9}$$

Note that from Eq. (4.7),

$$r^3 = \frac{(|z_3|^2 + c^2)(|z_4|^2 + c^2)}{c^2}, \tag{4.10}$$

and the radial coordinate $r$ has a non-zero minimal value $r_{\rm min} = c^{2/3}$ in the limit $|z_3|, |z_4| \to 0$, whereas $r$ goes to $\infty$ in the large radius limit $|z_3|, |z_4| \to \infty$. Here, the large $r$ implies that the cycle wrapped by the D7-brane extends to the inside of global Calabi-Yau manifold, which is not captured in the induced metric of D7-brane. Throughout this thesis, it is assumed that the volume of D7-brane is almost determined by the four-cycle in the near horizon limit. Therefore, we define the finite value $z_{\rm max}$ as the boundary of near-horizon limit. In the limit $|z_3|, |z_4| \longrightarrow |z_{\rm max}|$, the radial coordinate $r$ reaches $r_{\rm max} = L^3 = \dfrac{(|z_{\rm max}|^2 + c^2)^2}{c^2}$.[‡]

On the D7-brane, the Kähler form is given by pulling it back from the original one in 10D,

$$\begin{aligned} J &= i g_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}} \\ &= -\frac{1}{6} Q_1 \Omega_{11} - \frac{1}{6} Q_2 \Omega_{22} - \frac{1}{9} \cot\frac{\theta_1}{2} \cot\frac{\theta_2}{2} \left( \Omega_{12} + \Omega_{21} \right), \end{aligned} \tag{4.11}$$

where

$$g_{i\bar{j}} = \frac{c^2}{3(|z_i|^2 + c^2)^2} \delta_{i\bar{j}} + \frac{2}{9} \left( \frac{\bar{z}_i}{|z_i|^2 + c^2} \right) \left( \frac{z_j}{|z_j|^2 + c^2} \right), \tag{4.12}$$

[‡]Now it is assumed that the maxima of $z_3$ and $z_4$ are the same to each other, for simplicity.

and

$$\begin{aligned} Q_i &= \frac{3}{2} + \cot^2 \frac{\theta_i}{2}, \\ \Omega_{ij} &= d\theta_i \wedge \sin\theta_j d\phi_j. \end{aligned} \tag{4.13}$$

As discussed in Ch. 3, we introduce the two-form fluxes satisfying the Bianchi identity, i.e.,

$$F = M_1 Q_1 \Omega_{11} + M_2 Q_2 \Omega_{22}, \tag{4.14}$$

which are quantized on the basis of two-cycles $(\theta_i, \phi_i)$ as

$$\begin{aligned} \int_{(\theta_1,\phi_1)} F &= 2\pi M_1 \mathcal{V}_1 =: 2\pi N_1, \\ \int_{(\theta_2,\phi_2)} F &= 2\pi M_2 \mathcal{V}_2 =: 2\pi N_2, \end{aligned} \tag{4.15}$$

with the volumes $\mathcal{V}_1$ and $\mathcal{V}_2$ of local two-cycles in the D7-brane given by,

$$\mathcal{V}_1 = \frac{L^2}{9} \int_{\theta_1^{\min}}^{\pi} d\theta_1 \sin\theta_1 Q_1, \;\; \mathcal{V}_2 = \frac{L^2}{9} \int_{\theta_2^{\min}}^{\pi} d\theta_2 \sin\theta_2 Q_2, \tag{4.16}$$

Now, $\theta_1^{\min}$ and $\theta_2^{\min}$ are given by substituting $|z_3| = |z_4| = |z_{\max}|$ to Eq. (4.8), and the relations $M_i = N_i/\mathcal{V}_i$ ($N_i \in \mathbb{Z}$) with $i = 1, 2$ are required from the Dirac quantization condition. The field strengths in terms of holomorphic coordinates are expressed as

$$F_{z_3\bar{z}_3} = -2iM_1 \frac{|z_3|^2 + \frac{3}{2}c^2}{(|z_3|^2 + c^2)}, \;\; F_{z_4\bar{z}_4} = -2iM_2 \frac{|z_4|^2 + \frac{3}{2}c^2}{(|z_4|^2 + c^2)}, \tag{4.17}$$

these are given by

$$\begin{aligned} A = iM_1 &\left( -\frac{c^2}{2z_3(|z_3|^2 + c^2)} + \frac{1}{z_3} \ln(|z_3|^2 + c^2) \right) dz_3 \\ -iM_1 &\left( -\frac{c^2}{2\bar{z}_3(|z_3|^2 + c^2)} + \frac{1}{\bar{z}_3} \ln(|z_3|^2 + c^2) \right) d\bar{z}_3 \\ +iM_2 &\left( -\frac{c^2}{2z_4(|z_4|^2 + c^2)} + \frac{1}{z_4} \ln(|z_4|^2 + c^2) \right) dz_4 \\ -iM_2 &\left( -\frac{c^2}{2\bar{z}_4(|z_4|^2 + c^2)} + \frac{1}{\bar{z}_4} \ln(|z_4|^2 + c^2) \right) d\bar{z}_4. \end{aligned} \tag{4.18}$$

As for the two-form fluxes (4.14), there are two setups which satisfy the supersymmetric condition along the $D$-flat direction. First case is that supersymmtric condition $\int J \wedge F = 0$ holds without any vacuum expectation values (VEVs) of matter fields, that is, $M_1 = -M_2$ for the same size of local two-cycles $\mathcal{V}_1 = \mathcal{V}_2$ in the D7-brane. Second case is the fluxes with $\int J \wedge F \neq 0$, i.e., $M_1 \neq -M_2$ for $\mathcal{V}_1 = \mathcal{V}_2$, which break supersymmetry, but some non-vanishing

VEVs of charged scalar fields under the fluxed $U(1)$ symmetry restore it. In the following, we analyze wavefunctions with magnetic fluxes turned on, in these two supersymmetric cases. Before starting the detailed analysis, note that we will just abandon the modes which localize toward the opposite side to the tip of conifold hereafter, since the quantization of magnetic fluxes Eq.(4.15) is valid only in such a region inside the boundary of near horizon limit. In the following analysis, we focus on zero-modes localized toward the tip. In general, the fields localized at opposite side of the tip should be affected by the detailed structure of global Calabi-Yau space that includes conifold locally. Moreover, it might be also possible that the embedding gives rise to new zero-modes outside the boundary of near horizon limit. The structure of 4D $\mathcal{N} = 1$ supersymmetry is ensured by the above discussion, and it could be also captured by focusing on the massless bosons and fermions without identifying the explicit forms of twisting for the fields.

### 4.2.1 Bosons

**Transverse scalar modes**

First of all, let us study the wavefuntions of untwisted scalar modes without taking into account the curvature effects contained in $\nabla$, as discussed below Eq. (3.23). It corresponds to the shift of the number of fluxes as shown later. The scalar modes such as the extra-dimensional part on the D7-brane, transverse scalar mode and Minkowski vector mode obey the following Laplace equation,

$$-g^{mn} D_m D_n \Phi \;=\; -2 \sum_{i,j=3,4} g^{z_i \bar{z}_j} D_{z_i} D_{\bar{z}_j} \Phi - \sum_{i,j=3,4} g^{\bar{z}_i z_j} [D_{\bar{z}_i}, D_{z_j}] \Phi = m^2 \Phi, \qquad (4.19)$$

with the covariant derivative $D_{z_i} = \partial_{z_i} - iA_{z_i}$ and its Hermitian conjugate $D_{\bar{z}_i}$. Here, the eigenvalue $m^2$ denotes a mass-squared of the eigenmode. When the fluxes are inserted in a supersymmetric way, i.e., $M_1 = -M_2$, Eq. (4.19) becomes zero because of the structure of the commutator between $D_z$ and $D_{\bar{z}}$

$$\sum_{i,j=3,4} g^{\bar{z}_i z_j} [D_{\bar{z}_i}, D_{z_j}] \Phi = -i \sum_{i,j=3,4} g^{\bar{z}_i z_j} F_{z_j \bar{z}_i} \Phi = 0. \qquad (4.20)$$

By contrast, when the fluxes are inserted in a non-supersymmetric way, $M_1 \neq -M_2$, the term in Eq. (4.20) becomes non-zero. However, we assume that certain nonvanishing vacuum expectation values of some scalar fields cancel such a term at the Lagrangian level in the mode equation (4.19). As a result, the massless zero-mode obey the equation $D_{z_i}\Phi = 0$ or $D_{\bar{z}_i}\Phi = 0$ ($i = 3, 4$). (As discussed in Ref. [18], for the modes living on D7-brane on local $\mathbb{P}^2$ with supersymmetric fluxes, Eq. (4.20) is nonvanishing. After taking into account the twisting, single massive scalar mode appears.) We find the zero-mode solutions by solving the equations

$D_{z_3}\Phi_1 = 0$, $D_{\bar{z}_3}\Phi_2 = 0$, $D_{z_4}\Phi_3 = 0$ and $D_{\bar{z}_4}\Phi_4 = 0$:

$$\begin{aligned}
\Phi_1 &= f(\bar{z}_3) e^{M_1\left(-\ln c^2 \ln z_3 + \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)} \left(\frac{z_3}{|z_3|^2 + c^2}\right)^{\frac{M_1}{2}}, \\
\Phi_2 &= g(z_3) e^{-M_1\left(-\ln c^2 \ln \bar{z}_3 + \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)} \left(\frac{\bar{z}_3}{|z_3|^2 + c^2}\right)^{-\frac{M_1}{2}}, \\
\Phi_3 &= h(\bar{z}_4) e^{M_2\left(-\ln c^2 \ln z_4 + \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \left(\frac{z_4}{|z_4|^2 + c^2}\right)^{\frac{M_2}{2}}, \\
\Phi_4 &= k(z_4) e^{-M_2\left(-\ln c^2 \ln \bar{z}_4 + \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \left(\frac{\bar{z}_4}{|z_4|^2 + c^2}\right)^{-\frac{M_2}{2}},
\end{aligned} \tag{4.21}$$

where $g(z_3)$ and $k(z_4)$ ($f(\bar{z}_3)$ and $h(\bar{z}_4)$) are holomorphic (anti-holomorphic) functions determined by normalization condition of wavefunctions. $\mathrm{Li}_2(z)$ appearing in the exponents of obtained wavefunctions denotes the dilogarithm (Spence) function:

$$\mathrm{Li}_2(z) = -\int_0^1 \frac{\ln(1 - zt)}{t} dt, \tag{4.22}$$

with $z$ being a complex number and this function has a cut along the real axis of $z$ from 1 to $\infty$. When we evaluate the Feynman parameter integrals of a one-loop amplitude, we use this dilogarithm function in a similar manner [28]. For the solution of the scalar wavefunction, we take the two ansatzes, $\Phi = \Phi_1\Phi_3$ or $\Phi_2\Phi_4$ in terms of two of them in Eq. (4.21).

The forms of unknown functions in Eq. (4.21) are constrained by imposing the periodic boundary condition $\Phi(\theta_i, \phi_i + 2\pi) = \Phi(\theta_i, \phi_i)$. The $f, g, h, k$ in Eq. (4.21) then have to be proportional to the integer powers of $z_i$ and/or $\bar{z}_i$:

$$\begin{aligned}
f(\bar{z}_3) &= \sum_{p\in\mathbb{Z}} A_p^f \bar{z}_3^{p+M_1(-\ln c^2 + 1/2)}, \\
g(z_3) &= \sum_{p'\in\mathbb{Z}} A_{p'}^g z_3^{p'-M_1(-\ln c^2 + 1/2)}, \\
h(\bar{z}_4) &= \sum_{q\in\mathbb{Z}} A_q^h \bar{z}_4^{q+M_2(-\ln c^2 + 1/2)}, \\
k(z_4) &= \sum_{q'\in\mathbb{Z}} A_{q'}^k z_4^{q'-M_2(-\ln c^2 + 1/2)},
\end{aligned} \tag{4.23}$$

where $p$, $p'$, $q$, $q'$ are the integers and $A_{p,p',q,q'}^{f,g,h,k}$ are real constants determined by the normalization condition as shown later. By plugging Eq. (4.23) to Eq. (4.21), we find the explicit form of bosonic zero-mode wavefunctions $\Phi_{13,pq}\equiv\Phi_{1,p}\Phi_{3,q}$ and $\Phi_{24,p'q'}\equiv\Phi_{2,p'}\Phi_{4,q'}$ with

$$\begin{aligned}
\Phi_{1,p} &= A^f_p e^{M_1\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})}\left(|z_3|^2+c^2\right)^{-\frac{M_1}{2}}|z_3|^{2M_1\left(-\ln c^2+1/2\right)}\bar{z}_3^p,\\
\Phi_{2,p'} &= A^g_{p'} e^{-M_1\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})}\left(|z_3|^2+c^2\right)^{\frac{M_1}{2}}|z_3|^{-2M_1\left(-\ln c^2+1/2\right)}z_3^{p'},\\
\Phi_{3,q} &= A^h_q e^{M_2\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})}\left(|z_4|^2+c^2\right)^{-\frac{M_2}{2}}|z_4|^{2M_2\left(-\ln c^2+1/2\right)}\bar{z}_4^q,\\
\Phi_{4,q'} &= A^k_{q'} e^{-M_2\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})}\left(|z_4|^2+c^2\right)^{\frac{M_2}{2}}|z_4|^{-2M_2\left(-\ln c^2+1/2\right)}z_4^{q'}.
\end{aligned} \tag{4.24}$$

From the structure of periodicity, we find that wavefunctions $\Phi_{13,pq}$ and $\Phi_{24,p'q'}$ are orthogonal to each other, i.e., $\int dz_3^2dz_4^2\sqrt{g}\Phi^\dagger_{13,p_1q_1}\Phi_{13,p_2q_2}=0$ for $p_1\neq p_2$ or $q_1\neq q_2$ (similarly for $\Phi_{24,p'q'}$). They are thus independent zero-mode solutions for the Laplace equation (4.19) labeled by the integers $p$, $p'$, $q$ and $q'$. In the following, we consider the normalization conditions for the case of supersymmetric flux, non-supersymmetric flux and vanishing flux, step by step. Then, in both cases, the normalization condition restricts the possible integers for $p$, $p'$, $q$ and $q'$ and the validity conditions of zero-mode wavefunctions in the conifold region further constraint on $p$, $p'$, $q$ and $q'$. The degeneracies of zero-mode solutions labeled by the integers $p$, $p'$, $q$ and $q'$, are identified with the number of generations of bosonic mode.

- $M_1=-M_2\equiv M\neq 0$

First, we take into account the normalization of $\Phi_{13,pq}$ in the case of supersymmetric fluxes $M_1=-M_2\equiv M\neq 0$. The normalization condition

$$\int d^2z_3d^2z_4\sqrt{g}\Phi^\dagger_{13,p_iq_i}\Phi_{13,p_jq_j}=\delta_{p_ip_j}\delta_{q_iq_j}, \tag{4.25}$$

is explicitly written by

$$\begin{aligned}
1=\frac{16\pi^2c^2}{27}(A^f_p)^2(A^h_q)^2\int dr_3dr_4 e^{2M\mathrm{Li}_2(-\frac{r_3^2}{c^2})}e^{-2M\mathrm{Li}_2(-\frac{r_4^2}{c^2})}\frac{2r_3^2+2r_4^2+3c^2}{(r_3^2+c^2)^{M+2}(r_4^2+c^2)^{-M+2}}\\
\times r_3^{2p+1+4M(-\ln c^2+1/2)}r_4^{2q+1-4M(-\ln c^2+1/2)}.
\end{aligned} \tag{4.26}$$

By expanding the dilogarithm function in the limit $r\gg 1$,

$$\mathrm{Li}_2\left(-\frac{r^2}{c^2}\right)\simeq-\frac{\pi^2}{6}-\frac{1}{2}\left(\ln\left(\frac{r^2}{c^2}\right)\right)^2+\mathcal{O}\left(\frac{c^2}{r^2}\right), \tag{4.27}$$

it turns out that the exponential factor $e^{-2M\mathrm{Li}_2(-r_4^2/c^2)}$ ($e^{2M\mathrm{Li}_2(-r_3^2/c^2)}$) in the integrand of Eq. (4.26) diverges in the limit $r_4\gg c$ ($r_3\gg c$) if the flux $M$ is a positive (negative) value. As a result, the supersymmetric flux results in the non-normalizable bosonic wavefunction. The above procedure is applicable to the other wavefunction $\Phi_{24,p'q'}$, and we obtain the same result as $\Phi_{13,pq}$.

- $M_1=M_2\equiv M\neq 0$

In comparison with the supersymmetric fluxes, from now on we discuss the non-supersymmetric flux $M_1 = M_2 \equiv M \neq 0$ which gives rise to the nonvanishing Fayet-Iliopoulos (FI) term. Such an FI term is caused by the following $U(1)$ field strength:

$$F = MQ_1\Omega_{11} + MQ_2\Omega_{22}, \tag{4.28}$$

which corresponds to the supersymmetric flux in Eq. (4.14) by choosing the different sign of $M_2$. In this thesis, we assume that certain vacuum expectation values of charged scalar fields under the fluxed $U(1)$ symmetry cancel the non-zero FI term. This situation can be realized as follows. When the discussed $U(1)$ symmetry is anomalous, the Green-Schwarz field is the candidate for this charged fields. The bifundamental fields under the discussed $U(1)$ and another $U(1)$ induced by the other possible source also play a role of charged fields to cancel the FI term, although, in both cases, a backreaction of the present D7-brane configuration to such VEVs is assumed to be negligible. As shown below, one can obtain the normalizable bosonic wavefunction due to the sign flip $-M_2 \to M_2$ in contrast to the previous supersymmetric flux $M_1 = -M_2 \neq 0$.

To obtain the normalizable zero-mode solutions, we first derive the upper and lower bounds on the integers $p$ and $q$. In our analysis, we focus on the wavefunction inside the boundary of near horizon limit and it is then assumed that the tail of wavefunction outside the conifold does not give a significant effect. Therefore, the wavefunction has to be well localized inside the boundary of near horizon limit, otherwise it cannot be controlled. The asymptotic form of the bosonic wavefunction is evaluated in the limit $R = r_3 = r_4 \gg c$ as

$$\begin{aligned}\hat{\Phi}_{13,pq} \equiv 4\pi R(\sqrt{g})^{1/2}|\Phi_{13,pq}| &\propto R^{p+q-4M\ln c^2-2}e^{-M(\ln(R^2/c^2))^2} \\ &= R^{p+q-2M\ln(c^2R^2)-2}c^{2M\ln(R^2/c^2)},\end{aligned} \tag{4.29}$$

by employing Eq. (4.27) and then the extremal condition of $\hat{\Phi}_{13,pq}$ is realized at the following extremal point $R_*$ satisfying

$$p + q - 2 - 8M\ln R_* = 0. \tag{4.30}$$

Around this extremal point, the wavefunction of bosonic mode localizes. The localized wavefunction around the tip of cone requires the following inequality, $R_* \ll \sqrt{cL^{3/2} - c^2}$, [§] that is,

$$p + q < 8M\ln\sqrt{cL^{3/2} - c^2} + 2, \tag{4.31}$$

which justifies the localized bosonic mode in the conifold region. We call such a condition the validity condition of scalar mode.

To suppress the backreaction of D7-brane, we adopt the following illustrative parameters for the string coupling $g_s$ and the number of D3-branes $N_c$:

$$g_s = 0.1, \qquad N_c = 100, \tag{4.32}$$

[§] $r_{\max} = L^3 \gg \frac{(R_*^2+c^2)^2}{c^2}$.

which yields the horizon scale $L$ as

$$7.3 \leq L \leq 146, \tag{4.33}$$

in the unit of $M_{\rm Pl} = 1$, within the range of the string scale $M_s = 1/(2\pi\sqrt{\alpha'})$, $10^{16}\,\text{GeV} \leq M_s \leq 2 \times 10^{17}\,\text{GeV}$. For instance, by setting the string scale as $M_s \simeq 1.08 \times 10^{17}\,\text{GeV}$, the validity condition (4.31) is numerically given by

$$p + q < 2 + 0.8N \ \ (L \simeq 13.5), \tag{4.34}$$

with $c = 1$ and quantized integral fluxes $N = N_1 = N_2$. Accordingly, the value of $\theta_{\rm min}$ is determined by the value of $L$ and at the same time the fluxes $N$ are obtained from Eq. (4.15).

Next, we explore the convergence condition of the normalization factors provided by Eq. (4.26) with the flipped sign of $M_2$, which puts the lower bounds on $p$ and $q$. Around $r_3, r_4 \simeq 0$, the integrand of Eq. (4.26) is asymptotically evaluated as

$$c^{-4M-8}(2r_3^2 + 2r_4^2 + 3c^2) r_3^{2p+1+4M(-\ln c^2+1/2)} r_4^{2q+1+4M(-\ln c^2+1/2)}, \tag{4.35}$$

from which the normalization condition of wavefunction requires

$$p > -1 - 2M(-\ln c^2 + \frac{1}{2}), \quad q > -1 - 2M(-\ln c^2 + \frac{1}{2}). \tag{4.36}$$

For instance, the numerical values of the lower bounds (4.36) are provided by setting the string scale $M_s \simeq 1.08 \times 10^{17}$ GeV,

$$p > -1 - 0.05N, \quad q > -1 - 0.05N \ \ (L \simeq 13.5). \tag{4.37}$$

Thus, validity condition (4.34) and normalization condition (4.37) with the fixed number of flux $N$ allow us to constrain the value of $p$ and $q$. It results in the finite number of degenerate zero-modes which are distinguished by the choice of $p$ and $q$.

In order to check the above statements, we numerically estimate the bosonic wavefunction $\hat{\Phi}_{13,pq}$. In Fig. 4.1, the bosonic wavefunction $\hat{\Phi}_{13,pq}$ is plotted on the $R = r_3 = r_4$ hypersurface by setting the typical parameters (4.32) and

$$c = 1, \quad N = 1, \quad L \simeq 13.5, \quad M_s = 1.08 \times 10^{17}\,\text{GeV}, \tag{4.38}$$

in the unit $M_{\rm Pl} = 1$. The normalization factors of bosonic wavefunction are then computed numerically.

