A Money Demand Function Derived from a Pure Exchange Model

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Abstract

This paper explains Keynes's liquidity preference theory on the basis of standard microeconomic tools, i.e., the expected utility theory and a pure exchange model. Two individuals with initial endowments meet in the asset market, where bonds and money are exchanged to maximize their utilities. As a result, a unique price of bonds, which equilibrates the demand for bonds with the supply of bonds, is determined. Following the "Walras' law" the money market also clears at the same time. Hence the equilibrium rate of interest. The effectiveness of monetary policy and the liquidity trap are also discussed.

1. Introduction

As is well known, Keynes (1936) analyzed the demand for money by distinguishing it into three parts, i.e., transactions-motive, precautionary-motive, and speculative-motive. He assumed that the first two motives depend positively on income while the last motive negatively on the rate of interest. Needless to say, it is the introduction of the rate of interest into a money demand function that made his money demand function quite different from that of the classical school which depends only on income (the quantity theory of money).

Keynes asserted that the rate of interest adjusts such that the demand for money equals the money supply that is assumed to be exogenous. This is frequently expressed as "the rate of interest adjusts to equilibrate the money market" (e.g., Mankiw (1997, p.263)), though there does not exist a market where money is actually sold and bought. Anyway such a role of the rate of interest in the "money market" is crucial for Keynes's liquidity preference theory to hold.

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As is also well known, it is Hicks's (1937) deep insight which gave birth to the elegant formulation of such money market mechanism in the form of the *LM* curve. Since then various attempts have been made to give microeconomic foundations of the inverse relationship between the demand for money and the rate of interest. However, in my judgement, such attempts, after all, might be condensed into rather old papers such as Baumol (1952), Tobin (1956), and Tobin (1958, 1965).[1] If my judgement is not mistaken, there seems to be some room for consideration. The above-mentioned papers all deal with the demand for money depending on the rate of interest. Since the rate of interest is treated as given, however, it is not clear how the money market clears. The rate of interest must move to equilibrate the money market. Furthermore, it is important to pay attention to the bond market as well as the money market, because the rate of interest itself never exists independently, i.e., it is only an "inverse" of the price of bonds and the price of bonds is determined according to the demand for and the supply of bonds.

In this paper I derive a money demand function using the expected utility theory and a pure exchange model, and show that the money demand function depends negatively on the rate of interest (or positively on the price of bonds) and that the money market and the bond market clear at once due to the adjustment of the rate of interest (or the price of bonds). In Section 2 the asset market model with two individuals is presented. In Section 3 the expected utility theory is applies to the portfolio selection, while in Section 4 the result is connected to a pure exchange model to explain the adjustment mechanism of the asset market. In Section 5 a money demand function is derived and the effectiveness of monetary policy is considered. Section 6 includes some remarks.

2. The Model

The economy consists of *at least* two individuals (or groups). Let $\bar{n}$ and $\bar{M}$ denote respectively the supplies of bonds and money in the economy as a whole at the beginning of this period, and let $\bar{Y}$ and p denote respectively nominal national income and the price of bonds at this period. Bars mean fixed values. For simplicity assume that there are two individuals I and II, and that at the beginning of this period they hold respectively combinations ($\bar{n}^I$, $\bar{M}^I$) and ($\bar{n}^{II}$, $\bar{M}^{II}$) of bonds and money, where superscripts mean each individual.[2] Thus $\bar{n} = \bar{n}^I + \bar{n}^{II}$ and

[1] My judgement is based mainly on literature on the demand for money such as McCallum and Goodfriend (1987), Goldfeld and Sichel (1990), and Friedman (1987), and macroeconomic textbooks such as Mankiw (1997) and Barro (1997), and so on.

$\overline{M} = \overline{M}^I + \overline{M}^{II}$.

Suppose that each individual must retain a certain amount of money in terms of the transactions-motive and the precautionary-motive.[3] Write this amount as $k\overline{Y}^I$ and $k\overline{Y}^{II}$ with $\overline{Y}^I$ and $\overline{Y}^{II}$ as nominal income of each individual, and k as a positive constant. Then $\overline{Y} = \overline{Y}^I + \overline{Y}^{II}$, and wealth of I and II can be expressed respectively as follows:

$$W^I = p\overline{n}^I + \overline{m}^I, \tag{1}$$

and

$$W^{II} = p\overline{n}^{II} + \overline{m}^{II}, \tag{2}$$

where $\overline{m}^I = \overline{M}^I - k\overline{Y}^I$ (> 0) and $\overline{m}^{II} = \overline{M}^{II} - k\overline{Y}^{II}$ (> 0). Furthermore write the sum of $\overline{m}^I$ and $\overline{m}^{II}$ as $\overline{m}$. Then $\overline{m}$ represents the supply of money available for the speculative-motive.[4] From (1) and (2) budget constraints of I and II for the speculative-motive are respectively

$$pn^I + m^I = p\overline{n}^I + \overline{m}^I, \tag{3}$$

and

$$pn^{II} + m^{II} = p\overline{n}^{II} + \overline{m}^{II}. \tag{4}$$

Individual I chooses a combination (n^I, m^I) of bonds and money which maximizes his utility expressed by

$$U^I(n^I, m^I; p), \tag{5}$$

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2 It is not necessary to limit the number of individuals to two in order to analyze the adjustment mechanism of the asset market. As will be seen, the reason for two individuals is only to take advantage of a pure exchange model.

3 Since the transactions-motive and the precautionary-motive are supposed to lead to part of the demand for money, it would be correct to write that each individual *wants* to retain a certain amount of money. Hicks (1967) himself states that the transactions-motive does not lead to the demand for money.

4 Keynes (1936, p. 171) explains the liquidity preference theory as follows: "if the liquidity-preference due to the transactions-motive and the precautionary-motive are assumed to absorb a quantity of cash ... so that the total quantity of money, less this quantity, is available for satisfying liquidity-preferences due to the speculative-motive, the rate of interest and the price of bonds have to be fixed at the level at which the desire on the part of certain individuals to hold cash ... is exactly equal to the amount of cash available for the speculative-motive. Thus each increase in the quantity of money must raise the price of bonds ... ." Therefore the sum of $\overline{m}^I$ and $\overline{m}^{II}$ corresponds to "the total quantity of money, less this quantity," and in other words "the amount of cash available for the speculative-motive."

subject to his budget constraint (3). Similarly individual II chooses a combination (n^{II}, m^{II}) which maximizes his utility expressed by

$$U^{II}(n^{II}, m^{II}; p), \tag{6}$$

subject to his budget constraint (4). Note that utility functions (5) and (6) are different from usual utility functions in that they contain price in addition to the number of "goods." Such utility functions are based on the expected utility theory, which is considered in the next section.

3. Expected Utility Theory of Bonds and Money

Money bears no interest. Bonds here are consols, one unit of which bears interest equal to one unit of money per period in perpetuity. The relation between the price of bonds and the rate of interest are as follows:

$$i = 1/p, \tag{7}$$

where i is the rate of interest. The price of bonds may change at the next period, but one cannot know the price correctly. Let p^{eI} be the the price of bonds expected to hold at the next period by individual I, and g^{eI} be the corresponding expected rate of change of the price of bonds.[5] Then the expected rate of return on a bond for I, r^{eI}, is given by

$$r^{eI} = (p^{eI} - p + 1)/p = g^{eI} + i. \tag{8}$$

Everyone faces the same rate of interest, but each has his own expected rate of return on bonds.

For simplicity assume that $r^{eI} = a$ with probability $1-\pi$, and that $r^{eI} = b$ with probability π, where $a > 0 > b > -1$ and $0 < \pi < 1$. Then expected utility for I is

$$\begin{aligned} EU^I(c_a^I, c_b^I) &= (1-\pi)u^I(c_a^I) + \pi u^I(c_b^I), \\ c_a^I &= (1+a)pn^I + m^I, \\ c_b^I &= (1+b)pn^I + m^I, \end{aligned} \tag{9}$$

where c_a^I and c_b^I are wealth of I at the beginning of the next period for $r^{eI} = a$ and $r^{eI} = b$, respectively, and $u^I(c)$ is a utility function of wealth c (or implicitly consumption) for I with $u^{I\prime}(c) > 0$ and $u^{I\prime\prime}(c) < 0$.

Individual I maximizes his expected utility (9) subject to his budget con-

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[5] In what follows argument proceeds mainly concerning to individual I, but it similarly applies to individual II, superscript I being replaced by II.

straint

$$-bc_a^I + ac_b^I = W^I(a-b). \tag{10}$$

In what follows only interior solutions are considered, which implies $c_a^I > c_b^I$. An optimal choice is illustrated in Figure 1. When $p = p_1$, individual I chooses his optimal combination (c_{a1}^{I*}, c_{b1}^{I*}) at point A. Hence his demand for bonds n_1^{I*} and demand for money m_1^{I*} are determined uniquely.

