WASEDA UNIVERSITY DOCTORAL DISSERTATION

Structural Damage Identification based on the Characteristics of Interaction Field in Combination Data Space

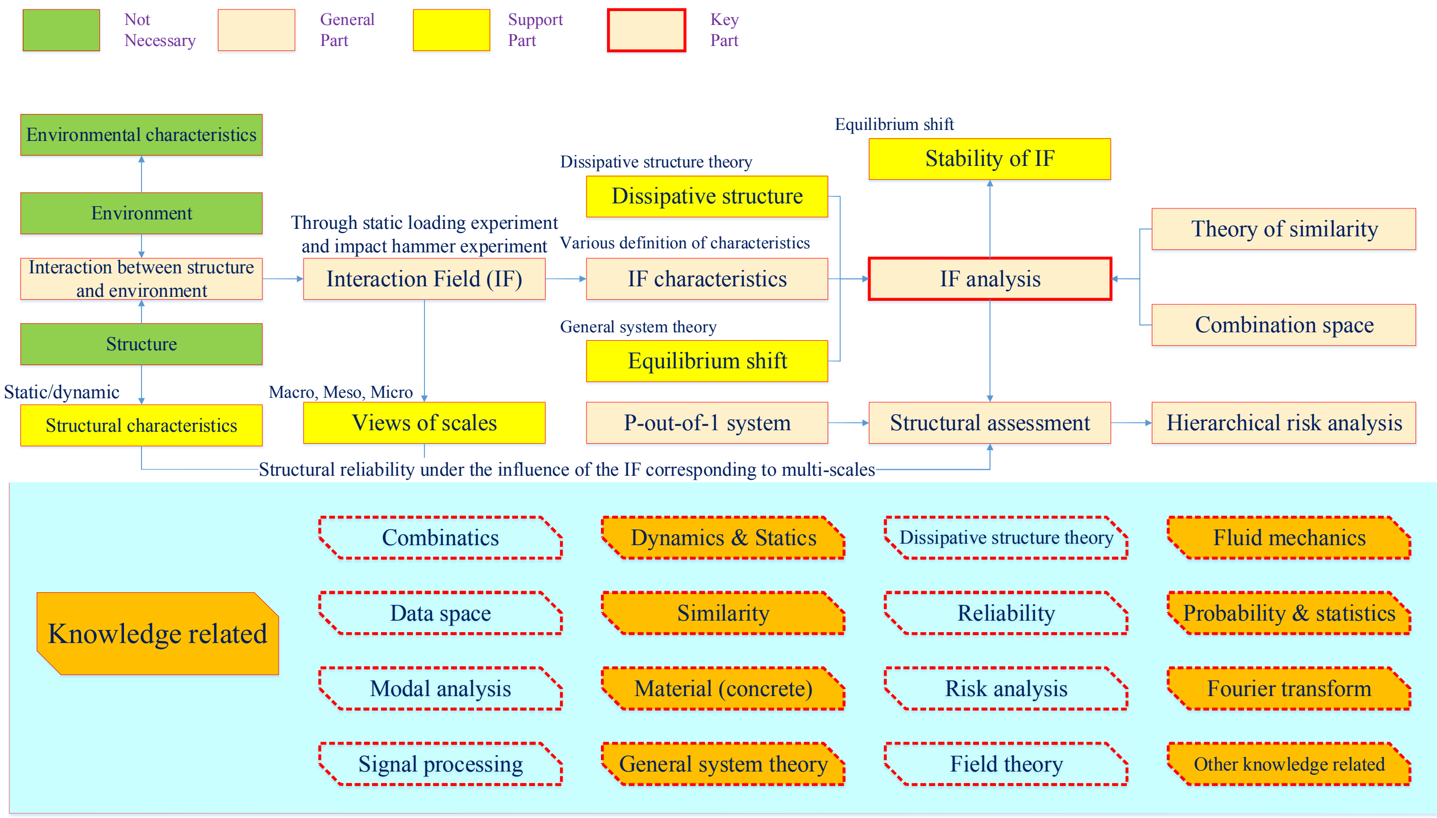
Xiong-Fei YE

Graduate School of Information, Production and Systems

Waseda University

February 2020

Thesis's Research Structure & Knowledge



Abstract

Nowadays, as infrastructure systems, particularly bridges, are rapidly aging all over the world, deterioration of structures is inevitable and may occur at any time. Concerning the environmental influences, the evolution of temperature, weather, humidity, and speed/depth of water, as well as the growth of corrosion, rust, scour, etc. change the internal force characteristics in structures. These random actions accelerate the structural deterioration and may lead to great structural damage, which decreases the lifespan and reliability of the systems and even results in the collapse of the bridge. To save expensive retrofitting, detecting damage at the early stage of the structures stands to be necessary retrofitting to prevent disasters.

Generally, a global structural system consists of three parts, the structure, the environment of the structure, and the interactions between structure and its environment as well as the interactions among the substructures. For some man-made infrastructures, in their lifetime accompanied by load effects of rains, winds, and earthquakes, etc., kinds of motion (e.g. wave) pass through them from entry to departure. Meanwhile, according to the health monitoring axioms, damage may happen inside the structure synchronously developing from micro to macro and eventually destroys the structure. There is a need to identify the health status of the structure.

To assess the status of the structure, often parameters considering the stress-strain relationship and the modal analysis on structural characteristics are the most considered, like elastic coefficient, natural frequency, damping ratio, etc. On the other hand, when the structure has damage, the equilibrium of the interaction filed will immediately be broken and updated, and its characteristics will change at the same time. This interaction processing, or named as interaction propagation, describing the transmission of static or dynamic, such as the force processing, and wave processing among countless elements, like a filed. In this way, the interaction field is formed to describe the interactions.

For the reinforced concrete structure, its mechanical behaviors are often nonlinear

and nonstationary. The structure often involves mixed materials (steel, concrete), and mixed substructures, which leads to the inconsistent force distribution, and has large material variability in its lifetime since the combined effect of prestressing. The indicators in structural characteristics-based damage detection may not accurately represent the change of the structural health status and the results of structural characteristics-based damage detection are often not satisfied. That is to say, only information on the structure is not enough for damage detection in a global structural system. Accordingly, if we focus on the interaction rather than the structure itself, maybe the result will be better.

In segmentation, concerning the indicators for damage identification, and risk of limited sensors used in measurement for the structural damage detection, this thesis starts 2 topics (A&B), structural assessment and risk evaluation. (A) The research firstly systematically proposes the method of interaction field analysis in combination data space for structural damage detection. (B) The hierarchical risk analysis is proposed since the structural system and the interaction itself is often hierarchical when assessing and maintaining the structures.

In the thesis, there are 6 chapters.

In chapter 1, the research background and great importance are attached to the necessity of structural safety and damage detection.

Chapter 2 mainly introduces the state-of-the-art literature and research objectives, that referring to the shortage of traditional damage detection, it asks to develop new research on interaction characteristics-based damage detection and conduct experiments or simulations to test our thought.

In chapter 3, the interaction field is imported into damage detection, which starts a new method, characteristics-based interaction field analysis, in short, interaction field analysis. Concerning the interaction processing (interaction filed), it has 4 categories of basic characteristics, the spread of interaction processing, the channel for interaction processing, the amount of interaction processing, and the expansion of interaction processing in the system. Based on these changing but distinguishable characteristics, it's possible to recognize the structure damaged or not. The interaction could be divided into two kinds, destructive interaction and non-destructive interaction; and the intensity, amount, distribution, and propagation time of the interaction could be simply mined from data records. So, in its initial research, the damage indicators turn out to be the intensity of displacement (IoD) for displacement data in statics, max-peak-time, and max-peak for acceleration data in dynamics.

Chapter 4 mainly introduces the experiment to test the interaction field analysis. In this research, in every stage of artificial damage experiment and evaluation, one destructive test is followed by one non-destructive test (stage 1 ~ stage 9). In the data processing, the combinatorial but evolutional description at multi-locations for interaction processing characteristics shows high sensitivity and efficient precision to distinguish the damage exacerbations in the structure by the variables indicating the similarity or difference in the timeline. To judge the change of interaction processing characteristics (data form: matrix) like IoD, max-peak-time, and max-peak, 2-D correlation coefficient (2D-CC), and distance representing similarity and difference between initial health status and considered status respectively, are tested. In static experiment (static loading and tendon cut), comparing result of the interaction field analysis and traditional mechanical structural characteristic, elastic coefficient, among all 11 stages, (1) for box girder 2, the elastic coefficient's effective proportion is 4/10; while the 2D-CC's effective proportion is 9/10, so it improves 50%; (2) for box girder 3, the elastic coefficient's effective proportion is 4/10; while the 2D-CC's effective proportion is 6/10, so it improves 20%. Also, in dynamic experiment (impact hammer experiment), comparing result of the characteristics of interaction field analysis and traditional mechanical structural characteristic from model analysis, among 9 stages, (1) referring the same data measured in experiment on box girder 2, the best right representation of variables and characteristics for interaction field analysis and modal analysis are 6/8 and 5/8, which shows the interaction field analysis is better; (2) referring the same data measured in the experiment on box girder 3, the best right representation of variables and characteristics for interaction field analysis and modal analysis are 7/8 and 6/8, which shows the interaction field analysis is better as well (improving 12.5%). Based on the experiment and data processing for both two almost full-size girders, this research starts the interaction field analysis, which broads a new viewpoint for damage identification and evaluation of the global structural system, and especially helps research community to have a deep understanding of the characteristics of interactions between structure and environment, or among substructures, besides general structural mechanical analysis on the characteristics of structure itself.

Importantly, in chapter 5, after discussing structural changes in the interaction field under artificial damages, and surveying the characteristics of the interaction filed, we have to face a problem that, the number of sensors used for measurement in engineering is far less than the number of particles of substances that interact in the actual structure. To recognize the influence of interaction processing on the structural assessment, the hierarchy is imported to describe the discrete relationship at different levels of the system. By the way, “fayer” is proposed as the unit of hierarchy (like the dimension in fractal). In the probability view, the assessment result is called hierarchical reliability, which surveying the survival probability of the whole system at different fayers. The hierarchical risk analysis concentrates on the information variation at different fayers of the same system. The local information is measured at different locations (substructures) and is combined to form the global information. In this process, the density of the sensors is the key factor. We need to learn the influence of different densities (number of sensors/elements to be measured in different fayers) to reach the final assessment, and then by calculating the hierarchical reliability at different fayers, we can set a standard for damage detection. Concerning the structural reliability systems in practice, since they are often designed to be redundant, a new reliability system, named as the P-out-of-1 reliability system is derived from the so-called K-out-of-N:F reliability systems, where P={Pi}, i=1,2,3,… is the failure probability of the whole system at every fayer, Pi=Ki/Ni, 1=Ni/Ni, i=1,2,3… And its analytical solution is given as follows. (1) Determine the elemental reliability at different fayers by hierarchical data iteration. (2) Calculate the reliability of the whole system at different fayers.

Herein, we can learn the relationships of reliability among different fayers. These relationships could be used to guide the measurements employing different numbers of sensors located at different substructures. And the series of hierarchical reliability provides the criteria for structural damage detection. By the way, according to the risk analysis, concerning the cost of labor, material etc. and referring to opportunity cost at different fayers, the total cost of maintenance design is reduced (18.16% for the case study of group pile foundation), by comparing the optimal cost to the randomly chosen cost. Based on measurement for interaction filed analysis, this research starts the hierarchical risk analysis for structural health assessment and damage detection within hierarchical interaction processing (in interaction filed analysis) for maintenance decision-making.

Finally, in chapter 6, it concludes the thesis by a summary of the main findings, the limitation of the recent research, and some possible future works.

Contents

1. Research Background

In the service life of infrastructure, kinds of damages appear and develop. On one aspect, the damage mainly occurs due to overload or fatigue[1]. The structure may also suffer the damages for the reason of environmental changes like the temperature[2], weather[3], and humidity[4], speed/depth of water[5,6]. Furthermore, the corrosion[7], rust[8], scour[9]… will aggravate the damage to the structure. On the other aspect, there are human factors like (a) oversights in the process of design, (b) severe service environment, (c) low-level construction/operation management, (d) improper maintenance, etc. And moreover, disasters like earthquakes or cyclones happening unexpectedly in lifetime of structure. Among kinds of infrastructures, the pre-stressed concrete bridges are ones widely used and used to be specifically concerned. From some surveys of bridges health[10,11,12] in Japan (Fig.1.1) a (Fig.1.2), damages exceedingly deteriorate structures forming microcosmical or local damage accumulation; then from micro to macro, it eventually and suddenly emerges to global damage as a social accident[13]. Actually, this problem is faced not only by Japan and the USA but also by almost all countries in the world. So, to guarantee the forecasting accuracy of the damage development and to avoid some sudden disasters, many diagnosis methods have been proposed for damage detection and health monitoring[14] to analyze or recognize the structural health status and play key roles in forecasting the lifespan and guaranteeing the safety of the infrastructures[15].

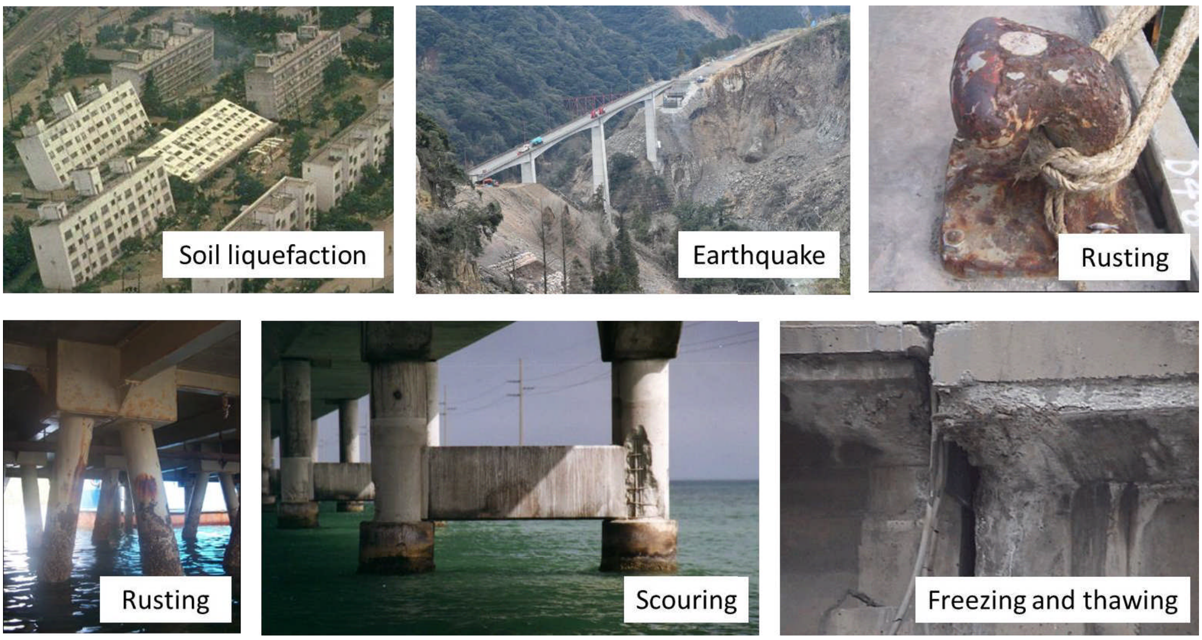


Figure 1.1. Kinds of damage caused by the soil liquefaction, earthquake, rusting, scouring, freezing and thawing, etc. in Japan.

Figure 1.2. Types of damage form to the beam-like structure surveyed in the USA.

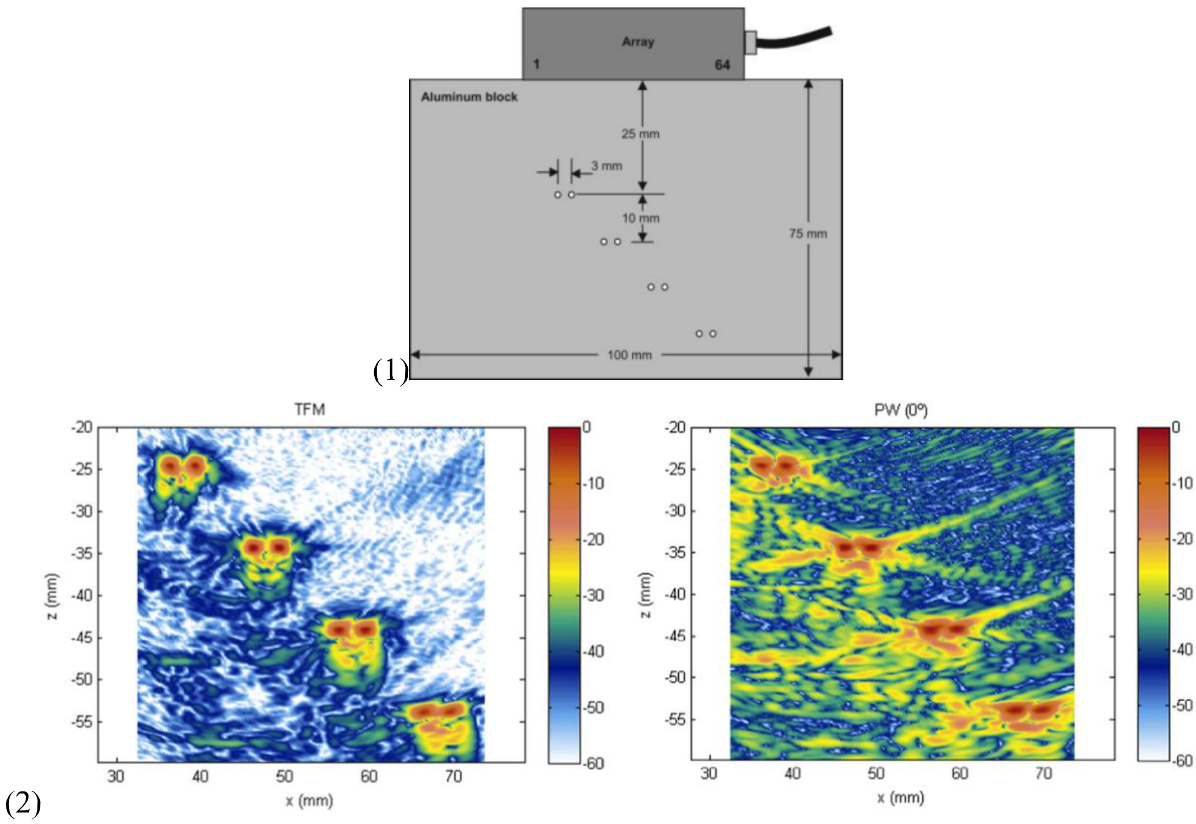


Figure 1.3. (1) Array in contact with an aluminum block with pairs of 1.5 mm diameter side-drilled holes (2) Imaging with a 64-elements array. Left: TFM image. Right: PWI with single emission at 0°. From the subfigure (2), we can clearly find that the color of locations with artificial damage (drilled holes) a

In terms of damage detection, usually, four problems are taken into consideration: the existence of damage, localization of damage, the evaluation of damage, and the prediction of residual load capacity[16]. The scales of detection can be divided into two sub-categories, local damage detection, and global damage detection[17]. When caring for the timeliness of detection, it can also be divided into online[18] and offline[19].

Among methods of damage detection, there are destructive examination (DE) a

(NDE)[20,21]. In recent years, decentralized structural health monitoring (SHM) is introduced into civil engineering[22] to detect the damage accompanying by NDE methods, like Ultrasonic Testing (UT) or named as Automated ultrasonic (AU), Radiographic Testing (RT), Infrared Thermography (IT), and Acoustic Emission (AE), etc. An example of the Automated Ultrasonic (AU) is introduced in Fig.1.3[23].

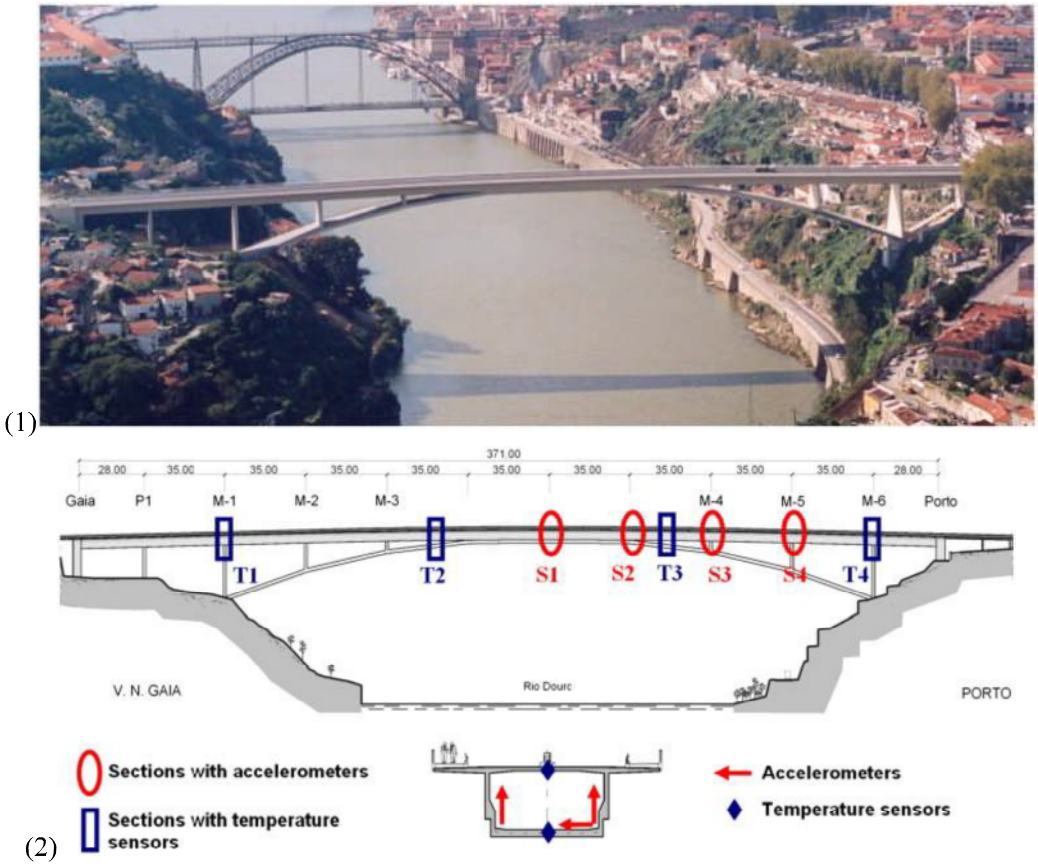


Figure 1.4 (1) The *Infante D. Henrique Bridge*, and (2) The position of the accelerometers and temperature sensors for health monitoring

Generally, beyond the scheduled or nonscheduled damage detection, the long-term damage assessment system or health monitoring system is very necessary, as it provides the original analyzing data in further structural evaluation in practice. For example, a structural health monitoring system for the *Infante D. Henrique Bridge* can be found in Fig.1.4[24]. Often, a bridge contains five parts (1) span structure, (2) support system, (3) pier and abutment, (4) cap, (5) well/pile foundation. Correspondingly, the damage assessment system or health monitoring system will be set at such locations. When doing damage detection and evaluation, these parts, especially the span structure and the foundation, are often specifically concerned. For this reason, we mainly concern about the girder used in span structure and the pile foundation in this research.

At present, the health assessment of the structure based on the data obtained by the monitoring systems or through other detection equipment is mainly focused on two aspects (1) Damage assessment, which mainly includes determining the existence of damage, the location of damage, and the extent of damage, for further carrying capacity. The evaluation lays the foundation; (2) Status assessment, which is based on the damage assessment, and comprehensively assesses the current working status of the structure in order to guide the daily maintenance and repair work.

A bridge health monitoring system usually employs multi-sensors at the key parts of the bridge to measure the response of the structure, stress, displacement, and dynamic characteristics, as well as temperature, humidity, etc. Then, the test data are transmitted to the central control system, and the health status of the bridge structure is evaluated in real/delayed time according to the predetermined evaluation method, and provide some guidelines for the decision making of maintenance & operation of the bridge. Meanwhile, the key part of the bridge is beam and theories of Timoshenko Beam[25], Euler-Bernoulli Beam[26] are very famous, besides the damage detection, as well as the health monitoring for beam-like structure, shall be specially taken into consideration with great determination from social society.

According to the above substantive definition, the structural health monitoring system is to diagnose the conditions and environmental factors that may lead the structural damages or a disaster, and to assess the signs and trends of structural performance degradation so that the measures for maintenance can be taken properly. The traditional detection methods along the bridge health monitoring system could indicate the development of damage, emanate the early warning, and emphasize the existence of the damage for the maintenance and reinforcement.

In a global structural system, 3 parts, structure, environment, and their interactions are often concerned. To understand the information of this global structural system, three aspects of information should be taken into consideration as well, structural information, environmental information, and information of interaction. In Fig.1.5, the structural information is described as the information on the structural inherent features/nature of the structure, the environmental information is the information to describe the environment around the structure, and the interaction information (interaction field information, when describing the interaction in the language of filed) is to describe the interaction between the structure and its environment, i.e. the motion propagating through the structure and its environment. In many instances, the vast majority of previous studies are more likely to concentrate on the structure itself to investigate the information like the resistance (as a distribution), natural frequency, damping ratio, etc. The stability of structural information is one of the most common and basic definitions in civil engineering.

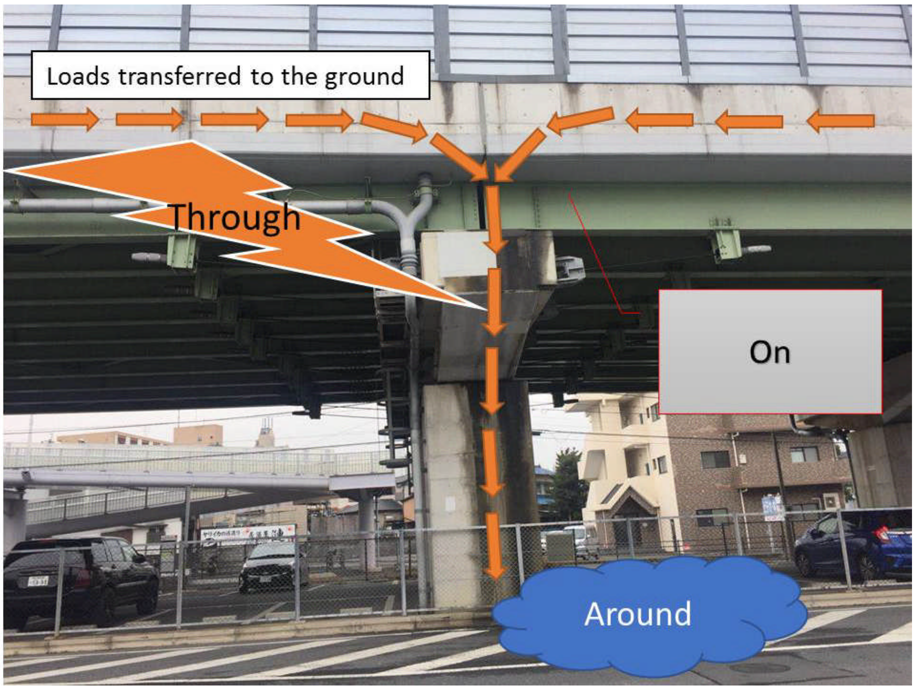


Figure 1.5. A typical continuous beam bridge. In the sense of space, there are three kinds of information for a structural system: the information on the structure system (inherent nature of the structure), information around the structure system (environment), and the information through the structure system (Interaction between structure and environment).

Though the research on the variation of environmental information is relatively scanty, it still attracts the attention of many researchers in ocean engineering, coastal engineering, environmental engineering, and research of corrosion, rust, weathering, etc. In damage detection, the research of environmental information often concerns the environmental impact on the structures or structural components under different environmental conditions and various structures/components. The variation of the environment is to survey the environmental characteristics to investigate its stability when interacting with the structures, i.e. the severity of the environment and the ability of the environment to regulate itself. In details, the activities of humans or other living organisms, or other natural factors, can cause environmental damage and various environmental information will change accordingly. On the one hand, environmental stability is about whether an environment can adapt it to a structure or human survival within a controlled range. On the other hand, it is about its ability to dissipate energy during earthquakes, typhoons, etc., to demonstrate whether it can effectively recover itself to reduce the impact on the structure and people after these natural factors. This kind of research often measures the environmental impact through the stability analysis of batch structures or components under different environmental conditions and indirectly determines the stability of environmental information.

The interaction information is a link between structural information and environmental information. The stationarity of interaction filed characteristics indicates the stability of the continuous interaction and in addition, it will directly determine the stability of the interaction or interaction field. In general, for a specific structure, often its basic environment is fixed and, in most cases, there is no need to discuss environmental stability. Moreover, there has been a lot of research discussing the ability of the structure to sustain the change of environment and it is the mainstream of reliability or the research of system equilibria (force, energy, etc.) nowadays. However, the research on the variation of interaction information has not yet been highlighted and expressed comprehensively, and interaction information even has not been treated as a separate piece of information besides structural information and environmental information. Based on the different sensitivities of the two, it can more effectively understand the changes of the structure itself, which provides a new way of thinking for the refinement of structural assessments, facilitating people to better control and maintain structural details.

2. Literature Review and Research Objectives

2.1 literature review

2.1.1 The Axioms of structural health monitoring

To have damage detection for the structures, we must refer to the Axioms of the structural health monitoring which are proposed by Worden, K. and his colleagues[27] firstly that:

Axiom 1: All materials have internal damage;

Axiom 2: The assessment of damage requires a comparison of the two states of the system;

Axiom 3: Non-reference studies can be used to determine the presence and location of damage, but a reference study model is needed to determine the type and extent of the damage.

Axiom 4a: The sensors alone cannot measure damage, and feature extraction in the statistical classification of data processing can convert sensor acquisition data into damage information.

Axiom 4b: in the absence of intelligent feature extraction methods, the test method is more sensitive to damage, the influence of operation and environmental factors on test results is greater.

Axiom 5: The length- and time-scales of the onset and development of damage dictate the properties required by the structural health monitoring sensor system;

Axiom 6: There is a trade-off between the sensitivity of the algorithm to damage and the ability to resist noise interference;

Axiom 7: The size of damage that can be measured by changes in the dynamic response of the system is inversely proportional to the size of the frequency range that can be excited.

According to these 7 axioms, influenced by the specific environment, any structure will have damage in different scales, and the damage evaluation with a standard will reflect the real situation compared with the ideal health condition of the structure.

2.1.2 The NDE techniques in structural damage detection

Right now, the NDE techniques are widely used in damage detection worldwide. Here, some common methods are introduced. As introduced in the research background, there are Ultrasonic Testing (UT), including Automated Ultrasonic (AU), Radiographic Testing (RT), Infrared Thermography (IT), and Acoustic Emission (AE), etc.

1) Radiographic Testing (RT)[28]

Radiography testing is often used to obtain a permanent image of the surface discontinuities at different periods in structural service life by comparing the changes in size and shape of these discontinuities. Common applications can be found in castings, forgings, corrosion mapping, reinforcing materials, etc. On the contrary, digital/computed radiography, and film radiography are the most often used test methods, especially the digital/computed radiography, which can be stored and compared faster and with higher accuracy.

2) Infrared Thermography (IT)[29]

Infrared thermography can be any equipment or method used to detect the energy inferring which is emitted from objects. It displays the image of temperature distribution. Accurately and differently to name the equipment and method, the equipment is called infrared thermograph and the method is called infrared thermography.

3) Ultrasonic Testing (UT)[30]

UT is a group of NDE techniques based on the ultrasonic wave propagation in the materials or structures. It utilizes some short ultrasonic pulse-wave with 0.1~15 MHz center frequencies and transmitted into materials occasionally to 50 MHz, to identify the wave characteristics in such materials. For example, ultrasonic thickness measurement is often tested to monitor some corrosion of the pipe. Automated ultrasonic testing usually incorporates computer software which aiding detecting discontinuities and reduces the inspection time in applications[31]. An example of the Automated Ultrasonic (AU) is introduced in Fig.1.3.

4) Acoustic Emission (AE)[32]

Acoustic emission (AE) means some phenomena of radiation of acoustic (elastic) waves occurring during the processes of mechanical loading in solid (structure), leading as a result of crack formation or plastic deformation of material aging, temperature gradients, etc. Particularly, AE is regarded as the consequence of structural changes generated when the accumulated elastic energy inside the structure or at the surface of the structure is released rapidly, that indicating the location of damage, and characterizing the acoustic when applied in SHM.

The comparison between technique 1, 2 and technique 3, 4 can be found in Table 2.1.

In infrastructural engineering, among kinds of damage detection methods, besides the force-related parameters, the vibrational parameters are often measured, and those characteristics, like the intensity, the frequency, etc. are often concerned simultaneously. For example, the hitting/tapping experiments produce some man-made stress waves. It generates mechanical stress on the structure by mechanical action, then measures the stress wave inside or on the surface of the structure through the deployed sensor, and determines the safety of the structure through certain technical means such as modal analysis at last.

Table 2.1 The comparison between RT, IT and UT, AE on some basic information.

Aspects of comparison	**RT, IT**	**UT, AE**
Basic introduction	Radon transform; Tomographic measurement; Reconstruction, one kind of algorithm.	Acoustic embryology; The phenomenon of radiation of acoustic/elastic waves.
Diagnosis Target	Mainly Local	Mainly Local
Experiment Subject	Structural characteristics	Structural characteristics
Experiment method	Indirectly, Deconstructing system changes and locate the damages	Directly, Locating the change of the system
Data using	Posterior (after the event)	Real-time ((very near the event))
Evaluation information	Filed of velocity etc.	AE event (time-series)
Computational complexity	high	low
Calculation indicator	Velocity filed	Frequency variation, b-value
Geometric parameter	Local information is needed	Local information may be needed

2.1.3 The data processing methods for signal processing in damage detection

Firstly, let me introduce the data space used for data collection in the experiment analysis in the thesis. In a data space, the selected mathematical objects are treated as points related to each other. The modern mathematics has proposed types of spaces[33], including Euclidean spaces, affine spaces, probability spaces, linear spaces, Hilbert spaces, Banach spaces, and topological spaces, etc. and these spaces will be used in various disciplines in mathematics and other departments. The data processing in damage detection usually concerns the general space, and also sometimes, the subspace, the subset of the space. If we treat the general space as “1”, and set the subspace as a “fraction” (in measure theory [0, 1]), maybe we will ask whether there is a need to have other positive integers (greater than 1) involved in the research. That’s why we want to propose a new notation, “combination space”, inspired by the subspace. Mathematically, a subspace refers to a partial space whose dimensions are smaller than the full space. As the name suggests, the combination space is the combination of the spaces, defined as the expansion of the set or the synthesis of sets by arithmetic operation, logical operation, or other operations. From a certain point of view, the combination can be treated as space mapping in topological space as well.

The general data processing of damage detection or health monitoring has the same framework as information identification in any data space which can be divided into four sub-processes. They are (1) practicality assessment, (2) data acquisition and purification, (3) feature extraction and data compression, (4) development of statistical models[34,35].

Let's return to the SHM, the basic problem of SHM is to identify the current state of the structure based on the dynamic response of the structure in health monitoring. The state of the structure could often be described by structural modal parameters (mainly on natural frequencies and modes) a (mainly stiffness parameters) in some specific data space. In this paper, they are both called the characteristics of the structure. The physical parameters of the structure are the direct expression of the state of the structural performance, and the parameters that need to be applied directly to the structural evaluation. Structural modal parameters are functions of physical parameters, which can directly reflect the changes of structural physical properties qualitatively or quantitatively. Therefore, modal parameter identification and physical parameter identification of structures are important parts of structural health monitoring for traditional damage detection.

Any structure can be regarded as a mechanical system consisting of physical parameters such as stiffness, mass, damping, etc. Once a structure is damaged, it will cause structural status to have a change and thus change the static/dynamic characteristics of the structure. The change in dynamic characteristics can be measured by a dynamic test. The vibration-based damage identification technology is based on the above principles and uses the changes in the dynamic characteristics of the measurement structure to diagnose structural damage.

2.1.4 Modal analysis

In traditional nondestructive examination (NDE), the modal parameters, like natural frequency, damping ratio, modal assurance criterion (MAC) or its modified version coordinate modal assurance criterion (COMAC) of the mode associated with each natural frequency[36], are often used in damage identification.

There are many kinds of damage detection methods in modal analysis, such as A. damage identification method based on natural frequency, B. damage identification method based on mode shape, C. damage identification method based on stiffness matrix and flexibility matrix, as well as damage identification method based on displacement and strain parameters, damage identification method based on artificial intelligence, wavelet-based analysis damage identification methods, and statistical damage identification methods, etc.

Here we choose the three most common methods of identification.

A. Damage identification method based on natural frequency and damping variation

Natural frequency variation is the most common indicator and has a long history to detect the structural damage; also, the natural frequency is one of the most easily obtained parameters in the structural modal analysis, and its test accuracy is high. Damage to the structure will result in a decrease in frequency, which directly promotes the application of frequency-dependent sensitive parameters in structural damage identification[37].

However, there are some shortages of this method. (1) Natural frequencies are sometimes insensitive to early damage to the structure, and often, only damage is found, and the location of damage cannot be determined. This is because damage at different locations may cause the same frequency change. (2) While the damage location is in the high-stress region of the structure, it is more reliable to use the change of the natural frequency for damage identification, but when the damage location is in the low-stress region of the structure, the damage identification cannot be performed by the change of the natural frequency. (3) As the degree of damage compared with the early stage of the structure decreases, the natural frequency shifts from low to high order, while changes of the high-order natural frequency are difficult to obtain. Therefore, the change of the natural frequency cannot identify the small damage in the structure. (4) Frequency is a global quantity of structural characteristics. It is insensitive to the local damage of the structure, and the use of frequency as a sensitive parameter cannot identify the damage of the symmetrical position of the structure. Therefore, if the structural frequency is used to identify the structural damage alone, a larger identification error will appear.

Also, pieces of research concern the damage-sensitive parameter of the damping ratio[38,39] corresponding to some specific natural frequency nowadays in the modal analysis. When using the damping for damage detection, people often collect and analyze some free vibration signals[40], especially when using different damping values for multi natural frequencies[41]. Nonetheless, there are concerns due to fluctuations of damping ratio in experimentally identified modal in vibration-based SHM that the precision of the damping ratio for damage detection is still a problem.

B. Damage identification method based on the change of vibration mode

Although the test accuracy of the mode shape is lower than the natural frequency, the mode shape contains more information about the structural state. Compared with the natural frequency, the change of mode shape is more sensitive to damage, and the location of the damage can easily be determined by this method. Many scholars proposed some related parameters such as MAC, MSF, COMAC, COMFS, and curvature modes on the basis of mode shapes, and developed a variety of damage identification methods based on mode shape changes such as modal confidence method, modality Orthogonal method, mode change method, etc.

However, studies have found that the use of numerical simulation and experimental methods to study the modal parameters of the specimen is due to damage changes. Also, the conclusion is that the MAC value is not sensitive to the emergence of the damage[42], thus although the MAC shows a regular decrease with increasing damage, the errors due to experiments and signal processing are often more pronounced than those caused by damage.

C. Damage detection technology based on structural stiffness matrix and flexibility matrix.

Generally, when the structure has damage, the stiffness matrix provides more information than the mass matrix. When large damage occurs in the structure, its stiffness will change significantly. Therefore, many scholars use the change of the stiffness matrix to study the damage. Structural damage will increase the flexibility of the structure. Therefore, the change of the structural flexibility matrix can be used as a detection indicator of damage[43].

For the disadvantages of this method. On the one hand, because the higher-order modes of the structure are difficult to accurately obtain in the test, it is difficult to obtain a more accurate structural stiffness matrix; on the other hand, the flexibility matrix is insensitive to changes in the structural quality relative to the stiffness matrix[44]. Since the elements of the structural flexibility matrix are few, the damage judgment using it directly is easily interfered with by the test noise. For modal analysis as a whole, there are also many disadvantages to the existing modal parameter identification methods.

a) The unknown excitation is generally assumed to be a stationary random excitation (w), and the actual engineering structure is usually subjected to a very complex unsteady signal, which cannot fully comply with the white noise assumption, making these methods limited in engineering applications.
b) For the actual engineering structure, the excitation often leads a nonlinear time-varying signal lacking adequate stability which can be recognized by naked-eyes. When the parameters are identified, the recognition results may appear unpredictable errors.

c) The response of the actual engineering structure in its environment is generally smaller and more random. Noise is an unavoidable problem. How to reduce noise or denoise needs new ideas and means.

d) Each existing identification method has its own scope of application and does not completely solve the problem of modal parameter identification in practice.

    1) The identification method in the frequency domain has the drawbacks that it is difficult to evaluate the characteristics of the unit. Then, it is difficult to consider the time-varying of the structural parameters, the accuracy is unreliable, and the identified characteristic index is not intuitive.

    2) For the time domain identification method, the accuracy of identifying low-order modal parameters is often not high, and the low-order frequency is precisely the engineering community's interest.

How to improve the measurement preciseness, and how to improve accuracy and efficiency in modal analysis to meet the needs of online monitoring, ask for further in-depth research.

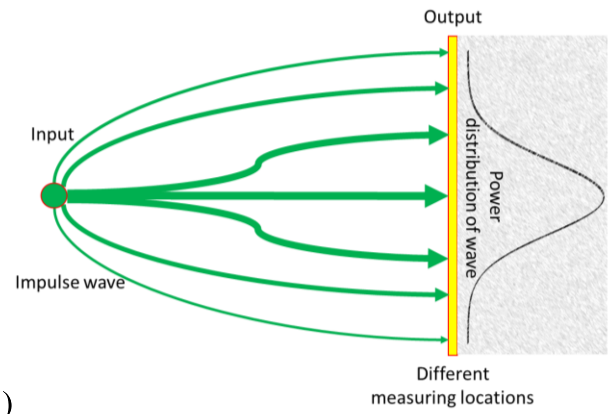
2.1.5 Inspirations from earthquake engineering and flood control project

In seismic studies, the epicenter of earthquakes, magnitude, intensity, propagation time, types of vibration waves, etc., are all critical research parameters. In order to understand the ground motions, we recognize geophysics, geological structures, fault zones, and even crack rock on slopes or mountains in the seismic process. People understand what is happening inside the earth through the waveform of seismic motion and the propagation of ground motion. In some sense, Acoustic emission (AE) used in damage detection is similar to the Earthquakes. Of course, sources of energy such as volcanic earthquakes may be more abundant. The wave transferring in the structure is similar to the seismic wave transmission in the Crust, where the characteristics of wave propagation can be sensed to detect damage. On the other hand, when we discuss disasters, we often mention the flood control (flood protection). In the process of flood development, when assessing the extent of the impact of the flood on the watercourse, the flood peak is the key parameter, and the peak movement is another key. When flood prevention work is carried out, the arrival time of the flood peak is very critical for the decision making for the managers. The movement rule of flood has different inflow volumes in different years, but it can maintain relative stability every year. The difference lays in the situation of the watercourse. The transmission time and intensity of water flow can also be different.

