Implementation of the Shapley Value of Games with Coalition Structures

Yoshio Kamijo*

The present paper studies non-cooperative bargaining models implementing a cooperative solution proposed by Kamijo (2005). The basic idea was inspired by the bidding approach of Perez-Castrillo and Wettstein (2001). They show that this mechanism achieves the Shapley value payoff vector in equilibrium. The solution concept considered in this paper is a generalized Shapley value applied to TU (transferable utility) games with coalition structures as well as the Aumann-Dreze value (Aumann and Dreze (1974)) and the Owen value (Owen (1977)). We would like to discriminate between two types of cooperation behind the characteristic function. Therefore we present two bargaining models for each.

JEL classification codes: C71; C72

Keywords: cooperative game; coalition structure; implementation; the Shapley value

1. Introduction

In this paper, we give the noncooperative bargaining foundations to the solution of cooperative games with coalition structures (henceforth, CSs) defined by Kamijo (2005). The solution is an extension of the Shapley value to cooperative game with CSs as well as the Aumann-Dreze value (Aumann and Dreze (1974)) and the Owen coalitional value (Owen (1977)).

The bargaining models are based on the bidding mechanism introduced by Perez-Castrillo and Wettstein (2001). This mechanism achieves the Shapley value as any equilibrium outcome. Other authors have also studied the bargaining models implementing the Shapley value (Gul (1989), Hart and MasColell (1992), Winter (1994), Evans (1996), Hart and Mas-Colell (1996) and Dasgupta and Chiu (1998)).

Roughly speaking, the bidding mechanism of Perez-Castrillo and Wettstein (2001) is defined as follows. If there is only one player, he obtains the worth of his stand-alone coalition and leaves the bargaining. Suppose that the rule of the bid-

* Graduate school of Economics Waseda University, 1-6-1, Nishi-waseda, Shinjuku, Tokyo, Japan. First version; November 2004 revised; February 2006.

The author wish to acknowledge the editor and the anonymous referees for their variable comments. Thanks are extended to Yukihiko Funaki and Yuan Ju for helpful comments and discussions.

ding mechanism is known when there are less than or equal to $n-1$ players, the bidding mechanism with n players is defined as follows. It consists of three steps:

(i) Every player i announces his bid b_j^i to any other j. The player who makes a maximum net bid, which measures the relative willingness of a player to be a proposer, is chosen as the proposer at the next step. Only he actually pays his bid to the other players after step (i).

(ii) The proposer, say α, makes an offer y_j to any other j.

(iii) Players other than the proposer respond sequentially in the predetermined order, say $(i_1,\cdots,i_n)$. Their response is either "accept" or "reject." If the offer is unanimously accepted by the responders, each responder j receives y_j and proposer α receives the worth of total cooperation minus the sum of y_j among the responders in addition to the payments of the bids. If there is a rejection, α leaves the bargaining table and obtains the worth of his stand-alone coalition. The other players proceeds the bidding mechanism for $n-1$ players.

They show that the above bargaining implements the Shapley value for any zero-monotonic games in any subgame perfect equilibrium.

We adopt the bidding mechanism as the basis of our bargaining models because of the following three points. First, it works well in sufficiently large domains (zero-monotonic games) compared to the other models (Gul (1989) and Evans (1996) for value additive games, Winter (1994) and Dasgupta and Chiu (1998) for convex games). Second, it achieves the Shapley value in every equilibrium outcome. Therefore we do not worry about the problem of multiple equilibria. Third, it implements the Shapley value as not only an expected value but also a realized one. This feature of the bidding mechanism has not been seen in other literature (Gul (1989)), Winter (1994), Evans (1996), Hart and MasColell (1996) and Dasgupta and Chiu (1998)).

In section 2, we explain our basic notations and definitions used in this paper and present the formula of the solution proposed by Kamijo (2005). We propose an extension of the bidding mechanism to games with CSs and prove one of our main theorems which shows that it implements the solution of Kamijo (2005) in equilibrium. Section 4 gives another extension of the mechanism to games with CSs.

The bargaining models considered in section 3 are based on an implicit assumption that the endowments producing the additional utility through cooperation are tradable among players like a resource exchange. We call this type of cooperation as "cooperation with resource trading." Under the assumption of this type of cooperation, a player who obtains the resource of others can behave like a coalition which consists of himself and other players even if they have already left. The bargaining model is a two stage bidding mechanism: (i) the bidding mechanism for each coalition to choose its *representative* and (ii) the mechanism for the representatives who collect the resources of their coalitions.

We show that this two stage bidding mechanism implements the solution by Kamijo (2005) for superadditive games in any subgame perfect equilibrium. Vidal-Puga and Bergantinos (2003) also consider the different two stage bidding mechanism from ours which implements the Owen's coalitional value under this assumption of cooperation.

The assumption is, however, not appropriate to describe the usual situation of economics in which each player has his own characteristics such as preference, ability, technology and so on. In this situation, it is required that to derive the cooperative outcome, the relevant players need to be actually together. The bargaining model in section 4 resolves this problem. In other words, we construct the mechanism implementing the solution of Kamijo (2005) without the assumption of cooperation with resource trading. The bargaining model considered in this section consists of three stages and one additional stage: (a) In stage A, a *negotiator* is chosen for each coalition through a bidding system, (b) in stage B, the negotiators play the bidding game for themselves, (c) in stage C, each negotiator proposes a payoff distribution for players in his coalition and the other players in the coalition respond it. In contrast to a representative, a negotiator is only a player who can meet and talk with the other negotiators from different coalitions. Therefore an agreement among negotiators is not executed until all the players other than the negotiators agree it. We show that the mechanism satisfying the requirement implements the solution of Kamijo (2005) for superadditive games.

The validity of the assumption of cooperation with resource trading is also discussed in the concluding remarks.

2. Definitions And Notations

We start with the definition of a *TU (transferable utility) game or a cooperative game*. A pair (N, v) is called a TU game where $N=\{1,\cdots,n\}$ is a finite set of players and v is a characteristic function which associates with each subset S of N a real number. A set $S\subseteq N$ is called a *coalition*. It is assumed by convention that $v(\emptyset)=0$. For $S\subseteq N$, $(S, v\backslash s)$ is a subgame of (N, v) to S where $v\backslash s(T)=v(T)$ for all $T\subseteq S$.

A game (N, v) is called superadditive if for all $S, T\subseteq N$ such that $S\cap T=\emptyset$, $v(S\cup T)\geqq v(S)+v(T)$ and strictly superadditive if strict equality holds. (N, v) is called zero-monotonic if for each $i\in N$ and each $S\subseteq N\backslash\{i\}$, $v(S\cup\{i\})\geqq v(S)+v(\{i\})$.

Let σ be a permutation on N. In other words, σ is a bijection on N. A set of all the permutations on N is denoted by $\Sigma(N)$. A set of players preceding to i at order σ is defined by $A_i^\sigma(N):=\{j\in N:\sigma(j)<\sigma(i)\}$. A marginal contribution of player i at order σ is defined by $a_i^\sigma(N, v):=v(A_i^\sigma(N)\cup\{i\})-v(A_i^\sigma(N))$. The Shapley value of TU games (Shapley (1953)) is defined as follows. For each $i\in N$,

$$\phi_i(N, v)=\frac{1}{|\Sigma(N)|}\sum_{\sigma\in\Sigma(N)}a_i^\sigma(N, v),$$

where $|\Sigma(N)|$ represents the cardinality of $\Sigma(N)$. The Shapley value is denoted by a symbol ϕ in the rest of the paper.

Next we define a *TU game with a coalition structure (CS) or a cooperative game* *with a CS*. A triple (N, v, C) is called a TU game with a CS where (N, v) is a TU game and $C=\{C_1,\cdots,C_m\}$ is called a *coalition structure*, where every two elements of C are disjoint and the union of all the elements of C is N. $\langle N\rangle$ denotes the CS of N in which everyone is separated. That is, $\langle N\rangle=\{\{1\},\cdots,\{n\}\}$.