As shown in Fig. 4.1, we cannot control the wavefunctions drawn by the blue dot-dashed curve ($p = 1, q = 2$) and the red-solid curve ($p = q = 2$), since they are not localized inside the boundary of near horizon limit, $R_* > \sqrt{cL^{3/2} - c^2} \simeq 7$. This structure is well captured in the analytical observation in Eq. (4.30). As illustrated in Fig. 4.1, we find the fifteen independent zero-mode solutions labeled by $(p, q) = (\pm 1, \pm 1)$, $(\pm 1, 0)$, $(0, \pm 1)$, $(0, 0)$, $(-1, 2)$, $(-1, 3)$, $(0, 2)$, $(2, -1)$, $(2, 0)$ and $(3, -1)$, in other words, fifteen generations of massless scalars. The above procedure can be applied to the wavefunction $\Phi_{24,p'q'}$ by replacing $M_2 \to -M_2$ and we obtain the same result as $\Phi_{13,pq}$.

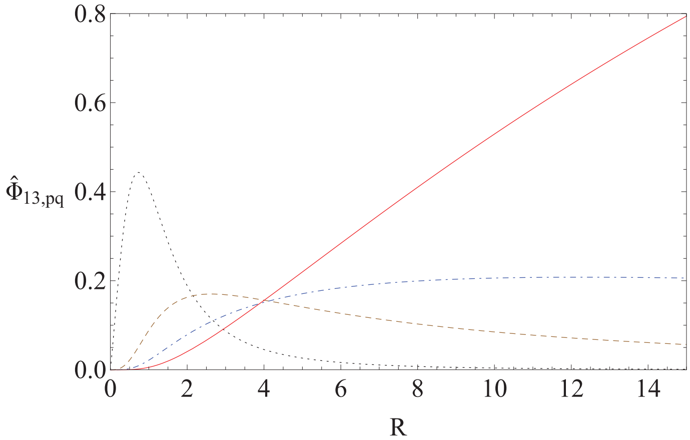


Figure 4.1: The typical bosonic wavefunction $\hat{\Phi}_{13,pq}$ on the $R = r_3 = r_4$ hypersurface for the non-supersymmetric fluxes $N_1 = N_2 = 1$. The black dotted ($p = q = 0$) and the brown dashed ($p = q = 1$) curves are localized at the point inside the boundary of near horizon limit, $R \lesssim 7$, whereas the blue dot-dashed ($p = 1, q = 2$) and red solid curves ($p = q = 2$) are away from the boundary.

Finally let us comment on the twisting. If the cycle wrapped by D7-brane has nontrivial normal bundle, it gives rise to the twisting to both the Laplace equation and the Dirac equation. In such a case, the twisting induces the shift of magnetic fluxes that the bifundamental matter fields feel as e.g., $N \to N+1$ in the case of a minimal twist in the conifold region.¶ Accordingly, such a twist changes the number of zero-modes and it is also applicable to the following zero-flux case.

¶Although the explicit value of shifting can be determined by the local topology of conifold region, we just assign the minimal twist for the magnetic flux as $N \to N+1$. In the case of D7-brane on $\mathbb{P}^2$, the twisting arises as $N \to N-3$ for bosons and $N \to N \pm 3/2$ for fermions [18], respectively.

- $M_1 = M_2 = 0$

Let us consider the vanishing flux, i.e., $M_1 = M_2 = 0$. The normalization condition of the bosonic wavefunctions becomes

$$\begin{aligned}\int d^2z_3 d^2z_4 \sqrt{g}\Phi^\dagger_{13,pq}\Phi_{13,pq} &= \frac{16\pi^2c^2}{27}(A^f_p)^2(A^h_q)^2\int dr_3dr_4\frac{2r_3^2+2r_4^2+3c^2}{(r_3^2+c^2)^2(r_4^2+c^2)^2}r_3^{2p+1}r_4^{2q+1}\\ &= (A^f_p)^2(A^h_q)^2\frac{4\pi^4c^{2(p+q)}(-pq-2p-2q)}{27\sin(p\pi)\sin(q\pi)},\end{aligned} \tag{4.39}$$

with $z_i = r_i\exp[i\theta_i]$ for $i = 3, 4$. The normalization factors are obtained as

$$A^f_pA^h_q = \sqrt{\frac{27\sin(p\pi)\sin(q\pi)}{4\pi^4c^{2(p+q)}(-pq-2p-2q)}}. \tag{4.40}$$

We remark that the integrals in Eq. (4.39) are performed over the radius coordinates $r_3$ and $r_4$ from 0 to infinity. Furthermore, we employ the following formula when the integral in Eq. (4.39) is evaluated:

$$\begin{aligned}\int dr\frac{r^{2K+1}}{(r^2+c^2)^{L+1}} &= c^{2K-2L}\int_0^{\frac{\pi}{2}}\sin^{2K+1}\theta\cos^{2L-2K-1}\theta\\ &= c^{2K-2L}\frac{\Gamma(K+1)\Gamma(L-K)}{2\Gamma(L+1)}.\end{aligned} \tag{4.41}$$

Thus, $p$ and $q$ are constrained as $-1 < p < 0$ and $-1 < q < 0$ such that the normalization factor does not diverge. However, the obtained range of $p$ and $q$ is contradicted with the integers $p$ and $q$. In such a vanishing flux case, we cannot obtain the normalizable modes. The result of $\Phi_{24,p'q'}$ is almost the same with $\Phi_{13,pq}$

Throughout this analysis, we concentrate on the localized wavefuncitons around the tip of cone. Thus, in contrast to the previous analysis, the integration in Eq. (4.39) should be performed over the radius coordinates from 0 to $r_{\rm max}$, and then it will expect the existence of normalizable modes inside the boundary of near horizon limit. However, our result implies that the obtained wavefuntions in Eq. (4.29) behave as $(p+q)$-th power of $R$ and any extremal point is not involved in them. The validity condition cannot be then applied in a simple way. Alternatively, when only monotonically decreasing functions, i.e., the wavefunctions with negative exponent, are counted for the normalizable zero-modes, we find that in the vanishing flux case, three independent zero-mode solutions, i.e., $(p, q) = (0, 0)$, $(0, 1)$ and $(1, 0)$, satisfy the normalization condition (4.36).

### Minkowski vector modes

Let us discuss the extra-dimensional wavefunctions of the 4D gauge fields on D7-branes (vector degrees of freedom in 4D Minkowski spacetime). Their solutions also obey Eq. (4.24) in a way similar to the transverse scalar modes. As a consequence of the real vector field in Minkowski spacetime, the wavefunction of each component should be real. Thus, the wavefunction with

$(p,q)=(0,0)$ is only allowed. We summarize that there exists the single zero-mode solution for the Minkowski vector mode and its wavefunction is a constant. Such a phenomena is coming from the structure that the 4D gauge field living on world-volume of D7-branes does not localize on it.

### Internal vector modes

We next study the wavefunction of the internal vector modes given in Eq. (3.24). For our purpose, we set the following gauge fixing condition,

$$\begin{aligned}\tilde{D}_m\Phi^m &= g^{z_3\bar{z}_3}\left(D_{z_3}\Phi_{\bar{z}_3}+D_{\bar{z}_3}\Phi_{z_3}\right)+g^{z_3\bar{z}_4}\left(D_{z_3}\Phi_{\bar{z}_4}+D_{\bar{z}_4}\Phi_{z_3}\right)\\ &\quad +g^{z_4\bar{z}_3}\left(D_{z_4}\Phi_{\bar{z}_3}+D_{\bar{z}_3}\Phi_{z_4}\right)+g^{z_4\bar{z}_4}\left(D_{z_4}\Phi_{\bar{z}_4}+D_{\bar{z}_4}\Phi_{z_4}\right),\end{aligned} \tag{4.42}$$

and in particular, we focus on the solutions satisfying both $\Phi_{z_3}=\Phi_{z_4}=0$ and $D_{z_3}\Phi_{\bar{z}_3}=D_{z_4}\Phi_{\bar{z}_3}=D_{z_3}\Phi_{\bar{z}_4}=D_{z_4}\Phi_{\bar{z}_4}=0$ (also flipping with $z_3\leftrightarrow\bar{z}_3$ and $z_4\leftrightarrow\bar{z}_4$). When the fluxes are inserted in a supersymmetric way, we find that vector modes satisfying the above conditions correspond to the zero-mode solutions as can be seen in Eq. (4.21),

$$\begin{aligned}
\Phi_{z_3} &= A^{(z_3)}(z_3,z_4)e^{-M_1\left(-\ln c^2\ln\bar{z}_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}\left(\frac{\bar{z}_3}{|z_3|^2+c^2}\right)^{-\frac{M_1}{2}}\\
&\quad\times\ e^{-M_2\left(-\ln c^2\ln\bar{z}_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)}\left(\frac{\bar{z}_4}{|z_4|^2+c^2}\right)^{-\frac{M_2}{2}}\qquad (M_1,M_2\le 0),\\
\Phi_{z_4} &= A^{(z_4)}(z_3,z_4)e^{-M_1\left(-\ln c^2\ln\bar{z}_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}\left(\frac{\bar{z}_3}{|z_3|^2+c^2}\right)^{-\frac{M_1}{2}}\\
&\quad\times\ e^{-M_2\left(-\ln c^2\ln\bar{z}_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)}\left(\frac{\bar{z}_4}{|z_4|^2+c^2}\right)^{-\frac{M_2}{2}}\qquad (M_1,M_2\le 0),\\
\Phi_{\bar{z}_3} &= A^{(\bar{z}_3)}(\bar{z}_3,\bar{z}_4)e^{M_1\left(-\ln c^2\ln z_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}\left(\frac{z_3}{|z_3|^2+c^2}\right)^{\frac{M_1}{2}}\\
&\quad\times\ e^{M_2\left(-\ln c^2\ln z_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)}\left(\frac{z_4}{|z_4|^2+c^2}\right)^{\frac{M_2}{2}}\qquad (M_1,M_2\ge 0),\\
\Phi_{\bar{z}_4} &= A^{(\bar{z}_4)}(\bar{z}_3,\bar{z}_4)e^{M_1\left(-\ln c^2\ln z_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}\left(\frac{z_3}{|z_3|^2+c^2}\right)^{\frac{M_1}{2}}\\
&\quad\times\ e^{M_2\left(-\ln c^2\ln z_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)}\left(\frac{z_4}{|z_4|^2+c^2}\right)^{\frac{M_2}{2}}\qquad (M_1,M_2\ge 0),
\end{aligned} \tag{4.43}$$

where

$$\begin{aligned}
A^{(z_3)}(z_3,z_4) &= \sum_{p',q'\in\mathbb{Z}} A^{(z_3)}_{p',q'} z_3^{p'-M_1(-\ln c^2+1/2)} z_4^{q'-M_2(-\ln c^2+1/2)}, \\
A^{(z_4)}(z_3,z_4) &= \sum_{p',q'\in\mathbb{Z}} A^{(z_4)}_{p',q'} z_3^{p'-M_1(-\ln c^2+1/2)} z_4^{q'-M_2(-\ln c^2+1/2)}, \\
A^{(\bar z_3)}(\bar z_3,\bar z_4) &= \sum_{p',q'\in\mathbb{Z}} A^{(\bar z_3)}_{p',q'} \bar z_3^{p+M_1(-\ln c^2+1/2)} \bar z_4^{q+M_2(-\ln c^2+1/2)}, \\
A^{(\bar z_4)}(\bar z_3,\bar z_4) &= \sum_{p',q'\in\mathbb{Z}} A^{(\bar z_4)}_{p',q'} \bar z_3^{p+M_1(-\ln c^2+1/2)} \bar z_4^{q+M_2(-\ln c^2+1/2)}.
\end{aligned} \tag{4.44}$$

Here, the normalization condition of wavefunction:

$$\int d^2z_3 d^2z_4 \sqrt{g} g^{i\bar{j}} \Phi_i^{\mathfrak{ab}} \Phi_{\bar{j}}^{\mathfrak{ba}} = 1, \tag{4.45}$$

allows us to determine the integers $p'$, $q'$ and real constants $A^{(z_3,z_4,\bar z_3,\bar z_4)}_{p',q'}$.

On the other hand, when the fluxes are inserted in a non-supersymmetric way, i.e., $M_1 = M_2 \equiv M$, $\Phi_{z_3} = \Phi_{z_4}$ and $\Phi_{\bar z_3} = \Phi_{\bar z_4}$ are expected from the structure of $A^{(z_3)} = A^{(z_4)} \equiv A^{(z)}$ and $A^{(\bar z_3)} = A^{(\bar z_4)} \equiv A^{(\bar z)}$ by considering the symmetry under the exchange of $z_3$ and $z_4$. When the flux vanishes, we obtain a single constant normalizable zero-mode wavefunction. Furthermore, when $M_1, M_2 \geq 0$, the normalization condition in Eq. (4.45) is deformed to

$$\begin{aligned}
&\int d^2z_3 d^2z_4 \left(g_{z_3\bar z_3} + g_{z_4\bar z_4} - g_{z_3\bar z_4} - g_{z_4\bar z_3}\right) \Phi_{\bar z_3}^2 \\
&= \int d^2z_3 d^2z_4 \left[\mathcal{R}_1(|z_3|,|z_4|) \bar z_3^{2p} \bar z_4^{2q} + \mathcal{R}_2(|z_3|,|z_4|)(z_3\bar z_4 + \bar z_3 z_4) \bar z_3^{2p} \bar z_4^{2q}\right],
\end{aligned} \tag{4.46}$$

where $\mathcal{R}_1$ and $\mathcal{R}_2$ denote the real-valued functions whose values are non-vanishing only if $p = q = 0$. By taking into account the above normalization condition, the asymptotic form of wavefunction of the vector mode is given in the limit $R = r_3 = r_4 \gg c$ as

$$\begin{aligned}
\hat\Phi_{\bar z_3,pq} \equiv 4\pi R \left(g_{z_3\bar z_3} + g_{z_4\bar z_4} - g_{z_3\bar z_4} - g_{z_4\bar z_3}\right)^{1/2} \Phi_{\bar z_3,pq} &\propto R^{-4M\ln c^2} e^{-M(\ln(R^2/c^2))^2} \\
&= R^{-2M\ln(c^2R^2)} c^{2M\ln(R^2/c^2)},
\end{aligned} \tag{4.47}$$

from which the extremal condition of $\hat\Phi_{z_3,pq}$ is realized at the following the extremal point $R_*$

$$R_* = 1, \tag{4.48}$$

under $c \ll 1$. Around this extremal point, the wavefunction of vector mode localizes. The localized wavefunction around the tip of cone requires the following inequality, $R_* \ll \sqrt{cL^{3/2} - c^2}$, i.e.,

$$1 < \sqrt{cL^{3/2} - c^2}, \tag{4.49}$$

which justifies the localized vector mode in the conifold region. We call such a condition the validity condition of the vector mode in a similar fashion. We conclude that the single internal vector zero-mode with $(p,q) = (0,0)$ is allowed in the local conifold region only if the parameters satisfy Eq. (4.49) and $c \ll 1$.

### 4.2.2 Fermions

To write down the Dirac equations, we rewrite the induced metric of D7-brane $ds^2_{D7} = e_1^2 + e_2^2 + e_3^2 + e_4^2$ on the vierbein bases,

$$\begin{aligned} e_1 &= \frac{1}{\sqrt{6(C_1^2+C_2^2)}}(C_2 d\theta_1 - C_1 d\theta_2), \\ e_2 &= \sqrt{\frac{1}{9}+\frac{1}{6(C_1^2+C_2^2)}}(C_1 d\theta_1 + C_2 d\theta_2), \\ e_3 &= \frac{1}{\sqrt{6(C_1^2+C_2^2)}}(C_2 \sin\theta_1 d\phi_1 - C_1 \sin\theta_2 d\phi_2), \\ e_4 &= \sqrt{\frac{1}{9}+\frac{1}{6(C_1^2+C_2^2)}}(C_1 \sin\theta_1 d\phi_1 + C_2 \sin\theta_2 d\phi_2), \end{aligned} \tag{4.50}$$

which are given by the holomorphic coordinates,

$$\begin{aligned} e_1 &= -\frac{|z_3||z_4|c}{\sqrt{6\rho}}\left[\frac{\bar z_3 dz_3 + z_3 d\bar z_3}{|z_3|^2(|z_3|^2+c^2)} - \frac{\bar z_4 dz_4 + z_4 d\bar z_4}{|z_4|^2(|z_4|^2+c^2)}\right], \\ e_2 &= -i\frac{|z_3||z_4|c}{\sqrt{6\rho}}\left[\frac{\bar z_3 dz_3 - z_3 d\bar z_3}{|z_3|^2(|z_3|^2+c^2)} - \frac{\bar z_4 dz_4 - z_4 d\bar z_4}{|z_4|^2(|z_4|^2+c^2)}\right], \\ e_3 &= -\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}\left[\frac{\bar z_3 dz_3 + z_3 d\bar z_3}{|z_3|^2+c^2} - \frac{\bar z_4 dz_4 + z_4 d\bar z_4}{|z_4|^2+c^2}\right], \\ e_4 &= -i\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}\left[\frac{\bar z_3 dz_3 - z_3 d\bar z_3}{|z_3|^2+c^2} - \frac{\bar z_4 dz_4 - z_4 d\bar z_4}{|z_4|^2+c^2}\right], \end{aligned} \tag{4.51}$$

with $\rho = |z_3|^2 + |z_4|^2$. The coefficients of these bases, i,e., the vierbein $e_{\alpha m}$ itself are represented by the subscripts like $\alpha = 1,2,3,4$ and $m = z_3, \bar z_3, z_4, \bar z_4$. In this subsection, the Greek indices label the local Lorenz frame, whereas the Roman indices denote the complex coordinates of the D7-brane worldvolume in extra dimensions. We definte the vierbeins in the dual basis as $\hat{e}_\alpha = e_\alpha^m \partial_m$ which is explicitly given by

$$\begin{aligned} \hat{e_1} &= -\frac{3|z_3||z_4|}{c\sqrt{6\rho}}\left[\frac{|z_3|^2+c^2}{|z_3|^2}(z_3\partial_{z_3} + \bar z_3 \partial_{\bar z_3}) - \frac{|z_4|^2+c^2}{|z_4|^2}(z_4\partial_{z_4} + \bar z_4 \partial_{\bar z_4})\right], \\ \hat{e_2} &= i\frac{3|z_3||z_4|}{c\sqrt{6\rho}}\left[\frac{|z_3|^2+c^2}{|z_3|^2}(z_3\partial_{z_3} - \bar z_3 \partial_{\bar z_3}) - \frac{|z_4|^2+c^2}{|z_4|^2}(z_4\partial_{z_4} - \bar z_4 \partial_{\bar z_4})\right], \\ \hat{e_3} &= -\frac{1}{2\rho\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}}\left[(|z_3|^2+c^2)(z_3\partial_{z_3} + \bar z_3 \partial_{\bar z_3}) + (|z_4|^2+c^2)(z_4\partial_{z_4} + \bar z_4 \partial_{\bar z_4})\right], \\ \hat{e_4} &= i\frac{1}{2\rho\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}}\left[(|z_3|^2+c^2)(z_3\partial_{z_3} - \bar z_3 \partial_{\bar z_3}) + (|z_4|^2+c^2)(z_4\partial_{z_4} - \bar z_4 \partial_{\bar z_4})\right]. \end{aligned} \tag{4.52}$$

When the background geometry has a non-zero curvature, the spin connections are involved in the zero-mode Dirac equation for the spinor field $\Psi$,

$$ie^m_\nu \tilde{\gamma^\nu}\left(\partial_m + \frac{1}{8}[\tilde{\gamma^\alpha}, \tilde{\gamma^\beta}]w_{m\alpha\beta} - iA_m\right)\Psi = 0, \tag{4.53}$$

where $\Psi = ((\psi_1^+), (\psi_2^-)) \equiv (\Psi_1, \Psi_2, \Psi_3, \Psi_4)$ with the extra-dimensional part of Majorana-Weyl fermions $\psi_1^+$ and $\psi_2^-$ given in Eq. (3.16). The Majorana condition $\lambda^* = B\lambda$ ($B = \Gamma^2\Gamma^4\Gamma^7\Gamma^9$) imposes that $\lambda_3$ ($\lambda_4$) is written by $\lambda_2$ ($\lambda_1$). Following the line of thoughts, we solve the Dirac equation (4.53) for $\psi_1^+$ and $\psi_2^-$ and at the same time, $\psi_3^-$ and $\psi_4^+$ are determined by them. We summarize the explicit form of spin connections in Appendix B in Ref. [7]. Since the 10D gamma matrices are defined as in Eq. (3.12), the explicit form of the Dirac equation (4.53) is obtained in this basis

$$\begin{pmatrix} 0 & 0 & \mathcal{D}_{+11} & \mathcal{D}_{+12} \\ 0 & 0 & \mathcal{D}_{+21} & \mathcal{D}_{+22} \\ \mathcal{D}_{-11} & \mathcal{D}_{-12} & 0 & 0 \\ \mathcal{D}_{-21} & \mathcal{D}_{-22} & 0 & 0 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} = 0, \tag{4.54}$$

which gives rise to the simultaneous differential equations,

$$\begin{aligned} \mathcal{D}_{+11}\Psi_3 + \mathcal{D}_{+12}\Psi_4 &= 0, \\ \mathcal{D}_{+21}\Psi_3 + \mathcal{D}_{+22}\Psi_4 &= 0, \\ \mathcal{D}_{-11}\Psi_1 + \mathcal{D}_{-12}\Psi_2 &= 0, \\ \mathcal{D}_{-21}\Psi_1 + \mathcal{D}_{-22}\Psi_2 &= 0. \end{aligned} \tag{4.55}$$