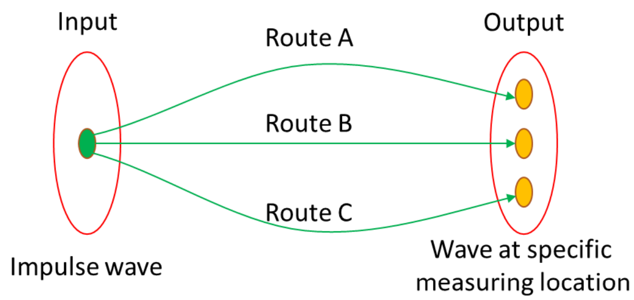
On the basis of inspiration from earthquake engineering and flood control projects, we discuss structural damage and hope to understand structural changes from the most basic propagation signals. In the measurement process, people often get time series. According to the survey in our experimental analysis, the similar indicators (max-peak, max-peak-time) of earthquake and flood can also be used to understand the structural changes in essence. Through a comprehensive analysis of the horizontal and vertical axes of the time series, the changes in the structure caused by the damage development can be quickly found through comparative analysis with the health status.

On the one side, the max-peak in the proposed method is generated by the impulse wave. The variation experiment used impact hammer to hit the box girder in different locations and recorded by the acceleration sensors in different locations. In Fig. 2.1 (1), the distribution of waves in the surface of the structure is different, then there is a distribution for different locations (which can be treated as one kind of extreme value distribution). The max-peak is related to the transmission process of the energy of the impulse wave. When there are damages in the structure, the distribution will change as well. On the other side, max-peak-time is another indicator mined from features of wave propagation for damage detection of the structure. When the damage is developing in the structure, the route of the wave will change. E.g., in Fig. 2.1 (2) at first, it is route B, then it may be route A or C when the damage of structure exacerbates.

The speed of the wave is often treated as constant. In a different route, the time used to reach the objective locations from the same location of input will be different. Moreover, the ratio between max-peak and max-peak-time will be another indicator, which is related to the max-peak and max-peak-time at the same time. It shows the speed of transfer between crest and trough as well as the cracks/voids growth in the system lifetime.



(1)



(2)

Figure 2.1. (1) The power distribution of wave for the propagation of the impulse wave in the system, (2) The wave route for different conditions of the system.

Table 2.2. Damage detection inspired by earthquake engineering and flood control.

Structural damage detection	**Earthquake engineering**	**Flood control project**
Max-peak	Earthquake magnitude	Flood crest
Max-peak-time	Arriving time of the earthquake	Arriving time of the flood crest
Structural condition	Geological conditions	River etc.
Non-Destructive	Seismic intensity	Damage evaluation
Wave route	Fracture & epicentral distance etc.	Path of the flood

When treating the interaction field as an Overall System Description, its characteristics should be studied; the interaction field characteristics (IFC) inspired by the earthquake engineering and the flood control can directly show the change of the structure with damage developing (Table 2.2) describing the overall interaction filed.

2.1.6 Hierarchy and K-out-of-N system

Concerning the interaction, the multi-scale and hierarchy in measurement must be taken into consideration. The research on the multi-scale has been studied very often[45], while the research on the hierarchy is seldom mentioned. But for discrete structural systems, or for the real world, this is quite necessary.

The hierarchical system is a fundamental concept from universe theory[46,47] and graph theory[48]. For example, "…proton, atom, and molecule…Earth-Moon system, the Solar system, and the Galaxy…" that, atoms have the protons as their own components while themselves as the components of the molecules, and the Solar system contains the Earth-Moon system while takes itself as a part of the Galaxy. Other cases like taxonomy, division of the administrative area, structural dismantling, as well as work breakdown structure, traffic network can also be categorized into the same framework of hierarchy. However, there are some misunderstandings about this concept. Hierarchy sometimes is regarded as the same idea as to scale[49] when comparing sub-system with the whole system.

However, the emphasis of the hierarchical system is focusing more on the subordination and logicality of components rather than the description of the scale. The scale can only be treated as one kind characteristic or the description of the hierarchy and the scale is often used in continuous situations. However, the scale and hierarchy often have the same system structure and the discrete system is the reality of this universe. A good selection of scale is quite important to form a reasonable hierarchy. As we all knew before, the research at different scales and system evaluation based on different scales, the results may be different.

Among many reliability problems[50,51], the K-out-of-N system is one kind of recurring system in practice, existing in a variety of engineering applications. There are two kinds of K-out-of-N system namely K-out-of-N: G[52] and K-out-of-N: F[53] , in which, the K-out-of-N: G system means at least K components surviving in the entire system of N components, while the K-out-of-N: F system requires if and only if at most K components failed for a system contains N components to fail. In this paper, the K-out-of-N system refers to K-out-of-N: F. Specially, the K-out-of-N system, when N=K=1, it turns out to be a general reliability problem.

Usually, the related studies on hierarchical reliability were unintentionally discussed. Research of K-out-of-N is usually not only focusing on the original purpose of this topic, the reliability improvement[54], efficiency[55], and repair capacity[56], but only caring about the parameter or parameter relationship between system and sub-system in dynamic changing K in the K-out-of-N system[57,58]and cases with incomplete information[59]. At the same time, research on the optimization with sub-systems to reach some high reliability of the system[60]and the K-out-of-N system in a serial-parallel style that the local systems (sub-systems) having an impact on the whole system's changing[61] is not discussed in details. Recently, some researchers tend to use the hierarchical model for analyzing the K-out-of-N system problem utilizing the artificial weighted method to solve such kind of problem[62] in engineering practice, including damage/disaster prevention and control.

2.2 Research idea, objectives and scope of the thesis

Concerning the interaction processing, in the sense of dissipative structure theory[63]for some real cases, like giant dams, super buildings, steel plant, etc. even for an ordinary girder, the motion will have dynamic states of balance and unbalance between input and output through such very likely non-linear, non-stationary system for the reason that it is highly impossible to provide a physical and chemical basis of evolution towards structures of increased complexity[64]. The examples of dissipative structure also exist in heavy machinery construction, nuclear reactors, and large span pre-stress concrete bridge…especially in those complex structures, the motion in the system is very hard to describe and represent, but it helps the structure realize new equilibrium in self-organization[65]. So, it often uses the method of black-box or grey-box with just input-output or accompanied by some special structure information, to understand the inner interactions of the system.

For structural and environmental characteristics, many researchers have done a variety of work, such as model analysis, hazard analysis[66], etc. However, for decision-making in civil engineering, generally, people will prefer to consider kinds of characteristics of structure[67] more to find indicators showing what happened and changed in the structure. The indicators of static and dynamic characteristics will be used; and they are not only of flexibility and stiffness matrix-based[68,69,70], of frequency change based[71], of damping ratio change based[72], of mode shape change based[73], or some hybrid damage detection methods[74,75,76], but also, general numerical evaluation methods[77,78,79], and some novel methods using intelligence calculation[80,81] and photographic tools/algorithms[82]. For better practice, usually, the research on characteristics in decision-making cares much more than the structural characteristics that it accounts for that a structure must stay in the environment and interacts with its environment. So, some aspects of the environment should be considered as well[83,84]. Regarding definitions, the kinds of features to describe the interaction are called interaction characteristics; and the method to analyze the change of interaction characteristic is named as the characteristics-based interaction field analysis. Once the characteristics of swarm behavior[85] of sub-structures (or swarm behavior in different locations of the structure, being similar to flocks or particle swarms.) ca

. In recent research, even some of the research more or less have the method of IFC-based involved in their researches, like AE[86], but they did not theoretically standardize this mechanism of interaction characteristics in engineering.

The first main purpose of this research is to clear the application of interaction processing in the structural damage detection and to develop a series of indicators for damage identification. Parameters used for experimental data processing inherits from general static and dynamic methods in traditional health monitoring or damage detection. In the experiment (Chapter 3), an offline case of global damage evaluation for girder with artificial damage is considered in the interaction processing of both statics and dynamics.

Moreover, the reliability for the complex system often means evaluating the capacity of risk defense to keep the eligible function[87]of the huge system which contains a large number of subsystems/modules/components with target cascading method [88]. However, since the measurement of interaction in/on the structure is hierarchical that one-time measurement is not able to determine the true state of the system that only a possibility of failure of the system is obtained. Also, for the need of safety and efficiency of the performance, functional surplus, ultra-stability, and failure-reservation are needed to be considered in the analysis. For complex systems, failure probability can be obtained by hierarchical clustering[89] with the principles of parallel and serial[90]. The hierarchical reliability has been used in various applications including circuit[91], software[92], and electric power system[93] and even in human reliability analysis[94]. However, all these former works are studies for a probability system whose structure is fixed. If the information is not fully known, model updating techniques should be introduced such as Bayesian logic[95], Bayesian networks[96]. Furthermore, in some reliability researches, the components are often treated as independent[97]. The components' failure events and interactions among these components are regarded as constant[98]. However, in order to obtain more accurate reliability value for subsequent decision-making, such intricate and sophisticated formulation of failure events should consider correlations among components[99].

So, the second main purpose of this research is to propose the hierarchical risk analysis (hierarchical reliability) for the structure in hierarchical interaction. In interaction field analysis, the damage caused by the interaction can be hierarchical, and on the other hand, the capacity of the structural resistance to the interaction, and this kind of resistance is also hierarchical. Until now, not too many studies have concentrated on the specific problem of the hierarchical system. The influence of hierarchy to the system risk assessment for the interaction distribution to the discrete situation, as well as its measurement, has not been studied yet.

2.3 Organization of thesis

This thesis is organized into 7 chapters as follows.

- As mentioned above, Chapter 1 outlines the motivation, research background, and Chapter 2 presents a concise literature review, research idea, objectives and scope of thesis for the health evaluation structure in circumstance of interactions, as well as the basic concepts of the interaction processing, the brief introduction of interaction field analysis, and hierarchical reliability of structure.
- Chapter 3 introduces the theory work of interaction field analysis as well as the interaction characteristics which describing the interaction field.
- Chapter 4 mainly applies the interaction filed analysis in damage detection, including the experiment (three almost full-size pre-stressed concrete box girders), and data processing, and damage detection. This chapter is the main part of damage detection, which introduces the data reconstruction for characteristics of the interaction field, variables of the interaction characteristics matrices, and the data processing for damage detection. Then the analysis, comparison with modal analysis, and engineering significance are discussed.
- Chapter 5 discusses the uncertainty within the measurement and structural evaluation when applying the interaction filed analysis in the circumstance of hierarchical interaction by proposing a new modal of redundancy reliability system (P-out-of-1 system) a .
- Chapter 6 concludes the thesis by a summary of the main findings, the limitation of the recent research, and lists some possible future works.

3. Method and Theory Work

3.1 Motion transferring and interaction processing

In structural statics, the interaction field analysis is a method to recognize the force (interaction processing of the varying force), or the varying displacement, etc. In structural dynamics, the interaction field analysis is a method to recognize the swarm-behavior of the waves (interaction processing of the varying waves) using different measuring types of equipment, the recorded data may be time series of displacement, acceleration, velocity, etc.).

Specifically, to simulate the interaction processing, there're three basic methods in different scales for different levels of fineness: method of Graph and method of Element (finite element method, boundary element method, etc.) responding to the macroscale, Lattice Boltzmann method[100] (LBM) responding to mesoscale, and method of Field responding to Microscale.

In general using, the accuracy of LBM method is between of graphic/element method and of field method; the graphic method is often used in the search for propagation path of interaction processing in structural systems, which have clear boundaries while element method often do not have; for higher preciseness, the method using field theory is more mathematically to describe the interaction processing in fluid mechanics, gravitation, or geomagnetism…

Here is an example of LBM to describe the interaction processing/filed in Fig.3.1, and suppose there is some damage or interspace in the structure; wherein Fig.3.1(A1~D1), there is a Line-Crack-barrier and in Fig.3.1(A2~D2) there is Round-Hole-barrier.

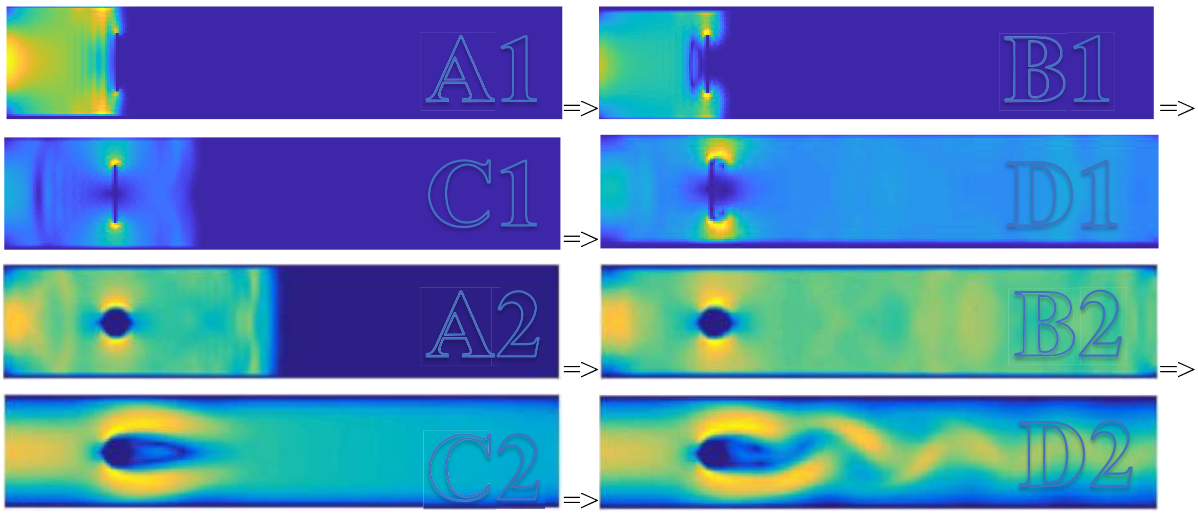


Figure 3.1. Simulation by LBM for the interaction processing in the space in different stages (A, B, C, & D), where A1, B1, C1, D1 simulate the Line-Crack-barrier, while A2, B2, C2, D2 simulate the Round-Hole-barrier. Interaction is imported into the system with a limited boundary continuously.

The interaction field is described as either the content of physical information exchanges between structure and environment (Lagrangian description[101]) or some dynamic field in mechanics (Eulerian description[102]) that the motion or interaction, transferred into the system or structure by the environmental influence. It will be flexibly applied in this research and based on common sense and research conventions; we don't have to make too many statements.

3.2 The method of characteristics-based interaction field analysis

Comprehensive theoretical research could be found on structural intensity measurements and energy conservation law[103] that the interaction processing will change when the structure changes. Also, the change of interaction processing shows the change of the system/structure in turn. In the infrastructure or any other open system[104], the structure has interactions with its environment, where the interaction processing within space distribution shows the relationships and differences among sub-systems at different locations, and the interaction processing updating in time axis shows the differences in different stages of system's lifetime. However, in engineering application, the input and output measured at both time and space are sometimes hard to obtain, especially for the overall investigation of output, and also in some systems, they are often nonlinear and nonstationary, so directly describe the interaction processing in the field to indicate the change of the system is usually impossible. Then the interaction field characteristics are suggested to approximately indicate the change in interaction processing.

Furthermore, for the detection and identification of damages, the characteristics-based method pays close attention to the damage spreading (e.g. the growth of crack), structural integrity (e.g. structural disintegration), signal propagation in the system (e.g. the power change of frequency, by fast Fourier transform), migration of equilibrium between load effects and resistance,… kinds of structural parameters used to describe the change of interaction processing in confined time or space in a limited universe model[105], such as the energy propagation or force distribution changing in a system. Sometimes in dynamics, the interaction processing often means waves.

On the other hand, traditional wave research mainly focuses on the department of communications, for example, Hertz wave in the optical fiber communications, where the frequency response, time-history response, are often applied to evaluate the information processing. To compare the wave in communication and structure, the difference of fidelity qualities for wave transmission between the fiber and the structure is how to grasp the extreme capability of variation. In the definition of the reliability of the communication system, like the optical fiber communication system, the reliability is to ensure the functionality of the system by providing high-quality signals in transmission. However, the same definition of wave reliability in the system is hard to guarantee for two reasons. First, the frequencies of waves in the system are miscellaneous and integrated. Second, the changes of waves in the structure are gigantic. So, to avoid the chaos of evaluation for different waves in different frequencies, the swarm behavior of waves in the structure is surveyed in interaction field analysis.

The interaction field analysis defined here is to evaluate the stability of transmission of waves in the system or the resistance to change of waves' swarm behavior influenced by the structure. For different systems that the waves go through, the change will be different. These systems for propagating interactions can be divided into three groups.

- Group 1, is the interactions that do not change or just have an infinitesimal change, including the optical fiber mentioned.
- Group 2, the interactions have a linear or nonlinear but stationary change, which can be described using known equations.
- Group 3, the interactions have a nonlinear and nonstationary change that cannot be described by any known equations.

For the second and third groups, the characteristics of the interaction field will have a continuous change along with the change of the waves itself.

But usually, in order to distinguish the interaction focused method from other methods, according to the description of a system, that some basic concepts like development (because of the interactions between system and its environment, among different sub-systems…), framework, components (elements, contents…), boundary, etc. will be considered[106,107], which turns out to have relationship with the damage diagnosis, for example, Appendix 1.

Concerning the division of characteristics of the interaction field, 4 categories of basic characteristics are introduced corresponding to these four concepts of the interaction processing:

- Category 1: Spread of the interaction processing, i.e. the characteristics to describe the spread of motion in the field. For kinds of interactions (described as a field), the spread of the interaction processing has many characteristics to reflect the status of structure in kinematics and kinetics according to the different descriptions (Lagrangian description and the Eulerian description).

- ⇨ Such as acceleration, speed, travel/propagation time, displacement, etc.
- Category 2: Channel of the interaction processing, i.e. the characteristics to describe the channel that the motion goes through the field. It is about the distribution of interaction processing in the system and the route of interaction processing in the system, where the natural optimal and most reasonable results can only be measured in field experiments and even the most state-of-the-a

-sectional area, etc.

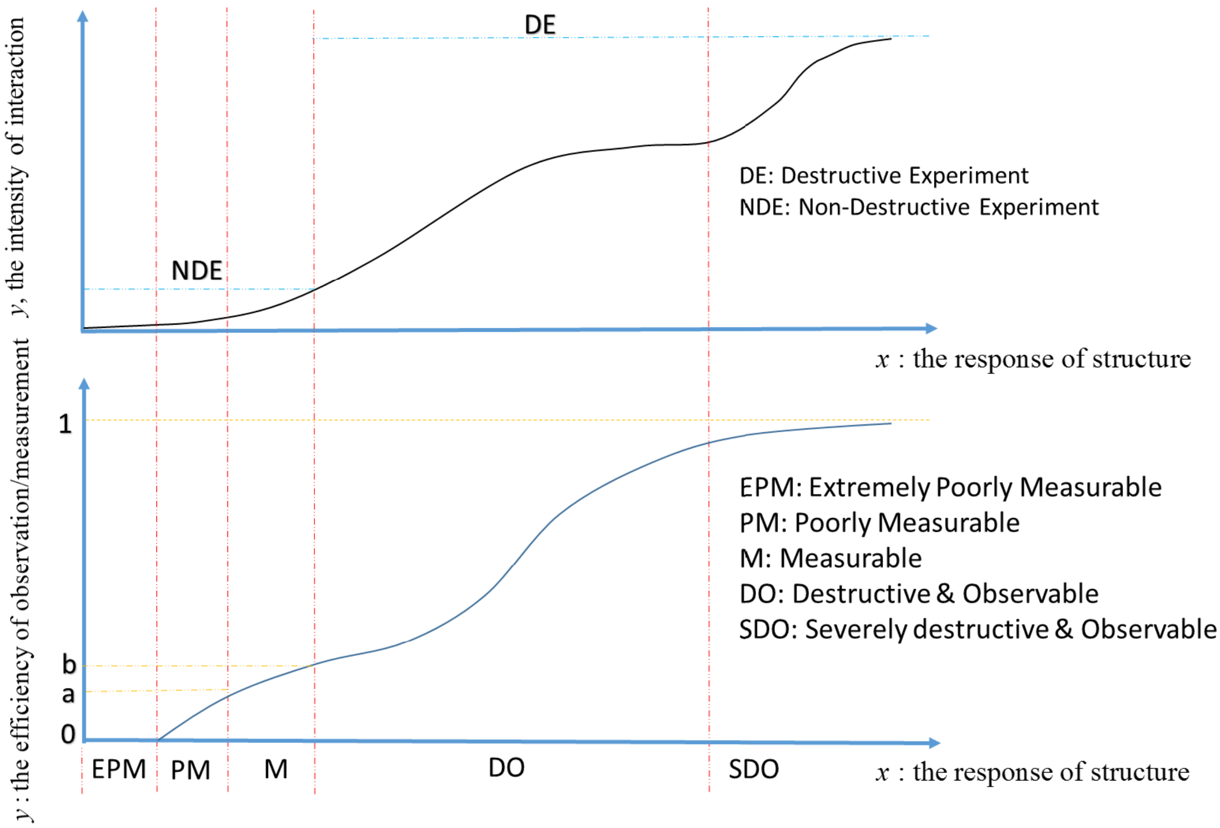
- Category 3: Amount of the interaction processing, i.e. the characteristics to describe the amount of interaction intensity in the field. The evaluation of the interaction amount can tell people the changes in flow propagation between carriers or interaction units. The amount of interaction processing may be measured at different locations (local interaction) a

(entirety) of the interaction filed.

- ⇨ Such as mass, pressure, quantity, intensity, strength, etc.
- Category 4: Expansion of the interaction processing, i.e. the characteristics to describe the expansion of the interaction processing in a specific system or systems without clear boundaries. There is an assumption that, in nature, any interaction cannot exist forever. It must have a life cycle or space boundary, carrying this interaction from its creation to its demise. In addition, under pure inertial conditions, interaction is still considered a system or the desired capability.
- ⇨ Such as lifetime, propagation region (depth, width, or boundary).

Usually, for a specific system, if the motion can stably go through the system, these characteristics will keep the same; in space view, the energy of interaction from input to output should be consistent; in time view, the speed of motion also should keep equivalent between input and output; for the channel of interaction processing, the basic topology & cross-sectional area should keep the same if the motion is stable; and moreover, the amount, lifetime, but also boundary, etc. of interaction's processing in the system should be the same synchronously.

The interaction field has four categories of characteristics, but their principles, standards, methods, ease, convenience, and accuracy… of measurement may have some differences. Of all related reasons, the intensity of interaction is one simple division standard (other methods may concern the quality of intermedia or the damping of interaction processing in the intermedia, etc.). As the Fig.3.2 shows, when the interaction processing's intensity increases, the structural response which directly influences the observability and measurability, the structural response will be more easily observed and measured that the efficiency of observation/measurement will increase, and the damage detection will also transition from NDE to DE. From small to great, the response of a structure can be divided into EPM (extremely poorly measurable), PM (poorly measurable), M (measurable), DO (destructive & observable), and SDO (severely destructive & observable). In



.2. The relationship of a possible diagrammatic sketch between the intensity of interaction processing and the structural response which directly influences the observability and measurability, or in other words, the efficiency.

In general understanding of Fig.3.2, for the DO and SDO, the response of the structure to the interaction processing can be linear and even exponential level, in such cases, the efficiency of measurement/observation can be much higher and even near 1 that the people can directly notice the state change of structure with minimal error in observation. While for the M and the PM, the response of a structure is much worse than linear, even the level of rooting or logarithm, that cannot be observed by the naked eye directly, and how to define the exact value or the value of boundary efficiency a, b is depending on the specific measurement or different standard. For the EPM, the efficiency often equals 0, and the general tools of measurement are useless. In fact, the increased accuracy, recognition and sensitivity of the experimental equipment will further expand the range of the observable and measurable. The method of acoustic emission is an example used in the EPM. However, in our general sense, it still belongs to the EPM.

Also, for the cases of the DO and the SDO, the change of the response can be directly measured, but for more information, the interaction processing will be taken into consideration. Also, of all kinds, for the case of the PM, the interaction processing will be much important that it can help to detect more changes in the structure. So, in the design of the experiment, there are two aspects should be specially considered. On one hand, for the reason of the interaction, the structural damage will be clearly observed along with the intensity of interaction processing increasing. On the other hand, some levels of interactions can be used to detect the change or the damage of the structure independently. In the following chapters, for the static loading experiment (of the DO and the SDO), the response of the structure is displacement, and for the impact hammer experiment, or named as vibration experiment or impact tests (of the PM), the response of the structure is acceleration.

Explanation of the interaction field characteristics.

For the intensity of distribution (IoD) (Fig.3.3 a&b), it means the displacement distribution in different locations for one step, in the analysis of the actual practical using, the reference step can be the IoD corresponding to the load of 1/50~1/10 design load. In the impact hammer experiment, examples are two characteristics when analyzing the data of acceleration(Fig.3.3.c)：

- Max-peak-time is from the first class of interaction field characteristics, inspired by flood control and earthquake engineering. From the acceleration time series, it means the arrival time of "flood" peak of acceleration data that the time from the time of hit (contact between hammer and structure) to the time of the max peak of every sensor's detection, in the research, we choose the positive maximum value.

- Max-peak is from the third class of interaction field characteristics. The max value is the max peak of the time series. It means the max value of the intensity of "flood", in this research, it means the positive maximum value.

There, the impact hammer experiment will be conducted for more than ten times at every hit point. After times of hit (Buffon's needle test[108]), there is a time distribution for those characteristics.

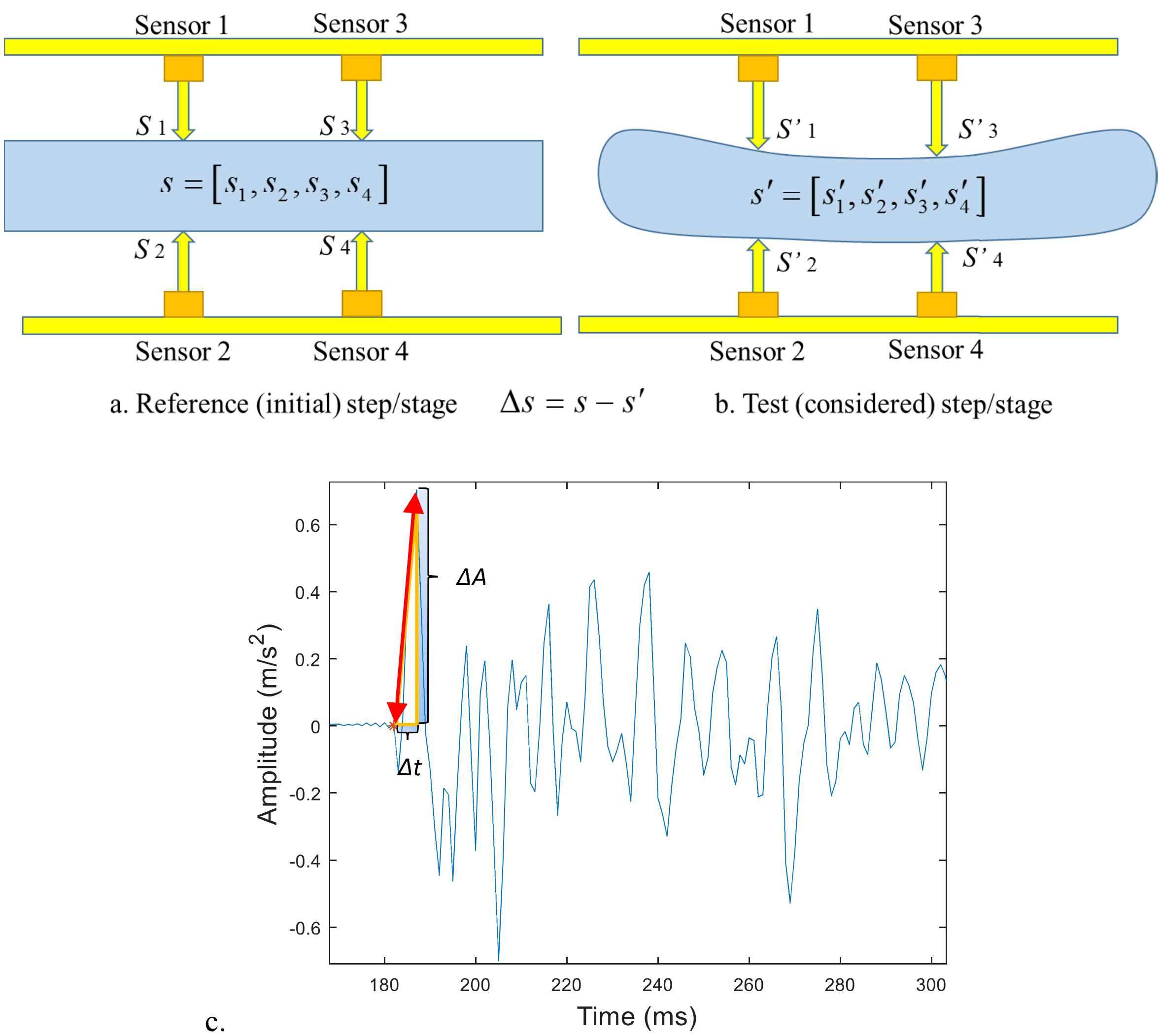


Figure 3.3. An example of the Intensity of Distribution (IoD) in displacement flow in (a) initial reference stage, s, and (b) test stage, s', and the change of the IoD is △s. In (c), as an example, it introduces The max-peak (Amplitude-a) a -peak-time (time-a), the maximum value and the arriving time for one response time series.

In summary, in this chapter, the definition of interaction field analysis was proposed. In the method of interaction field analysis, kinds of characteristics of interaction processing (interaction filed) a

4. Damage Detection by Recognizing Characteristics of Interaction Field

4.1 Experimental introduction

Since this research is to identify the structural healthy condition, as a need or for the convenience in designing the experiment, the process of adding structural damage is preferably increasing, or the structural residual capacity of resistance is preferably decreasing.

The whole experiment contains two sub-experiments, the static loading experiment, and the impact hammer experiment. In traditional static research, the relationship like stress-strain curve has been tested and verified by many researchers, and the change of displacement shows the response of the structure in the loading process. Also, in dynamics, model analysis shows that, one lower order natural frequency of the structure will approximately decrease[109] more or less at a high probability, while the damping ratio of this frequency will usually increase[110] when there are some damages that happened at a high probability on the contrary.

In this specific experiment design/conduction, there are 3 aspects considered, IoD, max-peak, and max-peak-time. And the structural characteristics in comparison for these researches include displacement, frequency, damping ratio, etc.

First, in order to get the necessary data for recognition of interaction field characteristics, and be convenient to compare interaction field analysis and modal analysis, the survey paid attention to the common and same data as other research, like loading (kN), displacement (mm) in static loading experiment, acceleration (m/s^2) time series in impact hammer experiment.

Second, the experiment tested 3 near full-scale pre-stressed reinforced concrete open-section box girders (in short, girder/box girder) in Public Works Research Institute (PWRI), named as box girder 1, 2, and 3 and only box girder 2 and box girder 3 (in same design in Fig. 4.1) were mainly concerned in this research. In the static loading experiment, the concentrated force was loaded in the middle of the structure. The impact hammer experiments are conducted on box girder 2 and box girder 3, both from the initial stage to the stage when there are large-scale cracks. Three girders were in the same design. The Fig.4.1, Fig.4.2 and Table 4.1 shows the example of box girder 2, 8500mm long, 2300mm wide, and 1000mm high. On this girder, there were 4 SDP-200 of displacement meter, 8 SDP-100 of displacement meter, and 10 acceleration sensors located according to the arrangement in the example of box girder 2 (Fig. 4.1 and Table 4.2). A stage here means a series of tests of static loading, the whole procedure of tendon cut, times of impact hammer experiments, or a mix of them. The series of loading steps start increasing from 0 to greatest loading, then decreasing to 0 at the end.

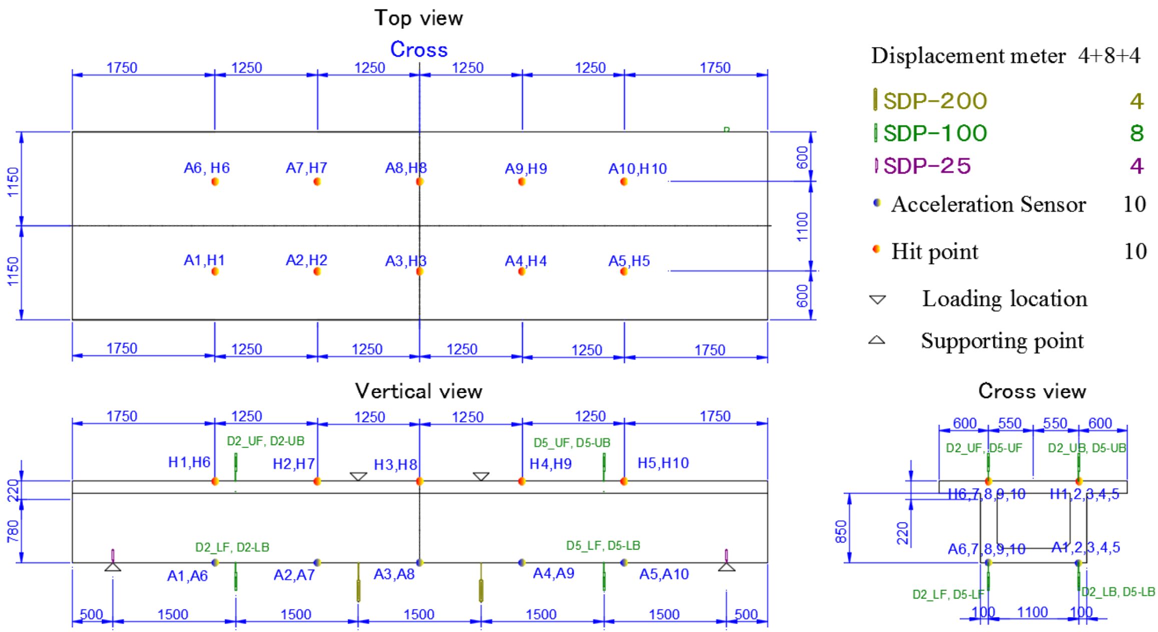


Figure 4.1. The experiment object, a prestressed box girder. The three-view drawing of the structure, and loading positions, supporting positions for the static-loading experiment, sensor locations and hit points arrangement and damage outside & inside.

Table 4.1. Loading of every stage for static loading and impact hammer experiment.

Object		**Pre-test**	**Initial load**			**Intermediate load**			**Damage load**			
Box girder 1	Stage	1	-	-	-	-	-	-	2	3	4	5
	Loading (kN)	-	-	-	-	-	-	-	971.1	1201.1	1541.2	2032.6
Box girder 2	stage	1	2	3	4	5	6	7	8	9	10	11
	Loading (kN)	-	816.1	-	-	840.1	-	-	838.4	973.9	1033.4	1427.3
Box girder 3	Stage	1	2	3	4	5	6	7	8	9	10	11
	Loading (kN)	-	804.0	-	-	776.5	-	-	816.3	980.4	1051.00	1351.7

Table 4.2. The introduction of the sensors.

Sensor	Type	Max Range	Purpose	Direction and Location	
displacement meter	SDP-200	200 mm	For Deflection	Vertical	Under the girder
	SDP-100	100 mm	For Deflection	Vertical	Both Under & Up the girder
	SDP-25	25 mm	Subsidence at the fulcrum	Vertical	The fulcrum of the girder
Acceleration sensor	type 8208	8000 m/s2	Vibration at the fulcrum	Vertical	Under the girder

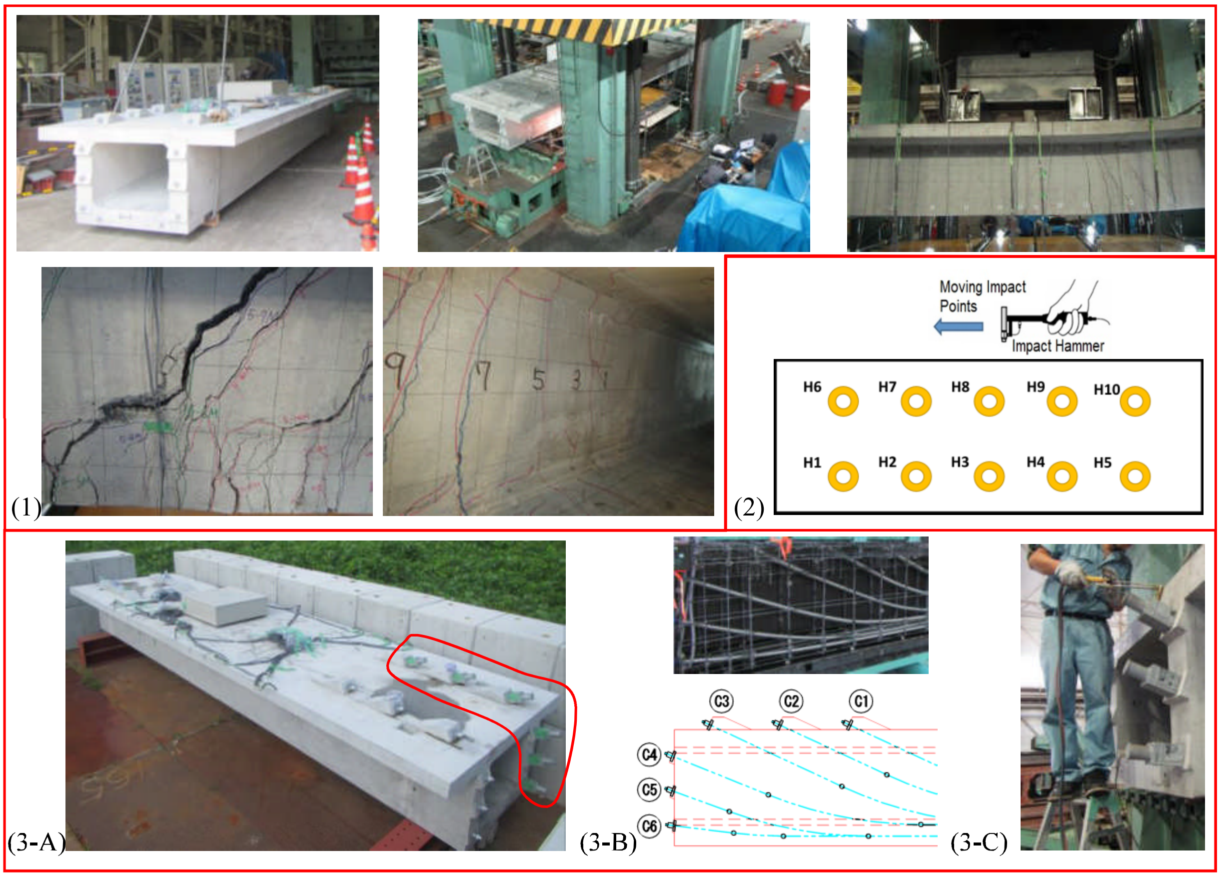


Figure 4.2. The real experiment object. (1) The almost full-scale girder for the experiment and damage after the static-loading experiment. (2) The schematic diagram of impact hammer experiment (from hit point H1 to hit point H10). (3-A) The real structure. (3-B) The tendon and the reinforcement situation of tendon inside of red line in (3-B), C1, C2, C3, C4, C5, and C6. (3-C) The real operation of tendon cut.

In the experiment, SDP-200, SDP-100, SDP-25means the displacement meter has the max range of measurement of 200 mm, 100 mm, and 25 mm respectively. The acceleration sensors, all are piezoelectric type ONOSOKKI NP-2120 with a maximum range of 8000 m/s2 and a sensitivity of 5pC/(m/s2)+/-2dB, and the impact hammer is Brüel & Kjær type 8208 with a maximum force compression of 44.4 KN and a sensitivity of 0.225 mV/N (the acceleration of the impulse is also record in the experiment). The sample rate of the acceleration sensor here is 2000. Then data is stored by the Multi-input Data Acquisition System Keyence Series NR-600.

This research analysis mainly covered 8 SDP-100 meters used in displacement measurement. The experiment contains 11 stages but a series of test steps (Fig. 4.3 for box girder 2, and the steps for box girder 3 has the same schedule but some differences in the value of force loaded). Meanwhile, the impact hammer experiment started from stage 1 (initial stage, no static loading) to stage 9 (when there were great cracks, we stopped the impact hammer experiment to protect the sensors). Every stage is a series of load increasing, for example, in the first stage for PC girder, loading from 0 kN to 30 kN in every 3 kN, and then from 30 kN to 0 kN in every 3 kN. Every step of loading, tendon-cut, and time interval after tendon-cut, it takes approximately 15 mins, 1 min, and 15 mins separately. For box girder 2 and 3, the tendon cut will be conducted at stage 3 and stage 6, and the recovery after tendon cut was at stage 4, and stage 7 corresponded for both box girder 2 and 3; Referring Fig. 4.2(3-A, B, C), for the tendon cut of box girder, only cut 6 tendons in total of same side (the first time C1, C2, C3, the second time C4, C5, C6) while for the tendon cut of box girder 3, 12 tendons of both sides were cut, that the first time cut C1, C2, C3 in one side and C1, C2, C3 in another side, the second time cut C4, C5, C6 in one side and C4, C5, C6 in another side.

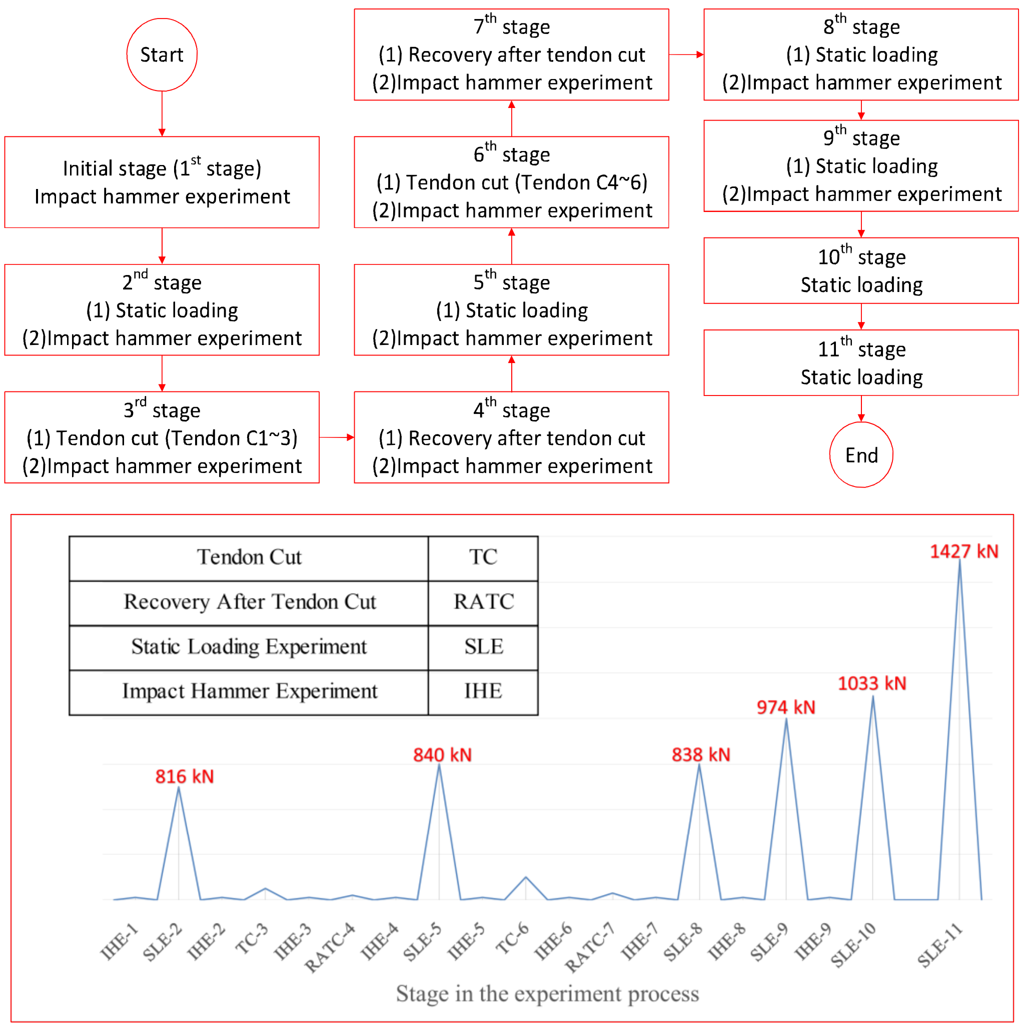


Figure 4.3. The experiment process corresponding to Table 3.1 for box girder 2.

