Essays on Mixed Oligopoly
（混合寡占に関する研究）

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Abstract

Chapter 1. Endogenous Timing in Mixed Duopoly with Increasing Marginal Costs

This chaper enlightens an importance of firms' order of moves in mixed duopoly. Esepically, it investigates the desirable roles of both public and private firms with increasing marginal cost technologies in mixed duopoly. In contrast to Pal (1998b) and Matsumura (2003a) — which use the constant marginal cost model — we show that it is possible for each firm to prefer the roles of the leader and the follower. Furthermore, this chapter analyzes the endogenization of the production timing of both types of firms by using the observable delay game of Hamilton and Slutsky (1990). We find that even in the increasing marginal cost model, we can obtain Pal's result — the two types of Stackelberg outcomes are in equilibrium.

Chapter 2. Mixed Duopoly, Privatization and Subsidization in an Endogenous Timing Framework

Based on the results of chapter 1, this chapter considers the endogenous timing in mixed duopoly with subsidization. Pal (1998) shows that the private leadership is always an equilibrium outcome in mixed duopoly without any subsidy. By considering the production subsidy, we find that private leadership could disappear from equilibrium and that Cournot and public leadership are likely to be equilibrium outcomes. Furthermore, we find that privatization under the optimal subsidy never enhances social welfare. Especially, when firms have identical technologies, the 'irrelevance result' à la White (1996) — the first-best allocation can be attained by the same subsidy before and after privatization — holds even though production timings of firms are endogenized. Finally, we examine privatization with lobbying activities and show that such privatization leads to deterioration of social welfare.

Chapter 3. An Endogenous Objective Function of a Partially Privatized Firm: A Nash Bargaining Approach

This chapter considers a mixed duopoly comprising a private firm and a partially privatized firm jointly owned by the welfare-maximizing government and a profit-maximizing private capitalist. Almost all the existing literature use Matsumura's (1998) model to express the partial privatization. In Matsumura's model, the privatized public firm maximizes the weighted average of social welfare and its profit respecting all the owners. In addition, the weight on welfare is assumed to be an increasing function with respect to the shares that the government holds. However, he does not consider how this weight is determined for a given shares that the government has. Then, this chapter incorporate this determination of the weight by interpreting the determination process as Nash bargaining between the government and the private capitalists. Interestingly, in such a model, we show that even when the government has more shares, it may attempt to reflect its objective in the partially privatized firm's objective, depending on the reservation utilities of both the owners.

Chapter 4. Interregional Mixed Duopoly

This chapter investigates an interregional mixed duopoly wherein a local public firm competes against a private firm. We employ a spatial model with price competition. The public firm is owned by the local government of the left half of the linear city called Region 1 and maximizes its welfare. We demonstrate that our two-stage game composed of location choice and price competition has two types of equilibria. In one equilibrium, the local public firm locates in Region 1 and the private firm locates outside the region. In the other equilibrium, both firms are located in Region 1.

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Preface

Despite the worldwide movement toward privatizing public enterprises, in many developed countries as well as developing countries and former communist transitional economies, many public firms still compete with private firms in some imperfectly competitive markets. Competition in such markets is called 'mixed oligopoly' in the field of Industrial Organization. For example, in the Norwegian oil industry, the state-owned Statoil competes with two other multinational corporations Esso Norge and Norske Shell. In Japan, Japan Broadcasting Corporation (NHK), which is one of the largest broadcasting enterprises in the world, competes some other private enterprises. The provision of health care and education are also examples of mixed oligopoly in most countries.

The first paper which modeled and formulated the mixed oligopoly is DeFraja and Delbono (1989). They regard the mixed oligopoly as competition wherein the firms having the different objective compete; profit-maximizing private firms and welfare-maximizing public firms. In addition, once the public firm is privatized, the firm is assumed to maximize its profit. In the mixed oligopoly defined as such, they explored the strategic interactions between private and public firms and the effect of privatization of public firms on welfare. They showed that the privatization improves (worsens) welfare if the market is more (less) competitive, that is, the number of private firms is large (small).

Their result is fairly suggestive, but their model might be somewhat restrictive and does not capture the reality from some points of view. First, their model does not sufficiently examine the effect of firms' order of moves on their actions and welfare. Second, they do not take into account the situation where the government uses some policies other than the privatization policy. Finally, the behaviors of various types of public enterprises such as local public firms are not accurately captured in their model.

The move structure and industrial policies in mixed oligopoly

From the first viewpoint, the previous literature concentrates on the following three points: (i) which move structure a public firm and a private firm prefer, Cournot, Stackelberg competition with public leadership, or that with private leadership, (ii) which move structure is likely to realize, and (iii) which firm should lead the mixed market, the public one or private one. The reason why these issues matters and have been focused is that an alternative order of moves gives rise to different welfare implications in the context of mixed oligopoly. In fact, in the sequential-move mixed duopoly model with constant marginal costs, welfare is larger when the public firm becomes a Stackelberg follower than when it becomes a leader.[1] Conscious of an importance of order of moves in mixed oligopoly, Pal (1998b) analyzed endogenous order of moves in mixed oligopoly by applying the observable delay game of Hamilton and Slutsky (1990), where all the firms simultaneously announce their production timings and then determine their outputs at their timings. In this game, the firms which choose earlier production timing can act like a Stackelberg leaders and the firms which choose later timing end up acting like followers. Pal (1998b) showed that in mixed *duopoly*, both public and private firms prefer a role of a Stackelberg follower and Stackelberg competition with both public and private leadership can be in equilibrium of the observable delay game. Interestingly, he also found that in mixed *oligopoly* where the number of private firms is more than two, the private leadership never appears in equilibrium and only the private leadership remains.[2]

This curious discontinuous result of divergence between duopoly and oligopoly is a matter of great concern to researchers in the mixed oligopoly. Some researchers have attempted to explain it by showing that public leadership is less robust than private leadership in mixed duopoly. Matsumura (2003a) uses a two-production-period model formulated by Saloner (1987) and shows that only private leadership is robust. Although he does not use the observable delay game, his findings appear to be sufficient for researchers to expect that public leadership might disappear if non-linear demand and/or increasing marginal costs are introduced. However, chapter 1 reveals

[1] For this point, see Beato and Mas-Colell (1984), Pal (1998b), and Matsumura (2003a).

[2] See also Jacques (2004) and Lu (2007a) who point out some forgotten equilibria in Pal (1998b). Further, with regard to the other models with endogenous timing in mixed oligopoly than Pal (1998b), see Matsumura (2003b) and Lu (2007b) who investigate endogenous timings in mixed markets consisting of a domestic public firm and foreign firms, Lu (2006) who investigates competition among domestic public and private firms and foreign firms, Bárcena-Ruiz (2007) who analyzes the endogenous models under price competition, and Matsumura and Matsushima (2003) and Ogawa and Sanjo (2007) who consider endogenous timings with product differentiation. Furthermore, Lu and Poddar (2009) point out the importance of endogenous timings when firms can choose not only their outputs but also other variables. Their model applies capacities of production as another variable. For discussions on capacities in mixed oligopoly, see Nishimori and Ogawa (2004), Lu and Poddar (2005), Lu and Poddar (2006), Ogawa (2006), and Bárcena-Ruiz and Garzón (2007).

that this is not the case. As in Pal (1998b), two types of leadership emerge as equilibria of the observable delay game, even in the mixed duopoly with increasing marginal costs and a general demand function. In short, public leadership is not an equilibrium peculiar to mixed markets with constant marginal costs and linear demand, and is relatively robust. Nevertheless, chapter 2 indicates a possibility that public leadership disappears from equilibria if the government provides all the firms with production subsidies. This proposes an importance of industrial policies applied to the mixed markets, in the sense that the governmental intervention affects not only firms' behaviors themselves but also the market structure in mixed oligopolistic industries. It adopts the second standpoint — a significance of industrial policies in mixed oligopoly other than a privatization policy — and it has been also overlooked in the literature. Interestingly, chapter 2 moreover shows that only the public leadership is in equilibrium when the subsidy level is in the middle range, whereas Cournot competition follows in equilibrium under the excessive subsidies. In the real world, we can observe that mixed oligopolistic industries come in a variety of forms, from relatively competitive markets resembling Cournot competition to the markets which is almost dominated by public enterprises. Chapter 2 should give one explanation to demonstrate such different market structures by introducing the subsidization policy.

The discussion on partial privatization

The celebrated work from the third standpoint is Matsumura (1998). The studies on mixed oligopoly before Matsumura (1998) was published have analyzed only the case of whole sales of public firms in considering privatization problems.[3] In other words, they have simply assumed that privatization implies that the government gives all the shares which it holds to the private sector and that this change in ownership transforms welfare-maximizing firms into profit-maximizing firms. However, in many cases, the government has usually held or even still holds a non-negligible proportion of shares in privatized firms. With the exception of the USA, we can observe many firms with a mixture of private and public ownership. For examples, the Bank of Iwate, whose largest shareholder is the government of Iwate prefecture, and Central Japan International Airport a part of which is owned by the government of Aichi prefecture, are representative of partially privatized firms in Japan. La Poste, which is a postal corporation of France, and Deutsche Bahn, which is a German railway corporation, are scheduled to be partially privatized.

Against such backdrops, Matsumura (1998) presented a model of partial privatization and

[3]The exception is Bös (1991) and Fershtman (1990).

investigated how many shares the government hold in terms of social welfare. In his paper, he pointed out that the privatized firms with mixed ownership are neither pure welfare-maximizers nor pure profit-maximizers since they must respect the interests of both private shareholders and the government. On the basis of this view, he assumed that these firms maximize the weighted average of the payoff of the government and its own profit and that the weight on the payoff of the government is positively related to the proportion of shares held by the government. Under these assumptions, partial privatization is shown to be desirable in mixed duopoly where one private firm competes against one privatized firm.[4]

Due to the simplicity of his model, many papers, which investigate the relationship between partial privatization and other policies in mixed oligopoly, follow Matsumura (1998).[5] Here some questions arise: (i) how is the weight, which is a function of the government's shares in the privatized firm, determined? and (ii) is it really plausible that a weight on welfare in the privatized firm's objective function is an increasing function of government's shares? Then, in chapter 3, we formulate the decision process of the weights as Nash bargaining between both the governmental and private owners. To investigate the relationship between the weights and shares which the government holds in diverse economic situations, two types of threat points are taken into account in that bargaining. First, even though the bargaining breaks down, the privatized firm is kept operated. In addition, the payoff of the largest shareholder becomes the objective function of the privatized firm. Second, after the breakdown of negotiation, owners immediately defund their firm and receive their money invested to the firm. Each owner invests the refunded money in other investment avenues and receives return from the investment. Under the former threat point it turns out, in chapter 3, that the weights and shares are irrelevant, which implies that full or partial privatization policies are fruitless. On the other hand, under the latter threat point, when the return from the investment conducted by the government is higher than that conducted by the private owners, the weight on welfare is positively related to shares which the government holds. Conversely, the weight is negatively related to them if the return for the government is lower than that for the private owners. In this case, Matsumra's (1998) assumption is completely reversed.

[4]This result is altered when free entry by private firms is taken into account. Matsumura and Kanda (2005) showed that full nationalization of the privatized firm is desirable under such entry.

[5]A lot of studies have investigated the desirability of partial privatization in various economic contexts and many of them have shown that it can be validated. For examples, Chang (2005) and Chang (2007) have shown that partial privatization can be optimal with strategic trade policies. Likewise, Jiang (2006) have shown it with wage bargaining, Ohori (2006) with environmental tax, and Han and Ogawa (2008) under the setting of an integrated market consisting of two countries.

The existence of local public enterprises

The crux of the matter from the fourth viewpoint is a need to capture the roles of various public firms in mixed oligopoly accurately and adequately. Of course, such a need does not imply that the existing works have not considered the existence of local public firms and have not explored competition between them and private firms. In fact, the previous literature on mixed oligopoly includes international trade models where domestic public firms compete against foreign private firms.[6] By regarding countries as regions or municipalities, these public firms in international mixed oligopoly is interpreted as local public firms in interregional competition. However, it may not too much to say that such models of local public firms cannot capture the roles of them completely. A typical example of local public firms is local public hospitals. They give their medical services to not only the residents who live in the relevant city or county but also those who live in other cities or counties. In other words, local public firms do not necessarily supply their goods or services only to the relevant region. In spite of the existence of such local public firms' behaviors, the existing papers, especially on mixed oligopoly with competition between domestic and foreign firms, have focused on the situation wherein public firms supply the goods to only the domestic market.

To fill this gap between contribution of the existing literature and the reality, and to incorporate the local public firms' behaviors into mixed oligopoly models accurately, chaper 4 establishes the new model which allows us to assess the roles of the local public firms. More specifically, we divide the Hotelling linear city into two parts and regard each part as one region. Each region is assumed to be reigned by one local government and one of local governments is also assumed to own its public firm. The local public firm competes against one private firm owned by profit-maximizing private owners. Like other papers which use the Hotelling model to analyze mixed oligopoly, we assume that these firms simultaneously choose their locations and then choose their prices. This setting reveals that two equilibria arise: (i) the local public firm is located near to the center of the relevant region, whereas the private firm is located in the end of the other region, and (ii) the local public firm is located in the relevant region which is near to the boundary, whereas the private firm is located in the end of the same region. In both equilibria, the local public firm

[6] Many studies about international mixed oligopoly have emerged and proliferated with progress in liberalization of trade and with the tide of deregulation. Fjell and Pal (1996) were the first to investigate the international mixed oligopoly, and analyzed behavior of a public firm competing with foreign firms and the effects of domestic public firms being purchased on domestic welfare. Fjell and Heywood (2002) examined Stackelberg competition with public leadership in international mixed oligopoly, Pal and White (1998), Pal and White (1998), and Serizawa (2000) analyzed the relationship between privatization and strategic trade policies. For other works on this topic, see Bernard, Dupere and Ronald (2003), Fujiwara (2006), Ohnishi (2008), and Long and Stähler (2009).

not only supplies the good to the residents in the associated region but also exports to the other region. In addition, the local public firm is always located in the relevant region.

Chapter 1

Endogenous Timing in Mixed Duopoly with Increasing Marginal Costs

1.1 Introduction

Despite the global trend toward privatization, public firms still exist and compete against private firms in a wide range of industries such as airlines, railways, natural gas, electricity, postal services, education, and hospitals. Such industries or markets are called "mixed" markets.[1,2] Many mixed markets are dominated by public firms with the first mover advantage, since these industries have a strong public nature and thus require particular cares that can only be provided by public institutions. On the other hand, some mixed markets are very competitive and resemble Cournot competition more closely than Stackelberg competition. This difference in type of competition inevitably generates differences not only in the profits of both public and private firms but also in social welfare, which influences the authority's regulations on competition structures or order of moves. Which firm should lead mixed markets — a public firm or a private firm? Or, is Cournot competition desirable? This problem is very significant when considering what mixed markets should be and when seeking to optimize the design of these markets. The first step in tackling this problem is the examination of the desirable role of public and private firms — Cournot competitor, Stackelberg leader, or Stackelberg follower.

Pal (1998b) and Matsumura (2003a) have already investigated this issue in a mixed duopoly with constant marginal costs. They have shown that the public firm prefers the role of a Stackelberg follower to that of a leader. Moreover, they have shown that the private firm also prefers the role of a follower to that of a leader when the marginal cost of the public firm is relatively

[1] This chapter is based on Tomaru and Kiyono (2009).

[2] Studies of mixed markets have become popular. They assume that public firms maximize social welfare, whereas private firms maximize their own profits. See DeFraja and Delbono (1990) and Nett (1993) for general reviews of mixed oligopoly.

high. Although the assumption of constant marginal costs is frequently used when analyzing mixed oligopoly, we should bear in mind that it restricts any analysis of a mixed market in the sense that the public firm must be more inefficient than the private firms. If this is not the case, the public firm will monopolize the market in any type of competition — Cournot competition, Stackelberg competition with public leadership, or Stackelberg competition with private leadership. This yields the first-best allocation. This is a trivial case, and thus, the assumption of constant marginal cost requires the public firm to be inefficient.[3]

However, not all empirical studies support this inefficiency and the results are conflicting (see Bös (1991) and Stiglitz (1988)). This implies that we should consider the case where the public firm is as efficient or more efficient than the private firms. In this chapter, we apply increasing marginal costs that allow for the efficiency of the public firm to be higher than that of its private counterpart. Much like the constant marginal cost models, this approach, too, is conventional in the literature on mixed oligopoly. For example, DeFraja and Delbono (1989) have analyzed the welfare implication of privatization. Fjell and Pal (1996) and Pal and White (1998) have investigated the effects of privatization and strategic trade policies in an international mixed oligopoly. Fjell and Heywood (2002) have examined the behavior of a public firm acting as a Stackelberg leader in international mixed oligopoly. All of these studies utilize the specific demand and cost functions. Although these specifications are useful when we desire some quantitative results, we should not ignore the fact that the manner in which the model is specified could critically affect the results. Then, we introduce more general functions to avoid some mistakes that arise when the functions are specified.

Of course, there are some works that have used the more general functions. For instance, Matsumura (1998) and Matsumura and Kanda (2005) have analyzed partial privatization in a mixed market with Cournot competition. Kato and Tomaru (2007) have examined the effects of subsidies in mixed oligopolies when all the firms simultaneously decide outputs and when a public firm is a Stackelberg leader. Even though the differences in the order of the firms' moves have a major impact on the equilibrium outcomes, the above studies do not compare the firms' payoffs in detail. As such, in this chapter, we explore the desirable roles of both the public and private firms in mixed duopoly. We find that in contrast to Pal (1998b) and Matsumura (2003a) (which assume constant marginal costs), both types of firms can prefer the roles of both the Stackelberg follower and the Stackelberg leader. Further, we reveal that our finding is also in

[3] Although the mixed duopoly with a homogeneous good is investigated in Pal (1998b), Matsumura (2003a) and this chapter, if the products are differentiated, then the public firm will not necessarily monopolize the market even when it is more efficient than private firms.

contrast with the results from the studies that assume a quadratic cost setting, such as DeFraja and Delbono (1989). Taking these findings together, we show that the results of the abovementioned existing researches are specific to their particular functions.

Although it is important to examine the roles prefered by public and private firms, it is also important to consider the roles they actually play. In other words, it is important to know the order of moves (Cournot competition, Stackelberg competition with public leadership, or Stackelberg competition with private leadership) when both the firms can choose their production timing. This is because each order of move has a different welfare implication. Many researches on mixed oligopoly have addressed this problem. Pal (1998b) analyzed the endogenous order of moves in a mixed oligopoly model with linear demand and constant marginal costs, by using the observable delay game of Hamilton and Slutsky (1990).[4] He found that both types of Stackelberg outcomes (public leadership and private leadership) are at equilibrium in mixed 'duopoly'. He also found that in mixed 'oligopoly' (i.e., when the number of private firms is more than two), the public leadership never appears in equilibrium.

This curious discontinuous result is a matter of great concern to researchers in this field. Some researchers have attempted to explain it by showing that public leadership is less robust than private leadership in mixed duopoly. Matsumura (2003a) uses a two-production-period model formulated by Saloner (1987) and shows that only private leadership is robust. Although he does not use the observable delay game, his findings appear to be sufficient for researchers to expect that public leadership might disappear if non-linear demand and/or increasing marginal costs are introduced. However, we show that this is not the case. As in Pal (1998b), two types of leadership emerge as the equilibria of the observable delay game, even in the mixed duopoly with increasing marginal costs.

The remainder of this chapter is organized as follows. Section 1.2 establishes the mixed duopoly model. Section 1.3 presents three subgames, that is, the Cournot duopoly and the two types of Stackelberg duopolies. This section also studies the properties of social welfare and the private firm's profits and compares each firm's payoffs in the three subgames. Furthermore, we present some examples to emphasize those analyses. Section 1.4 derives the equilibrium in our observable delay game. Section 1.5 concludes the study.

[4] With regard to the other models with endogenous timing in mixed oligopoly, see Matsumura (2003b) and Lu (2006) for quantity-setting models and Bárcena-Ruiz (2007) for a price-setting model.

1.2 The model

We consider a mixed duopoly model with one public firm and one private firm, both of which produce a single homogeneous good. Following existing research such as DeFraja and Delbono (1989) and DeFraja and Delbono (1990), we assume that the public firm maximizes social welfare and the private firm maximizes its own profit. The market price is determined by the inverse demand function $P = P(Q)$ where Q is the total output. We use "0" and "1" to denote the public firm and the private firm, respectively. Then, social welfare, W, is given by

$$W(q_0, q_1) := \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1),$$

where q_i and $C_i(\cdot)$ are firm i's output and cost function, respectively ($i = 0, 1$). Firm i's profit is given by

$$\Pi_i(q_0, q_1) := P(Q)q_i - C_i(q_i).$$

We assume the following.

Assumption 1.1. *For all* $Q \geq 0$, $P(Q)$ *is twice continuously differentiable,* $P'(Q) < 0$, *and* $P(Q) > 0$.

Assumption 1.2. *For all* $q_i \geq 0$, $C_i(q_i)$ *is twice continuously differentiable,* $C_i'(q_i) > 0$, *and* $C_i''(q_i) > 0$.

We allow both firms' marginal costs to be the same but not constant. Suppose that the public and private firms have constant marginal cost functions and the public firm's marginal cost is not higher than that of the private firm. In this case, the public firm monopolizes the market regardless of the competition type — Cournot competition or Stackelberg competition. Further, this public monopoly yields the first-best outcome in all the types of competition. Thus, such a situation is invalid in the analysis of mixed duopoly. Although it is of a particular interest in the analysis of mixed duopoly where the public firm has a lower constant marginal cost than the private firm, for simplicity of exposition, we assume that both firms' marginal costs are increasing.[5]

Note that Assumptions 1.1 and 1.2 jointly imply that the social welfare function, $W(q_0, q_1)$, is strictly concave, as shown by

$$\begin{aligned} & \frac{\partial^2 W(q_0, q_1)}{\partial q_0^2} \cdot \frac{\partial^2 W(q_0, q_1)}{\partial q_1^2} - \left(\frac{\partial^2 W(q_0, q_1)}{\partial q_0 \partial q_1} \right)^2 \\ = & \left(P'(q_0 + q_1) - C_0''(q_0)\right)\left(P'(q_0 + q_1) - C_1''(q_1)\right) - \left(P'(q_0 + q_1)\right)^2 > 0. \end{aligned} \tag{1.1}$$

[5] See Matsumura and Kanda (2005) for the importance of increasing marginal costs in mixed oligopoly. Instead of increasing marginal costs, there are many paper on mixed oligopoly with constant marginal costs, too. For example, see George and La Manna (1996), Mujumdar and Pal (1998), Pal (1998b), and Matsumura (2003a).

As stated in the Introduction, we consider the observable delay game of Hamilton and Slutsky (1990) in the context of output setting mixed duopoly where both firms choose the timing of production — either period 1 or 2 — before they actually determine their outputs. The competition between the two firms is formulated as a two-stage game. In the first stage, the firms simultaneously announce the period in which they will produce their outputs; the firms are then committed to this choice. Let $t_i \in \{1, 2\}$ be the time period chosen by firm i ($i = 0, 1$) in the first stage. In the second stage, after observing their decisions (t_0, t_1), the firms determine their outputs subject to their own pre-announced period of production. If both firms decide to select their outputs in the same time period, then Cournot duopoly follows. Otherwise, Stackelberg duopoly follows, in which the firm that chose period 1 becomes the output leader.

1.3 Fixed timing game

Before presenting the full equilibrium outcome in the observable delay game, we explore the properties of the three subgame equilibria, i.e., the Cournot and the two types of Stackelberg duopoly equilibria (the case where the private firm is a leader and the case where the public firm is a leader). Then, we compare each firm's payoffs in these three games.

1.3.1 Simultaneous-move subgame

First, we consider Cournot duopoly. Let $R_1(q_0)$ and $R_0(q_1)$ represent the reaction function of the private and public firms, respectively. $q_1 = R_1(q_0)$ is derived as a solution to the following first-order condition:

$$\frac{\partial \Pi_1(q_0, q_1)}{\partial q_1} = P(Q) + P'(Q)q_1 - C_1'(q_1) = 0. \tag{1.2}$$

To make this first order condition sensible for the private firm's optimization, we assume the following:

Assumption 1.3. *The marginal revenue of the private firm is a decreasing function of the public firm's output, i.e.,* $P'(Q) + P''(Q)q_1 < 0$.

The second-order condition,

$$\frac{\partial^2 \Pi_1(q_0, q_1)}{\partial q_1^2} = 2P'(Q) + P''(Q)q_1 - C_1''(q_1) < 0,$$

always holds by virtue of Assumptions 1.1, 1.2, and 1.3. The associated reaction curve of the private firm is shown by curve R_1R_1' in Figure 1.1. Similarly, $q_0 = R_0(q_1)$ is derived as a solution to the following first-order condition:

$$\frac{\partial W(q_0, q_1)}{\partial q_0} = P(Q) - C_0'(q_0) = 0, \tag{1.3}$$

where the second-order condition for this maximization,

$$\frac{\partial^2 W(q_0, q_1)}{\partial q_0^2} = P'(Q) - C_0''(q_0) < 0,$$

always holds by virtue of Assumptions 1.1 and 1.2. In view of (1.2) and (1.3), the marginal cost of the public firm, equated with the market price, exceeds the private firm's marginal cost, i.e., $C_0'(q_0) > C_1'(q_1)$. Further, from Assumptions 1.1, 1.2, and 1.3, we have the following:

$$R_1'(q_0) = -\frac{P'(Q) + P''(Q)q_1}{2P'(Q) + P''(Q)q_1 - C_1''(q_1)} \in (-1, 0) \quad \text{and} \tag{1.4}$$

$$R_0'(q_1) = -\frac{P'(Q)}{P'(Q) - C_0''(q_0)} \in (-1, 0). \tag{1.5}$$

Therefore, if a Cournot equilibrium exists, then it is globally stable,[6] and thus, unique.[7] In Figure 1.1, the Cournot equilibrium is shown by point C as the unique intersection point of the public firm's reaction curve, $R_0 R_0'$, and private firm's reaction curve, $R_1 R_1'$.