In the following, we focus on the upper right elements of the Dirac operator acting on $\Psi_3$ and $\Psi_4$ in Eq. (4.54). Their explicit forms are

$$\begin{aligned} \mathcal{D}_{+11} &= -\mathcal{A}\left[\partial_{z_3} + \frac{M_1}{z_3}\ln(|z_3|^2 + c^2)\right] + \mathcal{B}\left[\partial_{z_4} + \frac{M_2}{z_4}\ln(|z_4|^2 + c^2)\right] - \mathcal{E} + M', \\ \mathcal{D}_{+12} &= -i\mathcal{C}\left[\partial_{\bar{z}_3} - \frac{M_1}{\bar{z}_3}\ln(|z_3|^2 + c^2)\right] - i\mathcal{D}\left[\partial_{\bar{z}_4} - \frac{M_2}{\bar{z}_4}\ln(|z_4|^2 + c^2)\right] + i\mathcal{F}, \\ \mathcal{D}_{+21} &= -i\bar{\mathcal{C}}\left[\partial_{z_3} + \frac{M_1}{z_3}\ln(|z_3|^2 + c^2)\right] - i\bar{\mathcal{D}}\left[\partial_{z_4} + \frac{M_2}{z_4}\ln(|z_4|^2 + c^2)\right] + i\mathcal{F}, \\ \mathcal{D}_{+22} &= -\bar{\mathcal{A}}\left[\partial_{\bar{z}_3} - \frac{M_1}{\bar{z}_3}\ln(|z_3|^2 + c^2)\right] + \bar{\mathcal{B}}\left[\partial_{\bar{z}_4} - \frac{M_2}{\bar{z}_4}\ln(|z_4|^2 + c^2)\right] - \mathcal{E} - M', \end{aligned} \tag{4.56}$$

where

$$
\begin{aligned}
\mathcal{A} &= \frac{6|z_3||z_4|(|z_3|^2+c^2)}{c\sqrt{6\rho}\bar{z}_3}, \qquad \mathcal{B} = \frac{6|z_3||z_4|(|z_4|^2+c^2)}{c\sqrt{6\rho}\bar{z}_4}, \\
\mathcal{C} &= \frac{|z_3|^2(|z_3|^2+c^2)}{\rho\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}z_3}, \qquad \mathcal{D} = \frac{|z_4|^2(|z_4|^2+c^2)}{\rho\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}z_4}, \\
\mathcal{E} &= \frac{3\Big(|z_3|^2|z_4|^2(|z_3|^2-|z_4|^2)+c^2(|z_4|^4-|z_3|^4)\Big)}{2\rho c\sqrt{6\rho}|z_3||z_4|} + \frac{|z_3||z_4|c\left(|z_4|^2-|z_3|^2\right)}{4\rho^2\sqrt{6\rho}\left(\frac{1}{9}+\frac{c^2}{6\rho}\right)}, \\
\mathcal{F} &= \frac{2|z_3|^4+10|z_3|^2|z_4|^2+2|z_4|^4-3\rho c^2}{12\rho^2\sqrt{\frac{1}{9}+\frac{c^2}{6\rho}}} - \frac{c^2\left(\rho^2+6|z_3|^2|z_4|^2+3\rho c^2\right)}{72\rho^3\left(\frac{1}{9}+\frac{c^2}{6\rho}\right)^{3/2}}, \\
M' &= \frac{3c^2}{|z_3||z_4|c\sqrt{6\rho}}\left(M_1|z_4|^2-M_2|z_3|^2\right). \qquad (4.57)
\end{aligned}
$$

In comparison with Eq. (4.21) in the bosonic case, Eq. (4.55) suggests that the fermionic wavefunctions $\Psi_3$ and $\Psi_4$ have the following forms,

$$
\begin{aligned}
\Psi_3 &= F(z_3,z_4,\bar{z}_3,\bar{z}_4)e^{M_1\left(-\ln c^2\ln z_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}e^{M_2\left(-\ln c^2\ln z_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \\
&\quad\times\left(\frac{z_3}{|z_3|^2+c^2}\right)^{\frac{M_1}{2}}\left(\frac{z_4}{|z_4|^2+c^2}\right)^{\frac{M_2}{2}}, \\
\Psi_4 &= K(z_3,z_4,\bar{z}_3,\bar{z}_4)e^{-M_1\left(-\ln c^2\ln\bar{z}_3+\mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)}e^{-M_2\left(-\ln c^2\ln\bar{z}_4+\mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \\
&\quad\times\left(\frac{\bar{z}_3}{|z_3|^2+c^2}\right)^{-\frac{M_1}{2}}\left(\frac{\bar{z}_4}{|z_4|^2+c^2}\right)^{-\frac{M_2}{2}}, \qquad (4.58)
\end{aligned}
$$

where $F(z_3,z_4,\bar{z}_3,\bar{z}_4)$ and $K(z_3,z_4,\bar{z}_3,\bar{z}_4)$ are functions whose holomorphic and anti-holomorphic parts are fixed in the following. By plugging Eq. (4.58) into Eq. (4.55), we find

$$
\begin{aligned}
\left[\mathcal{A}\frac{\partial_{z_3}F}{F}-\mathcal{B}\frac{\partial_{z_4}F}{F}+\mathcal{E}\right]\Psi_3+i\left[\mathcal{C}\frac{\partial_{\bar{z}_3}K}{K}+\mathcal{D}\frac{\partial_{\bar{z}_4}K}{K}-\mathcal{F}\right]\Psi_4 &= 0, \\
i\left[\bar{\mathcal{C}}\frac{\partial_{z_3}F}{F}+\bar{\mathcal{D}}\frac{\partial_{z_4}F}{F}-\mathcal{F}\right]\Psi_3+\left[\bar{\mathcal{A}}\frac{\partial_{\partial z_3}K}{K}-\bar{\mathcal{B}}\frac{\partial_{\bar{z}_4}K}{K}+\mathcal{E}\right]\Psi_4 &= 0. \qquad (4.59)
\end{aligned}
$$

Let us consider the situation where all the coefficients of $\Psi_3$ and $\Psi_4$ vanish independently to each other, that gives rise to

$$
\begin{aligned}
\partial_{z_3} F &= \left[\frac{2}{3(|z_3|^2+c^2)} - \frac{1}{4|z_3|^2} - \frac{1}{2(2\rho+3c^2)}\right] \bar{z}_3 F, \\
\partial_{z_4} F &= \left[\frac{2}{3(|z_4|^2+c^2)} - \frac{1}{4|z_4|^2} - \frac{1}{2(2\rho+3c^2)}\right] \bar{z}_4 F, \\
\partial_{\bar{z}_3} K &= \left[\frac{2}{3(|z_3|^2+c^2)} - \frac{1}{4|z_3|^2} - \frac{1}{2(2\rho+3c^2)}\right] z_3 K, \\
\partial_{\bar{z}_4} K &= \left[\frac{2}{3(|z_4|^2+c^2)} - \frac{1}{4|z_4|^2} - \frac{1}{2(2\rho+3c^2)}\right] z_4 K.
\end{aligned} \tag{4.60}
$$

We obtain the functions $F$ and $K$ satisfying Eq. (4.60) as

$$
\begin{aligned}
F(z_3, z_4, \bar{z}_3, \bar{z}_4) &= G(\bar{z}_3, \bar{z}_4)(|z_3|^2+c^2)^{\frac{2}{3}}(|z_4|^2+c^2)^{\frac{2}{3}}(2\rho+3c^2)^{-\frac{1}{4}} z_3^{-\frac{1}{4}} z_4^{-\frac{1}{4}}, \\
K(z_3, z_4, \bar{z}_3, \bar{z}_4) &= H(z_3, z_4)(|z_3|^2+c^2)^{\frac{2}{3}}(|z_4|^2+c^2)^{\frac{2}{3}}(2\rho+3c^2)^{-\frac{1}{4}} \bar{z}_3^{-\frac{1}{4}} \bar{z}_4^{-\frac{1}{4}},
\end{aligned} \tag{4.61}
$$

where anti-holomorphic function $G(\bar{z}_3, \bar{z}_4)$ and holomorphic function $H(z_3, z_4)$ are constrained by the normalization conditions of fermions. In a similar way to the bosonic one, we impose their form as $G(\bar{z}_3, \bar{z}_4) = \bar{z}_3^a \bar{z}_4^b$ and $H(z_3, z_4) = z_3^a z_4^b$.

Finally, the explicit forms of the fermionic wavefunctions become

$$
\begin{aligned}
\Psi_3 &= G(\bar{z}_3, \bar{z}_4) e^{M_1\left(-\ln c^2 \ln z_3 + \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)} e^{M_2\left(-\ln c^2 \ln z_4 + \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \left(\frac{z_3}{|z_3|^2+c^2}\right)^{\frac{M_1}{2}} \left(\frac{z_4}{|z_4|^2+c^2}\right)^{\frac{M_2}{2}} \\
&\quad \times (|z_3|^2+c^2)^{\frac{2}{3}}(|z_4|^2+c^2)^{\frac{2}{3}}(2\rho+3c^2)^{-\frac{1}{4}} z_3^{-\frac{1}{4}} z_4^{-\frac{1}{4}}, \\
\Psi_4 &= H(z_3, z_4) e^{-M_1\left(-\ln c^2 \ln \bar{z}_3 + \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})\right)} e^{-M_2\left(-\ln c^2 \ln \bar{z}_4 + \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})\right)} \left(\frac{\bar{z}_3}{|z_3|^2+c^2}\right)^{-\frac{M_1}{2}} \left(\frac{\bar{z}_4}{|z_4|^2+c^2}\right)^{-\frac{M_2}{2}} \\
&\quad \times (|z_3|^2+c^2)^{\frac{2}{3}}(|z_4|^2+c^2)^{\frac{2}{3}}(2\rho+3c^2)^{-\frac{1}{4}} \bar{z}_3^{-\frac{1}{4}} \bar{z}_4^{-\frac{1}{4}},
\end{aligned} \tag{4.62}
$$

while $\Psi_1$ and $\Psi_2$ have no solution.

When we impose the periodic boundary conditions to the fermionic wavefunctions $\Psi_{3,4}(\theta_i, \phi_i + 2\pi) = \Psi_{3,4}(\theta_i, \phi_i)$, they are proportional to the integer power of $z_i$ and/or $\bar{z}_i$. As a result, the arbitrary functions in Eq. (4.62) are constrained in the following form

$$
\begin{aligned}
G(\bar{z}_3, \bar{z}_4) &= \sum_{a,b \in \mathbb{Z}} A^G_{a,b} \bar{z}_3^{a+M_1(-\ln c^2+1/2)-1/4} \bar{z}_4^{b+M_2(-\ln c^2+1/2)-1/4}, \\
H(z_3, z_4) &= \sum_{a',b' \in \mathbb{Z}} A^H_{a',b'} z_3^{a'-M_1(-\ln c^2+1/2)-1/4} z_4^{b'-M_2(-\ln c^2+1/2)-1/4},
\end{aligned} \tag{4.63}
$$

where the normalization conditions determine the integers $a$, $a'$, $b$, $b'$ and real constants $A^{G,H}_{a,b}$ in a way similar to the bosonic wavefunction. By substituting Eq. (4.63) to Eq. (4.62), each zero-mode fermionic wavefunction reduces to

$$\begin{aligned}\Psi_{3,ab} &= A^G_{a,b} e^{M_1 \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})} e^{M_2 \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})} (|z_3|^2+c^2)^{-\frac{M_1}{2}+\frac{2}{3}} (|z_4|^2+c^2)^{-\frac{M_2}{2}+\frac{2}{3}} (2\rho+3c^2)^{-\frac{1}{4}} \\ &\qquad \times |z_3|^{2M_1(-\ln c^2+1/2)-\frac{1}{2}} |z_4|^{2M_2(-\ln c^2+1/2)-\frac{1}{2}} \bar{z}_3^a \bar{z}_4^b, \\ \Psi_{4,a'b'} &= A^H_{a',b'} e^{-M_1 \mathrm{Li}_2(-\frac{|z_3|^2}{c^2})} e^{-M_2 \mathrm{Li}_2(-\frac{|z_4|^2}{c^2})} (|z_3|^2+c^2)^{\frac{M_1}{2}+\frac{2}{3}} (|z_4|^2+c^2)^{\frac{M_2}{2}+\frac{2}{3}} (2\rho+3c^2)^{-\frac{1}{4}} \\ &\qquad \times |z_3|^{-2M_1(-\ln c^2+1/2)-\frac{1}{2}} |z_4|^{-2M_2(-\ln c^2+1/2)-\frac{1}{2}} z_3^{a'} z_4^{b'},\end{aligned} \tag{4.64}$$

Since the different modes of wavefunctions $\Psi_{3,ab}$ ($\Psi_{4,a'b'}$) are orthogonal to each other, i.e., $\int dz_3^2 dz_4^2 \sqrt{g} \Psi^\dagger_{3,a_1b_1} \Psi_{3,a_2b_2} = 0$ for $a_1 \neq a_2$ or $b_1 \neq b_2$, (similarly for $\Psi_{4,a'b'}$) due to the periodicity, they are independent solutions for the Dirac equation (4.54) labeled by the integers $a$, $a'$, $b$ and $b'$. As discussed later, the normalization and validity conditions of wavefunctions constrain the integers for $a$, $a'$, $b$ and $b'$ in the conifold region. The number of allowed integers represents the degeneracy of zero-modes, that is the number of fermionic generations. In the following, we study the normalization conditions in three cases, the vanishing fluxes $M_1 = M_2 = 0$, the supersymmetric fluxes $M_1 = -M_2 \equiv M \neq 0$ and the non-supersymmetric fluxes $M_1 = M_2 \equiv M \neq 0$.

- $M_1 = -M_2 \equiv M \neq 0$

In the case of supersymmetric fluxes, the fermionic wavefunctions are non-normalizable as in the bosonic case. To understand this statement, let us consider the normalization condition of $\Psi_{3,ab}$

$$\int d^2 z_3 d^2 z_4 \sqrt{g} \Psi^\dagger_{3,a_ib_i} \Psi_{3,a_jb_j} = \delta_{a_ia_j} \delta_{b_ib_j}, \tag{4.65}$$

which is explicitly calculated as

$$\begin{aligned}1 = \frac{16\pi^2 c^2}{27} (A^G_{ab})^2 \int dr_3 dr_4 e^{2M \mathrm{Li}_2(-\frac{r_3^2}{c^2})} e^{-2M \mathrm{Li}_2(-\frac{r_4^2}{c^2})} \frac{(2r_3^2+2r_4^2+3c^2)^{1/2}}{(r_3^2+c^2)^{\frac{2}{3}+M} (r_4^2+c^2)^{\frac{2}{3}-M}} \\ \times r_3^{2a+4M(-\ln c^2+1/2)} r_4^{2b-4M(-\ln c^2+1/2)}.\end{aligned} \tag{4.66}$$

Since the dilogarithm function obeys asymptotically in the limit $r \gg 1$,

$$\mathrm{Li}_2\left(-\frac{r^2}{c^2}\right) \simeq -\frac{\pi^2}{6} - \frac{1}{2}\left(\ln\left(\frac{r^2}{c^2}\right)\right)^2 + \mathcal{O}\left(\frac{c^2}{r^2}\right), \tag{4.67}$$

the factor $e^{-2M \mathrm{Li}_2(-r_4^2/c^2)}$ ($e^{2M \mathrm{Li}_2(-r_3^2/c^2)}$) in the integrand of Eq. (4.66) diverges in the limit $r_4 \gg c$ ($r_3 \gg c$) under the positive (negative) flux $M$. It then turns out that the fermionic wavefunction $\Psi_{3,ab}$ is non-normalizable in the case of supersymmetric fluxes. For $\Psi_{4,a'b'}$, we find the same results as $\Psi_{3,ab}$ in a similar fashion.

- $M_1 = M_2 \equiv M \neq 0$

In the case of non-supersymmetric fluxes, one can obtain the normalizable fermionic wavefunction $\Psi_{3,ab}$ ($\Psi_{4,a'b'}$) under the positive (negative) flux $M$, since the non-supersymmetric fluxes (4.28) flip $M_2$ in the supersymmetric one to $-M_2$. As mentioned before, the Fayet-Ilipoulos D-term caused by non-supersymmetric fluxes is assumed to be canceled by some VEVs of charged scalar fields.

To justify our analysis in the local conifold region, we consider the validity condition of fermionic wavefunctions along the same step outlined in Sec. 4.2.1. We concentrate on $\Psi_{3,ab}$ with the non-supersymmetric fluxes without loss of generality. The asymptotic form of $\Psi_{3,ab}$ is yielded in the limit $R = r_3 = r_4 \gg c$

$$\begin{aligned}\hat{\Psi}_{3,ab} \equiv 4\pi R(\sqrt{g})^{1/2}|\Psi_3| &\propto R^{a+b-4M\ln c^2-\frac{1}{3}}e^{-M(\ln(-R^2/c^2))^2} \\ &= R^{a+b-2M\ln(c^2R^2)-\frac{1}{3}}c^{2M\ln(R^2/c^2)},\end{aligned} \tag{4.68}$$

by use of Eq. (4.27). The extremal condition of $\hat{\Psi}_{3,ab}$ is satisfied at the point $R_*$ satisfying

$$a + b - \frac{1}{3} - 8M\ln R_* = 0. \tag{4.69}$$

The localized wavefunction inside the tip of cone constraints the value of $R_*$ as $R_* \ll \sqrt{cL^{3/2} - c^2}$, which puts the tight bound for $a$ and $b$,

$$a + b < 8M\ln\sqrt{cL^{3/2} - c^2} + \frac{1}{3}. \tag{4.70}$$

This gives a validity condition of the fermionic wavefunction to validate our analysis in the local conifold region. In the same step outlined in the bosonic case, $a$ and $b$ are lower bounded by

$$a > -2M(-\ln c^2 + \frac{1}{2}), \quad b > -2M(-\ln c^2 + \frac{1}{2}). \tag{4.71}$$

By setting the following parameters

$$c = 1, \ \ N = 1, \ \ L \simeq 13.5, \ \ M_s = 1.08 \times 10^{17}\,\text{GeV}, \tag{4.72}$$

we plot the fermionic wavefunction $\hat{\Psi}_{3,ab}$ on the $R = r_3 = r_4$ hypersurface in Fig. 4.2 in which the normalization factors are numerically estimated. One cannot control the wavefunction drawn by the red-solid curve in Fig. 4.2, since it localizes outside the boundary of near horizon limit, $R_* > \sqrt{cL^{3/2} - c^2} \simeq 7$. Indeed, we can analytically check such an observation in Eqs. (4.69) and (4.70). In Fig. 4.2, there are three independent zero-mode solutions corresponding to $(a, b) = (0, 0), (1, 0)$ and $(0, 1)$. It is straightforward to extend this analysis for the other wavefunction $\Psi_{4,a'b'}$ with the negative flux $M < 0$, and the obtained wavefunction and its properties are almost the same as $\Psi_{3,ab}$.

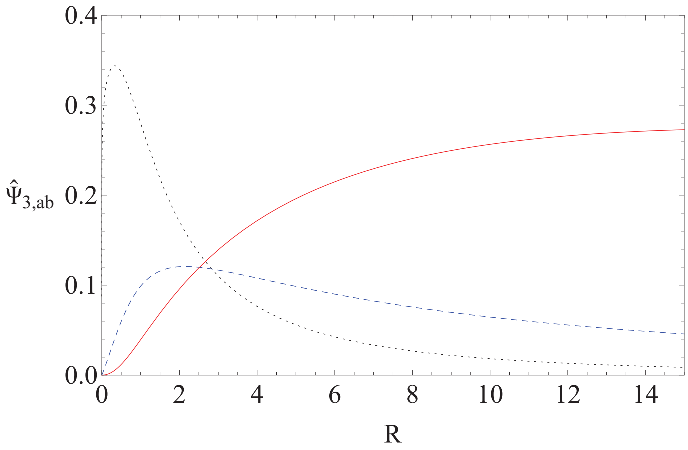


Figure 4.2: The fermionic wavefunction $\hat{\Psi}_{3,ab}$ on the $R = r_3 = r_4$ hypersurface in the case of nonvanishing Fayet-iliopoulos term and $N_1 = N_2 = 1$. The black dotted ($a = b = 0$) and the blue dashed ($a = 0, b = 1$) curves are localized at the point below the horizon scale $R \simeq 7$ in contrast to the red solid curve ($a = 1, b = 1$). The curve parametrized in $a = 1, b = 0$ is the same as the blue dashed curve ($a = 0, b = 1$).

- $M_1 = M_2 = 0$

When the fluxes are absent in the system, the normalization condition of $\Psi_{3,ab}$ is explicitly given by

$$
\begin{aligned}
1 &= \frac{16\pi^2 c^2}{27}(A^G_{a,b})^2 \int dr_3 dr_4 (r_3^2 + c^2)^{-\frac{2}{3}}(r_4^2 + c^2)^{-\frac{2}{3}} r_3^{2a} r_4^{2b} (2r_3^2 + 2r_4^2 + 3c^2)^{\frac{1}{2}} \\
&= \frac{2\sqrt{2}\pi^2 c^{\frac{2}{3}}}{27}(A^G_{a,b})^2 \int dr_3 \frac{r_3^{2a}}{(r_3^2 + c^2)^{\frac{2}{3}}} \left[ -\frac{4c^{2+2b}\csc(b\pi)\Gamma(-b-\frac{1}{3})}{\Gamma(\frac{2}{3})\Gamma(-b)} {}_2F_1\left(-\frac{1}{2}, -b-\frac{1}{3}, -b; \frac{r_3^2}{c^2} + \frac{3}{2}\right) \right. \\
&\qquad \left. + 8^{-b}\frac{(2r_3^2 + 3c^2)^{1+b}\Gamma(-b)\Gamma(2b+1)}{\Gamma(b+2)} {}_2F_1\left(\frac{2}{3}, b+\frac{1}{2}, b+2; \frac{r_3^2}{c^2} + \frac{3}{2}\right) \right],
\end{aligned} \tag{4.73}
$$

where the integral over the infinite region is convergent iff $-\frac{1}{2} < a, b < -\frac{1}{3}$ are satisfied in the same way with the bosonic case. Since these range of $a$ and $b$ are not consistent with the integers $a$ and $b$, we conclude that there are no normalizable zero-modes in the case of vanishing fluxes. For $\Psi_{4,a'b'}$, the obtained result is similar to $\Psi_{3,ab}$.