Third, in this experiment, the hit points were designed upwards the structure, and sensors were approximately located beneath the structure. The number of hit points and sensors were equal. And the location of every hit point was just right above every corresponding sensor. All hit points were evenly configured. For better understanding, here shows the data sensed by 10 acceleration sensors (corresponding to A1~A10) in Fig.4.4.

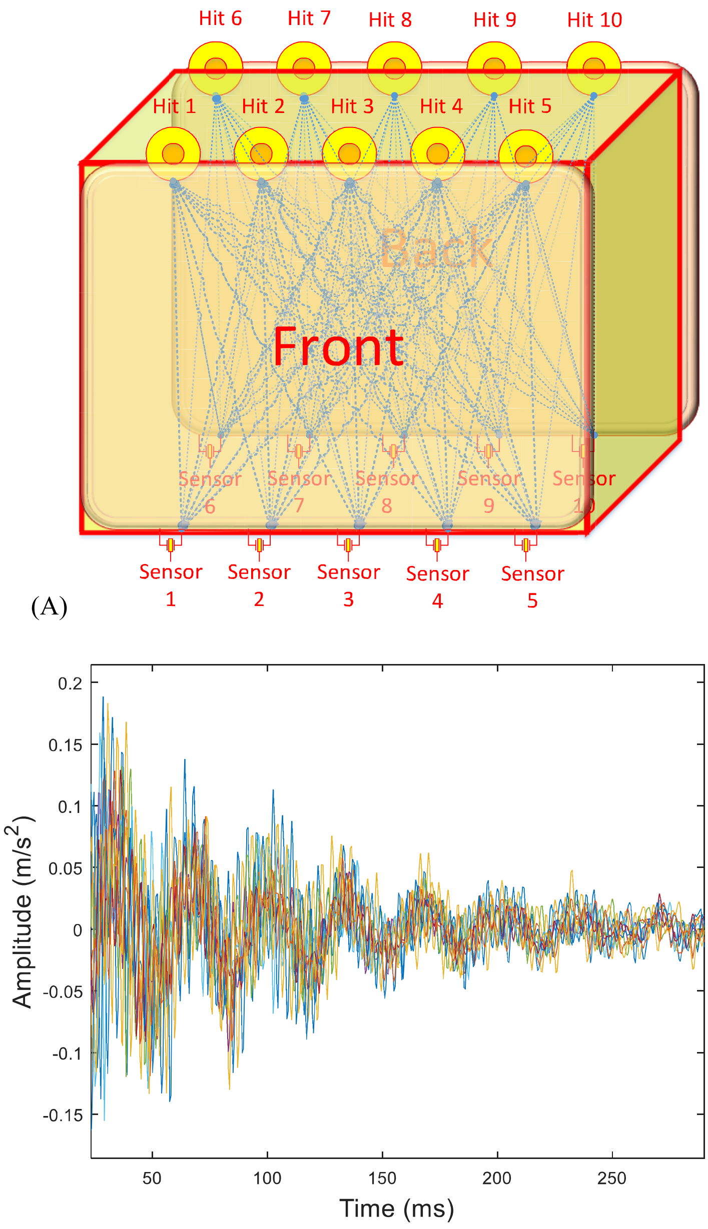


Figure 4.4. (A) Diagrammatic sketch of the 3-dimensional relationship of locations of hit points and sensors, referring Fig. 3.1. There are 10 hit points, 10 sensors. The lines between the hit point and the sensor just show the basic schematic of the relationships between the input (hitting) a (sensing). (B) An example of time series detected by 10 sensors.

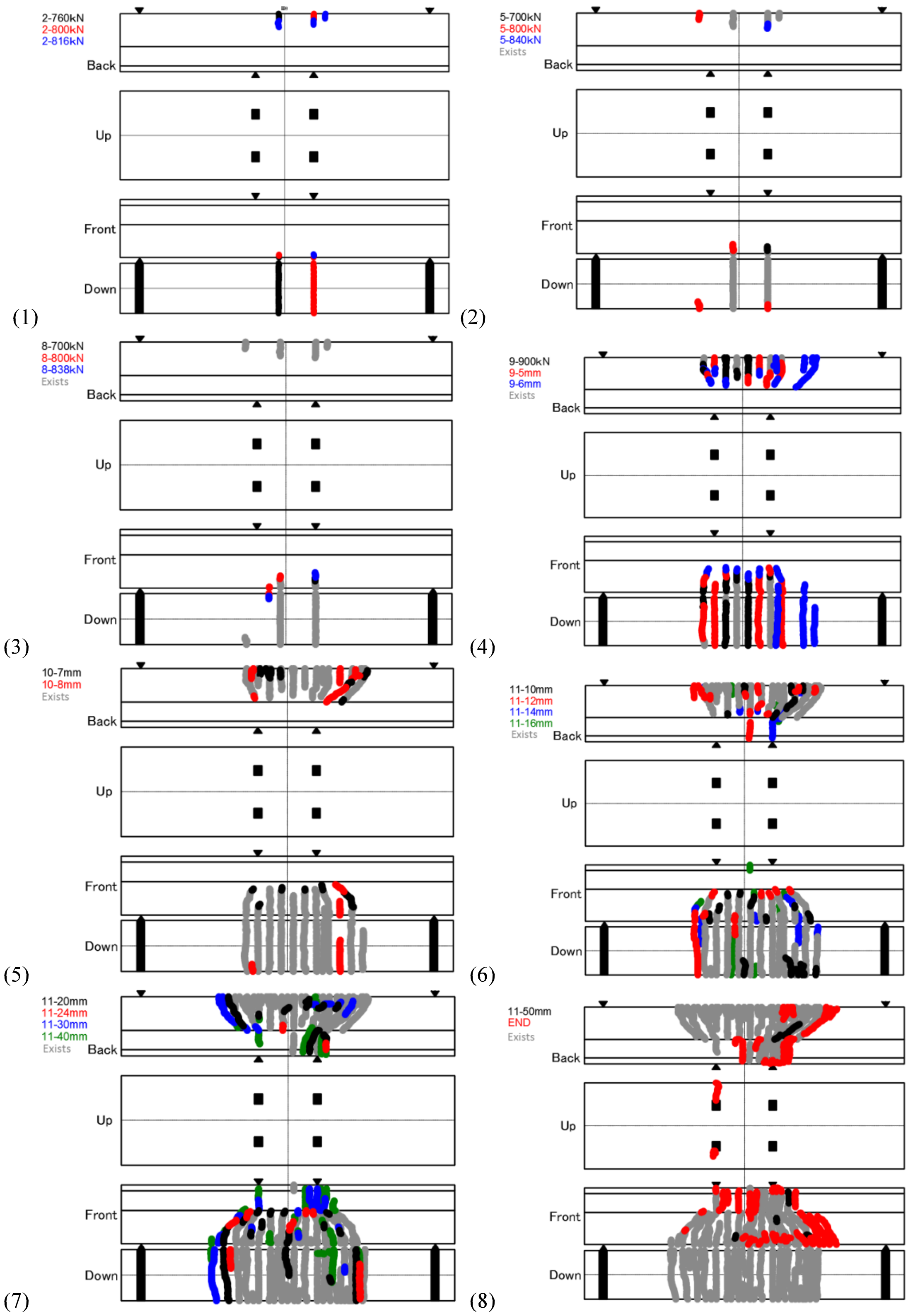


Figure 4.5. For girder 2, the subfigures show the evolution of the cracks, according to the real measurement of the cracks in the process of the experiment with box girder 2 in Table 1 at stage 2 (subfigure **1**), stage 5 (subfigure **2**), stage 8 (subfigure 3), stage 9 (subfigure **4**), stage 10 (subfigure **5**), and stage 11 (subfigure **6–8**).

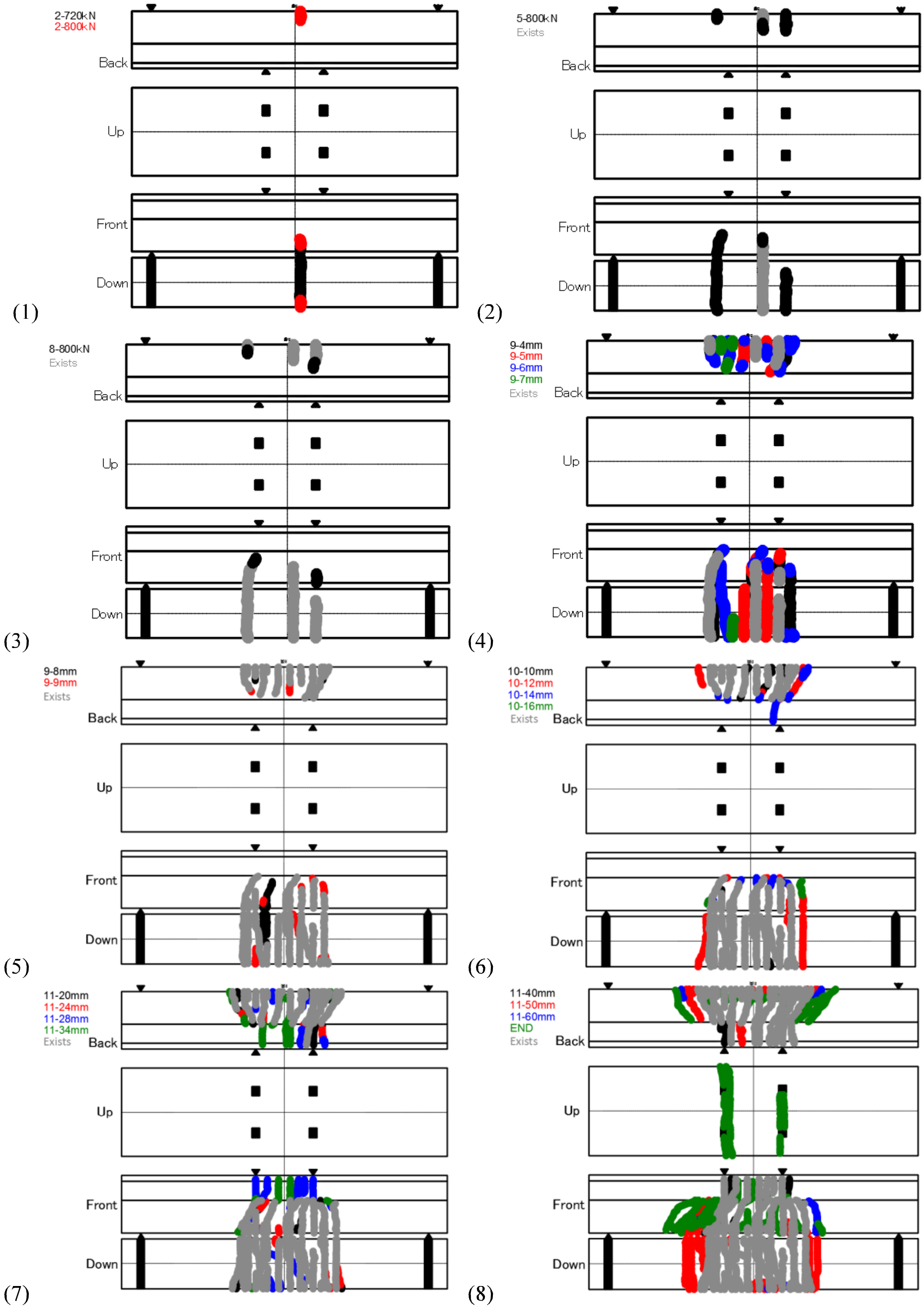


Figure 4.6. For girder 3, the subfigures show the evolution of the cracks, according to the real measurement of the cracks in the process of the experiment with box girder 3 in Table 1 at stage 2 (subfigure **1**), stage 5 (subfigure **2**), stage 8 (subfigure **3**), stage 9 (subfigure **4-5**), stage 10 (subfigure **6**), and stage 11 (subfigure **7–8**).

Fourthly, corresponding to the experiment, there are two kinds of damage: the damage caused by static loading and the damage caused by tendon cut. The crack development (Fig.4.5, the crack development in box girder 2 and Fig.4.6, the crack development in box girder 3) can be used to picture the damage development in the static loading. Note for the control of loading, there are two methods, the first, in the notation x-a kN for stage 2, stage 5, stage 8, stage 9, stage 10, and stage 11, x denotes the ID of stage and a denotes the loading magnitude at this stage. For example, 2-760 kN in subfigure 1. The second, in the notation x-b mm for stage 11, x denotes the ID of stage and b mm denotes the average displacement by meters at this stage: D3_F, D3_B, D4_F, D4_B (D: displacement, F: front, B: back).

For example, 10-7 mm in subfigure 5. Since the damage happens inside the structure, we can only know that there is damage caused by the tendon cut, but cannot observe it directly with the naked eye. But prior, we have known that the damages are developing in the structure from stage 1 to 11 (or from steps 1~995 for box girder 2, and from steps 1~1150 for box girder 3), that is to say, the health status of the structure is decreasing.

In this chapter, the experiment to test the interaction field analysis is conducted on two almost full-size girders with artificial life-cycle damages. Regarding the experiment design, four aspects are taken into consideration. Firstly, the design of the experiment was introduced. Secondly, the size of the structure and real structure were introduced and the process of experiment was proposed. Thirdly, in this experiment, the hit points were designed upwards the structure, and sensors were approximately located beneath the structure. Fourthly, the damage caused by static loading and the damage caused by tendon cut is reported.

4.2 Experimental data processing

4.2.1 Data reconstructed in combination space and variables in the calculation

In the cognition of things (including damage detection), all objects are described by different data in parallel. In the space, selected mathematical objects are treated as points, and there is one possible structure containing relationships among these points as well in this space[111]. In other words, a data space is the sum of mathematical objects and their relationships. As Pythagoras said, "all is number[112]". The (physical) space is made accessible for or occupied by data. These data can be superposed and collapsed on things in space, time and category, etc., then may use the content of (big) data can describe the laws and rules of the world[113]. From this philosophical perspective, we know that, through statistics and analysis of data, people can provide a description of the evolution of things and may find the constants in one data space by this description. Here, we are not trying to argue about the significance of numbers. However, we believe that all physical phenomena can be expressed in the data space, and the concept of data space here is not limited to data management, but more focused on the description of the objective things or the physical world.

The modern mathematics has proposed types of data spaces[114], including Euclidean spaces, affine spaces, probability spaces, linear spaces, Hilbert spaces, Banach spaces, and topological spaces, etc. as well as their subspaces. Meanwhile, these spaces will be used in various disciplines in mathematics and other departments. In our experiment, we record the data into a database, and then we analyze the data in a data space inevitably.

The data processing for damage detection usually concerns the general space, the subspace, as well as some other notations. Mathematically, a subspace refers to a partial space whose dimensions are smaller than the full space and a subspace can be a subset. In

(in measure theory, the interval can be [0, 1]), maybe we will ask whether there is a need to have other positive integers (greater than 1) involved in the research. That's the reason we want to propose a new notation, "combination (data) space", which is inspired by the subspace.

As the name of the combination space suggests, it is the combination of the spaces, which can be defined as the expansion of the set or the synthesis of sets by arithmetic operation, logical operation, or other operations. From a certain point of view, the combination can be treated as space mapping in topological form too.

Here is an assumption that, for any measurement data in structural damage assessment, it contains three kinds of information, (1) the information of the measuring object, the status of the structural health status, (2) the information of the environment, anything other than the measuring object, having relationships with the structure in the measurement, like the micro-disturbance, or noise, and (3) the information caused by interaction between measuring object and environment, like the strength and form of interaction. Kinds of information mixed together make the structural damage detection more complex. However, the information (2) & (3) a

(1) what we are most interested in is often about global information, i.e. the structural health status.

If the measurement is conducted in different locations of the measuring object, some public information (usually, we need to mine the global information among different locations) will be prominently obtained with some operation, like summation ("+"), subtraction ("-"), multiplication ("×"), and division ("÷") operations, etc. To put forward a theoretical framework and definition for the combination space for the public information.

Note that in primary research only the linear combination is recognized in the research. For a series, a vector or a series of vectors whose data describes/measures the same thing:

$$\mathbf{x} = \left\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_n\right\}, \quad \mathbf{x}_{\text{new}} = \text{operation}\left(\mathbf{x}\right) \tag{4.1}$$

For example, along with the combinatorics, the operation "+", and its inverse operation "-", the original information can be transferred into the new series,

$$\mathbf{x}_{\text{new}} = \ldots, \left\{-\mathbf{x}_i - \mathbf{x}_j - \mathbf{x}_k\right\}, \left\{-\mathbf{x}_i - \mathbf{x}_j\right\}, \left\{-\mathbf{x}_i\right\}, \left\{0\right\}, \left\{\mathbf{x}_i\right\}, \left\{\mathbf{x}_i + \mathbf{x}_j\right\}, \left\{\mathbf{x}_i + \mathbf{x}_j + \mathbf{x}_k\right\}, \ldots$$

as well as $\mathbf{x}_{\text{new}} = \left\{\ \right\}$ (the empty set).

So, there are $1 + 2\left(n + n^2 + n^3 + \ldots\right)$ kinds of combinations.

For the same operation "+" in the combination space, there are

(1) For any two elements in combination space, the result of the operation of the two elements is also in this combination space.

(2) For any three elements, they meet the law of associativity.

(3) There is an element $\{0\}$ for any other element having an operation with this element, the result will be itself.

(4) For any element, there exists another element that when having the operation with this element, the result is $\{0\}$.

So, it is a group. If it meets the law of commutation, it will be an Abel group.

Then a key issue in this research is the representing method for every characteristic i.e. how to organize the data structure appropriately. Three kinds of typical combination spaces are often applied in structural damage detection.

1). the information of difference/similarity

For $\mathbf{x}=\{\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\ldots,\mathbf{x}_i,\ldots,\mathbf{x}_j,\ldots,\mathbf{x}_n\}$, there is a difference/similarity matrix:

$$\rho=\begin{bmatrix}\rho_{(1,1)} & \rho_{(1,2)} & \cdots & \rho_{(1,j)} & \cdots & \rho_{(1,n)}\\ \rho_{(2,1)} & \rho_{(2,2)} & \cdots & \rho_{(1,j)} & \cdots & \rho_{(2,n)}\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ \rho_{(i,1)} & \rho_{(i,2)} & \cdots & \rho_{(i,j)} & \cdots & \rho_{(i,n)}\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ \rho_{(n,1)} & \rho_{(n,2)} & \cdots & \rho_{(n,j)} & \cdots & \rho_{(n,n)}\end{bmatrix}=\left[\rho_{(i,j)}\right], i,j=1,2,\ldots,n \tag{4.2}$$

where $\rho_{(i,j)}=f\left(\mathbf{x}_i,\mathbf{x}_j\right)$, which is the degree of difference/similarity between $\mathbf{x}_i$ and $\mathbf{x}_j$.

2). the combined structural information

This kind of information will be obtained from $\mathbf{x}=\{\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\ldots,\mathbf{x}_i,\ldots,\mathbf{x}_n\}$ that some data will be selected to describe some characteristic, that $\mathbf{x}_j=\{\mathbf{x}_{1,j},\mathbf{x}_{2,j},\mathbf{x}_{3,j},\ldots,\mathbf{x}_{i,j},\ldots,\mathbf{x}_{\mathbf{n},j}\}$. For example, $\mathbf{x}_{i,j}=\max\left(\mathbf{x}_i\right)$.

3). the denominator information

For $\mathbf{x}=\{\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\ldots,\mathbf{x}_i,\ldots,\mathbf{x}_n\}$, this kind of information often refers to not only some operations,

$$\text{ne} \qquad (\ ) \quad (\ ) \tag{4.3}$$

but also some transforms after the operation,

$$\mathbf{y}_{\text{new}}=\text{transforms}\left(\mathbf{x}_{\text{new}}\right)=g\left(\mathbf{x}_{\text{new}}\right), \tag{4.4}$$

like the Fourier transform, Hilbert transform, and Laplace transform…

So, for the second case, after the transform, there is

$$\mathbf{y}_{\text{new}} = \{f(\mathbf{x}_i)\}, \{f(\mathbf{x}_i + \mathbf{x}_j)\}, \{f(\mathbf{x}_i + \mathbf{x}_j + \mathbf{x}_k)\}, \ldots \quad (4.5)$$

We mainly concern the 1st kind of combination space. In the following introduction, there are four kinds of matrices selected as candidates for every characteristic and every value is calculated as $\frac{1}{n-1}\sum_{i=1}^{n} x_i$ for multiple times of measurements in the experiment, where x is one possible characteristic, and *n* is the maximum times for one location's tests in the experiment.

Concerning the displacement measured, there are 8 displacement meters of SDP-100 (in Table 4.2), i.e. D2_UF, D2_UB, D2_LF, D2_LB, D5_UF, D5_UB, D5_LF, D5_LB located in different places, while the static loading location is fixed. Design the "difference" matrix as:

$$S = \begin{bmatrix} s_{1,1} & s_{1,2} & s_{1,3} & \cdots & s_{1,j} & \cdots & s_{1,8} \\ s_{2,1} & s_{2,2} & s_{2,3} & \cdots & s_{2,j} & \cdots & s_{2,8} \\ s_{3,1} & s_{3,2} & s_{3,3} & \cdots & s_{3,j} & \cdots & s_{3,8} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ s_{i,1} & s_{i,2} & s_{i,3} & \cdots & s_{i,j} & \cdots & s_{i,8} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ s_{8,1} & s_{8,2} & s_{8,3} & \cdots & s_{8,j} & \cdots & s_{8,8} \end{bmatrix}$$

w − in Fig.4.3(a,b), s_i and s_j are the displacement at the location of meter *i* and *j* and $s_{i,j} = s_{j,i}$, means the absolute value of the difference between displacements detected by the -t *j*-th meter; $i, j = 1, 2, 3, \ldots, 8$ and $s_{i,j}=0$ when $i = j$.

For acceleration data, the reasonable configuration of matrices for the interaction field characteristics should be circumspect, anti-rolling out the position of sensors and hit-points. The impact hammer will strike on the structure from hit points 1 to 10, and for every hit, there are 10 sensors record the data and after 10 hits, there are two kinds of time matrices to evaluate the structure status, the max-peak-time (t) in combination matrices:

$$t = \begin{bmatrix} t_{1,1} & t_{1,2} & t_{1,3} & \cdots & t_{1,j} & \cdots & t_{1,10} \\ t_{2,1} & t_{2,2} & t_{2,3} & \cdots & t_{2,j} & \cdots & t_{2,10} \\ t_{3,1} & t_{3,2} & t_{3,3} & \cdots & t_{3,j} & \cdots & t_{3,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ t_{i,1} & t_{i,2} & t_{i,3} & \cdots & t_{i,j} & \cdots & t_{i,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ t_{10,1} & t_{10,2} & t_{10,3} & \cdots & t_{10,j} & \cdots & t_{10,10} \end{bmatrix}$$

where $t_{i,j}$ mean max-peak-time of the **-t** *j*-th sensor.

Meanwhile, max-peak's combination matrix:

$$m = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & \cdots & m_{1,j} & \cdots & m_{1,10} \\ m_{2,1} & m_{2,2} & m_{2,3} & \cdots & m_{2,j} & \cdots & m_{2,10} \\ m_{3,1} & m_{3,2} & m_{3,3} & \cdots & m_{3,j} & \cdots & m_{3,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{i,1} & m_{i,2} & m_{i,3} & \cdots & m_{i,j} & \cdots & m_{i,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{10,1} & m_{10,2} & m_{10,3} & \cdots & m_{10,j} & \cdots & m_{10,10} \end{bmatrix}$$

where $m_{i,j}$ means the max-peak of -t j-th sensor.

Furthermore, there is another data type to indicate the change of the matrix in the acceleration matrix named "order" converted from "value" (like the t & m). Each row of every matrix, $\begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3} & x_{i,4} & x_{i,5} & \cdots & x_{i,10} \end{bmatrix}$ means all 10 sensors' detection value at the -t -peak-time, there is an order for sensors to detect the signal one by one from the max value to min value (or form min value to max value, according to the requirement of the data processing in the experiment), the order of the combination matrix can be:

$$n = \begin{bmatrix} n_{1,1} & n_{1,2} & n_{1,3} & \cdots & n_{1,j} & \cdots & n_{1,10} \\ n_{2,1} & n_{2,2} & n_{2,3} & \cdots & n_{2,j} & \cdots & n_{2,10} \\ n_{3,1} & n_{3,2} & n_{3,3} & \cdots & n_{3,j} & \cdots & n_{3,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n_{i,1} & n_{i,2} & n_{i,3} & \cdots & n_{i,j} & \cdots & n_{i,10} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ n_{10,1} & n_{10,2} & n_{10,3} & \cdots & n_{10,j} & \cdots & n_{10,10} \end{bmatrix},$$

where $n_{i,j}$ means the order number of max-peak of -t j-th record. Here is an example. Following is a pair of order matrix for acceleration flow with 7 input locations and 7 output locations in the initial stage: real measurement vs. ideal situation in Fig.4.7 (Example).

The ideal Situation in Fig.4.7 is like this:

$$n = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1,3 & 3,1 & 4 & 5 & 6 & 7 \\ 3 & 2,4 & 4,2 & \mathbf{1,5} & \mathbf{5,1} & 6 & 7 \\ 4 & 3,5 & 5,3 & 2,6 & 6,2 & 1,7 & 7,1 \\ 5 & 4,6 & 6,4 & 3,7 & 7,3 & 2 & 1 \\ 6 & 5,7 & 7,5 & 4 & 3 & 2 & 1 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

where the hit is conducted from hit 1 to hit 7, and recorded by sensors, from sensor 1 to sensor 7, and the number in line means the order number to get the signal (interaction). For one input location, there may be two possible sensors firstly to detect the same prescriptive signal at the same time. For example, as the red number in matrix shows, for the input at the Hit 2, the Sensor 1 and 5 may detect this signal at the same time, but in real measurement, either Sensor 1 or 5 detected the signal earlier rather than two sensors detected at the same time.

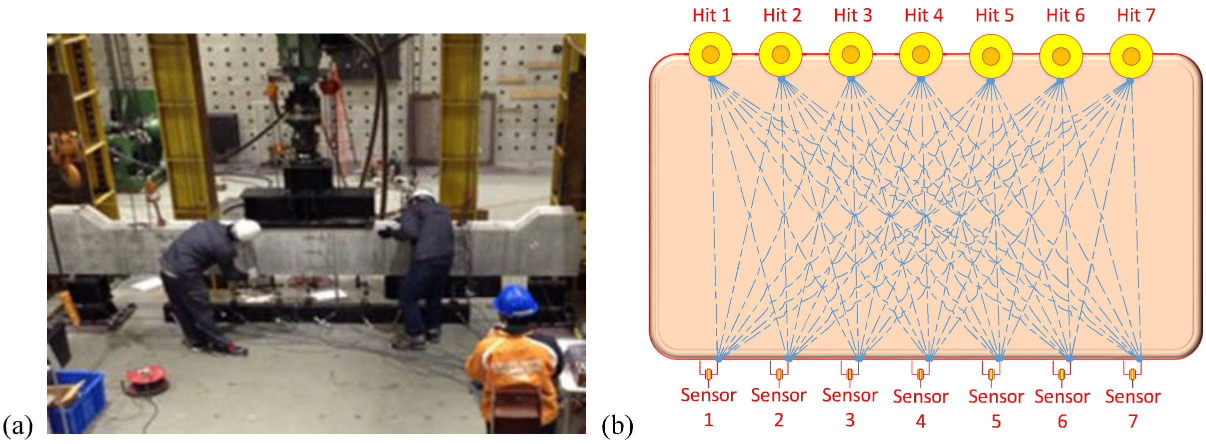


Figure 4.7. Example of (a) A real experiment in Kyoto. (b) 2-dimensional layout of 7 hit points and 7 sensors on a beam in the experiment.

For any matrix of characteristic value/order, like $s, t, m, n\ldots$, there, general matrix M:

$$M = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & \cdots & x_{1,j} & \cdots & x_{1,a} \\ x_{2,1} & x_{2,2} & x_{2,3} & \cdots & x_{2,j} & \cdots & x_{2,a} \\ x_{3,1} & x_{3,2} & x_{3,3} & \cdots & x_{3,j} & \cdots & x_{3,a} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{i,1} & x_{i,2} & x_{i,3} & \cdots & x_{i,j} & \cdots & x_{i,a} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{b,1} & x_{b,2} & x_{b,3} & \cdots & x_{b,j} & \cdots & x_{b,a} \end{bmatrix}$$

w -t j-th location of output.

To evaluate the interaction field characteristic matrices, two kinds of variables are used to show the difference from the initial stage of reference which is often selected as the first stage to the considered stage which represents the status of the specific status of the system.

1. 2-D correlation coefficient (2D-CC) between every considered status matrix and initial status matrix, which is indicating the similarity between the two matrices.

The 2D-CC is usually to distinguish the status matrix change between the initial state and the state considered in this research. The definition of this technology:

$$C=\frac{\sum\left(\mathbf{M}_{\text{initial}}^{T}-\bar{\mathbf{M}}_{\text{initial}}^{T}\right)\left(\mathbf{M}_{\text{considered}}-\bar{\mathbf{M}}_{\text{considered}}\right)}{\sqrt{\sum\left(\mathbf{M}_{\text{initial}}^{T}-\bar{\mathbf{M}}_{\text{initial}}^{T}\right)^{2}\sum\left(\mathbf{M}_{\text{considered}}-\bar{\mathbf{M}}_{\text{considered}}\right)^{2}}} \tag{4.6}$$

in which $\bar{\mathbf{M}}_{\text{initial}}=\text{mean2}\left(\mathbf{M}_{\text{initial}}\right)$ and $\bar{\mathbf{M}}_{\text{considered}}=\text{mean2}\left(\mathbf{M}_{\text{considered}}\right)$, mean2 means the expectation of all data in this matrix (data at both every row and every column).

2. Distance between every considered status matrix and initial status matrix, which is indicating the difference between the two matrices.

There are many methods to get the distance of two status matrices, here is an example:

$$\mathrm{D}=\sqrt[4]{\left(\sum_{j=1}\mathbf{M}^{i,j}{}_{\text{initial}}-\sum_{j=1}\mathbf{M}^{i,j}{}_{\text{considered}}\right)^{2}\left(\sum_{i=1}\mathbf{M}^{i,j}{}_{\text{initial}}-\sum_{j=1}\mathbf{M}^{i,j}{}_{\text{considered}}\right)^{2}} \tag{4.7}$$

where $\mathbf{M}^{i,j}$ means the data in the matrix of row i and column j.

4.2.2 Data processing

The data analysis processing of the experiment is shown in Fig. 4.8:

1) Start of the data analysis.
2) Choose the kinds of measuring data of interactions to do analysis (here, choose the displacement to represent the deformation caused by high-intensity interaction and acceleration to represent low intensity separately).
3) Choose the interaction field characteristics (Max-peak-time from class 1, max-peak from class 3 are chosen for acceleration, and IoD from class 3 is chosen for displacement).
4) Choose the variables (2D-CC and Distance) to represent the interaction processing's characteristic matrices (in which "2-D correlation coefficient, Distance" are for acceleration and "2-D correlation coefficient" is for displacement).
5) Data analysis. There are 11 stages (*max* = 9) for the static loading experiment and 9 stages (*max* = 9) for the dynamic experiment in this data processing. Calculate the expectation of characteristics from the times of records by different acceleration sensors. Construct the combination matrices. Calculate the variables to evaluate the characteristics.
6) Conduct comparative analysis with modal analysis. Compare the (acceleration) interaction field characteristics with the structural characteristics in modal analysis. These characteristics of both interaction and structure all are using the same original data.
7) End of the data analysis.

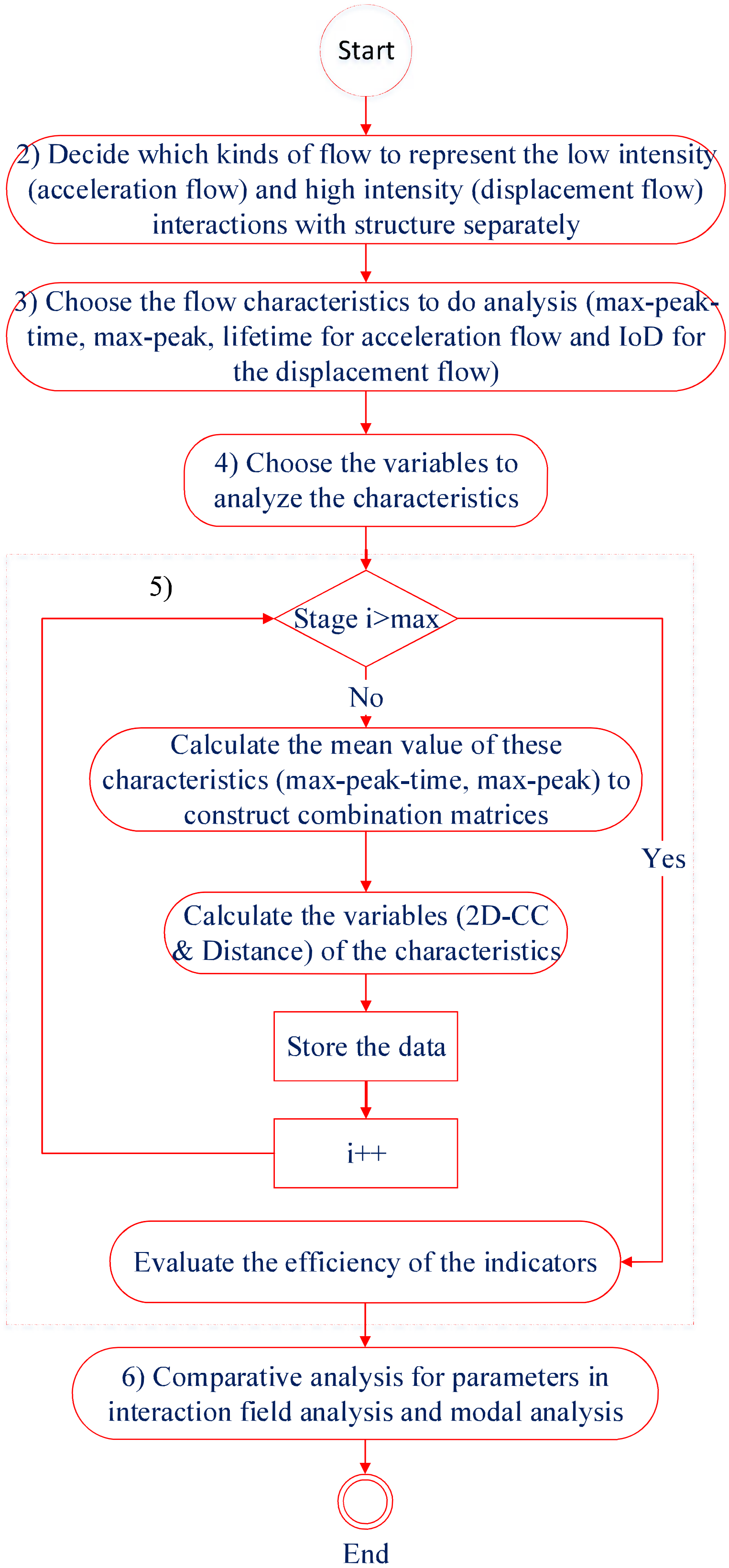


Figure 4.8. Flowchart of data analysis, in which, "max" means the maximum stages for acceleration changing/ maximum steps for displacement changing.

4.3 Damage detection by utilizing characteristics of the interaction field

4.3.1 Health status updating in the experiment

In this subchapter, 2D-CC is used to show the similarity[115] between the reference matrix (initial health stage) and some specifically considered matrix (considered stage), describing the change of the IoD within a very clear range of [-1,1]. Fig.4.9 & Fig.4.10 are the original data of "displacement VS step" of the box girder 2 and box girder 3 separately, and the Fig.4.11 & Fig.4.12 are the analysis results using 2D-CC for the difference matrices of IoD of displacements detected by meters (D2_UF, D2_UB, D2_LF, D2_LB, D5_UF, D5_UB, D5_LF, D5_LB) corresponding to the Fig.4.9a & Fig.4.10 separately. Comparing "Fig.4.9a" with "Fig.4.10" or "Fig.4.11a" with "Fig.4.12", the variable of 2D-CC is more sensitive than the displacement itself. In these figures, the reference matrix (initial) is set as some difference matrices at approximate 2% ~ 5% design load. From Fig. 4.9a and Fig.4.11a, the relationships between the loads and the structure response (displacement) are reflected very clear, that when increasing the loads on the structure, the displacement of the structure will increase. But for more detail information in the whole experimental process, it is a little harder. While in Fig. 4.10 and Fig.4.12, it shows more detail of structural change in the process of experiment.

For the stage 2, stage 5, and stage 8~11 of static loading, the difference matrix of displacement detected by different meters in different location of the structure will slowly change since the loading increases on the structure, the 2D-CC decreases, since the damages have occurred differently, the 2D-CC will still keep in a stable level after the static loading experiment. Once the loading disappears there is a sudden but great change. For the 3rd and 6th stage of tendon cut, the 2D-CC changes suddenly, it means the severe redistribution of internal force in structure. Since the damage increase from the 3rd stage to the 6th stage, the reaction of the 6th stage will be more severe. From the 2D-CC of the intensity distribution, every time, after the tendon cut, there is a short time for the structure to migrate to a new balance (a new development of the artificial damage and a new resistance condition); and the intensity of interaction processing in the system will change with a high uncertainty. For the 4th and 7th stage of the recovery period (almost 3 days) after the tendon cut. Comparing Fig.4.10 with Fig.4.12, it is very clear that different kinds of tendon cut will have different results. In the result of box girder 2, from the tendency in Fig.4.10, just after the tendon cut, the 2D-CC of the difference matrix increase immediately, which means the structure may have an enhancement within a short time, may the structure release some of its residual resistance at once. Then the structure will be in a series of changes of alternating cycles, enhanced or weakened, and then achieves new balances. There are two shapes of "W" in recovery periods of 3 days separately, and the 2D-CC will get through 5 recovery stages. For the reason that the difference matrix, S , shows the difference of displacement at the locations of every two meters, it can be used to a variety transferring equilibria of the interaction processing (detected by meters in different locations) in the structural artificial lifetime.

There are 3 equilibria transfers of "far from equilibrium state" corresponding to the asymmetric cut of C1, C2, and C3 in box girder 2 according to dissipative structure theory. For instance, from the change of recovery stage 2 to recovery stage 4 in Fig.4.10. Meanwhile, in Fig. 4.12, the box girder 3 is symmetrically cut, and from the original data, it will not easy to find such kinds of changes. About this phenomenon, from the perspective of interaction field analysis, it may contain the linear change near the real balance state, i.e. the so-called "near-equilibrium state". After the tendon cut, the change of the system is possibly influenced by the action of environmental micro perturbation[116,117], the crack or the weak part in the structure will slowly creep that the occurrence and development of structural damage exist in different locations, but due to the asymmetry of the tendon cut, the 2D-CC of difference matrix can clearly enlarge the influence of the indication of damage for that the reference matrix of the initial symmetry structural state. Comparing Fig.4.9b with Fig.4.10 and Fig.4.11b with Fig.4.12, it is concluded from the basic knowledge of structural mechanics that under a static load, if the basic shape is assumed to have not changed, the force applied to each part of the structure will be proportional to the force exerted by the loading device. Therefore, in Fig.4.9b and Fig.4.11b, the elastic coefficient is calculated by $k = F/s$, where F is the force of static loading and s is displacement, and it's not necessary to calculate the forces at different positions.

Because civil engineering structure has the characteristics of singularity and non-repeatability, and the internal force of prestressed concrete structure is complex, and the structure has non-linear and unstable changes in its life process, there is generally no analytical solution about damage changes. Under the condition of knowing the continuous development of structural damage in advance, only observation is needed to determine whether the damage factors are consistent with the actual damage development in each stage. If we observe the efficiency in both 10 stages (stage 2~11, except the initial stage, health), for box girder 2, the

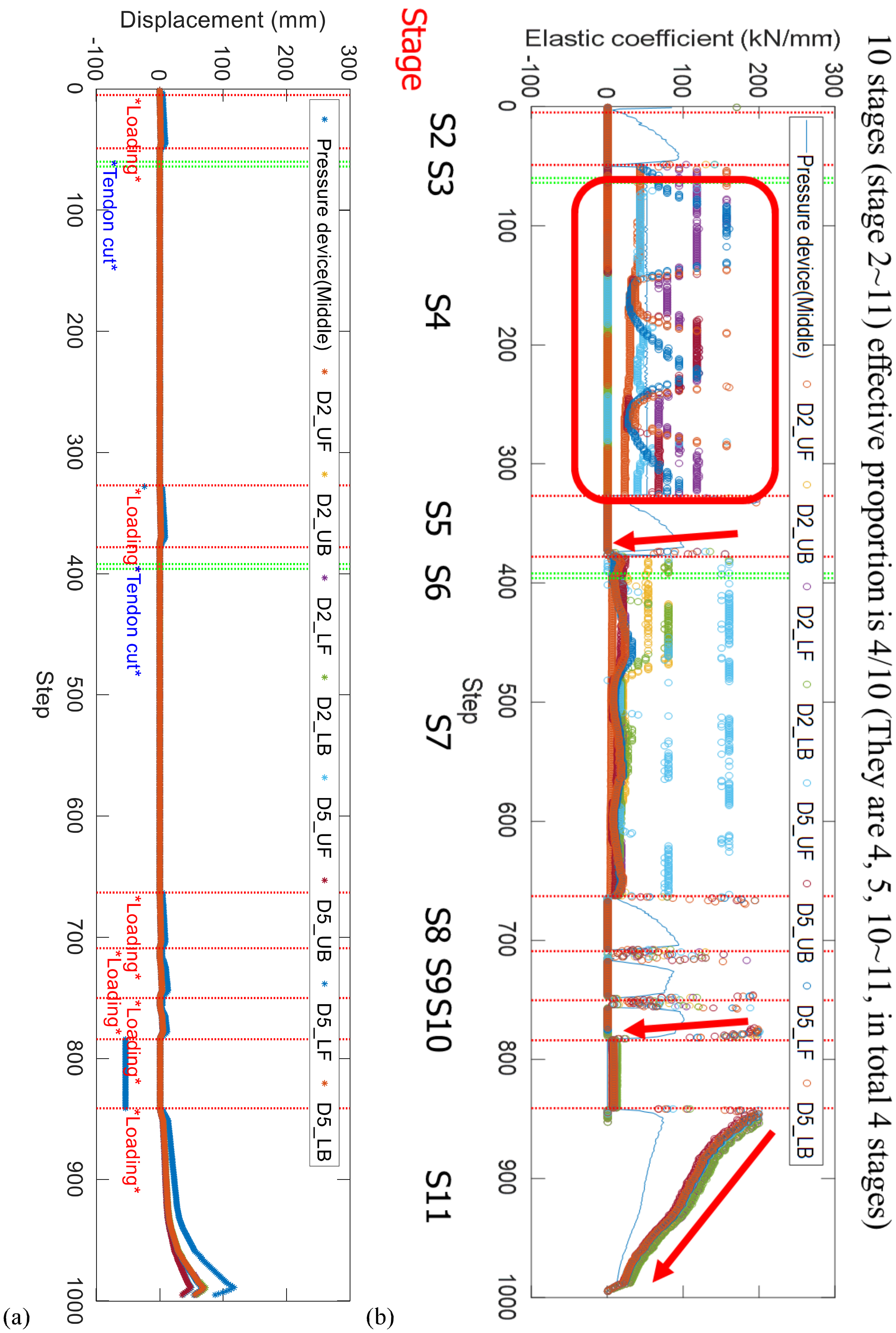


Figure 4.9. (a) The measured displacement and (b) elastic coefficient at displacement meters (loading device: D2_UF, D2_UB, D2_LF, D2_LB, D5_UF, D5_UB, D5_LF, and D5_LB) in every loading step of box girder 2. The green vertical line indicates tendon cut, and the red vertical line indicates the start to the end of static loadings.