The argument above in this section is not new at all. It only follows the expected utility theory. In fact useful relations can be exactly derived. As a typical example a change of the demand for money to a change of wealth is given by

$$dm^I / dW^I = 1 + [u^{I\prime}(c_a^I)/D][AR(c_a^I) - AR(c_b^I)], \tag{11}$$

$$AR(c) \equiv -u^{I\prime\prime}(c)/u^{I\prime}(c) > 0,$$

$$D = -au^{I\prime\prime}(c_a^I) - bdu^{I\prime\prime}(c_b^I) > 0,$$

$$d = b\pi / a(1-\pi) < 0.$$

Derivatives are evaluated at each optimal point. AR (c) is the degree of absolute risk aversion and the following is assumed as usual:

Assumption 1. $AR'(c) < 0$, i.e., the degree of absolute risk aversion for $u^I(c)$ is a decreasing function of wealth c.

This means $dm^I/dW^I < 1$, i.e., the demand for money increases less than wealth.

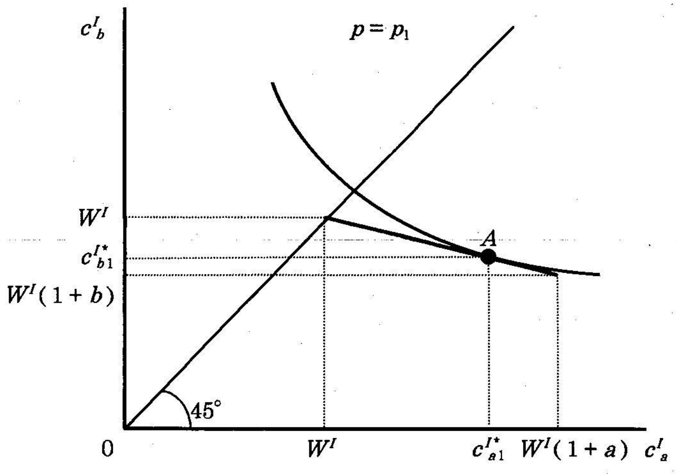

Fig. 1. Maximization of Expected Utility.

Furthermore the following is assumed:

Assumption 2. $1 + [u^{I\prime}(c_a^I)/D]\,[AR(c_a^I) - AR(c_b^I)] > 0.$

It is obvious under this assumption that $d\,m^I/d\,W^I > 0$, i.e., the demand for money always increases according as wealth becomes bigger. Then the following proposition is obtained.[6, 7]

Proposition 1. Under Assumptions 1 and 2, $0 < dm^I/dW^I < 1$ *holds.*

From the budget constraint (3) the following proposition is obtained immediately:

Proposition 2. Under Assumptions 1 and 2, $0 < d\,(pn^I)/dW^I < 1$ *holds.*

Here I like to point out that it is usual to take wealth as a parameter in such arguments. It implies in this case that the price of bonds p is regarded as a parameter with $\bar{n}^I$ and $\bar{m}^I$ as given for the moment. But such a price of bonds does not

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[6] The assumption that money bears no interest is not essential to Proposition 1. Let e be the rate of return on money with $a > e > b > -1$. Then (10) and (11) are respectively replaced by

$$-(b-e)c_a^I + (a-e)c_b^I = W^I(a-b)(1+e),$$

and

$$dm^I / dW^I = 1 + (1+e)\,[u^{I\prime}(c_a^I)/G]\,[AR(c_a^I) - AR(c_b^I)],$$
$$G = -(a-e)u^{I\prime\prime}(c_a^I) - (b-e)fu^{I\prime\prime}(c_b^I) > 0,$$
$$f = (b-e)\pi / (a-e)(1-\pi) < 0,$$

where $c_a^I = (1+a)pn^I + (1+e)m^I$ and $c_b^I = (1+b)pn^I + (1+e)m^I$. The case of $e = 0$, $e > 0$ and $e < 0$ can be regarded as that of cash, bank deposits and Gesell's stamped money (Keynes (1936, p. 357).

It can be easily shown that individual I wants to hold all of his wealth in the form of money if $e \geq (1-\pi)a + \pi b$, and he wants to hold at least part of his wealth in the form of bonds if $e < (1-\pi)a + \pi b$. The latter case is implicitly assumed to hold in (9). Inflation can be also treated by putting $c_a^I = [(1+a)pn^I + m^I]/(1+\pi)$ and $c_b^I = [(1+b)pn^I + m^I]/(1+\pi)$, where π is the rate of inflation.