As discussed in the end of Sec. 4.2.1, one cannot apply the original validity condition to the case of vanishing flux. Thus, we modify it such that the exponents of $R$ in the wavefunctions are restricted to be negative. When this condition and the convergence condition of the normalization factors (4.71) are satisfied, we find that there are no zero-mode solutions for any integers $a$ and $b$. However, the twisting could change this situation. Indeed, shifting the number of fluxes induces at least one fermionic zero-mode which satisfies the above conditions. As an example, a zero-mode solution appears in the case of $(a, b) = (0, 0)$ under the minimal shift $N \to N + 1/2$ where the double-valuedness of the spinor requires the half-integer twist.

## 4.3 Phenomenological aspects

Based on the localized zero-modes derived in the previous section, we in this section discuss their phenomenological consequences from the aspects of four and five dimensional low-energy effective theories.

Before addressing the phenomenological aspects of this model, let us mention about the supersymmetric structure for the obtained zero-modes. Since there is a 4D $\mathcal{N}=1$ supersymmetry around the local conifold region, bosons and fermions discussed in the previous section are encoded in the two types of 4D $\mathcal{N}=1$ supermultiplets. First is the single vector multiplet $\mathbf{V}=\{A_\mu,\eta_2\}$ with $\eta_2$ being the element of 10D Majorana-Weyl fermion $\lambda_2$. Second is the triple chiral multiplets $\mathbf{\Phi}_i=\{A_i,\eta_i\}$ $(i=1,3,4)$ with $\eta_i$ being the elements of 10D Majorana-Weyl fermion $\lambda_i$. Note that, $\lambda_{1,4}$ ($\lambda_{2,3}$) sector represents the fermionic partners of internal vector modes (transverse scalars and gauge bosons of 4D SYM), respectively. From the obtained matter wavefunction in Sec. 4.2 with the ansatz (4.32) and (4.38) without a flux, we find that the transverse scalar has the three generations and Minkowski vectors have the single generation. Other internal vectors have no zero-modes. Thus, the supersymmetry requires the four fermionic zero-modes. Since we consider the ansatz of twisting to match the number of fermions with that of bosons, it can be realized under the following twist, e.g., twist $N\to N+1$ for three fermions $\Psi_3$ and the twist $N\to N-1/2$ for a single fermion $\Psi_4$. The result is summarized in Table 4.1.

Under the twist, the bases of the Majorana-Weyl fermions $\Psi_i$ in 4D $\mathcal{N}=1$ supermultiplets are mixed. Thus, one can construct the supersymmetric models which contain the fermionic partners of internal vector modes. The Yukawa interactions are then given by $\lambda_1\lambda_1\phi$ for the transverse modes $\phi$ or $\lambda_1\lambda_2 A_m$ for the internal vector modes $A_m$. For the detail of the mixture of bases and corresponding Yukawa interactions, see, e.g., Ref. [18]. Following this line of thoughts, in Sec. 4.3.2, we exhibit the particular example of the triple overlap integrals of matter wavefunctions included in the above Yukawa interactions.

| | flux shift | untwisted | twisted | total zero-modes |
|---|---|---|---|---|
| transverse scalar | 0 | 3 | 3 | |
| Minkowski vecor | - | 1 | - | 4 |
| internal vector | - | 0 | - | |
| fermions($\Psi_3,\Psi_4$) | (1,-1/2) | (0,0) | (3,1) | 4 |

Table 4.1: The twist to achieve the supersymmetric spectrum under the ansatz (4.32), (4.38) for $(g_s,N_c,c,L,M_s)$ and $N=0$.

### 4.3.1 Wavefucntions on $AdS_5$

First, we account for the properties of matter wavefunction from the five-dimensional (5D) point of view. The type IIB action on $T^{1,1}$ reduces to 5D effective theory and the 5D background metric in the near-horizon limit becomes

$$ds_5^2 = \frac{r^2}{L^2} d^2 x_{1,3} + \frac{L^2}{r^2} dr^2. \tag{4.74}$$

Under the coordinate transformation $r = Le^{-y/L}$ with $L$ being the inverse of $AdS_5$ curvature, the 5D metric is given by

$$ds_5^2 = e^{-2y/L} d^2 x_{1,3} + dy^2. \tag{4.75}$$

The authors of Refs. [29] pointed out that this background geometry is just the Randall-Sundrum like model [30] where the infrared brane is placed at the tip of conifold, whereas the Planck brane is set at the proper cycle in the global Calabi-Yau manifold. In the following, we denote the locations of branes for $AdS_5$ as $y_H$ for the infrared brane and $y_{\rm pl}$ for the Planck brane, respectively, and $r_{\rm min}$ and $r_{\rm max}$ in the previous sections correspond to $y_{\rm max} = -L \ln\left(c^{2/3}/L\right)$ and $y_{\rm min} = -2L \ln L$. In this notation, the matter fields localize towards the position $y_{\rm max}$. If the structure of $AdS_5$ space also holds out of the near horizon region, the Planck brane can be located at any points including $y_{\rm pl} < y_{\rm min}$.

From the analyses in Secs. 4.2.1 and 4.2.2, we find that the normalizable matter wavefunctions with non-supersymmetric fluxes localize towards the tip of conifold. Indeed, the wavefunctions on the hypersurface $|z_3| = |z_4|$ have the following localized structure:

$$\begin{aligned}
\Phi_{13,pq} &\propto e^{2M{\rm Li}_2\left(1-\frac{L^{3/2}}{c}\exp\left(-\frac{3y}{2L}\right)\right)+\frac{3y}{2L}M} \left(L^{\frac{3}{2}} e^{-\frac{3y}{2L}} - c\right)^{2M\left(-\ln c^2+\frac{1}{2}\right)+\frac{1}{2}(p+q)}, \\
\Phi_{24,pq} &\propto e^{-2M{\rm Li}_2\left(1-\frac{L^{3/2}}{c}\exp\left(-\frac{3y}{2L}\right)\right)-\frac{3y}{2L}M} \left(L^{\frac{3}{2}} e^{-\frac{3y}{2L}} - c\right)^{-2M\left(-\ln c^2+\frac{1}{2}\right)+\frac{1}{2}(p+q)},
\end{aligned} \tag{4.76}$$

for bosons and

$$\begin{aligned}
\Psi_{3,ab} &\propto e^{2M{\rm Li}_2\left(1-\frac{L^{3/2}}{c}\exp\left(-\frac{3y}{2L}\right)\right)+\frac{3y}{2L}M-\frac{2y}{L}} \left(2Le^{-\frac{3y}{2L}}+1\right)^{-\frac{1}{4}} \left(L^{\frac{3}{2}} e^{-\frac{3y}{2L}} - c\right)^{2M\left(-\ln c^2+\frac{1}{2}\right)+\frac{1}{2}(a+b-1)}, \\
\Psi_{4,ab} &\propto e^{-2M{\rm Li}_2\left(1-\frac{L^{3/2}}{c}\exp\left(-\frac{3y}{2L}\right)\right)-\frac{3y}{2L}M-\frac{2y}{L}} \left(2Le^{-\frac{3y}{2L}}+1\right)^{-\frac{1}{4}} \left(L^{\frac{3}{2}} e^{-\frac{3y}{2L}} - c\right)^{-2M\left(-\ln c^2+\frac{1}{2}\right)+\frac{1}{2}(a+b-1)},
\end{aligned} \tag{4.77}$$

for fermions as a function of $y$. From Eqs. (4.76) and (4.77), the form of matter wavefunctions has the exponential structure determined by the $U(1)$ flux. This phenomena is numerically justified in Figs. 4.1 and 4.2. In the framework of $AdS_5$ supergravity [31], the exponential profile of wavefunctions for graviton [30] and matter [32] zero-modes is important to achieve a hierarchy between weak scale and Planck scale, and hierarchical structure of Yukawa couplings between quarks/leptons and Higgs fields. In our model, we also derive exponential forms of bosons and fermions due to the $U(1)$ magnetic flux in a different context. It is then possible to realize the hierarchical structures of overlap integrals between boson and fermions. The detail of them is discussed in the next subsection.

In particular, the approximate form of matter wavefunctions is given in the direction of $AdS_5$,

$$\Phi_{13,pq} \propto \begin{cases} e^{y\frac{3M}{L}\left(\ln(L^{\frac{3}{2}}c)-\frac{1}{4M}(p+q)\right)-y^2\frac{9M}{4L^2}} & (y \ll -L)\,, \\ e^{y\frac{9M}{2L}+(M\ln y)(-2\ln c^2+1+\frac{1}{2M}(p+q))} & (y \simeq y_{\max})\,, \end{cases}$$
$$\Phi_{24,pq} \propto \begin{cases} e^{-y\frac{3M}{L}\left(\ln(L^{\frac{3}{2}}c)+\frac{1}{4M}(p+q)\right)+y^2\frac{9M}{4L^2}} & (y \ll -L)\,, \\ e^{-y\frac{9M}{2L}-(M\ln y)(-2\ln c^2+1-\frac{1}{2M}(p+q))} & (y \simeq y_{\max})\,, \end{cases} \tag{4.78}$$

for bosons and

$$\Psi_{3,ab} \propto \begin{cases} e^{y\frac{3M}{L}\left(\ln(L^{\frac{3}{2}}c)-\frac{1}{4M}(a+b+\frac{7}{6})\right)-y^2\frac{9M}{4L^2}} & (y \ll -L)\,, \\ e^{y\left(\frac{9M}{2L}-\frac{2}{L}\right)+(M\ln y)(-2\ln c^2+1+\frac{1}{2M}(a+b-1))} & (y \simeq y_{\max})\,, \end{cases}$$
$$\Psi_{4,ab} \propto \begin{cases} e^{-y\frac{3M}{L}\left(\ln(L^{\frac{3}{2}}c)+\frac{1}{4M}(a+b+\frac{7}{6})\right)+y^2\frac{9M}{4L^2}} & (y \ll -L)\,, \\ e^{-y\left(\frac{9M}{2L}+\frac{2}{L}\right)-(M\ln y)(-2\ln c^2+1-\frac{1}{2M}(a+b-1))} & (y \simeq y_{\max})\,, \end{cases} \tag{4.79}$$

for fermions. Although, in the effective $AdS_5$ supergravity [31], the bulk masses of the matter fields (proportional to their graviphoton charges) determine the profile of matter wavefunctions, those are just parameters to realize the experimental values of quark and lepton masses. In our model, the approximate form of matter wavefunctions in Eqs. (4.78) and (4.79) suggests that the bulk masses are originating from the generation numbers associated with the $U(1)$ flux.

Next let us comment on the weak-Planck hierarchy. We consider the situation where the matter fields, in particular, the Higgs fields localize toward $y = y_H$ by the magnetic fluxes they feel, whereas the graviton localizes towards the Calabi-Yau manifold, i.e., $y = y_{\rm pl}$. As discussed in the original Randall-Sundrum model [30], the vacuum expectation values of 5D Higgs field $v_0$ and 4D effective Higgs field $v$ are closely related as

$$v = e^{-\frac{y_H - y_{\rm pl}}{L}} v_0. \tag{4.80}$$

In our case with $y_{\rm pl} = y_{\min}$ and $y_H = y_{\max}$, the smallness of distance between $y_H$ and $y_{\rm pl}$ leads to the mild hierarchy between the Higgs VEVs, $v \simeq (c^{2/3}v_0)/L^3$. However, when the effective $AdS_5$ description is valid even outside the conifold region, one can take $y_{\rm pl}$ as smaller values compared with the above sample values. Then, it is possible to achieve the hierarchy between electroweak scale and Planck scale. We again stress that this weak-Planck hierarchy is provided by only the localization profile of matter fields induced by the magnetic fluxes.

### 4.3.2 Overlap integrals

Based on the matter wavefunctions in Sec. 4.2.1 and 4.2.2, we study the 8D $U(N)$ SYM theory to consider the overlap integrals between matter fields. We in particular estimate the overlap integrals among the bifundamental fields appeared by introducing the following magnetic fluxes,

$$F_{\mathfrak{ab}} = \begin{pmatrix} M_{1(N_1)}\mathbb{1}_{N_1} & & \\ & M_{1(N_2)}\mathbb{1}_{N_2} & \\ & & \ddots \end{pmatrix} Q_1\Omega_{11} + \begin{pmatrix} M_{2(N_1)}\mathbb{1}_{N_1} & & \\ & M_{2(N_2)}\mathbb{1}_{N_2} & \\ & & \ddots \end{pmatrix} Q_2\Omega_{22}, \tag{4.81}$$

where $\mathbb{1}_{N_i}$ represents an $N_i \times N_i$ unit matrix. These magnetic fluxes break $U(N)$ into $U(N) \to U(N_1) \times U(N_2) \times \cdots U(N_n)$ with $\sum_{a=1}^{n} N_a = N$. As mentioned in Sec. 3.2, each bifundamental field $(\mathbf{N}_s, \bar{\mathbf{N}}_t), ((\bar{\mathbf{N}}_s, \mathbf{N}_t))$ for $s, t = 1, \ldots, n, (s \neq t)$ in the off-diagonal components of $U(N)$ feels $M_{1(N_s)} - M_{1(N_t)}$ and $M_{2(N_s)} - M_{2(N_t)}$ ( $M_{1(N_t)} - M_{1(N_s)}$ and $M_{2(N_t)} - M_{2(N_s)}$) units of fluxes, since their covariant derivative involves a commutation relation between gauge fields.

From now on, we study the overlap integrals among a single boson and two fermion wavefunctions. The overlap integrals could be originating from the Yukawa interaction as commented in Sec. 4.3 or certain operators beyond the leading SYM approximation.

The overlap integrals are given by integrating the wavefunctions in the extra-dimensional space over the internal cycles of D7-branes

$$Y_{(a_i,b_i)(a_j,b_j)(p_k,q_k)} = \int d^2 z_3 d^2 z_4 \sqrt{g} \Psi^{\mathcal{M}_1 \dagger}_{a_i b_i} \Psi^{\mathcal{M}_2}_{a_j b_j} \Phi^{\mathcal{M}_3}_{p_k q_k}, \tag{4.82}$$

where $\Psi^{\mathcal{M}}_{ab}$ and $\Phi^{\mathcal{M}}_{pq}$ correspond to the fermionic and bosonic wavefunctions (4.64) and (4.24) and they have bifundamental representations under a certain pair among the product subgroups of the original $U(N)$ gauge group. These wavefunctions are characterized by a pair of integer $(a, b) = (a_i, b_i), (a_j, b_j)$ and $(p, q) = (p_k, q_k)$ representing its generation, and a flux $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$. In Eq. (4.82), the subscripts $i$, $j$ and $k$ in matter fields denote their specific generation which is severely constrained by the normalization condition and validity condition for bosons (4.31), (4.36) and fermions (4.70), (4.71), respectively. Since the matter wavefunction of bifundamental field depends on a difference of two fluxes, the above flux number $\mathcal{M}$ carries this difference as $\mathcal{M}_u \equiv \{M_1^u, M_2^u\} = \{M_{1(N_{s_u})} - M_{1(N_{t_u})}, M_{2(N_{s_u})} - M_{2(N_{t_u})}\}$ for $s_u, t_u = 1, \ldots n$, where $(s_u, t_u) \neq (s_{u'}, t_{u'})$, $u \neq u'$ and $(u, u' = 1, 2, 3)$. From the phenomenological point of view, we in the following perform overlap integrals (4.82) for some choice of fluxes. We do not construct the explicit standard-like models and identify the standard model fields into the bifundamental representations. For such a direction, see, e.g., Refs. [17, 33] in the case of factorizable tori.

Let us consider the non-supersymmetric fluxes given in Eq. (4.28), that is, $M_1^s = M_2^s$ with $s = 1, 2$ and 3. The integrand in the overlap integrals between the boson $\Phi^{\mathcal{M}_3}_{24,p'_k q'_k}$ and the fermions $\Psi^{\mathcal{M}_1 \dagger}_{3,a_i b_i}$, $\Psi^{\mathcal{M}_2}_{3 a_j b_j}$ given by Eq. (4.82) has the following form

$$\sqrt{g} \Psi^{\mathcal{M}_1 \dagger}_{3,a_i b_i} \Psi^{\mathcal{M}_2}_{3 a_j b_j} \Phi^{\mathcal{M}_3}_{24,p'_k q'_k} = J(|z_3|^2, |z_4|^2) \times z_3^{a_i + p'_k} z_4^{b_i + q'_k} \bar{z}_3^{a_j} \bar{z}_4^{b_j}, \tag{4.83}$$

where $M_1^1, M_1^2 > 0$, $M_1^3 < 0$ and $J(|z_3|^2, |z_4|^2)$ is the real function. In Eq. (4.83), only real value contributes to non-vanishing overlap integrals. Therefore, we impose the following relation between the pair of integers

$$a_i + p'_k = a_j, \quad b_i + q'_k = b_j. \tag{4.84}$$

For the other combination of zero-modes, $\Psi^{\mathcal{M}_1 \dagger}_{3,a_i b_i} \Psi^{\mathcal{M}_2}_{4,a'_j b'_j} \Phi^{\mathcal{M}_3}_{13,p_k q_k}$, the overlap integrals with $M_1^1, M_1^3 > 0$ and $M_1^2 < 0$ are only yielded by the modes satisfying in a similar fashion

$$a_i + a'_j = p_k, \quad b_i + b'_j = q_k. \tag{4.85}$$

To estimate the overlap integrals (4.82) explicitly, we take the one particular example. Being aware of the construction of standard model, we just adopt the three fermionic wavefunctions drawn in Fig. 4.2, which are labeled by $i, j = 1, 2, 3$. For our purpose, we set the following parameters for the numbers of fluxes with the fixed volume $\mathcal{V}_1 = \mathcal{V}_2 \equiv \mathcal{V}$ of D7-brane

$$\begin{aligned} &c = 1, \;\; L \simeq 13.5, \;\; M_s = 1.08 \times 10^{17}\,\text{GeV}, \\ &\mathcal{M}_1\mathcal{V} = \mathcal{M}_2\mathcal{V} = \{1, 1\}, \;\; \mathcal{M}_3\mathcal{V} = \{-2, -2\}, \end{aligned} \tag{4.86}$$

under which Eqs. (4.70) and (4.71) allow us to consider the following pair of integers,

$$(a_i, b_i) = (0, 0), (0, 1), (1, 0), \;\; (a_j, b_j) = (0, 0), (0, 1), (1, 0). \tag{4.87}$$

The above pairs yield three generations for each fermionic zero-mode. In such a case, from Eqs. (4.31) and (4.36), there are twenty-one generations of bosons:

$$\begin{aligned} (p'_k, q'_k) =&(-1, -1), (-1, 0), (-1, 1), (-1, 2), (-1, 3), (-1, 4), \\ &(0, -1), (0, 0), (0, 1), (0, 2), (0, 3) \\ &(1, -1), (1, 0), (1, 1), (1, 2), \\ &(2, -1), (2, 0), (2, 1) \\ &(3, -1), (3, 0), \\ &(4, -1). \end{aligned} \tag{4.88}$$

Although it is generically possible to have non-zero overlap integrals between twenty-one generations of boson and three generations of fermions, it turns out that the overlap integrals for fourteen pairs of integers vanish. The remaining seven pairs,

$$(p'_k, q'_k) = (0, 0), (0, 1), (1, 0), (-1, 0), (-1, 1), (0, -1), (1, -1), \tag{4.89}$$

give rise to the non-vanishing overlap integrals. Here and in what follows, the seven generation numbers $k = 0, 1, 2, 3, 4, 5, 6$ ($i, j = 0, 1, 2$) correspond to (the first three of) the seven pairs of integers (4.89), respectively, e.g., $Y_{203} = Y_{(1,0)(0,0)(-1,0)}$ for $Y_{ijk}$ with $i = 2$, $j = 0$ and $k = 3$.

When we assume that bosons $\Phi^{\mathcal{M}_3}_{24,p'_k q'_k}$ have non-vanishing VEVs denoted by $\langle H_k \rangle$, the fermion masses are characterized by the eigenvalues of the following mass matrices,

$$M_{ij} = \sum_{k=0}^{6} Y_{ijk} \langle H_k \rangle, \tag{4.90}$$

where

$$\begin{aligned} Y_{ij0} &= \begin{pmatrix} Y_{000} & 0 & 0 \\ 0 & Y_{110} & 0 \\ 0 & 0 & Y_{220} \end{pmatrix}, \quad Y_{ij1} = \begin{pmatrix} 0 & Y_{011} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_{ij2} = \begin{pmatrix} 0 & 0 & Y_{022} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ Y_{ij3} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Y_{203} & 0 & 0 \end{pmatrix}, \quad Y_{ij4} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & Y_{214} & 0 \end{pmatrix}, \\ Y_{ij5} &= \begin{pmatrix} 0 & 0 & 0 \\ Y_{105} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_{ij6} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_{126} \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{4.91}$$

As an illustrative value, the following ratios between the VEVs of bosons,

$$\frac{\langle H_k \rangle}{\langle H_0 \rangle} = (1, 9, 7, 1, 7, 1, 8), \tag{4.92}$$

give rise to hierarchical fermion mass $(m_1, m_2, m_3)/m_3 = (1.13 \times 10^{-3}, 2.20 \times 10^{-2}, 1)$ which is similar to the structure of down-type quark mass ratios compared with the observed ones [34] $(m_d, m_s, m_b)\ /m_b = (1.17 \times 10^{-3}, 2.27 \times 10^{-2}, 1)$. As can be seen in this simple example, the quasi-localized matter wavefunctions are important to generate the hierarchical overlap integrals. It is also interesting to further discuss the phenomenological aspects of this model. We will construct the model building on this D7-branes in the conifold in a future work.