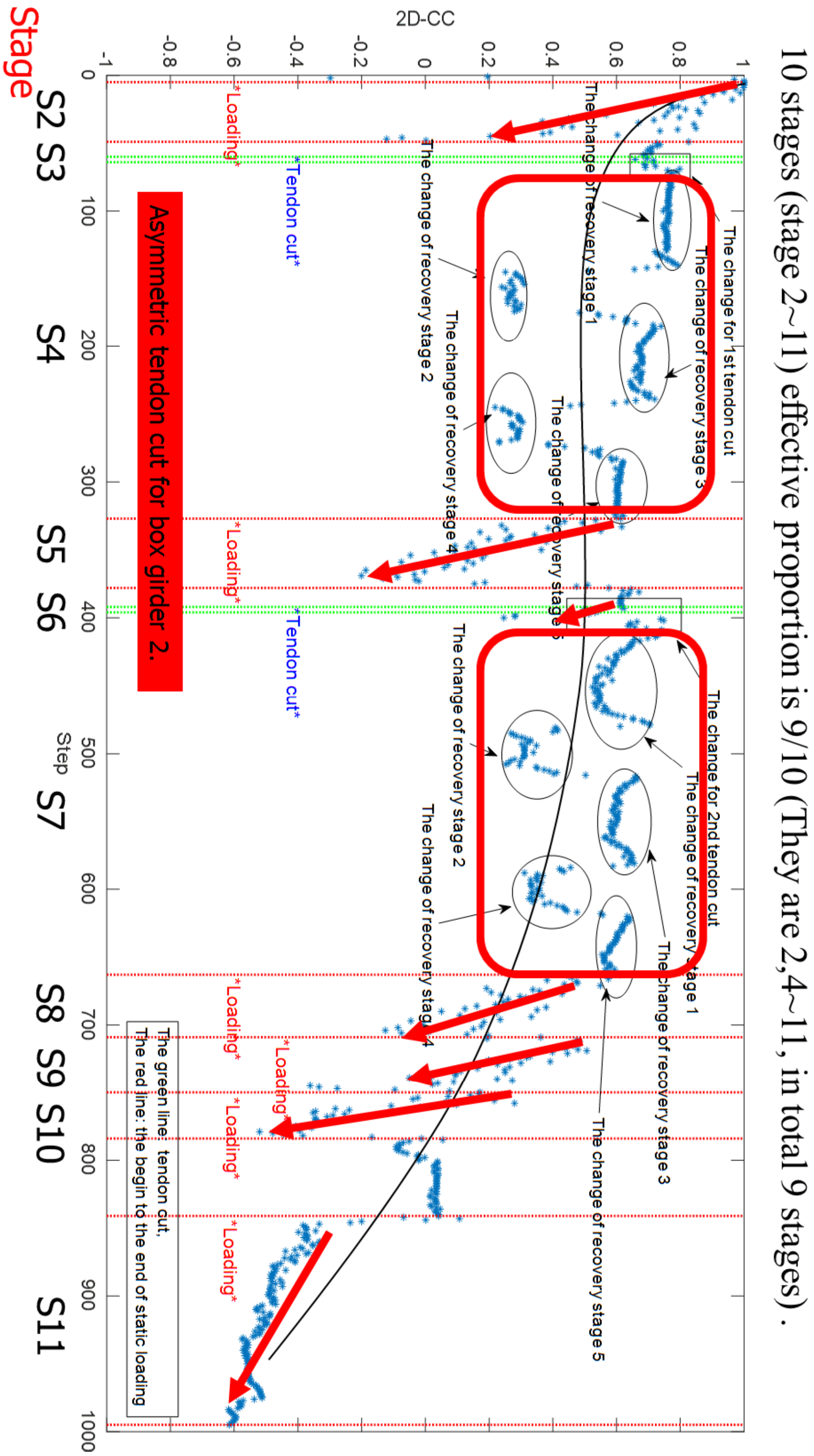


Figure 4.10. Max value of 2D-CC of IoD matrix of box girder 2 between every considered step to every initial step at approximate 2% ~ 5% design load, 51kN,74kN,97kN,118kN (step 4~7). The green vertical line shows tendon cut, and the red vertical line represents the beginning to the end of the static loading experiment.

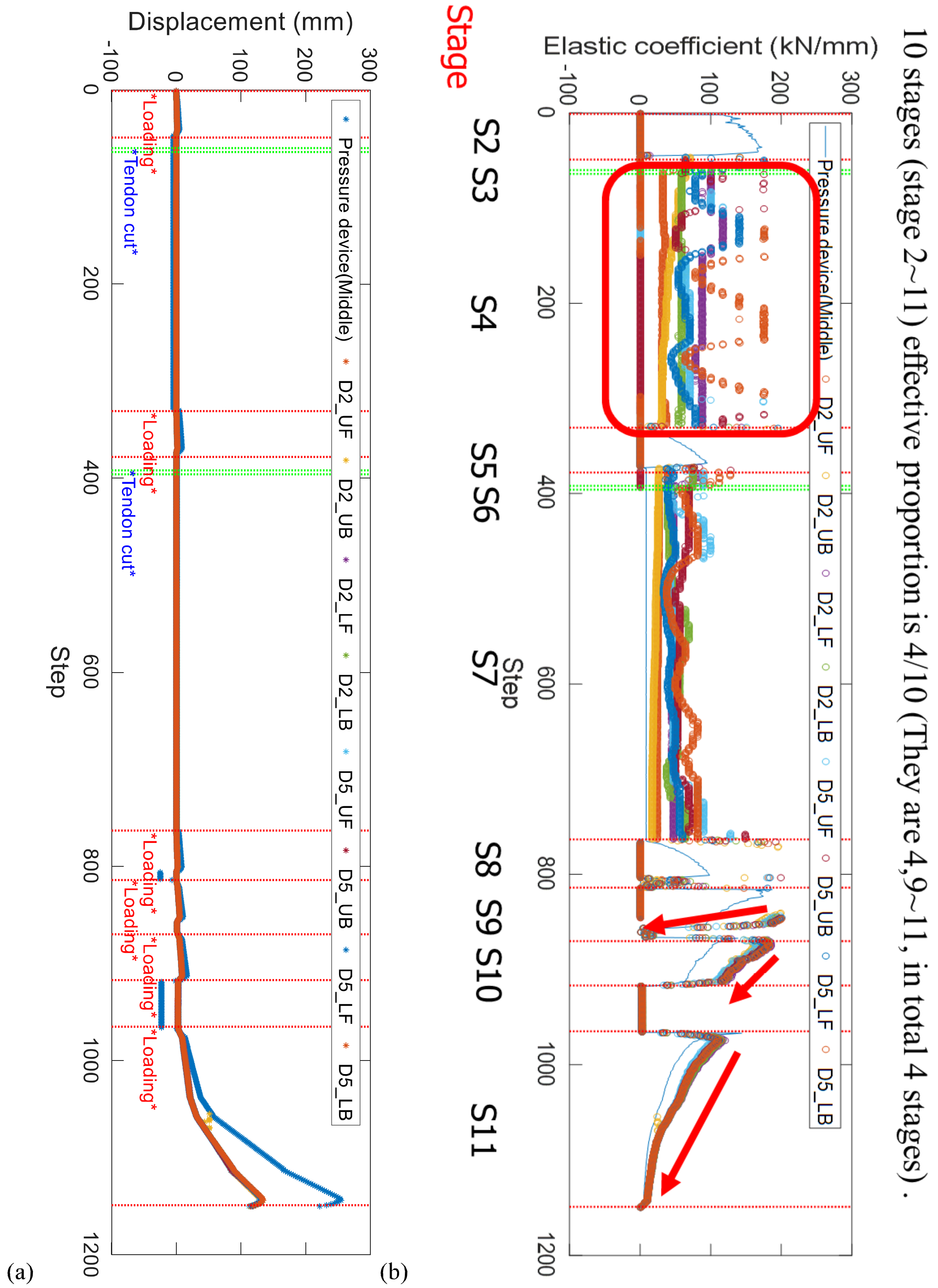


Figure 4.11. (a) The measured displacement and (b) elastic coefficient at displacement meters (loading device: D2_UF, D2_UB, D2_LF, D2_LB, D5_UF, D5_UB, D5_LF, and D5_LB) in every loading step of box girder 3. The green vertical line indicates tendon cut, and the red vertical line indicates the start to the end of static loadings.

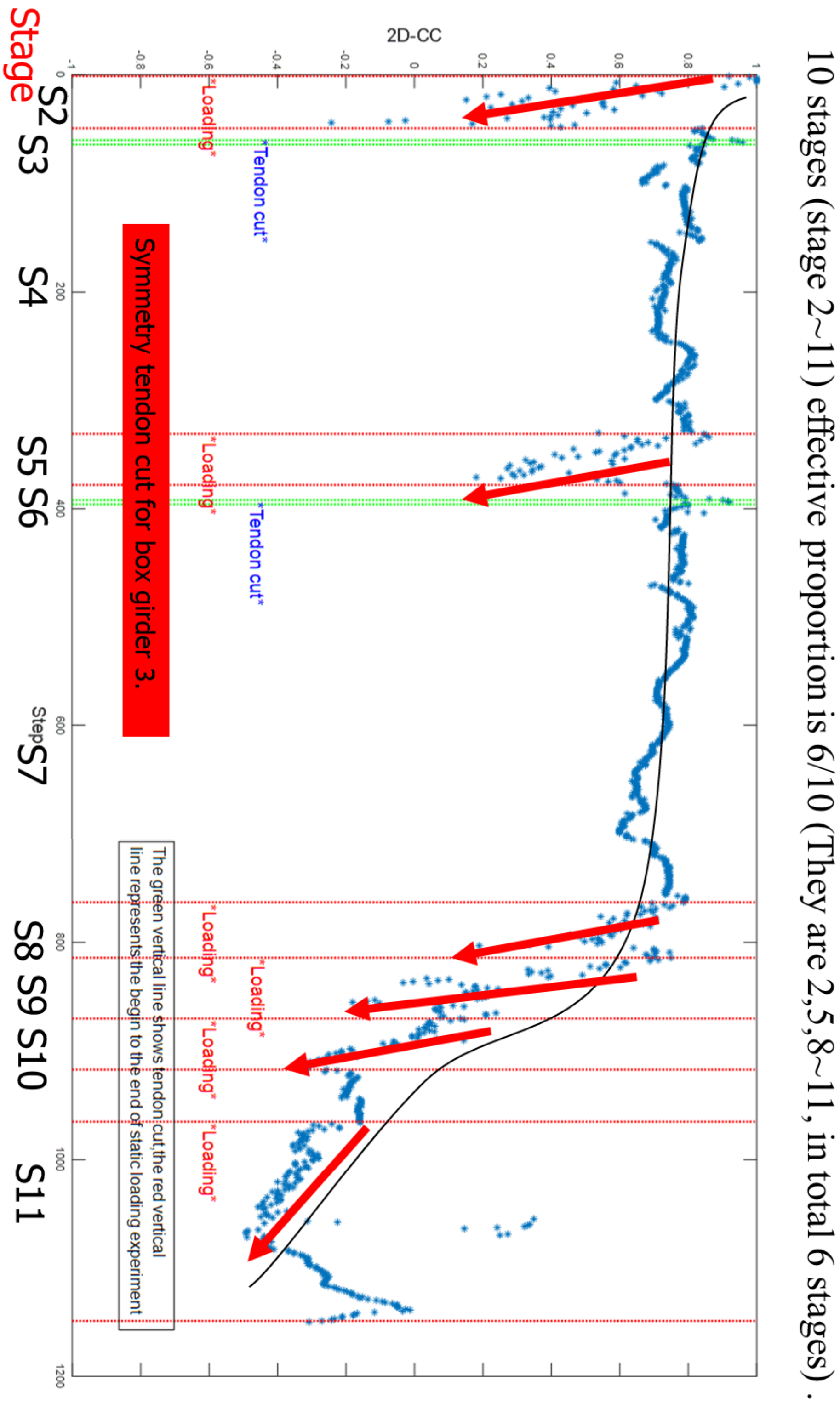


Figure 4.12. Max value of 2D-CC of IoD matrix of box girder 3 between every considered step to every initial step at approximate 2% ~ 5% design load, 50kN, 73kN, 87kN, 117kN (step 3~6). The green vertical line shows tendon cut, and the red vertical line represents the beginning to the end of the static loading experiment.

elastic coefficients are ambiguously effective at stage 4, stage 5, stage 10, and stage 11, and its effective proportion is 4/10; while the 2D-CC is effective at stage 3~11, and its effective proportion is 9/10, so it improves **50%** for the case of box girder 2. For box girder 3, the elastic coefficients are ambiguously effective at stage 4, stage 9, stage 10, and stage 11, and its effective proportion is 4/10; while the 2D-CC is effective at stage 2, stage 5, stage 8~11, and its effective proportion is 6/10, so it improves **20%** for the case of box girder 3.

Moreover, since the experimental objects have the general symmetrical size and also the most common style used in practice, the damage detection based on the deformation caused by the interaction could recognize the continuous damage caused by the static loading and the tendon cut; especially for the tendon cut, the special "W" is very clear and vivid. Even the asymmetry temperature can also influence the redistribution of structural internal force[118]that such a case may also be able to be analyzed in future research.

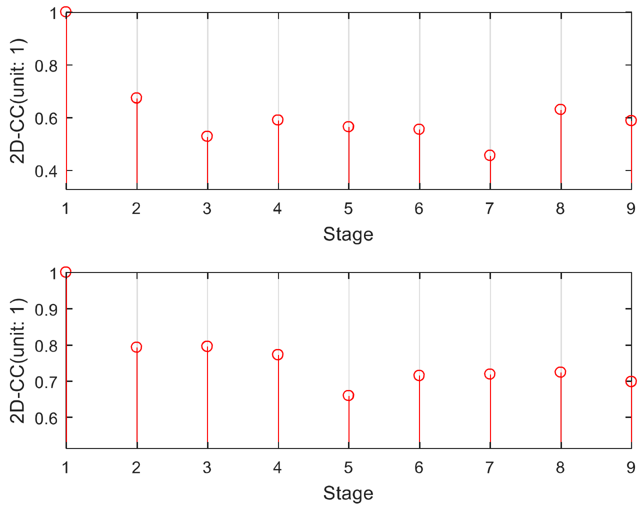
4.3.2 Comparative analysis with model analysis

For the dynamic impact hammer experiment, the case study shows the mean value of 2D-CC between considered stage and initial stage matrix for max-peak-time, max-peak in Fig. 4.13 for box girder 2 and Fig. 4.14 for the box girder 3, that the top of A, B is the characteristic's order matrix while the bottom is the characteristic's value matrix. In every figure, the y-label has no unit, i.e. (unit: 1), and the x-label is the ID of the stage. The red circle means the output in the data processing and the connecting line between every two data just shows the tendency of the whole experimental process. By analyzing Fig. 4.13 and Fig. 4.14, here we'd like to summary the result for two variables representing two characteristics.

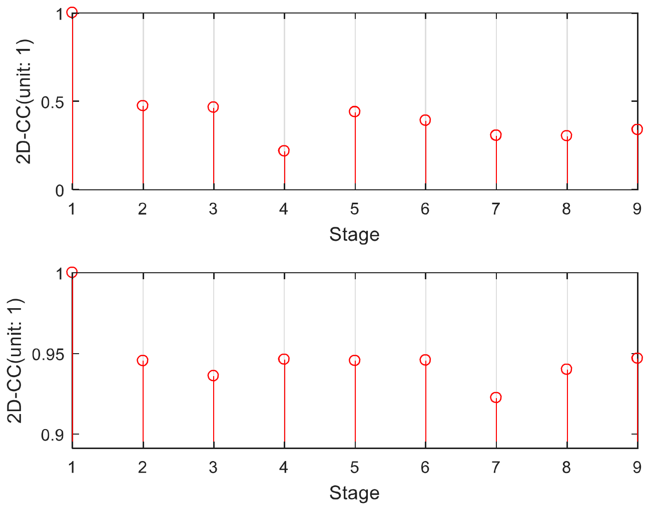
(1) 2D-CC: For the order matrix, it shows the accuracy rates of the (A) max-peak-time and (B) max-peak, represented by 2D-CC, are 6/8, 6/8 for box girder 2 respectively and 4/8, 4/8 for box girder 3. For the value matrix, it shows the accuracy rates of the (A) max-peak-time and (B) max-peak, represented by 2D-CC, are 5/8, 4/8 for box girder 2 respectively and 7/8, 5/8 for box girder 3.

(2) Distance: For the order matrix, it shows the accuracy rates of the (A) max-peak-time and (B) max-peak, represented by Distance, are 5/8, 5/8 for box girder 2 respectively and 6/8, 4/8 for box girder 3. For the value matrix, it shows the accuracy rates of the (A) max-peak-time and (B) max-peak, represented by Distance, are 5/8, 5/8 for box girder 2 respectively and 7/8, 6/8 for box girder 3.

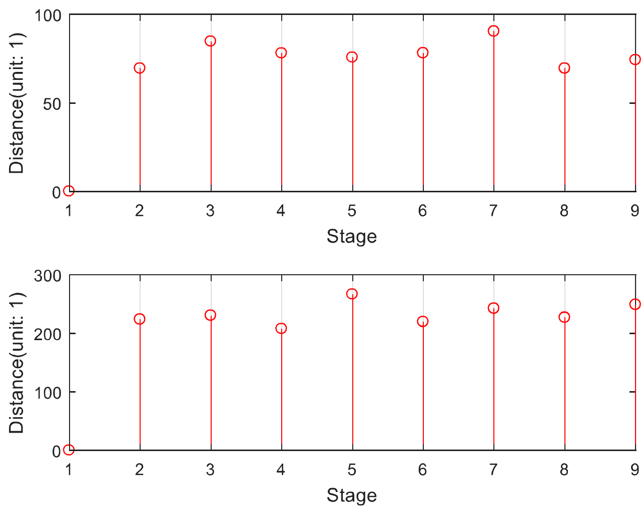
(A-1) right: 6 out of 8 (T) & 5 out of 8 (D)



(B-1) right: 6 out of 8 (T) & 4 out of 8 (D)



(A-2) right: 5 out of 8 (T) & 5 out of 8 (D)



(B-2) right: 5 out of 8 (T) & 5 out of 8 (D)

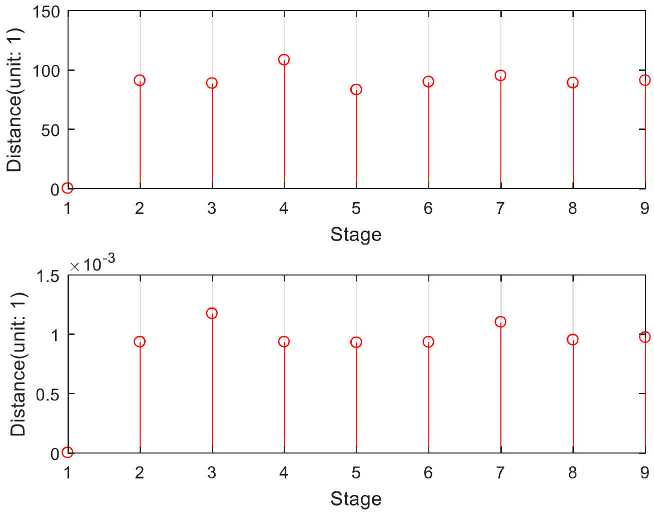


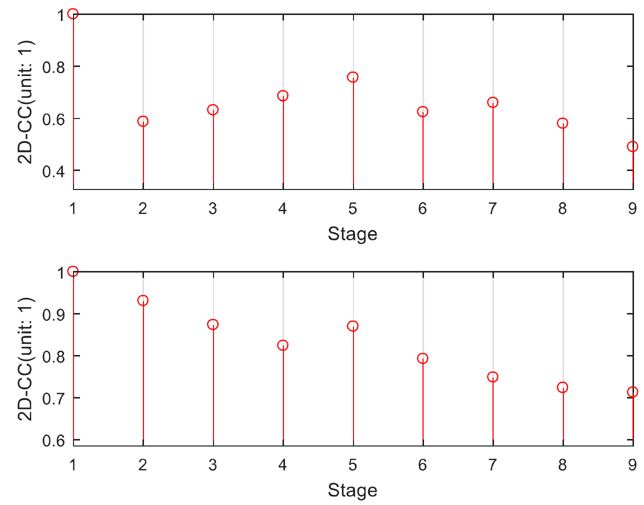
Figure 4.13. The mean value change of (1) 2-D correlation coefficient and (2) distance between considered stage and initial stage matrix (A, max-peak-time, top, T; B, max-peak, down, D) in 9 stages of box girder 2.

Table 4.3. The combination of the characteristic indices of box girder 2, where max-peak(A), max-peak-time(B), 2D-CC (1), Distance (2), order (O), value (V), Right representation (RR), for change between every two neighbor stages, right (o), wrong (x), no obvious change (-).

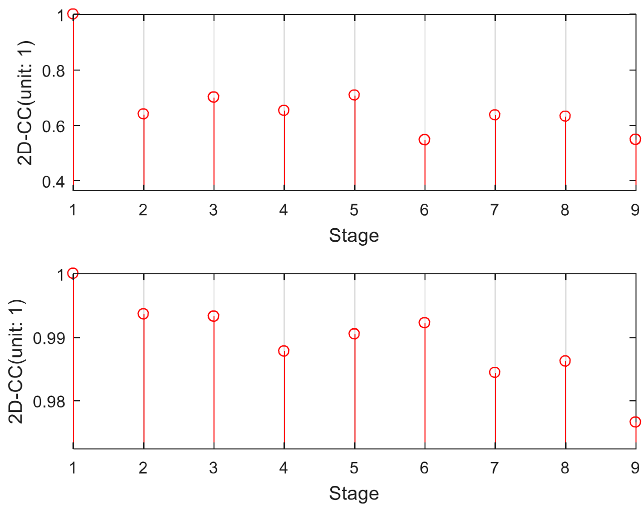
Indicators	**Stage 1→2**	**Stage 2→3**	**Stage 3→4**	**Stage 4→5**	**Stage 5→6**	**Stage 6→7**	**Stage 7→8**	**Stage 8→9**	**RR**
A-1-O	o	o	x	o	o	o	x	o	6/8
A-1-V	o	x	o	o	x	o	x	o	5/8
B-1-O	o	o	o	x	o	o	o	x	6/8
B-1-V	o	o	x	o	x	o	x	x	4/8
A-2-O	o	o	x	x	o	o	x	o	5/8
A-2-V	o	o	x	o	x	o	x	o	5/8
B-2-O	o	x	o	x	o	o	x	o	5/8
B-2-V	o	o	x	o	-	o	x	o	6/8
RR*	8/8	6/8	3/8	5/8	4/8	8/8	1/8	6/8	
Summary	o	o	x	o	x	o	x	o	5/8

* If RR is greater than 4/8, in summary, it is “o”, on the contrary, it is “x”; “a” out of “b” ~ a/b.

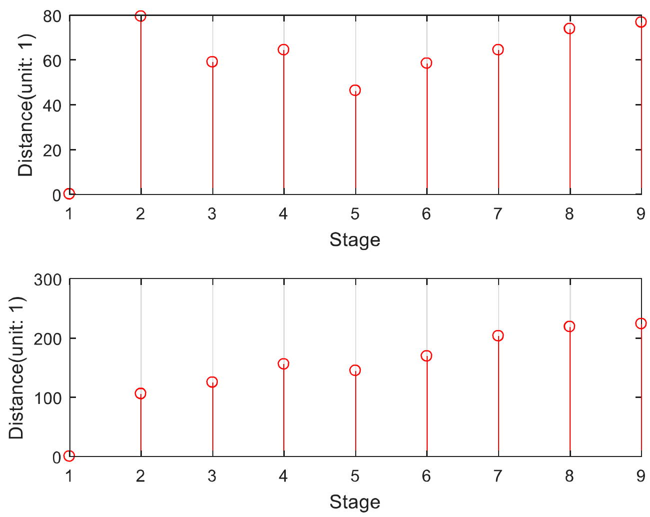
(A-1) right: 4 out of 8 (T) & 7 out of 8 (D)



(B-1) right: 4 out of 8 (T) & 5 out of 8 (D)



(A-2) right: 6 out of 8 (T) & 7 out of 8 (D)



(B-2) right: 4 out of 8 (T) & 6 out of 8 (D)

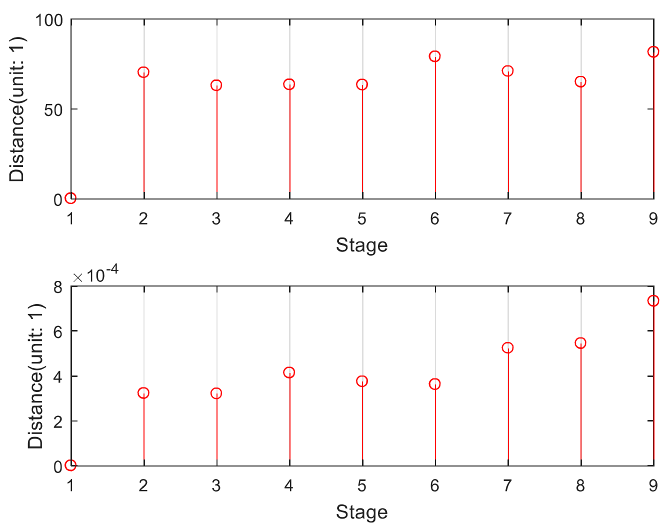


Figure 4.14. The mean value change of (1) 2-D correlation coefficient and (2) distance between considered stage and initial stage matrix (A, max-peak-time, top, T; B, max-peak, down, D) in 9 stages of box girder 3.

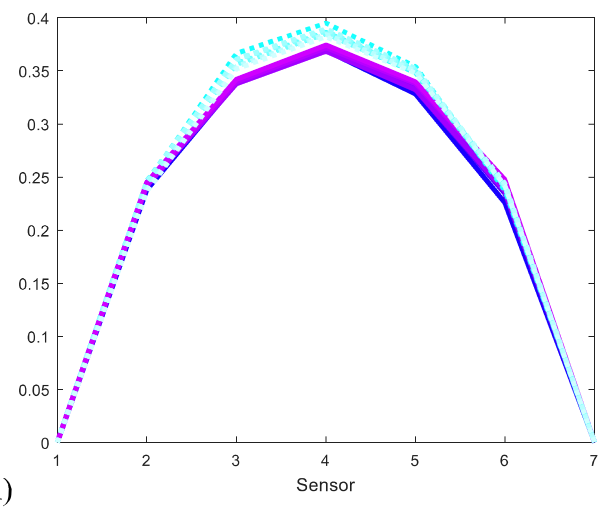
Table 4.3. The combination of the characteristic indices of box girder 2, where max-peak(A), max-peak-time(B), 2D-CC (1), Distance (2), order (O), value (V), Right representation (RR), for change between every two neighbor stages, right (o), wrong (x), no obvious change (-).

Indicators	**Stage 1→2**	**Stage 2→3**	**Stage 3→4**	**Stage 4→5**	**Stage 5→6**	**Stage 6→7**	**Stage 7→8**	**Stage 8→9**	**RR**
A-1-O	o	x	x	x	o	x	o	o	4/8
A-1-V	o	o	o	x	o	o	o	o	7/8
B-1-O	o	x	o	x	o	x	-	o	4/8
B-1-V	o	o	o	x	x	o	x	o	5/8
A-2-O	o	x	o	x	o	o	o	o	6/8
A-2-V	o	o	o	x	o	o	o	o	7/8
B-2-O	o	x	o	-	o	x	x	o	4/8
B-2-V	o	o	o	x	x	o	o	o	6/8
RR*	8/8	4/8	7/8	0/8	6/8	4/8	5/8	8/8	
Summary	o	x	o	x	o	x	o	o	5/8

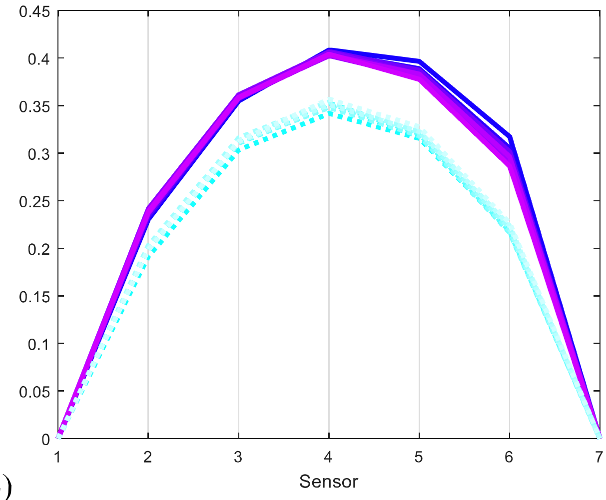
* If RR is greater than 4/8, in summary, it is “o”, on the contrary, it is “x”; “a” out of “b” ~ a/b.

However, from observation of the curve, the right representation represented by 2D-CC of displacement data is still superior to any characteristic of impact hammer experiment in numerical value and the reason may be the sampling in the impact hammer experiment cannot have a panoramic view of the whole life of the structure, and also the precision of measurement in the impact hammer experiment is influenced more by the environment. For different characteristics, they are sensitive in different cases that lead the curve of variables to represent these characteristics having many differences. Meanwhile, since the interaction field characteristics are used to recognize the change of the interaction processing in a structural lifetime rather than the structure, there are many balance states of interaction processing (such as stage 1~2, stage 2~7 and stage 7~9). At stage 2 & 7, there are large balance equilibria migrations, that various characteristics will have larger differences.

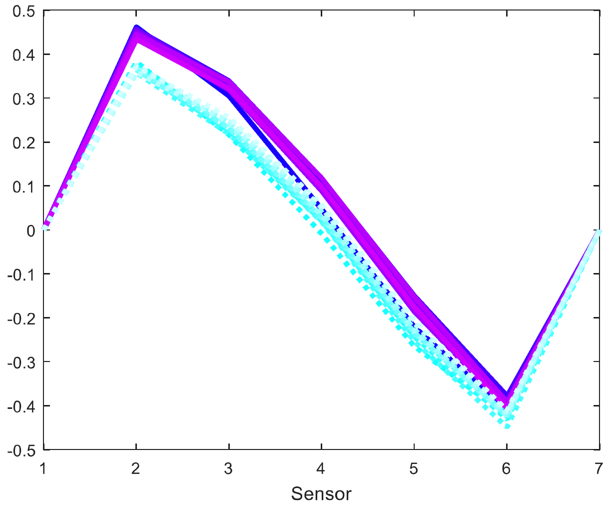
Moreover, to evaluate the efficiency of the interaction field analysis, we need to have some comparison. This research is an integrated study that is a technical integration of current non-destructive testing based on interaction. The research is mainly looking for a systematic interaction-based analysis method in addition to modal analysis. So, it is reasonable and fittable to have a comparison with the modal analysis. Since the model analysis and interaction field analysis can both find the inspiration from the sound (Table A1.1), but even using the same data, they tell two different stories for structure and interaction separately. The model analysis has the same data used as the interaction field analysis. According to the art-of-the-state (Chapter 2.1.2) of the model analysis, lower-order natural frequency, like the 1st and 2nd bending mode (29 HZ and 65HZ nearby, Fig.4.15 and Fig.4.16) a



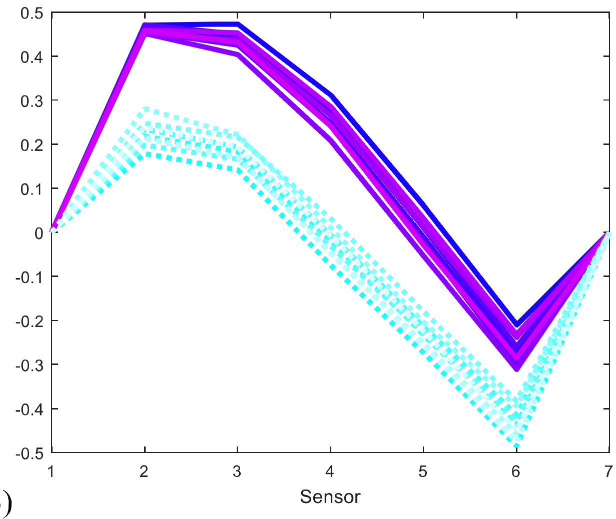
(1-A)



(1-B)



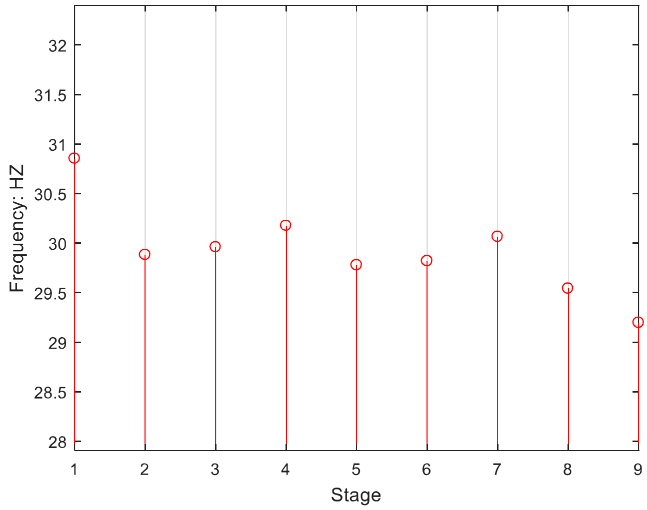
(2-A)



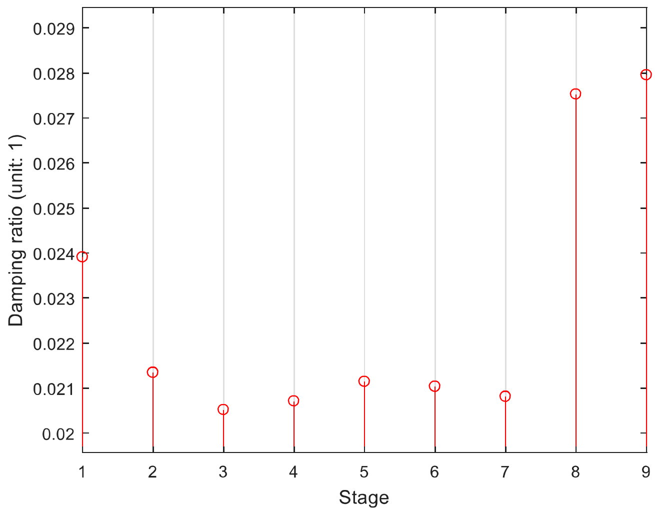
(2-B)

Figure 4.15. The 1st (≈29 HZ)& 2nd (≈65 HZ) bending mode of 2nd (A) a (B) box girders.

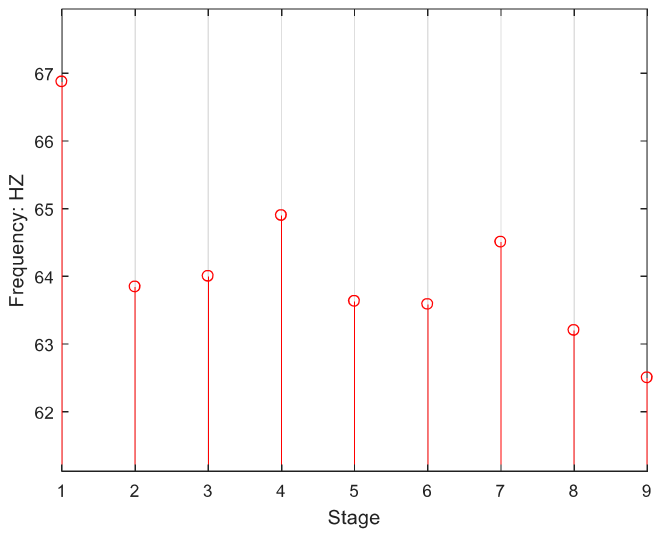
(1-A) right: 4 out of 8



(1-B) right: 3 out of 8



(2-A) right: 5 out of 8



(2-B) right: 3 out of 8

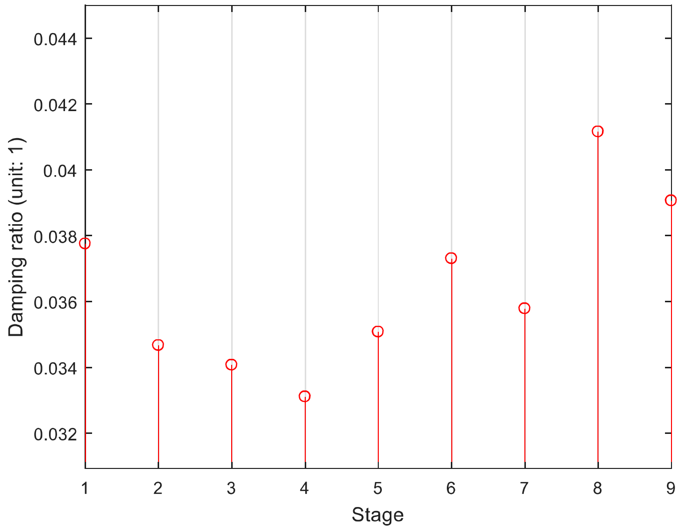


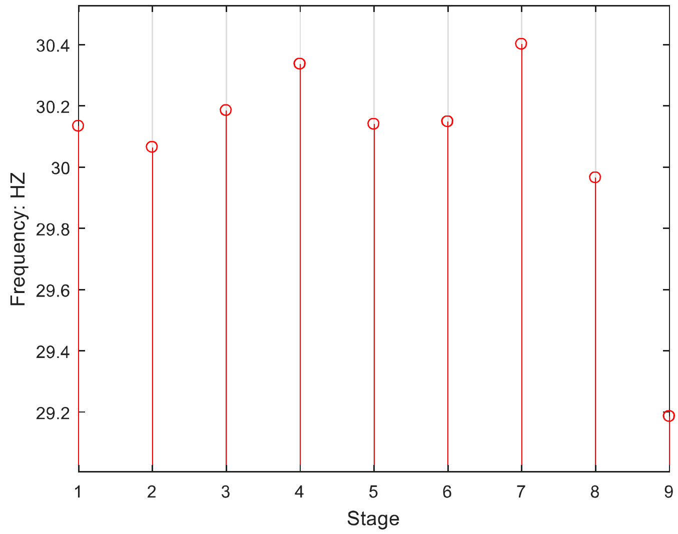
Figure 4.16. The mean value of (1-A) frequency and (1-B) damping ratio corresponding to the 1st bending mode (29~31 HZ). The mean value of (2-A) frequency and (2-B) damping ratio corresponding to the 2nd bending mode (64~66 HZ) for box girder 2.

Table 4.4. The combination of the characteristic indices of box girder 2, where Frequency (Fr), Damping ratio (DR), Right representation (RR), for change between every two neighbor stages, right (o), wrong (x), no obvious change (-).

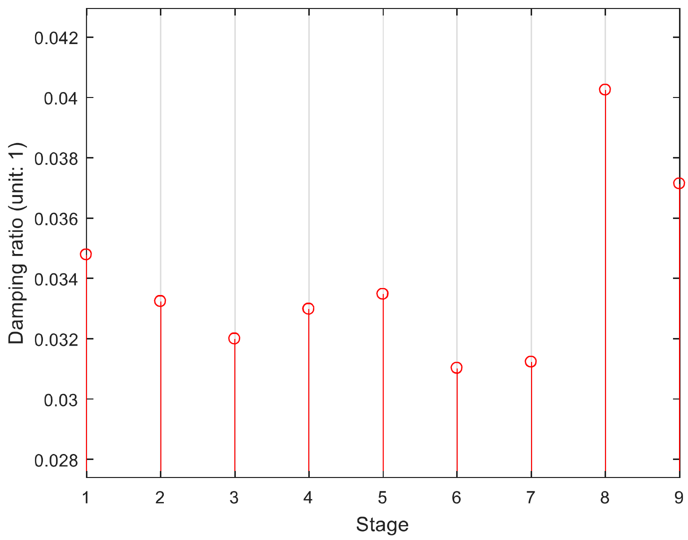
Indicators	**Stage 1→2**	**Stage 2→3**	**Stage 3→4**	**Stage 4→5**	**Stage 5→6**	**Stage 6→7**	**Stage 7→8**	**Stage 8→9**	**RR**
Fr_29	o	x	x	o	x	x	o	o	4/8
Fr_65	o	x	x	o	o	x	o	o	5/8
DR_29	x	x	o	o	x	x	o	o	4/8
DR_65	x	x	x	o	o	x	o	x	3/8
RR*	2/4	0/4	1/4	4/4	2/4	0/4	4/4	3/4	
Summary	x	x	x	o	x	x	o	o	3/8

* If RR is greater than 2/4, in summary, it is "o", on the contrary, it is "x"; "a" out of "b" ~ a/b.