Let σ be a permutation on N. It is said that σ is consistent with C if for any $i, j\in C_h\in C$, $k\in N$, $\sigma(i)<\sigma(k)<\sigma(j)$ implies $k\in C_h$. A set of all the permutations on N consistent with C is denoted by $\Sigma(N, C)$. Let $\sigma\in\Sigma(N, C)$, $M=\{1,\cdots,m\}$ and $k\in M$. Then an order $\sigma[k]$ on C_k induced by σ is defined by the condition that for any $i, j\in C_k$, $\sigma[k](i)<\sigma[k](j)$ if and only if $\sigma(i)<\sigma(j)$. An order $\sigma[M]$ on M induced by σ is defined by the condition that for any $k, h\in M$, $[M](k)\sigma[M](h)$ if and only if $\sigma(i)<\sigma(j)$ for any $i\in C_k$ and for any $j\in C_h$.

An extension of the Shapley value to TU games with CSs is offered by Owen (1977). The Owen's coalitional value ψ^O (henceforth, the Owen value) is defined as follows. For each $i\in N$,

$$\psi_i^O(N, v, C)=\frac{1}{|\Sigma(N, C)|}\sum_{\sigma\in\Sigma(N,C)} a_i^\sigma(N, v).$$

It is easily shown that $\psi^O(N, v, \{N\})=\psi^O(N, v, \langle N\rangle)=\phi(N, v)$.

For (N, v, C), a pair (M, v_C) is called an *intermediate game, or a quotient game of* (N,v,C) where $M=\{1,\cdots,m\}$ is a set of the coalitional indices of the elements in C and v_C is defined by $v_C(H)=v(\cup_{h\in H}C_h)$ for each $H\subseteq M$.

We now explain the solution proposed by Kamijo (2005). Let (N, v, C) be a cooperative game with a CS. Then ψ^G is defined as follows. For each $i\in C_k\in C$,

$$\psi_i^G(N, v, C) = \frac{1}{|\Sigma(N, C)|}\sum_{\sigma\in\Sigma(N,C)} b_i^\sigma(N, v, C).$$

where $b_i^\sigma(N, v, C)$ is defined by:

$$b_i^\sigma(N, v, C)=\begin{cases} a_i^{\sigma[k]}(C_k, v|_{C_k}) \\ \quad \text{if } i \text{ is not the last in } C_k \text{ at order } \sigma[k], \\ a_i^{\sigma[k]}(C_k, v|_{C_k})+a_k^{\sigma[M]}(M, v_C)-v(C_k) \\ \quad \text{if } i \text{ is the last in } C_k \text{ at order } \sigma[k]. \end{cases}$$

Then it is easily shown that $\psi^G(N, v, \{N\})=\psi^G(N, v, \langle N\rangle)=\phi(N, v)$ holds as well as the Owen value. The following lemma holds.

Lemma 1 *For each $k\in M$ and for each $i\in C_k$,*

$$\psi_i^G(N, v, C)=\frac{\phi_k(M, v_C)-v(C_k)}{|C_k|} + \phi_i(C_k, v|_{C_k}).$$

Proof. It is a natural consequence of the definition. □

The following proposition plays the important role in this paper.

Proposition 1 (**Myerson (1980)**) *The Shapley value ϕ satisfies the following conditions: for all $i\in N$ and $j\in N$,*

$$\phi_i(N, v)-\phi_i(N\backslash\{j\}, v) = \phi_j(N, v)-\phi_j(N\backslash\{j\}, v),$$

where $(N\backslash\{i\}, v)$ and $(N\backslash\{j\}, v)$ are subgame of (N, v) to $N\backslash\{i\}$ and $N\backslash\{j\}$ respectively.

This is called the *balanced contributions property*.

3. Bargaining Among Representatives

In this section, we consider situations where players can exchange their endowments (cooperative resources) for producing additional utility through cooperation with each other. In other words, a player obtaining the resources which players in coalition S originally possess is able to get $v(S)$ for himself and moreover $v(S \cup T)$ if he cooperates with coalition T.

The bargaining models presented in this section are a two-stage bidding mechanism: a bidding game within a coalition and a bidding game among coalitions. Let $C=\{C_1,\cdots,C_m\}$.

In the first stage, players in coalition C_k play the bidding game to decide its *representative*. The representative collects the resources of his coalition (or, sub coalition) in return to pay money (or, transferable utility). After the first stage bargaining, each representative is chosen from each coalition. In the second stage, the representatives who collect the resources of coalitions, play the bidding game again. Note that each representative acts like his coalition (or sub coalition) for himself. That is, cooperation between two representatives creates the amount equal to what the players in the two coalitions could create through their cooperation.

We now define the bargaining procedure formally. In this bargaining, players know all the past histories when they choose actions. Let (N, v, C) be a TU game with a CS. The bidding mechanism for (N, v) introduced by Perez-Castrillo and Wettstein (2001) is denoted by $\Gamma^{PW}(N, v)$. The definition of $\Gamma^{PW}(N, v)$ is also given in the following.

Stage 1 (1) Suppose we know the procedure of the bargaining model of stage 1 when the number of coalitions in C is less than $m (m \geqq 2)$. Then we define its rule with m coalitions.

Each coalition C_k plays a non-cooperative game in some predetermined order (e.g., $(C_1,\cdots,C_m)$). When there is one player in C_k, he is chosen as the representative of C_k. Assume that the rule of the game with at most $|C_k|-1$ players are defined. Then we define it when there are $|C_k|$ players.

Step (i) Each $i \in C_k$ announces his bid b_j^i to any other $j (j \in C_k \setminus \{i\})$.[1] Net bid B^i is calculated by

$$B^i = \sum_{j \in C_k \setminus \{i\}} b_j^i - \sum_{j \in C_k \setminus \{i\}} b_i^j \qquad (1)$$

for each $i \in C_k$. A player who makes a maximum net bid is chosen as proposer β_k at the next step. If there are many maximizers, one index is randomly selected. Once chosen as a proposer, β_k actually pays his bid $b_j^{\beta_k}$ to any other $j \in C_k \setminus \{\beta_k\}$. Notice that players other than the proposer need not to pay their bids.

Step (ii) Proposer β_k makes an offer $y_i \in \mathbf{R}$ to any other $j (j \in C_k \setminus \{\beta_k\})$.

Step (iii) Players other than β_k decide to accept or reject in some rotation, e.g., $(i_1,\cdots,i_{|C_k|})$.

If every $j \neq \beta_k$ accepts the offer, then it is said that *the offer is accepted*. If there is a rejection, it is said that *the offer is rejected*. Only after the acceptance of the offer in coalition C_k, the next coalition C_{k+1} plays the bargaining according to the above (i), (ii) and (iii) with the knowledge about the results of bargaining in the previous coalitions. If the offer is accepted in the last coalition C_m, the bargaining in the first stage is finished. Then, each proposer becomes a representative and obtains resources in his coalition in return for paying his offer (he chose at step (ii)) to the players in his coalition. As a result, player $i \in C_k$, who is not a proposer β_k, gets $b_i^{\beta_k} + y_i$ and leaves the bargaining table. On the other hand, the payoff of the representative $\beta_k \in C_k$ after the first stage is $p_k^1 = -\sum_{j \in C_k \setminus \{\beta_k\}} b_j^{\beta_k} - \sum_{j \in C_k \setminus \{\beta_k\}} y_j$.

After a rejection in C_k, the results of bargaining in the previous coalitions $C_h (h < k)$ are canceled without the transfer of the bids. Every coalition other than C_k proceeds to the bargaining from the top again. Therefore the non-cooperative game is played with $(N', v|_{N'}, C')$ where $N' = N \setminus C_k$ and $C' = C \setminus \{C_k\}$. On the other hand, coalition C_k is separated from the other coalitions and the members in C_k play $\Gamma^{PW}(C_k, v|_{C_k})$.

After an acceptance in the last coalition, the representatives $\beta_1,\cdots,\beta_k$ are selected and they go to stage 2 with their resources collected.

(2) When there is only one coalition, the bargaining is a little bid different from the above one. Let $C = \{N\} = \{C_k\}$. When $|N| = 1$, player $i \in N$ obtains $v(\{i\})$ and the bargaining is finished.

When $|N| \geqq 2$, coalition N plays step (i), (ii) and (iii) mentioned above. The difference lies in the following points. First, after an acceptance of an offer, proposer β_k obtains the worth of total cooperation (in this case, $v(N) = v(C_k)$) and pays his offer y_j to any other $j \in N \setminus \{\beta_k\}$. The bargaining is finished in the case of an acceptance. Second, after a rejection of some player in step (iii), proposer β_k leaves the bargaining with obtaining $v(\{\beta_k\})$ and the other players continue the same bargaining for $(N \setminus \{\beta_k\}, v|_{N \setminus \{\beta_k\}}, \{N \setminus \{\beta_k\}\})$ from the bargaining.