### 4.3.3 The possible embeddings of conifold into the global Calabi-Yau

In the previous discussions so far, the volume of internal cycle of D7-brane in the conifold is not finite, since the radial direction of $AdS_5$ is non-compact, i.e., $r \in [0, \infty]$. Correspondingly, the magnetic flux on our discussed D7-brane (4.14) diverges, although it is only valid in near-horizon limit in Klebanov-Witten background. Thus, we require the construction of the global compact Calabi-Yau space that involves the conifold geometry.

Let us briefly review the development of constructing such a global compact Calabi-Yau space. It is known that a compact Calabi-Yau space can be constructed as a hypersurface by using the technique of toric variety proposed by V. V. Batyrev in Ref. [35]. In this method, complex three-dimensional Calabi-Yau space is embedded in the ambient complex four-dimensional compact toric variety. In particular, Kreuzer and Skarke found that the number of such reflexive polytopes including the compact Calabi-Yau threefolds are estimated as 473800776 through a systematic method [36]. They released the software package to construct the compact Calabi-Yau threefold by the public code called PALP [37] by which the topological data of Calabi-Yau spaces is listed on the web page [38]. For a more detailed discussion of Calabi-Yau spaces, see a review, e.g., Ref. [39].

We expect that the conifold geometry discussed in the Part I of this thesis can be derived in a certain limit of global Calabi-Yau space. In Refs. [40, 41], they embedded the magnetized D7-branes in a global Calabi-Yau space by employing the 4D toric variety in a different context.

## 4.4 Summary

In this chapter, we have studied the matter wavefunctions living on local D7-branes wrapping the internal cycle in the conifold, where there are large number of D3-branes putted at the tip of conifold based on the Klebanov-Witten model [10]. The embedding of D7-branes into the conifold is determined in a kappa-symmetric way to ensure the stability of D7-branes and then the induced metric of D7-branes is determined. On this background of D7-brane, we have obtained analytical form of zero-mode wavefunctions of matter fields by solving the corresponding Dirac and Laplace equations.

To obtain the three-generation chiral zero-modes, we consider the magnetized D7-branes. As pointed out in Ref. [15], the inclusion of magnetic fluxes generates the degenerated charged fields which can be identified with the generation of quarks and leptons. By solving the Dirac and Laplace equations with fluxes, we have found that the degenerated zero-modes appear in the low-energy effective theory and at the same time, their wavefunctions are exponentially localized towards the tip of conifold. This quasi-localization of matter fields is interesting in the aspects of solving two types of problems in the standard model.

First is the gauge hierarchy problem in the standard model. In a similar way to the Randall-Sundrum model [30] on $AdS_5$ background, our background geometry is described by $AdS_5$ spacetime at the tip of conifold. In this 5D effective theory, we have found that the normalizable matter wavefunction localizes at the tip of conifold, whereas the graviton localizes towards the Calabi-Yau manifold as discussed in Sec. 4.3. The localization profile of matter fields depends on the number of magnetic fluxes. Thus, it is possible to explain the reason why the electroweak scale is small compared with the Planck scale on this conifold geometry. Furthermore, in the $AdS_5$ supergravity [31] compactified on $S^1/Z_2$, the bulk mass [32] of matter field is effectively considered as the the magnetic fluxes in the string setup. Indeed, such a bluk mass is quite interesting to realize the hierarchies of Yukawa coupling between quarks/leptons and Higgs fields in the model building on $AdS_5$ supergravity [42].

Second problem is the mass hierarchies between the generation of quarks and leptons. In Sec. 4.3.2, we have estimated the overlap integrals between single boson and two fermions. It is then found that the mild hierarchical structure of overlap integrals can be achieved by using the structure of quasi-localized matter wavefunctions.

Throughout the Part I of this thesis, the conifold geometry has been assumed to be glued to a certain global Calabi-Yau manifold, otherwise the volume of D7-brane is infinite and gravity is decoupled in our system. We expect that the corrections induced by the global Calabi-Yau manifold are not sensitive to the obtained matter wavefunctions. This is because our matter wavefunctions satisfying the validity conditions localize toward the tip of conifold and converge to zero in the direction of global Calabi-Yau space. In any cases, it is an important issue to consider the global embedding of the local results. As mentioned in Sec. 4.3.3, the Batyrev's method is useful to embed the conifold geometry into the global Calabi-Yau space by employing the toric geometry. Furthermore, it is possible to embed the cycles of D-branes in the global Calabi-Yau space. We leave this global embedding as a future work. Following the procedure of this chapter, it is straightforward to extend our conifold geometry to the warped deformed or resolved conifold.(For a detail of supersymmetric D7-branes on the warped deformed conifold, e.g., Ref. [43] in which they take into account the kappa-symmetry condition with H-flux.) Other types of Sasaki-Einstein manifolds, such as $AdS_5 \times Y^{p,q}$ and $AdS_5 \times L^{a,b,c}$, are also interesting from the phenomenological point of view. In such a case, kappa-symmetric embedding is discussed in Refs. [22, 44] and in a comprehensive review [45].

In this chapter, we have focused on the model building on D7-branes. However, there are several studies to construct the standard-like models on D3-branes at the orbifold singularity and at the bottom of a warped throat such as the conifold [23]. From the cosmological point of view, these D-brane configurations, in particular, D7-branes are quite important to discuss the stabilization of Kähler moduli. On these warped backgrounds, it was argued that the brane inflation and natural inflation can be possible on the warped deformed conifold [24] and the warped resolved conifold [25], respectively. As a future work, it is interesting to further study the various phenomenological and cosmological scenarios in terms of the D7-branes.

# Part II

# Cosmological aspects

# Chapter 5

# Axion inflation

In Part II, we study the cosmological aspects of axion which naturally appears in the low-energy effective theory of string theory on Calabi-Yau manifold. In the framework of string theory, the axions are obtained by integrating the higher-dimensional form fields such as Kalb-Ramond field, Ramond-Ramond fields, and internal metric over the inter cycles of extra-dimensional space.

From the fact that typical Calabi-Yau manifold predicts a lot of internal cycles, a lot of axions appear in the low-energy effective action through the compactification of extra-dimensional space. At the perturbative level, the potential of axion is constrained by the gauge symmetries of higher-dimensional form fields, whereas at the non-perturbative level, the non-perturbative effects generate the axion potential. However, the axion potential is still controlled by the discrete shift symmetries of axions fields which is a characteristic feature of string axion.

When we discuss the inflation scenario in an early Universe, the potential of inflaton field is highly constrained by the recent cosmological observations represented by the Planck. When the inflation scale is high enough, we have to control the inflaton potential by the quantum and gravitational effects. So far, there are several approaches to controll the inflaton dynamics by taking into account the quantum and gravitational effects. In particular, in string theory, one can discuss the gravitational and stringy effects to the inflaton dynamics, since the string theory is a candidate for the consistent quantum gravitational theory.

Indeed, string axion is a promising candidate for the inflaton field. The axionic shift symmetries of axion field protect the form of potential as well as quantum effects. First of all, we briefly review the inflation mechanism and well-known axion inflation scenarios based on string theory. After that, we propose the new type of axion inflation scenario within the framework of type IIB string theory on Calabi-Yau manifold. In this setup, we can exactly calculate the non-perturbative effects to the axion-inflaton by using the technique of topological string theory. Then, the non-perturbative effects bring the significant effects to the inflaton dynamics.

## 5.1 Introduction

First of all, we briefly review the inflation mechanism. Inflation, i.e., the accelerated expansion of an early Universe, solves the problems in the standard cosmology such as horizon and flatness problems. In addition, when there are unwanted relics in the early Universe such as monopole, cosmic strings, and topological defects, one can successfully dilute them by the inflation. We in particular focus on a slow-roll inflation where the accelerated expansion of the Universe can be achieved by the dynamics of single scalar field. Then, the quantum fluctuation of scalar field successfully produces the density perturbation of the Universe observed by the recent cosmological observations, WMAP and Planck.

The relevant action of a homogeneous scalar field (inflaton field) $\phi$ is

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right), \tag{5.1}$$

where the background metric is assumed to be flat Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2. \tag{5.2}$$

Here, $a(t)$ is the scale factor and $t$ is the cosmic time. When the material is described by the perfect fluid, the Einstein equation in FRW metric are

$$\begin{aligned} H^2 &= \frac{1}{3M_{\rm Pl}^2}\left[\frac{1}{2}(\dot{\phi})^2 + V(\phi)\right], \\ \frac{\ddot{a}}{a} &= \dot{H} + H^2 = \frac{1}{3}\left[-(\dot{\phi})^2 + V(\phi)\right], \end{aligned} \tag{5.3}$$

where dot denotes the derivative with respect to cosmic time and $H = \dot{a}/a$ is the Hubble-parameter. From Eq. (5.3), the accelerated expansion of the Universe $\ddot{a} > 0$ can be realized under

$$-\frac{\dot{H}}{H^2} < 1. \tag{5.4}$$

This condition can be satisfied if the velocity of $\phi$ is smaller than the energy density of scalar potential, i.e., $(\dot{\phi})^2 \ll V(\phi)$ as can be seen in Eq. (5.3). To realize the sufficiently accelerated expansion of the Universe, we first define the slow-roll parameters

$$\begin{aligned} \epsilon &= \frac{M_{\rm Pl}^2}{2}\left(\frac{d_\phi V}{V}\right)^2, \\ \eta &= M_{\rm Pl}^2\frac{1}{V}\frac{d^2V}{d\phi^2}. \end{aligned} \tag{5.5}$$

When these slow-roll parameters are much smaller than unity, we can realize the longed inflationary era. Indeed, in this slow-roll approximation, the equation of motion of $\phi$ can be solved as

$$\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} \simeq 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \\ &\to \dot{\phi} \simeq -\frac{1}{3H}\frac{dV}{d\phi}, \end{aligned} \tag{5.6}$$

where we use $\ddot{\phi} \ll 3H\dot{\phi}$. Under the slow-roll conditions $\epsilon \ll 1$ and $\eta \ll 1$, $\ddot{\phi} \ll 3H\dot{\phi}$ and $(\dot{\phi})^2 \ll V(\phi)$ are satisfied at the same time. It is then possible to realize the accelerated expansion of the Universe. The inflation ends when the slow-roll parameters are of order unity.

To estimate how long the Universe is accelerated, we introduce the $e$-folding number defined by,

$$N_e = \ln\left(\frac{a(t_{\rm end})}{a(t_*)}\right), \tag{5.7}$$

where $t_{\rm end}$ ($t_*$) is the time at which the inflation ends (pivot scale at which the mode with wave number $k$ crosses the horizon, $k = aH$). In typical inflation models, this $e$-folding number is within the range $50 \leq N_e \leq 60$. However, the exact value of $e$-folding number highly depends on the thermal history of the Universe after the inflation. Indeed, when there is no dilution effect after the inflation, $e$-folding number is determined by the energy density of scalar potential at the horizon exit (end of inflation), $V_*^{1/4}$ ($V_{\rm end}^{1/4}$) and the energy density of radiation $\rho_R^{1/4} = (\pi^2 g_*/30)T_R$ with $g_*$ being the effective degrees of freedom of the radiation at the reheating temperature $T_R$ [46],

$$N_e \simeq 62 + \ln\frac{V_*^{1/4}}{10^{16}\,{\rm GeV}} + \ln\frac{V_*^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\ln\frac{V_{\rm end}^{1/4}}{\rho_R^{1/4}}. \tag{5.8}$$

Thus, the reheating temperature is closely tied to the value of $e$-folding.

Finally, we show the cosmological observables predicted by the inflation scenario. During the inflation, the inflaton field fluctuates in space and consequently it gives the perturbations to the spacetime metric. The testable cosmological observables at the pivot scale are as follows [47]:

$$\begin{aligned} P_\xi &= \frac{1}{24\pi^2}\frac{V}{M_{\rm Pl}^4\epsilon} = 2.20 \pm 0.10 \times 10^{-9}, \\ n_s &= 1 + \frac{d\ln P_\xi}{d\ln k} = 1 + 2\eta - 6\epsilon = 0.9655 \pm 0.0062, \\ r &= 16\epsilon < 0.11, \end{aligned} \tag{5.9}$$

where $P_\xi$ is the power spectrum of curvature perturbation, $n_s$ is the tilt of curvature perturbation, and $r$ is the ratio between the power spectrum of tensor and curvature perturbations called the tensor-to-scalar ratio. These cosmological observables are highly constrained by the Planck data [47].

## 5.2 Natural inflation

We next summarize the well-known axion inflation scenarios. First is the simplest axion inflation known as a natural inflation where the axion-inflaton is the pseudo Nambu-Goldstone boson appeared in the spontaneous symmetry breaking. In the string context, there are several sources of non-perturbative effects to generate the axion potential such as gaugino condensation [48] on hidden D-branes, instanton effects on Euclidean branes, and world-sheet instanton effects. When there are certain non-perturbative effects, the axionic shift symmetry is broken down to the discrete one. Then, we can extract the following form of axion potential,

$$V = \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right), \tag{5.10}$$

where $\phi$ is the canonically normalized axion and $f$ is its decay constant. $\Lambda$ characterizes the scale of non-perturbative effects. When the Universe is inflated by the axion $\phi$, we call this class of inflation model the natural inflation. However, in order to fit with the recent Planck data, we require the trans-Planckian axion decay constant, $f \gg M_{\rm Pl}$.

To derive the axion potential, let us consider the type IIB string theory on Calabi-Yau manifold with D7-branes. The low-energy effective action of closed string moduli is described by the framework of $\mathcal{N} = 1$ supergravity. The moduli Kähler potential is

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) - \ln\left(\int_{\mathcal{M}} \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S}), \tag{5.11}$$

where $\xi = -\zeta(3)\chi/2(2\pi)^3$ with $\zeta(3) \simeq 1.2$ and $\chi$ being the Euler characteristic of Calabi-Yau manifold. $\mathcal{V}$ is the volume of Calabi-Yau manifold $\mathcal{M}$ as a function of Kähler moduli $T$ in the string unit; $g_s$ is the string coupling; $\Omega$ is the holomorphic three-form depending on the complex structure moduli $U$; and $S$ is the dilaton field. Here, this action is valid in the weak coupling and large volume limit, i.e., $\mathrm{Re}\, S \gg 1$ and $\mathcal{V} \gg 1$ in the string unit. The term proportional to the Euler characteristic of Calabi-Yau manifold is the leading $\alpha'$ correction which is important to achieve the LARGE volume scenario as explained below.

To stabilize the complex structure moduli $U$ and dilaton field $S$, we introduce the Ramond-Ramond three-form flux $F_3$ and Neveu-Schwarz three-form flux $H_3$ along the internal cycles of Calabi-Yau manifold. On this flux background, the fluxes induce the superpotential,

$$W_{\rm flux} = \int_{\mathcal{M}} G_3 \wedge \Omega, \tag{5.12}$$

with $G_3 = F_3 - iSH_3$. As pointed out in Ref. [49], we can stabilize all the complex structure moduli and dilaton fields by the inclusion of imaginary self-dual fluxes and they become massive at the compactification scale. However, this statement depends on the choice of fluxes. In Ch. 6, we discuss the scenario with light complex structure moduli.

In any case, when $U$ and $S$ are stabilized at this level, the superpotential is considered as constant $\langle W_{\rm flux}\rangle$ below the mass scales of $U$ and $S$. So far, we did not discuss the stabilization of Kähler moduli. As mentioned before, the potential of Kähler moduli is induced at the non-perturbative level. To simplify our analysis, let us assume that the volume of Calabi-Yau manifold is determined by single Kähler modulus $T$. The relevant Kähler potential of modulus $T$ becomes

$$K = -3\ln(T + \bar{T}). \tag{5.13}$$

When the gauginos of the pure $SU(N)$ super Yang-Mills living on D7-branes condensate, the non-perturbative superpotential is generated as

$$W_{\rm non} = \langle W_{\rm flux}\rangle + Ae^{-aT}, \tag{5.14}$$

where $A$ is the constant and $a = 8\pi^2/N$. From the Kähler potential and superpotential, it is possible to implement the stabilization of Kähler moduli by employing the $F$-term scalar potential,

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\right), \tag{5.15}$$

where $K^{I\bar{J}}$ is the inverse of Kähler metric $K_{I\bar{J}} = \partial^2 K/\partial\Phi^I\partial\bar{\Phi}^{\bar{J}}$ for $\Phi^I = S, T^a, U^i$ with $a$ being the number of Kähler moduli, $D_I W = W_I + K_I W$ is the Kähler covariant derivative for $W$ with $W_I = \partial W/\partial\Phi^I$ and $K_I = \partial K/\partial\Phi^I$.

The calculation of the moduli potential results in the following axion potential,

$$V \simeq \frac{1}{\mathcal{V}^2}\langle|W_{\rm flux}|\rangle Ae^{-2\pi{\rm Re}T}\cos\left(\frac{\phi}{f} + \theta_1\right) + \cdots, \tag{5.16}$$

where dots represent irrelevant terms to the axion potential and the phase of sinusoidal function $\theta_1$ is coming from the phase of $A$ and $\langle W_{\rm flux}\rangle$. Here, $\phi = \langle\sqrt{2K_{T\bar{T}}}\rangle{\rm Im}\,T$ is the canonically normalized axion. Following this procedure, we can read off the axion potential. By setting the certain uplifting sector such as the anti D-brane [50] and F-term uplifting scenario [51, 52], we obtain the potential form of natural inflation, although the realization of axoin-inflation requires the stabilization of the real part of Kähler modulus at a scale much larger than the axion mass. Furthermore, to achieve the trans-Planckian axion decay constant, we have to consider the alignment mechanism [53] by introducing the multiple axions. Then, the decay constant of specific linear combination of multiple axions is enhanced to be the trans-Planckian value.

## 5.3 Axion monodromy inflation

Next, we discuss another well-known axion inflation scenario, i.e., axion monodromy inflation [54, 55]. The potential of axion monodromy inflation is

$$V = \Lambda^{4-p}\phi^p, \tag{5.17}$$

where $\Lambda$ is the real constant, $\phi$ is the canonically normalized axion and $p$ is the rational number depending on the setup. In contrast to the natural inflation, the axion potential does not have an axionic shift symmetry. This structure is originating from the existence of branes such as D-brane, NS-brane and $[p, q]$-branes.

Let us consider the spacetime filling D5-brane whose low-energy effective action is given by the Dirac-Born-Infeld (DBI) action,

$$S = \frac{1}{(2\pi)^5 g_s(\alpha')^3}\int d^6x\sqrt{-\det(G+B)}, \tag{5.18}$$

where $g_s$ is the string coupling, $\alpha'$ is the regge slope, and $G$ is the 6D metric on D5-brane. The axion for our interest is the extra-dimensinoal component of Kalb-Ramond two-form $B$, i.e., $b = \int_\Sigma B$ with $\Sigma$ being the internal two-cycle D5-brane wraps. The 4D effective axion potential is obtained by dimensionally reducing the DBI action along the internal two-cycle $\Sigma$,

$$V \propto \frac{1}{(2\pi)^5 g_s (\alpha')^2}\sqrt{l^4 + b^2}, \tag{5.19}$$

where the overall scale depends on the non-trivial warp factor and $l$ is the volume of $\Sigma$ in string unit. When the value of axion field is larger than the volume of $\Sigma$, the above scalar potential linearly depends on the axion $b$. In this way, one can achieve the axion monodromy inflation as in Eq. (5.17) with $p = 1$.

When we extend this idea to the D4-brane on a twisted torus, one can realize the axion monodromy inflation with $p = 2/3$ [54]. Axion inflations with other rational numbers are proposed in Refs. [56, 57] and also in the field theoretical approach [58]. The lower $p$ is well consistent with the Planck data and at the same time, it predicts the detectable primordial gravitational wave.

## 5.4 Stabilization of Kähler moduli

In this section, we briefly review the stabilization of closed string moduli, in particular, the Kähler moduli, based on the type IIB superstring theory. As mentioned before, the authors of Ref. [49] pointed out that all the complex structure moduli and dilaton fields are stabilized by the $(2, 1)$ component of imaginary self-dual three-form fluxes through the flux-induced superpotential (5.12). Thus, we have to consider the stabilization of remaining closed string moduli, i.e., the Kähler moduli. In the following, we review the well-known Kähler moduli stabilization scenarios, the Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [50] and LARGE Volume Scenario (LVS) [59]. As shown below, since the potential of Kähler moduli is only induced by the loop and non-perturbative effects, one can discuss the stabilization of Kähler moduli without affecting that of complex structure moduli and dilaton, which are stabilized at the tree level.

### 5.4.1 KKLT scenario

As discussed in Ref. [50], one can achieve the stabilization of Kähler moduli and the realization of dS minimum, simultaneously. To simplify our analysis, let us consider the Calabi-Yau manifold with single Kähler modulus, that is, $h^{1,1} = 1$. Then, the Kähler potential of the Kähler modulus is described by

$$K = -3\ln(T + \bar{T}), \tag{5.20}$$

where the intersection number of the Calabi-Yau manifold is taken as unity. The superpotential is non-perturbatively generated by the instanton effects on the hidden Eucldean branes or the gaugino condensation on the hidden D7-branes,

$$W = \boldsymbol{c} + A_c e^{-a_c T}, \tag{5.21}$$

where $A_c$ is the complex constant, and $a_c$ is a real constant such as $a_c = 8\pi^2/N$ in the case of $SU(N)$ pure SYM on the hidden D7-branes wrapping the internal four cycle of Calabi-Yau manifold. The complex constant $\boldsymbol{c}$ is determined by the vacuum expectation value of the three-form fluxes.

By the above Kähler potential and superpotential, the Kähler modulus is stabilized at the minimum satisfying

$$D_T W = W_T + K_T W = -a_c A_c e^{-a_c T} - \frac{3}{T + \bar{T}} W = 0, \tag{5.22}$$

which leads to the extremal condition for the Kähler modulus, i.e., $\partial V/\partial T = 0$. It is remarkable that the current setup is justified in the large volume regime of Calabi-Yau manifold, $T \gg 1$ in string unit, and thus the supersymmetric condition (5.22) requires the tiny value of $\boldsymbol{c}$ in contrast to the LVS as explained below. However, since the obtained vacuum energy is negative, i.e., anti-de Sitter minimum, we have to uplift this minimum to the de Sitter one. The authors of Ref. [50] pointed out that the anti D3-brane is one of the candidate to give rise to the positive vacuum energy density. When this anti D3-brane is located at the warped throat, we obtain the tiny supersymmetry breaking term characterized by the following scalar potential:

$$V_{\rm up} = \frac{\mathcal{D}}{(T + \bar{T})^3}, \tag{5.23}$$

where $\mathcal{D}$ is a function of the possible warp factor and the number of anti D3-brane. By combining this uplifting potential and modulus potential, we can perform the stabilization of Kähler moduli at the de Sitter vacuum on the basis of the string theory.