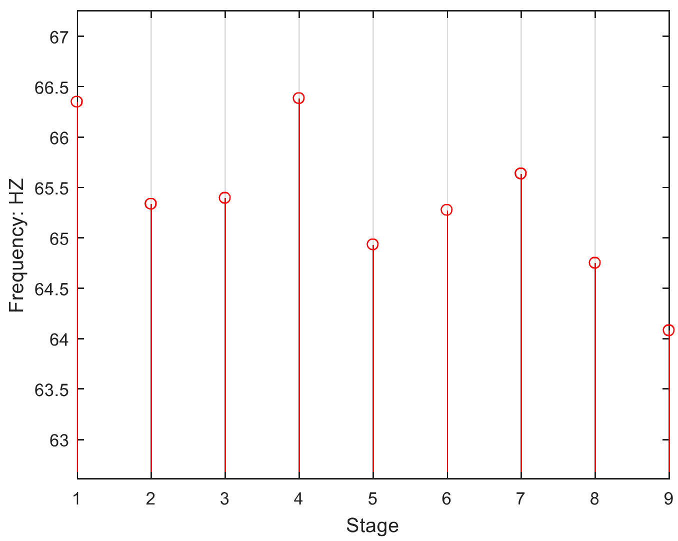
(1-A) right: 4 out of 8



(1-B) right: 4 out of 8



(2-A) right: 4 out of 8



(2-B) right: 6 out of 8

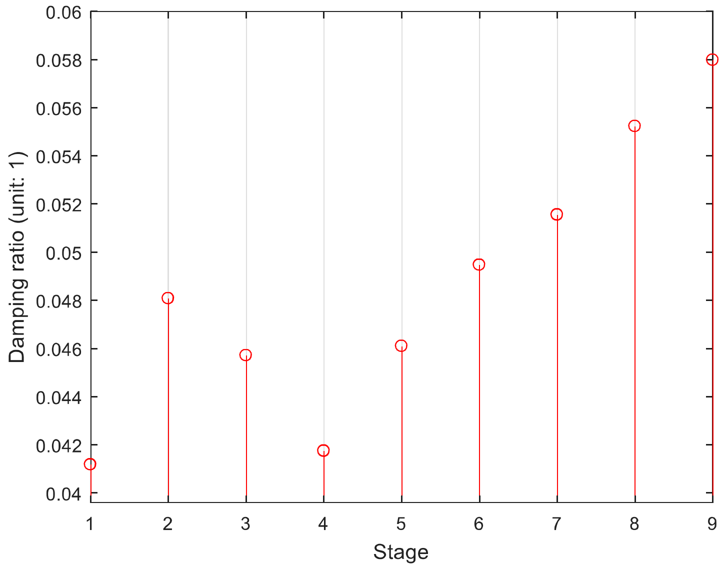


Figure 4.17. The mean value of (1-A) frequency and (1-B) damping ratio corresponding to the 1st bending mode (29~31 HZ). The mean value of (2-A) frequency and (2-B) damping ratio corresponding to the 2nd bending mode (64~66 HZ) for box girder 3.

Table 4.5. The combination of the characteristic indices of box girder 3, where Frequency (Fr), Damping ratio (DR), Right representation (RR), for change between every two neighbor stages, right (o), wrong (x), no obvious change (-).

Girder 3	**Stage 1→2**	**Stage 2→3**	**Stage 3→4**	**Stage 4→5**	**Stage 5→6**	**Stage 6→7**	**Stage 7→8**	**Stage 8→9**	**RR**
Fr_29	o	x	x	o	o	x	o	o	5/8
Fr_65	o	x	x	o	x	x	o	o	4/8
DR_29	x	x	o	o	x	o	o	x	4/8
DR_65	o	x	x	o	o	o	o	o	6/8
RR*	3/4	0/4	1/4	4/4	2/4	2/4	4/4	3/4	
Summary	o	x	x	o	x	x	o	o	4/8

* If RR is greater than 2/4, in summary, it is “o”, on the contrary, it is “x”; “a” out of “b” ~ a/b.

The right representation of both modes obtained by the Stochastic Subspace Identification (SSI) can be found in Fig.4.16 and Fig.4.17. If we want to know the changes in different stages of the model analysis and interaction field analysis, we need to know the reason for the change in these investigations. When the artificial damage gradually increases in the structure, because of the redistribution of internal forces, the cracks will be wider, deeper, and longer, the relationship between sub-structures will change, and the rate of energy dissipation will vary. The strength of associations among sub-structures will directly affect the occurrence and development of cracks. Then the interaction processing in the system will have different routes and rates of dissipation. There are some assumptions or presuppositions:

1. Once the crack is formed, it will not disappear, and the depth and length will not shrink.
2. The redistribution of internal forces will change the strengths of associations among sub-structures as well as the width of cracks.
3. In statics, the energy dissipation depends on the route length of interaction processing.
4. The natural frequency is determined by the structure itself, and the elastic modulus or coefficient of stiffness will directly influence the change of the natural frequency. Natural frequency in the experimental data processing using SSI is a holistic concept of the structure. Global change is determined by the sum of the local changes.

The interaction field analysis of acceleration data using the same original data with the model analysis. According to the results in Fig. 4.16, Fig.4.17, the best right presentation of the structural health status is 5/8 for box girder 2 and 6/8 for box girder 3, that some comparison conclusions between model analysis and interaction field analysis can be made.

In the experiment, the overall size of the structure does not change visually, while with the continuous development of the existing cracks or fresh cracks, its various sizes will have undergone tremendous changes. From the results, the interaction field characteristics are more sensitive to the cracks in depth and length, while the structural characteristics through model analysis are more sensitive to the redistribution of internal forces or the width of the cracks, which are also especially related to the topology of the interaction processing channel in class 2 of interaction field characteristics. Despite the topology of the channel is hard to get, its change will bring about the change of max-peak and max-peak-time in other categories of characteristics (1^{st} and 3^{rd}).

The energy dissipation in the structure from the location of input to the locations of output in the impact hammer experiment will spread along different paths. When there is max interaction intensity (the max power of interference), it means there is “flood” in some locations. Just as the natural frequency changes in Fig.4.16 shows, that the coefficient of stiffness has changed for the reason of cracks change.

From Fig.4.16, Fig.4.17, after the tendon cut (stage 3~4 and stage 6~7), the stiffness coefficient increases, that the storage capacity pre-stressed concrete is released (even there is small decrease at stage 6 in figures (A&B)) that the width of overall cracks decreases.

Since the interaction field characteristics of max-peak and max-peak-time are not sensitive to the width of the cracks, they will not change abnormally in these stages. Considering structural damping ratio and lifetime, they share the same idea that the intensity of vibration in space (wave in time) will gradually decrease. However, the damping of one specific natural frequency cannot clearly show the change of the whole structure change. The lifetime indicates the change in the time-axis of interaction processing that it is about the change of the overall interaction processing in the structure. Since the global change is determined by the sum of the local changes, if we care about all kinds of structural natural frequencies, maybe there is a chance to show that the lifetime of interaction processing is determined by the sum of all damping of natural frequencies.

Furthermore, if we combine all characteristics together for both interaction field analysis (Table 4.2, Table 4.3) a (Table 4.4, Table 4.5) with a simple logic calculation, the result will be much easy to find that the interaction field analysis is better than the modal analysis. For box girder 2, the ratios of the right representation are 5 out of 8, 3 out of 8 changes for the interaction field analysis and the modal analysis separately. For box girder 3, the ratios of the right representation are 5 out of 8, 4 out of 8 changes for the interaction field analysis and the modal analysis separately. By comparison, we can see that the interaction field analysis still has a great advantage over modal analysis in general.

In summary, through analysis of the order/value matrix, from the tendency of both intrinsic variables and the comparison variables in different stages are in better tendency than the characteristics (frequency and damping) in the model analysis.

4.4 Engineering significance of the application

To apply the indicators in interaction field analysis for damage identification in engineering, there are some general aspects to be concerned.

First, the characteristics (or damage indicators) when making the decision should be carefully chosen. For different kinds of interactions, how to select these characteristics properly is depending on the structural styles, materials, environment… and the convenience of detection. Second, the experiment (detection or monitoring) should be well organized that the significance of the data can cover enough information on the structure and interactions. Third, if conditions permit, multi kinds of interactions should be considered which can help to have better analysis and comparison in practice. Lastly, besides the interaction field analysis, other kinds of methods can also be conducted to help decision making.

When utilizing the interaction field analysis, once a structure is damaged, the state of structural force (including the moment of force) equilibrium will change. In the study of the displacement data for the experimental analysis, at stage 2, the structure has suffered a heavy loading, that the rebalance inner structure occurs; after the loading, a permanent change appears in curve 2D-CC vs step. Moreover, after static loading in stage 7, the 2D-CC is near or smaller than 0, so the system is totally different from its formal equilibrium that the structure has great damage befell in a high possibility. Also, for the acceleration data, from the qualitative study, there are 3 equilibria, since the initial change of the structure is caused by the capacity required for normal use from the initial state to the normal using state, but after the stage 2, the structure has suffered a series of changes that almost all the indicators tend to be in disorder of curve-changing complexly in a certain extent, and after stage 7, the change of all variables of all characteristics reunification, that the structure may experience a great damage. And there is a need to pay more attention to whether there is a need to maintain the structure.

In a . Change (fracture or damage) often occurs at a certain location. Our research about the tendon cut shows, an asymmetrical tendon cut will have a serial of rebalances in the structure that the interaction processing in the structure will have a large number of more far state equilibria in the vicinity of the equilibrium state, and finally, a new balance will be formed. This kind of change will be used to indicate the tendon destroyed or not. But more experiments should be done that there may exist different kinds of modes when the tendons are broken in different styles of pre-stressed structures.

Besides this research, to use the interaction field analysis with the data gotten by other methods, the data should be well organized into swarm data. Then, the difference matrix or combination matrix can be used in the interaction field analysis. Furthermore, in other kinds of analysis, if their local indicator (corresponding to the sub-structures) exists, such indicator can be used as one kind of specific "interaction processing" in the space of the structure, and then the interaction field analysis can also be adopted into such kind of analysis as well.

However, for the limitations of the experiment on concrete box girders, it cannot tell every change of all kinds of structural components, for other styles of structures, more experimental data should be obtained and analyzed. Fortunately, according to the dissipative structure theory, all structural changes follow the same laws of equilibrium state migration in system theory. Meanwhile, if there is a big change inside the structure, the efficient indicators can recognize such changes as well. The interaction field characteristics should have changes to confirm the changes in the structure which can also be used in health monitoring of bridge or other structures, that the curve of the interaction field characteristic may have a clearer and more continuous tendency. For the real bridge, measured by displacement, the tendon cut can be detected by a swarm of meters. For the acceleration data, it may use the data of vibration caused by wind, rain, or interactions between bridge and vehicles to show the change of interaction processing in different characteristics. The arrangement of sensors can be decided according to the specific problem, and the data style can be various in practice.

On the other hand, concerning the risk when survey the interaction field characteristics, the measurement is indeed having a very strong influence on the result of damage detection, so, to clear the structural real health status, some risk analysis is necessary.

4.5 Concluding remarks

In this chapter, the experiment is introduced briefly. Also, in this chapter, the combination space is proposed to reconstruct the interaction field characteristics. Then, in the data processing diagram, four kinds of interaction field characteristic matrices are selected. The interaction field characteristic matrices were built to fully show more information about the interaction processing's output-only/input-output, and through variables, the unique value was used to represent these matrices in different stages. From the result of the analysis, the interaction field analysis can have enough efficiency to detect the damage and evaluate the damage for both cracks development caused by the static loading, and tendon cut.

As assumed in this research, the data in measurement involves three kinds of information, the structural information, the environmental information (in the experiment, it is mostly noise), and information of interactions between structure and environment (or among substructures). Comparing the interaction field characteristics (through interaction field analysis) and structural characteristics (through modal analysis), the damage indicators, interaction field characteristics, have better sensitivity & efficiency when detecting/evaluating the damage. Also, we compared the result of displacement data from the static loading experiment and the result of acceleration data from the vibration experiment (impact hammer experiment), though the indicators' sequence in different stages has some differences, they still have enough strong correlations.

In summary, comparing with the modal analysis result of box girder 2 using the same data, the best right representation of interaction field analysis and modal analysis are **6 out of 8 changes** and **5 out of 8 changes** separately, which shows the results of interaction field analysis is better. Also, comparing with the modal analysis result of box girder 3 using the same data, it is very clear that the best indicator in interaction field analysis is **7 out of 8 changes**, while the best one in the modal analysis is only **6 out of 8 changes**. So, interaction field analysis is still better. Moreover, in the combined study, even doped with a lot of damage indicators that are not particularly good, it still performs better than traditional modal analysis. For box girder 2, the ratios of the right representation of the combination of the characteristic indices are **5 out of 8, 3 out of 8 changes** for the interaction field analysis and the modal analysis separately, and for box girder 3, the ratios are **5 out of 8, 4 out of 8 changes** separately. Then, we can learn that the interaction field analysis can have very good efficiency, high sensitivity, and enough precision which is convenient to detect and evaluate the damage in the structural lifetime.

5. Hierarchical Risk in Interaction Field Analysis

Nomenclature

$[\bullet]$ A function that rounds a value down to the nearest integer. $\bullet$ Placeholder for possible variable.

f The smallest/ lowest fayer/ root fayer.

$f-j$ or $(f-j)$ One specific fayer and j means the fayers between this fayer and the root fayer.

n The total number of elements in root fayer. $N_{(f-j)}$ The total elements in fayer $(f-j)$.

x For one element in fayer $(f-j)$, it has x components (elements in fayer $(f-j+1)$).

P_c The surviving ratio (Class 1), the passing ratio (Class 2), or

the synthesized ratio of surviving & passing (mixed class).

E The expectation of P_c in every fayer. $P_{\text{fail-}f}$ Failure probability of any element in root fayer.

$P_{\text{fail-}(f-j)}$ Failure probability of any element in fayer $(f-j)$.

$g_1(x)$ The function for the greatest integer which is smaller than x.

$g_2(x)$ The function for the smallest integer which is greater than x.

$$m_{(f-j)}=\begin{cases} g_1(\bullet)=\begin{cases} N_{(f-j)}P_c, & N_{(f-j)}P_c \text{is an integer} \\ \left[N_{(f-j)}P_c\right]+1, & N_{(f-j)}P_c \text{is not an integer} \end{cases}, \text{class 1} \\ g_2(\bullet)=\begin{cases} N_{(f-j)}P_c, & N_{(f-j)}P_c \text{is an integer} \\ \left[N_{(f-j)}P_c\right]+1, & N_{(f-j)}P_c \text{is not an integer} \end{cases}, \text{class 2，mixed-class} \end{cases}$$

a. In class 1, it means the maximum amount of failing elements for the system staying in health.
b. In class 2 and mixed-class, it means the minimum failing elements for the system failure.

$M_{(f-j)}$ Maximum failing elements in fayer $(f-j+1)$ of one element in fayer $(f-j)$ in class 1, or minimum failing elements in fayer $(f-j+1)$ of one element in fayer $(f-j)$, in class 2 and mixed class.

P_f The system's failure probability of the root fayer.

$P_{(f-j)}$ The system's failure probability of the fayer $(f-j)$.

$P_{i,j}$ The failure probability of i elements in the fayer $(f-j)$

y or Y_f The amount of failing elements in the root fayer.

$Y_{(f-j)}$ The amount of failing elements in the fayer $(f-j)$ of one case according to the random arrangement (combination) of elements in fayer $(f-j+1)$.

$Y_{(f-j)}^{\max}$ The max amount of failing elements in the fayer $(f-j)$ of all cases according to the random arrangement (combination) of elements in fayer $(f-j+1)$.

$C_{Y_{(f-j)}}$ The total number of the combinations for elements in fayer $(f-j)$.

$A_{Y_{(f-j)}-k_{(f-j)}}$ The possible combination of $Y_{(f-j)}-k_{(f-j)}$ failing elements.

$k_{(f-1)_1}$ The first kind of $k_{(f-1)}$, $0 \le k_{(f-1)_1} \le m_{(f-j)}$.

$k_{(f-1)_2}$ The second kind of $k_{(f-1)}$, $m_{(f-j)} < k_{(f-1)_2} \le Y_{(f-j)} - 1$.

$P_{Y_{(f-j)}-k_{(f-j)}}$ The failure probability of combination $A_{Y_{(f-j)}-k_{(f-j)}}$.

5.1 Hierarchy and hierarchical probability

5.1.1 Introduction of Hierarchy

As we have known that the measurement of the interaction processing is sometimes on different scales that the number of sensors will be different, and the precision of the global information obtained for the health status of the structure is different. So, it asks methods to estimate the risk when applying the interaction field analysis. To have a clear description of the structural system and to maintain the structure in its lifetime, hierarchy is imported from universe theory into the general probability theory and discrete structural systems. Meanwhile, in some universe theory, the scale is regarded as one kind of description of hierarchy.

In our research, the locality of the measurement itself and the incompleteness of the combination process make the method erroneous in a certain probability. On the other hand, according to the definition of risk, a probability or threat of damage, injury, liability, loss, or any other negative occurrence that is caused by external or internal vulnerabilities, and that may be avoided through preemptive action[119]. Such a certain probability turns out to be risk in maintenance. That is to say, the risk for decision-making based on the measurement of interaction field exists, since the interaction of the structure is hierarchical in such discrete engineering systems, and the maintenance costs of different levels of elements in hierarchy are not the same. Then we are required to adopt the viewpoint of probability theory and apply the hierarchy in recognizing the system from the perspective of reliability (one indicator for evaluating probabilistic damage and destruction of the structure[120]). To determine the reliability of all elements in hierarchy, if we can find a standard that can describe the phenomenon of hierarchy in this system, there is a chance to understand the system better.

According to people's common sense, observation systems at different levels should get similar evaluation results. However, in the past, the analysis of different scales has broken this possibility in the evaluation of continuous systems. Today, the understanding of discrete systems, that is, hierarchical systems, is also broken. Since the hierarchy has a very close relationship with the multi-scale[121,122], in the multi-scale analysis for structural reliability, through the finite element analysis, people can find that, in different scales, the reliability will be different. In an ideal case through the finite element simulation in Brick model[123], the reliability by analyzing the "resistance-load effects” shows the differences in multi-scales within a unified standard of evaluation. So, in this research, we especially concerned the hierarchical reliability for the structural healthy evaluation and mainly for the discrete system, like the infrastructures, and we have the confidence to foresee that the result of reliability calculation in hierarchy may be different.

On one side, the reliability assessment of the hierarchical model is through the analysis of the data describing the system at different levels[124]. The hierarchical probability of some parameters can be used to evaluate the information or sub-information, and also model the spatial data of the complex system[125]. The hierarchical probability can also be used in not only analysis but also algorithm[126,127]. Meanwhile, even though the hierarchy can describe the natural system, for a better understanding of the systems in engineering, there is a need for people to estimate the system's resistance to risk[128]. Among various kinds of performance evaluation indexes, reliability is of great significance. Therefore, in a hierarchical system, there is a need to propose the hierarchical reliability system with consideration of risk[129,130].

On the other side, people may measure the system and use kinds of parameters to calculate the reliability of the complex system from the perspective of probability (uncertainty). However, these parameters measured or analyzed are sometimes different in hierarchy. It is not the same parameters as local/global which is used at the same time even they are different in hierarchy. The multi-scale analysis is introduced in reliability calculation. Brick model[131] is one good example used in structural reliability, through the finite element analysis, people can find that there is a certain principle of change in the reliability of different scales. It is a case of structural reliability in artificial discretization (different levels) of continuum. In the finite element analysis, the "resistance-load effects" has become a single but unified evaluation.

So, just like the multi-scale analysis for reliability (often independent among scales), there are also different kinds of reliability in hierarchy (often not independent in hierarchy) to evaluate the complex system at the same time which is different from general reliability calculation with only one result.

In response to the interaction processing's multi scales in measurement using the different numbers of sensors, suppose the sensors are assumed to be evenly distributed on the structure. The reaction of the structure will also involve 3 kinds of scales, scopes of micro, meso, and macro. However, according one philosophy in quantum mechanics "one is born to be quantized" that things are discrete and hierarchical. The scale can be treated as one specific description of the hierarchy. Also, in decision-making for maintenance, the replacement or repair, number of elements or substructures are often integers. To have a deeper understanding of the system updating in the circumstance of the interactions, the hierarchy will be taken into consideration to substantively understand the system evolution in the reliability perspective.

However, on one hand, using "system" and "sub-system", sometimes is not enough in a real hierarchical system. There are "sub-sub-systems", and "sub-sub-sub-systems" which exist in systems of the integrated circuit, pile group foundation, machine, vehicle and etc. There is a need to propose the concept of the unit for hierarchy. On the other hand, to discuss the failure probability in hierarchy when the motion goes through the system, there is a need to define the unit for hierarchy in advance. Generally speaking, "level", "stage", "grade", and "layer", etc. are too ordinary to be a unit for system description, and they are not completely suitable for hierarchy to describe the relationship among subsystems in different scales of micro, meso, and macro as well. Also, subset, and subsystem, etc. are little inconvenient to be specifically used for a hierarchical system in practice. Here, a new arithmetical unit is recommended for hierarchy, "fayer". Fayer is a new word, which comes from the word "fay", meaning "to fit or join closely or tightly". Here in the paper, it means "the unit of subset's grade compared with the set".

In a canonical hierarchy, subsets in the same fayer should be of the same size, where the fayer is named as the normative fayer. In a non-canonical hierarchy (see Fig. 5.1), this constraint is not necessary, and the fayer is named as the non-normative fayer. The example shows the relationship among different fayers (the non- normative fayer).

The example Fig.5.1 shows the relationship among different fayers (non-standard fayer):

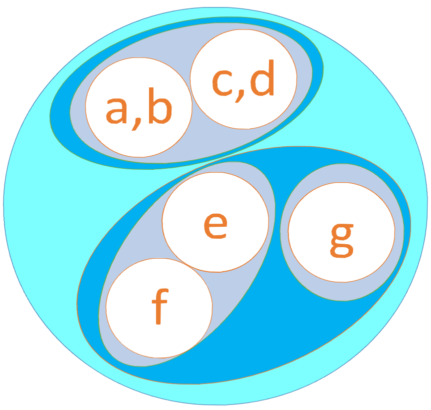


Figure 5.1. The schematic diagram of the hierarchical system

Fayer 0 (set): $\{\{\{\{a,b\},\{c,d\}\}\},\{\{\{e\},\{f\}\},\{\{g\}\}\}\}$, which has 1 element

Fayer 1 (subset): $\{\{\{a,b\},\{c,d\}\}\}$ $\{\{\{e\},\{f\}\},\{\{g\}\}\}$, which has 2 elements

Fayer 2 (sub-subset): $\{\{a,b\},\{c,d\}\}$ $\{\{e\},\{f\}\}$ $\{\{g\}\}$, which has 3 elements

Fayer 3 (sub-sub-subset): $\{a,b\}$ $\{c,d\}$ $\{e\}$ $\{f\}$ $\{g\}$, which has 5 elements

Then, for an easier understanding of the system in the circumscription of hierarchy, the parallel elements are divided into hierarchical frames in every fayer. The smallest hierarchical frames are called root elements corresponding to the fayer named as "root fayer" or "smallest fayer". Suppose that the higher / greater / outer fayer is the fayer close to the greatest fayer (fayer 0), while the lower / smaller / inner fayer is the fayer close to the smallest/root fayer (fayer ∞ or fayer f). "System within/of a specific fayer" means the description of system in a specific fayer. It is semantically equivalent to the expression of "all subsystems (in this system) of a specific scale". For the definition of standard fayer which is used in this paper, suppose the total elements in the whole system in the fayer f is n ; the number of sub-elements x in each element should keep the same, i.e. $x^f = n$.

5.1.2 Hierarchy and system-of-systems

To study the hierarchy, we may find another academic notation, System-of-systems.

System-of-systems (SoS) is a rather controversial research discipline, which has a close relationship with the hierarchy. Until now, the reference framework, thinking process, quantitative analysis, tools, and design methods, etc. of SoS are all incomplete[132], and there are many kinds of definitions for SoS. Generally, the SoS is a collection of limited numbers of independently operable systems that are linked together for a defined higher goal within a specific period of time (defined by Mohammad Jamshidi)[133]; moreover, these systems must obey two basic principles, operational independence and managerial independence[134]. Also, the methods of defining, abstracting, modeling, and analyzing SoS problems are often referred to as the System of Systems Engineering (SoSE). According to this definition, it should be pointed out that the formation of an SoS is not necessarily a permanent phenomenon, but integration of linkage systems formed for specific goals (e.g., robustness, cost, efficiency, etc.).

Contrasting hierarchy with SoS, although the purposes of both are to understand the relationship between the system and the subsystems, we can see that using SoS, people are more likely to longitudinally accomplish kinds of systems to finish some task (ISO/IEC/IEEE 15288:2015 by Annex G) [135] in logical relationship while using hierarchy, we mainly horizontally consider the system from the perspective of subsystems in the system within multi-fayers through the measurement. Logic (design or description), and measurement are two sub-disciplines of Mathematics.

More specifically, the relationship between SoS (cognition from systems to system-of-systems) a (cognition from system to subsystems) is similar to the relationship between integer and decimal, which can be transformed to each other through the movement of the decimal point. More vividly, SoS, like "addition (+)" in mathematics, forms a new system by combining multiple systems and then recognizes it. While hierarchy is more like "division (/)", it divides the origin system into different fayers of subsystems and then measures them.

On one hand, SoS is mainly based on the concept that "the sum of the parts is greater than the parts separately". And by understanding the various systems, it is impossible to fully understand the whole composite system that is made up of them. It is a kind of serial cognitive method, which is a kind of construction model and this kind of comprehension model is a bit like the **Gestalt in psychology**[136]. It understands things by studying the relationship between the whole and the parts. Simultaneously, hierarchy is more likely to focus on the parallel cognitive approach. Similar to **Cognitivism in psychology**[137], we parallelize the views of a system at varied fayers, and try to learn the characteristics of a thing by analyzing the parameters of every fayer. Learning the subsystems in all fayers, we can also understand the difference among these subsystems.

On the other hand, just as the Cognitive school itself is originated from the Gestalt school, the essence of SoS and hierarchy is unified. If the boundaries can be completely distinguished and defined, the subsets are independent of each other. At this time, SoS is equivalent to hierarchy. Moreover, the universe theory introduced in this paper can be regarded as an applied branch of the system theory, hierarchy model can be seen as a concrete form of understanding of the SoS in most instances. However, there is an obvious difference, that is, the Hierarchy method is set up as a local-global relationship artificially; thus, the subsystems are not necessarily independent. In other words, concerning the scope of application, the SoS must be hierarchical and not all of the hierarchical systems are SoS, i.e. (SoS ∩ Hierarchy = SoS).

The Hierarchy method emphasizes that people use the cognitive view of the tree-forest[138] in the process of cognition. Each person will measure the data of the system from different fayers by different measuring methods, and evaluate the system through these data. The related application can refer to Visual Hierarchy[139] with a surprising discovery. Furthermore, referring to Emergence of SoS[140], research of SoS focuses on the intrinsic mechanism (logic) of Emergent Behavior, while Hierarchy's research focuses on the discovering process (measurement) of Emergent Behavior. Even if the same kind of measurement is used based on the same standard, the evaluation results can be different. Then, people's understanding of things can also be different. People conducting the system investigation always start at a certain fayer, instead of giving a comprehensive description at all fayers, or conducting measurements from different fayers for one specific evaluation rather than one specific fayer. In addition to the risks of the system itself, different people may have puzzle understanding of the right data from the right fayer and often are mistakenly led astray among different fayers, which are also sources of risks.

5.1.3 The P-out-of-1 System and Hierarchical Probability

For the general *K*-out-of-*N* system (*K*, *N* both are integers), one well-known recurring system, the expansion in the SoS or other artificial designed complex systems in hierarchy (i.e. *K*-out-of-*N* system in hierarchy, that a hierarchical system composed of several fayers, where each fayer i is a K_i-out-of-N_i system and any element j in fayer i is a K_j-out-of-N_j subsystem), asks for the unified standard of system evaluation. So, there is a need to get a normalization for the *K*-out-of-*N* system. For a given normative situation, the total elements in the whole system in the root fayer f is determined as n, for short, (f, n), the *P*-out-of-1 system within (f, n), is designed as a new model of recurring reliability system updated from the traditional *K*-out-of-*N* system, i.e. $P=K/N$, where K and N are numbers of elements in one specific fayer. When concerning the non-normative situation, the *P*-out-of-1 system will involve the evaluation of every specific element in every fayer in the same standard like the normative situation. When talking about the *P*-out-of-1 system in following research, it just means the *P*-out-of-1 system within (f, n). In this new reliability system, suppose in every fayer, if it can meet the requirement risk defense capacity $P_{c_i} = (1 - K_i/N_i)$, this -t

If failing possibility of i-th fayer, P_i, is greater than or equal to $(1 - P_{c_i})$, this system of some fayer is failed to meet the standard. Usually, fix the ratio as a constant, P_c, i.e. for any fayer i, $P_{c_i} = P_c$, which unifies the standard of risk assessment in hierarchy. When doing research on hierarchical probability or hierarchical reliability, concerning the hierarchical probability of specific information, every fayer can be used to analyze, evaluate, and indicate the whole system's states.

For example, suppose that there are 1000000 apples equally but randomly put into bins, in which 2000 apples are deteriorated. When conducting the examination, the system's failure probability (or named as failing probability) in fayer i is P_i, i=0,1,2, and P_c =98.99% is the limit and i means the number of fayer. First of all, in a 4-fayer system (including fayer 0), its possible cases in every fayer (the failure probability of fayer 3 is 2‰):

Table.5.1 Example shows a 4-fayer probability system

	Fayer 1	**Fayer 2**	**Fayer 3**
Element scale	Big (B)	Middle (M)	Small (S)
Element	B-bin	M-bin	Apple
Number of elements	100	10000	1000000
Sub-element	M-bin	Apple	-

When choosing some special situations:

1) Suppose that the 2000 deteriorated apples are just in 20 M-bins equally; meanwhile these 20 deteriorated M-bins are in 1 B-bin. The other bin (B-bin, M-bin) a

$$< \quad - \qquad P_2 = 2\%_{00} < 1 - P_c$$

2) Suppose that the 2000 deteriorated apples are just in 100 M-bin equally; meanwhile these deteriorated 100 M-bins are in 2 B-bins equally. The other bin (B-bin, M-bin) are filled with other apples in good condition.

$$P_1 = 2\% > 1 - P_c \quad P_2 = 1\% < 1 - P_c$$

3) Suppose that some 1800 deteriorated apples are just in 150 M-bins equally, and other 200 deteriorated apples are in other 200 M-bins equally; at the same time, these 150 deteriorated M-bins are in 10 B-bins equally.

$$P_1 = 10\% > 1 - P_c \quad P_2 = 1.5\% > 1 - P_c$$

4) Suppose that some 1800 deteriorated apples are just in 150 M-bins equally and the other 200 deteriorated apples are in 200 M-bins, at the same time, these 150 deteriorated M-bins are in 51 B-bins in which one contains 100 M-bins and the other 50 only contain 1 M-bin equally.

$$P_1 = 1\% < 1 - P_c \quad P_2 = 1.5\% > 1 - P_c$$

So, for different cases, the possibilities of failure in different fayers are different. Also, since $P_0 = P_1$, there is no need to calculate P_0 specially, and for every root element, its failure probability can often be counted.

Since P-out-of-1 system is the normalized hierarchical system of K-out-of-N system which is a very important part of the research of redundant systems, and also, the hierarchy is one very common model to describe some general SoS that the SoS is applied in more and more departments[141], we want to find the differences or trends among multiple fayers under the same standard. Also, we want to know risks caused by hierarchy— "how does it influence the product's survival ratio, how does it vary the determination of the rate of qualification (passing ratio), whether the hierarchy in redundant design system is similar to the system scale in Brick model or not, and whether the parameter selection will influence the system evaluation, etc."

5.1.4 Central Limit Theorem of Hierarchical Probability

Let X_i, i=1,2,3, … , l, be identical independent random variables with uniform continuous density function $f_i(x_i)$. Let Y_j, j=1,2,3, … ,m, be identical independent elements of one random variable with uniform continuous density function$g_j(y_j)$. Suppose that the calculation method for each information in the system is "PLUS" ("+"). In hierarchical probability, its object is to obtain the distribution of combinations of infinite factors, i.e. the distribution of elements in a specific fayer for combined elements of the lower fayer. From traditional Central Limit theorem's proof, $S_l = \lim_{l\to\infty}\sum_{i=1}^{l} X_i$ will converge to normal distribution $N(\mu_X, \sigma_X)$.

Then if we get the same proof structure of traditional Central Limit theorem's proof on multi-variables, we say these proofs are of isomorphism. For operation "+" for the hierarchical variable, e.g., a 4-fayer system:

fayer 0, $A = \{A\}$;

fayer 1, $A = \{A_i \mid i = 1,2,\ldots,l\}$;

fayer 2, $A = \{A_{i,j} \mid i = 1,2,\ldots,l;\ j = 1,2,\ldots,m;\}$;

fayer 3, $A = \{A_{i,j,k} \mid i = 1,2,\ldots,l;\ j = 1,2,\ldots,m;\ k = 1,2,\ldots,n\}$.

Generally, $A = \{A_{i_1,i_2,\ldots,i_j,\ldots,i_n} \mid i_j = 1,2,\ldots,n_j\}$ for ***n***-th fayer in a system of (n+1) fayers.

If $A_{i_1,i_2,\ldots,i_j,\ldots,i_n}$ has one common continuous density function $f\left(x_{i_1,i_2,\ldots,i_j,\ldots,i_n}\right)$ then if $n \to \infty$

$$S_m = \lim_{\forall m_j\to\infty} \sum_{i_1}^{m_1}\sum_{i_2}^{m_2}\cdots\sum_{i_j}^{m_j}\cdots\sum_{i_n}^{m_n} Y_{i_1,i_2,\ldots,i_j,\ldots,i_n} \to N\ \mu_Y, \sigma_Y)$$

The 2nd hierarchical system: $S_m = \lim_{\exists m_j \to \infty} \sum_{i_1}^{m_1} \sum_{i_2}^{m_2} \cdots \sum_{i_j}^{m_j} \cdots \sum_{i_n}^{m_n} Y_{i_1, i_2, \ldots, i_j, \ldots, i_n} \to N(\mu_Y, \sigma_Y)$

The 3rd hierarchical system: $S_m = \lim_{\exists! m_j \to \infty, \prod_{j=1}^{n} m_j \to \infty} \sum_{i_1}^{m_1} \sum_{i_2}^{m_2} \cdots \sum_{i_j}^{m_j} \cdots \sum_{i_n}^{m_n} Y_{i_1, i_2, \ldots, i_j, \ldots, i_n} \to N(\mu_Y, \sigma_Y)$

It is easy to prove, all three kinds of hierarchical system can be transferred to isomorphism structure[142,143]:

$$S_m = \lim_{m \to \infty} \sum_{j=1}^{m} Y_j \to N(\mu_Y, \sigma_Y) \tag{5.1}$$

It is easy to prove the Central Limit Theorem of hierarchical probability since it has the same calculation structural form with the traditional sum of independent random variables[144]. Furthermore, generally, if we evaluate a hierarchical system, the density function of the parameters can be regarded as normal distributions especially for some unstated situations, and this will be predefined in the Monte Carlo simulation in the following sections.

5.1.5 Calculation of hierarchical probability

Generally, the problems of hierarchical probability for the P-out-of-1 system can be divided into 2 basic classes, the problems of surviving and the problems of passing, and also there are some mixed problems as well. Considering the system within fayer ($f-j$), calculate the elemental failure probability $P_{\text{fail-}(f-j)}$ within this fayer, then calculate the failure probability of the system in this fayer, $P_{(f-j)}$, and its reliability, $R_{(f-j)} = 1 - P_{(f-j)}$.

(1) Calculation of Hierarchical Failure Probability for First class of P-out-of-1 System

For the first class, the percent of surviving, or surviving ratio, which is used to evaluate the healthy sustainability the system's lifetime using system probability. Suppose that:

- There are n elements in the root fayer f.
- The number of elements in fayer $(f-j)$: $N_{(f-j)} = n / x^j$.
- The probability of every elemental failure to its function in the fayer f, $P_{\text{fa l-}f} = P_{\text{fail-}(f-0)} = P$.
- The probability of surviving ratio is P_c, and the expectation is $E_{(f-j)} = N_{(f-j)} (1 - P_c)$,
- $m_{(f-j)}$ is the greatest integer which is smaller than E, i.e. $m_{(f-j)} = \mathrm{g}_1 \left(E_{(f-j)} \right)$,

- $M_{(f-j)} = g_1\left(xP_{\text{fail-}(f-j+1)}\right)$ is the number of max failing elements in $\left(f-j+1\right)$ of one element in fayer $\left(f-j\right)$.
- Function g_1 means when $E_{(f-j)}$ is an integer, $m_{(f-j)} = E_{(f-j)} - 1$, and in other cases, $m_{(f-j)} = \left[E_{(f-j)}\right]$.

Then, the probability of system failure in fayer $\left(f-j\right)$:

$$P_{(f-j)} = 1 - \sum_{i=0}^{m_{(f-j)}} C_{n/x^j}^{i} \left(P_{\text{fail-}(f-j)}\right)^i \left(1 - P_{\text{fail-}(f-j)}\right)^{\left(n/x^j\right)-i} \tag{5.2}$$

w −j),

$$P_{\text{fail-}(f-j)} = 1 - \sum_{i=0}^{M_{(f-j)}} C_x^i \left(P_{\text{fail-}(f-j+1)}\right)^i \left(1 - P_{\text{fail-}(f-j+1)}\right)^{x-i}.$$

For this P-out-of-1:F system, it has the equivalence with K-out-of-N:F systems that varied K=$m_{(f-j)}$ out of varied N in every fayer.

(2) Calculation of Hierarchical Failure Probability for Second class of P-out-of-1 System

For the second class, the percent of passing (passing ratio) is used to evaluate the initial status of the system. Suppose that:

- There are y elements known to be failed, randomly distributed in n elements in fayer f.
- The number of elements in fayer $\left(f-j\right)$: $N_{(f-j)} = n/x^j$.
- The passing ratio is P_c.
- The min number of failing elements in fayer $\left(f-j\right)$: $m_{(f-j)}$=$g_2\left(N_{(f-j)}\left(1-P_c\right)\right)$.
- And the min number of failing elements in fayer $\left(f-j+1\right)$ of 1 element in fayer $\left(f-j\right)$: $M_{(f-j)} = g_2\left(xP_{\text{fail-}(f-j+1)}\right)$.
- The amount of failing elements is $Y_{(f-j)}$ in the fayer $\left(f-j\right)$, and the max, min amount of failing elements are $Y_{(f-j)}^{\max}$ a $Y_{(f-j)}^{\min}$ respectively, which are decided by $\text{g}_1\left(Y_{(f-j+1)}/M_{(f-j)}\right)$ a $\text{g}_1\left(Y_{f-j+1)}^{\max}/M_{(f-j)}\right)$, $\text{g}_1\left(Y_{(f-j+1)}^{\min}/M_{(f-j)}\right)$ separately, where

$g_2(\bullet)$ means the smallest integer which is greater than a number.

For example: the calculation of the probability of system failure in fayer$(f-1)$.

Firstly, choose $Y_{(f-1)}-k_{(f-1)}$ elements in fayer $(f-1)$, there are $C_{(n/x)}^{Y_{(f-1)}-k_{(f-1)}}$ kinds of combinations in total, then arrange these elements within $M_{(f-1)}\left(Y_{(f-1)}-k_{(f-1)}\right)$ elements from elements in fayer f.

Secondly, there are $k_{(f-1)}M_{(f-1)}$ elements in fayer f randomly arranged in all elements in fayer$(f-1)$. If $\left(k_{(f-1)}M_{(f-1)}-i\right), i=0,1,2,\ldots k_{(f-1)}M_{(f-1)}$ elements in fayer f are arranged in$\left(n/x-Y_{f-1}+k_{(f-1)}\right)$ elements in fayer$(f-1)$, and the rest i elements in fayer f will be arranged in $Y_{(f-1)}-k_{(f-1)}$ selected elements, since there are $N_{(f-1)}=n/x$, so in total $C_{(n/x)}^{Y_{(f-1)}-k_{(f-1)}}\sum_{i=0}^{k_{(f-1)}M_{(f-1)}}\left(C_{\left(n/x-Y_{(f-1)}+k_{(f-1)}\right)+\left(k_{(f-1)}m-i\right)-1}^{k_{(f-1)}m-i}C_{\left(Y_{(f-1)}-k_{(f-1)}\right)+i-1}^{i}\right)$ combinations will be referred in calculation for the number of combinations of failing sub-systems in fayer $(f-1)$.

Thirdly, the number of combinations of failing sub-systems in this fayer $(f-1)$:

$$A_{Y_{(f-1)}-k_{(f-1)}}=C_{(n/x)}^{Y_{(f-1)}-k_{(f-1)}}\left(\sum_{i=0}^{k_{(f-1)}M_{(f-1)}}C_{(n/x)-Y_{(f-1)}+k_{(f-1)}-1+k_{(f-1)}M_{(f-1)}-i}^{(n/x)-Y_{(f-1)}+k_{(f-1)}-1}C_{\left(Y_{(f-1)}-k_{(f-1)}\right)-1+i}^{Y_{(f-1)}-k_{(f-1)}-1}\right)-\sum_{a=0}^{k_{(f-1)}-1}C_{Y_{(f-1)}-a}^{Y_{(f-1)}-k_{(f-1)}}A_{Y_{(f-1)}-a}$$

;

and the total number of possible combinations is $C_{(n/x)+Y_f-1}^{(n/x)-1}$.

Then, the probability of system failure in fayer$(f-1)$ is $P_{(f-1)}=\dfrac{\sum_{k_{(f-1)}=0}^{Y_{(f-1)}-m_{(f-1)}}A_{Y_{(f-1)}-k_{(f-1)}}}{C_{(n/x)+Y_f-1}^{(n/x)-1}}$

Similarly, if $Y_{(f-j)}\geq m_{(f-j-1)}$ in fayer$(f-j+1)$, the total number of possible combinations, $C_{Y_{(f-j)}}=C_{N_{(f-j)}+Y_{(f-j+1)}-1}^{N_{(f-j)}-1}$, and set $P_{k_{(f-j)}}=\sum_{Y_{(f-j)}=Y_{(f-j)}^{\min}}^{Y_{(f-j)}^{\max}}\dfrac{\sum_{k_{(f-j)}=0}^{Y_{(f-j)}-m_{(f-j)}}\left(A_{Y_{(f-j)}-k_{(f-j)}}\right)}{C_{\left(N_{(f-j)}\right)+Y_{(f-j+1)}-1}^{\left(N_{(f-j)}\right)-1}}$, and $P_{Y_{(f-i)}}=\dfrac{A_{Y_{(f-i+1)}}}{C_{\left(N_{(f-i+1)}\right)+Y_{(f-i+2)}-1}^{\left(N_{(f-i+1)}\right)-1}}$,

$0 \le i \le j$. And the possible kinds of combination for $Y_{(f-j)} - k_{(f-j)}$ elements in fayer $(f-j)$ is calculated according to $A_{Y_{(f-j)}-k_{(f-j)}} = C_{N_{(f-j)}}^{Y_{(f-j)}-k_{(f-j)}} \left(\sum_{i=0}^{k_{(f-j)}M_{(f-j)}} C_{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)}-1+k_{(f-j)}M_{(f-j)}-i}^{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)}-1} C_{Y_{(f-j)}-k_{(f-j)}-1+i}^{Y_{(f-j)}-k_{(f-j)}-1} \right) - \sum_{a=0}^{k_{(f-j)}-1} C_{Y_{(f-j)}-a}^{Y_{(f-j)}-k_{(f-j)}} A_{Y_{(f-j)}-a}$.