[7] Let ω denote the ratio of the demand for money to wealth, i.e., $\omega = m^I / W^I$. Then it is known that ω depends on the degree of relative risk aversion defined as $RR(c) \equiv -c\,u^{I\prime\prime}(c) / u^{I\prime}(c) > 0$. In fact the following holds:

$$dw / dW^I = [u^{I\prime}(c_a^I) / D(W^I)^2]\,[RR(c_a^I) - RR(c_b^I)].$$

For example $RR'(c) = 0$ implies that individual I wants to hold money the ratio of which to wealth is constant irrespective of the amount of wealth.

necessarily clear the asset market. The price of bonds must adjust to equalize the aggregate demand for bonds with the aggregate supply of bonds. How can it be explained? Story is still on its way. It is the existence of two *different* individuals that are necessary to conclude the story of the liquidity preference theory. In the next section I try to show it.

4. Pure Exchange of Bonds and Money

What happens when p varies? An immediate effect for individual I is represented as $dW^I = \bar{n}^I dp$, i.e., the corresponding increase in his wealth. Substituting it into (11) yields

$$dm^I / dp = \bar{n}^I \{1 + [u^{I\prime\prime}(c_a^I) / D][AR(c_a^I) - AR(c_b^I)]\} > 0, \quad (12)$$

i.e., the demand for money rises as the price of bonds goes up under Assumption 2.

How about the demand for bonds in terms of unit, n^I, not that in terms of the amount, pn^I? A change of the demand for bonds to a change of p is given by

$$dn^I / dp = -(n^I / p) - (\bar{n}^I / p)[u^{I\prime\prime}(c_a^I) / D][AR(c_a^I) - AR(c_b^I)]. \quad (13)$$

It seems that a bond is an ordinary good under Assumption 2, but the following is assumed for accuracy:

Assumption 3. $1 + (\bar{n}^I/n^I)[u^{I\prime\prime}(c_a^I)/D]\ [AR\ (c_a^I) - AR\ (c_b^I)] > 0.$

Then the following proposition is obtained:[8]

Proposition 3. Under Assumption 3, $dn^I/dp < 0$, *i.e., the demand for bonds decreases (increases) as the price rises (declines).*

Figure 2 represents the n^I-m^I plane in which the above results are illustrated for $p_1 < p_2 < p_3$. Point A in Figure 2 corresponds to point A in Figure 1. In Figure 2 points A (n_1^{I*}, m_1^{I*}), B (n_2^{I*}, m_2^{I*}) and C (n_3^{I*}, m_3^{I*}) are respectively the optimal choices for p_1, p_2 and p_3. It follows from (12) and Proposition 3 that $n_1^{I*} > n_2^{I*} > n_3^{I*}$

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[8] From (13) the price elasticity of the bond demand is given by

$$1 + (\bar{n}^I / n^I)[u^{I\prime\prime}(c_a^I) / D][AR(c_a^I) - AR(c_b^I)],$$

which is less than one because of Assumption 1. This proves partly Proposition 2, i.e., $d\ (pn^I) / dW^I > 0$.

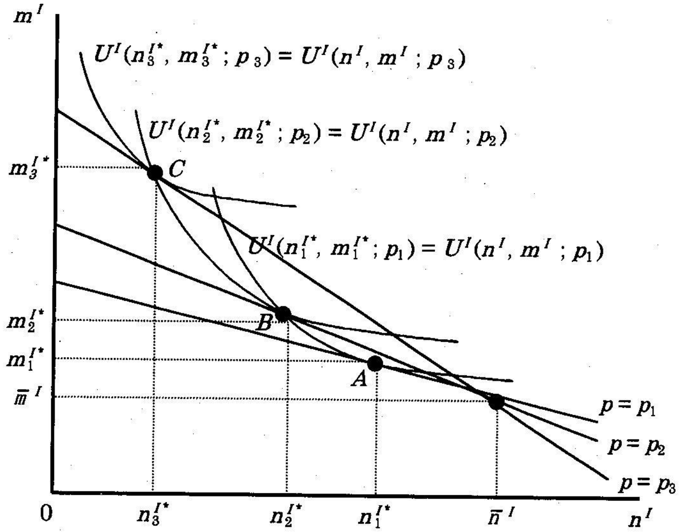

Fig. 2. Optimal Choices of Bonds and Money.

and $m_1^{I*} < m_2^{I*} < m_3^{I*}$. As has been already seen, these optimal choices can be obtained by maximizing (9) subject to (10).