Then, the bargaining for one coalition is well defined and this is the definition of $\Gamma^{PW}(N, v)$.

Stage 2 Let M' be a set of coalitional indices whose representatives participate in the bargaining of stage 2. Then players in $\{\beta_k : k \in M'\}$ play the bidding game for themselves. Note that each β_k has the resources in C_k. Therefore this corresponds to the bidding game of (M', v_C) which is a subgame of an intermediate game (M, v_C). As a result, each β_k obtains p_k^2 which is a payoff of player k in the bidding game for (M', v_C).

The final payoff of representative β_k is the sum of the payoff of stage 1, p_k^1, and the payoff of stage 2, p_k^2.

Remark 1 *We need some tie-breaking rule to choose one player as a proposer when there are many players making the maximal net bid at step (i) of stage 1. The random selection of a proposer from the tie players is not important in our bargaining game. For instance, the following tie-breaking rules are also possible.*

1. *The player with the highest number among tie players becomes a proposer.*
2. *After defining a power order of players independent of the model, the most powerful player among tie players becomes a proposer.*

Remark 2 *We need some predetermined sequential order in both the coalitions' play in stage 1 and the responders' responses at step (iii) in stage 1 to avoid a bad equilibrium. For instance, if the responders decide to accept or reject simultaneously, the situation where more than or equal to two players reject an offer is supported by Nash equilibrium even if all the responders are better off when the offer is accepted.*

We call the above bargaining a *two-stage bidding game (TSBG)*. When $|C|=1$, the TSBG coincides with the bidding mechanism of Perez-Castrillo and Wettstein (2001). They show the following result.

Theorem 1 (Perez-Castrillo and Wettstein (2001)) *$\Gamma^{PW}(N, v)$ implements the Shapley value of (N, v) in any subgame perfect equilibrium (SPE) if (N, v) is zero-monotonic.*

The next theorem shows that the TSBG implements solution defined by Kamijo (2005).

Theorem 2 *The outcome in any SPE of the TSBG for game (N, v, C) gives the same payoff vector as $\phi^G(N, v, C)$ when (N, v) is superadditive.*

Proof. When $|C|=1$, the theorem holds because the TSBG for (N, v, C) and $\phi^G(N, v, C)$ coincide with $\Gamma^{PW}(N, v)$ and $\phi(N, v)$ respectively and Γ^{PW} implements the Shapley value for zero-monotonic game due to Theorem 1 (of course, a superadditive game is zero-monotonic). Next we consider the case with $m (m \geqq 2)$ coalitions. The proof proceeds by the series of claims.

Claim 1: After a rejection of some player in coalition C_k at step (iii) of stage 1, the players in coalition C_k receive their Shapley value payoff of game $(C_k, v|_{C_k})$.

This is the same reason as the case $|C|=1$.

Claim 2: The following strategies in stage 1 and stage 2 constitute an SPE: for each C_k, $|C_k| \geqq 2$,

(i) at step (i) of stage 1, every $i \in C_k$ announces the following bids:

$$b_j^i = \frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} \text{ for any } j \in C_k \setminus \{i\},$$

(ii) at step (ii) of stage 1, proposer β makes an offer $y_j = \phi_j(C_k, v|_{C_k})$ for any other $j (j \in C_k \setminus \{\beta\})$,

(iii) at step (ii) of stage 1, responder j accepts the offer if $y_j \geqq \phi_j(C_k, v|_{C_k})$ and rejects it otherwise.

Let $L \subseteq M, |L| \geqq 2$ be a set of coalitional indices and $\{\beta_k : k \in L\}$ be a set of representatives who play the bargaining of stage 2. Then,

(iv) at step (i) of stage 2, each representative β_k makes a following bid a_h^k to β_h, $h \in L$,

$$a_h^k = \phi_h(L, v_C) - \phi_h(L \setminus \{k\}, v_C),$$

where (L, v_C) and $(L \setminus \{h\}, v_C)$ are subgames of (M, v_C) to L and $L \setminus \{h\}$ respectively.

Let β_k be a proposer at step (ii) of stage 2. Then,

(v) at step (ii) of stage 2, proposer β_k makes an offer $x_h = \phi_h(L \setminus \{k\}, v_C)$ for any other $\beta_h (h \in L \setminus \{k\})$,

(vi) at step (iii) of stage 2, representative β_h other than the proposer accepts the offer if $x_h \geqq \phi_h(L \setminus \{k\}, v_C)$ and rejects it otherwise.

First we confirm that following the strategies mentioned above, the outcome in fact gives the same payoff as $\phi^G(N, v, C)$

According to the strategies, M is a set of coalitional indices whose representatives participate in stage 2 since the offer is accepted in every coalition. The strategies described as (iv), (v) and (vi) are the ones that Perez-Castrillo and Wettstein (2001) constructs to prove their main theorem (Theorem 1 of this paper). Therefore representative $\beta_k \in C_k$ obtains the payoff of $\phi_k(M, v_C)$ from stage 2 by the same reason as Claim 1. Moreover because the offer is accepted in every coalition, his final payoff is:

$$\begin{aligned} p_k^1 + p_k^2 &= -\sum_{j \in C_k \setminus \{\beta_k\}} \frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} \\ &\quad - \sum_{j \in C_k \setminus \{\beta_k\}} \phi_j(C_k, v|_{C_k}) + \phi_k(M, v_C) \\ &= -(|C_k| - 1)\frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} \\ &\quad - v(C_k) + \phi_{\beta_k}(C_k, v|_{C_k}) + \phi_k(M, v_C) \\ &= \frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} + \phi_{\beta_k}(C_k, v|_{C_k}). \end{aligned}$$

On the other hand, the payoff of player $j \in C_k \setminus \{\beta_k\}$ is:

$$b_j^{\beta_k} + y_j = \frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} + \phi_j(C_k, v|_{C_k}).$$

We show that these actually constitute an SPE. First we consider the behavior of responder j at step (iii) of stage 1. If he rejects the offer, he obtains $\phi_j(C_k, v|_{C_k})$ by Claim 1. Let y_j be an offered by a proposer. If $y_j \geqq \phi_j(C_k, v|_{C_k})$, the payoff when he rejects the offer, $\phi_j(C_k, v|_{C_k})$ is less than or equal to the payoff when he accepts it (y_j or $\phi_j(C_k, v|_{C_k})$). On the other hand, if $y_j < \phi_j(C_k, v|_{C_k})$, the payoff when he accepts the offer (y_j or $\phi_j(C_k, v|_{C_k})$) is also less than or equal to the rejecting payoff. Therefore he can not improve his payoff by changing strategy from the one described in the claim after any history.

Next we consider the behavior of proposer β at step (ii) of stage 1. If he raises an offer to some player, the offer is accepted but his payoff will decrease. If he makes an offer less than $\phi_j(C_k, v|_{C_k})$ to player j, then the offer is rejected. In this case, the payoff of β will be $\phi_\beta(C_k, v|_{C_k})$ which is no greater than

$$\frac{\phi_k(M, v_C) - v(C_k)}{|C_k|} + \phi_{\beta_k}(C_k, v|_{C_k})$$

because $\phi_k(M, v_C) \geqq v_C(\{k\}) = v(C_k)$ by the individual rationality of the Shapley value for superadditive games. Therefore changing the offer does not improve proposer's payoff.

Finally we consider the behavior at step (i) of stage 1. Note that according to the above strategies, the payoff of player i is $\psi_i^G(N, v, C)$ with regardless to whether he is proposer or not, and net bid is 0 for any player. It means that if a player raises the bid for some other player by $\epsilon(>0)$, he will be a proposer in step (ii). However this is only to decrease his final payoff by ϵ compared with the above strategies. If he decrease his bid and loses the possibility to be a proposer, then his payoff is unchanged. Therefore the strategies of step (i) are in fact Nash equilibrium.

Therefore these strategies constitute an SPE.

Next we will show that $\psi^G(N, v, C)$ is achieved in any SPE. Suppose every player follows an SPE strategy. Then an outcome of the last coalition (C_m) is described as the following claims. Note that if $|C_m|=1$, then player $i \in C_m$ becomes a representative and he obtains $\phi_m(M, v_C)=\psi_i^G(N, v, C)$ in stage 2.