Hereafter, let the superscript "C" denote the equilibrium outcome of the Cournot game. Let q_1^C, q_0^C, and Q^C denote the private firm's output, the public firm's output, and the total output at equilibrium, respectively. The two outputs — q_0^C and q_1^C — satisfy $q_0^C = R_0(q_1^C)$ and $q_1^C = R_1(q_0^C)$. For subsequent analysis, we define $\Pi_1^C = \Pi_1(q_0^C, q_1^C)$ and $W^C = W(q_0^C, q_1^C)$.

1.3.2 Stackelberg competition with public leadership

Next, we consider a subgame where the public firm is the leader. First, the public firm chooses its output q_0 and the private firm then chooses its output q_1 after observing q_0. Since the public firm takes account of the private firm's reaction afterwards, its relevant payoff is given by the reduced-welfare function

$$\widehat{W}(q_0) := W(q_0, R_1(q_0)).$$

The public firm maximizes this welfare function $\widehat{W}(\cdot)$ with respect to q_0. The following assumption ensures that this optimization is sensible.

Assumption 1.4. *$\widehat{W}(q_0)$ is concave in q_0.*

[6] Under the standard continuous-timed Cournot adjustment process, it is sufficient for the stability of equilibrium that the slope of each firm's reaction function is less than unity.

[7] The existence of an equilibrium is assured when each firm's marginal cost at zero output is lower than the price set at either the private or public monopoly equilibrium by the other firm.

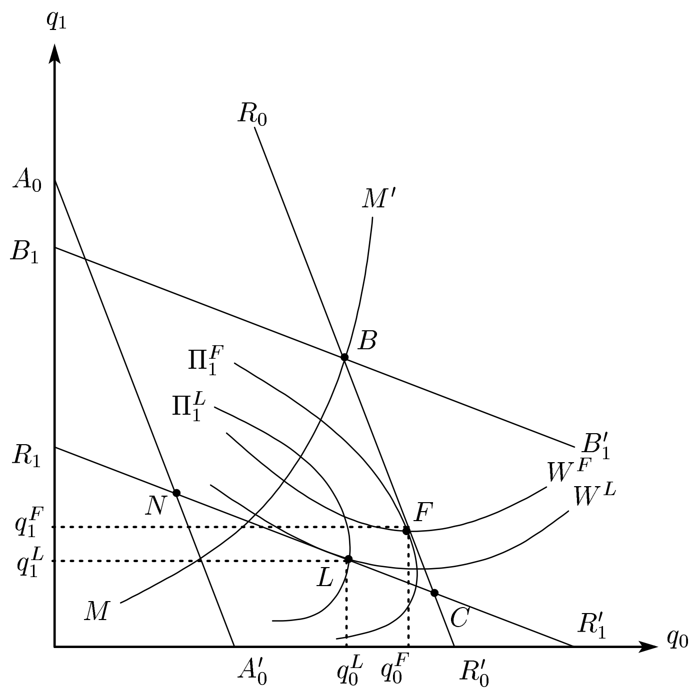

Figure 1.1: Comparison between Cournot and Stackelberg competition

Even with increasing marginal costs (Assumption 1.2), only one firm, either private or public, can be active in the market when the public firm is a leader. To eliminate such cases, we assume the following:[8]

Assumption 1.5. *$C_0'(0) \leq C_1'(q_1^M)$ and $P(\bar{q}_0) < C_0'(\bar{q}_0)$, where $\bar{q}_0 := \min_{q_0}\{q_0 | R_1(\bar{q}_0) = 0\}$ and $q_1^M := R_1(0)$.*

Appendix A demonstrates how Assumption 1.5 ensures that both firms are active. Under this assumption, the Stackelberg duopoly with the public firm as the leader has an interior solution in the sense that both firms produce strictly positive outputs at equilibrium. In this case, the equilibrium output of the public firm, q_0^L, should satisfy

$$\begin{aligned} 0 = \widehat{W}'(q_0^L) &= \frac{\partial}{\partial q_0} W(q_0^L, R_1(q_0^L)) + \frac{\partial}{\partial q_1} W(q_0^L, R_1(q_0^L)) R_1'(q_0^L) \\ &= P(q_0^L + R_1(q_0^L)) - C_0'(q_0^L) + \left[P(q_0^L + R_1(q_0^L)) - C_1'(R_1(q_0^L))\right] R_1'(q_0^L). \quad (1.6) \end{aligned}$$

Let $q_1^L = R_1(q_0^L)$ and Q^L represent the equilibrium output of the private firm and the equilibrium total output, respectively. We define $\Pi_1^L = \Pi_1(q_0^L, q_1^L)$ and $W^L = W(q_0^L, q_1^L)$. In Figure 1.1, the

[8] This assumption ensures that the public firm as a Stackelberg leader is always active in the market and does not prevent the private firm from being active. Our results seen in succeeding sections, however, hold even without this assumption, though a corner solution of public monopoly or private monopoly could follow. We make this assumption for reasons of brevity.

equilibrium is shown by point L, where the iso-welfare curve, W^L, touches the reaction curve of the private firm R_1R_1'. Note that this equilibrium is just the same as the optimal partial privatization equilibrium in the mixed duopoly discussed by Matsumura (1998).[9] When we endogenize the timing of the output decision, then we need not privatization but the public firm's becoming the output leader, in order to realize the second-best optimum by adjusting the government ownership of the public firm.

It should also be noted that the marginal cost of the public firm is again greater than that of the private firm. This is because the first-order condition for profit maximization by the private firm (1.2) implies that the price exceeds the marginal cost, which, coupled with $|R_1'(q_0)| < 1$ from (1.4) and (1.6), yields

$$\begin{aligned} P(Q^L) - C_0'(q_0^L) &= - R_1'(q_0^L) \left(P(Q^L) - C_1'(q_1^L)\right) \\ &< P(Q^L) - C_1'(q_1^L), \end{aligned}$$

which establishes $C_0'(q_0^L) > C_1'(q_1^L)$.

1.3.3 Stackelberg competition with private leadership

Finally, we consider the subgame where the public firm is a follower. First, the private firm chooses its output q_1 and the public firm then chooses its output q_0 after it observes q_1. In this subgame, the private firm can predict the reaction of the public firm in advance, and as such, its relevant payoff function is now given by

$$\widehat{\Pi}_1(q_1) := \Pi_1(R_0(q_1), q_1).$$

The private firm maximizes this profit function, $\widehat{\Pi}_1(\cdot)$, with respect to q_1. In order to make this optimization problem sensible, we assume the following.

Assumption 1.6. $\widehat{\Pi}_1(q_1)$ *is concave in* q_1 .

Let the superscript "F" represent the equilibrium outcome of the game where the public firm is a follower. Let q_1^F be the output chosen by the private firm in the equilibrium. As in the subgame with the public firm as the leader, we assume the following and preclude the trivial equilibria

[9]Matsumura (1998) allows partial privatization; the partially privatized firm's objective function is given by the convex combination of its own profit and social welfare. Here, full privatization implies that the firm maximizes only its own profit, while full nationalization implies that the firm maximizes only social welfare.

at which either firm produces nothing. The explanation for this assumption is also provided in Appendix A.[10]

Assumption 1.7. *$P(q_0^M) > C_1'(0)$ and $P(\bar{q}_1) + P'(\bar{q}_1)\bar{q}_1 < C_1'(\bar{q}_1)$.*

Thus, the associated equilibrium output of the private firm, q_1^F, should satisfy

$$0 = \widehat{\Pi}_1'(q_1^F) = P(q_1^F + R_0(q_1^F)) + \left[1 + R_0'(q_1^F)\right] q_1^F P'(q_1^F + R_0(q_1^F)) - C_1'(q_1^F). \tag{1.7}$$

Let $q_0^F = R_0(q_1^F)$ and Q^F denote the equilibrium output of the public firm and the equilibrium total output, respectively. We then define $\Pi_1^F = \Pi_1(q_0^F, q_1^F)$ and $W^F = W(q_0^F, q_1^F)$. In Figure 1.1, the equilibrium is shown by point F, where the iso-profit curve, Π_1^F, touches the reaction curve of the public firm R_0R_0'.

We again observe that the marginal cost of the public firm is greater than that of the private firm. In fact, the first-order condition for the public firm (1.3) implies that its marginal cost is the same as the market price, while the first-order condition for the private output-leader (1.7) implies that the market price exceeds the marginal cost of the private firm. Thus, in all three games (Cournot, Stackelberg with public leadership, and Stackelberg with private leadership), the public firm is less efficient than the private firm.

1.3.4 Comparisons among the three subgames

In this subsection, for the analysis of the first stage of our observable delay game, we make comparisons among the three subgames from the viewpoint of both firms' payoffs. For this purpose, we first present the results of the output comparisons among the three games. Although the results are well known in existing literature, studies in this field show them by using very specific cost and inverse demand functions (quadratic cost functions and linear demand functions). We show that the results obtained by the previous studies can be derived in a more general setting.

In Figure 1.1, two new curves, A_0A_0' and B_1B_1', are drawn. Here, A_0A_0' depicts the output profile satisfying $q_0 = argmax_{q_0}\Pi_0(q_0, q_1)$. A_0A_0' represents the reaction function of the privatized public firm maximizing its own private profit and not social welfare. On the other hand, B_1B_1' depicts the output profile satisfying $q_1 = argmax_{q_1}W(q_0, q_1)$. B_1B_1' represents the reaction function of a private firm that has been made public and thus seeks to maximize social welfare.

[10]This assumption is also made for exposition as in Assumption 5. In particular, this assumption excludes Pal's (1998b) corner solution — the threat of the entry of the public firm makes private firms choose larger outputs and improves welfare. Nevertheless, our results are not influenced by this assumption.

Let us compare the outputs of the three (subgames) equilibria. First, we compare C and L by examing the output adjustment incentive of the public firm as the Stackelberg leader at C.[11] The public firm has an incentive to reduce its own output, since an infinitesimal change in the public firm's output equating the market price with marginal cost does not affect social welfare but a decrease in the output induces the private firm with the downward-sloping reaction curve to expand the output, which certainly improves social welfare, because the market price exceeds the marginal cost of the private firm at C. Therefore, at the resulting equilibrium, $q_0^C > q_0^L$ and $q_1^C < q_1^L$ hold.

Next, we compare C and F. The private firm as the output leader takes account of the public firm's output decision, which is subject to the downward-sloping reaction curve R_0R_0', in advance, and strategically increases its own output to make the public firm's output smaller than that at the Cournot equilibrium.[12] Therefore, at the resulting equilibrium, $q_0^C > q_0^F$ and $q_1^C < q_1^F$ hold.

Finally, we compare the total outputs of the three subgames. Since the slope of the private firm's reaction curve is less than unity in the absolute value, based on (1.4), switching from C to L decreases the total output. On the other hand, we find that switching from C to F increases the total output, since the absolute value of the slope of the public firm's reaction curve is less than unity, based on (1.5). The above results yield Lemma 1.1.

Lemma 1.1. *The equilibrium output has the following relationships.*

$$(a)\ q_0^C > q_0^L,\ q_1^C < q_1^L, \quad (b)\ q_0^C > q_0^F,\ q_1^C < q_1^F, \quad and \quad (c)\ Q^L < Q^C < Q^F.$$

Proof: See Appendix B.

Lemma 1.1-(c) implies that the market price is the highest when the public firm is a leader and lowest when it is a follower. Therefore, consumer surplus is the highest when the public firm is a follower and lowest when it is a leader. There is another remark in order here. As stated in the above subsections, the public firm's marginal cost exceeds that of the private firm at all three equilibria. Thus, as shown in Figure 1.1, all the equilibria should lie below the upward-

[11] The discussion is similar to Matsumura (1998), wherein he proves that fully nationalizing the public firm is not socially optimal.

[12] At the Cournot equilibrium, the private firm equates its marginal revenue with its marginal cost, and as such, an infinitesimal increase in its own output does not affect the profit, but the resulting decrease in the public firm's output increases the profit via the higher market price.

sloping "equalized marginal cost" curve MM', depicting the locus of the outputs that equates the marginal costs between the two firms.[13]

We now compare the social welfare and the private firm's profit in the three subgames. The following lemma states that Cournot competition is not desirable for both firms.

Lemma 1.2. *Each firm's payoff in Cournot competition is related to that in Stackelberg competition, as follows:*

$$\text{(a) } W^L > W^C \text{ and } \Pi_1^L > \Pi_1^C \quad \text{and} \quad \text{(b) } W^F > W^C \text{ and } \Pi_1^F > \Pi_1^C.$$

Proof: See Appendix B.

We now provide the intuitive proof. $W^L > W^C$ and $\Pi_1^F > \Pi_1^C$ are obvious results from the first mover's advantage. $\Pi_1^L > \Pi_1^C$ follows, for $q_0^L < q_0^C$ and the private firm's profit becomes greater along with the smaller public firm's output.[14] And $W^F > W^C$ follows, for $q_1^F > q_1^C$ and the larger private firm's output improves the welfare along the public firm's reaction curve up to point B. This is because the public firm's marginal cost, equated with the market price along its reaction curve, exceeds that of the private firm, so that the increase in the private firm's output enhances social welfare.[15]

Lemma 1.2 implies that both public and private firms seek to avoid simultaneous production. This yields a problem: Which role is desirable for each firm, a Stackelberg leader or follower? By using the mixed duopoly with constant marginal costs, Pal (1998b) and Matsumura (2003a) show that the public firm prefers the role of a follower. Moreover, they show that the private firm also prefers the role of a Stackleberg follower if the marginal cost of the public firm is high. These results can also be derived in mixed duopoly with general increasing marginal costs, which is summarized in the following proposition.

Proposition 1.1. *Payoff functions in two Stackelberg equilibria have the following properties:*

$$\text{(a) } q_1^F - q_1^L \geq 0 \implies W^F > W^L \text{ and} \quad \text{(b) } q_0^F - q_0^L \geq 0 \implies \Pi_1^L > \Pi_1^F$$

[13]Curve MM' passes through the first-best point B, because social welfare maximization requires the market price to be the same as the equalized marginal costs between the firms. In addition, the curve is upward-sloping because the marginal cost is strictly increasing for both firms by Assumption 1.2.

[14]The change in the private firm's profit along its reaction curve is given by $d\Pi_1 = P'(q_0 + R_1(q_0))q_1 dq_0$.

[15]The change in the welfare along the public firm's reaction curve is given by $dW = (P(Q) - C_0'(q_0))\, dq_0 + (P(Q) - C_1'(q_1))\, dq_1$. The first term is zero along the public firm's reaction curve, while the second is strictly positive since F and C are below the equalized marginal cost curve, MM', as shown in Figure 1.1.

Proof: See Appendix B.

With regard to (a), $Q^F > Q^L$ (according to Lemma 1.1) implies that the market price is lower, and thus, the consumer surplus is greater at F than at L. Further, the greater marginal cost of the public firm implies that the higher output by the private firm lowers the social costs of production, thus enhancing the production efficiency. With regard to (b), consider a switch from F to L. If the public firm keeps its output at q_0^F even when it becomes the output leader, the private firm can increase its profit by adjusting the output from q_1^F to $R_1(q_0^F)$. However, the stated condition shows that $q_0^L \leq q_0^F$, so that this non-increase in the public firm's output raises the maximized profit of the private firm.[16]

Proposition 1.1 states that one firm prefers the role of the follower if the condition on the other firm's outputs is satisfied. Therefore, if this condition is violated, then that firm could desire the role of the leader. In fact, social welfare can be larger in Stackelberg competition with public leadership than in that with private leadership. This is shown in Proposition 1.2.

Proposition 1.2. *The following holds:*

$$q_1^L > q_1^F \ \ \textit{and} \ \ R_1'(q_0^L) \geq \frac{q_1^F - q_1^L}{q_0^F - q_0^L} \quad \Longrightarrow \quad W^L > W^F.$$

Proof: See Appendix B.

Figure 1.2 illustrates Proposition 1.2. For explanation, we define the upper contour set $W_L = \{(q_0, q_1) \mid W^L \leq W(q_0, q_1)\}$. From the strict concavity of welfare function W, this set is convex. In Figure 1.2, it is drawn as the area above the iso-welfare curve $W^L W^L$. This curve touches the line GG' at L. GG' also touches the reaction curve of the private firm, and thus, the slope of GG' is $R_1'(q_0^L)$. HH' is also drawn; its slope is equal to GG'. Further, HH' intersects F. This line lies below GG' due to the antecedent in Proposition 1.2. Then, take point M, where the normal vector in L encounters the line HH'. By the hyperplane theorem, the line GG', as a supporting hyperplane, separates upper contour set W_L and point M. Since the slope of HH' — equal to GG' — is negative, F is also separated from W_L. Therefore, welfare is lower at F as compared to L.

Intuitively, Proposition 1.2 can be explained as follows. A change from L to C decreases social welfare by Lemma 1.2, which follows from the fact that excess production by the public

[16] That is, $\Pi_1^F = \Pi_1(q_1^F, q_0^F) < \Pi_1(R_1(q_0^F), q_0^F) \leq \Pi_1\left(R_1(q_0^L), q_0^L\right) = \Pi_1^L$.

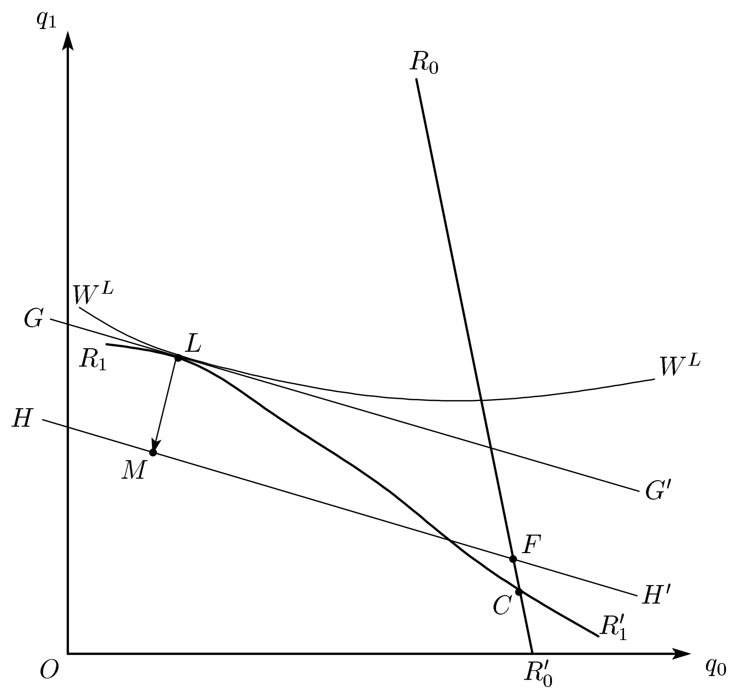

Figure 1.2: Illustration of Proposition 1.2

firm in Cournot competition is extremely detrimental to the cost efficiency. Lemma 1.2 also states that a change from C to F increases welfare because of improvements in cost efficiency by replacing the inefficient public firm's output with the efficient private firm's output. However, assumptions in Proposition 1.2 ensure that F lies to the southeast of L, which implies that the replacement of outputs is limited. Thus, an improvement in cost efficiency by a change from C to F cannot overcome the deterioration of cost efficiency by a change from L to C. This is why $W^L > W^F$.

Note that $\Pi_1^L > \Pi_1^F$ if $W^L \geq W^F$. By Proposition 1.1-(a), $W^L \geq W^F$ implies that $q_1^L > q_1^F$. Thus, from Lemma 1.1, $Q^F > Q^L$ implies that $q_0^F - q_0^L > q_1^L - q_1^F$, so that there holds $q_0^F > q_0^L > 0$, which yields $\Pi_1^L > \Pi_1^F$ by Proposition 1.1-(b). This relationship between Π_1 and W suggests that both firms cannot simultaneously desire to be Stackeberg leaders, and, of course, they cannot simultaneously desire to be followers. In contrast, as shown in Pal (1998b) and Matsumura (2003a), both firms can simultaneously desire to be Stackelberg followers in mixed duopoly with constant marginal costs.

As stated above, Pal (1998b) and Matsumura (2003a) show the somewhat counter-intuitive result that in a constant marginal cost model, private leadership always yields higher welfare than public leadership. However, Proposition 1.2 shows that this is not the case when the firms have increasing marginal costs. Although this proposition is important in that both the firms' desirable roles can be changed conditional on their outputs, it is slightly unclear what part the conditions play in determining these desirable roles. Moreover, we did not explicitly describe the discrepancy between a conclusion of our model with general increasing marginal cost functions and the well known facts of the previous studies with quadratic cost functions such as DeFraja and Delbono (1989). For resolving these problems, in the next subsection, we present some examples of specific demand and cost functions and compare the results from specification with those from our Lemmas and Proposition.

1.3.5 Examples

DeFraja and Delbono type

First, to compare the results from our Lemmas and Propositions with those from the literature with specific increasing marginal costs, we present an example and derive equilibrium outcomes for the three competition types — Cournot, Stackelberg competition with public, and Stackelberg competition with private leadership. For this purpose, we apply the model of DeFraja and Delbono (1989), which is frequently utilized in the literature, as an example.

In their model, the demand is linear and given by $P = a - Q$. Firms' cost function is quadratic and given by $C_i(q_i) = \frac{1}{2}kq_i^2$. Immediately, this formulation leads to the following equilibrium outcomes.

(Cournot)
$$q_0^C = \frac{a(1+k)}{1+3k+k^2}, \quad q_1^C = \frac{ak}{1+3k+k^2}, \quad Q^C = \frac{a(1+2k)}{1+3k+k^2},$$
$$\Pi_1^C = \frac{a^2k^2(2+k)}{2(1+3k+k^2)^2}, \quad \text{and} \quad W^C = \frac{a^2(1+5k+8k^2+2k^3)}{2(1+3k+k^2)^2};$$

(Public leadership)
$$q_0^L = \frac{a(1+3k+k^2)}{1+7k+5k^2+k^3}, \quad q_1^L = \frac{ak(2+k)}{1+7k+5k^2+k^3},$$
$$Q^L = \frac{a(1+5k+2k^2)}{1+7k+5k^2+k^3}, \quad \Pi_1^L = \frac{a^2k^2(2+k)^3}{2(1+7k+5k^2+k^3)^2},$$
$$\text{and} \quad W^L = \frac{a^2(1+6k+2k^2)}{2(1+7k+5k^2+k^3)}; \quad \text{and}$$

(Private leadership)
$$q_0^F = \frac{a(2+k)}{3+4k+k^2}, \quad q_1^F = \frac{a}{3+k}, \quad Q^F = \frac{a(3+2k)}{3+4k+k^2},$$
$$\Pi_1^F = \frac{a^2k}{6+8k+2k^2}, \quad \text{and} \quad W^F = \frac{a^2(9+10k+2k^2)}{2(1+k)(3+k)^2}.$$

Simple calculation yields (i) $q_0^C > q_0^L$ and $q_1^C < q_1^L$, (ii) $q_0^C > q_0^F$ and $q_1^C < q_1^F$, and (iii) $Q^L < Q^C < Q^F$, which are the same as in Lemma 1.1. Similarly, we can derive inequalities that are in Lemma 1.2. The result of Proposition 1.1-(b) is also derived from the above equilibrium outcomes. Indeed, $q_0^F \geq q_0^L$ if and only if $k \geq -1+\sqrt{2}$, $sgn\left(\Pi_1^L - \Pi_1^F\right) = sgn\left(-1+10k+9k^2+2k^3\right)$, and $g(k) = -1+10k+9k^2+2k^3$ is increasing in k for $k>0$. Since $g(-1+\sqrt{2}) = 2(1+\sqrt{2}) > 0$, $q_0^L \geq q_0^F \implies \Pi_1^L > \Pi_1^F$ holds. Thus, a particular model of the previous studies is synchronized with our model in that the same results follow when we compare outputs and profits. However, it is not analogous to ours with regard to a comparison of welfare. Welfare ranking is definitely determined in the linear demand and quadratic cost model, whereas Propositions 1.1 and 1.2 do not exculde the possibility that the relationship between W^L and W^F is reversed. In effect,

$$W^F - W^L = \frac{a^2 k(2+k)}{(3+k)^2(1+8k+12k^2+6k^3+k^4)} > 0,$$

and thus $W^F > W^L > W^C$. This result is an artifact of a particular model and Proposition 1.2 issues a warning to avoid the misunderstanding arising from the specification of the models.

Modified example

In what follows, in order to grasp the implications of Proposition 1.2, we present another example. Proposition 1.2 states that whether W^F is higher than W^L depends on the relative slope of the reaction curve of the private firm to that of the public firm. Then, to emphasize the importance of the relative slope, we modify the previous example of DeFraja and Delbono (1989). The demand remains the same but we change the cost functions, which are now given as

$$C_1(q_1) = \frac{1}{2}q_1^2 \quad \text{and} \quad C_0(q_0) = \begin{cases} q_0^2 & \text{if } q_0 \leq x, \\ \infty & \text{otherwise.} \end{cases} \tag{1.8}$$

The cost function of the public firm, up to the given parameter x, has a finite marginal cost if the output is finite. On the other hand, the marginal cost approaches infinity when the output is x. This type of cost function may be somewhat restrictive, but it could be rationalized if only the public firm is able to install capacity x. Dixit (1980) — who has shown that the installation of capacity by one private incumbent firm can be a strategic commitment device in competition with an entrant — uses similar cost functions as those used in this study. Similar to the private incumbent in Dixit (1980), the government might face a problem regarding the level of capacity that needs to be installed by the public firm in order to enhance welfare, when a private firm can be allowed to enter the market which a public firm as an incumbent monopolizes.[17] The energy industry might be a typical example. Earlier, such industries were monopolized by public firms

[17]For discussions on capacity in mixed oligopoly, see Wen and Sasaki (2001), Nishimori and Ogawa (2004), Lu and Poddar (2005), and Tomaru, Nakamura and Saito (2009).