There is another way of finding them out. Consider the following expected utility function with p as a parameter:

$$U^I(n^I, m^I; p) = (1-\pi)u^I((1+a)pn^I + m^I) + \pi u^I((1+b)pn^I + m^I). \quad (14)$$

This is just the utility function $U^I(n^I, m^I; p)$ mentioned in Section 2. It is of course the same as the expected utility function (9). Thus maximizing (14) subject to

$$pn^I + m^I = p\overline{n}^I + \overline{m}^I, \quad (3)$$

must yield the same optimal choice (n^{I*}, m^{I*}) as can be obtained by maximizing (9) subject to (10). This can be analyzed graphically as follows. First fix p. Second draw the budget line (3) on the n^I-m^I plane. Third depict "indifference curves" for (14) on the same plane. As can be easily seen, such "indifference curves" are downward sloping.[9] Finally find a point on the budget line tangent to

[9] In general it would be reasonable to assume that a utility function $U^I(n^I, m^I; p)$ with p as a parameter has the same properties as that in microeconomic consumer theory, e.g., a utility function of "bananas and oranges." This simile comes from Professor Niehans who suggested the possibility of a utility function like $U^I(n^I, m^I; p)$. See Hahn and Brechling (1965, p. 285).

such an "indifference curve." The point of contact represents a unique optimal choice for fixed p.[10]

In Figure 2 are also shown three "indifference curves" $U^I\ (n_1^{I*},\ m_1^{I*};\ p_1) = U^I\ (n^I,\ m^I;\ p_1)$, $U^I\ (n_2^{I*},\ m_2^{I*};\ p_2) = U^I\ (n^I,\ m^I;\ p_2)$ and $U^I\ (n_3^{I*},\ m_3^{I*};\ p_3) = U^I\ (n^I,\ m^I;\ p_3)$. They are so depicted that $U^I\ (n_1^{I*},\ m_1^{I*};\ p_1) = U^I\ (n^I,\ m^I;\ p_1)$ goes through point B and $U^I\ (n_2^{I*},\ m_2^{I*};\ p_2)$ goes through point C. "Indifference curves" never cross each other for the same p but cross each other for different p's. In other words one and the same point on the plane represents different utility levels for different p's.

It should be stressed that "indifference curves" like these are not so important for the analysis of optimal choices. Without relying on "indifference curves," as has been already seen, an optimal combination of bonds and money can be completely determined on the basis of (9) and (10). But it is important that pairs of an "indifference curves" and a budget line remind us of an offer curve and furthermore the Edgeworth box diagram representative of a pure exchange since one of the essence of the asset market is just an exchange of money (bonds) for bonds (money) among *different* individuals.

An "offer curve" for individual I is defined as the locus of the optimal points for various p's like points A, B and C in Figure 2. Such an "offer curve" $F^I\ F^I$ for individual I is drawn in Figure 3a, while an "offer curve" $F^{II}\ F^{II}$ for individual II is pictured in Figure 3b.[11] For each value of p a budget line shares the only point with an "offer curve." Such a point is usually a point of intersection, but a point of tangency at an endowment point. In both cases an appropriate "indifference curve" is tangent to the budget line on the point.

Now the assertion that the rate of interest clears the money market can be explained using a pure exchange model. To do so the Edgeworth box diagram is constructed, as usual, by rotating Figure 3b and joining it to Figure 3a. The outcome is Figure 4, which can be interpreted as follows. Two individuals I and II with initial endowments $(\bar{n}^I, \bar{m}^I)$ and $(\bar{n}^{II}, \bar{m}^{II})$ meet in the asset market, where bonds and money are exchanged to maximize their utilities. "The market price [of bonds] will be fixed at the point at which the sales of the 'bears' and the purchases of the 'bulls' are balanced." (Keynes (1936, p. 170)) In Figure 4 individual I is a 'bear,' while individual II is a 'bull.' As the result of the adjustment mecha-

10 The "marginal rate of substitution" for (14) is given by

$$-p - p[a(1-\pi)u^{I\prime}(c_a^I) + b\pi u^{I\prime}(c_b^I)]/[(1-\pi)u^{I\prime}(c_a^I) + \pi u^{I\prime}(c_b^I)],$$

which is equal to the slope of the budget line at each optimal point.

11 Assumtion 3 prohibits these "offer curves" from bending backward.

nism through the price of bonds, a unique price of bonds p^*, which equilibrates the demand for bonds with the supply of bonds, is determined. Equivalently, following the "Walras's law," the rate of interest adjusts to equilibrate the money market. Hence the equilibrium rate of interest ($i^* = 1/p^*$). Let the equilibrium point denote E as in Figure 4. Then for p^* such an equilibrium point lies on a "contact curve" and represents Pareto optimality for portfolio selection.