Claim 3: Let $|C_m| \geqq 2$. At step (ii) and (iii) of stage 1, if $\phi_m(M, v_C)>v(C_m)$, a proposer makes an offer $y_j=\phi_j(C_m, v|_{C_m})$ to any other j and each responder accepts it in any SPE.

Let $y=(y_j)_{j \in C_m \setminus \{\beta_m\}}$ be the offers made by proposer β_m.

When $y_j<\phi_j(C_m, v|_{C_m})$ for some $j \in C_m \setminus \{\beta_m\}$, then the offer is rejected by some player.

In fact, even if all the players except for j accept the offers, player j has the incentive to reject it by Claim 1.

When $y_j>\phi_j(C_m, v|_{C_m})$ for all $j \in C_m \setminus \{\beta_m\}$, the offer is accepted. To prove this fact, first consider the response of the last player in C_m, say $j_{|C_m|}$. Then if he rejects the offer, he obtains $y_{j|C_m|}$ and otherwise $\phi_{j|C_m|}(C_m, v|_{C_m})$ by Claim 1. Therefore an acceptance of the offer is his best response. Second, consider the response of the second last player in C_m, say $j_{|C_m|-1}$. Since he knows that after his acceptance of the offer, the last player also accepts the offer, his best response is also an acceptance of the offer. Repeating these arguments, we obtain that every responder accepts the offer.

Given $\epsilon>0$, consider an offer $y^\epsilon=(y_j^\epsilon)_{j \in C_m \setminus \{\beta_m\}}$ such that $y_j^\epsilon=\phi_j(C_m, v|_{C_m})+\epsilon$ for any $j \in C_m \setminus \{\beta_m\}$. Then y^ϵ is accepted because of the argument mentioned in the previous paragraph. Note that after an acceptance, β_m obtains $\phi_m(M, v_C)$ in stage 2. Then, the payoff of β_m for an acceptance is greater than $\phi_{\beta_m}(C_m, v|_{C_m})$, his payoff for a rejection, for sufficiently small $\epsilon>0$ because

$$\begin{aligned}\phi_m(M, v_C)-\sum_{j \in C_m \setminus \{\beta_k\}} \phi_j(C_m, v|_{C_m})-(|C_m|-1)\epsilon \\ =\phi_m(M, v_C)-v(C_m)+\phi_{\beta_m}(C_m, v|_{C_m}) \\ -(|C_m|-1)\epsilon \\ >\phi_{\beta_m}(C_m, v|_{C_m})\end{aligned}$$

by the assumption of the claim and the efficiency of the Shapley value. For any $\epsilon>0$, offer y^ϵ of the proposer is, however, not the best response to the behaviors of the responders since he can improve his payoff by choosing smaller $\epsilon'>0, (\epsilon'<\epsilon)$. Therefore proposer makes offer y^0 and all the responders accept it in any SPE because if some responder rejects the offer, proposer deviates to offer y^{ϵ} for some $\epsilon >0$.

When $\phi_m(M, v_C)=v(C_m)$ in addition to the equilibrium behaviors mentioned above, all the strategies which lead a rejection of some responder are equilibrium. Note that although there are two types of equilibrium, each outcome leads to the same payoff of players, for proposer β,

$$\phi_m(M, v_C)-\sum_{j\in C_m\setminus\{\beta\}}\phi_j(C_m, v)=\phi_m(M, v_C)-v(C_m)+\phi_\beta(C_m)+\phi_\beta(C_m, v|_{Cm}),$$

and for responder j, $\phi_j(C_m, v|_{Cm})$.

We consider the behaviors at step (i) of stage 1.

Claim 4: For all $i\in C_m$, net bid B^i is 0 in equilibrium.

Let $\Omega := argmax\{B^i : i\in B_m\}$. If $\Omega = C_m$, this claim holds since $\sum_{i\in Cm}B^i=0$.

We consider the case where $\Omega\neq C_m$. Then we can take two players $i\in C_m$ and $j\in C_m$ such that $i\in\Omega$ and $j\in C_m\setminus\Omega$ since Ω is not empty. Let $\delta>0$ and consider the following bid c^i of player i:

$$c_k^i=\begin{cases}\dfrac{\delta}{|\Omega|}+b_k^i & \text{if } k\in\Omega\setminus\{i\}\\ -\delta+b_k^i & \text{if } k=j\\ b_k^i & \text{otherwise}\end{cases}$$

Given this bid, the net bid of $k\in\Omega\setminus\{i\}$ is $B^j-\dfrac{\delta}{|\Omega|}$ and net bid of i is.

$$B^i-\delta+\frac{\delta(|\Omega|-1)}{|\Omega|}=B^i-\frac{\delta}{|\Omega|}.$$

If δ is sufficiently small, the following inequalities hold.

$$B^i-\frac{\delta}{|\Omega|}>B^k\,\forall k\in C_m\setminus\Omega,\ k\neq j, \text{ and}$$
$$B^i-\frac{\delta}{|\Omega|}>B^j+\delta.$$

So player i can decrease the total bid $\sum_{k\in C_m\setminus\{i\}}b_k^i$ with unchanging Ω. By doing so, his expected payoff strictly increases. Therefore this situation is not in any SPE.

Claim 5: The payoff of each player in C_m is the same regardless of who is chosen as a proposer at step (ii) in stage 1.

We already know that all B^i are the same. If player i would strictly prefer to be a proposer (representative), he could improve his payoff by slightly increasing one of his bid b_j^i. Similarly, if player i would strictly prefer that some other player j was the proposer, he could improve his payoff by decreasing b_j^i. The fact that player i does not do so in equilibrium means that he is indifferent to the proposer's identity.

Claim 6: A final payoff of player $i\in C_m$ is $\phi_i^C(N, v, C)$.

We denote by u_i^j the final payoff of player $i\in C_m$ when $j\in C_m$ is chosen as a proposer (representative). Then, for $i\in C_m$,

$$u_i^j=b_i^j+\phi_i(C_m, v|_{Cm}) \text{ for any } j\in C_m\setminus\{i\},$$
$$u_i^i=-\sum_{j\in C_m\setminus\{i\}}b_j^i+\phi_m(M, v_C)-v(C_m)+\phi_i(C_m, v|_{Cm}).$$

Since $u_i^j = u_i^k$ holds for all $j \neq i$, $k \neq i$ by Claim 5, we set $b_i^j = \bar{b}_i$ for all $j \neq i$. Because $B^i = 0$ by Claim 4,

$$\sum_{j \in C_m \setminus \{i\}} \bar{b}_j - \sum_{j \in C_m \setminus \{i\}} \bar{b}_i = 0 \Longleftrightarrow |C_m| \bar{b}_i = \sum_{j \in C_m} \bar{b}_j \tag{2}$$

Since $u_i^j = u_i^i$ holds, we obtain the following equality:

$$-\sum_{j \in C_m \setminus \{i\}} \bar{b}_j + \phi_m(M, v_C) - \sum_{j \in C_m \setminus \{i\}} \phi_j(C_m, v|_{C_m}) = \bar{b}_i + \phi_i(C_m, v|_{C_m}).$$

It implies

$$\sum_{j \in C_m} \bar{b}_j = \phi_m(M, v_C) - \sum_{j \in C_m} \phi_j(C_m, v|_{C_m}) = \phi_m(M, v_C) - v(C_m) \tag{3}$$

Equations (2) and (3) mean

$$\bar{b}_i = \frac{\phi_m(M, v_C) - v(C_m)}{|C_m|}.$$

Then,

$$u_i^j = u_i^i = \frac{\phi_m(M, v_C) - v(C_m)}{|C_m|} + \phi_i(C_m, v|_{C_m}) = \phi_i^G(N, v, C).$$

Therefore we have shown that the bargaining in last coalition C_m gives the same payoff as $\phi^G(N, v, C)$ in equilibrium if the offers are accepted in all the previous coalitions.