Then, the probability of system failure in fayer $(f-j)$:

$$P_{(f-j)} = \sum \left(\prod P_{Y_{(f-S_{(f-j)})}} P_{k_{(f-j)}} \right)$$

$$= \sum_{Y_f=y}^{y} P_{Y_f} \left(\sum_{Y_{(f-1)}=Y_{(f-1)}^{\min}}^{Y_{(f-1)}^{\max}} P_{Y_{(f-1)}} \left(\cdots \sum_{Y_{(f-i)}=Y_{(f-i)}^{\min}}^{Y_{(f-i)}^{\max}} P_{Y_{(f-i)}} \left(\cdots \sum_{Y_{(f-j)}=Y_{(f-j)}^{\min}}^{Y_{(f-j)}^{\max}} P_{Y_{(f-j)}} \left(\sum_{k_{(f-j)}=0}^{Y_{(f-j)}-m_{(f-j)}} \frac{A_{Y_{(f-j)}-k_{(f-j)}}}{C_{Y_{(f-j)}}} \right) \cdots \right) \cdots \right) \right) \quad (5.3)$$

In this P-out-of-1:F system, it has the equivalence with K-out-of-N:F systems that varied $K=m_{(f-j)}$ out of varied N in every fayer. Also, the method used in first class can be used to solve the problem of the second class for the results with the not very precise data provided for reliability calculation with hierarchical probability in different fayer; if there are thousands of components in a system, these components usually belong to some determined sub-system. But there are some cases they are not determined since some sub-systems are independent.

(3) Calculation of Hierarchical Failure Probability for mixed class of P-out-of-1 System

In the real world, the percent of passing and percent of surviving cannot be easily distinguished from each other in risk assessment, though the original definition of reliability is about the percent of surviving. When talking about the definition of reliability, it is usual to hypothesize the percent of passing can be satisfied.

But when conducting the experiment to get data for reliability, it may often be easy to lose sight of the difference between passing ratio and surviving ratio. That is to say, the research of reliability may often mix both of them together. Suppose that:

- The number of elements in fayer $(f-j)$ is $N_{(f-j)} = n/x^j$;
- the elemental failing probability corresponding to its surviving ratio in the fayer f is $P_{\text{fail-}f} = P$;
- the number of prior failing elements is y, which is corresponding to its passing ratio in the fayer f; and the synthesized ratio of surviving & passing is P_c.

- The amount of failing elements is $Y_{(f-j)}$ in fayer $(f-j)$, and the possible max, min amount of failing elements are $Y_{(f-j)}^{\max}$ a $Y_{f-j)}^{\min}$ respectively, which are decided by $g_1\left(Y_{(f-j+1)}/M_{(f-j)}\right)$ a $g_1\left(Y_{f-j+1)}^{\max}/M_{(f-j)}\right)$, $g_1\left(Y_{(f-j+1)}^{\min}/M_{(f-j)}\right)$ separately.
- The mini failing element in fayer$(f-j)$ is $m_{(f-j)}$=$g_2\left(N_{(f-j)}P_c\right)$,

  when $N_{(f-j)}P_c$ is an integer, $m_{(f-j)} = N_{(f-j)}\left(1-P_c\right)$,

  else $m_{(f-j)} = g_1\left(N_{(f-j)}\left(1-P_c\right)\right)+1 = [N_{(f-j)}\left(1-P\right)_c]+1 = [N_{(f-j)}\left(1-P\right)_c]+1$.
- The min failing elements in fayer $(f-j+1)$ of one element in fayer$(f-j)$ is $M_{(f-j)} = g_2\left(xP_{\text{fail-}(f-j+1)}\right)$.
- Set $k_{(f-j)} = \{k_{(f-j)_1}, k_{(f-j)_2}\}$, $0 \le k_{(f-j)_1} \le m_{(f-j)}$ and $m_{(f-j)} < k_{(f-j)_2} \le Y_{(f-j)} - 1$.
- Define

  $$P_{\text{fail-}(f-j)} = \sum_{i=M_{(f-j)}-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)}^{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)} C^{i}_{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)}\left(P_{\text{fail-}(f-j+1)}\right)^{i}\left(1-P_{\text{fail-}(f-j+1)}\right)^{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)-i}$$ as

  the failure probability of any element in fayer$(f-j)$.

Then, set $P_{Y_{(f-i)}} = \dfrac{A_{Y_{(f-i+1)}}}{C^{\left(N_{(f-i+1)}\right)-1}_{\left(N_{(f-i+1)}\right)+Y_{(f-i+2)}-1}}$, $0 \le i \le j$,

$$P_{Y_{(f-j)}-k_{(f-j)}} = \frac{A_{Y_{(f-j)}-k_{(f-j)}}}{C_{Y_{(f-j)}}} \bullet \begin{cases} 1, & k_{(f-j)} = k_{(f-j)_1} \\ P_{m_{(f-j)}-\left(Y_{(f-j)}-k_{(f-j)}\right)}, & k_{(f-j)} = k_{(f-j)_2} \end{cases},$$

$$P_{m_{(f-j)}-\left(Y_{(f-j)}-k_{(f-j)}\right)} = \sum_{i=m_{(f-j)}-\left(Y_{(f-j)}-k_{(f-j)}\right)}^{N_{(f-j)}} C^{i}_{N_{(f-j)}-\left(Y_{(f-j)}-k_{(f-j)}\right)}\left(P_{\text{fail-}(f-j)}\right)^{i}\left(1-P_{\text{fail-}(f-j)}\right)^{\left(N_{(f-j)}-\left(Y_{(f-j)}-k_{(f-j)}\right)\right)-i},$$

and also

$$P_{\text{fail-}(f-j)} = \sum_{i=M_{(f-j)}-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)}^{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)} C^{i}_{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)}\left(P_{\text{fail-}(f-j+1)}\right)^{i}\left(1-P_{\text{fail-}(f-j+1)}\right)^{x-\left(Y_{(f-j+1)}-k_{(f-j+1)}\right)-i}.$$

In which, the possible kinds of combination for $Y_{(f-j)}-k_{(f-j)}$ elements in fayer $(f-j)$ is

$$
\begin{cases}
A_{Y_{(f-j)}} = C_{N_{(f-j)}}^{Y_{(f-j)}} \\
A_{Y_{(f-j)}-1} = C_{N_{(f-j)}}^{Y_{(f-j)}-1}\left(\sum_{i=0}^{M_{(f-j)}} C_{N_{(f-j)}-Y_{(f-j)}+0+M_{(f-j)}-i}^{N_{(f-j)}-Y_{(f-j)}+0} C_{Y_{(f-j)}-2+i}^{Y_{(f-j)}-2}\right) - C_{Y_{(f-j)}}^{Y_{(f-j)}-1} A_{Y_{(f-1)}} \\
\cdots \\
A_{Y_{(f-j)}-k_{(f-j)_1}} = C_{N_{(f-j)}}^{Y_{(f-j)}-k_{(f-j)_1}}\left(\sum_{i=0}^{k_{(f-j)_1}M_{(f-j)}} C_{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)_1}-1+k_{(f-j)_1}M_{(f-j)}-i}^{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)_1}-1} C_{Y_{(f-j)}-k_{(f-j)_1}-1+i}^{Y_{(f-j)}-k_{(f-j)_1}-1}\right) - \sum_{a=0}^{k_{(f-j)_1}-1} C_{Y_{(f-j)}-a}^{Y_{(f-j)}-k_{(f-j)_1}} A_{Y_{(f-j)}-a} \\
\cdots \\
A_{m_{(f-j)}} = C_{N_{(f-j)}}^{m_{(f-j)}}\left(\sum_{i=0}^{\left(Y_{(f-j)}-m_{(f-j)}\right)M_{(f-j)}} C_{N_{(f-j)}-m_{(f-j)}-1+\left(Y_{(f-j)}-m_{(f-j)}\right)M_{(f-j)}-i}^{N_{(f-j)}-m_{(f-j)}-1} C_{m_{(f-j)}-1+i}^{m_{(f-j)}-1}\right) - \sum_{a=0}^{\left(Y_{(f-j)}-m_{(f-j)}\right)-1} C_{Y_{(f-j)}-a}^{m_{(f-j)}} A_{Y_{(f-j)}-a} \\
\cdots \\
A_{Y_{(f-j)}-k_{(f-j)_2}} = C_{N_{(f-j)}}^{Y_{(f-j)}-k_{(f-j)_2}}\left(\sum_{i=0}^{k_{(f-j)_2}M_{(f-j)}} C_{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)_2}-1+k_{(f-j)_2}M_{(f-j)}-i}^{N_{(f-j)}-Y_{(f-j)}+k_{(f-j)_2}-1} C_{Y_{(f-j)}-k_{(f-j)_2}-1+i}^{Y_{(f-j)}-k_{(f-j)_2}-1}\right) - \sum_{a=0}^{k_{(f-j)_2}-1} C_{Y_{(f-j)}-a}^{Y_{(f-j)}-k_{(f-j)_2}} A_{Y_{(f-j)}-a} \\
\cdots \\
A_1 = C_{N_{(f-j)}}^{1}\left(\sum_{i=0}^{\left(Y_{(f-j)}-1\right)M_{(f-j)}} C_{N_{(f-j)}-2+\left(Y_{(f-j)}-1\right)M_{(f-j)}-i}^{N_{(f-j)}-2} \times 1\right) - \sum_{a=0}^{Y_{(f-j)}-2} C_{Y_{(f-j)}-a}^{1} A_{Y_{(f-j)}-a}
\end{cases}
.
$$

Finally, the probability of system failure in fayer $(f-j)$ is

$$
(\ -\ )\ \sum\left(\prod\ (\ -(\ -\ ))\ \ (\ -\ )^{-}\ (\ -\ )\right)
$$

$$
= \sum_{=}\left(\sum_{(\ -\ )=(\ -\ )}^{m\ (\ -\ )} (\ -\ )\left(\cdots \sum_{(\ -\ )=(\ -\ )}^{(\ -\ )} P_{Y_{(f-j)}}\left(\cdots \sum_{(\ -\ )=(\ -\ )}^{(\ -\ )} P_{Y_{(f-j)}}\left(\sum_{(\ -\ )=}^{(\ -\ )}\left(P_{Y_{(f-j)}-k_{(f-j)}}\right)\right)\cdots\right)\cdots\right)\right)
$$

(5.4)

For this *P*-out-of-1 system, it is equal to *K*-out-of-*N* systems with varying $K=m_{(f-j)}$ out of *N* in every fayer. Corresponding to the brief introduction before and back to the simple assumptions and problem classification, Class 1, Class 2 and Mix-Class. here are two kinds of problems, independent problems (Problem A, B, and C).

For the independent problems, the calculation can just accord to the equations in section 3, but for the correlative problems, the calculation should care the multi-dimensional Pearson product-moment correlation coefficient (PPCC), among elements in every fayer[145]. Also, the distribution of the parameter, in principle, should be a normal distribution.

Problem A, B for Class 1 and Class 2 separately

About the transportation/storage loss and screening problems (Problem A, B), take the fruit as examples (Table 5.2), which can also fit for some building materials, machine components. Here are some assumptions and standards:

(1) Suppose that in a batch of apples, there are 10000 apples.
(2) Divide these apples randomly into 100 bins, each bin filled with 100 apples.
(3) Then do inspections for this batch of apples. If in one bin, there are more than 5 apples are deteriorated, this bin is not qualified. Also, if there are more than 5 bins are defined unqualified, it is not qualified as well.

Table.5.2 Example shows the 2 fayer probability system

	Fayer 1	**Fayer 2**
Element scale	Big (B)	Small (S)
Element	Bin	Apple
Number of elements	100	10000

Problem A: the problem of surviving ratio (of the 1st class)

Every apple has a probability $P_{\text{fail}} = 0.005$ to be deteriorated in the process of transportation; calculate the probability of failure to meet the requirement in the assumption (3) of fayer 1 and fayer 2.

Problem B: the problem of passing ratio (of the 2nd class)

There are 50 deteriorated apples mixed in this batch blindly (Prior); calculate the probability of failure to meet the requirement in the assumption (3) of fayer 1 and fayer 2.

Both two problems show the equivalent to the problem of a *P*-out-of-1 system problem if using the *K*-out-of-*N* system description, in the example of problem B, suppose that surviving ratio or passing ratio is $P_c = 95\%$, so, in fayer 1, K=5 bins from N=100 bins, in fayer 2, K means 500 apples from N (10000 apples).

For the **problem A**, from calculation method in class 1 and problem description, there are: The elemental failure probability in fayer 2, $P_{\text{fail-2}}$=0.005; The elemental failure probability in fayer 1, $P_{\text{fail-1}}=1-\sum_{i=0}^{5} C_{100}^{i}\left(P_{\text{fail-2}}\right)^{i}\left(1-P_{\text{fail-2}}\right)^{100-i} \approx 1.246\times10^{-5}$.

Then: The probability of failure in fayer 2, $P_2=1-\sum_{i=0}^{500} C_{10000}^{i}\left(P_{\text{fail-2}}\right)^{i}\left(1-P_{\text{fail-2}}\right)^{10000-i}$;

The probability of failure in fayer 1, $P_1=1-\sum_{i=0}^{5} C_{100}^{i}\left(P_{\text{fail-1}}\right)^{i}\left(1-P_{\text{fail-1}}\right)^{100-i}$.

For the **Problem B**, from calculation method in class 2 and problem description, there are:

The prior failing elements in fayer 2, 50;

The max prior failing elements in fayer 1, $Y_1 = 50/5 = 10$;

The total combination of fayer 1, $C_{100+50-1}^{100-1} = C_{149}^{99}$.

Then: The probability of failure in fayer 2: $50/10000 = 0.5\% < 0.05$, so $P_2 = 0$;

The probability of failure in fayer 1: $P_1 = \left(A_{10} + A_9 + A_8 + A_7 + A_6 + A_5\right) / C_{149}^{99}$, where

$$\begin{cases} A_{10} = C_{100}^{10} \\ A_9 = C_{100}^{9}\left(\sum_{i=0}^{5} C_{95-i}^{90} C_{8+i}^{8}\right) - C_{10}^{9} A_{10} \\ A_8 = C_{100}^{8}\left(\sum_{i=0}^{10} C_{101-i}^{91} C_{7+i}^{7}\right) - C_{10}^{8} A_{10} - C_{9}^{8} A_9 \\ A_7 = C_{100}^{7}\left(\sum_{i=0}^{15} C_{107-i}^{92} C_{6+i}^{6}\right) - \sum_{a=0}^{2} C_{10-a}^{7} A_{10-a} \\ A_6 = C_{100}^{6}\left(\sum_{i=0}^{20} C_{113-i}^{93} C_{5+i}^{5}\right) - \sum_{a=0}^{3} C_{10-a}^{6} A_{10-a} \\ A_5 = C_{100}^{5}\left(\sum_{i=0}^{25} C_{119-i}^{94} C_{4+i}^{4}\right) - \sum_{a=0}^{4} C_{10-a}^{5} A_{10-a} \end{cases}.$$

Problem C for Mixed Class

There are some similar applications in industry, especially in the microelectronic field (Problem C), suppose that there are 10000 transistors, randomly but uniformly assembled into 100 groups, every group, there are 100 transistors, which forms a *P*-out-of-1 system. In this hierarchical system, there are 3 fayers (fayer 0, 1, 2).

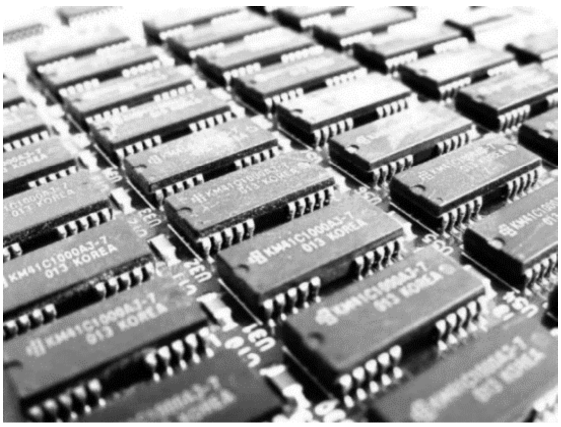


Figure 5.2. Example, the Integrated circuit used in CMOS (Version: KM41C10004J-7)

Suppose that failing possibility of every root element is P_{fail}=0.005, and there are 50 root elements as prior failed elements randomly distributed in the system before the operation period of system lifetime. In fayer 1, 2, 3, the elemental failing possibility is P_{f_1} , P_{f_2} , where $P_{f_2}=P_{\text{fail}}$. Calculate the probability of failure in fayer 1, 2 (i.e. P_1, P_2). Also, the qualified probability is P_c=95%. From calculation method and problem description above, the prior failing elements in fayer 2: 50; The failure probability of elements in fayer 2: $P_{\text{fail-2}}$=0.005; The max prior failing elements in fayer 1: Y_1=50/5=10; The total combinations of fayer 1 is $C_{100+50-1}^{100-1}=C_{149}^{99}$, and then:

$$
\left\{
\begin{aligned}
&A_{10}=C_{100}^{10}\\
&A_9=C_{100}^{9}\left(\sum_{i=0}^{5}C_{95-i}^{90}C_{8+i}^{8}\right)-C_{10}^{9}A_{10}\\
&A_8=C_{100}^{8}\left(\sum_{i=0}^{10}C_{101-i}^{91}C_{7+i}^{7}\right)-C_{10}^{8}A_{10}-C_{9}^{8}A_9\\
&A_7=C_{100}^{7}\left(\sum_{i=0}^{15}C_{107-i}^{92}C_{6+i}^{6}\right)-\sum_{a=0}^{2}C_{10-a}^{7}A_{10-a}\\
&A_6=C_{100}^{6}\left(\sum_{i=0}^{20}C_{113-i}^{93}C_{5+i}^{5}\right)-\sum_{a=0}^{3}C_{10-a}^{6}A_{10-a}\\
&A_5=C_{100}^{5}\left(\sum_{i=0}^{25}C_{119-i}^{94}C_{4+i}^{4}\right)-\sum_{a=0}^{4}C_{10-a}^{5}A_{10-a}\\
&A_4=C_{100}^{4}\left(\sum_{i=0}^{30}C_{125-i}^{95}C_{3+i}^{3}\right)-\sum_{a=0}^{5}C_{10-a}^{4}A_{10-a}\\
&A_3=C_{100}^{3}\left(\sum_{i=0}^{35}C_{131-i}^{96}C_{2+i}^{2}\right)-\sum_{a=0}^{6}C_{10-a}^{3}A_{10-a}\\
&A_2=C_{100}^{2}\left(\sum_{i=0}^{40}C_{137-i}^{97}C_{1+i}^{1}\right)-\sum_{a=0}^{7}C_{10-a}^{2}A_{10-a}\\
&A_1=C_{100}^{1}\left(\sum_{i=0}^{45}C_{143-i}^{98}C_{0+i}^{0}\right)-\sum_{a=0}^{8}C_{10-a}^{1}A_{10-a}\\
&A_0=0\ \text{for the reason that } 5/100=0.05=1-P_c
\end{aligned}
\right.
\Rightarrow
\left\{
\begin{aligned}
&P_{1,10}=\left(A_{10}/C_{149}^{99}\right)\bullet 1\\
&P_{1,9}=\left(A_9/C_{149}^{99}\right)\bullet 1\\
&P_{1,8}=\left(A_8/C_{149}^{99}\right)\bullet 1\\
&P_{1,7}=\left(A_7/C_{149}^{99}\right)\bullet 1\\
&P_{1,6}=\left(A_6/C_{149}^{99}\right)\bullet 1\\
&P_{1,5}=\left(A_5/C_{149}^{99}\right)\bullet 1\\
&P_{1,4}=\left(A_4/C_{149}^{99}\right)\bullet\sum_{k=1}^{96}C_{96}^{i}\left(P_{\text{fail-1}}^{i}\bullet(1\text{-}P_{\text{fail-1}})^{96-i}\right)\\
&P_{1,3}=\left(A_3/C_{149}^{99}\right)\bullet\sum_{k=2}^{97}C_{97}^{i}\left(P_{\text{fail-1}}^{i}\bullet(1\text{-}P_{\text{fail-1}})^{97-i}\right)\\
&P_{1,2}=\left(A_2/C_{149}^{99}\right)\bullet\sum_{k=3}^{98}C_{98}^{i}\left(P_{\text{fail-1}}^{i}\bullet(1\text{-}P_{\text{fail-1}})^{98-i}\right)\\
&P_{1,1}=\left(A_1/C_{149}^{99}\right)\bullet\sum_{k=4}^{99}C_{99}^{i}\left(P_{\text{fail-1}}^{i}\bullet(1\text{-}P_{\text{fail-1}})^{99-i}\right)\\
&P_{1,0}=0\bullet\sum_{k=5}^{100}C_{100}^{i}\left(P_{\text{fail-1}}^{i}\bullet(1\text{-}P_{\text{fail-1}})^{100-i}\right)=0
\end{aligned}
\right.;
$$

where $P_{\text{fail-1}}=1-\sum_{i=0}^{[100(1-P_c)]}C_{100}^{i}\left(P_{\text{fail-2}}\right)^{i}\left(1-P_{\text{fail-2}}\right)^{100-i}=1-\sum_{i=0}^{5}C_{100}^{i}\left(P_{\text{fail-2}}\right)^{i}\left(1-P_{\text{fail-2}}\right)^{100-i}$.

The probability of failure in fayer 2:

$$P_2=\sum_{i=g_2(10000(1-P_c))-50}^{10000\text{-}50}C_{10000\text{-}50}^{i}\left(0.005^{i}\bullet 0.995^{10000\text{-}50-i}\right)=\sum_{i=450}^{9950}C_{9950}^{i}\left(0.005^{i}\bullet 0.995^{9950-i}\right).$$

The probability of failure in fayer 1: $P_1=\sum_{i=0}^{10}P_{1,i}$. So, the reliability: $R_1=1-P_1, R_2=1-P_2$.

5.2 The hierarchical reliability problem of a foundation system (from Class 1)

In engineering structures, the foundation system is the most general system interacting with kinds of environment impacts (motions imported into the structure). For example, the earthquake, the loads from the structure, the action caused by underwater, the pressure of earth and rock. Such kinds of interactions will influence the health of sub-structural in lower fayers, and then change the whole structure. We need to concern the structural reliability in different fayers for maintenance that which part should be replaced and which part should be repaired.

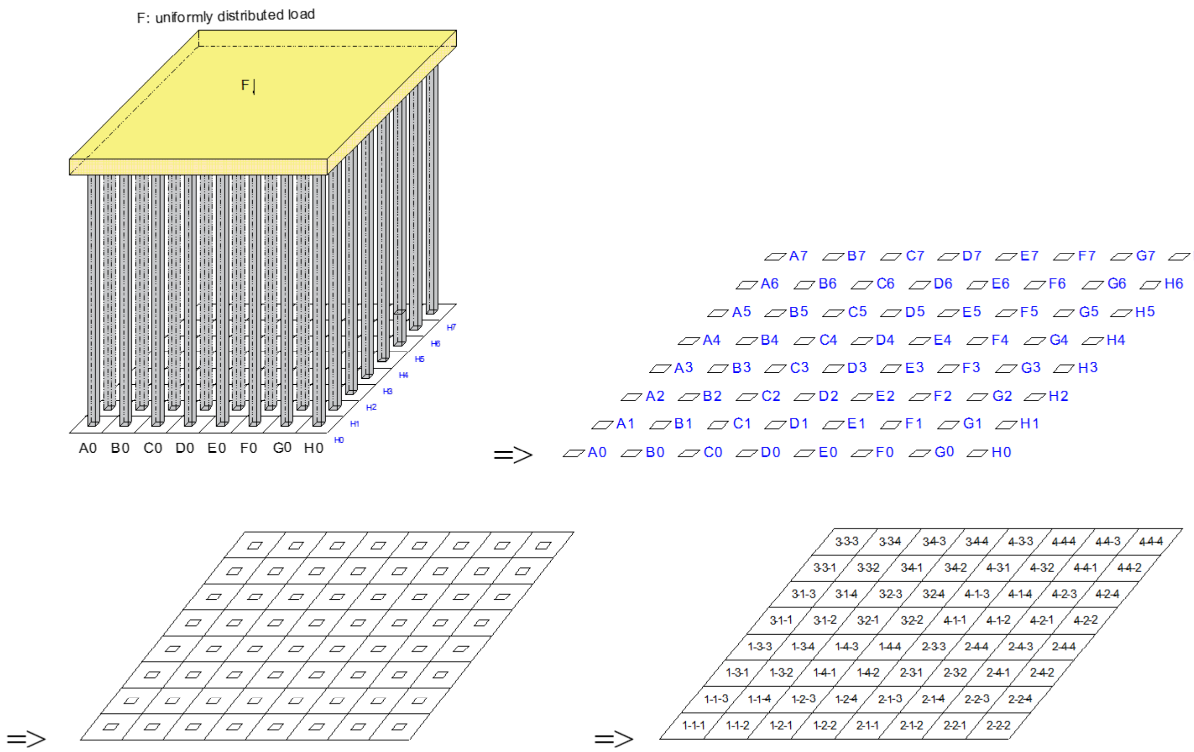


Figure 5.3. The equivalent system in concrete structure for the foundation system

As the definition of the structural reliability is based on the Joint distribution function $\mathrm{F}(r,s)$ but also Joint density function $f(r,s)$ of Load $(S, S=\{s\})$ a $(R, R=\{r\})$, $f_R(r)$ and $f_S(s)$ are the functions of the probability distribution of the load effects and resistance separately, then the failure probability at fayer f can be:

$$\begin{cases} f(r,s), \mathrm{F}(r,s) \\ P_f = P_{\text{fail}} = P\left((R-S)<0\right) = P\left(\dfrac{R}{S}<1\right) \end{cases} \tag{5.5}$$

$$P_{\text{fail}}=\iint_{r<s} f(r,\mathrm{s})d_r d_s=\int_0^{\infty}\left(\int_0^{s} f(r,\mathrm{s})d_r\right)d_s=\int_0^{\infty}\left(\int_0^{s} f_R(r)f_S(\mathrm{s})d_r\right)d_s$$
$$=\int_0^{\infty} f_S(\mathrm{s})\left(\int_0^{s} f_R(r)d_r\right)d_s=\int_0^{\infty} f_S(\mathrm{s})F_R(s)d_s \tag{5.6}$$

For the independent problems, the calculation can just accord to the equations in the previous subchapter 5.1, but for the correlative problems, the calculation should care the multi-dimensional PPCC among elements in every fayer[146]. Also, the distribution of the parameter, in principle, it should be normal distribution. Here is the example in civil engineering is the foundation system in Fig.5.3.

Elements in this example have indispensable inner associations. The Pile foundations are interconnected by soil and rock. Meanwhile, the influence is 3-dimensional. And also, every pile foundation's resistance and load effects of the environment are unknown. Also, these pile groups influence with each, they are non-independent but correlative.

Table 5.3. The failure probability between resistance and load effects in different fayers

P: failure probability between resistance and load effects		**Resistance (R)**				
		Fayer 0	Fayer 1	Fayer 2	…	Fayer f
Load effects (S)	Fayer 0	P(0,0)	P(0,1)	P(0,2)	…	P(0,f)
	Fayer 1	P(1,0)	P(1,1)	P(1,2)	…	P(1,f)
	Fayer 2	P(2,0)	P(2,1)	P(2,2)	…	P(2,f)
	…	…	…	…	…	…
	Fayer f	P(f,0)	P(f,1)	P(f,2)	…	P(f,f)

According to the hierarchical probability and same normalized limit, the calculation result will be different for different fayer. The calculations in different fayers in the system, the results will be different (Table 5.3). When doing decision-making based on reliability, there may be different values of the reliability of different elements in different fayer is different. In the same standard, the element chosen for replacement or maintenance in different fayer will be totally different. Then, just like the example given before in Fig.6.2, there are 4 fayers (fayer 0,1,2,3) in this concrete structural system, suppose the failing distribution of every root element and correlation coefficient (Pearson product-moment correlation coefficient, PPCC) a elements is the known.

Fayer 0: the whole system (only one element in this fayer);

Fayer 1: $\{1,2,3,4\}$;

Fayer 2: $\{1\text{-}1, 1\text{-}2, 1\text{-}3, 1\text{-}4, 2\text{-}1, 2\text{-}2, 2\text{-}3, 2\text{-}4, 3\text{-}1, 3\text{-}2, 3\text{-}3, 3\text{-}4, 4\text{-}1, 4\text{-}2, 4\text{-}3, 4\text{-}4\}$;

Fayer 3: $\{i\text{-}j\text{-}k \mid i, j, k \in \{1,2,3,4\}\}$, i.e. $\{1\text{-}1\text{-}1, 1\text{-}1\text{-}2, 1\text{-}1\text{-}3, 1\text{-}1\text{-}4, \ldots\}$ in total 4*4*4=64 elements.

Suppose every elemental failure probability distribution is Normal distribution and Rosenblatt transform, Orthogonal transform, or Nataf transform, etc. are used for the case the distribution of the variable is not normal distribution which is not considered in this research[147]. The object is to get the reliability of the system in fayer 3, $R(3,3)$, in fayer 2, $R(2,2)$, and in fayer 1, $R(1,1)$. The first issue for this problem is how to calculate the PPCC in different fayer. Here is an example, there is a 3-fayer system $\{\{x,y\},\{z,w\}\}$ (fayer 0, only one element, PPCC in fayer 1 is 1). The elements of fayer 1 are $\{x,y\},\{z,w\}$. The elements of fayer 2 are x,y,z,w. Every element has its own distribution or multi-dimensional joint distribution.

The PPCC in fayer 1 is $\rho_1 = \begin{bmatrix} \rho_{(x,y),(x,y)} & \rho_{(x,y),(z,w)} \\ \rho_{(z,w),(x,y)} & \rho_{(z,w),(z,w)} \end{bmatrix}$ using multidimensional scaling[148,149].

The PPCC in fayer 2 is $\rho_2 = \begin{bmatrix} \rho_{x,x} & \rho_{x,y} & \rho_{x,z} & \rho_{x,w} \\ \rho_{y,x} & \rho_{y,y} & \rho_{y,z} & \rho_{y,w} \\ \rho_{z,x} & \rho_{z,y} & \rho_{z,z} & \rho_{z,w} \\ \rho_{w,x} & \rho_{w,y} & \rho_{w,z} & \rho_{w,w} \end{bmatrix}$ using the general method[150], and also for the situation of non-normality[151].

Suppose in fayer F, the every elemental failure probability distribution is $X_k \sim N(\mu,\delta)$, $k \in \{1,2,3,\ldots,K\}$, its PPCC, of every two elements, X_{k_1} and X_{k_2} ($X_{k_1}, X_{k_2} \in \{X_k\}$) is ρ_{k_1,k_2}, in which $k_1 \in \{1,2,3,\ldots,K_1\}$, $k_2 \in \{1,2,3,\ldots,K_2\}$, $K_1 = K_2 \le K$, so there is:

$$\rho_F = \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \cdots & \rho_{1,K_2} \\ \rho_{2,1} & \rho_{2,2} & \cdots & \rho_{2,K_2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{K_1,1} & \rho_{K_1,1} & \cdots & \rho_{K_1,K_2} \end{bmatrix}.$$

Secondly, suppose $E^{(i)}, E^{(j)}$ $i = 1,2,3,\ldots$ $j = 1,2,3,\ldots$ are two arbitrary elements in fayer (F-1), which are structured by a certain rule of calculation of some elements in fayer F separately, to dilute the correlation coefficient between every two elements in fayer F; and in this paper, it uses weight accumulation.

$$U_i = a_1^{(i)} X_1^{(i)} + a_2^{(i)} X_2^{(i)} + \cdots + a_P^{(i)} X_P^{(i)} \triangleq \mathbf{a}^{(i)\prime} \boldsymbol{E}^{(i)},$$

$$V_j = b_1^{(j)} X_1^{(j)} + b_2^{(j)} X_2^{(j)} + \cdots + b_q^{(j)} X_q^{(j)} \triangleq \mathbf{b}^{(j)\prime} \boldsymbol{E}^{(j)}, \tag{5.7}$$

In w

$$\boldsymbol{E}^{\ i} = (X_1^{(i)}, X_2^{(i)}, \cdots, X_p^{(i)}), \mathbf{a}^{(i)} = \left[a_1^{(i)}, a_2^{(i)}, a_3^{(i)}, \ldots, a_p^{(i)} \right]',$$

$$\boldsymbol{E}^{(j)} = (X_1^{(j)}, X_2^{(j)}, \cdots, X_q^{(j)}), \mathbf{b}^{(i)} = \left[b_1^{(i)}, b_2^{(i)}, b_3^{(i)}, \ldots, b_q^{(i)} \right]'. \tag{5.8}$$

Then

$$\begin{cases} D(U) = D(\boldsymbol{a}'\boldsymbol{E}^{(i)}) = \boldsymbol{a}'\mathrm{Cov}(\boldsymbol{E}^{(i)}, \boldsymbol{E}^{(i)})\boldsymbol{a} = \boldsymbol{a}'\boldsymbol{\Sigma}_{ii}\boldsymbol{a} \\ D(V) = D(\boldsymbol{b}'\boldsymbol{E}^{(j)}) = \boldsymbol{b}'Cov(\boldsymbol{E}^{(j)}, \boldsymbol{E}^{(j)})\boldsymbol{b} = \boldsymbol{b}'\boldsymbol{\Sigma}_{jj}\boldsymbol{b} \\ \mathrm{Cov}(U,V) = \boldsymbol{a}'\mathrm{Cov}(\boldsymbol{E}^{(i)}, \boldsymbol{E}^{(j)})\boldsymbol{b} = \boldsymbol{a}'\boldsymbol{\Sigma}_{ij}\boldsymbol{b} \end{cases}$$
$$\Rightarrow \rho(U,V) = \frac{\mathrm{Cov}(U,V)}{\sqrt{D(U)}\sqrt{D(V)}} = \frac{\boldsymbol{a}'\boldsymbol{\Sigma}_{12}\boldsymbol{b}}{\sqrt{\boldsymbol{a}'\boldsymbol{\Sigma}_{11}\boldsymbol{a}}\sqrt{\boldsymbol{b}'\boldsymbol{\Sigma}_{22}\boldsymbol{b}}},$$
$$\Rightarrow \rho(U_i, V_j) = \max\left(\rho(U_i, V_j) \middle| \mathbf{a}^{(i)}, \mathbf{b}^{(j)} \right)$$

So

$$\rho_{\mathrm{F}} = \left\{ \rho(U_i, V_j) \middle| i, j = 1, 2, 3, \ldots \right\}, \tag{5.9}$$

In which $D(\bullet)$ means the variance of vectors and matrices, $\mathrm{Cov}(\bullet)$ means covariance, and $\rho(\bullet)$ means the PPCC.

Since the joint distribution of load effects S, and resistance R, suppose this joint distribution is the normal distribution, failure probability can be described as Equation (6.2).

Then, suppose this problem is in the 1st class of hierarchical reliability, the root elements have the failure probability $\mathbf{P}_{\text{fail-}(f-j)}$ a $\rho_{\text{fail-}\ f-j)}$, the failing reliability $P_{(f-j)}$ in the fayer $(f-j)$ with the standard limit P_c, $\mathrm{Pr}_{N_{(f-j)}}$ is the function for the

$P_{(f-j)}$ for $N_{(f-j)}$ elements in the fayer $(f-j)$:

$$P_{(f-j)} = \Pr_{N_{(f-j)}}\left(\mathbf{P}_{\text{fail-}(f-j)}, \rho_{\text{fail-}(f-j)}\right) = \left(1 - \sum_{i=0}^{g_1\left(P_c \bullet N_{(f-j)}\right)} C_{N_{(f-j)}}^{i} \mathbf{P}_{\text{fail-}(f-j)}^{i} \left(1 - \mathbf{P}_{\text{fail-}(f-j)}\right)^{N_{(f-j)}-i}\right)\Bigg|_{\rho_{\text{fail-}(f-j)}} \quad (5.10)$$

From Equation (5.10) and problem description, in fayer 3, the elemental failure probability is $\mathbf{P}_{\text{fail-3}}$, in which elements' failure probability often does not equal with each other, and the correlation coefficient of failing element i and element j is $\rho_{\text{fail-3},(i,j)}$. Define, the failure probability's PPCC of fayer 3:

$$\rho_{\text{fail-3}} = \begin{bmatrix} \rho_{\text{fail-3},(1,1)} & \rho_{\text{fail-3},(1,2)} & \cdots & \rho_{\text{fail-3},(1,j)} & \cdots & \rho_{\text{fail-3},(1,64)} \\ \rho_{\text{fail-3},(2,1)} & \rho_{\text{fail-3},(2,2)} & \cdots & \rho_{\text{fail-3},(1,j)} & \cdots & \rho_{\text{fail-3},(2,64)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{\text{fail-3},(i,1)} & \rho_{\text{fail-3},(i,2)} & \cdots & \rho_{\text{fail-3},(i,j)} & \cdots & \rho_{\text{fail-3},(i,64)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{\text{fail-3},(64,1)} & \rho_{\text{fail-3},(64,2)} & \cdots & \rho_{\text{fail-3},(64,j)} & \cdots & \rho_{\text{fail-3},(64,64)} \end{bmatrix} = \left[\rho_{\text{fail-3},(i,j)}\right], i, j = 1, 2, ..., 64.$$

In the fayer 2: $\mathbf{P}_{\text{fail-2}} = \Pr_4\left(\mathbf{P}_{\text{fail-3}}, \rho_{\text{fail-3}}\right)$, every element in this fayer has 4 elements of fayer 3, and for elements in fayer 2, define the correlation coefficient of fayer 2:

$$\rho_{\text{fail-2}} = \begin{bmatrix} \rho_{\text{fail-2},(1,1)} & \rho_{\text{fail-2},(1,2)} & \cdots & \rho_{\text{fail-2},(1,j)} & \cdots & \rho_{\text{fail-2},(1,16)} \\ \rho_{\text{fail-2},(2,1)} & \rho_{\text{fail-2},(2,2)} & \cdots & \rho_{\text{fail-2},(1,j)} & \cdots & \rho_{\text{fail-2},(2,16)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{\text{fail-2},(i,1)} & \rho_{\text{fail-2},(i,2)} & \cdots & \rho_{\text{fail-2},(i,j)} & \cdots & \rho_{\text{fail-2},(i,16)} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \rho_{\text{fail-2},(16,1)} & \rho_{\text{fail-2},(16,2)} & \cdots & \rho_{\text{fail-2},(16,j)} & \cdots & \rho_{\text{fail-2},(16,16)} \end{bmatrix} = \left[\rho_{\text{fail-2},(i,j)}\right], i, j = 1, 2, ..., 16,$$

in which $\rho_{\text{fail-2},(i,j)} = f\left(\rho_{\text{fail-3}}\right)$ means the correlation coefficient of element i and element j in fayer 2, which is the function of the failure probability's PPCC of elements in fayer 3.

In the fayer 1: $\mathbf{P}_{\text{fail-1}} = \Pr_4\left(\mathbf{P}_{\text{fail-2}}, \rho_{\text{fail-2}}\right)$, every element in this fayer has 4 elements of fayer 2, and for elements in fayer 1, define the correlation coefficient of fayer 2:

$$\rho_{\text{fail-1}}=\begin{bmatrix}\rho_{\text{fail-1},(1,1)} & \rho_{\text{fail-1},(1,2)} & \rho_{\text{fail-1},(1,3)} & \rho_{\text{fail-1},(1,4)}\\ \rho_{\text{fail-1},(2,1)} & \rho_{\text{fail-1},(2,2)} & \rho_{\text{fail-1},(2,3)} & \rho_{\text{fail-1},(2,4)}\\ \rho_{\text{fail-1},(3,1)} & \rho_{\text{fail-1},(3,2)} & \rho_{\text{fail-1},(3,3)} & \rho_{\text{fail-1},(3,4)}\\ \rho_{\text{fail-1},(4,1)} & \rho_{\text{fail-1},(4,2)} & \rho_{\text{fail-1},(4,3)} & \rho_{\text{fail-1},(4,4)}\end{bmatrix}=\left[\rho_{\text{fail-1},(i,j)}\right], i,j=1,2,...,4,$$

in which $\rho_{\text{fail-1},(i,j)}=f\left(\rho_{\text{fail-2}}\right)$ means the correlation coefficient of element i and element j in fayer 1, which is the function of the failure probability's PPCC of elements in fayer 2.

For different fayers, their failure probability and reliability are:

For fayer 3: $P(3,3)=\Pr_{64}\left(\mathbf{P}_{\text{fail-3}},\rho_{\text{fail-3}}\right)$, $R(3,3)=1-P(3,3)$,

where, in total 64 elements (matrix of 64*64);

For fayer 2: $P(2,2)=\Pr_{16}\left(\mathbf{P}_{\text{fail-2}},\rho_{\text{fail-2}}\right), R(2,2)=1-P(2,2)$,

where in total 16 elements (matrix of 16*16);

For fayer 1: $P(1,1)=\Pr_{4}\left(\mathbf{P}_{\text{fail-1}},\rho_{\text{fail-1}}\right)$, $R(1,1)=1-P(1,1)$,

where, in total 4 root elements (matrix of 4*4).

Furthermore, suppose in the fayer $(f-j)$, there are n elements, the P_{fail} can be calculated for some special cases by formulas[152]:

$$P_{(f-j)}=\Phi_n\left(-\beta;\rho\right) \tag{5.11}$$

in which, $\beta=\left(\beta_1,\beta_2,\beta_3,\ldots,\beta_i,\ldots,\beta_n\right)$, in short $\beta=\left(\beta_i\right)$, $\beta_i=\frac{\text{Expectation}}{\sqrt{\text{variance}}}$ $i=1,2,\ldots,n$ is the Reliability index, n is the number of sub-elements.

Suppose $\beta_i=\beta_e$, $\rho=\begin{bmatrix}1 & \rho_{1,2} & \cdots & \rho_{1,n}\\ \rho_{2,1} & 1 & \cdots & \rho_{2,n}\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{n,1} & \rho_{n,2} & \cdots & 1\end{bmatrix}=\left[\rho_{i,j}\right]; i,j=1,2,3\ldots n.$

If $\rho_{i,j}$ is constant, there is[153]:

$$P_{(f-j)} = \Phi_n(-\beta;\rho) = \int_{-\infty}^{+\infty} \varphi(t) \prod_{i=1}^{n} \Phi\left(\frac{\beta_i - t\sqrt{\rho}}{\sqrt{1-\rho}}\right) dt = \int_{-\infty}^{+\infty} \varphi(t) \left(\Phi\left(\frac{\beta_i - t\sqrt{\rho}}{\sqrt{1-\rho}}\right)\right)^n dt \qquad (5.12)$$

$\varphi(t)$ is the failure probability density function of normal distribution.