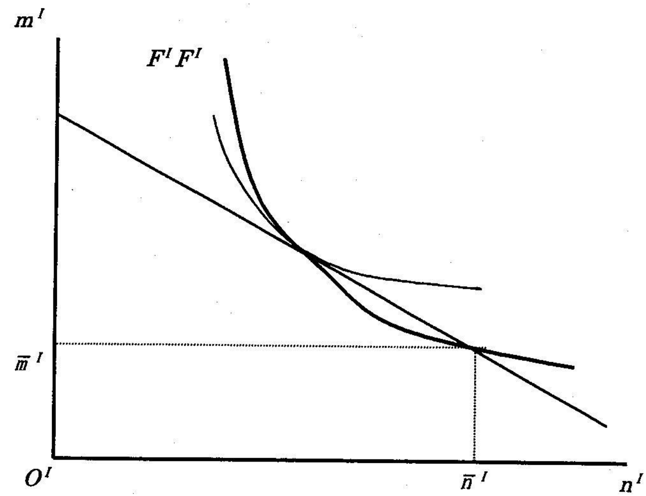

Fig. 3a. Offer Curve $F^I F^I$ for I.

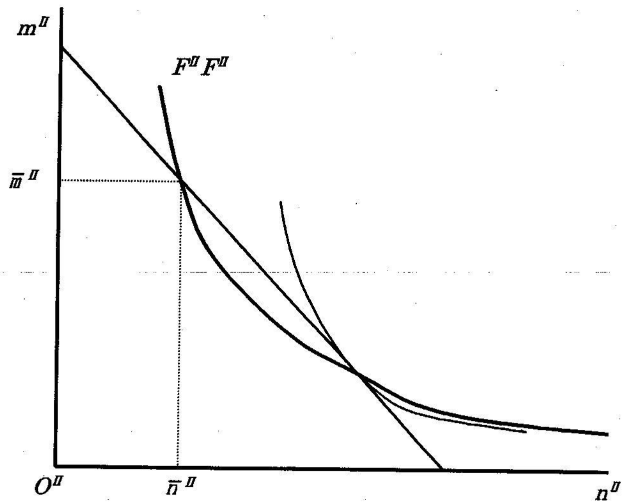

Fig. 3b. Offer Curve $F^{II} F^{II}$ for II.

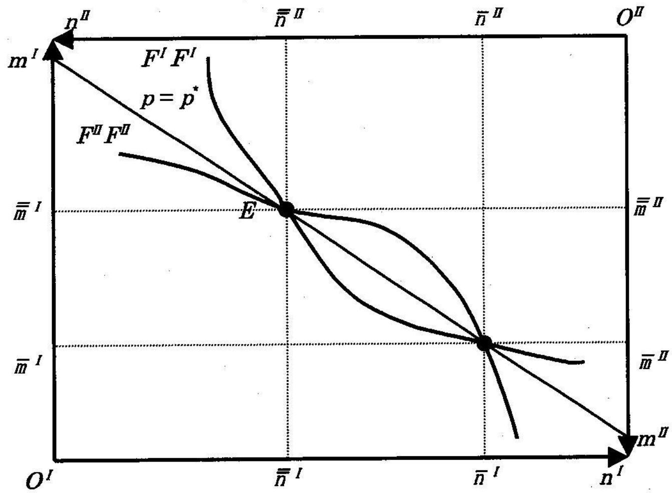

Fig. 4. Equilibrium in the Asset Market.

5. Money Demand Function and Monetary Policy

Let us derive a money demand function as well as a "bond demand function." New budget constraints for individuals I and II at point E are respectively given by

$$pn^I + m^I = p\bar{\bar{n}}^I + \bar{\bar{m}}^I, \tag{15}$$

and

$$pn^{II} + m^{II} = p\bar{\bar{n}}^{II} + \bar{\bar{m}}^{II}, \tag{16}$$

where $(\bar{\bar{n}}^I, \bar{\bar{m}}^I)$ and $(\bar{\bar{n}}^{II}, \bar{\bar{m}}^{II})$ are new endowments of bonds and money for I and II corresponding to point E.