Next we consider the bargaining of stage 1 for coalition C_{m-1}. Suppose that every player follows an SPE strategy. If the offer is accepted in coalition C_m according to these strategies, we can show that the payoff of players in C_{m-1} is ϕ^G in equilibrium by the same arguments as Claim 3, 4, 5 and 6. However if there is a rejection in coalition C_m, then the behaviors that $b_j^i = 0$ for all $i \in C_{m-1}$, $j \in C_m$, $i \neq j$ and an offer is accepted in coalition C_{m-1} lead to an equilibrium. A rejection in C_m occurs by SPE strategies only if C_m is a null coalition because of the argument of the proof of Claim 3. It means that $\phi_m(M, v_C) = v_C(\{m\}) = v(C_m)$ and $\phi_i^G(N, v, C) = \phi_i^G(N \setminus C_m, v|_{N \setminus C_m}, C \setminus \{C_m\})$ holds for each $i \in C_k$, $k \neq m$ by the properties of the Shapley value. Thus, even if there exists a rejection in coalition C_m, their equilibrium payoff are ϕ^G.

Repeating these arguments by $m-2$ times, we conclude that the payoff of player $i \in N$ is $\phi_i^G(N, v, C)$. Therefore in any SPE and in any coalition, ϕ^G is implemented by the TSBG. □

Remark 3 *The bidding stage at step (i) of stage 1 to select a proposer is important to achieve ϕ^G as an "actual value". Notice that, even if one player is randomly selected as a proposer in equilibrium by Claim 4, the payoff of a player is independent of whether he is chosen as a proposer or not. It is worth mentioning that what happens if a proposer is selected at random instead of the bidding stage. Then, it also implements ϕ^G but it is an "expected value".*

Remark 4 *The definition of the TSBG says that after a rejection in some coalition C_k, $N \setminus \{C_k\}$ continue the TSBG for $(N \setminus C_k, v|_{N \setminus C_k}, C \setminus \{C_k\})$. The structure that the remaining coalitions continue the bargaining is, however, not so important to implement ϕ^G. This is because each coalition plays the bargaining in stage 1 separately.*

Therefore there is no way for players in $N\setminus C_k$ to prevent some responder in C_k from rejecting an offer. Moreover, the rational responder who concerns only his own payoff does not see an outcome of the other players. Therefore our result is irrelevant to the bargaining process of the remaining coalitions after rejection. In fact, if after a rejection in C_k, all the coalition C_h, $h\in M$ play $\Gamma^{PW}(C_h, v|C_h)$, ϕ^G is implemented.

Remark 5 *When there are more than or equal to two coalitions, the TSBG works well if we select a proposer at step (ii) of stage 1 by "total bids" instead of net bids because the proof of Theorem 2 shows that all the players in a coalition make equal bids in any SPE. In fact, this non-cooperative game leads to the payoff of ϕ^G in some SPE. In this case, we may not, however, assure the uniqueness of equilibrium of the bidding stage.*

A two stage bargaining model based on the bidding game is also considered in Vidal-Puga and Bergantinos (2003). Their model differs from ours only in the bargaining process after a rejection at step (iii) of stage 1. After a rejection of a player in coalition C_k, in our model, members in coalition C_k is separated and play the bidding game for themselves and the other coalitions continue the two stage bidding game from the top, whereas in their bargaining model, only proposer β_k whose offer is rejected is isolated and leaves the bargaining with obtaining the worth of his stand alone coalition (i.e., $v(\{\beta_k\})$) and all the coalitions in which C_k is replaced by $C_k\setminus\{\beta_k\}$ continue the bargaining from the beginning. Therefore the bidding game in stage 2 is played for (M, w) where w is defined by:

$$w(H)=v(\bigcup_{h\in H} C'_h), \quad \text{for any } H\subseteq M,$$

where $C'_h\subseteq C_h$ is a set of members in coalition C_h whose resource the representative collects. They show that this modified two stage bidding game (the modified TSBG) implements the Owen value in some SPE.

Example 1 *Consider the TSBG for a three-person game (N, v, C) where $N=\{1,2,3\}$, $C=\{C_1, C_2\}=\{\{1\},\{2,3\}\}$, $M=\{1,2\}$* and *$v(\{1,3\})=1$, $v(N)=2$* and *$v(S)=0$ otherwise. After a rejection in coalition $\{2,3\}$, they obtain their Shapley value of $(\{2,3\}, v|_{\{2,3\}})$. That is, both players obtain 0 payoffs. On the other hand, $\phi(M, v_C)=(1,1)$. Therefore if a player becomes a proposer (representative) in coalition $\{2,3\}$, he gets payoff 1 from stage 2. Then, by Claim 4 and 5, two players' bids, b^2_3 and b^3_2 satisfy the following equations:*

$$b^2_3=b^3_2 \text{ and } 1-b^2_3=b^3_2.$$

Therefore we obtain $b^2_3=b^3_2=\frac{1}{2}$ and their final payoff is $(1,\frac{1}{2},\frac{1}{2})$. This is exactly $\phi^G(N, v, C)$.

Next we consider the modified TSBG for the same game. Notice that the modified TSBG as well as the TSBG coincide with the bidding mechanism by Perez-Castrillo and Wettstein (2001) when there is a singleton coalition structure. Therefore after a rejection of player 3 (resp. 2) in coalition $\{2,3\}$, he obtains $\phi_3(\{1,3\}, v|_{\{1,3\}})=\frac{1}{2}$ (resp.

$\phi_2(\{1,2\}, v|_{\{1,2\}})=0$). *Therefore if player* 2 (*resp.* 3) *becomes a proposer* (*representative*) *in coalition* {2,3}, *he gets* 1 (*the payoff of second stage*) *minus* $\frac{1}{2}$ (*resp.* 0) (*the payoff which he should pays to player* 3(*resp.* 2)). *Then, by Claim* 4 *and* 5 *again* (*as is mentioned in step B and step C in the proof of Theorem 1 of Vidal-Puga and Bergantinos* (*2003*), *these statements also hold for the modified TSBG. The proofs are almost similar to the ones of Claim 4 and 5*), *two players' bids,* b_3^2 *and* b_2^3 *satisfy the following equations:*

$$b_3^2 = b_2^3 \text{ and } \frac{1}{2} - b_3^2 = b_2^3 (\text{or } 1 - b_2^3 = b_3^2 + \frac{1}{2}).$$

Therefore we obtain $b_3^2 = b_2^3 = \frac{1}{4}$ *and their final payoff is* $(1, \frac{1}{4}, \frac{3}{4})$. *This is the Owen value for* (N, v, C).

Remark 6 *In contrast to Theorem 2, if a game is superadditive but not strictly superadditive, there exists some SPE in the modified TSBG that gives the different payoff vector of players from the Owen value* (*See, Example 1 in Vidal-Puga and Bergantinos* (*2003*)).

Compared to the modified TSBG, the TSBG does not allow members in coalition C_k to contact with the players in the other coalitions if there is a rejection of an offer made by a proposer in C_k. It means that after a rejection, not only a proposer making the rejected offer but also players in his coalition can not cooperate with the other coalitions. Therefore we may state that the restriction of the CS of the TSBG is stronger than that of the modified TSBG. This is the same conclusion as Kamijo (2005) in which the relationship between ϕ^G and the Owen value is analyzed through axiomatization.

4. Bargaining Among Negotiators

In this section, we consider a noncooperative game which satisfies the requirement that to derive outcome through cooperation, the relevant players must be together. It means that players cannot act like a coalition in contrast to the previous section. Therefore the TSBG and the modified TSBG, in which the players collecting the resources of their coalitions (i.e., representatives) play the bidding game in the second stage, does not work well.

The bargaining model in this section consists of four stages -stage A, B, C and D. Let (N, v, C) be a TU game with a CS and $C = \{C_1, \cdots, ,C_m\}$. In stage A, for each coalition, each player makes bids to other players to decide a *negotiator* for his coalition. A player who makes a maximum net bid is chosen as the negotiator of the coalition. Only negotiators go to stage B.

Stage B consists of three steps, step (i), (ii) and (iii) and are similar to the bidding mechanism. At step (i), each negotiator makes a bid to any other negotiators to decide a proposer at step (ii). The proposer is determined in the same manner as stage A. At step (ii), the proposer makes an offer x_k, which the proposer actually pays if agreements are made in every relevant coalition in stage C, to the negotiator of coalition C_k. At step (iii), the other negotiators sequentially decide to accept or reject the offer. If the offer is accepted by every negotiator, then all the coalitions go to stage C. Otherwise, a coalition which the proposer belongs to, goes to stage D and the negotiators from the other coalitions continue with stage B, starting from the beginning.