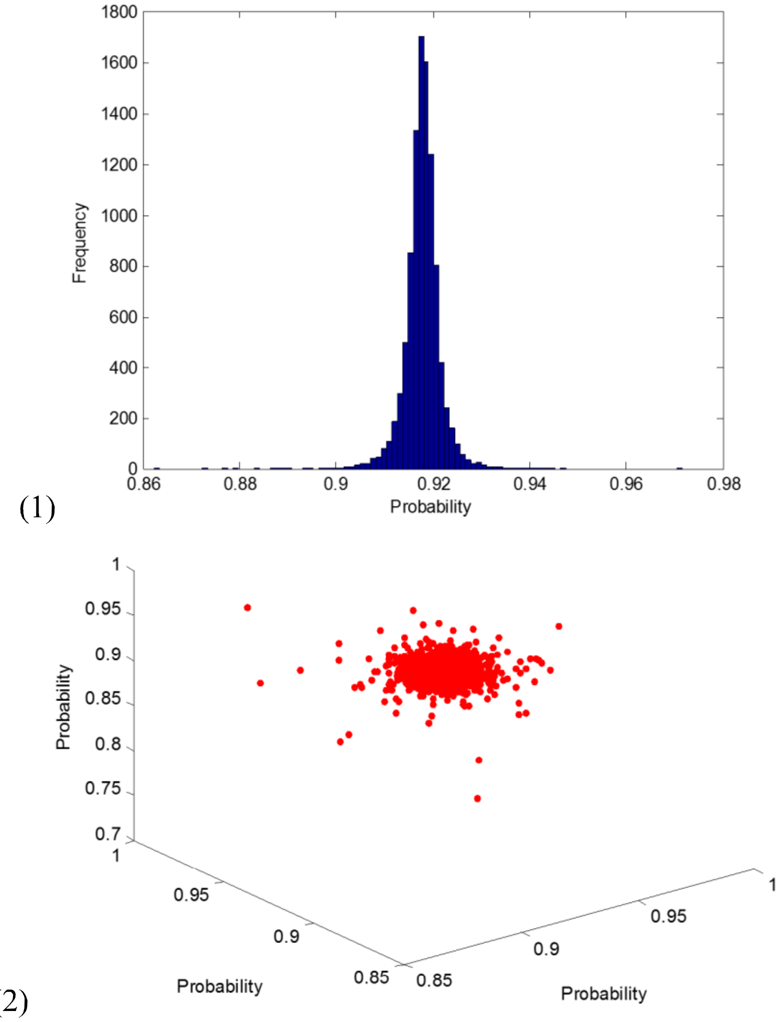


Figure 5.4. (1) Monte Carlo simulation for normal distribution of one element (expectation≈0.92), (2) Monte Carlo simulation for Joint Normal distribution for randomly selected three elements (3D)

Monte Carlo simulation: Experiment environment, Windows 8.1, MATLAB R2017a X64.

Since the precision of calculation for failing reliability $P_{(f-j)}$ in the fayer $(f-j)$ with PPCC, $\rho_{\text{fail-}(f-j)}$ sometimes will be hard to guarantee in practice, so Monte Carlo simulation (10000 times) is used to do simulation. As the problem description shows there are 64 elements in fayer 3, 16 elements in fayer 2, and 4 elements in fayer 1.

For every element in fayer 2, it contains 4 elements in fayer 3. For every element of fayer 1, it contains 4 elements of fayer 2. Suppose the capacity of risk defense is $P_c = 0.90$, and the expectation of every elemental failure probability is 0.92. For example, suppose the PPCC in every fayer refer to Table A2.1~A2.3 in appendix 2.

Example of one specific element, it has a distribution for reliability, e.g. Fig.5.4(1). If randomly choose 3 elements, there is joint distribution, e.g. Fig.5.4(2). After the simulation, the reliability of every fayer:

$R(3,3) = 0.9987$;

$R(2,2) = 0.9992$;

$R(1,1) = 0.9997$

5.3 Risk estimate for decision-making and engineering significance

Firstly, let's discuss the risks in decision making for these worked examples concerning the assessment itself.

In the problem A, the apples will get deteriorated in the process of the transportation. In general terms, with the increasing transportation distance or as time goes by in warehouse, the risk of the deterioration will increase. According to the calculation previously, the risk in different fayer will be different in this process.

For deeply understanding, in the ideal risk analysis for surviving ratio in hierarchy, some problems can be treated as independent and stationary incremental process (in problem A, suppose the number of the apples is large enough that the deterioration of every apple is independent, while the process of deterioration can be stationary that the distribution of deteriorated apples is independent identical distribution in every moment), which can be comprehended in the model of hierarchical Poisson process.

Meanwhile, hierarchical Poisson process is different from the conventional compound Poisson process that in hierarchical Poisson process, when describing/evaluating the same object, the same random variable in different fayers will influence the distribution with each other; while in compound Poisson process, the different random variables are independent but influence the same object as compound at the same time.

According to the relationship between the Binomial distribution and Poisson distribution[154], referring to the calculation in Section 5.1.3 (t), if n is great enough, usually, more than 100, while $\lambda = nP$ is not too big, usually less the Binomial distribution can be approximated as Poisson distribution.

$$C_n^m P^m (1-P)^{n-m} \approx \frac{\lambda^m}{m!} e^{-\lambda} \tag{5.13}$$

For failure of m element in root fayer in one moment, its distribution:

$$P^f \{M(t_0+t) - M(t_0) = m\} = P_m^f = \frac{(\lambda t)^m}{m!} e^{-\lambda t} \tag{5.14}$$

Whose expectation is λt .

In every fayer, these expectations will be different. If we want to get a reasonable decision-making for the cost of the loss in transportation, the risk caused by the hierarchy should not be ignored. If we set this expectation (λt) in of this distribution, that the increasing failing elements are incremental over time, as a constant in Problem B (pre-deteriorated apples) t to say, in the process of the apples' transportation, for every screening, there is $Y_f = \lambda t$, predetermined in the calculation in Section 5.1.3 (t). Then there is a no-aftereffect process (in such case, the surviving ratio is only related to the previous moment, while the loss is increasing), and the model will be transferred to model of hierarchical Markov process but not Martingale. And in this case, it belongs static Markov process.

If we do not use the expectation of this distribution, this problem will be transferred a new case, which is similar to Problem C (Section 5.1.3, type 3). Such kinds of research will expand the application for the influence of hierarchy on engineering or industry. This problem can be named as dynamic Markov process in hierarchy.

For every screening (survey for the passing ratio) in this process, there is a chance to get the result of failing to pass. Suppose the state in Markov process is corresponding to the number of the failing elements (Y_f). Then there are state i and state j :

i : there are $Y_f = m_i$ elements are failing to pass the screening. And the former states are $\{i_2, i_3, \ldots, i_k\}$ from the nearest state i_2 to the furthest reference state i_k in the past.

j : there are $Y_f = m_j$ elements are failing to pass the screening, and there is $m_j > m_i$.

Then, the probability of change from the state i to state j is:

$$P_{i \to j} = P\left(j \middle| i, i_2, i_3, \ldots, i_k\right) = \frac{P\left(j, i, i_2, i_3, \ldots, i_k\right)}{P\left(i, i_2, i_3, \ldots, i_k\right)} \tag{5.15}$$

When do the decision-making on the maintenance of the structure concerning the equilibrium of force flow, energy flow or other kinds of flow, which fayer to be considered, and which part to be maintained or replaced… these problems will be taken into consideration within the cost-benefit analysis.

In the example, since the hierarchical reliability can be used to help the decision-making in maintenance, different fayers have different risks under the unified standard and the maintenance of elements will be different. For example, in Fig.5.5, it shows elements not meeting the standard in every fayer, in which, the decision of element maintenance in Fayer 0 means the whole system is rebuilt or not.

Fayer 1: $\{3\}$;

Fayer 2: $\{$1-1, 2-3, 3-2, 3-3, 4-4$\}$;

Fayer 3: $\{$1-1-2, 1-1-3, 1-4-1, 2-2-2, 2-3-2, 2-3-4, 3-2-3, 3-2-4, 3-3-3, 3-3-4, 3-4-1, 4-4-2, 4-4-3$\}$

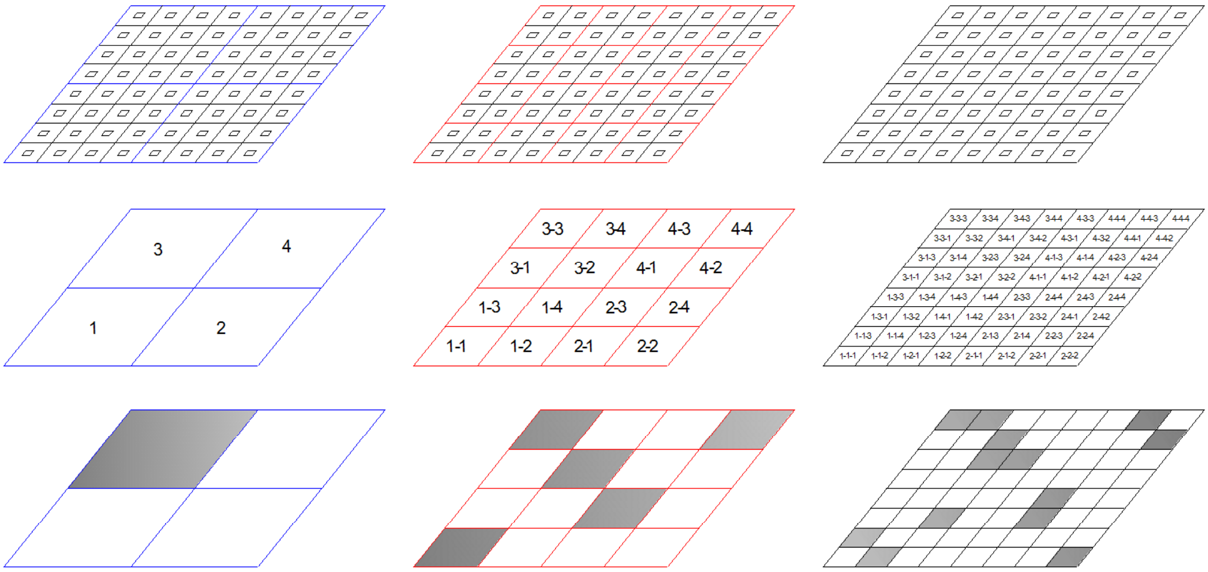


Figure 5.5. The 4-fayer hierarchical system, every element in fayer 1 contains 16 root elements, in fayer 2, it contains 4 root elements, and in fayer 3, it contains 1 root element (2nd layer); and the lattices (3rd layer) a

(unreliable)

Secondly, besides the risk assessment and reliability calculation, using various parameters in different fayers will also bring risks in the process of hierarchical risk assessment using hierarchical probability. Since the reliability evaluation will involve kinds of parameters in practice, and the investigations are often in inappropriate count.

For example, in the decision-making of maintenance for a hierarchical system with multi-failure modes [155] whose elements are independent. Suppose failure $P_{(f-j)}^{(i)} = g_i\left(x_i\big|_{(f-j)}\right), \mathbf{x} = \{x_1, x_1, x_1, \ldots, x_i, \ldots, x_n\}$ is caused by event x_i in fayer $(f-j)$. Generally, the result of the system failing in this fayer: $P_{(f-j)} = F\left(\mathbf{x}\big|_{(f-j)}\right)$. However, usually, in the investigation of parameters, it is hard to distinguish which parameter belongs to which fayer, so the calculating is often not according to its real fayer.

In a possible example in the diagrammatic sketch in Fig.5.6 (1), it shows, when the fayer changes from the higher to the lower, the reliability will decrease. One possible explanation is shown in Fig.5.6 (2), when the fayer changes from the higher to the lower, the overlapping of the distribution the load effects (S) a   (R) will increase, and reliability will also decrease. In details, according to the central limit theorem used in hierarchical probability; when the number of random variables increases, the distribution will become closer to a normal distribution, and the variance of this normal distribution is getting smaller and smaller simultaneously. In other words, in our study, as the system's fayer changes from the higher to the lower, the uncertainties of element are increased. As the system's fayer changes from the lower to the higher, the uncertainties of element are reduced. So, if both distributions are normal, when the fayer changes from the higher to the lower, the distribution of R and S will become flatter, suppose that the distance between expectations $\mu(s)$ and $\mu(r)$ will not change, the variances $\delta(s)$ and $\delta(r)$ both will increase and the overlap area will increase, so the reliability will decrease.

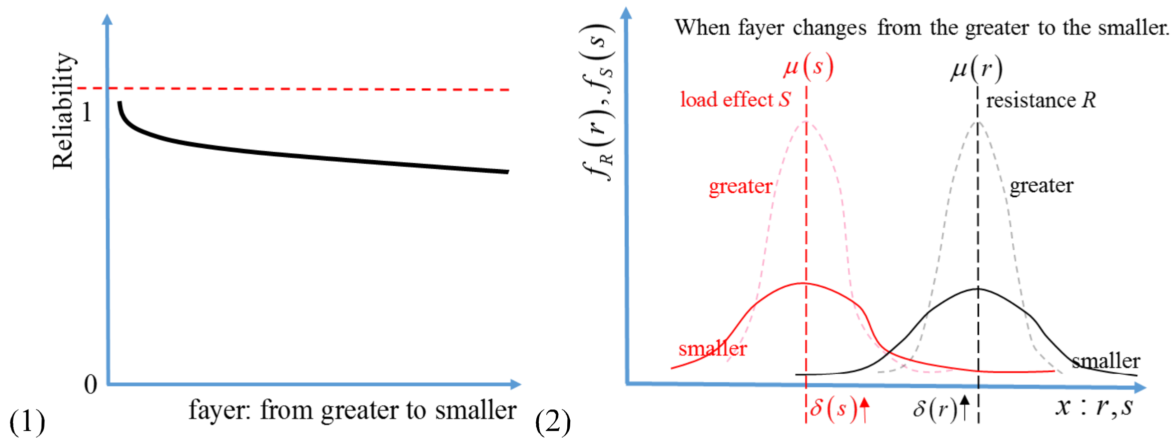


Figure 5.6. Diagrammatic sketch of a possible situation. In sub-figure (1) when the fayer grows from greater to smaller, the reliability will decrease. In sub-figure (2) when the fayer grows from greater to smaller, the load effects (S) a

In the example like problem D, for an element's failure probability in fayer $(f-j)$, $P_{(f-j)}$, if one parameter x_i, ($1 \le i \le n$) is not recognized in the right fayer$(f-j)$ but other fayer, then there are two situations:

Case 1: the x_i belongs to the greater fayer, and the reliability calculated is greater than the actual reliability, it will decrease the cost of maintenance but underestimate the real risk.

Case 2: the x_i belongs to the smaller fayer, and the reliability calculated is smaller than the actual reliability, it will increase the cost of maintenance by over-valuation of the actual risk.

Often, people would like to focus on one specific analysis method on some specific parameters in early warning system, and usually, the damage has risen to a certain extent, these parameters in different fayers are not consistent in prediction. Since the decision-making should care both security and economy at the same time, if use parameters in the fayer near but smaller than the fayer $(f-j)$, the balance between risk and maintenance cost will be reasonable. And if there is a process of hierarchical adjustment, the precision of the calculation result will be high with an optimization cost.

Thirdly, another risk exists in the maintenance program selection.

Since in every fayer, its cost for the replace or repair of different failed elements are different and the cost of the maintenance in different fayers are also different. The works for maintenance may contain the recognition of the failed elements and the replacement or repair. The risk of the maintenance will lead to an uncertain cost which asks for the staff's experience of operating/managing/evaluating….

Suppose the maintenance cost avoiding system failure in fayer $(f-j)$ is $Co_{(f-j)}$, and reliability improved (failing probability decreased) of the system is $\Delta P_{(f-j)} = P_{(f-j)} - P'_{(f-j)}$, $P'_{(f-j)}$ is the failure probability after maintenance. Usually, $P'_{(f-j)} < P_{(f-j)} \le 1 - P_c$.

In value engineering, suppose the benefit is $B\left(\Delta P_{(f-j)}\right)$, then the value is

$$v_{(f-j)} = \frac{B\left(\Delta P_{(f-j)}\right)}{Co_{(f-j)}}. \tag{5.11}$$

$$Co_{(f-j)} = f_{(f-j)}\left(N_{(f-j)}\right) + N_{(f-j)} c_{(f-j)}. \tag{5.12}$$

Where, in fayer $(f-j)$, $f_{(f-j)}\left(N_{(f-j)}\right)$ means the function of labor and other costs on the number of units replaced/installed/…, $N_{(f-j)}$ means the number of units replaced/installed/…, $c_{(f-j)}$ is the unit cost of element purchased. By comparing the values of maintenance in different fayers, there is an optimal choice.

For the case of Fig.5.4, if the benefit $B\left(\Delta P_{(f-j)}\right)$ is the same that the system can continue working healthily, there will be a solution for maintenance (replacement). Suppose:

$$f_1(N_1)=5N_1,\ f_2(N_2)=3N_2,\ f_3(N_3)=N_3;$$

$$N_1=1,\ N_2=5,\ N_3=13;\ c_1=16,\ c_2=3,\ c_3=1.$$

Then $Co_1=5\times1+1\times16=21, Co_2=3\times5+5\times3=30, Co_3=1\times13+13\times1=26$.

According to this calculation, the maintenance of the system in fayer 1 is the best choice for decision making. However, the risk for each kind of maintenance will be totally different that usually, the $B\left(\Delta P_{(f-j)}\right)$ will be different. $B\left(\Delta P_{(f-j)}\right)$ includes economic benefits, social benefits, environmental benefits, etc. It is so complex that often only the economic benefits will be covered in the calculation. On the other hand, according to the conservative estimate, the opportunity cost will increase substantially. Furthermore, $v_{(f-j)}$ itself asks for the functional analysis, and usually, the decision-making is based on incomplete information.

Lastly, about the application of risk avoidance in engineering, there are some corresponding countermeasures.

1. Different types of data should be categorized and summarized into different fayers to reduce the mixed-use of data due to human factors.
2. For the systematic assessment of a specific fayer influenced by the hierarchical interaction processing, the data measured should be appropriately near/in this fayer.
3. The decision-making should take the accuracy of the sampling test into account for the

reason that detection methods, instruments, and even the randomness or errors in measurement may sometimes change the result.

4. The setting of evaluation criteria should base on the specific condition of interaction processing for damage assessment. Under different circumstances and conditions, or different fayers, the eligibility criteria for different research targets should be set by statistics. Specially, in practice, for evaluation of *P*-out-of-1 system (*K*-out-of-*N* in hierarchy system), according to the requirement of a conservative estimate for the failure probability, comparing with the root fayer, in the higher fayer, *K* should be set slightly smaller than **[***P*N***],** where the values of *P* and *N* are known.
5. Avoid impulsive decision making. In the process of decision-making for risk estimation in maintenance, e.g. maintenance of the infrastructure, the comparison to find the highest risk item within a specific standard should be conducted by measuring data from multiple fayers.
6. Since that we do not know which level suited best as a level of decision making with only one fayer, in practice, maximally allowed by the conditions of measurement (limited by the equipment and environment, etc.), choose the measurable highest fayer and lowest fayer, as well as some other representative medium fayers between the highest fayer and the lowest fayer where the data can be relatively easily measured; synthesize them together to provide an overall evaluation of a system or obtain a comprehensive identification of local/global damage for decision making of maintenance that whether the repair or replacement of components will be conducted in different fayers in the system.
7. The maintenance process should focus on the minimization of total costs or avoiding opportunity cost when choosing components for repair/replacement/reconstruction (for components/subsystems/system separately) from the lower fayer to the higher fayer, that an objective function should be considered and algorithms should be developed as well.

5.4 Concluding remarks

When evaluating the influence of the interaction field on the structure, we found that interaction processing in different scales and in hierarchy, and the research on hierarchy has not been fully imported into the risk analysis for structural damage detection. Also, when conducting the maintenance, the structure will be analyzed in hierarchy (substructures, sub-sub-structures …), and the design of the structure is often of redundancy design. Then, the P-out-of-1 system is proposed as a new redundancy system for the evaluation in decision-making for maintenance. The problems related to P-out-of-1 system is concluded into three classes, and three typical examples and example related to the interaction field analysis are introduced.

Concerning the risk will help reduce their cost in maintenance, in the example, among 3 solutions, costs are 21, 30, 26, separately, if we do not care about the risk, it may be 21*1/3+30*1/3+26*1/3=25.66, which increase the **18.16%** in this case.

The K-out-of-N system in the hierarchy, or named as P-out-of-1 system can be acquired. In this research, there are 2 classes of hierarchical probability, (Class 1) surviving ratio and (Class 2) passing ratio, and mixed situation of two classes will be studied. Then, for the risk assessment using hierarchical reliability for decision-making, unreasonable parameter selection will bring a certain risk of loss, which is the second main purpose of this research. Then some suggestions for engineering application of hierarchical reliability are proposed when considering such kind of risk.

6. Conclusion and Future Work

In the lifetime of a structure, three kinds of information, structural information representing the change of the structure, environmental information indicating the variation of influence of environment to the structure, and interaction processing information reflecting the relationship between structure and its environment, could be recognized. High intensity of the interaction processing will lead the structure to get damaged, while low intensity can be used to detect the existence and the level of damage.

Concerning the interaction field analysis is mainly about the stability when the interaction propagation is in the structure. In

(Lagrangian description) or some dynamic field in mechanics (Eulerian description) ——that the motion or interaction, transferred into the system or structure by the environmental influence. By recognizing the characteristics of interaction processing and concentrating on its developing tendency, it is possible to indicate the system changes, which can be used to assess the damage of the structure. When the level of structural damage is high, the stability of the interaction processing through the system goes down, vice versa.

The main experiment objects are two nearly full-scale girders, investigations of which have been rarely conducted in past studies. In the experiment, the interaction field characteristics were investigated in data analysis of displacement and acceleration. The continuous aggravating damage was applied to the structure in different steps and stages. Static loading experiment and impact hammer experiment were conducted in sequence, which reduced bearing capacity (increased deformation) a

-locations.

Furthermore, in static experiment (static loading experiment and tendon cut), comparing result of the interaction field analysis and mechanical analysis on elastic coefficient, among the 9 stages, for box girder 2, the elastic coefficient's effective proportion is 4/10; while the 2D-CC's effective proportion is 9/10, so it improves **5/10**; for box girder 3, the elastic coefficient's effective proportion is 4/10; while the 2D-CC's effective proportion is 6/10, so it improves **2/10**. Also, in dynamic experiment (impact hammer experiment), comparing result of the interaction field analysis and model analysis, in kinds of characteristics of interaction processing, referring the same data measured in experiment on box girder 2, the best right representation of variables and characteristics for interaction field analysis and modal analysis for **6/8** and **5/8**, shows the interaction processing is better. Also, referring to the same data measured in the experiment on box girder 3, the best right representation of variables and characteristics for interaction field analysis and modal analysis **7/8** and **6/8**, shows the interaction field analysis is better. In the combined study, unfortunately, doped with a lot of damage indicators that are not particularly good, but the interaction field analysis still performs better than traditional modal analysis. For box girder 2, the combined results of all indicators have the right representation, **5 out of 8, 3 out of 8 changes,** for the interaction field analysis and the modal analysis separately. For box girder 3, the combined results of all indicators have the right representation, **5 out of 8, 4 out of 8 changes,** for the interaction field analysis and the modal analysis separately.

Moreover, in this research, hierarchical reliability is proposed to evaluate the risk for the measurement of hierarchical interaction. Since the interaction has its description of macro, meso, and micro, its influence on the structure has different scales corresponded. In this expanded study, the hierarchical probability is introduced into the reliability theory, concerning the evaluation of the complex system. For convenience in research, the unit, "fayer" is suggested as the unit of the hierarchy. Also, the P-out-of-1 system is proposed from the inspiration of the general K-out-of-N system, which describes the surviving capacity of the hierarchical system according to its definition of probability. In this system, the combinatorics is adopted to help people to understand the relationship among different fayers and to provide the calculation for the probability.

In this work, it was found the estimation of the failure probability of the whole system is greatly depending on the structure of hierarchy. The determination of the system reliability is related to the fayer defined by the analyst. In the view of hierarchical probability, even the same standard is given to all fayers, the results are different as well in every fayer.

There is a tendency in the complex system from fayer 1 to the root fayer, and according to some reference in scale analysis for reliability, it is monotonously declining. In different fayers, the individuals' associations are not the same. The correlation should be concerned with the calculation of hierarchical reliability for a complex system. Based on the hierarchical reliability, the decision-making for maintenance needs to provide kinds of strategies for the combination of maintaining different elements in different fayers using the benefit-cost analysis. In decision-making, there are three kinds of risks that should be considered. (1) The risk caused by the hierarchical system itself. (2) The risk caused by the parameter investigation. (3) The risk caused by incomplete information and the opportunity cost. The example shows, surveying the optimal cost will significantly reduce costs as much as **18.16%**.

This research strongly concerns the interaction processing in damage identification. It improves or clears three topics in structural damage detection and evaluation, i.e. the interaction field analysis on the motion propagation, characteristics developing tendency, and the hierarchical structural reliability on the response of the structure in interactions for maintenance.

For the author's academic plan, this research is the premier research for a theory on the information evaluation, named as Value of Information (VOI) for the global structural system.

There are some limitations in the research, firstly, it doesn't apply too much knowledge about the structural characteristics and environmental characteristics that have been studied for hundreds of years. Also, this research only considers a limited number of interaction field characteristics and it is hard to have a very comprehensive description of all characteristics of the global structural system that may benefit the research community, or even contribute to the whole society of science and engineering. Unless we have finished the multiple kinds of information of all structural, environmental and interaction information, it is able to have a comprehensive understanding of the global structural system.

About the future research on interaction processing (analysis, reliability, and response from the structure), there are some suggestions:

(1) For future researches, the local damage detection, system assessment in structural health monitoring using interaction field analysis should be tested. Or, the data detected by other methods of NDE or other experiments could be used in interaction field analysis to find more structural changing information. Through further study of interaction field analysis, the fluid and solid mechanics may have a unified framework that the application of methods for research fields from both kinds of mechanics can be crossed, and the discipline barriers between them could be reduced. Moreover, besides the qualitative analysis in IFC-based interaction field analysis, for better criterion, maybe the Analytic Hierarchy Process (AHP)

can be used in calculating.

(2) Moreover, for the case of hierarchical interactions, if the passing ratio is low, usually the surviving ratio is also low, and vice versa. In further research, the problem of correlative surviving ratio and passing ratio in the hierarchical system would be taken into consideration. Also, for the future work, different parameters used in different fayers or scales may build the relationships according to the principle of similitude and give a reasonable evaluation of the real system using the same standard.

(3) Furthermore, for the case of unfixed and even dynamic risk assessment, more in-depth discussions are needed.

As long-term research in the future, a theoretical paradigm for the framework of VOI-based analysis (method) should be proposed. The research in the thesis compared with its anticipatory framework only concerns a limited but typical case study on interaction processing information, hierarchical structure, experiment, etc. More effort and hard work are needed in the future.

There are still many things to do in the research. The flow field, or action field, is a very useful tool and is of great significance for summarizing, innovating, and summarizing the theory of non-destructive testing. As Mr. Sun Yat-sen said, "The revolution has not been successful, and comrades still need to work hard." To realize my dreams in science and engineering, I will do my best in the future!

Appendix

Appendix 1

Table A1.1 Inspiration and characteristics of the wave for damage diagnosis.

Characteristics of wave		**Damage diagnosis**	
Sound (voice), song	**Wave**	**Analyzing Methods**	**Data processing**
Loudness	Amplitude/wavelength	In	The data in
Acoustic pressure	Distance, sound pressure or In		interaction field analysis is often in the matrix, evaluated by
Auditory impression	Direction and Speed of sound (velocity)		2-D correlation coefficient, and
Tone	Frequency	-history	Distance;
Musical sound or noise	Harmonic wave	Response; …	For statistics: Expectation,
Timbre	Wave pattern	Extreme Analysis;	Variance, Mode,
Pitch interval	Wave interval	Pattern recognition;	Median;
		Analysis of similarity/difference …	Etc.
Concerto, Symphony		Principal component analysis; Association analysis …	

Appendix 2

Table A2.1. The correlation coefficient among elements (E1~E64) of fayer 3

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	E14	E15	E16	E17	E18	E19	E20	E21	E22
E1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E2	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.13
E3	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E4	0.00	0.00	0.00	1.00	0.10	0.89	0.32	0.00	0.00	0.65	0.52	0.00	0.57	0.93	0.00	0.00	0.41	0.28	0.00	0.71	0.00	0.27
E5	0.00	0.00	0.00	0.10	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	0.00	0.27	0.94	0.91	0.00	0.98	0.25	0.48	0.00
E6	0.00	0.00	0.00	0.89	0.00	1.00	0.33	0.00	0.00	0.58	0.36	0.00	0.47	0.89	0.00	0.00	0.26	0.11	0.00	0.61	0.00	0.10
E7	0.00	0.00	0.00	0.32	0.00	0.33	1.00	0.00	0.00	0.12	0.00	0.00	0.11	0.32	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00
E8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E10	0.00	0.00	0.00	0.65	0.00	0.58	0.12	0.00	0.00	1.00	0.50	0.00	0.30	0.64	0.16	0.00	0.24	0.35	0.00	0.48	0.00	0.33
E11	0.00	0.00	0.00	0.52	0.00	0.36	0.00	0.00	0.00	0.50	1.00	0.00	0.23	0.42	0.36	0.00	0.29	0.95	0.00	0.60	0.00	0.94
E12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E13	0.00	0.00	0.00	0.57	0.80	0.47	0.11	0.00	0.00	0.30	0.23	0.00	1.00	0.48	0.21	0.70	0.89	0.00	0.75	0.54	0.33	0.00
E14	0.00	0.00	0.00	0.93	0.00	0.89	0.32	0.00	0.00	0.64	0.42	0.00	0.48	1.00	0.00	0.00	0.29	0.17	0.00	0.63	0.00	0.16
E15	0.00	0.00	0.00	0.00	0.27	0.00	0.00	0.00	0.00	0.16	0.36	0.00	0.21	0.00	1.00	0.26	0.30	0.40	0.25	0.14	0.00	0.39
E16	0.00	0.00	0.00	0.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.00	0.26	1.00	0.84	0.00	0.93	0.17	0.47	0.00
E17	0.00	0.00	0.00	0.41	0.91	0.26	0.00	0.00	0.00	0.24	0.29	0.00	0.89	0.29	0.30	0.84	1.00	0.21	0.87	0.48	0.39	0.21
E18	0.00	0.12	0.00	0.28	0.00	0.11	0.00	0.00	0.00	0.35	0.95	0.00	0.00	0.17	0.40	0.00	0.21	1.00	0.00	0.46	0.00	0.99
E19	0.00	0.00	0.00	0.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00	0.25	0.93	0.87	0.00	1.00	0.20	0.48	0.00
E20	0.00	0.00	0.00	0.71	0.25	0.61	0.22	0.00	0.00	0.48	0.60	0.00	0.54	0.63	0.14	0.17	0.48	0.46	0.20	1.00	0.00	0.45
E21	0.00	0.00	0.00	0.00	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.00	0.47	0.39	0.00	0.48	0.00	1.00	0.00
E22	0.00	0.13	0.00	0.27	0.00	0.10	0.00	0.00	0.00	0.33	0.94	0.00	0.00	0.16	0.39	0.00	0.21	0.99	0.00	0.45	0.00	1.00
E23	0.00	0.00	0.00	0.23	0.76	0.00	0.00	0.00	0.00	0.19	0.60	0.00	0.66	0.00	0.46	0.70	0.83	0.63	0.73	0.46	0.33	0.63
E24	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E25	0.00	0.12	0.00	0.28	0.00	0.11	0.00	0.00	0.00	0.35	0.95	0.00	0.10	0.17	0.39	0.00	0.23	0.99	0.00	0.46	0.00	0.98
E26	0.14	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E27	0.00	0.12	0.00	0.30	0.00	0.13	0.00	0.00	0.00	0.35	0.96	0.00	0.11	0.19	0.39	0.00	0.22	0.99	0.00	0.47	0.00	0.98
E28	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E29	0.00	0.00	0.00	0.11	0.40	0.00	0.00	0.00	0.00	0.10	0.36	0.00	0.34	0.00	0.29	0.37	0.44	0.39	0.38	0.27	0.16	0.39
E30	0.00	0.13	0.00	0.26	0.00	0.00	0.00	0.00	0.00	0.33	0.93	0.00	0.00	0.17	0.38	0.00	0.19	0.98	0.00	0.43	0.00	0.97
E31	0.10	0.00	0.00	0.37	0.00	0.35	0.22	0.00	0.00	0.26	0.00	0.17	0.20	0.36	0.00	0.00	0.11	0.00	0.00	0.27	0.00	0.00
E32	0.00	0.11	0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.27	0.80	0.00	0.00	0.12	0.31	0.00	0.13	0.85	0.00	0.36	0.00	0.84
E33	0.00	0.00	0.00	0.61	0.00	0.60	0.29	0.00	0.00	0.42	0.17	0.00	0.28	0.63	0.00	0.00	0.11	0.00	0.00	0.35	0.00	0.00
E34	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.11	0.35	0.31	0.00	0.33	0.00	0.18	0.00
E35	0.00	0.00	0.00	0.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00	0.16	0.70	0.67	0.00	0.72	0.16	0.37	0.00
E36	0.00	0.00	0.00	0.60	0.00	0.57	0.26	0.00	0.00	0.35	0.12	0.00	0.26	0.59	0.00	0.00	0.00	0.00	0.00	0.39	0.00	0.00
E37	0.00	0.00	0.00	0.26	0.43	0.15	0.00	0.00	0.00	0.25	0.42	0.00	0.43	0.17	0.36	0.39	0.51	0.41	0.40	0.37	0.16	0.42
E38	0.00	0.00	0.00	0.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00	0.27	0.87	0.83	0.00	0.90	0.19	0.45	0.00
E39	0.00	0.00	0.00	0.27	0.00	0.24	0.00	0.00	0.00	0.22	0.00	0.00	0.17	0.25	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00
E40	0.00	0.14	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.33	0.93	0.00	0.00	0.15	0.38	0.00	0.19	0.98	0.00	0.44	0.00	0.97
E41	0.00	0.13	0.00	0.24	0.00	0.00	0.00	0.00	0.00	0.31	0.91	0.00	0.16	0.13	0.41	0.00	0.28	0.95	0.00	0.44	0.00	0.94
E42	0.00	0.00	0.00	0.86	0.00	0.82	0.30	0.00	0.00	0.56	0.39	0.00	0.43	0.85	0.00	0.00	0.25	0.16	0.00	0.59	0.00	0.15
E43	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.11	0.00	0.15	0.00
E44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.12	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E46	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.16	0.40	0.38	0.12	0.41	0.15	0.20	0.12
E47	0.00	0.00	0.00	0.00	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00	0.17	0.38	0.33	0.00	0.38	0.00	0.18	0.00
E48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E49	0.00	0.00	0.00	0.36	0.00	0.36	0.21	0.00	0.00	0.24	0.00	0.00	0.22	0.38	0.00	0.00	0.11	0.00	0.00	0.17	0.00	0.00
E50	0.00	0.12	0.00	0.29	0.00	0.12	0.00	0.00	0.00	0.35	0.96	0.00	0.12	0.18	0.41	0.00	0.25	1.00	0.00	0.47	0.00	0.99
E51	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.25	0.15	0.00	0.27	0.00	0.10	0.00
E52	0.00	0.12	0.00	0.28	0.00	0.12	0.00	0.00	0.00	0.35	0.94	0.00	0.00	0.18	0.39	0.00	0.20	0.98	0.00	0.45	0.00	0.97
E53	0.00	0.11	0.00	0.29	0.18	0.11	0.00	0.00	0.00	0.34	0.94	0.00	0.24	0.17	0.44	0.14	0.38	0.98	0.15	0.49	0.00	0.97
E54	0.00	0.00	0.00	0.17	0.13	0.00	0.00	0.00	0.00	0.18	0.43	0.00	0.15	0.12	0.21	0.12	0.24	0.44	0.11	0.22	0.00	0.44
E55	0.00	0.00	0.00	0.17	0.91	0.00	0.00	0.00	0.00	0.10	0.36	0.00	0.74	0.00	0.40	0.85	0.91	0.38	0.88	0.39	0.41	0.38
E56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E57	0.00	0.12	0.00	0.21	0.12	0.17	0.00	0.00	0.00	0.16	0.16	0.00	0.20	0.15	0.00	0.10	0.18	0.10	0.00	0.18	0.00	0.11
E58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.12	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E59	0.00	0.00	0.00	0.55	0.00	0.57	0.24	0.00	0.00	0.35	0.00	0.00	0.27	0.57	0.00	0.00	0.12	0.00	0.00	0.29	0.00	0.00
E60	0.00	0.00	0.00	0.76	0.19	0.75	0.31	0.00	0.00	0.46	0.17	0.00	0.56	0.74	0.00	0.13	0.39	0.00	0.14	0.50	0.00	0.00
E61	0.00	0.00	0.00	0.40	0.00	0.41	0.19	0.00	0.00	0.20	0.00	0.00	0.27	0.41	0.00	0.00	0.14	0.00	0.00	0.24	0.00	0.00
E62	0.00	0.00	0.00	0.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.00	0.30	0.90	0.88	0.00	0.93	0.25	0.43	0.00
E63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E64	0.00	0.00	0.00	0.85	0.46	0.80	0.27	0.00	0.00	0.50	0.21	0.00	0.80	0.81	0.00	0.36	0.64	0.00	0.39	0.61	0.14	0.00

	E23	E24	E25	E26	E27	E28	E29	E30	E31	E32	E33	E34	E35	E36	E37	E38	E39	E40	E41	E42	E43
E1	0.00	0.00	0.00	0.14	0.00	0.11	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E2	0.00	0.12	0.12	0.00	0.12	0.00	0.00	0.13	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.13	0.00	0.00
E3	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E4	0.23	0.00	0.28	0.00	0.30	0.00	0.11	0.26	0.37	0.21	0.61	0.00	0.00	0.60	0.26	0.00	0.27	0.25	0.24	0.86	0.00
E5	0.76	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.00	0.00	0.35	0.74	0.00	0.43	0.92	0.00	0.00	0.00	0.00	0.11
E6	0.00	0.00	0.11	0.00	0.13	0.00	0.00	0.00	0.35	0.00	0.60	0.00	0.00	0.57	0.15	0.00	0.24	0.00	0.00	0.82	0.00
E7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.29	0.00	0.00	0.26	0.00	0.00	0.00	0.00	0.00	0.30	0.00
E8	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E9	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E10	0.19	0.00	0.35	0.00	0.35	0.00	0.10	0.33	0.26	0.27	0.42	0.00	0.00	0.35	0.25	0.00	0.22	0.33	0.31	0.56	0.00
E11	0.60	0.00	0.95	0.00	0.96	0.00	0.36	0.93	0.00	0.80	0.17	0.00	0.00	0.12	0.42	0.00	0.00	0.93	0.91	0.39	0.00
E12	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E13	0.66	0.00	0.10	0.00	0.11	0.00	0.34	0.00	0.20	0.00	0.28	0.25	0.61	0.26	0.43	0.69	0.17	0.00	0.16	0.43	0.00
E14	0.00	0.00	0.17	0.00	0.19	0.00	0.00	0.17	0.36	0.12	0.63	0.00	0.00	0.59	0.17	0.00	0.25	0.15	0.13	0.85	0.00
E15	0.46	0.00	0.39	0.00	0.39	0.00	0.29	0.38	0.00	0.31	0.00	0.11	0.16	0.00	0.36	0.27	0.00	0.38	0.41	0.00	0.00
E16	0.70	0.00	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.00	0.00	0.35	0.70	0.00	0.39	0.87	0.00	0.00	0.00	0.00	0.12
E17	0.83	0.00	0.23	0.00	0.22	0.00	0.44	0.19	0.11	0.13	0.11	0.31	0.67	0.00	0.51	0.83	0.00	0.19	0.28	0.25	0.00
E18	0.63	0.00	0.99	0.00	0.99	0.00	0.39	0.98	0.00	0.85	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.98	0.95	0.16	0.00
E19	0.73	0.00	0.00	0.00	0.00	0.00	0.38	0.00	0.00	0.00	0.00	0.33	0.72	0.00	0.40	0.90	0.00	0.00	0.00	0.00	0.11
E20	0.46	0.00	0.46	0.00	0.47	0.00	0.27	0.43	0.27	0.36	0.35	0.00	0.16	0.39	0.37	0.19	0.17	0.44	0.44	0.59	0.00
E21	0.33	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	0.00	0.18	0.37	0.00	0.16	0.45	0.00	0.00	0.00	0.00	0.15
E22	0.63	0.00	0.98	0.00	0.98	0.00	0.39	0.97	0.00	0.84	0.00	0.00	0.00	0.00	0.42	0.00	0.00	0.97	0.94	0.15	0.00
E23	1.00	0.00	0.64	0.00	0.63	0.00	0.57	0.61	0.00	0.50	0.00	0.22	0.50	0.00	0.59	0.72	0.00	0.61	0.68	0.00	0.00
E24	0.00	1.00	0.00	0.11	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E25	0.64	0.00	1.00	0.00	0.99	0.00	0.39	0.98	0.00	0.85	0.00	0.00	0.00	0.00	0.42	0.00	0.00	0.98	0.95	0.16	0.00
E26	0.00	0.11	0.00	1.00	0.00	0.11	0.00	0.00	0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E27	0.63	0.00	0.99	0.00	1.00	0.00	0.39	0.98	0.00	0.85	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.98	0.95	0.18	0.00
E28	0.00	0.00	0.00	0.11	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E29	0.57	0.00	0.39	0.00	0.39	0.00	1.00	0.38	0.00	0.31	0.00	0.13	0.27	0.00	0.36	0.42	0.00	0.38	0.41	0.00	0.00
E30	0.61	0.00	0.98	0.00	0.98	0.00	0.38	1.00	0.00	0.83	0.00	0.00	0.00	0.00	0.38	0.00	0.00	0.96	0.93	0.14	0.00
E31	0.00	0.11	0.00	0.29	0.00	0.00	0.00	0.00	1.00	0.00	0.20	0.00	0.00	0.39	0.00	0.00	0.14	0.00	0.00	0.33	0.00
E32	0.50	0.00	0.85	0.00	0.85	0.00	0.31	0.83	0.00	1.00	0.00	0.00	0.00	0.00	0.37	0.00	0.00	0.83	0.80	0.10	0.00
E33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	1.00	0.00	0.00	0.37	0.00	0.00	0.20	0.00	0.00	0.55	0.00
E34	0.22	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00	1.00	0.26	0.00	0.17	0.31	0.00	0.00	0.00	0.00	0.00
E35	0.50	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.00	0.00	0.00	0.26	1.00	0.00	0.26	0.67	0.00	0.00	0.00	0.00	0.12
E36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00	0.37	0.00	0.00	1.00	0.00	0.00	0.17	0.00	0.00	0.59	0.00
E37	0.59	0.00	0.42	0.00	0.41	0.00	0.36	0.38	0.00	0.37	0.00	0.17	0.26	0.00	1.00	0.41	0.00	0.38	0.42	0.13	0.00
E38	0.72	0.00	0.00	0.00	0.00	0.00	0.42	0.00	0.00	0.00	0.00	0.31	0.67	0.00	0.41	1.00	0.00	0.00	0.11	0.00	0.12
E39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.20	0.00	0.00	0.17	0.00	0.00	1.00	0.00	0.00	0.27	0.00
E40	0.61	0.00	0.98	0.00	0.98	0.00	0.38	0.96	0.00	0.83	0.00	0.00	0.00	0.00	0.38	0.00	0.00	1.00	0.94	0.14	0.00
E41	0.68	0.00	0.95	0.00	0.95	0.00	0.41	0.93	0.00	0.80	0.00	0.00	0.00	0.00	0.42	0.11	0.00	0.94	1.00	0.12	0.00
E42	0.00	0.00	0.16	0.00	0.18	0.00	0.00	0.14	0.33	0.10	0.55	0.00	0.00	0.59	0.13	0.00	0.27	0.14	0.12	1.00	0.00
E43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.12	0.00	0.00	0.00	0.00	1.00
E44	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E46	0.39	0.00	0.12	0.00	0.12	0.00	0.25	0.11	0.00	0.00	0.00	0.16	0.30	0.00	0.15	0.38	0.00	0.12	0.15	0.00	0.00
E47	0.29	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00	0.15	0.28	0.00	0.14	0.36	0.00	0.00	0.00	0.00	0.00
E48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.22	0.12	0.00	0.15	0.00	0.00	0.34	0.00
E50	0.66	0.00	1.00	0.00	1.00	0.00	0.40	0.98	0.00	0.85	0.00	0.00	0.00	0.00	0.43	0.00	0.00	0.98	0.96	0.16	0.00
E51	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.26	0.00	0.00	0.00	0.00	0.00
E52	0.61	0.00	0.97	0.00	0.98	0.00	0.37	0.96	0.00	0.83	0.00	0.00	0.00	0.00	0.40	0.00	0.00	0.96	0.93	0.17	0.00
E53	0.76	0.00	0.98	0.00	0.98	0.00	0.46	0.96	0.00	0.83	0.00	0.00	0.00	0.00	0.48	0.19	0.00	0.96	0.96	0.15	0.00
E54	0.38	0.00	0.44	0.00	0.44	0.00	0.17	0.43	0.00	0.42	0.00	0.00	0.00	0.00	0.26	0.15	0.00	0.44	0.43	0.11	0.00
E55	0.94	0.00	0.39	0.00	0.38	0.00	0.52	0.36	0.00	0.28	0.00	0.30	0.64	0.00	0.55	0.85	0.00	0.37	0.45	0.00	0.00
E56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E57	0.15	0.00	0.10	0.00	0.11	0.00	0.10	0.10	0.26	0.00	0.00	0.00	0.00	0.16	0.00	0.12	0.00	0.00	0.11	0.18	0.00
E58	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
E59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.39	0.00	0.00	0.43	0.00	0.00	0.14	0.00	0.00	0.51	0.00
E60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.53	0.00	0.20	0.51	0.11	0.11	0.23	0.00	0.00	0.69	0.00
E61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00	0.28	0.00	0.00	0.31	0.00	0.00	0.00	0.00	0.00	0.38	0.00
E62	0.77	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.00	0.00	0.00	0.33	0.70	0.00	0.43	0.88	0.00	0.00	0.16	0.00	0.00
E63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E64	0.30	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.34	0.00	0.54	0.13	0.38	0.54	0.27	0.35	0.28	0.00	0.00	0.75	0.00