Let n^d and m^d be the demand for bonds and the demand for money in the aggregate. Then $n^d = n^I + n^{II}$ and $m^d = m^I + m^{II}$, where (n^I, m^I) and (n^{II}, m^{II}) are optimal combinations for I and II, respectively, for each p, and they correspond with $(\bar{\bar{n}}^I, \bar{\bar{m}}^I)$ and $(\bar{\bar{n}}^{II}, \bar{\bar{m}}^{II})$ for $p = p^*$ (or $i = i^*$). It follows that a bond demand function and a money demand function can be written respectively as $n^d = n^d(p; p^*)$ and $m^d = m^d(i; i^*)$, where $n^d(p^*; p^*) = \bar{\bar{n}}^I + \bar{\bar{n}}^{II} = \bar{n}$, and $m^d(i^*; i^*) = \bar{\bar{m}}^I + \bar{\bar{m}}^{II} = \bar{m}$. The shapes of a bond demand function and a money demand function are shown in Figures 5a and 5b. If the price of bonds is lower than the equilibrium value p^*, an excess demand for bonds occurs, and the price of bonds rises. Conversely, if the price of bonds is higher than the equilibrium value, an excess supply of bonds

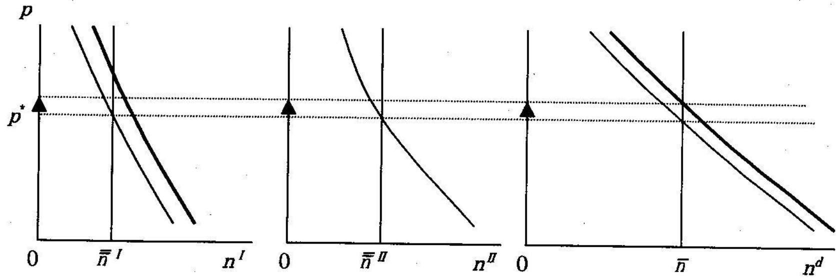

Fig. 5a. Demand for Bonds.

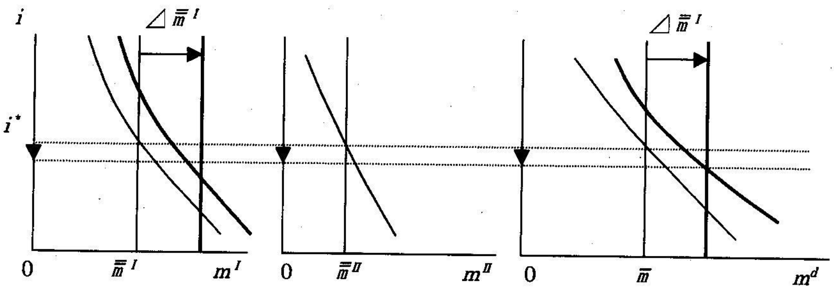

Fig. 5b. Demand for Money.

occurs, and the price of bonds falls. In other words the demand for money surely depends negatively on the rate of interest, and the rate of interest adjusts to equilibrate the asset market.

Then what determines the equilibirium values p^* and i^*? Given u^I and u^{II}, the equilibium values of the price of bonds and the rate of interest are respectively

$$p^* = B(\bar{n}^I, \bar{n}^{II}, \bar{m}^I, \bar{m}^{II}, \pi, a, b),$$

and

$$i^* = M(\bar{n}^I, \bar{n}^{II}, \bar{m}^I, \bar{m}^{II}, \pi, a, b)\,(= B(\bar{n}^I, \bar{n}^{II}, \bar{m}^I, \bar{m}^{II}, \pi, a, b)^{-1}), \quad (17)$$

where $\bar{n} = \bar{n}^I + \bar{n}^{II}$, $\bar{m} = \bar{m}^I + \bar{m}^{II}$, $\bar{m}^I = \bar{M}^I - k\bar{Y}$, $\bar{m}^{II} = \bar{M}^{II} - k\bar{Y}^{II}$, $\bar{M} = \bar{M}^I + \bar{M}^{II}$ and $\bar{Y} = \bar{Y}^I + \bar{Y}^{II}$.[12] Furthermore it can be said that any combination of bonds and

[12] It should be noted that the equilibrium rate of interest depends on the quantity of bonds as well as that of money, as Keynes (1936, p. 213) asserts as follows: "the current rate of interest depends ... on the strengths of the desires to hold it [i.e., wealth] in liquid and in illiquid forms respectively, coupled with the amount of the supply of wealth in the one form relatively to the supply of it in the other."

money on the budget line through point E and the (old) initial endowment point pictured in Figure 4 leads to the same equilibrium price of bonds and the same rate of interest.