### 5.4.2 LARGE volume scenario

Next, we consider another well-known scenario of Kähler moduli stabilization, called the LARGE volume scenario (LVS). For illustrative purposes, let us consider the Calabi-Yau manifold defined by the degree 18 hypersurface in the weighted projective space $C\mathbb{P}_{1,1,1,6,9}[18]$ in which the numbers of moduli fields are $h^{1,1} = 2$ and $h^{2,1} = 272$. Many complex structure moduli and dilaton fields are supposed to be stabilized at the supersymmetric minimum due to the three-form fluxes, in a similar way to the KKLT scenario. The low-energy effective action of the Kähler moduli on this Calabi-Yau is described by the following Kähler potential,

$$K = -2 \ln \left( \frac{1}{9\sqrt{2}} (\tau_b^{3/2} - \tau_s^{3/2}) + \frac{\xi}{2 g_s^{3/2}} \right), \tag{5.24}$$

where $\tau_b = {\rm Re} T_b$ and $\tau_s = {\rm Re} T_s$. The term proportional to the Euler characteristic of Calabi-Yau manifold is the leading $\alpha'$ correction which is important to achieve the LVS. On the other hand, the superpotential is assumed to be the following form:

$$W = \boldsymbol{c} + A_s e^{-a_s T_s}, \tag{5.25}$$

where $\boldsymbol{c}$ is the complex constant determined by the three-form fluxes and the second term is generated by the D3-instantons on the cycle $T_s$. Note that, in comparison with the KKLT scenario, the magnitude of $\boldsymbol{c}$ is of $\mathcal{O}(1)$ which is natural in the flux compactification. Then, the scalar potential is simplified as

$$V \simeq \frac{c_1 a_s^2 |A_s|^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{c_2 a_s \boldsymbol{c} A_s \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{c_3 \xi |\boldsymbol{c}|^2}{g_s^{3/2}\mathcal{V}^3}, \tag{5.26}$$

where $\mathcal{V} = \frac{1}{9\sqrt{2}}(\tau_b^{3/2} - \tau_s^{3/2})$ is the volume of Calabi-Yau manifold and $c_{1,2,3}$ are constants of order unity. Now, we consider the large volume regime, i.e., $\mathcal{V} \simeq \tau_b^{3/2}$. In this regime, the extremal conditions of both the Kähler modulus $\tau_b$ and $\tau_s$ lead to the following vacuum expectation values:

$$\mathcal{V} \propto e^{a_s\tau_s}, \qquad \tau_s \propto \frac{\xi^{2/3}}{g_s}, \tag{5.27}$$

and the mass of the lightest Kähler modulus is described by $\boldsymbol{c}/\mathcal{V}^{3/2}$. However, the obtained minimum still remains at the anti-de Sitter minimum, and thus we need the certain uplifting scenario in the same way with the KKLT scenario as discussed in the previous section.

# Chapter 6

# New potentials for string axion inflation

In this section, we discuss a cosmological aspect of type IIB superstring theory, in particular, new type of axion inflation based on type IIB superstring theory compactified on Calabi-Yau orientifold, proposed in Ref. [8]. In contrast to the well-known axion inflation model, our axion-inflaton is one of complex structure moduli which determine the shape of Calabi-Yau manifold. The inflaton potential is derived from the geometric corrections corresponding to the instanton corrections in type IIA superstring theory. Thanks to the mirror symmetry, the inflaton potential is well-controlled under the $\alpha'$-and $g_s$-corrections. Since an axionic shift symmetry of complex structure modulus is broken by the instantonic correction and certain three-from fluxes, the axion potential is the mixture of polynomial and sinusoidal functions. We also discuss the stabilization of Kähler moduli and a typical reheating temperature for the complex structure moduli inflation.

## 6.1 Introduction

The cosmic inflation is an attractive scenario which solves the fine-tuning problems such as the flatness and horizon problems and gives the density perturbations of our Universe. Since the inflation scenario considers the early Universe, the higher-dimensional operators are relevant to the inflation models. A fundamental theory such as superstring theory will be expected to realize a cosmic microwave background data by the successful inflation scenario.

In superstring theory, there are many moduli fields associated with the higher-dimensional form fields in the low-energy effective theory, since the typical Calabi-Yau manifold predicts a lot of internal cycles. Furthermore, the moduli potentials are prohibited by the Lorentz and gauge symmetries at the perturbative level. In this sense, the moduli fields are identified as one of the candidates of inflaton fields. In the type IIB superstring theory on Calabi-Yau manifold, the potentials of complex structure moduli and dilaton fields are induced by the inclusion of perturbative three-form fluxes along with the internal three-cycles of Calabi-Yau manifold [49]. On the other hand, the potential of Kähler moduli can be only realized at the non-perturbative level by the Euclidean instanton effects and gaugino condensation effects on the hidden D-branes. So far, one of the Kähler moduli has discussed in the context of moduli inflation.(For details of moduli inflation, see, e.g., Ref. [60].)

In particular, the imaginary part of the Kähler moduli, i.e, Kälb-Ramond B-field, has been identified with the inflaton field. When such an axion potential is originating from the non-perturbative effects on the hidden D-branes, the scalar potential takes the form of natural inflation [61], whereas the Dirac-Born-Infeld action induces the form of axion-monodromy inflation [55, 54]. By contrast, the complex structure moduli are not so focused on the aspects of inflation, since the complex structure moduli have been generically stabilized by the three-form fluxes [49]. However, it depends on the ansatz of three-form fluxes. When one of the complex structure moduli is lighter than the Kähler moduli, it can play a role of inflaton field through the natural inflation by using the threshold-corrections [62] and instanton effects [63]. As discussed in Refs. [64, 65], the backreaction of the Kähler moduli will be dangerous for the inflation with complex structure moduli, since the Kähler moduli could be destabilized during inflation due to the positive cosmological constant.

To realize the successful axion inflation with complex structure moduli, we begin with the low-energy effective action of type IIB string theory on Calabi-Yau manifold which is described by the 4D $\mathcal{N} = 1$ supergravity. Since the action of complex structure moduli is governed by the $\mathcal{N} = 2$ SUSY, the prepotential of Calabi-Yau manifold exactly determines the potential of complex structure moduli in the context of topological string [66, 67]. It is found that the geometric corrections for the prepotential appear in the low-energy effective field theory. It can be calculated by solving the Picard-Fuchs equation of period vector on Calabi-Yau manifold. According to it, the superpotential induced by the three-form fluxes along with internal three-cycles of Calabi-Yau manifold receives the non-perturbative effects through the period vector of Calabi-Yau manifold. When one of the complex structure moduli is stabilized by the non-perturbative effects through the period vector of Calabi-Yau manifold, it is then possible to consider the situation that one of the complex structure moduli is sufficiently lighter than the other complex structure moduli, dilaton and the Kähler moduli. Thus, one can achieve the inflation scenario with lightest complex structure moduli after the stabilization of Kähler moduli. The scalar potential of inflaton field is derived from the non-perturbative corrections calculated on mirror Calabi-Yau manifold. In contrast to the result of Ref. [63], we focus on the general ansatz of three-form fluxes which induces more general class of large-field inflation, that is, a mixture of polynomial and sinusoidal functions. The proposed new axion inflation models predict the favorable spectral tilt of curvature perturbation and tensor-to-scalar ratio reported by the recent results of Planck [47].

The remaining part of this chapter is organized as follows. In Sec. **??**, we give a review of the quantum corrected Kähler potential and flux-induced superpotential, i.e., Gukov-Vafa-Witten (GVW) superpotential [68]. Along with the moduli stabilization procedure in Ref. [63], we present the new class of F-term axion monodromy inflation and its prediction is shown to be consistent with the cosmological observables reported by Planck in Sec. **??**. Its inflation mechanism is interesting from the inflationary point of view and the thermal history of the Universe after the inflation, in particular, the reheating process has a characteristic feature compared with a usual Kähler moduli inflation. Finally, Sec. 6.4 is devoted to the summary of this chapter.

## 6.2 Quantum corrected period vector in type IIB string theory

In this section, we give a brief review of the quantum corrected Kähler potential and GVW superpotential [68] based on the type IIB superstring theory on Calabi-Yau orientifold. The potential of the complex structure moduli ($U$) and dilaton ($S$) can be induced by inserting the three-form fluxes $G_3 = F_3 - iSH_3$, that is a linear combination of Ramond-Ramond (R-R) $F_3$ and Neveu-Schwarz (NS) three-forms $H_3$, on the internal three-cycles of Calabi-Yau manifold [49],*,

$$W_{\rm GVW}(S,U) = \int G_3(S) \wedge \Omega(U), \tag{6.1}$$

where $\Omega$ is the unique holomorphic three-form of Calabi-Yau manifold depending on the complex structure moduli. The Dirac quantization requires that these three-form fluxes are quantized on the cycles of Calabi-Yau manifold. The GVW superpotential is then rewritten by

$$W_{\rm GVW}(S,U) = \sum_{\alpha=1}^{2h_-^{1,2}+2} (N_F - iSN_H)^\alpha \Pi_\alpha, \tag{6.2}$$

where $N_F^\alpha$ and $N_H^\alpha$ are integers of R-R and NS-NS fluxes, respectively and $\alpha = 1, \ldots, 2h_-^{1,2} + 2$ with $h_-^{1,2}$ being the number of surviving complex structure moduli under the orientifold involution. Now, the period vector $\Pi_\alpha$ encodes the contribution of complex structure moduli.

In addition to the superpotential, the Kähler potential is provided in the 4D $\mathcal{N} = 1$ supergravity,

$$K = K(U, \bar{U}) - \ln(S + \bar{S}) + K(T, \bar{T}), \tag{6.3}$$

where $K(T, \bar{T})$ represents the Kähler potential of Kähler moduli $T$ in the large volume limit, $T \gg 1$ in the string unit. The Kähler potential (6.3) and superpotential (6.2) give rise to the $F$-term scalar potential,

$$V = e^K \left( K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right), \tag{6.4}$$

where $K^{I\bar{J}}$ is the inverse of Kähler metric $K_{I\bar{J}} = \partial^2 K/\partial\Phi^I \partial\bar{\Phi}^{\bar{J}}$ for $\Phi^I = S, T^a, U^i$ with $a$ being the number of Kähler moduli, $D_I W = W_I + K_I W$ is the Kähler covariant derivative for $W$ with $W_I = \partial W/\partial\Phi^I$ and $K_I = \partial K/\partial\Phi^I$. Since the Kähler moduli satisfy the so-called no-scale structure, $K^{a\bar{b}} K_a K_{\bar{b}} = 3$, the scalar potential is simplified as

$$V = e^K \left( \sum_{S,U^i} K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} \right), \tag{6.5}$$

*Throughout this chapter, we adopt the reduced Planck unit, $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV $= 1$.

which implies that the extremal conditions of complex structure moduli and axion-dilaton are achieved at the minimum, $\langle D_I W \rangle = 0$ with $\Phi^I = S, U^i$. On the contrary, the stabilization of Kähler moduli is achieved by including the string loop and $\alpha'$-corrections [69] and/or non-perturbative effects represented by Kachru-Kallosh-Linde-Trivedi (KKLT) scenario [50] or Large Volume Scenario (LVS) [59].

So far, we show the well-discussed moduli stabilization procedure for the Kähler moduli, dilaon and complex structure moduli. In the following, we show the details of the quantum corrections for the period vector which induces the light complex structure moduli. Guided by the results of mirror symmetry and $\mathcal{N} = 2$ special geometry within the context of topological string and gauged linear sigma model (For details of topological string, see, Ref. [70].), the geometric corrections for the complex structure moduli are exactly calculated. The Kähler potential including the geometric corrections is provided by

$$\begin{aligned} e^{-K(U,\bar{U})} &= i\Pi^\dagger \cdot \Sigma \cdot \Pi \\ &= i|X^0|^2\Big(2(F - \bar{F}) - (U^i - \bar{U}^i)(F_i + \bar{F}_i)\Big), \end{aligned} \tag{6.6}$$

where $\Sigma$ is the symplectic matrix,

$$\Sigma = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}. \tag{6.7}$$

$F$ is the prepotential in the language of $\mathcal{N} = 2$ SUSY and its derivative $F_i$ with respect to $X^\zeta$ with $\zeta = 0, i$ which are represented as [71]

$$\begin{aligned} &F = -\frac{1}{3!}\kappa_{ijk}U^iU^jU^k - \frac{1}{2}\kappa_{ij}U^iU^j + \kappa_i U^i + \frac{1}{2}\kappa_0 - \frac{1}{(2\pi i)^3}\sum_\beta n_\beta \mathrm{Li}_3(q^\beta), \\ &\mathcal{F} = (X^0)^2 F, \qquad \mathcal{F}_\zeta = \frac{\partial}{\partial X^\zeta}\mathcal{F}, \qquad F_i = \frac{\partial}{\partial U^i}F. \end{aligned} \tag{6.8}$$

Here, $\mathrm{Li}_s(z) = \sum_{n=1} \frac{z^n}{n^s}$ is the polylogarithm function as a function of $q^\beta = e^{2\pi i d_i \beta_i}$ and the integers $n_\beta$ are the genus zero Gromov-Witten invariants labeled by $\beta$, i.e., $\beta = d_i\beta_i$ with $d_i$ and $\beta_i$ being the integers and the elements in cohomology $H_2(\tilde{M}_{\mathrm{CY}}, \boldsymbol{Z})\backslash\{0\}$ of mirror Calabi-Yau manifold $\tilde{M}_{\mathrm{CY}}$, respectively. The period vector $\Pi = (X^0, X^i, \mathcal{F}_0, \mathcal{F}_i)^T$ including the geometric corrections becomes

$$\begin{aligned} \Pi &= X^0 \begin{pmatrix} 1 \\ U^i \\ 2F - U^i\partial_i F \\ \partial_i F \end{pmatrix} \\ &= X^0 \begin{pmatrix} 1 \\ U^i \\ \frac{1}{3!}\kappa_{ijk}U^iU^jU^k + \kappa_i U^i + \kappa_0 - \sum_\beta n^0_\beta\big(\frac{2}{(2\pi i)^3}\mathrm{Li}_3(q^\beta) - \frac{d_i}{(2\pi i)^2}U^i\mathrm{Li}_2(q^\beta)\big) \\ -\frac{1}{2}\kappa_{ijk}U^jU^k - \kappa_{ij}U^j + \kappa_i - \frac{1}{(2\pi i)^2}\sum_\beta n^0_\beta d_i \mathrm{Li}_2(q^\beta) \end{pmatrix}, \end{aligned} \tag{6.9}$$

where $q^{\beta_i} = e^{2\pi i d_i U^i}$ and $\kappa_{ijk}, \kappa_{ij}, \kappa_i, \kappa_0$ are the topological quantities,

$$\kappa_{ijk} = \int_{\tilde{M}_{\rm CY}} J_i \wedge J_j \wedge J_k, \qquad \kappa_{ij} = -\frac{1}{2}\int_{\tilde{M}_{\rm CY}} J_i \wedge J_j \wedge J_j,$$
$$\kappa_j = \frac{1}{4\,3!}\int_{\tilde{M}_{\rm CY}} c_2(\tilde{M}_{\rm CY}) \wedge J_j, \qquad \kappa_0 = \frac{\zeta(3)}{(2\pi i)^3}\int_{\tilde{M}_{\rm CY}} c_3(\tilde{M}_{\rm CY}) = \frac{\zeta(3)}{(2\pi i)^3}\chi(\tilde{M}_{\rm CY}), \tag{6.10}$$

with $\zeta(3) \simeq 1.2$ and $\chi(\tilde{M}_{\rm CY})$ being the Euler characteristic of Mirror Calabi-Yau manifold.

By plugging Eqs. (6.7), (6.8) and (6.9) into (6.6), the Kähler potential for the complex structure moduli is written by setting $X^0$ as unity,

$$\begin{aligned} e^{-K(U,\bar{U})} &= \frac{i}{6}\sum_{ijk}\kappa_{ijk}(U^i - \bar{U}^i)(U^j - \bar{U}^j)(U^k - \bar{U}^k) - \frac{\zeta(3)}{4\pi^3}\chi(\tilde{M}_{\rm CY}) \\ &+ \frac{i}{(2\pi i)^2}\sum_{\beta} d_i n_\beta (U^i - \bar{U}^i)\Big(\mathrm{Li}_2(q^\beta) + \mathrm{Li}_2(\bar{q}^\beta)\Big) \\ &- \frac{2i}{(2\pi i)^3}\sum_\beta n_\beta\Big(\mathrm{Li}_3(q^\beta) + \mathrm{Li}_3(\bar{q}^\beta)\Big), \end{aligned} \tag{6.11}$$

where the $(\alpha')^3$ correction corresponds to the term proportional to the Euler characteristic. The third and fourth terms in Eq. (6.11) encode the contributions from the geometric corrections, which enjoy the discrete axionic shift symmetries.

For sake of simplicity, the normalization for the complex structure moduli $U^i$ is changed as $iU^i$ and consequently the Kähler potential reduces to

$$\begin{aligned} e^{-K(U,\bar{U})} &= \frac{1}{6}\sum_{ijk}\kappa_{ijk}(U^i + \bar{U}^i)(U^j + \bar{U}^j)(U^k + \bar{U}^k) - \frac{\zeta(3)}{4\pi^3}\chi(\tilde{M}_{\rm CY}) \\ &+ \frac{2}{(2\pi)^2}\sum_\beta\sum_{n=1}^{\infty} d_i n_\beta (U^i + \bar{U}^i)\frac{1}{n^2}\cos\left(-i\pi n\sum_j d_j(U^j - \bar{U}^j)\right) e^{-\pi n\sum_k d_k(U^k + \bar{U}^k)} \\ &+ \frac{4}{(2\pi)^3}\sum_\beta\sum_{n=1}^{\infty} n_\beta\frac{1}{n^3}\cos\left(-i\pi n\sum_j d_j(U^j - \bar{U}^j)\right) e^{-\pi n\sum_k d_k(U^k + \bar{U}^k)}. \end{aligned} \tag{6.12}$$

As demonstrated above, when some of the complex structure moduli are stabilized through not the perturbative corrections in the period vector, but the non-perturbative effects involved in the period vector, one can consider the situation that these complex structure moduli become lighter than the Kähler moduli. Such a scenario can be realized when the geometric corrections for the period vector are much suppressed than the non-perturbative effects to stabilize the Kähler moduli. In the next section, we extract the scalar potential for the lightest complex structure moduli for the several Kähler moduli stabilization scenarios.

## 6.3 New type of axion inflation

In this section, we first show the moduli stabilization to achieve the light complex structure moduli and one of the axion of such complex structure moduli becomes a candidate for the axion-inflaton. Depending on the setup of Kähler moduli stabilization, one can obtain the several potentials of axion-inflaton. The geometric corrections for the period vector of Calabi-Yau manifold lead to the characteristic feature of axion potential which is a mixture of polynomial and sinusoidal functions. In the following, we discuss the two types of Kähler moduli stabilization represented by the KKLT scenario and LVS scenario.

### 6.3.1 Moduli stabilization

To obtain the light complex structure modulus, we first denote the two complex structure moduli $U_1$ and $U_2$, while the other $(n-2)$ complex structure moduli and axion-dilaton are represented by $z$. By choosing certain three-form fluxes Eq. (6.2) in the large complex structure limit ($|z|, |U_1|, |U_2| \gg 1$), we consider the following ansatz of their Kähler potential and superpotential based on Ref. [63],

$$\begin{aligned} K(S,U) &= -\ln\left[f_0\left(\mathrm{Re}\, z, \mathrm{Re}\, U_1, \mathrm{Re}\, U_2\right)\right], \\ W(S,U) &= g_0(z) + g_1(z)(U_2 + NU_1), \end{aligned} \tag{6.13}$$

where $f_0$ corresponds to the first line of Eq. (6.12), i.e., the part at the tree- and loop-level, $g_{0,1}(z)$ are third-order polynomial function of only $z$ and $N$ is the integer. Here, we choose the specific ansatz of three-form fluxes (6.2) to extract the light complex structure modulus as shown later. and $N$ is the integer. When we change the base of complex structure moduli as

$$\begin{aligned} \Psi &\equiv U_2 + NU_1, \\ \Phi &\equiv U_2, \end{aligned} \tag{6.14}$$

the Kähler potential and superpotential become

$$\begin{aligned} K(S,U) &= -\ln\left[f_0\left(\mathrm{Re}\, z, (\mathrm{Re}\,\Psi - \mathrm{Re}\,\Phi)/N, \mathrm{Re}\,\Phi\right)\right], \\ W(S,U) &= g_0(z) + g_1(z)\Psi, \end{aligned} \tag{6.15}$$

which manifests that the imaginary part of $\Phi$ does not appear in the potential. Thus, $\mathrm{Im}\,\Phi$ remains massless, whereas the other moduli fields $\Psi$, $z$ and $\mathrm{Re}\,\Phi$ could be stabilized at the minimum, $D_I W = 0$ with $I = \Psi, z$ and $K_\Phi = 0$, respectively. The mass of $\mathrm{Re}\,\Phi$ is induced by the nonvanishing superpotential, i.e., SUSY-breaking effects. It is then possible to extract the light axion $\mathrm{Im}\,\Phi$, which can be considered as the inflaton field by taking into account the non-perturbative corrections through the period vector.