The difference between a negotiator and a representative in the previous section lies in two points. One is that the negotiator cannot obtain the resources of his coalition and therefore he cannot act like his coalition. The other is that he does not have the right to enforce the cooperation of his coalition. Therefore an acceptance at step (iii) of stage B does not imply cooperation among the players in the coalitions. Cooperation between players in different coalitions is not achieved until bargaining in stage C succeeds.

In stage C, for each coalition, players in the coalition follow the procedure which consists of step (i) and (ii). At step (i), the negotiator, who is determined in stage A and participates in stage B, makes an offer to any other members in his coalition. Players other than the negotiator sequentially decide to accept or reject at step (ii). If every player in the coalition accepts the offer, the offer is accepted in this coalition and otherwise the offer is rejected. When the offer is accepted in every coalition, the bargaining succeeds and all the players agree with producing the worth of their total cooperation. When there is a coalition in which a rejection occurs, they do not agree.

If bargaining succeeds, the proposer in stage B collects the worth of their total cooperation and pays his offer to the other negotiators. Then each negotiator pays his offer in stage C to the members in his coalition. These exchanges are done all at once. After the exchanging, they leave the non-cooperative game.

In stage D, each coalition plays the bidding mechanism separately.

We explain this bargaining model formally. Let (N, v, C) be a TU game with a CS, $C = \{C_1, \cdots, C_m\}$, and $M = \{1, \cdots, m\}$.

(i) If $|C| = 1$, the players who belong to coalition $C_k \in C (C_k = N)$ play $\Gamma^{PW}(N, v)$.
(ii) The bargaining procedure with $m (m \geqq 2)$ coalitions is defined as follows. It consists of four stages.

Stage A Each coalition plays the following procedure sequentially in some predetermined order (e.g., $(C_1, \cdots, C_m)$). For C_k, if $|C_k| = 1$, then $i \in C_k$ is automatically chosen as a negotiator. When $|C_k| \geqq 2$, each $i (i \in C_k)$ makes his bid b^i_j to any other $j (j \in C_k \setminus \{i\})$. Net bid is calculated and negotiator n_k is chosen in the same manner as step (i) of stage 1 of TSBG. Once chosen as a negotiator, n_k actually pays his bid $b^{n_k}_j$ to any other j.
After a bidding in the last coalition C_m, all the negotiators are selected. Only they go to stage B.

Stage B Let $L \subseteq M$ and $\{n_k : k \in L\}$ be a set of negotiators who participate in stage B. When $|L| = 1$, then the negotiator and the players in his coalition go to stage D. Suppose that we know the rule of stage B for less than $|L|$ negotiators. The rule of it for $|L|$ negotiators consists of three steps.

Step (i) Each n_k makes bid a_h^k to any other negotiator $n_h (h \in L \setminus \{k\})$. Net bid A^k is calculated and proposer index α (i.e., proposer n_α) is determined by the same manner as step (i) of stage 1 of TSBG. Once chosen, n_α actually pays his bid a_j^α to any other negotiator n_h.

Step (ii) Proposer n_α makes an offer $x_k \in \mathbf{R}$ to any other n_k.

Step (iii) Each negotiator other than n_α sequentially decides to accept or reject the offer. If the offer is accepted by all the negotiators, then it is said that *an agreement among negotiators is formed*. Otherwise it is said that the *agreement among negotiators is not formed*. After forming an agreement among negotiators in stage B, the negotiators and their coalitions go to stage C. On the other hand, if there is a rejection, coalition C_α, in which proposer n_α belongs to, goes to stage D and the other negotiators continue the stage B from the beginning. For notational convenience, after an agreement in stage B, we set $x_\alpha = v_C(L) - \sum_{k \in L \setminus \{\alpha\}} x_k$.

Stage C Let $L \subseteq M$ be a set of coalition whose negotiators form an agreement in stage B. The following procedures are played for each $C_k, k \in L$ in some predetermined order.

Step (i) Negotiator n_k makes an offer y_j to any other $j (j \in C_k \setminus \{n_k\})$.

Step (ii) Each player in C_k other than the negotiator sequentially decides to accept or reject the offer.

If the offer is accepted by any players in C_k, it is said that *the offer is accepted in coalition* C_k. Otherwise it is said that *the offer is rejected in coalition* C_k. Only if the offer is accepted in coalition C_k, the next coalition, say C_{k+1}, [follows the procedures] mentioned above. When the offer is accepted in any coalition, then it is said that *cooperation among* L *is achieved*. Otherwise it is said that *cooperation among* L *is not achieved*.

If cooperation among L is achieved, then player n_α, who is a proposer in stage B when an agreement among negotiators is formed, collects the worth of cooperation among L (i.e., $v_C(L) = v(\bigcup_{l \in L} C_l)$), and pays his offer x_k to any other negotiator $n_k (k \in L \setminus \{\alpha\})$. Then negotiator n_k pays his offer y_j to any other player $j (j \in C_k \setminus \{n_k\})$. These trading are simultaneously done and after that, they leave the bargaining.

If cooperation among L is not achieved, then every negotiator and their coalitions go to stage D.

Stage D For each coalition C_k, they play $\Gamma^{PW}(C_k, v|_{C_k})$ for themselves.

We call the above bargaining a *bargaining among negotiators*. The following theorem holds.

Theorem 3 *The bargaining among negotiators for superadditive game* (N, v, C) *gives the same payoff as* $\phi^G(N, v, C)$ *in any SPE.*

Proof. We will show this theorem by the series of claims.

Claim (a): The subgame of stage D for $(C_k, v|_{C_k})$ gives the same payoff as the Shapley value of $(C_k, v|_{C_k})$ for any SPE.

This is the same reason as Claim 1 in Theorem 2.

Claim (b): In the subgame starting from stage C with L, $\{n_k\}_{k\in L}$ and $\{x_k\}_{k\in L}$, where $L(|L|\geqq 2)$ is a set of coalition indices participating in stage C, n_k is a negotiator of coalition C_k and x_k is the agreed offer of C_k in stage B, then

(i) if there exists $k\in L$ such that $x_k < v(C_k)$ holds, then an offer is rejected in coalition C_k. Therefore every coalition goes to stage D,

(ii) if $x_k \geqq v(C_k)$ holds for all $k\in L$, there exists the following SPE outcome: for each C_k, the negotiator n_k proposes $y_j=\phi_j(C_k, v|_{C_k})$ for every $j\in C_k\backslash\{n_k\}$, and every $j(j\in C_k\backslash\{n_k\})$ accepts it, and

(iii) if $x_k > v(C_k)$ holds for all $k\in L$ the outcome of (ii) is the one in any SPE.

Consider the bargaining for coalition C_k such that $x_k < v(C_k)$ holds. If $j\neq n_k$ rejects an offer made by n_k, he receives $\phi_j(C_k, v|_{C_k})$ because after an rejection, players in coalition C_k go to stage D and the outcome of the stage D is the Shapley value of subgame $(C_k, v|_{C_k})$ by Claim (a). To be accepted by all the members, offer y must satisfy $y_j\geqq\phi_j(C_k, v|_{C_k})$ for all $j\in C_k\backslash\{n_k\}$. Gathering $x_k<v(C_k)$ and $y_j\geqq\phi_j(C_k, v|_{C_k})$ for all $j\neq n_k$ leads to the fact that the payoff of n_k is less than $\phi_{n_k}(C_k, v|_{C_k})$ since $y_{n_k}=x_k-\sum_{j\in C_k\backslash\{n_k\}}y_j < v(C_k)-\sum_{j\in C_k\backslash\{n_k\}}\phi_j(C_k, v|_{C_k})=\phi_{n_k}(C_k, v|_{C_k})$. Therefore n_k does not have the incentive to make an offer accepted.

The (ii) and (iii) are easily shown by the similar arguments of Claim 3 in Theorem 2.

Claim (c): In the subgame starting from stage B with L and $\{n_k\}_{k\in L}$, where L is a set of coalition indices participating in stage B and n_k is a negotiator of coalition C_k, the payoff of player n_k is $\phi_k(L, v_C)+\phi_{n_k}(C_k, v|_{C_k})-v_C(\{k\})$ for any SPE, where (L, v_C) is a subgame of (M, v_C) to L.