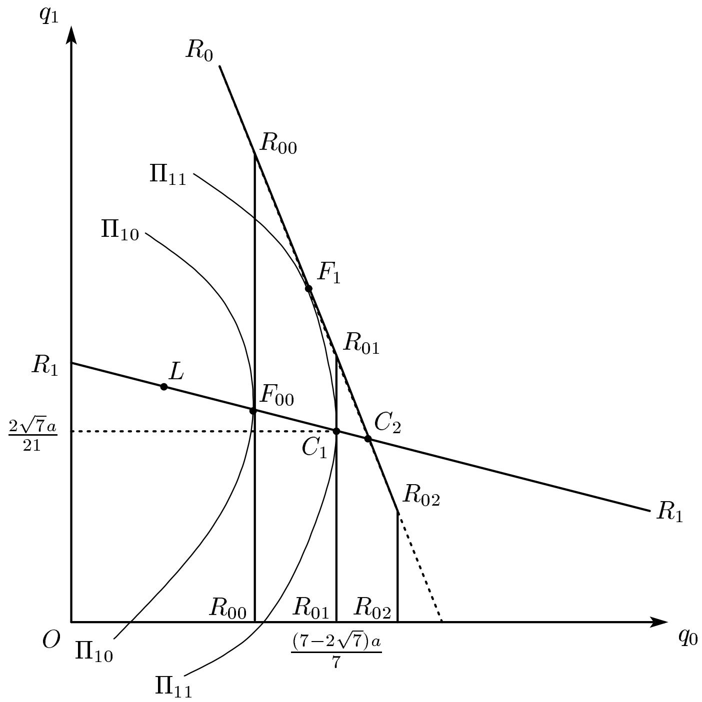

Figure 1.3: Example

with large capacities. Lately, private firms have been able to enter these industries because of deregulation.

In the above setting, both firms' reaction functions are given as follows.

$$R_1(q_0) = \frac{a-q_0}{3} \quad \text{and} \quad R_0(q_1) = \begin{cases} \dfrac{a-q_1}{3} & \text{if } q_1 \geq a-3x, \\ 0 & \text{otherwise.} \end{cases}$$

Figure 1.3 illustrates the relationships between reaction functions and parameter x. Since the output of the public firm cannot be more than x, the public firm's reaction curve is kinked at $q_0 = x$, as in Dixit (1980). Under this reaction curve, equilibrium can be changed according to x. Suppose that $x = (7-2\sqrt{7})a/7$. The public firm's reaction curve in this case is drawn as $R_0R_{01}R_{01}$. As seen in Figure 1.3, the iso-profit curve $\Pi_{11}\Pi_{11}$ is tangent to this reaction curve at Fu_1; it is also tangent to $R_{u1}R_{u1}$ at C_1 on the private firm's reaction curve R_rR_r. Thus, F_1 is indifferent to C_1 when the private firm is the leader. This implies that it prefers the intersection between both firms' reaction curves to F_1 if $x < (7-2\sqrt{7})a/7$, whereas it prefers F_1 if $x > (7-2\sqrt{7})a/7$.

We first consider the case where $x > (7-2\sqrt{7})a/7$. In this case, equilibrium outputs and welfares in the three competition types are as follows:

$$q_0^C = \frac{1}{4}a, \;\; q_1^C = \frac{1}{4}a, \;\; q_0^L = \frac{5}{23}a, \;\; q_1^L = \frac{6}{23}a, \;\; q_0^F = \frac{5}{21}a, \;\; q_1^F = \frac{2}{7}a,$$
$$W^C = \frac{9}{32}a^2, \qquad W^L = \frac{13}{46}a^2, \quad \text{and} \quad W^F = \frac{85}{294}a^2.$$

A simple calculation yields $q_1^F > q_1^L$ and $W^F > W^L$. This implies that Proposition 1.1-(a) holds. On the other hand, when $x < (7-2\sqrt{7})a/7$, we have

$$q_0^C = q_0^F = x, \quad q_1^C = q_1^F = \frac{a-x}{3}, \quad q_0^L = \frac{5}{23}a, \quad q_1^L = \frac{6}{23}a,$$
$$W^C = W^F = \frac{4a^2 + 10ax - 23x^2}{18}a^2, \quad \text{and} \quad W^L = \frac{13}{46}a^2.$$

In this case, the equilibrium in Cournot and Stackelberg competition with private leadership is along the private reaction curve, which is linear. Thus, we obtain

$$R_1'(q_0^L) = \frac{q_1^F - q_1^L}{q_0^F - q_0^L} = -\frac{1}{3}.$$

Furthermore, we also have $q_1^L > q_1^F$, considering that $x > 5a/23$. As Proposition 1.2 states, $W^L - W^F = (5a - 23x)^2/414 > 0$.

In summary, the public firm prefers the role of a Stackelberg leader over that of a follower if its marginal cost shows significant increases at the lower output levels. If not, the public firm will desire to be a Stackelberg follower.[18]

1.4 Equilibrium in the observable delay game

In the previous section, we compared and analyzed both the firms' payoffs in three subgames (Cournot, Stackelberg with private leadership, and Stackelberg with public leadership). Now, we investigate the equilibria of the first stage in our observable delay game. In this stage, both the public and private firms select the production timings anticipating their payoffs determined in the succeeding stages.

When two private firms compete in the market (or the public firm is privatized), $(t_0, t_1) = (1, 1)$ is achieved in the equilibrium, that is, Cournot competition occurs in a private duopoly. This is because each firm wants to be a Stackelberg leader and tries to produce in advance.

[18] Readers may wonder why simple quadratic cost functions (for example, $C_i(q_i) = \frac{1}{2}k_i q_i^2$) are used as an example here. We can easily show that for any pair (k_0, k_1), $W^F > W^L$ always holds. Thus, in order to present the situation in which $W^L > W^F$, we use the special cost functions in the context. In this sense, we might be able to assert that $W^F > W^L$ is robust.

Private / Public	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	$W^C, \ \Pi_1^C$	$W^L, \ \Pi_1^L$
$t_0 = 2$	$W^F, \ \Pi_1^F$	$W^C, \ \Pi_1^C$

Table 1.1: Payoff matrix in the observable delay game

However, as shown by Lemma 1.2, in mixed duopoly, $W^C < \min\{W^L, W^F\}$ and $\Pi_1^C < \min\{\Pi_1^L, \Pi_1^F\}$ imply that the public firm and private firm avoid announcing the same production periods. Furthermore, as implied by the discussion stated below Proposition 1.2, when the public firm strictly prefers becoming the leader (i.e., $W^L > W^F$), the private firm also strictly prefers becoming the follower; conversely, when the private firm strictly prefers the role of a leader, then the public firm strictly prefers the role of a follower. This is a critical difference between mixed and private duopoly.

Proposition 1.3. *In mixed duopoly, pure strategy equilibria entail* $(t_0, t_1) = (1, 2)$ *or* $(2, 1)$. *Furthermore, the following results hold.*

(*i*) *When* $W^L > W^F$, *both firms strictly prefer that the public firm becomes the leader.*

(*ii*) *When* $\Pi_1^F > \Pi_1^L$, *both firm strictly prefer that the private firm becomes the leader.*

Thus, when the condition in either (i) or (ii) holds, the two possible equilibria are ranked by the Pareto principle. Then, it would be plausible to say that the Pareto superior outcome is realized in mixed duopoly.

In addition, Propostion 1.3 shows the robustness of Pal's (1998b) multiple equilibria result in the sense that we can obtain this multiplicity even in mixed duopoly with increasing marginal cost technologies. Pal (1998b) has shown that, in mixed duopoly, both Stackelberg outcomes are equilibrium outcomes; he also shows that in mixed oligopoly (i.e., when the number of private firms is more than two), the public leadership never appears in equilibrium. Some researchers have tried to explain this discontinuous result by showing that the public leadership is less robust than the private leadership in duopoly. Matsumura (2003a) uses a two-production period model formulated by Saloner (1987) and shows that only private leadership is robust. Although Matsumura (2003a) does not use the observable delay game, it may be sufficient for researchers to expect that Pal's (1998b) discontinuous result is dependant on the specificity of his model (linear demand and constant marginal costs) and does not hold if non-linear demand and/or increasing marginal cost are introduced. However, Proposition 1.3 states that this is not the case.

Questions arise as to what is the reason behind Pal's discontinuous result. Since the discussion on this is far beyond the scope of this chapter, we wish only to give a brief explanation. In order to understand why the number of private firms matters when it comes to determining the equilibrium of the observable delay game, we focus on the case of Stackelberg competition with public leadership. As shown in Section 1.3, the public firm with leadership commits to produce less than in Cournot mixed duopoly (and oligopoly), and thus, the inefficient production of the public firm is replaced by the efficient production of private firm(s). This replacement expands the outputs of the private firms through strategic substitution. These firms gain from such increases in market shares. If one private firm deviates from the present situation and decides to produce at period 1, then the firm acts as a Stackelberg leader and commits to a higher output. These two positive effects on the market shares of the private firms affect the existence of Stackelberg competition with private leadership in equilibrium. In the case where the number of the private firms is relatively large, the former effect is likely to be small and dominated by the latter effect, because an increase in the output per private firm by strategic substitution is small. Thus, private firms have an incentive to deviate from Stackelberg competition with public leadership in markets where there are many private firms. Hence, Pal's discontinuity result is obtained.[19]

In sum, the essence behind Pal's discontiuity result is (i) the fact that private firms have an incentive to deviate public leadership when the market is competitive and (ii) the fact that there are always two equilibria (public leadership and private leadership) in mixed duopoly because both a public firm and a private firm prefer the roles of a leader and a follower to a Cournot competitor regardless of functional forms (of demand and costs).

It might be of an interest to investigate privatization of the public firm and its effect on social welfare. Unfortunately, the effect of privatization on social welfare is ambiguous. Nevertheless, we can present two conditions for privatization to enhance social welfare. One is that welfare in private leadership is higher than that in public leadership. Note that the equilibria in mixed

[19]Indeed, under the setting of linear demand $P = a - Q$ and quadratic costs $C_i(q_i) = \frac{1}{2}q_i^2$, private firms attempt to deviate when the number of private firms is more than one.

duopoly are the two Stackelberg outcomes and the equilibrium in private duopoly is the Cournot outcome. Since private duopoly equilibrium occurs on private firm 1's reaction curve, the level of welfare under public leadership exceeds that in private duopoly. Thus, both equilibria in mixed duopoly result in higher welfare than those in private duopoly. The other condition is that welfare in mixed Cournot duopoly is higher than that in private Cournot duopoly. This is because two Stackelberg outcomes in mixed duopoly surpass private Cournot duopoly in social welfare, in accordance with Lemma 1.2.

1.5 Concluding remarks

In this chapter, we analyzed the three competition types in mixed duopoly involving public and private firms — Cournot competition and Stackelberg competition with public and private leadership. We consider the problem of the roles that are desired by both firms. Pal (1998b) and Matsumura (2003a) have already investigated this problem by using a model with constant marginal costs. They show that both public and private firms prefer the role of the follower. We find that their result is crucially dependent on the specificity of cost functions, because in a setting with general increasing marginal cost functions, it is also possible that the firms prefer the role of the leader. Further, we endogenize the role of moves to detect the role that is in equilibrium by using the observable delay game of Hamilton and Slutsky (1990). We find that in our observable game, two Stackelberg outcomes (Stackelberg with public leadership and with private leadership) are in equilibrium. This implies that the result of Pal's (1998b) model with linear demand and constant marginal costs is quite robust and does not depend on demand and cost functions.

Throughout this chapter, we assume that the reactions of firms are strategic substitutes. However, as seen from (1.4) and (1.5), only the reaction of the private firm can be strategic complements. For example, this can be the case when the price elasticity of demand is constant. In such a case, some of our results are altered: (i) $q_0^L > q_0^C > q_0^F$ and $q_1^L > q_1^C$ and $q_1^F > q_1^C$ and (ii) $\Pi_1^L < \Pi_1^C < \Pi_1^F$. Interestingly, the welfare ranking shown in Lemma 1.2 does not change, but our proposition on welfare (Proposition 1.2) is slightly altered: $R_1'(q_0^L) \leq (q_1^F - q_1^L)/(q_0^F - q_0^L) \implies W^L > W^F$. Furthermore, the Stackelberg competition with private leadership is the only equilibrium of the observable delay game.[20] Thus, changing the characteristics of product market competition overturn our key results.

Although, in this chapter, we have clarified the desirable role from the viewpoint of social

[20]The proofs of these results are available on request.

welfare and analyzed which competition follows in equilibrium, there exist some problems. First, we use quantity-setting mixed duopoly. In the literature on mixed oligopoly, there are many papers that are not based on quantity-setting competition. Anderson, de Palma and Thisse (1997) analyze price competition in the Dixit-Stiglitz type differentiated goods model. Cremer, Marchand and Thisse (1991) also analyze price competition in the Hotelling-type spatial model. Matsumura and Matsushima (2004) take into account cost-reducing R&D competition in their model. It is of interest to consider the roles that are desirable for public and private firms and to examine endogenous production timing in these settings. The second problem pertains to the firms' ownership. In reality, the public firm is owned not only by the government but also by private capitalists. It is necessary to examine how the results will change if the public firm in our model is such a partially privatized firm. These problems are left for future research.

Appendix A

About Assumption 1.5

To preclude the case in which the public firm produces nothing, it suffices to prove $\widehat{W}'(0) > 0$ in view of Assumption 1.4. Here,

$$\begin{aligned}\widehat{W}'(0) &= P(q_1^M) - C_0'(0) + \left[P(q_1^M) - C_1'(q_1^M)\right] R_1'(0),\\ &> P(q_1^M) - C_0'(0) - P(q_1^M) + C_1'(q_1^M),\\ &= C_1'(q_1^M) - C_0'(0),\end{aligned}$$

where the first inequality holds by virtue of (1.4); the second, by virtue of Assumption 1.5.

On the other hand, to preclude the case in which the public firm keeps the private firm from producing anything, it suffices to prove $\widehat{W}'(\bar{q}_0) < 0$, i.e., the minimum output of the public firm to let the private firm produce nothing. Here holds

$$\widehat{W}'(\bar{q}_0) = P(\bar{q}_0) - C_0'(\bar{q}_0) + [P(\bar{q}_0) - C_{r1}'(0)]\, R_1'(\bar{q}_0) = P(\bar{q}_0) - C_0'(\bar{q}_0),$$

which is again strictly positive by virtue of Assumption 1.5.

About Assumption 1.7

The marginal profit of the private firm acting as the output leader is $\widehat{\Pi}_1'(q_1) = P(q_1 + R_0(q_1)) + (1 + R_0'(q_1))\, P'\,(q_1 + R_0(q_1))\, q_1 - C_1'(q_1)$. Then the private firm chooses a strictly positive output when $\widehat{\Pi}_1'(0) = P(q_0^M) - C_1'(0) > 0$ in view of Assumption 1.6, for the public firm produces its monopoly output when the private firm produces nothing, i.e., $q_0^M := R_0(0)$. This condition holds by virtue of the first part of Assumption 1.7.

To prove that the public firm produces a strictly positive output at the equilibrium, first define $\bar{q}_1 := \min_{q_1}\{q_1 | R_0(\bar{q}_1) = 0\}$, i.e., the minimum output of the private firm to let the public firm produce nothing. Then in view of Assumption 1.6 it suffices to demonstrate that $\widehat{\Pi}_1'(\bar{q}_1) < 0$. This inequality holds, for

$$\widehat{\Pi}_1'(\bar{q}_1) = P(\bar{q}_1) + [1 + R_0'(\bar{q}_1)]\, P'(\bar{q}_1)\bar{q}_1 - C_1'(\bar{q}_1) < P(\bar{q}_1) + P'(\bar{q}_1)\bar{q}_1 - C_1'(\bar{q}_1),$$

by virtue of (1.5).

Appendix B

Proof of Lemma 1.1

(a) First, we show that $q_0^C > q_0^L$ and $q_1^C < q_1^L$. Evaluating (1.6) at $q_0 = q_0^C$, we have

$$\begin{aligned}\widehat{W}'(q_0^C) &= P(q_0^C + R_1(q_0^C)) - C_0'(q_0^C) + \left[P(q_0^C + R_1(q_0^C)) - C_1'(R_1(q_0^C))\right] R_1'(q_0^C),\\ &= \left\{P(q_0^C + R_1(q_0^C)) - C_1'(R_1(q_0^C))\right\} R_1'(q_0^C),\\ &< 0.\end{aligned}$$

The second equality follows from the first order condition in the Cournot duopoly (1.3). Since $\widehat{W}$ is concave in q_0, we obtain $q_0^C > q_0^L$. Further, $q_1^C < q_1^L$ (from $q_1^C = R_1(q_0^C)$, $q_1^L = R_1(q_0^L)$ and $R_1'(q_0) < 0$).

(b) Next, we show that $q_0^C > q_0^F$ and $q_1^C < q_1^F$. Evaluating (1.7) at $q_1 = q_1^C$, we have

$$\begin{aligned}\widehat{\Pi}_1'(q_1^C) &= P(q_r^C + R_u(q_r^C)) + q_r^C P'(q_r^C + R_u(q_r^C))\left[1 + R_u'(q_r^C)\right] - C_r'(q_r^C)\\ &= P'(q_r^C + R_u(q_r^C))q_r^C R_u'(q_r^C)\\ &> 0.\end{aligned}$$

The second equality follows from (1.2). Since $\widehat{\Pi}_1$ is concave in q_1, we obtain $q_1^C < q_1^F$. Further, $q_0^C > q_0^F$ (from $q_0^C = R_0(q_1^C)$, $q_0^F = R_0(q_1^F)$ and $R_0'(q_1) < 0$).

(c) Finally, we show that $Q^L < Q^C < Q^F$. We define $\widehat{Q}(q_0) = q_0 + R_1(q_0)$ and $\widetilde{Q}(q_1) = q_1 + R_0(q_1)$. Note that $Q^C = \widehat{Q}(q_0^C)$ and $Q^L = \widehat{Q}(q_0^L)$. We can obtain $\widehat{Q}'(q_0) = 1 + R_1'(q_0) > 0$.

Since we know that $q_0^C > q_0^L$ (from Lemma 1.1 (a)), we can obtain $Q^C > Q^L$. Similarly we can observe that $\widetilde{Q}'(q_1) = 1 + R_0'(q_1) > 0$. Since we know $q_1^C < q_0^F$ (from Lemma 1.1 (b)), we can obtain $Q^C < Q^F$. ■

Proof of Lemma 1.2

First, we show that $W^L > W^C$. Since the public firm as a leader maximizes social welfare and it can choose $q_0 = q_0^C$, we obtain $W^L \geq W^C$. Further, since $q_0^C > q_0^L$ (from Lemma 1.1 (a)), we have $W^L \neq W^C$. Hence, we obtain $W^L > W^C$. Similarly, we can prove $\Pi_1^F > \Pi_1^C$.

Next, we show that $W^F > W^C$. $W(R_0(q_1), q_1)$ is increasing in q_1. From Lemma 1.1 (a), we have $q_1^C < q_1^F$. Since $q_0^C = R_0(q_1^C)$ and $q_0^F = R_0(q_1^F)$, we obtain $W^F > W^C$. Similarily, $\Pi_1(q_0, R_1(q_0))$ is decreasing in q_0. Also, $q_0^L < q_0^C$ (from Lemma 1.1 (b)). Since $q_1^C = R_1(q_0^C)$ and $q_1^L = R_1(q_0^L)$, we obtain $\Pi_1^L > \Pi_1^C$. ■

Proof of Proposition 1.1

(a) Since $W(q_0, q_1)$ is strictly concave, we have

$$\begin{aligned} W^L - W^F &< \frac{\partial W(q_0^F, q_1^F)}{\partial q_0}(q_0^L - q_0^F) + \frac{\partial W(q_0^F, q_1^F)}{\partial q_1}(q_1^L - q_1^F) \\ &= \frac{\partial W(q_0^F, q_1^F)}{\partial q_1}(q_1^L - q_1^F). \end{aligned}$$

Since $\partial W(q_0^F, q_1^F)/\partial q_1 > 0$, we derive Proposition 1.1 (a).

(b) We define $\widetilde{\Pi}_1(q_0) = \Pi_1(q_0, R_1(q_0))$. $\widetilde{\Pi}_1(q_0)$ is decreasing in q_0. Thus, if $q_0^F \geq q_0^L$, then $\widetilde{\Pi}_1(q_0^L) \geq \widetilde{\Pi}_1(q_0^F)$. From the definition of the reaction function $R_1(q_0)$, we obtain

$$\Pi_1^F = \Pi_1(q_0^F, q_1^F) < \Pi_1(q_0^F, R_1(q_0^F)) = \widetilde{\Pi}_1(q_0^F).$$

This implies that $\widetilde{\Pi}_1(q_0^L) \geq \widetilde{\Pi}_1(q_0^F) > \Pi_1^F$. Since $\widetilde{\Pi}_1(q_0^L) = \Pi_1^L$, Proposition 1.1 (b) is proved. ■

Proof of Proposition 1.2

Strict concavity of welfare function W yields

$$
\begin{aligned}
W^F - W^L &< \frac{\partial W(q_0^L, q_1^L)}{\partial q_0} \cdot (q_0^F - q_0^L) + \frac{\partial W(q_0^L, q_1^L)}{\partial q_1} \cdot (q_1^F - q_1^L), \\
&= \left[P(Q^L) - C_0'(q_0^L)\right](q_0^F - q_0^L) + \left[P(Q^L) - C_1'(R_1(q_0^L))\right](q_1^F - q_1^L), \\
&= -\left[P(Q^L) - C_1'(R_1(q_0^L))\right] \cdot R_1'(q_0^L) \cdot (q_0^F - q_0^L) \\
&\quad + \left[P(Q^L) - C_1'(R_1(q_0^L))\right](q_1^F - q_1^L), \quad \text{(from (1.6))}, \\
&= \left[P(Q^L) - C_1'(q_1^L)\right] \cdot \left[-R_1'(q_0^L)(q_0^F - q_0^L) + q_1^F - q_1^L\right].
\end{aligned}
$$

Further, the inequality $q_0^F > q_0^L$ holds if $q_1^F \geq q_1^L$; this is derived from $Q^L < Q^F$ in Lemma 1.1. Using this fact and the assumption of Proposition 1.2, the right-hand side of the above equation can be rewritten as

$$
\left[P(Q^L) - C_1'(q_1^L)\right](q_0^L - q_0^F)\left[R_1'(q_0^L) - \frac{q_1^F - q_1^L}{q_0^F - q_0^L}\right].
$$

We find that this term is negative. Thus, $W^L > W^F$. ■

Chapter 2

Mixed Duopoly, Privatization and Subsidization in an Endogenous Timing Framework

2.1 Introduction

This chapter demonstrates how subsidization affects firms' behaviors in a mixed market or mixed duopoly in which public firms compete against private firms.[1][2] In particular, we focus on the importance of the order of their moves. Despite the large body of theoretical literature that examines mixed oligopoly, the existing works have not conducted minute analyses on how this order of moves changes the effects of subsidization on firms' behaviors, profits and welfare. This chapter aims to fill this gap by introducing subsidization into a mixed oligopoly model and by shedding light on how both public and private firms' order of moves influences their payoffs for various levels of subsidies.[3]

Many studies suggest the importance of endogenous timing in mixed duopoly and oligopoly without any subsidy. Using the observable delay game formulated by Hamilton and Slutsky

[1]This chapter is based on Tomaru and Saito (2010).

[2]Mixed oligopoly can be seen in many countries and there are rich examples such as the packaging and delivery services, airlines, railways, natural gas and electricity. See DeFraja and Delbono (1990) and Nett (1993) for general reviews of mixed oligopoly models. In addition, an importance of the mixed oligopoly is implied by the recent surge of works on it. For example, Nishimori and Ogawa (2002), Matsumura and Matsushima (2004), and Tomaru (2007) analyze the inefficiency of the public firms. Matsumura (1998), Matsumura and Kanda (2005) and Fujiwara (2007) discuss the model of partial privatization. Fjell and Pal (1996), Pal and White (1998), and Chao and Yu (2006) consider an international mixed oligopoly.

[3]The subsidies for private firms could be justified in removing or alleviating the inefficiencies caused by imperfections in markets. For the theoretical properties of subsidies, see Flam, Persson and Svensson (1983), Phelps (1994), Snower (1994), and Picard (2001). They investigate them in the markets without public firms. Even in the mixed market, subsidies play important roles and, in effect, are provided for firms in some mixed industries. For example, in Japan, the Small and Medium Enterprise Agency provides private firms with subsidies in order to encourage distribution services and enhance their efficiency. Yamato Transport, which is one of major delivery enterprises and competes against Japan Post which is a semi-public firm, is subsidized.

(1990), Pal (1998b) shows that Stackelberg competition with private leadership and that with public leadership are equilibrium outcomes in mixed duopoly with constant marginal costs. Tomaru and Kiyono (2009) also demonstrate this in a setting where both public and private firms have increasing marginal costs. Moreover, Matsumura (2003a) shows that in a different endogenous timing game, the private leadership can always be an equilibrium outcome while public leadership can never be one.[4]

Their results indicate that private leadership is plausible in a mixed duopoly. However, these results are drastically altered when we consider the production subsidy. We find that if the government subsidizes public and private firms and the level of subsidy is not very low, then public leadership becomes an equilibrium outcome and private leadership never. Industries such as telecommunications, electricity, natural gas, airline, and, increasingly, the postal sector, are dominated by former public monopolies with a first-mover advantage. These industries more closely resemble Stackelberg competition with public leadership. The subsidy could be an explanation for this reality, and our findings can be said to fill the gap between the reality and the results from the existing works such as Pal (1998b), Tomaru and Kiyono (2009) and Matsumura (2003a).