Before taking a closer look at the potential for $\mathrm{Im}\,\Phi$, let us comment on the stabilization of Kähler moduli. As discussed before, perturbative flux-induced potential generates the potential of $\mathrm{Re}\,\Phi, z, \Psi$ fields and $\mathrm{Im}\,\Phi$ remains massless. To achieve the successful inflation with complex structure axion, the mass scale of $\mathrm{Im}\,\Phi$ should be lighter than that of Kähler moduli. To stabilize the Kähler moduli, we have to add the non-perturbative effects for the Kähler moduli in the superpotential and the Kähler moduli should be stabilized at the high-energy scale compared with the mass-scale of remaining complex structure modulus $\mathrm{Im}\,\Phi$. We expect that if the stabilization of Kähler moduli is irrelevant to the that of $\mathrm{Im}\,\Phi$, Kähler moduli can be parametrically larger than the inflaton field $\mathrm{Im}\,\Phi$. In the following sections, we consider the KKLT scenario and LVS with some uplifting scenario in order to stabilize the Kähler moduli. For the KKLT scenario, the magnitude of flux-induced superpotential $w = \langle W(S, U)\rangle$ has to be compatible with that of non-perturbative effects, i.e., $w \ll 1$ in the Planck unit, while for the LVS, we can stabilize the Kähler moduli even when $w$ is of order unity.

From now on, we show the non-perturbative corrections calculated in the framework of topological string. In the large complex structure regime, i.e., the large field value of complex structure moduli $\mathrm{Re}\,z, \mathrm{Re}\,\Phi, \mathrm{Re}\,\Psi \gg 1$, there are exponential terms in the Kähler potential and superpotential

$$\begin{aligned}\Delta K &\simeq -\frac{1}{\langle f_0\rangle}\sum_{i=1}^{2} f_1^{(i)}\left(\frac{2}{\pi} + (U_i + \bar{U}_i)\right)\cos(-i\pi(U_i - \bar{U}_i))e^{-\pi(U_i+\bar{U}_i)},\\ \Delta W &\simeq \sum_{i=1}^{2}(g_2^{(i)} + g_3^{(i)}U_i)e^{-2\pi U_i},\end{aligned} \tag{6.16}$$

where the integer $d_i$ has been taken as unity and we set the heavy moduli-dependent parameters $g_{2,3}^{(i)}, f_1^{(i)}$ as real constants, for simplicity. In the usual moduli stabilization procedure, these corrections terms are enough suppressed. However, they give a characteristic potential for the axion associated with the complex structure moduli as shown later.

According to Ref. [63], we consider the following moduli space,

$$e^{-2\pi\langle\mathrm{Re}U_2\rangle} \ll e^{-2\pi\langle\mathrm{Re}\,U_1\rangle} \ll 1, \tag{6.17}$$

under which the exponential term for $U_2$ is negligible and the Kähler potential and superpotential are simplified as

$$\begin{aligned}\Delta K &\simeq -\frac{f_1^{(1)}}{\langle f_0\rangle}\left(\frac{2}{\pi} + \frac{\Psi + \bar{\Psi} - \Phi - \bar{\Phi}}{N}\right)\cos\left(-i\pi\frac{\Psi - \bar{\Psi} - \Phi + \bar{\Phi}}{N}\right)e^{-\pi\frac{\Psi+\bar{\Psi}-\Phi-\bar{\Phi}}{N}},\\ \Delta W &\simeq \left(g_2^{(1)} + \frac{g_3^{(1)}}{N}(\Psi - \Phi)\right)e^{-2\pi\frac{\Psi-\Phi}{N}},\end{aligned} \tag{6.18}$$

on the field basis $(\Psi, \Phi)$.

In the following, the imaginary part of $\Phi$ is assumed to be much lighter than the other complex structure moduli which are already stabilized at the minimum by the perturbative flux-induced superpotential. In addition, the Kähler moduli are also heavier than the $\mathrm{Im}\,\Phi$ and the fluctuation of Kähler moduli can be negligible during and after the inflation. This is because, in our setup, the inflation potential is originating from the nonvanishing $\Delta W$ which can be taken to be parametrically smaller than the tree-level part of GVW superpotential $w = \langle W \rangle$ as well as the non-perturbative superpotential for the Kähler moduli $W_{\rm non}(T)$. Although the backreaction from the Kähler moduli has to be take into account, we left it for a future work. When we discuss the dynamics of $\mathrm{Im}\,\Phi$, one can consider the situation that the superpotential behaves effectively constant $W = w + W_{\rm non}(\langle T \rangle) \equiv w_0$. Furthermore, Kähler moduli could be stabilized at the vacuum realized by KKLT or LVS, irrelevant to the dynamics of lightest complex structure modulus, when there is no coupling between Kähler moduli and $\Phi$ in the superpotential.

### Inflaton potential based on KKLT scenario

We first begin with the KKLT scenario by including the non-perturbative effects such as gaugino condensation on hidden D7-branes or world-sheet instanton effects $W_{\rm non}(T)$. Then, we can stabilize the Kähler moduli at the minimum $D_T W = 0$. By assuming the prefactors of Kähler moduli-dependent non-perturbative effects do not depend on $\Phi$, the superpotential is effectively considered as constant $w_0 = w + W_{\rm non}(\langle T \rangle)$.

From now on, we move onto the potential of lightest complex structure modulus $\mathrm{Im}\,\Phi$, on the fixed moduli space of Kähler moduli, heavy complex structure moduli and dilaton fields. From the Kähler potential and superpotential at the tree Eq. (6.15) and loop-levels Eq. (6.18), we find that the superpotential and its covariant derivative are yielded in the limit of $w_0 \gg \Delta W$,

$$
\begin{aligned}
W &\simeq w_0 + \Delta W, \\
D_\Phi W &\simeq \Delta W_\Phi + \Delta K_\Phi W \\
&\simeq e^{-2\pi \langle \mathrm{Re}\, U_1 \rangle} \Bigg[ e^{-2\pi i \frac{\langle \mathrm{Im}\,\Psi \rangle - \mathrm{Im}\,\Phi}{N}} \left( -\frac{g_3^{(1)}}{N} + \frac{2\pi}{N} \left( g_2^{(1)} + \frac{g_3^{(1)}}{N} (\langle \Psi \rangle - \Phi) \right) \right) \\
&\quad - \frac{w_0 f_1^{(1)}}{\langle f_0 \rangle} \left( -\frac{1}{N} \cos \left( 2\pi \frac{\langle \mathrm{Im}\,\Psi \rangle - \mathrm{Im}\,\Phi}{N} \right) + \frac{\pi}{N} \left( \frac{2}{\pi} + 2 \langle \mathrm{Re}\, U_1 \rangle \right) e^{-2\pi i \frac{\langle \mathrm{Im}\,\Psi \rangle - \mathrm{Im}\,\Phi}{N}} \right) \Bigg].
\end{aligned}
\tag{6.19}
$$

By substituting the above into the $F$-term scalar potential, the scalar potential for $\tilde{\phi} = \mathrm{Im}\,\Phi$

is extracted as[†]

$$
\begin{aligned}
e^{-K-\Delta K}V &\simeq K^{\Phi\bar{\Phi}}D_\Phi W D_{\bar{\Phi}}\bar{W} - 3|W|^2 \\
&\simeq -3|W|^2 + K^{\Phi\bar{\Phi}}e^{-4\pi\langle \mathrm{Re}\, U_1\rangle}\Biggl[A^2 + B^2(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})^2 \\
&+ (C^2+2AC)\cos^2\left(2\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right) + BC(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})\sin\left(4\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right)\Biggr] \\
&\simeq -3w_0^2 - 6w_0 e^{-2\pi\langle \mathrm{Re}\, U_1\rangle}\Biggl[\left(g_2^{(1)} + \frac{g_3^{(1)}}{N}\langle \mathrm{Re}\, U_1\rangle\right)\cos\left(2\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right) \\
&+ \frac{g_3^{(1)}}{N}(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})\sin\left(2\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right)\Biggr] \\
&- 3\,e^{-4\pi\langle \mathrm{Re}\, U_1\rangle}\Biggl[\left(g_2^{(1)} + \frac{g_3^{(1)}}{N}\langle \mathrm{Re}\, U_1\rangle\right)^2 + \left(\frac{g_3^{(1)}}{N}\right)^2(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})^2\Biggr] \\
&+ K^{\Phi\bar{\Phi}}e^{-4\pi\langle \mathrm{Re}\, U_1\rangle}\Biggl[A^2 + B^2(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})^2 \\
&+ (C^2+2AC)\cos^2\left(2\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right) + BC(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})\sin\left(4\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right)\Biggr],
\end{aligned}
\tag{6.20}
$$

where $K = K(S,U) + \Delta K + K(T,\bar{T})$ is the Kähler potential of all the (closed string) moduli and the other parameters are determined by the stabilized heavy moduli fields

$$
\begin{aligned}
A &= -\frac{g_3^{(1)}}{N} + \frac{2\pi g_2^{(1)}}{N} + \frac{2\pi g_3^{(1)}}{N}\langle \mathrm{Re}\, U_1\rangle - \frac{w_0 f_1^{(1)}2\pi}{\langle f_0\rangle N}\left(\frac{1}{\pi} + \langle \mathrm{Re}\, U_1\rangle\right), \\
B &= \frac{2\pi g_3^{(1)}}{N^2}, \quad C = \frac{w_0 f_1^{(1)}}{\langle f_0\rangle N}.
\end{aligned}
\tag{6.21}
$$

Since the vacuum energy is negative, i.e., AdS minimum, we introduce a certain uplifting sector such as the anti D-brane [50] and F-term uplifting scenario [51, 52]. By canonically normalizing the lightest complex structure modulus at this uplifted minimum, we obtain the inflaton potential under the large complex structure regime $\langle \mathrm{Re}\, U_1\rangle \gg 1$,

$$
V_{\rm inf} \simeq \Lambda_1\left(1 - \cos\frac{\phi}{M_1}\right) + \Lambda_2\phi\sin\frac{\phi}{M_1},
\tag{6.22}
$$

where $\phi \equiv k_1(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})$ is the canonically normalized axion with $k_1 = \mathcal{O}(\langle \mathrm{Re}\, U_2\rangle)$ being the relevant normalization factor, and $M_1 = Nk_1/2\pi$ and $\Lambda_{1,2} \simeq \mathcal{O}(e^{\langle K\rangle}w_0 e^{-2\pi\langle \mathrm{Re}\, U_1\rangle})$ are the

[†]As discussed in LVS in the next section 6.3.1, the negative term in the scalar potential $-3|W|^2$ vanishes due to the extended no-scale structure of Kähler moduli [72].

real constants. Although the first term is just the potential of natural inflation, the obtained total potential is the mixture of sinusoidal function and polynomial function. When the second term in Eq. (6.22) is absent, the potential is similar to the natural inflation with modulation as discussed in Refs. [73, 74, 75]. However, in our setup, the two terms in Eq. (6.22) are almost of the same order and one cannot neglect the second term in Eq. (6.22).

In the previous discussion so far, we have not taken into account the higher harmonic terms such as $e^{-4\pi\langle \mathrm{Re} U_1\rangle}$, since the superpotential is dominated by the constant $w_0$. On the other hand, we can consider the Kallosh-Linde model [76], where the Kähler moduli are stabilized at the minimum $w_0 \sim 0$ by the cancellation between tree-level GVW superpotential $w$ and the non-perturbative superpotential for Kähler moduli $W_{\rm non}(\langle T\rangle)$. From Eq. (6.20), we then extract the inflaton potential at the order $e^{-4\pi\langle \mathrm{Re}\, U_1\rangle}$,

$$V_{\rm inf} \simeq \Lambda_3 \phi^2, \tag{6.23}$$

where $\phi \equiv k_2(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})$ is the canonically normalized axion with $k_2$ being the relevant normalization factor of $\mathcal{O}(\langle \mathrm{Re}\, U_2\rangle)$ and $\Lambda_3 \simeq \mathcal{O}(e^{\langle K\rangle}e^{-4\pi\langle \mathrm{Re}\, U_1\rangle})$ is a real constant. Here, we set the certain uplifting sector along the same step outlined in Eq. (6.22). In this way, the potential of chaotic inflation can be obtained.

**Inflaton potential based on Large Volume Scenario**

Next, we perform the stabilization of Kähler moduli in the LVS by the contributions from the $\alpha'$ corrections in the Kähler potential and non-perturbative effects in the superpotential.

Since the Kähler moduli has the extended no-scale structure [72], $K^{a\bar{b}}K_a K_{\bar{b}} = 3$, up to loop-level, the negative term in the scalar potential, $-3|W|^2$, vanishes. Thus, the scalar potential in Eq. (6.20) reduces to

$$\begin{aligned} V &\simeq e^{K+\Delta K} K^{\Phi\bar{\Phi}} D_\Phi W D_{\bar{\Phi}}\bar{W} \\ &\simeq e^{K+\Delta K} K^{\Phi\bar{\Phi}} e^{-4\pi\langle \mathrm{Re}\, U_1\rangle}\Bigg[A^2 + B^2(\mathrm{Im}\,\Psi - \tilde{\phi})^2 \\ &+ (C^2+2AC)\cos^2\left(2\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right) + BC(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi})\sin\left(4\pi\frac{\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}}{N}\right)\Bigg]. \end{aligned} \tag{6.24}$$

By including the certain uplifting sector to obtain the Minkowski minimum, The scalar potential for the canonically normalized axion $\phi \equiv k_3\left(\langle \mathrm{Im}\,\Psi\rangle - \tilde{\phi}\right)$ with $k_3$ being the canonically normalization factor of $\mathcal{O}(\langle \mathrm{Re}\, U_2\rangle)$, is simplified as

$$V_{\rm inf} \simeq \Lambda_4\phi^2 + \Lambda_5\phi\sin\left(\frac{\phi}{M_3}\right) + \Lambda_6\left(1-\cos\left(\frac{\phi}{M_3}\right)\right), \tag{6.25}$$

where $M_3 = Nk_3/4\pi$ and $\Lambda_{4,5,6} \simeq \mathcal{O}(e^{\langle K\rangle}e^{-4\pi\langle \mathrm{Re}\, U_1\rangle})$ . The scalar potential in LVS is a mixture of sinusoidal and polynomial functions. Note that the scale of these three terms are almost the same order.

Let us comment on the backreaction from the Kähler moduli. In LVS, the potential of the Kähler moduli is approximately given by

$$V_{\rm LVS} \sim e^{K(S,U)} \frac{|W|^2}{\mathcal{V}^3}, \tag{6.26}$$

with $\mathcal{V}$ being the volume of Calabi-Yau manifold. The potential of axion-inflaton $\phi$ appears from Eq. (6.26) at the order $\mathcal{O}(e^{\langle K(S,U)\rangle} w_0 e^{-2\pi\langle {\rm Re}\, U_1\rangle}/\mathcal{V}^3)$. When the following inequality is satisfied

$$e^{\langle K(S,U)\rangle} \frac{w_0 e^{-2\pi\langle {\rm Re}\, U_1\rangle}}{\mathcal{V}^3} < e^{\langle K(S,U)\rangle} \frac{e^{-4\pi\langle {\rm Re}\, U_1\rangle}}{\mathcal{V}^2}, \tag{6.27}$$

we can suppress the effect induced by the backreaction of Kähler moduli. As an example, it is possible to realize this situation when the volume of Calabi-Yau manifold is mildly large, $\mathcal{V} \simeq 10^2$, ${\rm Re}\, U_1 \simeq 0.7$ and $w_0 \simeq 1$. In this situation, the mass of lightest Kähler modulus is also larger than the inflaton field. When the backreaction from the Kähler moduli gives a sizable effect to the inflaton potential, it generates the scalar potential as in Eq. (6.22). The total scalar potential takes a form of Eq. (6.25) with $\Lambda_{5,6} > \Lambda_4$. Although it is interesting to discuss the detailed study of the backreaction on the concrete Calabi-Yau manifold, we leave it for a future work.

### 6.3.2 Numerical analyses

Guided by the previous results, we study the details of three types of inflation scenarios given by Eqs. (6.22), (6.23) and (6.25). However, the inflaton potential in Eq. (6.23) is excluded by the recent result of Planck data. Furthermore, the inflaton potential (6.22) is similar to the potential of natural inflation with sinusoidal term. In our setup, the value of axion decay constant is enhanced to be trans-Planckian, since the axion-inflaton develops the winding trajectory on the $(\Psi, \Phi)$-plane, that is, $M_2 \gg 1$. As reviewed in Ch. 5, the scalar potential of natural inflation without the sinusoidal term (6.22) is in agreement with the Planck data [47] under its large axion decay constant.

From now on, we set the parameters in the scalar potential to realize the observed magnitude of power spectrum of curvature perturbation. From the inflaton potential in Eq. (6.22), the slow-roll parameters are found as

$$\begin{aligned}
\epsilon &= \frac{1}{2}\left(\frac{\partial_\phi V}{V}\right)^2 = \frac{1}{2}\left(\frac{\left(\frac{\Lambda_1}{M_1}+\Lambda_2\right)\sin\frac{\phi}{M_1} + \Lambda_2\frac{\phi}{M_1}\cos\frac{\phi}{M_1}}{\Lambda_1\left(1-\cos\frac{\phi}{M_1}\right) + \Lambda_2\phi\sin\frac{\phi}{M_1}}\right)^2 \simeq \frac{2}{\phi^2} - \frac{1}{3M_1^2}\frac{\Lambda_1+4M_1\Lambda_2}{\Lambda_1+2M_1\Lambda_2}, \\
\eta &= \frac{\partial_\phi\partial_\phi V}{V} = \frac{\frac{1}{M_1}\left(\frac{\Lambda_1}{M_1}+2\Lambda_2\right)\cos\frac{\phi}{M_1} - \Lambda_2\frac{\phi}{M_1^2}\sin\frac{\phi}{M_1}}{\Lambda_1\left(1-\cos\frac{\phi}{M_1}\right) + \Lambda_2\phi\sin\frac{\phi}{M_1}} \simeq \frac{2}{\phi^2} - \frac{5}{6M_1^2}\frac{\Lambda_1+4M_1\Lambda_2}{\Lambda_1+2M_1\Lambda_2},
\end{aligned} \tag{6.28}$$

and consequently the cosmological observables such as $n_s$ and $r$ are evaluated as discussed in Sec. 5.1,

$$
\begin{aligned}
n_s &= 1 - 6\epsilon + 2\eta \simeq 1 - \frac{8}{\phi^2} + \frac{1}{3M_1^2}\left(1 + \frac{2M_1\Lambda_2}{\Lambda_1 + 2M_1\Lambda_2}\right), \\
r &= 16\epsilon \simeq \frac{32}{\phi^2} - \frac{16}{3M_1^2}\left(1 + \frac{2M_1\Lambda_2}{\Lambda_1 + 2M_1\Lambda_2}\right). \qquad (6.29)
\end{aligned}
$$

It then turns out that the negative value of $\Lambda_2$ gives rise to small (large) $n_s$ ($r$). The parameters in the scalar potential are chosen such that the inflaton mass is positive at least at a vacuum, i.e., $\partial^2 V/\partial\phi^2 = (\Lambda_1 + 2M_1\Lambda_2)/M_1^2 > 0$ in the following numerical analyses. In Fig. 6.1, we plot the cosmological observables ($n_s, r$) for the inflaton potential (6.22). Then, the prediction of ($n_s$,$r$) is similar to that of natural inflation without sinusoidal term and the trans-Planckian axion decay constant is favored by the Planck data [47] from the aspect of spectral tilt of curvature perturbation. By setting the illustrative parameters in the scalar potential, we summarize the typical values of cosmological observables with a sufficient $e$-folding number in Table 6.2.

Finally, we study the scalar potential in Eq. (6.25) which is the linear combination of sinusoidal and polynomial functions. In a similar fashion, the slow-roll parameters are explicitly written by

$$
\begin{aligned}
\epsilon &= \frac{1}{2}\left(\frac{2\Lambda_4\phi + (\Lambda_5 + \frac{\Lambda_6}{M_3})\sin\frac{\phi}{M_3} + \frac{\Lambda_5}{M_3}\phi\cos\frac{\phi}{M_3}}{\Lambda_4\phi^2 + \Lambda_5\phi\sin\frac{\phi}{M_3} + \Lambda_6(1-\cos\frac{\phi}{M_3})}\right)^2 \simeq \frac{2}{\phi^2} - \frac{4M_3\Lambda_5 + \Lambda_6}{3M_3^2\left(2M_3(M_3\Lambda_4 + \Lambda_5) + \Lambda_6\right)} + \mathcal{O}\left(\frac{1}{M_3^4}\right), \\
\eta &= \frac{2\Lambda_4 - \frac{\Lambda_5}{M_3^2}\phi\sin\frac{\phi}{M_3} + \frac{1}{M_3}(2\Lambda_5 + \frac{\Lambda_6}{M_3})\cos\frac{\phi}{M_3}}{\Lambda_4\phi^2 + \Lambda_5\phi\sin\frac{\phi}{M_3} + \Lambda_6(1-\cos\frac{\phi}{M_3})} \simeq \frac{2}{\phi^2} - \frac{5(4M_3\Lambda_5 + \Lambda_6)}{6M_3^2\left(2M_3(M_3\Lambda_4 + \Lambda_5) + \Lambda_6\right)} + \mathcal{O}\left(\frac{1}{M_3^4}\right),
\end{aligned}
\qquad (6.30)
$$

which give rise to the $n_s$ and $r$,

$$
\begin{aligned}
n_s &\simeq 1 - \frac{8}{\phi^2} + \frac{4M_3\Lambda_5 + \Lambda_6}{3M_3^2\left(2M_3(M_3\Lambda_4 + \Lambda_5) + \Lambda_6\right)} + \mathcal{O}\left(\frac{1}{M_3^4}\right), \\
r &\simeq \frac{32}{\phi^2} - \frac{16(4M_3\Lambda_5 + \Lambda_6)}{3M_3^2\left(2M_3(M_3\Lambda_4 + \Lambda_5) + \Lambda_6\right)} + \mathcal{O}\left(\frac{1}{M_3^4}\right). \qquad (6.31)
\end{aligned}
$$

It then turns out that in the case of $M_3 \gg 1$, the large value of $\Lambda_5$ in comparison with $\Lambda_4$ allows us to obtain the large $n_s$ and small $r$. The parameters in the scalar potential are chosen such that the inflaton mass is positive at least at a vacuum, i.e., $\partial^2 V/\partial\phi^2 = (2M_3(M_3\Lambda_4 + \Lambda_5) + \Lambda_6)/M_3 > 0$. By setting the illustrative parameters, we summarize the typical values of cosmological observables with a sufficient $e$-folding number in Table 6.2, from which the prediction of this inflaton potential is well in agreement with the recent Planck data [47]. In Fig. 6.2, we plot the prediction of ($n_s, r$) within the rage of $e$-folding number, $50 \leq N_e \leq 60$ by setting $\Lambda_4/\Lambda_6 = 1$ and $\lambda_5/\lambda_6 = 5$. We find that the several values of axion decay constant lead to both the small- and large-field inflations without depending on the value of spectral tilt of curvature perturbation. Note that in the case of small-field inflation, a flat plateau appears in the scalar potential.

| $M_1$ | $\Lambda_2/\Lambda_1$ | $N_e$ | $n_s$ | $r$ | $dn_s/d\ln k$ |
|---|---|---|---|---|---|
| 8 | 5 | 50 | 0.96 | 0.04 | $-0.0005$ |
| 8 | 5 | 60 | 0.964 | 0.03 | $-0.0003$ |
| 10 | 5 | 50 | 0.964 | 0.055 | $-0.0006$ |
| 10 | 5 | 60 | 0.969 | 0.041 | $-0.0004$ |
| 12 | 5 | 50 | 0.966 | 0.063 | $-0.0006$ |
| 12 | 5 | 60 | 0.971 | 0.049 | $-0.0004$ |
| 15 | 5 | 50 | 0.968 | 0.07 | $-0.0006$ |
| 15 | 5 | 60 | 0.973 | 0.056 | $-0.0004$ |

Table 6.1: For the input parameters $M_1$ and $\Lambda_2/\Lambda_1$, we obtain the cosmological observables, spectral tilt of curvature perturbation $n_s$, tensor-to-scalar ratio $r$ and the running of spectral index $dn_s/d\ln k$ at the $e$-folding number $N_e$.