We prove this claim by induction on the number of $|L|$. We first consider the case $|L|=1$. Let $L=\{k\}$. Then coalition C_k goes to stage D. By Claim 1, the payoff of the negotiator becomes $\phi_{n_k}(C_k, v|_{C_k})=\phi_k(L, v_C)+\phi_{n_k}(C_k, v|_{C_k})-v_C(\{k\})$ because $\phi_k(L, v_C)=v_C(\{k\})=v(C_k)$.

We assume that the claim is true when the number of coalitions is less than $|L|$. We now show that it is satisfied when there are $|L|$ coalitions.

We consider the subgame after step (i) and let $\alpha\in L$ be a proposer at step (ii) as a result of step (i).

Let x_k be an offer made by a negotiator n_α to other negotiator $n_k(k\neq\alpha)$. While after a rejection negotiator n_k obtains $\phi_k(L\backslash\{\alpha\}, v_C)+\phi_{n_k}(C_k, v|_{C_k})-v_C(\{k\})$ by the assumption of the induction, he obtains $x_k - \sum_{j \in C_k \setminus \{n_k\}} \phi_j(C_k, v|_{C_k}) = x_k + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})$ when the offer is accepted in step (iii) and cooperation among L is achieved in stage C. Therefore the condition that $x_k \geqq \phi_k(L \setminus \{\alpha\}, v_C)$ is necessary for each negotiator n_k to accept the offer.

Next we have to consider the following three cases.

Case (a): $v_C(L) > v_C(L \setminus \{\alpha\}) + v_C(\{\alpha\})$. Suppose that either an offer made by n_α is rejected or it is accepted but cooperation among L is not achieved in stage C. Then player n_α obtains $\phi_{n_\alpha}(C_\alpha, v|_{C_\alpha})$ in both cases. However consider the following strategy of proposer α: he makes an offer x such that $x_k = \phi_k(L \setminus \{\alpha\}, v_C) + \frac{\varepsilon}{|L|}$ for each $k \in L \setminus \{\alpha\}$ and $x_\alpha = v_C(L) - v_C(L \setminus \{\alpha\}) - \frac{|L|-1}{|L|}\varepsilon$ where $\varepsilon = v_C(L) - v_C(M \setminus \{\alpha\}) - v_C\{\alpha\} > 0$. Note that

$$x_k = \phi_k(L \setminus \{\alpha\}, v_C) + \frac{\varepsilon}{|L|} > v(C_k),$$

for all $k \in L \setminus \{\alpha\}$,

and

$$x_\alpha = v_C(L) - v_C(L \setminus \{\alpha\}) - \frac{|L|-1}{|L|}\varepsilon$$
$$= v_C(\{\alpha\}) + \frac{\varepsilon}{|L|} > v(C_\alpha),$$

hold because of the individual rationality of the Shapley value in the domain of superadditive games. Therefore offer x is accepted at step (iii) and cooperation among L is achieved in stage C by Claim (b). Then player n_α gets

$$v_C(\{\alpha\}) + \frac{\varepsilon}{|L|} + \phi_{n_\alpha}(C_\alpha,) - v(C_\alpha)$$
$$= \frac{\varepsilon}{|L|} + \phi_{n_\alpha}(C_\alpha, v).$$

Therefore these two cases can not be in SPE.

On the other hand, if $x_k > \phi_k(L \setminus \{\alpha\}, v_C)$ for some $k \neq \alpha$, player n_α can increase his payoff by making offer x'_k such that $v(C_k) < x'_k < x_k$. Therefore in any SPE, proposer n_α offers $\phi_k(L \setminus \{\alpha\}, v_C)$ to each $n_k (k \in L \setminus \{\alpha\})$ and each responder $n_k \neq \alpha$ accepts it in step (iii) and cooperation among L is achieved in stage C. Moreover player n_α obtains,

$$v_C(L) - v_C(L \setminus \{\alpha\}) + \phi_{n_\alpha}(C_\alpha, v) - v_C(\{\alpha\}), \quad (4)$$

and n_k, $k \neq \alpha$ obtains

$$\phi_k(L \setminus \{\alpha\}, v_C) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\}). \quad (5)$$

in any SPE.

Case (b): $v_C(L) = v_C(L \setminus \{\alpha\}) + v_C(\{\alpha\})$ and $v_C(L) > \sum_{k \in L} v_C(\{k\})$. When an agreement is formed among negotiators in stage B and cooperation among L is achieved in stage C, n_α obtains $\phi_{n_\alpha}(C_\alpha, v|_{C_\alpha})$ because the conditions that $x_k = \phi_k(L \setminus \{\alpha\})$ for all $k \in L \setminus \{\alpha\}$ are necessary for the acceptance of the offer by the argument of case (a) and the condition that $y_\alpha \geqq v(C_\alpha)$ is necessary for achieving cooperation among L by Claim (b). On the other hand, if either a rejection is encountered at step (iii) or cooperation among L is not achieved in stage C, he obtains $\phi_{n_\alpha}(C_\alpha, v)$. Therefore his payoff is irrelevant to whether the offer is accepted or rejected, and whether cooperation among L is achieved or not.

On the other hand, player $n_k, k \neq \alpha$ obtains $\phi_k(L \setminus \{\alpha\}) + \phi_{n_k}(C_k, v|_{C_k}) - v(C_k)$ if either the offer is accepted and cooperation among L is achieved, or the offer is rejected in step (iii). He obtains $\phi_{n_k}(C_k, v|_{C_k})$ if the offer is accepted but cooperation among L is not achieved in stage C.

Therefore an SPE does not support the situation that the agreement is formed among negotiators in stage B and cooperation among L is not achieved in stage C because there is some $k \in L \setminus \{\alpha\}$ such that his payoff is better off by rejecting the offer in step (iii) of stage B.[2] Therefore an SPE supports either the situation that someone rejects the offer in step (iii) or the offer is accepted in step (iii) and cooperation among L is achieved in stage C. Moreover in both cases, their payoffs are equal to (4) and (5).

Case (c): $v_C(L) = \sum_{k \in L} v_C(\{k\})$. Then, each negotiator obtains the payoff of (4) if he is a proposer and otherwise obtains that of (5) regardless of whether an offer is accepted or rejected in step (iii) and whether cooperation among L is achieved in stage C or not.

Next, we have to consider the behaviors at step (i). For the same reason of Claim 4 and 5 in Theorem 2, we conclude that $A^k = 0$ for all $k \in L$ and each negotiator's payoff is the same regardless of who is chosen as a proposer in any SPE. Let q_k^h be the payoff of n_k when $n_h \in L$ is a proposer. Then,

$$\begin{aligned}\sum_{h \in L} q_k^h &= \sum_{h \in L \setminus \{k\}} (\phi_k(L \setminus \{h\}, v_C) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})) \\ &\quad + v_C(L) - v_C(L \setminus \{k\}) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\}) \\ &= \sum_{h \in L \setminus \{k\}} \phi_k(L \setminus \{h\}, v_C) + v_C(L) - v_C(L \setminus \{k\}) \\ &\quad + |L|(\phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})) \\ &= \sum_{h \in L \setminus \{k\}} \phi_k(L \setminus \{h\}, v_C) + \sum_{h \in L} \phi_h(L, v_C) \\ &\quad - \sum_{h \in L \setminus \{k\}} \phi_h(L \setminus \{k\}, v_C) + |L|(\phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})) \\ &= \sum_{h \in L \setminus \{k\}} (\phi_k(L \setminus \{h\}, v_C) + \phi_h(L, v_C) - \phi_h(L \setminus \{k\}, v_C)) + \phi_k(L, v_C) \\ &\quad + |L|(\phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})) \\ &= \sum_{h \in L \setminus \{k\}} \phi_k(L, v_C) \\ &\quad + \phi_k(L, v_C) + |L|(\phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})) \\ &= |L|(\phi_k(L, v_C) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})).\end{aligned}$$

The second last equality is by the balanced contributions property of the Shapley value.

Since $q_k^h = q_k^l$ for any $h, l \in L$, we conclude that $q_k^h = \phi_k(L, v_C) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\}) \forall h \in L$. Hence Claim (c) holds when there are $|L|$ coalitions, we finish the proof of this claim.