In broad literature, there are works that analyze mixed oligopolies in the context of government subsidies designed to promote an increase in the outputs of private firms. White (1996) shows that the government can realize the first-best allocation by utilizing the subsidization policy in a Cournot mixed oligopoly. Surprisingly, he also shows that the first-best allocation is achievable by the same subsidy as in mixed oligopoly even after the privatization of a public firm. Since White (1996), a series of "irrelevance results" has been generated. Poyago-Theotoky (2001) demonstrates that the optimal subsidy is identical and that profits, outputs and welfare are also identical irrespective of whether (i) all the public and private firms move simultaneously or (ii) the public firm acts as a Stackelberg leader or (iii) all firms are privatized and maximize profits. Myles (2002) proves this series of results in a setting where there are more general cost and demand functions.[5]

The conclusion of these results is that privatization is fruitless in terms of social welfare as long as the subsidization policy is feasible for the government. However, this conclusion relies

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[4]For other papers on endogenous timing in mixed oligopoly, see Bárcena-Ruiz (2007) who investigates price competition and Matsumura (2003b) who analyzes competition between a domestic public firm and a foreign private firm. See also Lu (2006) who considers the situation in which a public firm competes against both domestic and foreign private firms.

[5]For other studies on the irrelevance results, see Tomaru (2006) and Kato and Tomaru (2007). Tomaru (2006) examines robustness of the irrelevance results from the view of partial privatization formulated by Matsumura (1998). Kato and Tomaru (2007) show that the irrelevance results hold even when private firms have objectives other than profits.

critically on the assumption of the given timing of moves after privatization. Fjell and Heywood (2004) show that when the public leader is privatized and becomes the private leader, the optimal subsidy and welfare are reduced. Their result suggests the need to examine what move structures are likely to arise in mixed and private oligopoly when we consider privatization along with subsidization. For this examination, we consider a stage of firms' selecting the production timing right after the stage of decision of the level of subsidies by the government and then compare the results from mixed and private duopoly with endogenous timing and subsidy. We find that public leadership and Cournot are equilibrium outcomes in mixed duopoly and that Cournot is an equilibrium outcome in private duopoly. Along with the results of Poyago-Theotoky (2001) and Myles (2002), our results imply that the irrelevance results hold when the production timing is endogenized.

The findings of the literature mentioned above and ours, however, are dependent on the fact that the government has a discretion over the subsidy. It might lose its discretion if interest groups lobby, and the political process is highly complicated. In this case, the government cannot set the optimal subsidy. Many papers on lobbying activities and campaign contribution show that the production subsidies and export subsidies are likely to be excessive. Following this, we focus on the welfare and profits at a given subsidy level that is higher than the optimal subsidy level and analyze the effects of privatization. The result of this analysis is that under such subsidies, privatization decreases both the profits of the private firm and welfare.

The remainder of this chapter is organized as follows. Section 2.2 presents our model for comparing three types of competition, namely, Cournot competition and Stackelberg competition with public leadership and Stackelberg competition with private leadership. In addition, it explains how the subsidy level influences welfare and the profits of both private and public firms; it also investigates the rankings of welfare and profits in the three types of competition. Section 2.3 discusses optimal subsidy when the production timing is endogenized. Section 2.4 investigates the effect of privatization. Section 2.5 explores privatization with lobbying activities. Finally, Section 6 concludes this chapter.

2.2 The model

We analyze mixed duopoly with public firm 0 and private firm 1 producing a single homogeneous good. The private firm maximizes its own profits. On the other hand, the public firm is owned by the welfare-maximizing government, and thus, firm 0 maximizes the welfare. The output of firm i is q_i $(i = 0, 1)$, such that $Q = q_0 + q_1$ represents the total output. Let $P(Q)$ be the inverse demand function; further, each firm has technology represented by the cost function $C_i(q_i)$ $(i = 0, 1)$. Throughout this chapter, we assume the following:

Assumption 2.1. *For any $Q \geq 0$, the inverse demand function $P(Q)$ is twice-continuously differentiable, where $P'(Q) < 0$ and $P''(Q) \leq 0$.*

Assumption 2.2. *For any $q_i \geq 0$, firm i's cost function $C_i(q_i)$ is twice-continuously differentiable, where $C_i'(q_i) > 0$ and $C_i''(q_i) > 0$.*[6]

Social welfare $W(q_0, q_1)$ and each firm's profit $\Pi_i(q_0, q_1, s)$, $(i = 0, 1)$ are given by

$$\begin{aligned} W(q_0, q_1) &:= \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1), \\ \Pi_i(q_0, q_1, s) &:= P(Q)q_i - C_i(q_i) + sq_i, \end{aligned} \tag{2.1}$$

respectively, where s is the production subsidy. When s is negative, firms face production taxes. Note that both firms' profits rely on subsidies while social welfare is not directly affected by the subsidies. This is because the subsidies for the firms are just lumpsum transfers.

To meet the aims of our chapter—analyzing endogenous timing in mixed duopoly with subsidy—we need to explore how both firms' payoffs are influenced by subsidies under fixed move structures: Cournot competition and Stackelberg competition with public and private leadership. For this purpose, we start by deriving both the private and public firms' reaction functions. The first-order conditions of public firms 0 and 1 are given as

$$\frac{\partial W}{\partial q_0} = P(Q) - C_0'(q_0) = 0, \tag{2.2}$$

$$\frac{\partial \Pi_1}{\partial q_1} = P(Q) + P'(Q)q_1 - C_1'(q_1) + s = 0. \tag{2.3}$$

The second-order conditions for both firms' maximization problems are satisfied by virtue of Assumptions 2.1 and 2.2. Equations (2.2) and (2.3) yield firm i's reaction function R_i, which satisfies

$$\begin{aligned} &\frac{\partial R_0}{\partial q_1} = -\frac{P'(Q)}{P'(Q) - C_0''(q_0)} \in (-1, 0), \quad \frac{\partial R_1}{\partial q_0} = -\frac{P'(Q) + P''(Q)q_1}{2P'(Q) + P''(Q)q_1 - C_1''(q_1)} \in (-1, 0), \\ &\frac{\partial R_1}{\partial s} = -\frac{1}{2P'(Q) + P''(Q)q_1 - C_1''(q_1)} > 0. \end{aligned} \tag{2.4}$$

[6]If both public and private firms have constant marginal costs, the public firm's cost must be higher than the private firm's in order to preclude public monopoly. We consider the optimal subsidy in the later sections. Then, it is absolutely obvious that the private monopoly yields the first best outcome and that time structure (either Cournot, Stackelberg or endogenous timing) does not matter. Thus, we assume increasing marginal costs to avoid such an obvious outcome. For further discussion on the importance of increasing marginal costs in mixed oligopoly, see Matsumura and Kanda (2005).

Hence, if a Cournot equilibrium exists, then it is globally stable[7] and is thus uniquely determined.[8]

2.2.1 Three types of move structures

First, we derive Cournot equilibrium under mixed duopoly. Let the superscript 'mC' denote Cournot equilibrium under mixed duopoly. The equilibrium outputs in Cournot competition are characterized by the first-order conditions (2.2) and (2.3). Then, we define them as $q_i^{mC}(s)$ $(i = 0, 1)$ and $Q^{mC}(s) = q_0^{mC}(s) + q_1^{mC}(s)$. For analysis, we examine the comparative statics under Cournot competition. Simple calculation yields

$$\Delta \cdot q_0^{mC\prime}(s) = R_0'(q_1) \cdot \frac{\partial R_1}{\partial s} < 0, \;\; \Delta \cdot q_1^{mC\prime}(s) = \frac{\partial R_1}{\partial s} > 0, \;\; \Delta \cdot Q^{mC\prime}(s) = \frac{\partial R_1}{\partial s}\{1 + R_0'(q_1)\} > 0,$$

where $\Delta = 1 - R_0'(q_1) \cdot (\partial R_1/\partial q_0) > 0$. Production subsidies increase the output of private firm 1 as well as total outputs, while they decrease the output of public firm 0.

Second, we consider Stackelberg competition with public and private leadership. Since a Stackelberg leader chooses its output anticipating the output of the follower, the public firm with leadership maximizes $\widehat{W}(q_0, s) := W(q_0, R_1(q_0, s))$ while the private firm with leadership maximizes $\widehat{\Pi}_1(q_1, s) := \Pi_1(R_0(q_1, s), q_1, s)$. We assume that these objective functions are concave, which yields the following first-order conditions;

$$\begin{aligned} \frac{\partial \widehat{W}}{\partial q_0} &= P(q_0 + R_1(q_0, s)) - C_0'(q_0) + [P(q_0 + R_1(q_0, s)) - C_1'(R_1(q_0, s))] \cdot \frac{\partial R_1}{\partial q_0} = 0, \\ \frac{\partial \widehat{\Pi}_1}{\partial q_1} &= P(R_0(q_1) + q_1) - C_1'(q_1) + [1 + R_0'(q_1)]\, P'(R_0(q_1) + q_1)q_1 + s = 0. \end{aligned} \tag{2.5}$$

The equilibrium outputs in public and private leadership are derived from these equations and the reaction functions of the followers. Let the superscripts 'mL' and 'mF' denote public and private leadership, respectively. We define the equilibrium outputs in leadership structure 'mj' as $q_i^{mj}(s)$ $(j = L, F,\ i = 0, 1)$. Equilibrium total output, in turn, is given as $Q^{mj}(s) = q_0^{mj}(s) + q_1^{mj}(s)$. In addition, the payoffs of public firm 0 and private firm 1 are, respectively, as follows: $W^{mj}(s) = W(q_0^{mj}(s), q_1^{mj}(s))$ and $\Pi_1^{mj}(s) = \Pi_1(q_0^{mj}(s), q_1^{mj}(s))$.

In this chapter, for tractability of analysis and exposition we exclude the possibilities of public and private monopolies. To ensure that all the above equilibrium outputs q_i^{mj} $(i = 0, 1$ and

[7] This assumption is the standard Cournot adjustment process in duopoly. Under this process, a sufficient condition for the stability of the equilibrium is that the absolute value of the slope of each firm's reaction function is less than 1.

[8] The existence of unique equilibrium is assured when each firm's marginal cost at zero output is lower than the price set at either private or public monopoly equilibrium by the other firm.

$j = C, L, F$) are positive, we should restrict the range of subsidy levels. We, therefore, define set S as follows: $S = \{s \mid q_i^{mj}(s) > 0,\ i = 0, 1\ and\ j = C, L, F\}$. Hereafter, we concentrate on the analysis of subsidized mixed duopoly for $s \in S$. Further, we make an assumption on welfare functions, W^{mC}, W^{mL} and W^{mF}, for analysis in the later sections. This assumption ensures that the welfare maximization problem with respect to subsidy level s sensible.

Assumption 2.3. *Three welfare functions, W^{mC}, W^{mL}, and W^{mF}, are concave in $s \in S$.*

2.2.2 Comparison among the Cournot equilibrium and two Stackelberg equilibria

Some existing works analyze the effect of subsidy on welfare in mixed duopoly. White (1996) shows that the government can attain the Pareto-efficient allocation by utilizing the optimal subsidy in Cournot competition. Poyago-Theotoky (2001) and Myles (2002) have shown that, even in public leadership, the government can also attain the allocation at the same level of subsidy as in Cournot. This is called the 'irrelevance result'. In this subsection, we compare the three different games, namely, Cournot, public leadership and private leadership and reexamine the irrelevance result in our general setting with heterogeneous costs. Consider the following two-stage game; at stage 1 the government selects the level of subsidy and at stage 2 public and private firms choose the output in each timing of a given time structure.

Surprisingly, we find that the irrelevance result of Poyago-Theotoky and Myles can be derived from our general setting with cost heterogeneity. Furthermore, we find that the Pareto-efficient allocation is attainable even in private leadership whereas the optimal subsidy is different from that of White (1996).

Proposition 2.1. *Pareto-efficient allocation is achievable in all the three games, Cournot, public leadership, and private leadership, by the optimal subsidies. Moreover, the optimal subsidies in Cournot, s^C, and public leadership, s^L, are the same (i.e. $s^C = s^L := s^*$), while that in private leadership, s^F, is lower than s^*.*

Proof: See Appendix.

In Japan, Japan Post was a major public firm which provided postal and delivery services until it was privatized in 2007. Japan Post had a small market share in the delivery service industry, which was dominated by private firms such as Yamato Transport and Nippon Express. As is the case with the delivery industry in Japan, some industries might more closely resemble

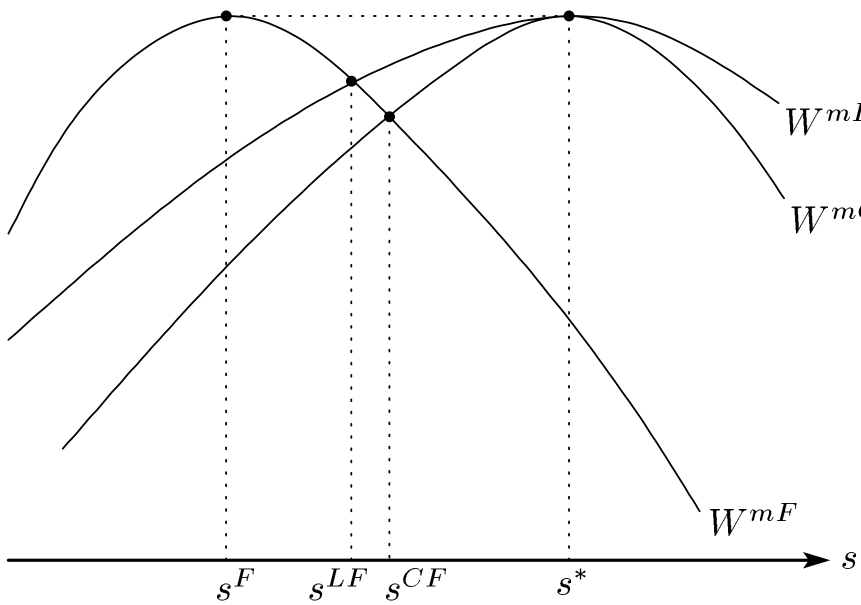

Figure 2.1: Welfare curves of the three types of competition

Stackelberg competition with private leadership. Proposition 2.1 shows the idiosyncrasy of such a competition and industries.

The result of Proposition 2.1, new to the literature, shows that the irrelevance result does not hold in private leadership but that the difference of results in private leadership from that in Cournot and public leadership is restricted to the level of optimal subsidy. This seems surprising but the underlying intuition is clear. In the case of private leadership, the private firm has an incentive to expand its production for any given level of subsidy in order to increase market share. In addition, the public firm tends to produce excessively because it takes into account not only its own profit but also consumer surplus. Thus, a lower subsidy level yields high levels of outputs in a Pareto-optimal allocation.

The difference in optimal subsidies in the given move structures illustrates the importance of the relationships between the level of subsidy and welfare in these structures. To understand these relationships, we compare them to an any assumed level of subsidy. Figure 2.1 illustrates the comparisons. In this figure, the welfare in each game is drawn as a hump-shaped curve. As stated in Proposition 2.1, the maximal of the curves in Cournot and public leadership is reached at s^* while that of the curve in private leadership is reached at s^F which is smaller than s^*. All the three welfare curves are increasing in s when $s \leq s^F$ and decreasing in s when $s > s^*$. On the other hand, when $s^F < s \leq s^*$, welfare curves in Cournot and public leadership, W^{mC} and

W^{mL}, are increasing whereas that in private leadership, W^{mF}, is decreasing, i.e. relationships among the welfare of three games are switched.[9] These results give the relations among the welfare functions.

Lemma 2.1. *Suppose that s^{jF} is the subsidy level such that $W^{mj}(s)$ equals to $W^{mC}(s)$ ($j = C, L$). Then, we have*

$$\begin{aligned}
&(a) \quad W^{mL}(s) \geq W^{mC}(s) > W^{mF}(s) && \textit{if } s > s^{CF}, \\
&(b) \quad W^{mL}(s) > W^{mF}(s) \geq W^{mC}(s) && \textit{if } s^{CF} \geq s > s^{LF}, \\
&(c) \quad W^{mF}(s) \geq W^{mL}(s) > W^{mC}(s) && \textit{if } s^{LF} \geq s > s^{F}, \\
&(d) \quad W^{mF}(s) > W^{mC}(s), \;\; W^{mL}(s) > W^{mC}(s) && \textit{otherwise.}
\end{aligned}$$

As compared to the ranking of welfare, the ranking of the private firm's profits is relatively simple, which is given in the following lemma.

Lemma 2.2. *Depending on the subsidy level, the following results are obtained with regard to the profits of the private firm.*

$$\begin{aligned}
&(a) \quad \Pi_1^{mL}(s) \leq \Pi_1^{mC}(s) < \Pi_1^{mF}(s) && \textit{if } s \geq s^*, \\
&(b) \quad \Pi_1^{mC}(s) < \Pi_1^{mL}(s), \;\; \Pi_1^{mC}(s) < \Pi_1^{mF}(s) && \textit{if } s < s^*.
\end{aligned}$$

Proof: See Appendix.

Since it is obvious that the profits in private leadership is not less than that in Cournot, we only explain the intuition behind the relationship between the profits in public leadership and Cournot. Suppose that the subsidy is relatively low ($s < s^*$). In this case, the private firm produces less whereas the public firm produces more in Cournot, which means that the public firm is inefficient due to its increasing marginal cost. Thus, an inefficient public firm as a Stackelberg leader can improve social welfare by transfering its production to the efficient private firm. Since this transfer increases the market share of the private firm, $\Pi_1^{mC}(s) < \Pi_1^{mL}(s)$ for $s < s^*$. On the other hand, if the level of subsidy is relatively high ($s \geq s^*$), the private firm produces excessively and is, thus, inefficient. Therefore, the public leader has an incentive to substitute its output for the output of the private firm, which leads to a decrease in the market share of the private firm and, thus, $\Pi_1^{mC}(s) \geq \Pi_1^{mL}(s)$.

[9] Although not drawn in Figure 2.1, there is some possibility that curve W^{mF} intersects curve W^{mL} at a certain level of subsidy $s < s^F$. However, this possibility never influences the later discussions. In addition, we can show that curve W^{mF} lies above curve W^{mC} for any $s < s^F$, and the reverse is true for any $s > s^*$ as long as $s \in S$.

2.3 Endogenous timing game

As discussed in the introduction, some studies investigate endogenous timing in mixed duopoly without any subsidy and then show that private leadership is likely to be an equilibrium outcome. In this section, we attempt to examine how their result could be altered if the government provides both public and private firms with production subsidy. Following Pal (1998b), we apply the observable delay game of Hamilton and Slutsky (1990), in which firms simultaneously choose the production timing, and thereafter, produce their output at their production timing.

Our game considered in this section proceeds as follows. At stage 1, the government sets a unit production subsidy for firms. At stage 2, the firms simultaneously announce the period in which they will produce their output and are commited to this choice. Let $t_i \in \{1, 2\}$ be the time period chosen by firm i ($i = 0, 1$) at stage 2. Finally, at stage 3, each firm chooses the output level q_i at the period decided at stage 2. More precisely, if both the firms announce the same production period at stage 2, Cournot competition emerges at stage 3. Otherwise, when each firm selects a different period, Stackelberg competition appears in stage 3. We solve the subgame perfect equilibrium in this game by using backward induction.

Now, we proceed to stage 2, because stage 3 was described in section 2.2. At stage 2, public and private firms determine their production timings for any given level of subsidy. Pal (1998b) and Tomaru and Kiyono (2009) show that private leadership is always an equilibrium outcome of observable delay game in mixed duopoly without subsidy policy. Matsumura (2003a) also shows that in a different endogenous timing game, private leadership can always be an equilibrium outcome while public leadership can never be one. However, we find that their results completely change once the government subsidizes firms.

Proposition 2.2. *The following equilibria hold at stage 2:*

$$\begin{aligned}
&(a)\ (t_0, t_1) = (1, 1), && \textit{if } s > s^*, \\
&(b)\ (t_0, t_1) = (1, 1), (1, 2), && \textit{if } s = s^*, \\
&(c)\ (t_0, t_1) = (1, 2), && \textit{if } s^{CF} < s < s^*, \\
&(d)\ (t_0, t_1) = (1, 2), (2, 1), && \textit{otherwise.}
\end{aligned}$$

Proof: See Appendix.

Surprisingly, Proposition 2.2 states that contrary to Pal (1998b), Tomaru and Kiyono (2009) and Matsumura (2003a), the private leadership never appears as an equilibrium outcome of stage 2 when the level of subsidy is not low.

Let us explain the intuition behind Proposition 2.2. In case (a) of Proposition 2.2, a large amount of subsidy promotes the excess production by the private firm. To mitigate total production costs in the whole industry due to this excess production, the public firm wants to reallocate production from the private firm to itself by acting aggressively if this action is commitable. This is the same situation as that of private duopoly. Hamilton and Slutsky (1990) show that in the observable delay game of private duopoly, firms select period 1 and, thus, Cournot competition follows. Accordingly, in our mixed duopoly, only Cournot competition becomes an equilibrium outcome. In case (b), that is $s = s^*$, we know that at this subsidy level, Cournot and public leadership are irrelevant in the sense that the first-best allocation prevails and Cournot and public leadership are indifferent for both firms. Thus $(t_0, t_1) = (1, 2)$ is added to equilibrium outcomes.

In case (c), the subsidy is in the middle range in which the private firm with a leader advantage produces more than in the Pareto-efficient allocation, but that without the advantage produces less. In order to avoid such overproduction by the private firm with leadership, the public firm tries to produce in advance. As a result, only $(t_0, t_1) = (1, 2)$ becomes an equilibrium outcome. Finally, we explain case (d) in which the subsidy is too low and the private firm does not produce as much. This implies that a transfer of production by the overproducing public firm to the underproducing private firm decreases total costs and increases social welfare due to increasing marginal costs. Hence, the public firm acts so as to realize either public leadership or private leadership.

Now, we explore the analysis of stage 1. In this stage, the government sets the subsidy to maximize social welfare. In Section 2.2, we considered the two-stage game in three different games and showed that in the Cournot and public leadership structures the subsidy is the same (s^*) but in the private leadership structure it is not (s^F). The difference of optimal subsidies implies that whether or not the irrelevance result holds in our endogenous timing framework depends on the time structure realized as the equilibrium. From Propositions 2.1 and 2.2, we immediately establish the following result.

Proposition 2.3. *Suppose that $s^* \in S$. There exists a subgame perfect equilibrium under mixed duopoly given as follows:*

$$(q_0, q_1, s) = (q_0^{mC}(s^*), q_1^{mC}(s^*), s^*) = (q_0^{mL}(s^*), q_1^{mL}(s^*), s^*).$$

Readers might think that s^F can also be an equilibrium subsidy. Certainly, the government can achieve the first-best allocation by setting the subsidy s^F when Stackelberg competition with private leadership follows. However, since multiple equilibria, private leadership and public leadership, follow at stage 2 for any level of subsidy $s \leq s^{CF}$, if the government sets s^F, it could end up lowering social welfare than under the first-best. Thus, the government never has any reason for setting s^F. Henceforth, we regard only s^* as an equilibrium subsidy in mixed duopoly with endogenous timing.

Proposition 2.3 states that when the government optimally chooses the subsidy, only Cournot and/or private leadership are the equilibrium outcome of endogenous timing and private leadership is not. In the real world, there are many situations where Cournot and Stackelberg with public leadership are suitable. Industries such as telecommunications, electricity, and postal sector, are dominated by former public monopolies with a first-mover advantage.[10] In addition, the result of Proposition 2.3 amplifies the importance of the irrelevance result of Poyago-Theotoky (2001) and Myles (2002), in the sense that Proposition 2.3 shows that Cournot and public leadership are likely to arise in mixed duopoly with subsidization, and private leadership in cases where the irrelevance result does not hold is not likely.

2.4 Privatization

White (1996) discusses the other irrelevance result other than that of Poyago-Theotoky (2001) and Myles (2002). He shows that the government is able to realize the Pareto-efficient allocation in Cournot mixed oligopoly and Cournot private oligopoly by setting the same optimal subsidy. In this section, we examine whether or not this irrelevance holds in our endogenous timing model.

For this purpose, we first derive the equilibrium of the endogenous timing model in private duopoly. Because firm 0 maximizes its own profits (1) after privatization, the first-order condition for firm 0's profit maximization in Cournot competition yields the reaction function of firm 0, $R_0^p(q_1, s)$. This reaction function satisfies

$$\frac{\partial \Pi_0}{\partial q_0} = P(R_0^p(q_1, s) + q_1) + P'(R_0^p(q_1, s) + q_1)R_0^p(q_1, s) - C'(R_0^p(q_1, s)) + s = 0.$$

Firm 1 also maximizes its profits, and thus, its reaction function still remains $R_1(q_0, s)$. As in section 2, we define firms' equilibrium outputs in Cournot competition in private duopoly as follows: $q_i^{pC}(s)$ and $Q^{pC}(s) = q_0^{pC}(s) + q_1^{pC}(s)$. Further, equilibrium outputs in Stackelberg competition with firm 0's leadership (pL) and with firm 1's leadership (pF) are given as $q_i^{pj}(s)$ and $Q^{pj}(s) = q_0^{pj}(s) + q_1^{pj}(s)$ $(i = 0, 1, j = L, F)$. Then, we define firm i's profits as $\Pi_i^{pj}(s) := \Pi_i(q_0^{pj}(s), q_1^{pj}(s), s)$ $(i = 0, 1, j = C, L, F)$. As is well known, the following result is derived in private duopoly.

[10] For detailed examples of such industries, see Fjell and Heywood (2002).

Lemma 2.3. *For all subsidies, each profit function of privatized firm 0 and private firm 1 in the private duopoly satisfies the following relationships:*

$$\Pi_0^{pF}(s) < \Pi_0^{pC}(s) < \Pi_0^{pL}(s), \quad \Pi_1^{pL}(s) < \Pi_1^{pC}(s) < \Pi_1^{pF}(s).$$

We now examine the decision of the production timing at stage 2, that is, each firm announces the production period at stage 3.

Lemma 2.3 implies that each firm has the incentive to be the leader. Thus, each firm always chooses the period $t_i = 1$ $(i = 0, 1)$ in this stage, such that for any subsidy, $(t_0, t_1) = (1, 1)$ is realized as the equilibrium in this observable delay game. Therefore, Cournot competition occurs at $t_i = 1$ $(i = 0, 1)$ in stage 3.