One thing to be considered here is a change of the equilibrium interest rate to an increase in money supply (expansionary monetary policy). For simplicity assume an increase in $\bar{\bar{m}}^I$ by $\Delta \bar{\bar{m}}^I$.[13] How the price of bonds and the rate of interest change is illustrated in Figure 5 under Assumption 1. The result is that $\partial i^*/\partial \bar{\bar{m}}^I < 0$, i.e., expansionary monetary policy (an increase in $\bar{\bar{m}}^I$) depresses the equilibrium rate of interest. The same applies to the case of an increase in $\bar{\bar{m}}^{II}$, i.e., $\partial i^*/\partial \bar{\bar{m}}^{II} < 0$. So monetary policy is effective.[14]

6. Concluding Remarks

Apart from its evaluation in modern macroeconomics, I have been much dissatisfied with rationalizations of the liquidity preference theory found at least in literature I have seen. In this paper, therefore, I have attempted to give a consistent explanation as much as possible. This paper paid particular attention to more than one individual since only one person cannot exchange his asset. It has turned out that both the expected utility theory and a pure exchange model make it possible to clarify the implications of the theory.

To conclude I like to mention the following:

1) The liquidity preference theory or the *LM* curve simply claims that $\partial i^*/\partial \bar{M} < 0$, i.e., an increase in money supply as a whole lowers the rate of interest. But it is necessary to note how the increase in money supply is distributed over individuals. Even under $\partial i^*/\partial \bar{\bar{m}}^I < 0$ and $\partial i^*/\partial \bar{\bar{m}}^{II} < 0$, whether the rate of interest goes down is not determined if one holds more money but the other holds less money. So the sign of $\partial i^*/\partial \bar{M}$ is not always negative.

2) The liquidity trap is usually explained as the situation where the elasticity of a money demand with respect to the rate of interest is infinite. Is it really correct? First of all such a money demand should refer to $m^d = m^I + m^{II}$ from the definition of elasticity. To see whether this is true, adding (15) and (16) yields

$$n^d + im^d = \bar{n} + i\bar{m}, \tag{18}$$

because of (7). The liquidity trap is supposed to occur for a relatively low but pos-

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13 An increase in $\bar{\bar{m}}^I$ can be also interpreted as a decrease in $\bar{Y}$, in which case (17) is none other than the *LM* curve, the relation between i^* and $\bar{Y}$ with $\bar{M}$ as fixed.

14 Similarly $\partial i^*/\partial \bar{\bar{n}}^I > 0$ and $\partial i^*/\partial \bar{\bar{n}}^{II} > 0$ hold under Assumption 2. This becomes important taking open market operations into account.

itive value of the rate of interest. Let $n^d = 0$ in (18). Even in this case i must tend to zero in order for m^d to become infinite. This is not the liquidity trap. In fact under Assumption 2 the graph of $m^d = m^d\ (i;\ i^*)$ is downward sloping, not horizontal.

The liquidity trap could be defined as $dm^I/d\bar{\bar{m}}^I = dm^{II}/d\bar{\bar{m}}^{II} = 1$, i.e., the situation where the whole of an increase in $\bar{\bar{m}}^I$ and $\bar{\bar{m}}^{II}$ is absorbed as an increase in the money demand. For simplicity assume that only $\bar{\bar{m}}^I$ increases. Then this definition means $AR'\ (c) = 0$ due to (11), the violation of Assumption 1. Furthermore $AR'\ (c) = 0$ implies that the LM curve (17) is horizontal. The horizontal LM curve is certainly regarded as the liquidity trap case. But the elasticity of a money demand with respect to the rate of interest for I in this case is given by

$$-(i/m^I)(dm^I/di) = (p\bar{n}^I)/m^I,$$

which is not infinite. To sum up, the liquidity trap does not imply infinite elasticity.

3) Finally I would like to pose a fundamental question briefly: what is the rate of interest? It is known that in the classical school it is the marginal productivity of capital, which is independent of the quantity of money. In the liquidity preference theory, as is often cited, "the rate of interest is the reward for parting with liquidity for a specified period." (Keynes (1936, p. 167)) But here I pay attention to the following: "*the rate of time-discounting, i.e. ... the ratio of exchange between present goods and future goods.* ... As an approximation, ... we can identify this [i.e., the rate of time-discounting] with the rate of interest." (Keynes (1936, p. 93)) If expansionary monetary policy is effective, the increase in the quantity of money can also lower the rate of time-discounting. This means that money has the power of changing people's mind, i.e., that of lessening the distinction between the present and the future, though Keynes himself was doubtful of the effectiveness of monetary policy.

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