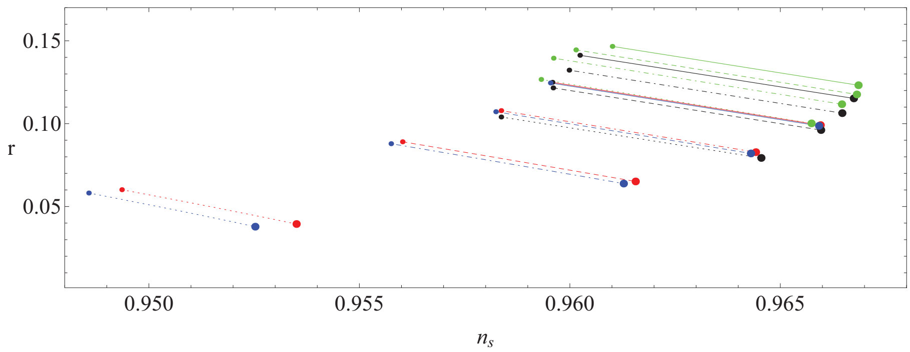


Figure 6.1: The inflaton potential (6.22) with and without sinusoidal term predicts $(n_s, r)$ in the range of $e$-folding number, $50 \leq N_e \leq 60$, where $N_e = 50$ ($N_e = 60$) corresponds to the leftmost (rightmost) circle on each line. The black line corresponds to the prediction of inflaton potential (6.22) without sinusoidal term, i.e., natural inflation. On the other hand, the red, blue and green lines represent the prediction of inflaton potential (6.22) with sinusoidal term, $\Lambda_2/\Lambda_1 = 1, 5, -1/6M_1$, in particular, the solid, dotdashed, dashed and dotted lines denote the axion decay constants $M_1 = 15, 12, 10, 8$, respectively.

| $M_3$ | $\Lambda_4/\Lambda_6$ | $\Lambda_5/\Lambda_6$ | $N_e$ | $n_s$ | $r$ | $dn_s/d\ln k$ |
|---|---|---|---|---|---|---|
| 5 | 1 | 5 | 60 | 0.969 | 0.05 | −0.0007 |
| 5 | 1 | 5 | 55 | 0.965 | 0.06 | −0.0008 |
| 5 | 1 | 5 | 50 | 0.962 | 0.07 | −0.0009 |
| 3 | 1 | 5 | 60 | 0.97 | 0.008 | 0.0009 |
| 3 | 1 | 5 | 55 | 0.964 | 0.097 | 0.0009 |
| 3 | 1 | 5 | 50 | 0.956 | 0.012 | 0.0009 |
| 5 | 1/5 | 1 | 60 | 0.968 | 0.05 | −0.0007 |
| 5 | 1/5 | 1 | 55 | 0.965 | 0.06 | −0.0008 |

Table 6.2: For the input parameters $M_3$ and $\Lambda_4/\Lambda_6$, $\Lambda_5/\Lambda_6$, we obtain the cosmological observables such as spectral tilt of curvature perturbation $n_s$, tensor-to-scalar ratio $r$ and the running of spectral index $dn_s/d\ln k$ at the $e$-folding number $N_e$.

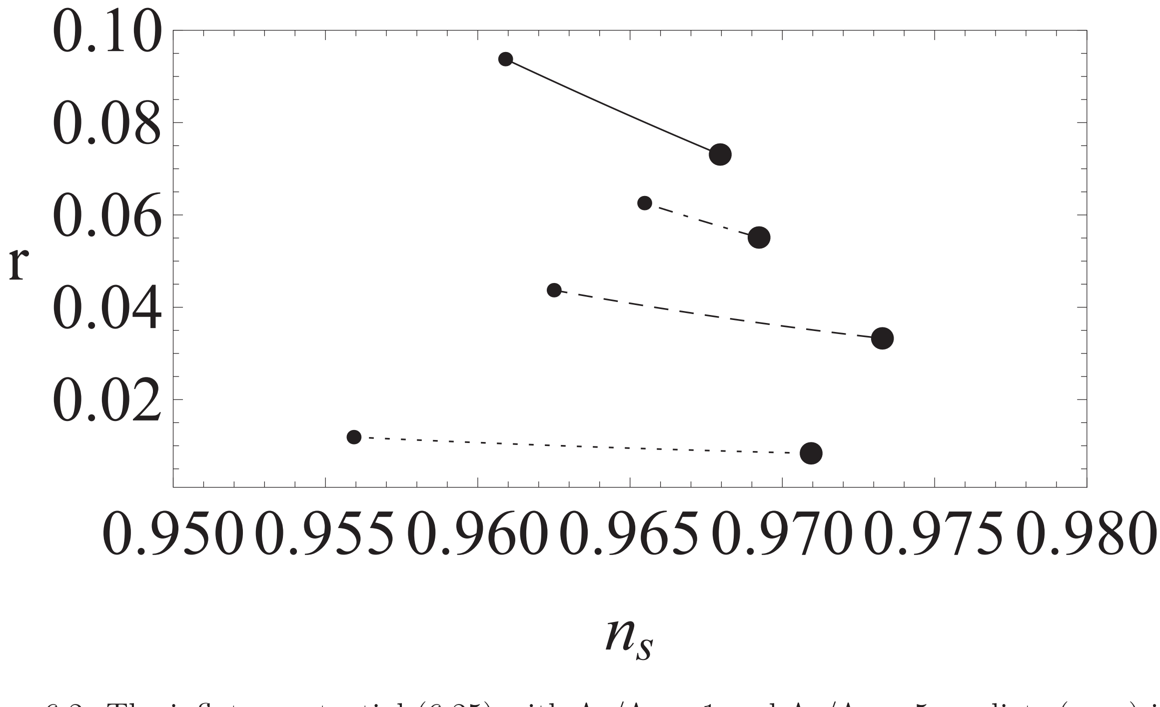


Figure 6.2: The inflaton potential (6.25) with $\Lambda_4/\Lambda_6 = 1$ and $\Lambda_5/\Lambda_6 = 5$ predicts $(n_s, r)$ in the range of $e$-folding number, $50 \leq N_e \leq 60$, where $N_e = 50$ ($N_e = 60$) corresponds to the leftmost (rightmost) circle on each line. The solid, dotdashed, dashed and dotted lines correspond to decay constants of axion $M_3 = 6, 5, 4, 3$, respectively.

### 6.3.3 Reheating temperature

In the numerical analyses in Sec. 6.3.2, we estimate the cosmological observables within the range of $e$-folding number $50 \leq N_e \leq 60$. However, the exact value of $e$-folding number highly depends on the thermal history of the Universe after the inflation. Indeed, when there is no dilution effect after the inflation, $e$-folding number is determined by the energy density of scalar potential at the horizon exit (end of inflation), $V_*^{1/4}$ ($V_{\rm end}^{1/4}$) and the energy density of radiation $\rho_R^{1/4} = (\pi^2 g_*/30)T_R$ with $g_*$ being the effective degrees of freedom of the radiation at the reheating temperature $T_R$ [46],

$$N_e \simeq 62 + \ln\frac{V_*^{1/4}}{10^{16}\,{\rm GeV}} + \ln\frac{V_*^{1/4}}{V_{\rm end}^{1/4}} - \frac{1}{3}\ln\frac{V_{\rm end}^{1/4}}{\rho_R^{1/4}}. \tag{6.32}$$

Thus, the reheating temperature is closely tied to the value of $e$-folding.

In the following, we analyze the typical reheating process of axion-inflaton associated with the lightest complex structure modulus. After the inflation, inflaton oscillates around the minimum and it decays into the matter fields in the standard model an/or hidden sectors. When the inflaton field is the axion associated with the Kähler moduli, the main decay channels of axion-inflaton is the decay of inflaton to gauge boson, gauginos, Higgs fields due to a dimensional counting.(For details of the decay processes in the LVS, see, e.g., Refs. [77].) However, in contrast to the Kähler moduli, the gauge kinetic function involves the dependence of complex structure moduli at the loop-level, rather than the at tree-level. In the following, we proceed to study the detail of decay process of complex structure moduli. To discuss the decay process, we assume that the SUSY is broken at a scale larger than the inflation scale. The main source of SUSY-breaking is originating from the Kähler moduli or matter fields. Below the inflation scale, the matter contents are composed of those of standard model.

First of all, we estimate the mass scale of Weyl fermion paired with the inflaton field, $\psi$. From the mass formula of Weyl fermions in the 4D $\mathcal{N} = 1$ supergravity,

$$m_{IJ} = e^{G/2}\left(\nabla_I G_J + \frac{1}{3}G_I G_J\right), \tag{6.33}$$

where $\nabla_I G_J = \partial G_J/\partial\Phi^I - \Gamma^K_{IJ}G_K$, $G = K + \ln(\bar{W}W)$, $G_I = \partial G/\partial\Phi^I$ and $\Gamma^K_{IJ}$ is the Christoffel symbol constructed by the metric $K_{IJ}$, it turns out that the mass scale of $\psi$ is almost of the same order of the gravitino mass, $m_{3/2} = e^{G/2}$, since the real part of $\Phi$ is satisfied in $G_\Phi \simeq 0$,

$$m_{\psi\psi} \simeq e^{G/2}G_{I\bar{J}}. \tag{6.34}$$

In this way, the inflaton does not decay into $\psi$. Consequently, the leading decay channels are classified into the following three types of them.

First of all, the inflaton decays into the gauge bosons in the standard model. As mentioned before, the complex structure modulus appears in the gauge kinetic function at the one-loop level. On the toroidal background, the authors of Refs. [78] calculated such one-loop threshold corrections for $\Phi$, whereas such corrections on Calabi-Yau background are not well understood. In this chapter, we assume the existence of Φ-dependent corrections in the gauge kinetic function,

$$\mathcal{L} = -\frac{1}{4g_a^2}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}\frac{\Delta(\Phi)}{16\pi^2}F^a_{\mu\nu}F^{a\mu\nu} \tag{6.35}$$

where $\Delta(\Phi)$ is the corresponding threshold correction for Φ [79], and $a = 1, 2, 3$ represent the standard model gauge groups, $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. From the above axionic couplings, the decay width is estimated as

$$\Gamma^{(1)}_\phi \equiv \sum_{a=1}^{3}\Gamma(\phi \to g^{(a)} + g^{(a)}) = \sum_{a=1}^{3}\frac{N^a_G}{128\pi}\left(\frac{\partial_\phi(\Delta(\Phi))g_a^2}{16\pi^2 d}\right)^2\frac{m_\phi^3}{M_{\rm Pl}^2} \simeq 2.8\times 10^{-3}\left(\frac{m_\phi}{10^{13}\,{\rm GeV}}\right)^2, \tag{6.36}$$

where $\sum_a N^a_G = 12$, $d \simeq \mathcal{O}(\sqrt{K_{\Phi\bar{\Phi}}}) \simeq \mathcal{O}(1)$, $m_\phi$ is the inflaton mass, and $(g_a)^{-2} \simeq 3.7$ is the gauge coupling at the grand unification scale $2\times 10^{16}$ GeV. Here, we implicitly assume the matter contents of minimal supersymmetric standard model at the grand unification scale. However, the following analysis is not relevant to the value of gauge couplings. Similarly to the case of toroidal background [78], we further approximate the form of threshold correction $\Delta(\Phi)$ as the linear term of Φ in the large complex structure limit, i.e., $\Delta(\Phi) = \Phi$. When the above decay channel is the total decay channel, we estimate the reheating temperature in an instantaneous approximation. By equaling the Hubble parameter at the reheating and the total decay width, the reheating temperature $T_R$ is found as

$$\begin{aligned}&\Gamma^{(1)}_\phi \simeq H(T_R),\\ &\Rightarrow T_R = \left(\frac{\pi^2 g_*}{90}\right)^{-1/4}\sqrt{\Gamma^{(1)}_\phi M_{\rm Pl}} \simeq 4.4\times 10^7\left(\frac{m_\phi}{10^{13}\,{\rm GeV}}\right)^2\,{\rm GeV},\end{aligned} \tag{6.37}$$

with $g_* = 106.75$ being the effective degrees of freedom of the radiation at the reheating in the standard model. Note that we consider the situation where the gauginos are heavier than the inflaton due to the SUSY-breaking and the decay from the inflaton into the gaugino pairs is prohibited. Anomaly-induced inflaton decays are also suppressed because of $K_\Phi \simeq 0$ [80].

Next, let us take a closer look at the inflaton decay into the matter fields $Q$ in the standard model through the Kähler potential. Using the same notation $\phi$ for its supermultiplet, the relevant couplings between $\phi$ and $Q$ is expected to appear in the shift-symmetric way

$$K \simeq f(T+\bar{T})e^{-2\pi{\rm Re}\,U_1}\cos\left(\frac{\phi}{M}\right)|Q|^2, \tag{6.38}$$

where $M$ is the decay constant of axion-inflaton, and $f(T+\bar{T})$ is the shift-symmetric function of Kähler moduli. Since the shift symmetries of axions protect the form of potential, the axion-inflaton does not appear in the Kähler potential at the tree-level. At the non-perturbative level, the axionic shift symmetry is broken by the instantonic effects. When the axionic coupling in Eq. (6.38) is dominant in the Kähler potential, the axion-inflaton does not decay into the matter fields in the visible sector. This is because the inflaton is lighter than the supersymmetric particles.

Finally, we explore the inflaton decay into the matter fields through the superpotential. Although the couplings between moduli and matter fields are associated by higher-dimensional operators, they could give a sizable effect. Their relevant interactions are captured in the Yukawa couplings,

$$W = Y_{ijk}(\Phi) Q^i Q^j Q^k, \tag{6.39}$$

where $Y_{ijk}(\Phi)$ denotes the physical Yukawa coupling and the matter chiral multiplets $Q_i$ in the visible sector are canonically normalized. (For details of moduli-dependent Yukawa coupling, see, e.g., Ref. [15] in the case of toroidal background.) The decay width from the axion-inflaton to the standard model particles becomes

$$\Gamma_\phi^{(2)} \equiv \Gamma(\phi \to \psi^i \psi^j Q^k) \simeq \frac{1}{64(2\pi)^3} \left\langle \frac{\partial Y_{ijk}}{\partial \phi} \right\rangle^2 m_\phi, \tag{6.40}$$

where the fermions $\psi^i$ and bosons $Q^i$ are the elements of the chiral multiplet $Q^i$, respectively. When the above decay channel is the dominant one, the reheating temperature is obtained in an instantaneous approximation,

$$\begin{aligned} &\Gamma_\phi^{(2)} \simeq H(T_R), \\ &\Rightarrow T_R = \left(\frac{\pi^2 g_*}{90}\right)^{-1/4} \sqrt{\Gamma_\phi^{(2)} M_{\rm Pl}} \simeq 2.1 \times 10^8 \left(\frac{\langle \partial_\phi Y_{ijk} \rangle}{10^{-5}}\right)^2 \left(\frac{m_\phi}{10^{13}\,{\rm GeV}}\right) {\rm GeV}, \end{aligned} \tag{6.41}$$

with $g_* = 106.75$. The first derivative of Yukawa coupling with respect to axion-inflaton is important to estimate the reheating temperature, although the explicit form of Yukawa coupling depends on the D-brane models.

As a result, when the reheating process is determined by the gauge boson decay, the typical reheating temperature is lower than that of Kähler moduli. It gives an significant effect to the dark matter abundance. The detail of phenomenological aspects based on this axion inflation will be worked out in future.

## 6.4 Summary

In this chapter, we have discussed the new type of axion inflation in terms of the complex structure moduli, on the basis of type IIB string theory on Calabi-Yau manifold.

In contrast to the well-known axion inflation model, our axion-inflaton is one of complex structure moduli which determines the shape of Calabi-Yau manifold. The inflaton potential is derived from the geometric corrections corresponding to the instanton corrections in type IIA superstring theory. Thanks to the mirror symmetry, the inflaton potential is well-controlled under the $\alpha'$- and $g_s$-corrections. Since an axionic shift symmetry of complex structure modulus is broken by the instantonic correction and certain three-from fluxes, the axion potential is the mixture of polynomial and sinusoidal functions. In particular, when the Kähler moduli are stabilized in the LVS scenario, the predictions of obtained axion inflation are in agreement with the recent Planck data. Furthermore, the masses of Kähler moduli are parametrically taken lager than that of the axion-inflaton.

The complex structure moduli inflation has the characteristic feature for the reheating process after the inflation. The couplings between the complex structure moduli and the gluon fields are one-loop suppressed in contrast to the Kähler axion. Thus, the typical reheating temperature is lower than that of Kähler axion inflaton. It is quite interesting to combine this new type of axion inflation and realistic D-brane model building in a future work.

# Chapter 7

# Conclusions and discussions

Throughout this thesis, we have discussed about the phenomenological and cosmological aspects of type IIB string theory compactified on some specific Calabi-Yau manifold. In particular, we focus on the geometrical structure of Calabi-Yau space to reveal the high-energy physics predicted by the string theory in terms of the low-energy effective theory. The main topics of Part I are the phenomenological aspects of low-energy effective theory, with an emphasis on the hierarchical structure of four-dimensional matter wavefunctions among quarks, leptons and Higgs fields, which capture the geometrical structure of the extra-dimensional space. In particular, we identified extra-dimensional space with the conifold locally, that is known as one of non-compact Calabi-Yau spaces with a singularity. In such a model, $AdS_5 \times T^{1,1}$ spacetime appears around the conifold singularity as a supersymmetric classical solution of type IIB string theory, i.e., Klebanov-Witten model. From the phenomenological point of view, the merit to consider $AdS_5 \times T^{1,1}$ model is that it is easier to be treated than the other Calabi-Yau space since i) the explicit form of its metric is known, and ii) at least $\mathcal{N} = 1$ supersymmetry is preserved. In addition to them, this background geometry is theoretically interesting, since the conical singularity appears not only through the backreaction of a large number of D3-branes discussed in the AdS/CFT correspondence, but also in the landscape of string theory suggested via a statistical approach. After adding the certain D7-branes to the Klebanov-Witten background in a supersymmetric way, we explored the detail of matter wavefunctions living on these D7-branes. When we consider the magnetized D7-branes wrapping the internal cycle of Klebanov-Witten background, the matter wavefunction is exponentially localized around the tip of conifold. Such an exponential form of wavefunction is controlled by the magnetic flux, which is also realized in the matter wavefunction in $AdS_5$ spacetime discussed in the phenomenological model building. In the future, it is interesting to analyze the matter wavefunction on another curved background in a similar way.

In Part II, we studied the cosmological aspects of type IIB string theory on Calabi-Yau manifolds, in particular, the inflation mechanism with an axion. Since a period vector on Calabi-Yau manifold can be derived by the notion of $\mathcal{N} = 2$ special geometry, we can compute the explicit form of the non-perturbative or geometrical correction in the axion potential. These corrections can lead to the model where the complex structure moduli are lighter than the other moduli field. The action of axion associated with the complex structure moduli is depending on the metric of the Calabi-Yau manifold. The shift symmetry of such an axion is originated from the Lorentz symmetry on the extra-dimensional space. By turning on the three-from fluxes on the internal three-cycles of Calabi-Yau manifold, we find that the new type of axion potential can be realized, which is applicable for a wider range of Calabi-Yau manifold. When we identify the axion of the lightest complex structure modulus with the inflaton, the prediction of cosmological observables is different from those of the other axion inflation models such as natural inflation and axion monodromy inflation. Furthermore, the couplings among the complex structure moduli and matter fields are constrained by the geometrical symmetry on the extra-dimensional space. Indeed, the complex structure moduli couple to the gauge bosons at the one-loop level. Since the reheating process after the inflation is thus different from the analysis of axion associated with the Kähler moduli, the reheating temperature is quite lower than those of other axion inflation models. In order to study the phenomenological features of this axion inflation model with the complex structure moduli, the explicit configuration for D-brane embeddings should be considered and the detailed analysis is required for studying them in a future work.

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# 早稲田大学　博士（理学）　学位申請　研究業績書

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|---|---|
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