In stage 1, the government sets the subsidy level to maximize social welfare. Following this, it recognizes that Cournot competition appears as the equilibrium, and its objective function becomes

$$\widetilde{W^p}(s) = W(q_0^{pC}(s), q_1^{pC}(s)) = \int_0^{Q^{pC}(s)} P(z)dz - C_0\big(q_0^{pC}(s)\big) - C_1\big(q_1^{pC}(s)\big).$$

The first-order condition for $\widetilde{W^p}(s)$ leads to the following optimal subsidy s^{**}:

$$s^{**} = \arg\max_{\{s\}} \ \widetilde{W^p}(s). \tag{2.6}$$

Thus, the subgame perfect equilibrium in the private duopoly after the privatization of firm 0 is characterized in the following proposition.

Lemma 2.4. *Suppose that $s^{**} \in S$. Under privatization, the following subgame perfect equilibrium is realized:*

$$(q_0, q_1, s) = (q_0^{pC}(s^{**}), q_1^{pC}(s^{**}), s^{**}).$$

We now focus on the comparison between the subgame perfect equilibria derived under mixed and private duopolies. In mixed duopoly, Cournot and public leadership appear in equilibrium. As shown in Proposition 2.1, in these market structures, one control variable of uniform production subsidy does well for the Pareto-efficient allocation. On the other hand, due to asymmetry of cost functions, uniform subsidy does not always yield Pareto-efficient allocation in private Cournot duopoly. In this case, the irrelevance result à la White (1996) does not hold. Without any heterogeneity of cost functions, this irrelevance is recovered, since Cournot competition in both mixed and private duopoly follows in endogenous timing, similar to White (1996).

Proposition 2.4. *Suppose that public and private firms face the same cost functions. Then, even if we consider each firm's endogenous production timing, when the government utilizes output subsidization, whether this situation is that of mixed duopoly or private duopoly, the optimal subsidy that yields the first-best allocation is identical.*

Fjell and Heywood (2004) demonstrate that if the privatized firm is still a Stackelberg leader, then the optimal subsidy of private oligopoly is different from that of mixed oligopoly, and moreover, privatization reduces social welfare. This suggests that after privatization, the first-best allocation may require a subsidy other than that in mixed oligopoly when a change in the market and competition structures accompanying privatization is taken into account. However, Proposition 2.4 states that the results of White (1996) hold even though both the private and public firms choose their own production timings.

2.5 Subsidization policy and privatization with lobbying

Although the above discussion on optimal subsidy and privatization may attract our interest, we should bear in mind that it presumes that the omniscient government has a free discretion over the determination of the level of subsidy. Past literature on subsidized mixed oligopoly assumes that the government has perfect controllability over setting subsidy and thus can set the optimal subsidy. Yet, this may not be the case when there is lobbying by interest groups and a highly complicated political process prevails. Many papers on lobbying activities and campaign contribution have shown that production subsidies and export subsidies are likely to be excessive.[11] This section attempts to examine the effect of privatization for a given level of subsidy that is higher than the optimal one.

Unfortunately, in our present setting, what we can say is limited due to its generality of demand and cost functions. To make our discussion clear, we specify these functions. The inverse demand is assumed to be linear and is given by $P = a - Q$. Following DeFraja and Delbono (1989) and other existing works, we also assume that the firms face the symmetric quadratic cost functions, $C_i(q_i) = \frac{1}{2}kq_i^2$. The simple calculation yields $s^* = a/(k+2)$.

As stated in the previous section, symmetry of cost functions equalizes the optimal subsidy in mixed Cournot duopoly s^* with that in private Cournot duopoly s^{**} and this level of subsidy leads to the Pareto-optimal allocation in these two types of duopoly, which implies that when the

[11]Another reason for such excessive subsidies is policy makers' limited knowledge of firms' technologies and demand. Accordingly, the policy makers would unexpectedly end up setting a level of subsidy higher than s^*.

government can set the subsidy $s^* = s^{**}$, privatization does not make any change in welfare and the profit of private firm.

Suppose that the government is forced to set the level of subsidy at $s > s^* = s^{**}$ due to the lobbying activities of the owners of the private firm. For this level of subsidy, before and after privatization of the public firm, Cournot competition is the only equilibrium outcome of our endogenous timing game. Based on this, the difference between welfare of private and mixed duopoly is

$$\widetilde{W}^p(s) - W^{mC}(s) = -\frac{(k^3+3k^2+k+1)\left\{a-(k+2)s\right\}^2}{2(k+3)^2(k^2+3k+1)^2} < 0.$$

This is because the excessive subsidy stimulates not only the existing private firm but also the privatized firm, which results in a large amount of total production costs. Consequently, privatization worsens social welfare. Similarily the difference of private firm's profits is

$$\Pi_1^{pC}(s) - \Pi_1^{mC}(s) = -\frac{(k+2)\left\{(k+2)s-a\right\}\left\{(2k^2+7k+4)s+(2k^2+6k+1)a\right\}}{(k+3)^2(k^2+3k+1)^2} < 0.$$

Thus, privatization with lobbying activities decreases social welfare as well as the profit of the private firm. This decrease in profits gives owners of the private firm incentives to oppose to and to hamper privatization. The results are summarized in Proposition 2.5, below.

Proposition 2.5. *Suppose that lobbying activities result in excessive subsidy ($s > s^*$). Then, privatization decreases not only the profit of private firm but also social welfare.*

We should notice that our results in this proposition rely on symmetry and specificity in functions. In fact, we cannot confirm whether or not privatization deteriorates social welfare for $s > s^*$ in general models. One of the reasons is the indeterminancy of whether s^* is larger than s^{**}. Suppose that s^* is smaller than s^{**}. In this case, welfare in mixed duopoly is decreasing in s for $s > s^*$ and that in private duopoly is increasing in s for $s < s^{**}$. We cannot exclude the case where the maximal of welfare in private duopoly is larger than that in mixed duopoly for $s = s^{**}$. Nevertheless, we can easily show that the profit of the existing private firm always decreases for the relevant range of subsidy in a more general setting as long as both firms have identical technologies. This implies that privatization in a subsidized mixed duopoly with lobbying activities is likely to be opposed even if it improves social welfare.

2.6 Concluding remarks

In this chapter, we investigate the endogenous timing in mixed duopoly with subsidization by using the observable delay game by Hamilton and Slutsky (1990). First, we find that for a level of subsidy that is not very low, Stackelberg competition with public leadership and Cournot are likely to appear as equilibrium outcomes and that with private leadership does not become an equilibrium outcome. This is contrary to the results of Pal (1998b), Tomaru and Kiyono (2009) and Matsumura (2003a). Second, we show that the government can achieve the Pareto-efficient allocation in Stackelberg competition with private leadership by providing firms with subsidy that is less than optimal in Cournot mixed duopoly. This result implies that if private leadership is in a subgame perfect equilibrium of our endogenous timing game with subsidy, there is some possibility that irrelevance result à la White (1996) does not hold. However, we show that public leadership and Cournot are in equilibrium of mixed duopoly and that Cournot is in equilibrium of private duopoly. Along with the results of Poyago-Theotoky (2001) and Myles (2002), these findings indicate that the irrelevance results à la White (1996) hold even when we consider the endogenous timing. Finally, we examine the effect of privatization on profits of the private firm and social welfare for a subsidy level that is higher than the optimal subsidy. It is shown that such privatization always decreases both.

We make some remarks on our model and findings. This chapter assumes that public and private firms compete in quantity. As Bárcena-Ruiz (2007) presents the endogenous timing model where these firms compete in price, we can easily extend our model to price competition. In the linear demand model of differentiated goods, simple calculation shows that welfare curves W^{mL} and W^{mC} lie to the left of curve W^{mF}, which alters the outcomes of Proposition 2.2; (a) $(t_0, t_1) = (1, 1)$ for $s < s^*$, (b) $(t_0, t_1) = (1, 1), (1, 2)$ for $s = s^*$, (c) $(t_0, t_1) = (1, 2)$ for $s^* < s < s^{CF}$ and (d) $(t_0, t_1) = (1, 2), (2, 1)$ for $s \geq s^{CF}$. Further, we assume that the number of private firms is one. In Pal (1998b), which investigates a mixed oligopoly without any subsidy, the result depends crucially on the number of private firms. He shows that public and private leadership are in equilibrium of mixed oligopoly with a small number of private firms and that only the private leadership is in equilibrium with a large number of private firms. In the model of linear demand and symmetric quadratic cost, we can show that one private firm always has an incentive to deviate from Cournot and public leadership under the optimal subsidy level. It indicates that there is no symmetric equilibrium on the private firms.

Finally, we discuss the possibility of extension. we adopted the observable delay game by Hamilton and Slutsky (1990). There may be circumstances under which this game is inadequate to examine the endogenization of the production timings. Saloner (1987) and Matsumura (2003a) use the two-period model to analyze the manner in which each firm decides how much output to produce in each period. It is of interest that we investigate how formulations other than that of

Hamilton and Slutsky (1990), such as Saloner (1987) and Matsumura (2003a), change the results. In addition, we consider only full privatization. In reality, many privatized firms are owned by private and public sector entities. It might also be interesting to examine how our results such as Proposition 2.2 are altered when we apply Matsumura's (1998) approach that models such partial privatization. These problems are left for future research.

Appendix

Proof of Proposition 2.1

First, we prove that the Pareto-efficient allocation can be realized in all three games. For this, we show that there exist subsidies such that both firm's marginal costs are tantamount to price in Cournot competition and two types of Stackelberg competition. Let us consider the case where the government sets the production subsidy at $s^C = -q_1^{mC}(s^C)P'(Q^{mC}(s^C))$ in a Cournot game. Under this level of subsidy, the first order condition of private firm is given by

$$\begin{aligned}\frac{\partial \Pi_1}{\partial q_1} &= P(Q^{mC}(s^C)) + P'(Q^{mC}(s^C))q_1^{mC}(s^C) - C_1'(q_1^{mC}(s^C)) + s^C q_1^{mC}(s^C),\\ &= P(Q^{mC}(s^C)) - C_1'(q_1^{mC}(s^C)).\end{aligned}$$

Thus, along with the fact that the public firm is a welfare-maximizer, the Pareto-optimal allocation can be attained in a Cournot game.

In the case where the public firm is a Stackelberg leader, that is, the case of Stackelberg competition with public leadership, we apply subsidy $s^L = -q_1^{mL}(s^L)P'(Q^{mL}(s^L))$.

$$\begin{aligned}0 = \frac{\partial \widehat{W}}{\partial q_0} &= P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)) + \left\{P(Q^{mL}(s^L)) - C'(q_1^{mL}(s^L))\right\} \cdot \frac{\partial R_1}{\partial q_0},\\ &= P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)).\end{aligned}$$

In the case of Stackelberg competition with private leadership, suppose that the government selects $s^F = -P'(Q^{mF}(s^F))q_1^{mF}(s^F)[1 + R_0'(q_1^{mF}(s^F))]$. Then both the public and private firms' first-order conditions (2.2) and (2.5) are given as

$$P(Q^{mF}(s^F)) - C_0'(q_0^{mF}(s^F)) = P(Q^{mF}(s^F)) - C_1'(q_1^{mF}(s^F)) = 0.$$

Hence, in any of the two Stackelberg games, the Pareto-optimal allocation is achieved.

Next, we prove that $s^F < s^C = s^L$. For convenience, we define the output level q_i^* as

$$P(q_0^* + q_1^*) = C_i'(q_i^*), \qquad i = 0, 1. \tag{2.7}$$

From the definition of s^C and s^L, we obtain

$$s^C = -q_1^{mC}(s^C)P'(Q^{mC}(s^C)) = -q_1^* P'(q_0^* + q_1^*) = -q_1^{mL}(s^L)P'(Q^{mL}(s^L)) = s^L.$$

In addition, $s^F = -P'(q_0^* + q_1^*)q^* \left[1 + R_0^{m\prime}(q_1^*)\right]$. Since $R_0'(\cdot) < 0$, we have $s^F < s^C = s^L$. ■

Proof of Lemma 2.2

To prove (a) and (b), we first show that $q_0^{mL}(s) \leq q_0^{mC}(s)$ if $s \leq s^*$ and $q_0^{mL}(s) > q_0^{mC}(s)$ if $s > s^*$. Define $f(s) := s + P(Q^{mC})q_1^{mC}(s)$. Notice that $f(s^*) = 0$ and the differential of this function f is positive. In fact,

$$\begin{aligned}
f'(s) &= 1 + \left[P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)\right] q_1^{mC\prime}(s) + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial s} \cdot \left[P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)\right]}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial q_0}}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&= \frac{1 + (1 - R_0'(q_1^{mC}(s))) \cdot \frac{\partial R_1}{\partial q_0}}{1 - R_0'(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC\prime}(s), \\
&> 0.
\end{aligned}$$

Thus, by the private firm's first-order condition (2.3), we obtain the following fact:

$$s \gtreqless s^* \iff P(Q^{mC}(s)) - C'(q_1^{mC}(s)) \lesseqgtr 0.$$

Further, evaluating $\partial\widehat{W}/\partial q_0$ at $q_0^{mC}(s)$, we find

$$\begin{aligned}
\left.\frac{\partial \widehat{W}}{\partial q_0}\right|_{q_0 = q_0^{mC}(s)} &= P(Q^{mC}(s)) - C'(q_0^{mC}(s)) + \left[P(Q^{mC}(s)) - C'(q_1^{mC}(s))\right] \cdot \frac{\partial R_1}{\partial q_0}, \\
&= \left[P(Q^{mC}(s)) - C'(q_1^{mC}(s))\right] \cdot \frac{\partial R_1}{\partial q_0},
\end{aligned}$$

Thus, the second-order condition of the public firm as a leader yields

$$s \gtreqless s^* \iff q_0^{mC}(s) \lesseqgtr q_0^{mL}(s).$$

We now proceed to the proof of Lemma 2.2 (a) and (b). Since private firm 1 as a Stackelberg leader can choose its output to prevent its profit from becoming lower than Π_1^{mC} and since $\widehat{\Pi}_1$ is strictly concave, we obtain $\Pi_1^{mC}(s) < \Pi_0^{mF}(s)$ for any s. In order to prove the relationship between $\Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s)$, we define $\bar{\Pi}_1(q_0, s) := \Pi_1(q_0, R_1(q_0, s), s)$. Note that $\bar{\Pi}_1(q_0^{mC}(s), s) = \Pi_1^{mC}(s)$ and $\bar{\Pi}_1(q_0^{mL}(s), s) = \Pi_1^{mL}(s)$. Definition of R_1 yields,

$$\frac{\partial \bar{\Pi}_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} + \frac{\partial \Pi_1}{\partial q_1} \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} + 0 \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} < 0.$$

Since $q_0^{mL}(s) \leq q_0^{mC}(s)$ if $s \leq s^*$ and $q_0^{mL}(s) > q_0^{mC}(s)$ if $s > s^*$, we get $\Pi_1^{mL}(s) \leq \Pi_1^{mC}(s)$ if $s \leq s^*$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$ if $s > s^*$. ■

Proof of Proposition 2.2

Consider the following four cases: (a) $s > s^*$, (b) $s = s^*$, (c) $s^{CF} < s < s^*$ and (d) $s < s^{CF}$.

(a) $s > s^*$

In this case, we know that $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) > \Pi_1^{mL}(s)$. Thus, an act of production at period 1, i.e. $t_i = 1$ ($i = 0, 1$), is a dominant strategy for both the firms. For $s > s^*$, the equilibrium is $(t_0, t_1) = (1, 1)$.

(b) $s = s^*$

In this case, we find that $W^{mL}(s) = W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) = \Pi_1^{mL}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ and $t_0 = 2$ for $t_1 = 2$. On the other hand, those of the private firm are $t_1 = 1$ and $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 1),\ (1, 2)$.

(c) $s^{CF} < s < s^*$

In this case, social welfare and private firm's profits satisfy the inequalities $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 2)$.

(d) $s < s^{CF}$

In this case, we find that $W^{mF}(s) > W^{mC}(s)$, $W^{mL}(s) > W^{mC}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mL}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 2$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Hence, the equilibrium is $(t_0, t_1) = (1, 2),\ (2, 1)$. ■

Chapter 3

An Endogenous Objective Function of a Partially Privatized Firm: A Nash Bargaining Approach

3.1 Introduction

In this chapter, we demonstrate how a firm's objective function is determined when each owner has a different interest. [1] In particular, we use a "mixed duopoly" model where a profit maximizing private firm competes against a partially privatized firm. The privatized firm is owned by two types of owners: one is a private capitalist and the other is the government. The private capitalist usually expects the firm to maximize its own profits Π_0 whereas the government, the social welfare W. This implies that the owners have contradictory interests, and thus, it is not easy for them to set the privatized firm's objective function. Against this backdrop, this chapter aims to explain the process of setting the function as a bargaining process.

Since the 1980s, many public firms have been privatized, and the private sector has owned such firms fully or partially.[2] DeFraja and Delbono (1989) examine the effect of privatization of a public firm on social welfare and show that in some situations, privatizing a public firm enhances social welfare despite it not involving an improvement in production efficiency but only a change in the firm's objective and behavior. This result is extended to partial privatization by Matsumura (1998). A partially privatized firm is a mixed joint stock company owned by a profit maximizing private capitalist and the welfare maximizing public sector (or the government). In his model, a

[1] This chapter is based on Kamijo and Tomaru (2008).

[2] We can see such privatized firms in a wide range of industries such as the airlines, gas, electricity, telecommunications, banking, and education industries. The Japanese government established four corporations in Japan — Japan Post Network Corporation, Japan Post Service Corporation, Japan Bank Corporation and Japan Post Insurance Corporation — and made Japan Post Holdings Corporation (JP) have these corporations as subsidiaries, in October 2007. By 2017, the Japanese government intends to sell two-thirds of its shares in JP. Thus, Japan Post will be a typical partially privatized firm in Japan.

partially privatized firm is assumed to maximize $\alpha W+(1-\alpha)\Pi_0$, $\alpha \in [0, 1]$, the weighted average of owners' interests. It is also assumed that this weight increases with the corresponding owner's shareholding ratio (i.e., α is an increasing function of the public sector's shareholding ratio). In other words, if an owner increases shares in the firm, then the firm gives extra consideration to the owner's concern. Matsumura shows that partial privatization is always a more effective means for achieving high social welfare than both full nationalization and full privatization.

These works can also be analyzed from the viewpoint of what objective a player should pursue in strategic environments. The possibility that a player who complies with some behavioral principle distinct from his objective receive better returns than when he acts so as to maximize the real objective is already known in several contexts.[3] A problem that arises for a player who recognizes that changing his objective is beneficial for him pertains to how he credibly reports the change in the objective or the utility function to his rivals. As Schelling (1980) indicates, the useful way to credibly change the objective is to lose or restrict the power of the player in a legal manner. Thus, privatization and partial privatization constitute credible means to change the objective of a public firm because the rivals believe that the firm now concerns the interests of both the owners and behaves so as to harmonize their contradicting interests.

The problem discussed here is related to how two parties in a partially privatized firm agree on an objective of the firm. In the growing literature on mixed oligopoly, Matsumura's model and its variations are intensively used to analyze the market outcome in various conditions, without considering how a partially privatized firm makes decisions.[4] Moreover, in Matsumura (1998), it is assumed that the owner who has a larger part of shares of the firm strongly reflects his objective in the partially privatized firm's behavior. However, it can so happen that the majority may not pretend to reflect its objective in the partially privatized firm's objective because as we explained in the previous paragraph, the pursuit of a different objective by a player can prove to be beneficial to his true objective. One example is the Bank of Iwate, whose largest stockholder is Iwate prefecture and which is one of representative partially-privatized firms in Japan. In

[3]For instance, Crawford and Varian (1979) and Sobel (1981) show that in the Nash bargaining problem, distorting the player's utility function might benefit the player. In the context of strategic delegation, it is known that hiring agents who participate in the game on behalf of its real player gives the player (called the principal) a first mover or other advantage over the opponents (e.g., Vickers 1985, Fershtman 1985, Fershtman and Judd 1987, Sklivas 1987, Fershtman, Judd and Kalai 1991). However, when a contract between the principal and the agent can not be observed by the opponents, using such delegation does not change the equilibrium outcome from the one when the principal himself plays the game (Katz 1991).

[4]For example, Matsumura and Kanda (2005) show that when firms are allowed to enter in the market freely, full nationalization is desirable in terms of social welfare, unlike Matsumura (1998). Further, some research studies the relationship between the partial privatization policy and other policies. Chao and Yu (2006) show that the partial privatization policy is substitutable for import tariff as a trade policy.

2006 the bank has made the mid-term business plan under which great importance is attached to profits and introduction of highly-advanced management system (*the 124th general meeting of shareholders*, June 24. 2006). This example shows that even though the enterprises whose largest shareholder is the government acts like the profit-maximizing firm, the government might not oppose the firm's action. To study such behavoirs of owners, in this chapter, we provide a model where the objective of a partially privatized firm is endogenously determined through bargaining between the two sectors. Further, we examine the validity of the assumption adopted by Matsumura (1998). We also consider the welfare implications of the endogenously determined objective model.

To explore how a partially privatized firm makes decisions or how two parties determine the objective of the firm, we consider a two-stage game described as follows. In the first stage, the public and private sectors discuss the management policy of the firm, which is well represented by the parameter $\alpha \in [0, 1]$. This parameter indicates the weight attached to the management policy by the two sectors. In the process of reaching an agreement through bargaining, this information becomes public, and in the next stage, the privatized firm competes against the other private firms in Cournot fashion. On the other hand, when they fail to reach an agreement through negotiation, they play the *defund game* to decide to either continue operating the business of the partially privatized firm or defund and liquidate it. When both sectors choose in favor of the continuation of the firm, the majority party asserts the total control over the firm by resorting to a shareholder meeting. Thereafter, the firm acts so as to maximize the majority's objective. In contrast, when one of them chooses to defund the firm, each party is returned funds in proportion its shareholding ratio, which it then uses to invest in their other opportunities.

We first conduct a comparative statics of the agreed value of α with respect to the share $s \in (0, 1)$ of the public sector. We find that this crucially depends on the outcome of the defund game. Specifically, when the continuation of the firm is chosen in the defund game, an increment of s does not affect the agreed value of the weight of the public sector, α^*. On the other hand, when the defunding the firm is decided on, the effect of an infinitesimal increment in s on α^* relies on the difference between the return rates of public and private investments. If the former rate is higher than the latter, then the weight on social welfare in the privatized firm's objective function becomes larger as the government's share increases; if the return rate of public investment is lower than that of private investment, the result is reversed. Thus, our endogenous determined objective model indicates that it might be difficult to support Matsumura's assumption. Moreover, we obtain different implications pertaining to the effectiveness of privatization or partial privatization from DeFraja and Delbono (1989) and Matsumura (1998). We find that when the marginal cost of the public firm is higher than that of the private firm, with the difference not being substantial, and the outcome of the defund game is liquidation of the firm, not privatizing the firm is the optimal choice for the government that is concerned with social welfare.

To conclude the introduction, we note a few characteristics of our approach that are derived from existing literature. First, we analyze a bargaining situation of the first stage using a cooperative game framework similar to that employed by Aoki (1980, 1982) to analyze modern corporations as coalitions of several stakeholders. We use the *Nash solution* as our solution concept for the first stage game. Second, we do not characterize the partially privatized firm as one that chooses its output so as to maximize the Nash product of the two parties, given the output of the other private firm. Instead, we adopt the two stage-game where in the first stage, the two parties determine the objective of the partially privatized firm because it is difficult to imagine that the owners of the firm make decisions on the daily output determination. This point is one of the critical difference of our model from De Donder and Roemer (2009) that also consider endogenous determination of the objectives of a firm in which there are related stake holders having different interests.[5] Third, even though the majority party can always resort to the general shareholders' meeting to control the firm, we assume that it cooperatively bargains with the minority to determine the firm's objective for as long as there is scope for mutual benefit through bargaining. Therefore, resorting to the general shareholders' meeting is one of possible threats posed by the majority party in order to obtain a better outcome from negotiations. Finally, we do not consider the problem of delegation because it totally changes the context and makes it difficult to set the comparison in our research against that in existing works such as DeFraja and Delbono (1989) and Matsumura (1998) and other studies in this field (for research considering delegation in the mixed oligopoly, see White 2001).

This chapter proceeds as follows. In section 3.2, we explain the standard mixed duopoly where a private firm competes against a partially privatized firm jointly owned by a profit maximizing private capitalist and the welfare maximizing government. We find that the discrepancy between their interests gives rise to some room for bargaining over what the partially privatized

[5]De Donder and Roemer (2009) consider a vertically differentiated market where two firms simultaneously choose the quality and price of the good and firms are controled by both profit-motivated agent and revenue-motivated agent. To analyze this market, they define a new equilibrium concept, Firm Unanimity Nash Equilibrium, which corresponds to Nash equilibrium between two firms when there is efficient bargaining between profit-motivated agent and revenue-motivated agent. With some assumption on the profit and revenue function of the firms, Firm Unanimity Nash Equilibrium becomes the one such that each firm maximizes the weighted Nash product of the profit and the revenue given the other firm's strategic variables. Moreover, they also consider the case that the government takes a participation in one firm.

firm should maximize. In section 3.3, we provide the model of bargaining between the two sectors and conduct a comparative statics of α^* on s. In section 3.4, we consider the welfare implications obtained from our endogenously determined objective model. Section 3.5 presents the conclusion.

3.2 Model

We consider an industry where a partially privatized firm (firm 0) and a private firm (firm 1) are engaged in Cournot competition. These firms produce a homogeneous commodity, and demand for this commodity is given by the inverse demand function $P = P(Q) = 1 - Q$. Here, P represents the price, $Q = q_0 + q_1$, the total quantity produced by the two firms; and q_i represents the output of the firm i ($i = 0, 1$). Let the cost functions of these firms be given by $C_i(q_i) = F + c_i q_i$. Since issues of entry are not considered in this chapter, we assume that $F = 0$.