The Claim 4 and Claim 5 in the proof of Theorem 2 hold in stage A for every coalition. By Claim (c), after stage A, negotiator n_k is going to obtain $\phi_k(L, v_C) + \phi_{n_k}(C_k, v|_{C_k}) - v_C(\{k\})$ and player $i \in C_k, i \neq n_k$ is going to obtain $\phi_i(C_k, v|_{C_k})$ in addition to the payment or the receiving of bids in stage A. By the same arguments as the proof of Claim 6 in Theorem 2, these facts imply that equilibrium bid of player $i \in C_k$ (to player $j \in C_k, j \neq i$ is) $\frac{\phi_k(M, v_C) - v(C_k)}{|C_k|}$. Therefore we obtain the desired result. □

Theorem 3 says that the bargaining among negotiators satisfies the requirement that in deriving the cooperative outcome, the relevant players must be together and implements ψ^G. Then the next question arises: What modification of the bargaining enables the Owen value to be achieved? From our observation in section 3, one may suppose that if after a rejection in stage C, only a negotiator who makes the rejected proposal is isolated and the other players in his coalition and all the players in the other coalitions continue the same bargaining for $n-1$ players, the Owen value might be implemented. However it is not so easy. We illustrate this point in the following discussion.

Given (N, v, C), we define the modified version of the bargaining among negotiators as follows. When there is only one player, he obtains the value of his stand alone coalition. Suppose that its rule is defined for $n-1$ players case. Then we define its rule when there are n players.

The difference between the bargaining through negotiators and modified one is only after a rejection in stage C. After a rejection of a player in coalition C_k at stage C, negotiator n_k is isolated and all the players in $N\backslash\{n_k\}$ continue the same bargaining for $(N\backslash\{n_k\}, v|_{N\backslash\{n_k\}}, C\backslash\{C_k\}\cup\{C_k\backslash\{n_k\}\})$ instead of all the coalitions being separated and going to stage D. The following example shows that there is an SPE outcome which gives the different payoff of players from the Owen value in this modified bargaining model.

Example 2 *Consider three-person game (N, v, c) in Example 1 again. Then,*

$$\phi^O(N, v, C) = \left(1, \frac{1}{4}, \frac{3}{4}\right).$$

Note that if either there is only one coalition in the CS or all the coalition in the CS are one player coalition, the modified version is the same as the bidding game in Perez-Castrillo and Wettstein (2001).

Suppose that in the bargaining for sub coalition $S\subseteq N$, players in S follow the strategy described in Perez-Castrillo and Wettstein (2001) which leads to the Shapley value payoff of players in S. Then we constitute the equilibrium strategy for N.

1. *In stage A, player 2 (resp. 3) makes bid $b_3^2 = \frac{1}{4}\left(\text{resp. } b_3^2 = \frac{1}{4}\right)$.*
2. *Let $n_1 = 1$ and $n_2 = 2$ or 3 be negotiators in stage B. Then, n_1 (resp. n_2) makes bid $a_2^1 = \frac{3}{4}$ (resp. $a_1^2 = \frac{3}{4}$) at step (i). If n_1 becomes a proposer at step (ii), he makes an offer $x_2 = \frac{1}{2}$ and n_2 accepts it if $x_2 \geqq \frac{1}{2}$ and rejects it otherwise. If n_2 becomes a proposer at step (ii), he makes an offer $x_1 = 0$ and n_1 accepts it if $x_1 \geqq 0$ and rejects it otherwise.*
3. *In stage C, if player 2 (resp. 3) is a negotiator, he makes an offer $y_3 = \frac{1}{2}$ (resp. $y_2 = \frac{1}{2}$) and player 3 (resp. 2) accepts it if $y_3 \geqq \frac{1}{2}$ (resp. $y_2 \geqq \frac{1}{2}$) and rejects it otherwise.*

We can easily check that the strategy profile described above is an SPE and leads $\left(\frac{3}{4}, \frac{5}{8}, \frac{5}{8}\right)$ as expected payoff of players. This is different from the Owen value.

5. Concluding remarks

We discriminate between the two types of cooperation behind the characteristic function and present two bargaining models implementing ϕ^G for each type of cooperation (in section 3 and 4 respectively). Thus, in the final section of this paper, we will give some economic examples behind the characteristic function to illustrate the importance of classifying these types of cooperation.

First consider the following n-person TU game (N, v), which is served as an example of cooperation with resource trading. Player i has his resource vector $w_i \in \mathbf{R}^l$ which he can input for himself or jointly with other members into the common accessible technology $f : \mathbf{R}^l \rightarrow \mathbf{R}$ which can produce the amount of utility transferable among players. If members in $S \subseteq N$ cooperate, they can jointly input their resources into this technology. Therefore the worth of coalition S is defined by $v(S) := f(\sum_{i \in S} w_i)$. A particular feature of this kind of cooperation is that player's own character such as preference, ability and so on has no significance. The player who obtains all the resources of members in S can get exactly $v(S) = f(\sum_{i \in S} w_i)$ for himself even when the players other than him has already left away. This point enables the bargaining models of Gul (1989), Vidal-Puga and Bergantinos (2003) and Vidal-Puga (2005) to work well. This is, however, very special case in economics because each agent has his own nonexchangeable character such as preference, ability and so on in most economic situation. Following example shows this point.

Next consider the following modified version of the above story. The productivity of the technology depends on the ability of players who manage it. Henceforth $v(S)$ is redefined by $f(\sum_{i \in S} w_i ; S)$ which now depends not only on the resource input but also on the members in the cooperation. Then, game (N, v) can not be explained by the cooperation with resource trading because even if player i obtains all the resource in coalition S, he can not get the amount of $f(\sum_{i \in S} w_i ; S)$, but $f(\sum_{i \in S} w_i ; \{i\})$. In this game, to achieve the cooperation among S, all the members in S in fact work together. This is an usual interpretation of cooperation in cooperative game theory.

Notes

1 Here, we do not consider the resource constraint and thus the restriction of the strategy space because we treats a transferable utility game. If we allow the existence of the resource constraint, this non-cooperative game is well defined when all the payments in this non-cooperative game are done after the game has been finished with producing the total output.

2 This is because $v_C(L) = v_C(L \backslash \{a\}) + v_C(\{a\})$ and $v_C(L) > \sum_{k \in L} v_C(\{k\})$ imply that $v_C(L \backslash \{a\}) > \sum_{k \in L \backslash \{a\}} v_C(\{k\})$.

References

Aumann, R. J., and J. H. Dreze (1974), "Cooperative games with coalition structures," *International Journal of Game Theory*, 3, 217–237.

Dasgupta, A., and Y. S. Chiu (1998), "On implementation via demand commitment games," *International Journal of Game Theory*, 27, 161–189.

Evans, R. A. (1996), "Value, consistency and random coalition formation," *Games and Economic Behavior*, 12, 68–80.

Gul, F. (1989), "Bargaining foundations of Shapley value," *Econometrica*, 57, 81–95.

Hart, S., and A. Mascolell (1992), "A model of n-person noncooperative game," Discussion Paper 7, Harvard Institute of Economic Research, Harverd University.

—— (1996), "Bargaining and value," *Econometrica*, 64, 357–380.

Kamijo, Y. (2005), "An Axiomatization of the S-S value of a cooperative game with a coalition structure," mimeo.

Myerson, R. B. (1980), "Conference structures and fair allocation rules," *International Journal of Game Theory*, 9, 169–182.

Owen, G. (1977), "Values of games with a priori unions," in *Essays in Mathematical Economics and Gawe Theory*, ed. by R. Henn, and O. Moeschlin, vol. 37, pp. 76–88. Springer-Verlag.

Perez-Castrillo, D., and D. Wettstein (2001), *"Bidding for the surplus: A noncooperative approach to the Shapley value," Journal of Economic Theory*, 100, 274–294.

Shapley, L. S. (1953), "A value for n-person game," in *Contributions to the Theory of Games*, ed. by H.Kuhn, and A.Tucker, vol. 2, pp. 307, 317. Princeton University Press, Princeton, NJ.

Vidal-Puga, J. J. (2005), "Implementation of the levels structure value," *Annals of Operations Researach*, 137, 191–209.

Vidal-Puga, J. J., and G. Bergantinos (2003), "An implementation of the Owen value," *Games and Economic Behavior*, 44, 412–427.

Winter, E. (1994), "The demand commitment bargaining and snowballing cooperation," *Economic Theory*, 4, 255–273.