	E44	E45	E46	E47	E48	E49	E50	E51	E52	E53	E54	E55	E56	E57	E58	E59	E60	E61	E62	E63	E64
E1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E2	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.12	0.11	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E4	0.00	0.00	0.00	0.00	0.00	0.36	0.29	0.00	0.28	0.29	0.17	0.17	0.00	0.21	0.00	0.55	0.76	0.40	0.00	0.00	0.85
E5	0.00	0.00	0.40	0.38	0.00	0.00	0.00	0.26	0.00	0.18	0.13	0.91	0.00	0.12	0.00	0.00	0.19	0.00	0.95	0.00	0.46
E6	0.00	0.00	0.00	0.00	0.00	0.36	0.12	0.00	0.12	0.11	0.00	0.00	0.00	0.17	0.00	0.57	0.75	0.41	0.00	0.00	0.80
E7	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.31	0.19	0.00	0.00	0.27
E8	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
E9	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	0.00	0.00	0.00	0.00	0.00	0.00
E10	0.00	0.00	0.00	0.00	0.00	0.24	0.35	0.00	0.35	0.34	0.18	0.10	0.00	0.16	0.12	0.35	0.46	0.20	0.00	0.00	0.50
E11	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.00	0.94	0.94	0.43	0.36	0.00	0.16	0.00	0.00	0.17	0.00	0.00	0.00	0.21
E12	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E13	0.00	0.00	0.32	0.24	0.00	0.22	0.12	0.11	0.00	0.24	0.15	0.74	0.00	0.20	0.00	0.27	0.56	0.27	0.75	0.00	0.80
E14	0.00	0.00	0.00	0.00	0.00	0.38	0.18	0.00	0.18	0.17	0.12	0.00	0.00	0.15	0.00	0.57	0.74	0.41	0.00	0.00	0.81
E15	0.00	0.00	0.16	0.17	0.00	0.00	0.41	0.00	0.39	0.44	0.21	0.40	0.00	0.00	0.11	0.00	0.00	0.00	0.30	0.00	0.00
E16	0.00	0.00	0.40	0.38	0.00	0.00	0.00	0.25	0.00	0.14	0.12	0.85	0.00	0.10	0.00	0.00	0.13	0.00	0.90	0.00	0.36
E17	0.00	0.00	0.38	0.33	0.00	0.11	0.25	0.15	0.20	0.38	0.24	0.91	0.00	0.18	0.00	0.12	0.39	0.14	0.88	0.00	0.64
E18	0.00	0.00	0.12	0.00	0.00	0.00	1.00	0.00	0.98	0.98	0.44	0.38	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E19	0.00	0.00	0.41	0.38	0.00	0.00	0.00	0.27	0.00	0.15	0.11	0.88	0.00	0.00	0.00	0.00	0.14	0.00	0.93	0.00	0.39
E20	0.00	0.00	0.15	0.00	0.00	0.17	0.47	0.00	0.45	0.49	0.22	0.39	0.00	0.18	0.00	0.29	0.50	0.24	0.25	0.00	0.61
E21	0.00	0.00	0.20	0.18	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.41	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00	0.14
E22	0.00	0.00	0.12	0.00	0.00	0.00	0.99	0.00	0.97	0.97	0.44	0.38	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E23	0.00	0.00	0.39	0.29	0.00	0.00	0.66	0.13	0.61	0.76	0.38	0.94	0.00	0.15	0.00	0.00	0.00	0.00	0.77	0.00	0.30
E24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E25	0.00	0.00	0.12	0.00	0.00	0.00	1.00	0.00	0.97	0.98	0.44	0.39	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E26	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00	0.00	0.00
E27	0.00	0.00	0.12	0.00	0.00	0.00	1.00	0.00	0.98	0.98	0.44	0.38	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E29	0.00	0.00	0.25	0.20	0.00	0.00	0.40	0.00	0.37	0.46	0.17	0.52	0.00	0.10	0.00	0.00	0.00	0.00	0.43	0.00	0.12
E30	0.00	0.00	0.11	0.00	0.00	0.00	0.98	0.00	0.96	0.96	0.43	0.36	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E31	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.22	0.34	0.31	0.00	0.00	0.34
E32	0.00	0.00	0.00	0.00	0.00	0.00	0.85	0.00	0.83	0.83	0.42	0.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E33	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.53	0.28	0.00	0.00	0.54
E34	0.00	0.00	0.16	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.13
E35	0.00	0.00	0.30	0.28	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.64	0.00	0.00	0.00	0.00	0.20	0.00	0.70	0.00	0.38
E36	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.43	0.51	0.31	0.00	0.00	0.54
E37	0.00	0.00	0.15	0.14	0.00	0.12	0.43	0.00	0.40	0.48	0.26	0.55	0.00	0.00	0.10	0.00	0.11	0.00	0.43	0.00	0.27
E38	0.00	0.00	0.38	0.36	0.00	0.00	0.00	0.26	0.00	0.19	0.15	0.85	0.00	0.12	0.00	0.00	0.11	0.00	0.88	0.00	0.35
E39	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.23	0.00	0.00	0.00	0.28
E40	0.00	0.00	0.12	0.00	0.00	0.00	0.98	0.00	0.96	0.96	0.44	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E41	0.00	0.00	0.15	0.00	0.00	0.00	0.96	0.00	0.93	0.96	0.43	0.45	0.00	0.11	0.00	0.00	0.00	0.00	0.16	0.00	0.00
E42	0.00	0.00	0.00	0.00	0.00	0.34	0.16	0.00	0.17	0.15	0.11	0.00	0.00	0.18	0.00	0.51	0.69	0.38	0.00	0.00	0.75
E43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E44	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
E45	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E46	0.00	0.00	1.00	0.17	0.00	0.00	0.13	0.10	0.12	0.19	0.00	0.42	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.12
E47	0.00	0.00	0.17	1.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00	0.00
E48	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E49	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.35	0.15	0.00	0.00	0.39
E50	0.00	0.00	0.13	0.00	0.00	0.00	1.00	0.00	0.98	0.99	0.45	0.41	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E51	0.00	0.00	0.10	0.13	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00
E52	0.00	0.00	0.12	0.00	0.00	0.00	0.98	0.00	1.00	0.96	0.41	0.36	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E53	0.00	0.00	0.19	0.00	0.00	0.00	0.99	0.00	0.96	1.00	0.46	0.55	0.00	0.12	0.00	0.00	0.00	0.00	0.24	0.00	0.00
E54	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00	0.41	0.46	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00
E55	0.00	0.00	0.42	0.35	0.00	0.00	0.41	0.20	0.36	0.55	0.30	1.00	0.00	0.15	0.00	0.00	0.13	0.00	0.90	0.00	0.37
E56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E57	0.00	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.11	0.12	0.00	0.15	0.00	1.00	0.00	0.00	0.17	0.00	0.13	0.00	0.19
E58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
E59	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.50	0.27	0.00	0.00	0.54
E60	0.00	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.17	0.00	0.50	1.00	0.41	0.14	0.00	0.79
E61	0.00	0.00	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.41	1.00	0.00	0.00	0.42
E62	0.00	0.00	0.40	0.38	0.00	0.00	0.00	0.26	0.00	0.24	0.14	0.90	0.00	0.13	0.00	0.00	0.14	0.00	1.00	0.00	0.39
E63	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
E64	0.00	0.00	0.12	0.00	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.37	0.00	0.19	0.00	0.54	0.79	0.42	0.39	0.00	1.00

Table A2.2. The correlation coefficient among elements (E1~E16) of fayer 2

	E1	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**	**E9**	**E10**	**E11**	**E12**	**E13**	**E14**	**E15**	**E16**
E1	1.00	0.45	0.29	0.45	0.58	0.00	0.87	0.25	0.35	0.35	0.50	0.50	0.00	0.35	0.50	0.67
E2	0.45	1.00	0.52	0.60	0.52	0.00	0.52	0.22	0.63	0.63	0.45	0.45	0.45	0.32	0.00	0.60
E3	0.29	0.52	1.00	0.52	0.67	0.00	0.33	0.00	0.41	0.82	0.29	0.58	0.58	0.82	0.00	0.52
E4	0.45	0.60	0.52	1.00	0.52	0.00	0.52	0.22	0.63	0.63	0.45	0.45	0.45	0.32	0.00	0.60
E5	0.58	0.52	0.67	0.52	1.00	0.00	0.33	0.00	0.41	0.82	0.29	0.58	0.58	0.41	0.58	0.77
E6	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E7	0.87	0.52	0.33	0.52	0.33	0.00	1.00	0.29	0.41	0.41	0.58	0.58	0.00	0.41	0.00	0.52
E8	0.25	0.22	0.00	0.22	0.00	0.00	0.29	1.00	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.45
E9	0.35	0.63	0.41	0.63	0.41	0.00	0.41	0.00	1.00	0.50	0.00	0.71	0.00	0.50	0.00	0.32
E10	0.35	0.63	0.82	0.63	0.82	0.00	0.41	0.00	0.50	1.00	0.35	0.71	0.71	0.50	0.00	0.63
E11	0.50	0.45	0.29	0.45	0.29	0.00	0.58	0.25	0.00	0.35	1.00	0.00	0.50	0.00	0.00	0.45
E12	0.50	0.45	0.58	0.45	0.58	0.00	0.58	0.00	0.71	0.71	0.00	1.00	0.00	0.71	0.00	0.45
E13	0.00	0.45	0.58	0.45	0.58	0.00	0.00	0.00	0.00	0.71	0.50	0.00	1.00	0.00	0.00	0.45
E14	0.35	0.32	0.82	0.32	0.41	0.00	0.41	0.00	0.50	0.50	0.00	0.71	0.00	1.00	0.00	0.32
E15	0.50	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.45
E16	0.67	0.60	0.52	0.60	0.77	0.00	0.52	0.45	0.32	0.63	0.45	0.45	0.45	0.32	0.45	1.00

Table A2.4. The correlation coefficient among elements (E1~E4) of fayer 1

	E1	**E2**	**E3**	**E4**
E1	1.00	0.67	0.68	0.82
E2	0.67	1.00	0.54	0.68
E3	0.68	0.54	1.00	0.50
E4	0.82	0.68	0.50	1.00

Acknowledgements

I would like to express my sincere gratitude to my supervisors, Prof. Harutoshi OGAI a

-Woo KIM for their kindly and excellent guidance, as well as Prof. Sato Tadano for his kind suggestions and help throughout my doctoral study. Their critical comments and warm encouragement in discussion helped me to refine and promote my research tremendously. With their assistance in my fantastic age, I find I'm closer to my dream as a scientist than ever.

I would like to thank Dr. Nayan Tyoti BAISHYA, Dr. Ge YU, Dr. Cheng Lu, and Dr. Xiao LUO from WASEDA University, and Dr. Kai-Chun CHANG, Dr. Xuzhao LU, Dr. Oscar LUNA, Dr. Sudanna BORJIGIN, and Dr. Yoshinao GOI from KYOTO University, as well as A/Prof. Tomonori NAGAYAMA and Dr. Di SU from The University of Tokyo, for their kindly discussion and generously sharing with me their knowledge.

I like to share my joy in completing the thesis with my friends, especially Mr. Guangyi NI, Miss. Huayu LIANG, Mr. Wenwei CHEN, Mr. Yunpeng ZHAO, Mr. Zhuochao GU, Mr. Xixun WANG, Mr. Haoyang DENG in WASEDA university, and Mr. Takuya MIMASU, Mr. Syunsuke NAKAJIMA, Miss. Jiamin LIN, Mr. Xinda MA, Miss. Shiyuan ZHAO, Mr. Daigo KAWABE, Mr. Shun KOHDERA, Mr. Zhan XU in KYOTO University.

I wish to acknowledge the heartwarming support provided by my parents, Jiajun YE and Biyun DENG. I also would like to express my thanks to my Godparents, Academician Dongming GUO and Prof. Xiangqun SONG, to my enlightenment mentor Academician Zhongtuo WANG, my undergraduate supervisor Academician Jinping OU and master supervisor Prof. Shigeru FUJIMURA, my cooperative advisors Prof. Dagang LU, Prof. Zhongfu LI, Prof. Zhixun SU, A/Prof. Min XU, and A/Prof. Yi ZHANG. Without them, I would have missed the science and technology and missed the chances of approaching the natural truth.

I'd like to impress my great appreciation to the WASEDA University, KYOTO University, Public Works Research Institute, Japanese Society for Promotion of Science (JSPS), to provide the pivotal research conditions. I also would like to express my appreciation for the support from the China Scholarship Council in both life and research.

The last and important, I a /Prof. Shigeyuki TATENO, Prof. Hiroshi INUJIMA and other professors in the Production Systems of IPS, Waseda University for their kindly and valuable suggestions in the review process.

I a

References

[1] Ha . (2012). Fa
. A
-413.

[2] Xia Y, Xu Y L, Wei Z L, et al. (2011). Variation of structural vibration characteristics versus non-uniform temperature distribution. Engineering Structures, 33(1):146-153.

[3] Lindemann K, Odland J, Strengtheagen J. (1977). On the Application of Hull Surveillance Systems for Increased Safety and Improved Structural Utilization in Rough Weather. Instrumentation.

[4] Fujihira M. (2013). Da
. Mokuzai Hozon, 39(3):125-139.

[5] Fang H C, Jia X L. (1990). A Novel Fatigue Life Estimation Method of Great Water Depths Structure Members Using Damage Mechanics.

[6] Kobayashi R, Arai T, Yamada H. (2008). St -Speed Water Jet and the Damage Process of Metals in Jet Cutting Technology. Nihon Kikai Gakkai Ronbunshu B Hen/transactions of the Japan Society of Mechanical Engineers Part B, 53(489):1539-1542.

[7] Zhu H, Luo H, Ai D, et al. (2016). Mechanical impedance-based technique for steel structural corrosion damage detection. Measurement, 88:353-359.

[8] Nishiyama I, Okada T, Kamura H, et al. (1999) Damage Evaluation of Structural Steel by Damage of Rust Prevention Paint, Architectural Institute of Japan:315-316.

[9] Papanicolaou T P D, Admiraal D M, Wilson C P D, et al. (2012) Monitoring the Effects of Knickpoint Erosion on Bridge Pier and Abutment Structural Damage Due to Scour. Physica A Statistical Mechanics & Its Applications, 168(3):1-6.

[10] Enright M P, Frangopol D M. Survey and evaluation of damaged concrete bridges. Journal of Bridge Engineering, 2000, 5(1): 31-38.

[11] Okeil A M, Cai C S. Survey of Short and Medium-Span Bridge Damage Induced by Hurricane Katrina . Journal of Bridge Engineering, 2008, 13(4):377-387.

[12] Watanabe E, Furuta H, Yamaguchi T, et al. On longevity and monitoring technologies of bridges: a survey study by the Japanese Society of Steel Construction. Structure & Infrastructure Engineering, 2014, 10(4):471-491.

[13] Ko J M, Ni Y Q. Technology developments in structural health monitoring of large-scale bridges. Engineering structures, 2005, 27(12): 1715-1725.

[14] Pei Q, Guo X, Zhang M Z. (2003) A review of health monitoring and damage detection of bridge structures. Earthquake Engineering & Engineering Vibration, 23(2):61-67.

[15] Shang X. Study on Bridge Damage Identification based on Dynamic Detection. Thesis, Chang'an University, 2014.

[16] Yan Y J, Cheng L, Wu Z Y, et al. Development in vibration-based structural damage detection technique . Mechanical Systems and Signal Processing, 2007, 21(5): 2198-2211.

[17] Doebling S W, Farrar C R, Prime M B. A summary review of vibration-based damage identification methods. Shock and vibration digest, 1998, 30(2): 91-105.

[18] Capellari G, Azam S E, Mariani S. Online damage detection in plates via vibration measurements, Model Validation and Uncertainty Quantification, Vol 3. Springer International Publishing, 2015: 85-91.
[19] Park J H, Kim J T, Hong D S, et al. Sequential damage detection approaches for beams using time-modal features and artificial neural networks. Journal of Sound and Vibration, 2009, 323(1): 451-474.
[20] Adams R D, Cawley P. A review of defect types and nondestructive testing techniques for composites and bonded joints. NDT international, 1988, 21(4): 208-222.
[21] Ph Papaelias M, Roberts C, Davis C L. A review on non-destructive evaluation of rails: state-of-the-a Institution of Mechanical Engineers, Part F: Journal of Rail and rapid transit, 2008, 222(4): 367-384.
[22] Yun C B, Min J. Smart sensing, monitoring, and damage detection for civil infrastructures. KSCE Journal of Civil Engineering, 2011, 15(1): 1-14.
[23] Cruza, J. F., Camacho, J., & Fritsch, C. (2017). Plane-wave phase-coherence imaging for NDE. NDT & E In -37.
[24] Magalhães, F., Cunha, A., & Caetano, E. (2012). Vibration based structural health monitoring of an arch bridge: from automated OMA to damage detection. Mechanical Systems and Signal Processing, 28, 212-228.
[25] Bahaadini, R., Saidi, A. R., & Hosseini, M. (2019). Flow-induced vibration and stability analysis of carbon nanotubes based on the nonlocal strain gradient Timoshenko beam theory. Journal of Vibration and Control, 25(1), 203-218.
[26] Śniady, P., Podwórna, M., & Idzikowski, R. (2019). Stochastic vibrations of the Euler–Bernoulli beam based on various versions of the gradient nonlocal elasticity theory. Probabilistic Engineering Mechanics, 56, 27-34.
[27] Worden, K., Farrar, C. R., Manson, G., & Park, G. (2007, June). The fundamental axioms of structural health monitoring. In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, Vol. 463, No. 2082, pp. 1639-1664.
[28] https://lmats.com.au/services/non-destructive-testing/radiographic-test-r
[29] // /en/products/ir-thermo/what-thermo.html
[30] https://en.wikipedia.org/wiki/Ultrasonic_testing
[31] Bettayeb, F., Rachedi, T., & Benbartaoui, H. (2004). An improved automated ultrasonic nde system by wavelet and neuron networks. Ultrasonics, 42(1), 853-858.
[32] https://en.wikipedia.org/wiki/Acoustic_emission
[33] Gray, Jeremy. Ideas of Space-Euclidean, Non-Euclidean, and Relativistic. 1989.
[34] Farrar, C. R., Doebling, S. W., & Nix, D. A. (2001). Vibration–based structural damage identification. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 359(1778), 131-149.
[35] SOHN, H. (2004). A review of structural health monitoring literature: 1996-2001. Los Alamos National Laboratory Report.

[36] Ay, E., Ediz, B., Sönmezay, B., & Çetin, S. T. (2019). Modal Analysis of Lenses Used in Automotive Lighting Industry and Obtaining MAC Matrix. International Journal of Engineering and Applied Sciences, 11(2), 401-413.
[37] Salawu, O. S. (1997). Detection of structural damage through changes in frequency: a review. Engineering structures, 19(9), 718-723.
[38] Curadelli, R., Riera, J., Ambrosini, D. & Amani, M., (2008) “Damage detection by means of structural damping identification.” Engineering Structures, V. 30, pp. 3497-3504.
[39] Frizzarin, M. et al., (2010) “Damage detection based on damping analysis of ambient vibration data,” Structural Control and Health Monitoring, V. 17, pp. 368-385.
[40] Chen, Y. , Qian, Y. , Li, H. , He, H. , & Yan, W. . (2017). Da
. Journal of Vibration & Shock, 36(13), 208-213 and 257.
[41] Frizzarin, M. , Feng, M. Q., Franchetti, P. , Soyoz, S. a . (2010), Damage detection based on damping analysis of ambient vibration data. Struct. Control Health Monit., 17: 368-385.
[42] Fox, C. H. J. (1992). The location of defects in structures-A comparison of the use of natural frequency and mode shape data. In 10th International Modal Analysis Conference (Vol. 1, pp. 522-528).
[43] Salawu, O. S., & Williams, C. (1993, February). Structural Damage Detection Using Experimental Modal Analysis: A Comparison of Some Methods. In Proceedings of The In . 254-254). SEM Society for Experimental Mechanics Inc.
[44] Berman, A., & Flannelly, W. G. (1971). Theory of incomplete models of dynamic structures. AIAA journal, 9(8), 1481-1487.
[45] Liu, R. , Guo, H. , & Yuan, X. . (2014). Seismic structure extraction based on multi-scale sensitivity analysis. Springer-Verlag New York, Inc.
[46] Treder H J. Boltzmann's cosmogony and the hierarchical structure of the universe. Astronomische Nachrichten, 1976, 297(20):117-126.
[47] Grabinska T. The hierarchical structure of the Universe. ESA Special Publication. ESA Special Publication, 1986:303-308.
[48] Sugiyama K, Tagawa S, Toda M. Methods for visual understanding of hierarchical system structures. IEEE Transactions on Systems Man & Cybernetics, 2007, 11(2):109-125.
[49] Kutyniok, Gitta, ed. Shearlets: Multiscale analysis for multivariate data. Springer Science & Business Media, 2012.
[50] Ebeling C E. An introduction to reliability and maintainability engineering. Tata McGraw-Hill Education, 2004.
[51] Billinton R, Allan R N. Reliability evaluation of engineering systems. US: Plenum press, 1992.
[52] Kuo W, Zhang W, Zuo M. A consecutive-k-out-of-n: G system: The mirror image of a consecutive-k-out-of-n: F system. IEEE Transactions on Reliability, 1990, 39(2): 244-253.

[53] Chiang D T, Niu S C. Reliability of consecutive-k-out-of-n: F system. IEEE Transactions on Reliability, 1981, 30(1): 87-89.
[54] Rutenberg M. Method and system for improving the operational reliability of electronic systems formed of subsystems which perform different functions: US, US4740887. 1988.
[55] Hausken K, Levitin G. Efficiency of even separation of parallel elements with variable contest intensity. Risk Analysis, 2008, 28(5): 1477-1486.
[56] Smidt-Destombesa K S D, Harten A V. On the interaction between maintenance, spare part inventories and repair capacity for a -out-of- system with wear-out. European Journal of Operational Research, 2006, 174(1):182-200.
[57] Coit D W, Chatwattanasiri N, Wattanapongsakorn N, et al. Dynamic k-out-of-n system reliability with component partnership. Reliability Engineering & System Safety, 2015, 138: 82-92.
[58] Sooktip T, Wattanapongsakorn N, Coit D W. System reliability optimization with k-out-of-n subsystems and changing k. In
IEEE, 2011:1382-1387.
[59] Zhou Y, Fu L, Zhang J, et al. A Reliability Test of a Complex System Based on Empirical Likelihood. PloS one, 2016, 11(10): e0163557.
[60] David W. Coit, Jia Chen Liu. System Reliability Optimization with k-out-of-n Subsystems. International Journal of Reliability Quality & Safety Engineering, 2011, 7(02):129-142.
[61] Boddu P, Xing L. Reliability evaluation and optimization of series–parallel systems with k-out-of-n: G subsystems and mixed redundancy types. Proceedings of the Institution of Mechanical Engineers Part O Journal of Risk & Reliability, 2013, 227(2):187-198.
[62] Ding Y, Zuo M J, et al. The Hierarchical Weighted Multi-State, k-out-of-n, System Model and Its Application for Infrastructure Management. IEEE Transactions on Reliability, 2010, 59(3):593-603.
[63] Prigogine I. Moderation et transformations irreversibles des systemes ouverts . Bull. Cl. Sci. Acad. R. Belg, 1945, 31(600606.11).
[64] Prigogine I, Lefever R. Theory of Dissipative Structures . Synergetics. Vieweg Teubner Verlag, 1973:124-135.
[65] Nicolis G, Prigogine I. Self-organization in nonequilibrium systems. Wiley, New York, 1977.
[66] Manuele F A. Hazard Analysis and Risk Assessment. On the Practice of Safety, Third Edition, 251-271.
[67] Jeong S, Hou R, Lynch J P, et al. An information modeling framework for bridge monitoring. Advances in Engineering Software, 2017. 114: 11-31
[68] Baruch M. Optimization procedure to correct stiffness and flexibility matrices using vibration tests. AIAA journal, 1978, 16(11): 1208-1210.
[69] Pandey A K, Biswas M. Damage detection in structures using changes in flexibility. Journal of sound and vibration, 1994, 169(1): 3-17.

[70] Ya

-1761.

[71] Ralbovsky M, Flesch S D R. Frequency changes in frequency-based damage identification. Structure & Infrastructure Engineering, 2010, 6(5):611-619.

[72] Dammika A J, Kawarai K, Yamaguchi H. Experimental-a

-based structural health monitoring of bridges. Research Report of Department of Civil and Environmental Engineering, 2015, 41.

[73] Kim J T, Ryu Y S, Cho H M, et al. Damage identification in beam-t

-based method vs mode-shape-based method. Engineering structures, 2003, 25(1): 57-67.

[74] Radzieński M, Krawczuk M, Palacz M. Improvement of damage detection methods based on experimental modal parameters. Mechanical Systems & Signal Processing, 2011, 25(6):2169-2190.

[75] Mousavi M, Gandomi A H. A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error. Engineering Structures, 2016, 111:425-434.

[76] Hou Z, Noori M, Amand R S. Wavelet-based approach for structural damage detection. Journal of Engineering mechanics, 2000, 126(7): 677-683.

[77] Doebling S W. Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review. Shock & Vibration Digest, 1996, 30(11):2043-2049.

[78] Sohn H, Farrar C R, Hemez F M, et al. A review of structural health monitoring literature: 1996–2001. Los Alamos National Laboratory, 2003. (pages undefined)

[79] Moughty J J, Casas J R. A State of the Art Review of Modal-Based Damage Detection in Bridges: Development, Challenges, and Solutions. Applied Sciences, 2017, 7(5): 510.

[80] Holford K M, Eaton M J, Hensman J J, et al. A new methodology for automating acoustic emission detection of metallic fatigue fractures in highly demanding aerospace environments: An overview. Progress in Aerospace Sciences, 2017. (pages undefined)

[81] de Oliveira M A, Inman D J. Performance analysis of simplified Fuzzy ARTMAP and Probabilistic Neural Networks for identifying structural damage growth. Applied Soft Computing, 2017, 52: 53-63.

[82] Langone R, Reynders E, Mehrkanoon S, et al. Automated structural health monitoring based on adaptive kernel spectral clustering. Mechanical Systems and Signal Processing, 2017, 90: 64-78.

[83] Fadeyi M O. The role of building information modeling (BIM) in delivering the sustainable building value. In

[84] -concrete Composite Bridge Deck. Procedia Engineering, 2016, 156: 32-39.

[85] Armbruster D, Martin S, Thatcher A. Elastic and inelastic collisions of swarms. Physica D: Nonlinear Phenomena, 2017, 344: 45-57.

[86] Sagasta F, Zitto M E, Piotrkowski R, et al. Acoustic emission energy b-value for local damage evaluation in reinforced concrete structures subjected to seismic loadings. Mechanical Systems and Signal Processing, 2018, 102: 262-277.
[87] Barlow R E, Proschan F. Statistical theory of reliability and life testing: probability models. Florida State Univ Tallahassee, 1975.
[88] Zhang X L, Huang X Z, Liu Y. A hierarchical decomposition approach for large system reliability allocation. Eksploatacja i Niezawodnosc - Maintenance and Reliability, 2009, 43(3):32-37.
[89] Yin C, Kareem A. Computation of failure probability via hierarchical clustering. Structural Safety, 2016, 61: 67-77.
[90] Li D Q, Yang Z Y, Cao Z J, et al. System reliability analysis of slope stability using generalized Subset Simulation. Applied Mathematical Modelling, 2017, 46:650-664.
[91] Riege S P, Thompson C V, Clement J J. A hierarchical reliability analysis for circuit design evaluation. IEEE Transactions on Electron Devices, 1998, 45(10): 2254-2257.
[92] Wang L, Bai X, Zhou L, et al. A hierarchical reliability model of service-based software system. Computer Software and Applications Conference, 2009. COMPSAC'09. 33rd Annual IEEE International. IEEE, 2009, 1: 199-208.
[93] Billinton R, Satish J, Goel L. Hierarchical reliability evaluation in an electric power system. Athens Power Tech, 1993. APT 93. Proceedings. Joint In
IEEE, 1993, 2: 616-621.
[94] Musharraf M, Hassan J, Khan F, et al. Human reliability assessment during offshore emergency conditions. Safety science, 2013, 59: 19-27.
[95] Dedecius K, Ettler P. Hierarchical modelling of industrial system reliability with probabilistic logic, In Informatics in Control, Automation and Robotics. 2014:133-139.
[96] Nishijima K. Optimal reliability of components of complex systems using hierarchical system models. Institute of Structural Engineering, ETH Zurich, 2007.
[97] Sarhan A. Reliability equivalence of independent and non-identical components series systems. Reliability Engineering & System Safety, 2000, 67(3): 293-300.
[98] Zhang T, Horigome M. Availability and reliability of system with dependent components and time-varying failure and repair rates. IEEE Transactions on reliability, 2001, 50(2): 151-158.
[99] Hendawi S, Frangopol D M. System reliability and redundancy in structural design and evaluation. Structural Safety, 1994, 16(1-2): 47-71.
[100] Aidun C K, Clausen J R. Lattice-Boltzmann method for complex flows . Annual review of fluid mechanics, 2010, 42: 439-472.
[101] Faraoni, Valerio. "Lagrangian description of perfect fluids and modified gravity with an extra force." Physical Review D 80.12 (2009): 124040.
[102] Liu, Chun, and NOEL J. WALkINGTON. "An Eulerian description of fluids containing visco-elastic particles." Archive for rational mechanics and analysis 159.3 (2001): 229-252.

[103] Pascal J C, Carniel X, Li J F. Characterisation of a dissipative assembly using structural intensity measurements and energy conservation equation. Mechanical systems and signal processing, 2006, 20(6): 1300-1311.

[104] Von Bertalanffy L. The theory of open systems in physics and biology. Science, 1950, 111(2872): 23-29.

[105] Haag M G, Haag L C. The Reconstructive Aspects of Class Characteristics and a Limited Universe . Shooting Incident Reconstruction. Elsevier Inc. 2011:35–54.

[106] Von Bertalanffy L. General system theory. New York, 1968, 41973(1968): 40.

[107] Blanchard B S, Fabrycky W J, Fabrycky W J. Systems engineering and analysis. Englewood Cliffs, NJ: Prentice Hall, 1990.

[108] Mugford S T, Mallon E B, Franks N R. The accuracy of Buffon's needle: a rule of thumb used by ants to estimate area. Behavioral Ecology, 2001, 12(6): 655-658.

[109] Hassiotis S. Identification of Stiffness Reductions Using Natural Frequencies. Journal of Engineering Mechanics, 1995, 121(10):1106-1113.

[110] Yamaguchi H, Matsumoto Y, Kawarai K, et al. Damage detection based on modal damping change in bridges, In Proceedings of the ICSBE'12, Kandy, Sri Lanka,14–16 December 2013.

[111] Tsirelson, Boris. "Spaces in mathematics." WikiJournal of Science 1.1 (2018): 1.

[112] Zhmud, Leonid Ja. "" All Is Number"?" Basic Doctrine" of Pythagoreanism Reconsidered." Phronesis (1989): 270-292.

[113] Barnaghi, Payam, Amit Sheth, and Cory Henson. "From data to actionable knowledge: big data challenges in the web of things." IEEE Intelligent Systems 6 (2013): 6-11.

[114] Gray, Jeremy. Ideas of space: Euclidean, non-Euclidean, and relativistic. Oxford: Clarendon Press, 1979.

[115] Tao L, Wang H B. Detecting and locating human eyes in face images based on progressive thresholding, IEEE International Conference on Robotics and Biomimetics. 2008:445-449.

[116] Dems K, Mroz Z. Stability Conditions for Brittle-Plastic Structures with Propagating Damage Surfaces*. Journal of structural mechanics, 1985, 13(1): 95-122.

[117] Jagota A, Hui C Y, Dawson P R. The determination of fracture toughness for a porous elastic-plastic solid. International journal of fracture, 1987, 33(2): 111-124.

[118] Xu Z D, Wu Z. Simulation of the effect of temperature variation on damage detection in a long-span cable-stayed bridge. Structural Health Monitoring, 2007, 6(3): 177-189.

[119] Schlegel, Gregory L., and Robert J. Trent. Supply chain risk management: An emerging discipline. Crc Press, 2014.

[120] Moses, Fred, and John D. Stevenson. "Reliability-based structural design." Journal of the Structural Division 96.2 (1970): 221-244.

[121] Burnett, Carolyn, and Thomas Blaschke. "A multi-scale segmentation/object relationship modelling methodology for landscape analysis." Ecological modelling 168.3 (2003): 233-249.

[122] Li, Jinghai, et al. "Multi-scale methodology for complex systems." Chemical Engineering Science 59.8-9 (2004): 1687-1700.
[123] Akula V M K. Multiscale reliability analysis of a composite stiffened panel. Composite Structures, 2014, 116(9):432-440.
[124] Gelman A, Hill J. Data analysis using regression and multilevel hierarchical models. New York, NY, USA: Cambridge University Press, 2007.
[125] Banerjee S, Carlin B P, Gelfand A E. Hierarchical modeling and analysis for spatial data. Crc Press, 2014.
[126] Bazin P L, Weiss M, Dinse J, et al. A computational framework for ultra-high resolution cortical segmentation at 7 Tesla. Neuroimage, 2014, 93: 201-209.
[127] Yuan Y, Jiang Z, Qi W. HDPA: Hierarchical deep probability analysis for scene parsing. IEEE In
-318.
[128] Cowlagi R V, Saleh J H. Coordinability and consistency in accident causation and prevention: formal system theoretic concepts for safety in multilevel systems. Risk analysis, 2013, 33(3): 420-433.
[129] Palazzo M, Evans R. Logistic regression analysis of fixed patient factors for postoperative sickness: a model for risk assessment. BJA: British Journal of Anaesthesia, 1993, 70(2): 135-140.
[130] Thornton D. Constructing and testing a framework for dynamic risk assessment. Sexual Abuse, 2002, 14(2): 139-153.
[131] Akula V M K. Multiscale reliability analysis of a composite stiffened panel. Composite Structures, 2014, 116(9):432-440.
[132] Popper S W, Bankes S C, Callaway R, et al. System of systems symposium: Report on a summer conversation. Potomac Institute for Policy Studies, Arlington, VA, 2004, 320.
[133] Pyster, A., Olwell, D. H., Hutchison, N., Enck, S., Anthony Jr, J. F., & Henry, D. Guide to the systems engineering body of knowledge (SEBoK) v. 1.4., June 29, 2015.
[134] M.W. Maier, “Architecting principles for systems-of-systems,” Syst. Eng., vol. 1, no. 4, pp. 267–284, 1998.
[135] https://standards.ieee.org/standard/15288-2015.html
[136] Köhler, Wolfgang. Gestalt psychology: An introduction to new concepts in modern psychology. WW Norton & Company, 1970.
[137] Wallace, Brendan, et al. The mind, the body and the world: Psychology after cognitivism?. Andrews UK Limited, 2015.
[138] L Chen, Topological structure in visual perception, Science 12 November 1982: Vol. 218 no. 4573 pp. 699-700
[139] Lu, Yiliang et al. Revealing Detail along the Visual Hierarchy: Neural Clustering Preserves Acuity from V1 to V4. Neuron , 2018.
[140] McCarter, B. G., & White, B. E. (2008). Emergence of SoS, sociocognitive aspects. In Systems of Systems Engineering (pp. 83-117). New York, NY, USA: Taylor & Francis, 2008.

[141] Ja . M., Keating, C. B., and Bradley, J. M.. A histogram analysis for system of systems. International Journal of System of Systems Engineering, 2014, 5(3), 193–227.

[142] Borel A. Linear algebraic groups. Springer Science & Business Media, 2012.

[143] Morse N, Sacksteder R. Statistical Isomorphism. Matematika, 1966, 37(1):147–160.

[144] Hoeffding W, Robbins H. The central limit theorem for dependent random variables. Duke Mathematical Journal, 1948, 15(3): 773-780.

[145] Noda I. Two-Dimensional Correlation Analysis of Spectra Collected without Knowing Sampling Order. Journal of Molecular Structure, 2017.

[146] Noda I. Two-Dimensional Correlation Analysis of Spectra Collected without Knowing Sampling Order. Journal of Molecular Structure, 2017.

[147] Chen C, Wu W, Zhang B, et al. Correlated probabilistic load flow using a point estimate method with Nataf transformation. International Journal of Electrical Power & Energy Systems, 2015, 65: 325-333.

[148] Borg I, Groenen P J F, Mair P. A . Springer Science & Business Media, 2012.

[149] Borg I, Groenen P J F. Modern multidimensional scaling: Theory and applications. Springer Science & Business Media, 2005.

[150] Benesty J, Chen J, Huang Y, et al. Pearson correlation coefficient. Noise reduction in speech processing. Springer Berlin Heidelberg, 2009: 1-4.

[151] Kowalski C J. On the effects of non-normality on the distribution of the sample product-moment correlation coefficient. A -12.

[152] Thoft-Christensen P, Murotsu Y. A . Application of structural systems reliability theory. Springer-Verlag, 1986.

[153] Dunnett C W, Sobel M. Approximations to the Probability In -Distribution. Biometrika, 1955, 42(1-2):258-260.

[154] Takeuchi, Kei, and Y. Fujino. "Binomial distribution and Poisson distribution." (1981).

[155] Wang W, Banjevic D, Pecht M. A multi-component and multi-failure mode inspection model based on the delay time concept. Reliability Engineering & System Safety, 2010, 95(8): 912-920.

Publication List

Journal paper

X. F. Ye, H. Ogai, and C. W. Kim, "Discrepancy Analysis of Load–Displacement in the Combination Space for Concrete Box Girder Assessment." Strength of Materials, Vol.50, NO.5, pp.695-701, November 2018.
h   //doi.org/10.1007/s11223-018-0014-6

Xiong-Fei Ye, Kai-Chun Chang, Chul-Woo Kim, Harutoshi Ogai, Yoshinobu Oshima, and O.S. Luna Vera, "Flow Analysis and Damage Assessment for Concrete Box Girder Based on Flow Characteristics," Sustainability, Vol. 11, NO.3, pp.1-32, January 2019.
h   //doi.org/10.3390/su11030710

Xiong-Fei YE, Yi ZHANG, Harutoshi OGAI, Chul-Woo KIM, "Hierarchical Probability and Risk Assessment for K-out-of-N System in Hierarchy", Reliability Engineering & System Safety, Vol.189, NO.9, pp.242-260, April 2019.
h   //doi.org/10.1016/j.ress.2019.04.026

X. F. Ye, C. W. Kim, and H. Ogai, "Fast Damage Detection Using Max-Peak and Max-Peak-Time for PC Box Girder in Vibration Experiment", (Accepted by) Measurement, October 2019
https://doi.org/10.1016/j.measurement.2019.107178

International Conference

Xiong-Fei YE, Harutoshi OGAI, and Chul-Woo KIM "Diagnosing damage of a PC box girder based on max-peak-time in impact hammer experiment", The 13th International Workshop on Advanced Smart Materials and Smart Structures Technology, 2017, The University of Tokyo, Japan.

Xiong-Fei YE, Harutoshi OGAI, and Chul-Woo KIM, "Health evaluation for Prestress Concrete Box Girder Based on Flow Analysis of Max-peak in Dynamic Experiment", The 3rd In