Further, we assume that the partially privatized firm's marginal cost c_0 is higher than the private firm's marginal cost c_1. For simplicity, we suppose that $c_0 = c > 0 = c_1$. This assumption of the partially privatized firm's inefficiency is standard in a mixed oligopoly with linear costs and guarantees that the private firm is active in the market.[6]

Private firm 1 maximizes its profit:

$$\Pi_1(q_1, q_0) = (P(Q) - C_1(q_1))\, q_1 = (1 - q_0 - q_1)q_1.$$

Firm 0 is a partially privatized firm which is jointly owned by a profit maximizing private capitalist and the welfare maximizing government. Since the privatized firm with mixed ownership must respect both owners, it cannot be either a pure welfare maximizer or a pure profit maximizer. Therefore it should take into consideration its own profit, given by

$$\Pi_0(q_0, q_1) = (P(Q) - C_0(q_0))\, q_0 = (1 - q_0 - q_1 - c)q_0,$$

as well as social welfare, given by

$$W(q_0, q_1) = \int_0^Q P(z)dz - C_0(q_0) - C_1(q_1) = (q_0 + q_1) - \frac{1}{2}(q_0 + q_1)^2 - cq_0.$$

Following Matsumura (1998), we assume that firm 0 maximizes the weighted average of social welfare and its own profit that is given by

$$V_0(q_0, q_1, \alpha) = \alpha W(q_0, q_1) + (1 - \alpha)\Pi_0(q_0, q_1),$$

[6]This inefficiency is supported by the empirical studies such as Mizutani (2004) and Megginson and Netter (2001). In addition, some theoretical papers prove such inefficiency by showing that public firms strategically adopt a lower level of cost-reducing R&D investment. For example, see Matsumura and Matsushima (2004), Nishimori and Ogawa (2002), and Tomaru (2007).

where $\alpha \in [0,1]$ denotes the weight of the payoff of the government in firm 0's objective. An interpretation of this parameter is that it represents the power of the government to reflect its objective in the partially privatized firm's objective function. In fact, if this power is very strong such that the government can set α to 1, then the partially privatized firm becomes a welfare maximizer. On the other hand, if the power is very weak such that the other owner, the private capitalist, can set α to 0, the firm becomes a profit maximizer.

The first-order conditions for maximizing V_0 and Π_1 with respect to q_0 and q_1 yield the equilibrium outcomes:

$$q_0^*(\alpha) = \frac{1-2c}{3-2\alpha}, \quad q_1^*(\alpha) = \frac{1-\alpha+c}{3-2\alpha}, \quad \text{and} \quad Q^*(\alpha) = \frac{2-\alpha-c}{3-2\alpha}, \tag{3.1}$$

$$\Pi_0^*(\alpha) = \frac{(1-\alpha)(1-2c)^2}{(3-2\alpha)^2}, \quad \Pi_1^*(\alpha) = \frac{(1-\alpha+c)^2}{(3-2\alpha)^2}, \tag{3.2}$$

$$W^*(\alpha) = \frac{(11-8\alpha)c^2 - 2(4-3\alpha)c + 8 - 10\alpha + 3\alpha^2}{2(3-2\alpha)^2}. \tag{3.3}$$

When the weight on welfare α becomes larger, the total output and the output of the privatized firm increase, whereas that of the private firm decreases. The profit of the private firm is monotonically decreasing with α. On the other hand, that of the partially privatized firm is concave and maximized at $\alpha = 1/2$. For our convenience, we define this level of α as α_p. As easily seen from the continuity and comparison among the extremum and the value at the end points, the social welfare is maximized at $\alpha = (1-5c)/(1-4c)$ if $c \leq 1/5$ and at $\alpha = 0$ if $c > 1/5$. We also define the level of α as α_g.

It may be regarded that welfare maximizing α_g is higher than profit maximizing α_p because α is the weight attached to welfare; however, the relationship between α_g and α_p is dependent on c. In effect,

$$\alpha_g - \alpha_p = \frac{1-5c}{1-4c} - \frac{1}{2} = \frac{1-6c}{2(1-4c)}, \qquad \forall\, c \leq \frac{1}{5},$$

and this implies that α_p is higher than α_g if $c > 1/6$. The result that $\alpha_p > \alpha_g$ is convincing. The less aggressive behavior by the highly inefficient public firm enhances the quantity supplied by the more efficient private firm, which leads to an improvement of welfare. Thus, our model does not exclude the possibility that α_p is higher than α_g. Nevertheless, we assume that the government has an incentive to make the partially privatized firm produce more than the private capitalist, that is, $\alpha_g > \alpha_p$. In other words,

Assumption 3.1. *The partially privatized firm's marginal cost is sufficiently low, that is,* $c < 1/6$.[7]

[7] In the succeeding sections, we will consider the bargaining problem between the government and a private

It should be noted that under this assumption, we have

$$W^{*\prime}(\alpha) \gtreqless 0 \iff \alpha \lesseqgtr \alpha_g \quad \text{and} \quad \Pi_0^{*\prime}(\alpha) \gtreqless 0 \iff \alpha \lesseqgtr \alpha_p. \tag{3.4}$$

The latter is obvious since Π_0^* is concave. W^* is also a hump-shaped curve whose maxima occurs at $\alpha = \alpha_g$. The relationship reveal that both owners' desirable outcomes are different, which leaves some scope for bargaining between them over α as will be seen in the next section. If each owner can control α freely without the other owner's approval, then he gains a higher payoff than when he is the sole owner. However, it might be difficult for one owner to select α by ignoring the other owner's interest.

We would like to mention another remark here. The above discussion suggests that each owner's payoff becomes larger when the concerned firm has an objective function other than the owner's objective. Fershtman and Judd (1987) and Sklivas (1987) consider the model where ownership and management of firms are separated. In their model, the owner presents to the manager an incentive contract in which the manager is paid at the margin in proportion to a linear combination of profits and sales. This incentive contract works as a type of commitment device to deviate the private firm's objective function from a function other than profit, which results in higher profits than in the case of the integration of ownership and management. The objective function of the partially privatized firm in our model V_0 can be also reinterpreted as such an incentive contract presented by the government and private capitalist. In short, they make a contract with their manager in which the larger a linear combination of welfare and profits becomes, the more he is paid.[8] As described above, however, the decision of the details of an contract (i.e., V_0 or α) might not go well, because one owner's interest does not always coincide with the other owner's. In the next section, as one way of deciding the objective function of the partially privatized firm (or the detail of the incentive contract), we consider the bargaining over α between the government and private capitalist.

capitalist. This bargaining problem is well defined under Assumption 3.1. In fact, this assumption assures the convexity of the bargaining set, which will be proved in Lemma 3.1 of section 3.1. Without this assumption, the bargaining set is not always convex and the analysis becomes difficult. Then, we impose this assumption.

[8] We should consider this interpretation with a special attention. If the government can delegate the management of the full nationalized firm to the manager with some incentive contract, then it loses an incentive to privatize the firm, since such incentive contracts allow the desirable level of α for the govenment, α_g, to be realized in the absence of bargaining between both owners. However, the aim of this chapter is to reconsider the partial privatization. Then, in the case of interpreting our model as managerial delegation, we might have to assume that an incentive contract with managers or civil servants, which makes feasible the obejetive function of the public firm other than welfare, is infeasible.

3.3 Bargaining between the government and the private capitalist

In the previous section, we saw how a certain weight α influences the privatized firm's profits and social welfare. The results in the previous section demonstrate that the governmental owner of the firm prefers some intermediate value α_g to $\alpha = 1$ which implies that the government can totally control the firm. This leads to an important welfare implication, which has been already pointed out by Matsumura (1998) and Bennett and Maw (2003), that social welfare could be higher if the government partly loses its power in the management of the public firm. However, these studies do not explicitly consider the process of how this weight is determined. Thus, in this section and the following section, we establish a model wherein the governmental owner and the private capitalist engage in negotiations over the parameter α in the firm's objective function, which is assumed to represent the management policy of the firm, in order to answer (i) how each owner's share in the firm affects the bargaining outcome and (ii) whether or not the (partial) privatization of the public firm contributes to enhancing social welfare.

Before explaining the components of our bargaining model in detail, it is useful to confirm the reason why the government and the private capitalist have to bargain. We assume that the government owns a share of $s \in (0, 1)$ in the privatized firm 0 and that the private capitalist owns a share of $1 - s$. At the moment, the share s is assumed to be an exogenous parameter for the governmental owner and the private capitalist. In proportion to their shares, the two owners receive their dividends from the profit of the firm: $s\Pi_0^*(\alpha)$ and $(1 - s)\Pi_0^*(\alpha)$ for the government and the private capitalist respectively. Thus, both owners' payoffs are given by $U_g(\alpha) = W^*(\alpha)$ and $U_p(\alpha; s) = (1 - s)\Pi_0^*(\alpha)$, where subscripts g and p represent the government and the private capitalist respectively.

As mentioned in the previous section, under Assumption 3.1, the welfare maximizing level of α, i.e., $\alpha_g = (1 - 5c)/(1 - 4c)$, is higher than the profit maximizing level of α, i.e., $\alpha_p = 1/2$. Therefore, for $\alpha \in (0, 1/2)$, both the owners agree to an increase in α. Similarly, for $\alpha \in ((1-5c)/(1-4c), 1)$, they agree to a decrease in α. In contrast, when $\alpha \in [1/2, (1-5c)/(1-4c)]$, the government approves an increase in α, but the private capitalist opposes it. Thus, in this interval of the value of α, the owners' interests are conflicting, and thus, they have to agree on some value of α through bargaining in order to continue operating the firm. Through the negotiations between these owners, they decide on α in the range $[1/2, (1 - 5c)/(1 - 4c)]$.

We construct the following multistage game including the stage of bargaining between the government and the private capitalist over the management policy α.

Stage 1: The two parties engage in negotiation over weight $\alpha \in [0, 1]$. If they reach an agreement on the value of α, Stage 2a follows; otherwise, they play the game in Stage 2b.

Stage 2a: The partially privatized firm, with the agreed weight α in Stage 1, and the private firm compete in Cournot fashion.

Stage 2b: The two parties play a defund game in order to determine whether they should continue to operate the firm or defund and liquidate it.

We assume that the bargaining process in Stage 1 can be well described as the bargaining problem by Nash (1950, 1953) and thus characterized by two components: the feasible set of players' payoffs and their payoffs in the case of disagreement. The outcome of Stage 2a, which varies according to the value of α determined in Stage 1, defines the feasible payoffs of the players. This was solved in the previous section, and the outcomes were given in equations (3.1), (3.2), and (3.3). On the other hand, the payoff in the case of disagreement in the negotiation is determined through the defund game in Stage 2b, which is detailed later.

In the following part of this section, we describe the bargaining situation in game-theoretic fashion. The bargaining model is characterized by the feasible set of their payoffs as well as the payoffs in the case of failure of negotiations. We assume that the bargaining environment satisfies Nash's four axioms. Thus, we use the Nash solution as a solution concept for the Stage 1 bargaining problem.

3.3.1 Feasible set

One of the essential components of the bargaining problem is the feasible set of the players' payoffs when all the possibilities of coordination has been considered. Here, we assume that the players can coordinate and negotiate the management policy of the firm, which is well represented by the value of α, and that they have full knowledge regarding the market outcome after agreeing on the management policy. Thus, with the basic assumption of the *free disposal of utility*, the feasible set of payoffs through bargaining is defined as follows:

$$\begin{aligned} A :&= \{(u_g, u_p) \in \mathbb{R}^2 \ : \exists \alpha \in [0,1] \text{ such that } U_g(\alpha) \geq u_g \text{ and } U_p(\alpha, s) \geq u_p\} \\ &= \{(u_g, u_p) \in \mathbb{R}^2 \ : \exists \alpha \in [\alpha_p, \alpha_g] \text{ such that } U_g(\alpha) \geq u_g \text{ and } U_p(\alpha, s) \geq u_p\} \end{aligned} \tag{3.5}$$

The second equality holds because of the fact that the *strong* pareto frontier of the payoffs U_g and U_p is realized at $\alpha \in [\alpha_p, \alpha_g]$.

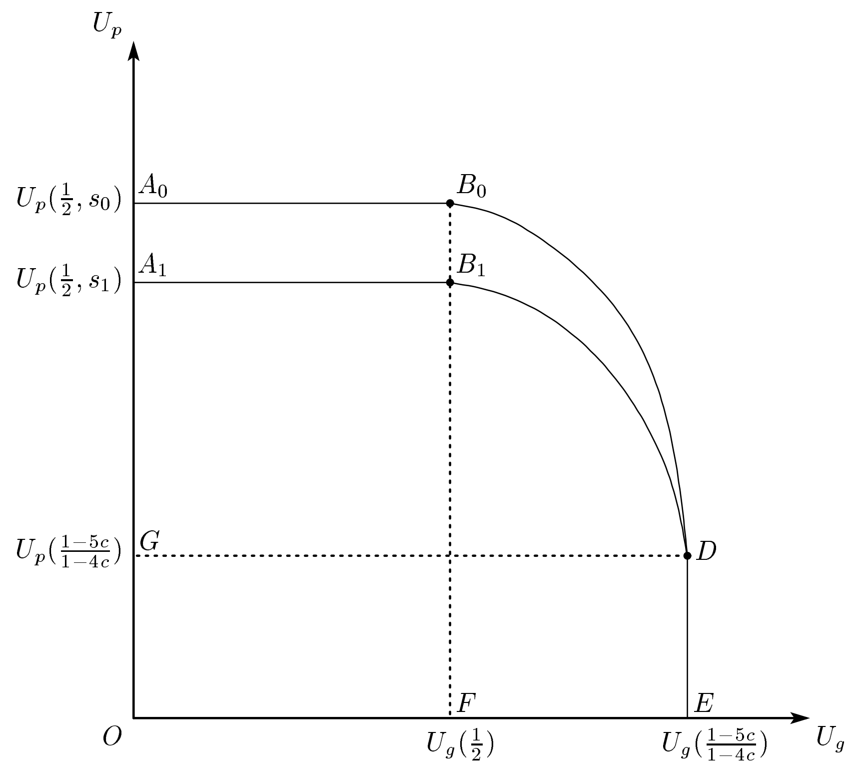

Figure 3.1: Feasible set A ($s_1 > s_0$)

As the following lemma shows, the feasible set A has some desirable property in the context of the bargaining problem.

Lemma 3.1. *The feasible set of our bargaining problem, A, is a convex set under Assumption 3.1.*

Proof: See Appendix.

The frontier of the feasible set A is attained when $\alpha \in [1/2, (1-5c)/(1-4c)]$. The slope of the frontier is smooth not only in the interior of the interval $[1/2, (1-5c)/(1-4c)]$ but also at the endpoint of the interval since $dU_p/dU_g(1/2) = 0$ and $dU_p/dU_g \to -\infty$ as $\alpha \nearrow (1-5c)/(1-4c)$ (see Figure 3.1). Note that an increase in s from s_0 to s_1 contracts the feasible set A in the vertical direction. This is because an increase in s implies that the private capitalist receives less dividend whereas an allocation of dividends does not influence social welfare.

3.3.2 The defund game

The other component of the bargaining problem pertains to players' payoffs when the negotiation breaks down. These payoffs are determined in the defund game formulated as follows. After the breakdown of negotiations, the government and the private capitalist face a problem regarding whether they should continue operating the business of the partially privatized firm or defund and liquidate it. In the case of defunding the firm, the partially privatized firm is wound up and the money invested is returned to both owners. Subsequently, the owners invest the refunded money in other investment avenues. In this case, the private capitalist obtains

$$b_p = \hat{b}_p(s) = r_p(1-s)K, \tag{3.6}$$

where K represents the total amount of investment in the firm and r_p, the return rate on other investments. Since the firm is liquidated, the remaining private firm 1 monopolizes the market. Therefore, social welfare is the sum of the welfare in private monopoly and the returns from the investments for both parties. The government's payoff b_g is given by

$$b_g = \hat{b}_g(s) = W_M + r_g sK + r_p(1-s)K = \frac{3}{8} + [r_g s + r_p(1-s)]\,K, \tag{3.7}$$

where $W_M = 3/8$ represents welfare in private monopoly and r_g, the return rate on public investment. We do not make any assumptions on the relationships between two return rates, r_p and r_g. In short, in this chapter, r_p and r_g are not always equalized.[9]

In the case that they decide to continue operating the firm, the majority party totally controls the management of the firm by resorting to the majority rule of the general shareholders meeting because he or she has already failed to coordinate with his opponent on the management of the firm. Thus, if the private capitalist is the majority party ($s < 0.5$), the payoffs e_i^p for party $i = p, g$ are

$$e_p^p = U_p(0) \quad \text{and} \quad e_g^p = U_g(0)$$

respectively.[10] On the other hand, when the government is the majority party ($s > 0.5$), the

[9]Generally, public investment in infrastructure projects and public utilities is less profitable than private investment; however, it is important in facilitating industries or securing people's lives. Thus, even if the return rate on public investment r_g is lower than that on private investment, public investment must be persisted with as long as the government has funds for investment. Moreover, r_g can be higher than r_p, because people might attach a higher value on a public investment, and this appraisal might raise the value measured in money, i.e., r_g. Of course, r_p is tantamount to r_g when the government can trade its share in the firm freely. Such trade may be feasible if the government sees its investment in perspective of profitability.

[10]From the theoretical viewpoint, considering the cases where $e_p^p = U_p(\alpha_p)$ and $e_p^g = U(\alpha_g)$ implies that the disagreement point exists on the bargaining frontier. Then, such negotiation always breaks down. In addition, the government's choice $\alpha = \alpha_g$ when the negotiation breaks down means that only the government can make a managerial incentive contract *à la* Fershtman and Judd (1987) and Sklivas (1987) with a manager. In the case where such managerial delegation is feasible, however, the government does not have any incentive to privatize its public firm. One of the purposes of our chapter is to investigate the plausibility of the assumption of Matsumura (1998) who analyzes parial privatization. Thus, to be consistent with Matsumura's model, we preclude the case where $e_p^g = U(\alpha_g)$. For the similar reason, we preclude the case where $e_p^p = U_p(\alpha_p)$, too.

payoffs e_i^g for party $i = p, g$ are

$$e_p^g = U_p(1) \quad \text{and} \quad e_g^g = U_g(1).$$

We now explain how the defund game is played between the two parties. In the defund game, each player simultaneously chooses either to continue operating the firm or to defund it. To simplify the exposition, we assume that when either of the players chooses to defund the firm, another player inevitably follows his partner's decision.[11]

Case I: $s < 0.5$

	Continue (C)	Defund (D)
Continue (C)	$e_p^p,\ e_g^p$	$b_p,\ b_g$
Defund (D)	$b_p,\ b_g$	$b_p,\ b_g$

Table 3.1: Defund Game: $s < 0.5$

The payoff matrix for the defund game in the case of $s < 0.5$ is described in Table 3.1. Only in the case when both parties choose to continue (C) do they obtain the payoffs of continuation of the firm; otherwise, they obtain the payoffs of defunding. For simplicity, we assume that when a player is indifferent between selecting (C) and (D), he chooses (C). Then, (C, C) is an equilibrium when the following two conditions hold:

$$e_p^p \geqq b_p \iff \frac{1}{9}(1-2c)^2 \geqq r_p K \tag{3.8}$$

$$e_g^p \geqq b_g \iff \frac{1}{18}(8-8c+11c^2) \geqq \frac{3}{8} + (sr_g + (1-s)r_p)\, K \tag{3.9}$$

On the other hand, if either of the conditions is not satisfied, the equilibrium payoffs are (b_p, b_g).

Case II: $s > 0.5$

Table 3.2 presents the payoff matrix for the defund game with $s > 0.5$. Similar to the defund game with $s < 0.5$, (C, C) is an equilibrium only if $e_p^g \geqq b_p$ and $e_g^g \geqq b_g$ hold. In fact, (C, C) does not become an equilibrium because $e_p^g = 0 < b_p$ always holds (as long as $s < 1$), and one of the conditions is not satisfied. Thus, in this case, the equilibrium payoffs are (b_p, b_g).

[11] Suppose that one owner chooses to defund and the other chooses to continue. In this case, only the former owner has to start his business with only his share of the capital, because the money that the latter owner invested should be returned. This might make it impossible for his firm to produce goods with the same technology as before; in other words, the firm might encounter extremely high marginal costs. As a result, it might not be able to continue production anymore. In order to exclude such an extreme case, we adopt this assumption.

	Continue (C)	Defund (D)
Continue (C)	$e_p^g,\ e_g^g$	$b_p,\ b_g$
Defund (D)	$b_p,\ b_g$	$b_p,\ b_g$

Table 3.2: Defund Game: $s > 0.5$

Case III: $s = 0.5$

In the case of each party having an equal share, even when both the players choose to continue operating the firm, they cannot reach an agreement regarding the management of the firm and neither of the parties can enforce its objective. Thus, we assume that they inevitably defund the firm, and the equilibrium payoffs are (b_p, b_g).

Let the disagreement point of the bargaining be the equilibrium payoff of the defund game described above and denoted by $d = (d_p, d_g)$. From the observation of the above three cases, we obtain the following lemma.

Lemma 3.2. *A disagreement point $d = (d_g, d_p)$ of the bargaining is given as follows:*

$$(d_g, d_p) = \begin{cases} (e_p^p, e_g^p) & \text{if } s < 0.5 \text{ and if (3.8) and (3.9) are satisfied} \\ (b_p, b_g) & \text{otherwise.} \end{cases}$$

3.3.3 A bargaining problem and the Nash solution

The two parties bargain over which point in A they realize, where each point in the frontier of A has a one-to-one correspondence with the value of weight α, with disagreement payoff $d = (d_g, d_p)$ being their returns in the case of failure of negotiations. In other words, this is a situation where when each of them can enforce the payoffs of d, they explore a better outcome through their coordination. Thus, when there does not exist a bargaining outcome that is more beneficial to both as compared to their respective disagreement payoffs, there is no room for bargaining.

When the disagreement point is (e_p^p, e_g^p), it can be easily verified that $d \in A$ because A is a convex set and $0 < \alpha_p < \alpha_g < 1$. Thus, in this case, bargaining between the two parties takes place. On the other hand, when the disagreement point is (b_p, b_g), whether or not d is included in A depends on the selection of the parameters. However, the following lemma shows that this is achieved only by the restriction on the value of the capital K.

Lemma 3.3. *When K is relatively small in the sense that K is smaller than some upper bound $\bar{K} > 0$, $(b_p, b_g) \in A$.*

Proof: See Appendix.

A pair (A, d) represents a *bargaining problem for the partially privatized firm's objective*. In order to make this bargaining problem plausible, we assume that the capital K is smaller than the upper bound $\bar{K}$ in Lemma 3.3. We use the Nash bargaining solution defined below as our solution concept for the bargaining problem.

Definition 3.1. *The Nash bargaining solution (U_g^*, U_p^*) is defined by the solution for the following maximization problem:*

$$\max\ (u_g - d_g)(u_p - d_p)\ \textit{ s.t. } (u_g, u_p) \in A \textit{ and } (u_g, u_p) \geq d. \tag{3.10}$$

Lemmas 3.1 and 3.3 on the feasible set and the disagreement point assure the existence and the uniqueness of the Nash solution. The Nash solution (U_g^*, U_p^*) is simply connected to the agreed value of α. Let α^* denote the solution of the following maximization problem:

$$\max\ (U_g(\alpha) - d_g)(U_p(\alpha, s) - d_p)\ \text{ s.t. } \alpha \in [1/2, (1-5c)/(1-4c)]. \tag{3.11}$$

Since (U_g^*, U_p^*) is located in the frontier of A due to the strong pareto efficiency of the Nash solution, $(U_g(\alpha^*), U_p(\alpha^*, s)) = (U_g^*, U_p^*)$ holds. Thus, α^* is the agreed value of the management policy of the firm through negotiations and is affected by the feasible set A and the disagreement point d.

In our setting, maximization problem (3.11) has an interior solution. Thus, the first-order condition yields

$$U_g'(\alpha)(U_p(\alpha, s) - d_p) + \frac{\partial U_p}{\partial \alpha}(U_g(\alpha) - d_g) = 0 \tag{3.12}$$

at $\alpha = \alpha^*(s)$.

3.3.4 The comparative statics of α^*

In this subsection, we examine how the agreed value α^* is affected by the variations in the feasible set A and the disagreement point d caused by the change in share s. The reason for focusing on the parameter s is that it is extensively considered in literature as the device that controls the objective of the partially privatized firm. Specifically, Matsumura (1998) demonstrates that partial privatization is better than both full privatization and full nationalization, and further shows that welfare maximization is attained by controlling the share s, under the assumption that α is positively correlated with s. Thus, the purpose of this subsection is to check the validity of the assumption of Matsumura (1998) in our bargaining model.

Recall that some parameters — r_p, r_g, K, and s — change the disagreement point d, as seen in Lemma 3.2. Then, we focus on how different the results are under two disagreement points $d = (e_p^p, e_g^p)$ and $d = (b_p, b_g)$. First, the result under the former disagreement point is presented as Proposition 3.1.

Proposition 3.1. *Under Assumption* 1, $s < 0.5$, *(3.8), and (3.9), there holds*

$$\alpha^{*\prime}(s) = 0,$$

and the agreed value of α *is*

$$\alpha_0 := \alpha^*(s) = \frac{31 - 146c - \sqrt{97 - 1084c + 3076c^2}}{2(18 - 76c)}.$$

Proof: See Appendix.

We should note that α_0 is decreasing in c. Differentiating α_0 with respect to c,

$$\frac{d\alpha_0}{dc} = -\frac{2\left(-149 + 886c + 17\sqrt{97 - 1084c + 3076c^2}\right)}{(9 - 38c)^2\sqrt{97 - 1084c + 3076c^2}} < 0, \quad \text{for } c \in \left(0, \frac{1}{6}\right).$$

An improvement in the unit cost results in large marginal benefits from the expansion of the privatized firm's market share, as compared to the marginal loss. As a result, the private capitalist agrees to privatized firm's more aggressive actions.

Proposition 3.1 states that the government does not have the discretion to control α through buying or selling its shares if the size of its capital in the privatized firm is relatively small and the private capitalist still holds the majority of shares. Therefore, in this case, further privatization cannot influence the privatized firm's managerial policy and thus its profits and social welfare. This result stems from the fact that the capital received by government after the breakdown of the negotiations is reallocated to consumers in a lump-sum manner.

Indeed, the disagreement point need not be independent of s if this capital is used for another investment, and thereafter, the return is redistributed to consumers. Disagreement point $d = (b_p, b_g)$ corresponds to this situation. The business in which public or privatized firms engage is often strongly public in nature, and thus, it might be required that the size and scale of these firms be relatively large for some reasons such as sustaining perpetual business and securing universality of services. For such privatized firms, assumptions with respect to $d = (b_p, b_g)$ are satisfied. Further, the following proposition suggests that the government can have control over α.

Proposition 3.2. *Assume that* $(d_p, d_g) = (b_p, b_g)$. *Under Assumption* 1, *there holds*

$$r_g \gtreqqless r_p \quad \Longleftrightarrow \quad \alpha^{*\prime}(s) \gtreqqless 0.$$

Proof: See Appendix.

Proposition 3.2 shows that a buyback by the government (i.e., an increase in s or partial nationalization) raises the weight on welfare α when the return rate of public investment r_g is higher than that of investment by the private capitalist r_p. Conversely, if the public investment is less beneficial than the private investment, then partial nationalization lowers a government's influence on the objective function of the partially privatized firm. Matsumura (1998) assumes that α is positively related to s. However, our proposition implies that when negotiations between the government and the private capitalist are considered, the assumption need not hold.

3.4 Welfare implications

The results obtained in the previous section demonstrate that based on our bargaining model, it is difficult to support the assumption posed by Matsumura (1998) wherein α is positively related to s. It seems that our bargaining model merely allows the relationship between α and s to head in a different direction than that in Matsumura (1998). However, it plays an important role in examining the welfare implication. In our model, bargaining between the two parties occurs only when firm 0 is partially privatized, i.e., $s \in (0, 1)$. This implies that the welfare function is discontinuous at $s = 0$ and $s = 1$. In this section, we argue whether or not this discontinuity changes the optimal privatization policy.

The model considered here is a multistage game similar to the one analyzed in the previous section. The difference is that we add a governmental choice stage of partial privatization before proceeding to the multistage game outlined in the previous section. Thus, in the first stage, the government chooses the portion of the share of the public firm that is sold to the private capitalist. In other words, the government chooses its ratio s in the partially privatized firm. Therefore, given the share s, the multistage game considered in the previous section follows. Thus, the government in the first stage selects some ratio of partial or full privatization instead of full nationalization, only when such a choice is beneficial with respect to social welfare. For analysis, we consider the following assumption.

Assumption 3.2. *The capital K satisfies the following condition:*

$$K \leq \frac{(3-14c)(1-2c)}{32\ \max\{r_p, r_g\}}.$$

This assumption implies that $U_p(\alpha_p) \geq b_p$ and $U_g(\alpha_p) \geq b_g$.[12] $U_p(\alpha_p) \geq b_p$ is an individual rationality condition. If this condition is violated, the private capitalist loses the incentive to hold any share in the privatized firm. On the other hand, $U_g(\alpha_p) \geq b_g$ is more restrictive than the government's individual rationality condition. Nevertheless, the alleviation of competition accompanied by the liquidation of the privatized firm can deteriorate social welfare drastically, and the return of public investment might not be able to compensate this drastic welfare loss. Thus, it appears natural to consider that competition provides sufficient welfare even though production by the partially privatized firm is small due to a lower α.

In Figure 3.1, the disagreement point $d = (b_p, b_g)$ is included in A_0B_0FO (or A_1B_1FO) under Assumption 2. This area is involved with the private capitalist's higher payoffs. In this advantageous situation for the private capitalist, the optimal policy is given in Proposition 3.3.

Proposition 3.3. *Under Assumptions 1 and 3.2, the following hold:*

(i) *the government chooses partial privatization when* $1/10 < c < 1/6$*;*

(ii) *the government does not privatize the public firm at any level when* $\frac{\sqrt{33}-5}{8} < c \leq 1/10$*;*

(iii) *when* $c \leq \frac{\sqrt{33}-5}{8}$*, if (3.8) and (3.9) are satisfied, the government sells more than half its shares, whereas if not, then the government does not privatize the public firm at any level.*

Proof: See Appendix.

Suppose that the unit cost of the privatized firm is relatively high. In this case, the marginal benefits from a decrease in price due to higher production by the privatized firm is lower than marginal losses from an increase in total costs. Hence, the government partially privatizes the

[12] $U_p(\alpha_p) \geq b_p$ and $U_g(\alpha_p) \geq b_g$ are respectively given as

$$K \leq \frac{(1-2c)^2}{8r_p} \quad \text{and} \quad K \leq \frac{(3-14c)(1-2c)}{32\{sr_g + (1-s)r_p\}}.$$

Thus, Assumption 3.2 implies the latter condition. Moreover, simple calculation yields

$$\frac{(1-2c)^2}{8r_p} - \frac{(3-14c)(1-2c)}{32\ \max\{r_p, r_g\}} > 0.$$

firm and reallocates the output of the firm to that of the other firm, which, in turn, enhances the social welfare. In the case where the unit cost is low, it is possible that full nationalization is more welfare enhancing than certain levels of partial privatization. This is true if the firm has enough capital. If not, the government can achieve higher welfare by selling half its shares than by fully nationalizing the firm. In this case, the bargaining solution α^* is α_0. Since α_0 is decreasing in c, α^* is close to the most desirable level of the government α_g, when the unit cost is very low.

It may be plausible that this result relies largely on Assumption 3.2, since this assumption provides the private capitalist with some advantage in the disagreement payoff and thus in the bargaining; this, in turn, lowers α^*. Then, let us consider the following assumption.

Assumption 3.3. *The capital K satisfies the following condition:*

$$K \leq \frac{(1-4c)^2}{8\ \max\{r_p, r_g\}}.$$

This assumption implies that $U_p(\alpha_g) \geq b_g$ and $U_g(\alpha_g) \geq b_g$. In Figure 3.1, the disagreement point $d = (b_p, b_g)$ is in $GDEO$, which provides the government with an advantage in bargaining. However, we can obtain the same result as in Proposition 3.3 even if we impose Assumption 3.3, instead of Assumption 3.2. This is summarized in Proposition 3.4.

Proposition 3.4. *Under Assumptions 1 and 3.3, all the claims in Proposition 3.3 hold.*

Proof: See Appendix.

As shown in Propositions 3.3 and 3.4, in contrast with Matsumura (1998) and other papers, except for Matsumura and Kanda (2005), partial privatization is not always desirable, depending on the partially privatized firm's marginal cost and the disagreement point. If the marginal cost is relatively high, then partial privatization is desirable. However, the government should not sell any shares in the public firm (fully nationalized firm) if the marginal cost is in the middle range. Further, provided that the marginal cost is relatively low, two possibilities can be considered. One is that the firm should be partially privatized, when the size of the firm's capital is low. The other is that it should be fully nationalized if the capital is not low.

3.5 Concluding remarks

In this chapter, we examine the behavioral principle of a firm owned by different types of owners, and in particular, we analyze how this principle is determined. For this analysis, we utilize a mixed duopoly where a private firm competes against a partially privatized firm jointly owned by the welfare maximizing government and a profit maximizing private capitalist. This model is employed in many existing studies. Such studies usually assume that the government can control the objective function of the partially privatized firm by adjusting its shares in the firm, ignoring the possibility of the private capitalist opposing the government's claims and the process of determination of the firm's objective function. Further, existing studies also assume that if the government increases its shares, it can more strongly reflect its objective, that is, social welfare, in the objective function of the partially privatized firm. However, we show that these assumptions need not be adequate when both owners negotiate over the objective function of the firm. Specifically, the effect of an increment in the shares that the government holds on the objective function of the privatized firm relies on the difference between the return rates of public and private investments. If the former rate is higher than the latter, then the weight on social welfare in the privatized firm, α, becomes larger as the government's share s becomes large. Interestingly, if the return rate of public investment is lower than that of private investment, the result is reversed.

In addition, we find that in contrast with Matsumura (1998), partial privatization is not always desirable, depending on the partially privatized firm's marginal cost and the disagreement point. If the marginal cost is relatively high, then partial privatization is desirable. However, the government should not sell any shares in the public firm (fully nationalized firm) if the marginal cost lies in the middle range. Further, provided that the marginal cost is relatively low, two possibilities can be considered. One is that the firm should be partially privatized when the size of the firm's capital is low. The other is that it should be fully nationalized if the capital is not low.

Although our model includes many insights, we must admit that the model depends crucially on some queer assumptions. First, we regard the shareholders' meetings in the partially privatized firm as a Nash bargaining process. Then, we assume that both public and private sectors determine a parameter α in the process. This means that one owner contends with the other over the real number from 0 to 1. This situation seems unrealistic, and considering the setting might be nonsense. One way to avoid such a somewhat ridiculous situation is that both owners negotiate over the outputs of the partially privatized firm for each output of the private firm. Unfortunately, the formulation requires the highly intricate and complex caluculation, thus it is difficult to solve it. Nevertheless, if we consider only the solvability, we could model such a negotiation as follows; the private capitalist remains the profit-maximizer, but the public sector is assumed to be total-output-maximizer.[13] This formulation would warrant the solvability of the model. Our fo-

[13]For the paper which assumes that public enterprises are total output maximizers, see Nett (1994). However, the

cus is, however, on the consistency with Matsumura's (1998) model. To this end, we apply the setting presented in the previous section without selecting such a solvable model.

Second, we assume that the partially privatized firm's objective function is represented as the weighted average of owners' objectives, and that both owners determine the weight α. These assumptions would be rebutted by the readers who think that such determination is feasible in the private firm. Such a misgiving would emanate from the presumption that both owners agree to the prior arrangement that the partially privatized firm maximizes the weighted average of each owners' objective. It appears weird, in particular, in the sense that our aim is to endogenize the partially privatized firm's objective function, but the form of the objective function is restricted in advance. This restriction provokes and highlights the peculiarity of asymmetry between partially privatized and private firms with regard to the way to determine the objective function. One possible means to elude this kind of peculiarity and to conserve the consistency with the existing works may be voting. Specifically, owners, including public and private sector, vote the outputs which are desirable for them as the rival firm's output and the voting rights represented as the shares in the partially privatized firm are given. However, it may not be so easy to incorporate voting into the model, with preserving the possibility of partial privatization. Indeed, under simple models, the shareholders, who have the shares, win by the majority rule, and thus partial privatization cannot be supported. Therefore, some ideas should be required to discuss the partial privatization by using a vote.

Third, we posit that the privatized firm can liquidate when the negotiation breaks down. This assumption may be of a paucity of validity from the viewpoint of the reality. Of course, we can consider the case where the firm becomes inactive at the threat point, as assumed by many existing studies on labor-management negotiation or other contexts. However, this case boils down to the affinity to the result of Proposition 3.1, that is, the consequence that the disposal of shares which the public sector holds does not affect weight α. Thus, taking such a threat point seems invalid and inappropriate when we expolre the effectiveness of partial privatization.

Although there are some defects as described above, our model possesses some contribution to the literature in that it can be extended in many directions as follows. The first direction pertains to the market structure. We assume that there is one private firm in the market. This is a slightly restrictive assumption. Matsumura and Kanda (2005) analyzes mixed oligopoly where the free entry of private firms is allowed and shows in their study that the government should fully nationalize the public firm. It would be interesting to examine how the results of Matsumura

alalysis using such a setting is rare in the mixed oligopoly theory.

and Kanda (2005) change if bargaining between the government and a private capitalist is taken into consideration. Secondly, we neglect an incentive for the private capitalist to sell or buy shares in the privatized firm. The effectiveness of the privatization policy would be limited if the private capitalist does not want to acquire shares more than a certain level below a given price. This would require the introduction of a stock market and a model of how different owners may exchange shares in their firm. Finally, our model can be applied to the merger between a private firm owned by the profit maximizing private sector and a public firm owned by the welfare maximizing government.

Appendix

Proof of Lemma 3.1

U_g and U_p satisfy

$$U_g'(\alpha) = \frac{(1-2c)\{1-\alpha-c(5-4\alpha)\}}{(3-2\alpha)^3} \quad \text{and} \quad \frac{\partial U_p}{\partial \alpha} = \frac{(1-2c)^2(1-s)(1-2\alpha)}{(3-2\alpha)^3}$$

respectively. The slope of the bargaining frontier is

$$\frac{dU_p}{dU_g} = \frac{\partial U_p/\partial \alpha}{dU_g/d\alpha},$$

if $dU_g/d\alpha \neq 0$. Moreover,

$$U_g''(\alpha) = \frac{(1-2c)(3-18c-4\alpha+16c\alpha)}{(3-2\alpha)^4} \quad \text{and} \quad \frac{\partial^2 U_p}{\partial \alpha^2} = -\frac{8(1-2c)^2(1-s)\alpha}{(3-2\alpha)^4}.$$

Then, as we know, U_g and U_p have the following relationship:

$$\frac{d^2U_p}{dU_g^2} = \frac{1}{(dU_g/d\alpha)^2}\left[d^2U_p/d\alpha^2 - \frac{(dU_p/d\alpha)(d^2U_g/d\alpha^2)}{dU_g/d\alpha}\right].$$

Based on the above relationships, we obtain

$$\frac{d^2U_p}{dU_g^2} = -\frac{1}{U_g'(\alpha)^3} \cdot \frac{(1-6c)(1-2c)^3(1-s)}{(3-2\alpha)^6}.$$

The sign of this second derivative is opposite to that of $dU_g/d\alpha$. Thus, from (3.4), dU_p/dU_g is positive and $\partial^2U_p/\partial U_g^2$ is negative if $\alpha \in [0, 1/2)$. If $\alpha \in (1/2, (1-5c)/(1-4c))$, dU_p/dU_g and $\partial^2U_p/\partial U_g^2$ are negative. Finally, if $\alpha \in ((1-5c)/(1-4c), 1]$, dU_p/dU_g and $\partial^2U_p/\partial U_g^2$ are positive. Moreover, $dU_p/dU_g \to \infty$ as $\alpha \searrow (1-5c)/(1-4c)$ and $dU_p/dU_g \to -\infty$ as $\alpha \nearrow (1-5c)/(1-4c)$, and $dU_p/dU_g(1/2) = 0$.

Define a function $f : (-\infty, U_g((1-5c)/(1-4c))] \to \mathbb{R}$ as follows. For $x \in (U_g(1/2), U_g((1-5c)/(1-4c))]$,

$$f(x) = U_p(\alpha(x)), \text{ where } \alpha(x) \text{ is such that } U_g(\alpha(x)) = x \text{ and } 1/2 \leqq \alpha(x) \leqq (1-5c)/(1-4c)$$

and for $x \in (-\infty, U_g(1/2)]$,

$$f(x) = U_p(1/2).$$

By the definition of A, the feasible set of the bargaining problem is characterized by the function f as follows:

$$A = \{(x, y) \in \mathbb{R} : x \leqq U_g((1-5c)/(1-4c)), y \leq f(x)\}$$

Since dU_p/dU_g and $\partial^2 U_p/\partial U_g^2$ are negative when $\alpha \in (1/2, (1-5c)/(1-4c))$, $f' < 0$ and $f'' < 0$ when $x \in (U_g(1/2), U_g((1-5c)/(1-4c)))$. Thus, we have the desired result. ■

Proof of Lemma 3.3

First, the disagreement payoffs must be less or equal to their maximum payoff. Thus, the following conditions hold:

$$b_p \leqq U_p(\alpha_p) = U_p(1/2) \iff 8r_pK \leqq (1-2c)^2 \tag{3.13}$$

$$b_g \leqq U_g(\alpha_g) = U_g(\frac{1-5c}{1-4c}) \iff 8\left(sr_g + (1-s)r_p\right)K \leqq 1 - 8c + 16c^2. \tag{3.14}$$

In addition to these, one of the loose sufficient conditions that d is included in A is that d is located at a position in the area under the line intersecting $(U_p(\frac{1-5c}{1-4c}), U_g(\frac{1-5c}{1-4c}))$ and $(U_p(1/2), U_g(1/2))$. Thus, we obtain

$$\begin{aligned} b_g <& \frac{U_g(\frac{1-5c}{1-4c}) - U_g(1/2)}{U_p(\frac{1-5c}{1-4c}) - U_p(1/2)} \left(b_p - U_p(1/2)\right) + U_g(1/2) \\ \iff& \quad 8\left(sr_g + (1-s)r_p\right)K + 2r_pK < 1 - 6c + 8c^2. \end{aligned} \tag{3.15}$$

■

Proof of Proposition 3.1

This proposition can be easily derived. From Lemma 3.2 and the definitions of U_g, U_p, e_g^p, and e_p^p, the maximization problem for our bargaining can be rewritten as

$$\begin{aligned} \max \quad & (1-s)(W^*(\alpha) - W^*(0))(\Pi_0^*(\alpha) - \Pi_0^*(0)) \\ \text{s.t.} \quad & \alpha \in [1/2, (1-5c)/(1-4c)]. \end{aligned}$$

The first-order condition for this problem is given as

$$0 = \frac{(1-2\alpha)^3\alpha\,\{2(9-38c)\alpha^2-(31-146c)\alpha+12(1-5c)\}}{18(3-2\alpha)^5}.$$

The agreed value α^*, which satisfies this equation and is included in $[\alpha_p, \alpha_g]$, is α_0. ■

Proof of Proposition 3.2

Provided that $d = (b_p, b_g)$. For convenience, we define

$$\widehat{V}(\alpha, s) := (U_g(\alpha) - \hat{b}_g(s))(U_p(\alpha, s) - \hat{b}_p(s)).$$

Then, by implicit function theorem, we have $sgn\{\alpha^{*\prime}(s)\} = sgn\{\partial^2\widehat{V}/\partial s\partial\alpha\}$. Notice that

$$\frac{\partial U_p}{\partial s} = -\Pi_0^*(\alpha) = -\frac{1}{1-s}\cdot U_p(\alpha, s) \quad \text{and} \quad \frac{\partial^2 U_p}{\partial s\partial\alpha} = -\Pi_0^{*\prime}(\alpha) = -\frac{1}{1-s}\cdot\frac{\partial U_p}{\partial\alpha}.$$

By using these, we can rewrite $\partial^2\widehat{V}/\partial s\partial\alpha$ evaluated at $\alpha = \alpha^*(s)$ as follows:

$$\begin{aligned}
\left.\frac{\partial^2\widehat{V}}{\partial s\partial\alpha}\right|_{\alpha=\alpha^*(s)} &= U_g'(\alpha^*(s))\left(\frac{\partial U_p}{\partial s} + \hat{b}'(s)\right) + \frac{\partial^2 U_p}{\partial s\partial\alpha}\left(U_g(\alpha^*(s)) - \hat{b}_g(s)\right) - \frac{\partial U_p}{\partial\alpha}\cdot\hat{b}_g'(s),\\
&= -\frac{1}{(1-s)}\Big\{U_g'(\alpha^*(s))\left[(1-s)\Pi_0^*(\alpha^*(s)) + (1-s)r_pK\right]\\
&\quad + (1-s)\cdot\Pi_0^{*\prime}(\alpha^*(s))\left(U_g(\alpha^*(s)) - \hat{b}_g(s)\right)\Big\} + \frac{\partial U_p}{\partial\alpha}\cdot\hat{b}_g'(s),\\
&= -\frac{1}{1-s}\cdot\left[U_g'(\alpha^*(s))\left(U_p(\alpha^*(s), s) - \hat{b}_p(s)\right) + \frac{\partial U_p}{\partial\alpha}\left(U_g(\alpha^*(s)) - \hat{b}_g(s)\right)\right]\\
&\quad + \frac{\partial U_p}{\partial\alpha}\cdot K(r_g - r_p),\\
&= \frac{\partial U_p}{\partial\alpha}\cdot K(r_g - r_p), \qquad \text{based on (3.12)}
\end{aligned}$$

Since $\partial U_p/\partial\alpha \geq 0$ for $\alpha \in [\alpha_p, \alpha_g]$, we obtain $sgn\{\alpha^{*\prime}(s)\} = sgn(r_g - r_p)$. ■

Proof of Proposition 3.3

First of all, we prove that partial privatization is desirable in the case where $1/10 < c < 1/6$. Simple calculation yields

$$\begin{aligned}
&W^*(\alpha_g) > W^*(1) \geq W^*(\alpha_p) > W^*(0), && \text{if } c \leq \frac{1}{10}, && (3.16)\\
&W^*(\alpha_g) > W^*(\alpha_p) > W^*(1) \geq W^*(0), && \text{if } \frac{1}{10} < c \leq \frac{1}{8},\\
&W^*(\alpha_g) > W^*(\alpha_p) > W^*(0) > W^*(1), && \text{if } \frac{1}{8} < c < \frac{1}{6}.
\end{aligned}$$

Thus, from the fact that $\alpha^*(s) \in [\alpha_p, \alpha_g]$ and $W^{*\prime}(\alpha) > 0$ for any $\alpha \in [\alpha_p, \alpha_g]$, we can find that partial privatization gives rise to higher welfare than does full nationalization or privatization.

However, for $c \leq 1/10$, we cannot conclude that partial privatization is desirable based on (3.16). Therefore, by using another approach, we show that full nationalization is the best policy for firm 0's marginal cost in the relevant range. Since the welfare function W^* is continuous, there exists $\hat{\alpha} \in [\alpha_p, \alpha_p]$ such that $W^*(1) = W^*(\hat{\alpha})$, and this α is equal to $(1-8c)/(1-6c)$. We prove the desirability of full nationalization (i.e., $W^*(\alpha^*(s)) < W^*(1)$) for $d = (b_p, b_g)$ by showing that $\alpha^*(s) < \hat{\alpha}$ for any $s \in (0, 1)$. By the same procedure, we also prove the results in the case where $d = (e_p^p, e_g^p)$. For this purpose, we rearrange the first-order condition (3.12) and obtain

$$A + B\alpha^*(s) + C\alpha^*(s)^2 + D\alpha^*(s)^3 = 0, \tag{3.17}$$

where

$$\begin{aligned}
A &= 13 - 114c + 300c^2 - 248c^3 - 72(1-s)(2-7c)Kr_p - 72s(1-2c)Kr_g,\\
B &= -30 + 252c - 648c^2 + 528c^3 + 8(1-s)(51-156c)Kr_p + 240s(1-2c)Kr_g,\\
C &= 16 - 128c + 320c^2 - 256c^3 - 32(1-s)(11-31c)Kr_p - 224s(1-2c)Kr_g,\\
D &= 32(1-s)(3-8c)Kr_p + 64s(1-2c)Kr_g.
\end{aligned}$$

For convenience, we define the following function:

$$F(\alpha, s) := A + B\alpha + C\alpha^2 + D\alpha^3.$$

(i) When $d = (b_p, b_g)$ and $r_g \geq r_p$

In this case, from Proposition 3.2, we know that $\alpha^{*\prime}(s) \geq 0$. Then, we now show that $\lim_{s\to 1} \alpha^*(s) < \hat{\alpha}$. Converging s in $F(\alpha, s)$ to 1 and evaluating this at $\alpha = \hat{\alpha}$, we obtain

$$\lim_{s\to 1} F(\hat{\alpha}, s) = \frac{E_0}{(1-6c)^3}, \tag{3.18}$$

where

$$E_0 = 8Kr_g(1-2c)^2(1-10c) - (1-6c)(1 - 18c + 336c^2 - 3400c^3 - 12048c^4 + 8960c^5).$$

The sign of (3.18) relies on the sign of the numerator E_0 since the denominator $(1-6c)^3$ is positive in the relevant range of c.

$$\begin{aligned} E_0 &= 8Kr_g(1-2c)^2(1-10c) - (1-6c)(1-18c+336c^2-3400c^3-12048c^4+8960c^5), \\ &\leq 8 \cdot \frac{(3-14c)(1-2c)}{32r_g} \cdot r_g(1-2c)^2(1-10c) \\ &\quad - (1-6c)(1-18c+336c^2-3400c^3-12048c^4+8960c^5), \quad \text{(by Assumption 3.2)} \\ &= -\frac{1}{4}(1-28c+1212c^2-19392c^3+124976c^4-319808c^5+212800c^6). \end{aligned}$$

The right-hand side is negative for any $c \in (0, 1/10)$. Accordingly, from the second-order condition for our bargaining model, we have $\lim_{s\to 1}\ \alpha^*(s) < \hat{\alpha}$.

(ii) When $d = (b_p, b_g)$ and $r_p > r_g$

In this case, from Proposition 3.2, we know that $\alpha^{*\prime}(s) < 0$, and thus, we show that $\lim_{s\to 0}\ \alpha^*(s) < \hat{\alpha}$. Applying a procedure similar to that employed in (i), we find that

$$\lim_{s\to 0} F(\hat{\alpha}, s) = \frac{E_1}{(1-6c)^3},$$

where

$$E_1 = 8(1-2c)^3(1-11c)Kr_p - (1-6c)(1-18c+336c^2-3400c^3+12048c^4-8960c^5).$$

Further, based on Assumption 3.2, we find that

$$\begin{aligned} E_0 &\leq -\frac{1}{4}(1-25c+1174c^2-19208c^3+124544c^4+319312c^5-212576c^6), \\ &< 0. \end{aligned}$$

This implies that $\lim_{s\to 0}\ \alpha^*(s) < \hat{\alpha}$.

(iii) When $d = (e_p^p, e_g^p)$

Comparing $\hat{\alpha}$ and α_0 directly, we have

$$\begin{aligned} \hat{\alpha} - \alpha_0 &= \frac{5-108c+340c^2+(1-6c)\sqrt{97-1084c+3076c^2}}{4(1-6c)(9-38c)} \gtreqqless 0 \\ &\iff \quad c \lesseqqgtr \frac{\sqrt{33}-5}{8}. \end{aligned}$$

■

Proof of Proposition 3.4

We prove Proposition 5 using the same procedure as that used in the proof of Proposition 4. We only show the case where $d = (b_p, b_g)$, since the proof for the case wherein $d = (e_p^p, e_g^p)$ is independent of Assumptions 2 and 3.

(i) When $r_g \geq r_p$

Converging s in $F(\alpha, s)$ to 1 and evaluating this at $\alpha = \hat{\alpha}$, we obtain

$$\lim_{s\to 1} F(\hat{\alpha}, s) = \frac{E_2}{(1-6c)^3},$$

where

$$E_2 = 8(1-2c)^3(1-10c)Kr_g + (1-6c)(-1+18c-336c^2+3400c^3-12048c^4+8960c^5),$$

and based on Assumption 3.3,

$$\begin{aligned} E_2 &\leq 4c^2(57-1114c+7548c^2-19640c^3+13120c^4), \\ &< 0. \end{aligned}$$

This implies that $\lim_{s\to 1}\ \alpha^*(s) < \hat{\alpha}$.

(ii) When $r_p > r_g$

Converging s in $F(\alpha, s)$ to 0 and evaluating this at $\alpha = \hat{\alpha}$, we have

$$\lim_{s\to 1} F(\hat{\alpha}, s) = \frac{E_3}{(1-6c)^3},$$

where

$$E_3 = 8(1-2c)^3(1-11c)Kr_p - (1-6c)(1-18c+336c^2-3400c^3+12048c^4-8960c^5),$$

and based on Assumption 3.3,

$$\begin{aligned} E_3 &\leq -c(1+214c-4380c^2+29992c^3-78304c^4+52352c^5), \\ &< 0. \end{aligned}$$

This implies that $\lim_{s\to 0}\ \alpha^*(s) < \hat{\alpha}$. ■

Chapter 4

Interregional Mixed Duopoly

4.1 Introduction

During the recent wave of privatization, not only state-owned firms but also local public firms have been privatized. [1]Nevertheless, local public firms still exist in many developing countries as well as in developed countries. This is because they usually provide essential services such as natural gas, electricity, water, medical facilities, and education. In most cases, such goods and services are also provided by private firms. The purpose of this chapter is to investigate mixed markets wherein private and local public firms compete.

Competition among public and private firms has been studied in literature on mixed oligopolies (*e.g.,* DeFraja and Delbono, 1989). It usually assumes one country or one market in which one public firm and several private firms compete, and it examines the effect of the privatization of the public firm on social welfare. Thus, the literature has not established an appropriate model reflecting the behaviors of local public firms in a country comprising a number of regions or provinces. Certainly, a few previous studies such as Fjell and Pal (1996), Pal and White (1998), and Matsushima and Matsumura (2006) have investigated the effect of imports from foreign firms on the domestic mixed market. If we regard the domestic and foreign countries as provinces or counties, it appears that some works analyzed mixed markets, which include a local public firm. However, in the real world, local public firms in one region often supply goods and services to consumers in other regions. In fact, state (or city) universities, local airports, and city hospitals supply services to residents of other regions. For example, in Japan, Yokohama City University, which is owned and managed by Yokohama City, admits not only students who live in Yokohama but also those hailing from the other regions. Another example is Kobe Airport owned by Kobe City, which is a representative airport in the Kansai area of Japan. In this chapter, we establish a

[1]This chapter is based on Inoue, Kamijo and Tomaru (2009).

model wherein a local public firm in a region competes against a private firm and supplies goods and/or services to consumers who live outside the region.

For this purpose, we employ a Hotelling-type spatial model (Hotelling, 1929) in which the population is dispersed and each consumer has a specific personal address on the line with a length of unity (hence, the so-called linear city). In this model, a firm locates at a point on the line, and the purchase of goods from one of them involves transportation costs that vary according to the consumer's location. Since consumers have to incur the transportation costs of goods, they select a firm to purchase goods from, taking into account the transportation costs in addition to prices. Studies on mixed oligopolies using a spatial model have been conducted earlier (see, *e.g.*, Cremer et al., 1991; Matsumura and Matsushima, 2003, 2004; Matsushima and Matsumura, 2003, 2006). Cremer et al. (1991) conducted a pioneering work on spatial mixed oligopoly, in which they assumed that the state-owned and private firms exist in a linear city and decide their own locations and prices. We extend their model by dividing the city into two symmetric districts, Regions 1 and 2, each of which is run by a local government, and thus, a firm owned by the government is regarded as a local public firm. We assume that the local government of Region 1 owns the public firm, and the owners of the private firm reside in Region 2. In addition, we assume that the local public firm aims at maximizing local welfare in Region 1 and that the local welfare does not include the profit of the private firm.

In the above setting, we find that our model of location choice and price competition has multiple equilibria. In one equilibrium (henceforth, we refer to this equilibrium as E_1), the local public firm locates in the region run by the government, and the private firm locates outside the region. This equilibrium well fits hospitals, which exist ubiquitously regardless of whether they are owned by the public or private sector. In this equilibrium, the local public firm supplies goods and services to all the residents in Region 1. Moreover, this firm also provides goods to some of the residents in Region 2. In the other equilibrium (E_2), both firms locate in Region 1. In Japan, various universities including private and local public universities agglomerate in large cities such as Tokyo, Osaka, and Kobe. Such universities present an example of equilibrium E_2. Furthermore, in contrast to E_1, goods are supplied to a large number of the consumers in Region 1 by the private firm, and the local public firm monopolizes the demand of the residents in Region 2.

The results of our chapter are very peculiar compared to those of the existing works. d'Aspremont, Gabszewicz and Thisse (1979) show that in private duopoly, one firm is located at one endpoint of the linear city, and the other firm is located at the other endpoint. Cremer et al. (1991) investigated the mixed duopoly model wherein a private firm competes against a state-owned firm that maximizes the social welfare of the entire linear city. They show that one firm is located at point $1/4$ and the other is located at point $3/4$ of the city, which indicates that competition between the state-owned and private firms yields the first-best locational configuration. The difference between the result of our chapter and those of the existing works arises from the fact that the local public firm in our model takes into account only the benefits of residents in one region (Region 1). This implies that the local public firm has two incentives; one is to decrease the transportation costs of the residents in Region 1 and the other is to increase its profits from Region 2. Due to these two incentives, which do not appear in the existing works, our result differs from those of the existing works.

Other interesting features of the multiple equilibria are that equilibrium E_2 payoff-dominates equilibrium E_1, and that the social welfare of the entire city is larger in E_1 than in E_2. The reasons for these occurrences are as follows. In equilibrium E_2, the local public firm sets a higher price to earn higher profits from Region 2 since it monopolizes the demand of Region 2. As a result, not only the local public firm but also the private firm enjoys higher profits due to strategic complementarity in the price-setting stage. Since the profit of the public firm in E_2 is so large that it should increase the local welfare to a level higher than that in E_1, E_2 is payoff-dominant to E_1. Moreover, the residents in Region 2 incur higher transportation costs in E_2 due to the one-sided location of both firms, which results in lower social welfare in E_2 than in E_1.

As shown in Matsumura, Ohkawa and Shimizu (2005), in the context of the spatial competition, in the sequential-move model, the efficient equilibrium is chosen from among the multiple equilibria in the simultaneous-move model. We investigate the sequential location choice game in our setting. Similar to Matsumura et al. (2005), we find that E_2 is chosen in the sequential-move game, although it is not efficient from the social welfare viewpoint. Further, this is robust in the sense that E_2 is chosen regardless of whether the public firm is a leader or a follower.

Our results related to the order of moves have the following significances in the literature on mixed oligopoly as well as that on pure oligopoly. First, whether a public firm should become a leader or a follower has been discussed by several researchers in the context of mixed oligopoly such as Matsumura and Matsushima (2003) and Ogawa and Sanjo (2007). They show that the equilibrium location pattern is different between the public leadership and the private leadership in the location choice. In contrast to these studies, we show that the same equilibrium location pattern arises, regardless of whether the public firm is a leader or a follower. Second, the multiplicity of equilibria in the simultaneous location choice case are resolved in the case of sequential location choice in our model. That is, the sequential location choice serves as an equilibrium selection between E_1 and E_2. Similar results are observed in Matsumura et al. (2005) who consider a shipping model with quantity-setting in circular markets. Finally, we also consider the issue of the endogenous order of moves in mixed oligopoly by using the observable delay game of Hamilton and Slutsky (1990), for example, Pal (1998b) for Cournot competition and Bárcena-Ruiz (2007) for Bertrand competition. We show that in equilibrium, two types of Stackelberg competition (public leadership and public followership) arise.

There exist some studies that also consider public firms that supply goods to consumers outside their region without using a spatial model. Bárcena-Ruiz and Garzón (2005b) analyze the model where two regions (or countries) trade with one another and their governments strategically decide whether to privatize their public firms. They show that the decision of privatization and the trade patterns are determined by the difference in the marginal costs between the local public and private firms. Using a similar model, Bárcena-Ruiz and Garzón (2005a) analyze whether national governments should decide whether to privatize public firms or whether this decision should be delegated to a supra national authority. Since these two studies use non spatial models, they do not discuss firms' location patterns, which can be abundantly analyzed in our spatial model. Meanwhile, some works investigate a spatial model with plural regions. Tharakan and Thisse (2002) analyze the model in which two regions are divided by a boundary point on the linear city, and each region has a private firm. However, they assume that each private firm locates at the center of its region, although in our model, firms' locations are determined endogenously, and they can locate in either country.[2]

The remainder of this chapter proceeds as follows. In Section 4.2, we explain the basic framework of the spatial model. In Section 4.3, we first explore the subgame perfect equilibrium for the two-stage game: In the first stage, a local public firm and a private firm choose their location, and in the second stage, they compete in price. We then discuss the properties of the two types of equilibria. In Section 4.4, we extend the basic model to a sequential-move game. In Section 4.5, we offer some concluding remarks and discuss possibilities for future research.

4.2 Model

A linear city represented by the interval $[0, 1]$ exists, and consumers are uniformly distributed with a unit density in the city. We assume two regions that divide this city into two symmetric

[2] Ohsawa (1999) also considers the regional division of the linear city in the context of tax competition.

areas. These areas $[0, 1/2)$ and $[1/2, 1]$ are referred to as Regions 1 and 2, respectively.

There are two firms — A and B — that produce a homogeneous good at the same constant marginal production cost. Here, we introduce the assumption of zero marginal production cost to simplify the analysis because our results do not depend on it. Each consumer purchases one unit of the good from the firm offering the lowest full price, defined as the mill price charged by the firm plus the transportation cost between the firm and the consumer.[3] Thus, the demand is perfectly inelastic. Let $a \in [0, 1]$ and $b \in [0, 1]$ denote the locations of Firms A and B, respectively. The mill price of Firm i is $P_i \in [0, \infty)$ ($i = A, B$), and the transportation cost is quadratic in distance. Then, for example, the full price the consumer residing at point y bears equals the mill price P_A plus the transportation cost $t(y - a)^2$ when he purchases the good from Firm A. The value of t (> 0) does not affect the results obtained from our analysis.

We assume that each region is ruled by a local government, and Firm A is owned by the local government of Region 1, that is, Firm A is the local public firm of Region 1. The other firm, Firm B, is a private firm owned by private shareholders in Region 2.[4] Since our mixed duopoly represents the competition between a local public firm and a private firm from outside the region, we describe it as an *interregional mixed duopoly*.

When $a \neq b$, for a consumer residing at

$$x = \frac{a + b}{2} + \frac{P_A - P_B}{2(a - b)t}, \tag{4.1}$$

the full price of purchasing from either of the two firms is the same. Thus, this point denotes the boundary of the demand for each firm. If Firm A is located at the left of Firm B, that is, $a < b$, the consumers who live on the left-hand side of x purchase from Firm A, whereas those living on the right-hand side of x purchase from Firm B, and vice versa. Accordingly, Firms A and B face the demands given by

$$D_A(P_A, P_B, a, b) = \begin{cases} x & \text{if} \quad a < b, \\ 1 - x & \text{if} \quad a > b, \\ 0 & \text{if} \quad a = b \text{ and } P_A > P_B, \\ \frac{1}{2} & \text{if} \quad a = b \text{ and } P_A = P_B, \\ 1 & \text{if} \quad a = b \text{ and } P_A < P_B, \end{cases}$$
$$D_B(P_A, P_B, a, b) = 1 - D_A(P_A, P_B, a, b),$$

where, in the case of both firms locating at the same point ($a = b$), all the consumers purchase from the firm offering a lower price because the transportation costs are the same for both firms.

[3]We implicitly assume that each consumer derives a surplus from consumption equal to s, which is so large that every consumer consumes one unit of the product. However, the value of s is irrelevant to the result. Thus, we omit the surplus as with Cremer et al. (1991).

[4]As described later, the local welfare of Region 1 does not include the profit of Firm B due to this assumption.

			Π_A	C_1	T_1
Case 1	$a < b$	$x \geq 1/2$	$P_A x$	$P_A/2$	$\int_0^{1/2} t(a-z)^2 dz$
Case 2	$a < b$	$x < 1/2$		$P_A x + P_B(1/2 - x)$	$\int_0^x t(a-z)^2 dz + \int_x^{1/2} t(b-z)^2 dz$
Case 3	$a > b$	$x \geq 1/2$	$P_A(1-x)$	$P_B/2$	$\int_0^{1/2} t(b-z)^2 dz$
Case 4	$a > b$	$x < 1/2$		$P_B x + P_A(1/2 - x)$	$\int_0^x t(b-z)^2 dz + \int_x^{1/2} t(a-z)^2 dz$
		$P_A < P_B$	P_A	$P_A/2$	
Case 5	$a = b$	$P_A = P_B$	$P_A/2$	$P_A/2\ (= P_B/2)$	$\int_0^{1/2} t(a-z)^2 dz\ (= \int_0^{1/2} t(b-z)^2 dz)$
		$P_A > P_B$	0	$P_B/2$	

Table 4.1: Classification of W_1

When both firms set the same price, we assume that the total demand is equally divided between them.

Social welfare is defined by

$$
\begin{aligned}
W &= \Pi_A + \Pi_B - P_A D_A(P_A, P_B, a, b) - P_B D_B(P_A, P_B, a, b) - T \\
&= \begin{cases} -\int_0^x t(a-z)^2 dz - \int_x^1 t(b-z)^2 dz & \text{if} \quad a \leq b, \\ -\int_0^x t(b-z)^2 dz - \int_x^1 t(a-z)^2 dz & \text{otherwise,} \end{cases}
\end{aligned}
$$

where $P_i D_i$ denotes the sum of the burden of the mill price from Firm i; T, the total transportation cost; and Π_i, the profit of each firm, which is given by

$$\Pi_i = P_i D_i(P_A, P_B, a, b) \quad i = A, B.$$

Since individual demands are perfectly inelastic, positive prices (*i.e.*, the prices above marginal costs) do not distort the allocation of resources. Thus, the maximization of social welfare is equivalent to the minimization of the total transportation cost.

We assume that the local public firm maximizes the local welfare of its own region, whereas the private firm maximizes its own profit. Thus, Firm A maximizes the following local welfare of Region 1.

$$W_1 = \Pi_A - C_1 - T_1,$$

where C_1 denotes the sum of their price burden, and T_1, the sum of the transportation costs borne by the residents of Region 1. Note that Π_A, C_1, and T_1 vary with the locations of the two firms and the corresponding boundary x. In addition, when both firms locate at the same point ($a = b$), the local welfare of Region 1 also depends on the prices set by the firms. Thus, we describe these relations in Table 1. Henceforth, W_1 in Case j is denoted by F_j $(j = 1, 2, 3, 4, 5)$.

Since social welfare represents the sum of the local welfare of the two regions, the local welfare of Region 2 is given by

$$W_2 = W - W_1.$$

We consider the following two-stage game: In the first stage, each firm chooses its location simultaneously, and in the second stage, the firms choose their prices simultaneously, having observed their locations chosen in the first stage. We assume that each firm can locate at any point in the interval $[0, 1]$ without any restriction.[5] We use a subgame perfect equilibrium as our solution concept, and thus, the game is solved backwards.

4.3 Results

4.3.1 Price-setting

In the second stage, Firms A and B compete with respect to price in the given locations. Since their objectives vary with their locations, to analyze the equilibrium, we should separate them into three cases: (I) $a > b$, (II) $b < a$, and (III) $a = b$. Then, we obtain the equilibrium prices as shown in the following lemma.

Lemma 4.1. *The equilibrium prices in the second stage are as follows:*

$$\begin{array}{lll} \textit{(I)} & a < b: & P_A^1 = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1 = -\frac{(a-b)(3-a-b)t}{3}. \\ \textit{(II)} & a > b: & \begin{cases} P_A^3 = \frac{(a-b)(4-a-b)t}{3}, & P_B^3 = \frac{(a-b)(2+a+b)t}{3} \quad \textit{if} \quad a+b > 1, \\ P_A^4 = (a-b)t, & P_B^4 = \frac{(a-b)(1+a+b)t}{2} \quad \textit{otherwise}. \end{cases} \\ \textit{(III)} & a = b: & P_A^5 = 0, \qquad P_B^5 = 0. \end{array}$$

Superscript j of P_i^j corresponds to Case j in Table 1 ($j = 1, 3, 4, 5$).

Proof: See Appendix.

Here, we present some remarks on Lemma 4.1. First, when Firm A locates in Region 1 and to the left of Firm B ($a < b$), Firm A becomes a tough player in the price-setting game because most of its customers are Region 1 residents, and it has no incentive to increase its profit from Region 1, which is offset by the decrease of the consumer surplus of Region 1. Thus, it does not hesitate to charge a low price. In fact, $P_A^1 - P_B^1 = (a - b)(3 - 2a - 2b)t/3 < 0$ when $a \leq 1/2$. On the other side, however, Firm A may charge a higher price than Firm B when $a > b$. In fact,

[5]This assumption allows the local public firm to locate outside its home region. However, Firm A does not locate in the outside region in equilibrium. This is because the firm has an incentive to reduce the transportation costs of the residents in Region 1, as will be described in detail later.

$P_A^4 - P_B^4 = (a-b)(1-a-b)t/2 > 0$. This is because, in this case, Firm A attempts to earn a large amount of profit from Region 2.

Second, except for (III), Firm A consistently has customers in Region 2. It is notable that, in (I), the boundary x is greater than $1/2$ (Case 2 does not realize in equilibrium). The reason is that since, in contrast to the profit earned from Region 1, the increase of its profit from Region 2 improves the local welfare of Region 1, Firm A sets a low price to capture the demand from Region 2.

Finally, similar to other spatial models such as those presented by d'Aspremont et al. (1979) and Cremer et al. (1991), the equilibrium prices represents the increasing functions of the distance between the two firms. For example, P_A^1 and P_B^1 are decreasing functions of a, whereas they are increasing functions of b.

In the next subsection, we consider the location problem in the first stage.

4.3.2 Location choice

Now, we consider the location choices of the two firms in the first stage. In this stage, the objective functions of both firms change according to their locations. Thus, we represent Figure 4.1 based on Lemma 4.1 (Case 5 is on the line $a = b$). This figure shows the location pair that establishes each case in the second-stage equilibrium.

If Firm B is located at $[0, 1/2)$, as Firm A moves toward the right (that is, a increases), the price of Firm A changes from P_A^1 to P_A^4 at $a = b$ and from P_A^4 to P_A^3 at $a = 1-b$. If Firm B is located at $[1/2, 1]$, it changes from P_A^1 to P_A^3 at $a = b$ as a increases.[6] Accordingly, in order to obtain the reaction function of Firm A, we need to distinguish between the two cases. When $b \in [0, 1/2)$, the objective of Firm A is given by

$$W_1 = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \leq b, \\ F_4 = \frac{P_A^4}{2} - P_B^4 x^4 - \int_0^{x^4} t(b-z)^2 dz - \int_{x^4}^{1/2} t(a-z)^2 dz & \text{if} \quad b < a \leq 1-b, \\ F_3 = P_A^3(1-x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise}, \end{cases} \tag{4.2}$$

where x^j denotes the boundary in Case j ($j = 1, 3, 4$).[7] When $b \in [1/2, 1]$, it is given by

$$W_1 = \begin{cases} F_1 = P_A^1(x^1 - \frac{1}{2}) - \int_0^{1/2} t(a-z)^2 dz & \text{if} \quad a \leq b, \\ F_3 = P_A^3(1-x^3) - \frac{P_B^3}{2} - \int_0^{1/2} t(b-z)^2 dz & \text{otherwise}. \end{cases} \tag{4.3}$$

[6] Henceforth, we consider P_i^5 as P_i^1 when $a = b$ ($i = A, B$). This does not affect our result.

[7] The boundaries in the second-stage equilibrium are as follows:

$$x^1 = \frac{3+a+b}{6}, \quad x^3 = \frac{2+a+b}{6}, \quad x^4 = \frac{1+a+b}{4}.$$

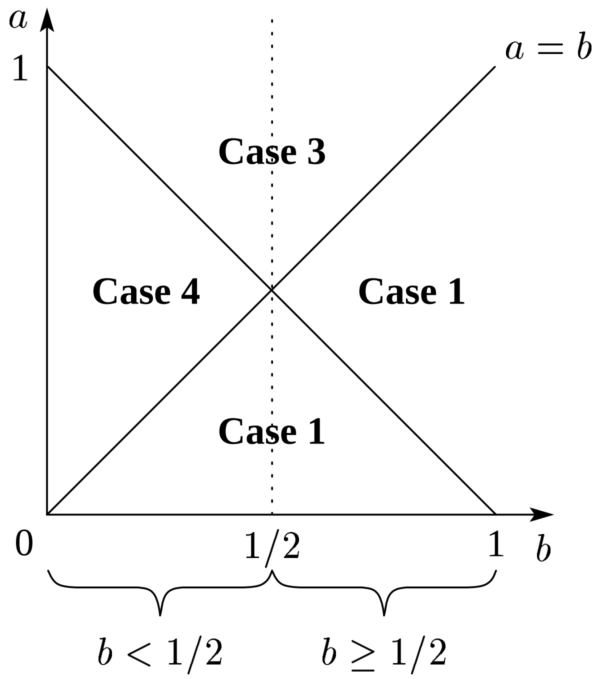

Figure 4.1: The ranges of cases in the second stage equilibrium

From Equations (4.2) and (4.3), we can obtain the reaction function of Firm A as follows:

$$R_A(b) = \begin{cases} \frac{10-b-\sqrt{73-20b+4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18-2b+\sqrt{378+72b+16b^2}}{6} & \text{otherwise,} \end{cases} \tag{4.4}$$

where $\bar{b} \approx 0.366$. Following the same procedure, we can also obtain the reaction function of Firm B as follows:

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise,} \end{cases} \tag{4.5}$$

where $\bar{a} \approx 0.380$. See Appendix for the derivation of reaction functions (4.4) and (4.5).

Figure 4.2 describes the reaction functions $R_A(b)$ and $R_B(a)$. $R_A(b)$ is jumped at $b = \bar{b}$ and $R_B(a)$ is jumped at $a = \bar{a}$. As shown in this figure, our model has two subgame perfect equilibria, E_1 and E_2. Let (a_i^*, b_i^*) denote the pair of equilibrium location points in E_i $(i = 1, 2)$. We have the following proposition.

Proposition 4.1. *There are two subgame perfect equilibria, E_1 and E_2, in the two-stage game. The location points, prices, and the boundaries in equilibrium are as follows:*

$$E_1 \begin{cases} a_1^* = \frac{-20+\sqrt{466}}{6} \approx 0.265 \\ b_1^* = 1 \\ P_A(a_1^*, b_1^*) = \frac{(-14+\sqrt{466})(26-\sqrt{466})t}{108} \approx 0.310t \\ P_B(a_1^*, b_1^*) = \frac{(32-\sqrt{466})(26-\sqrt{466})t}{108} \approx 0.426t \\ x_1^* = \frac{4+\sqrt{466}}{36} \approx 0.711 \end{cases} \quad E_2 \begin{cases} a_2^* = \frac{10-\sqrt{73}}{3} \approx 0.485 \\ b_2^* = 0 \\ P_A(a_2^*, b_2^*) = \frac{(10-\sqrt{73})t}{3} \approx 0.485t \\ P_B(a_2^*, b_2^*) = \frac{(13-\sqrt{73})(10-\sqrt{73})t}{18} \approx 0.360t \\ x_2^* = \frac{13-\sqrt{73}}{12} \approx 0.371 \end{cases}$$

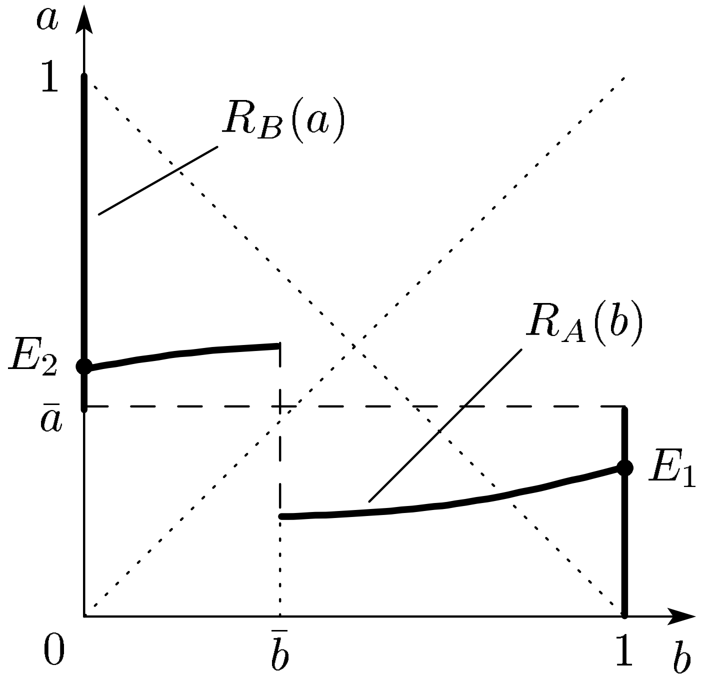

Figure 4.2: The reaction curves in the first stage

where x_k^* denotes the boundary in equilibrium E_k ($k = 1, 2$).

Proposition 4.1 shows that our two-stage game has two types of equilibria. Each firm is located in its home region in E_1, whereas in E_2, both firms are located in Region 1 whose government owns Firm A.

As shown by d'Aspremont et al. (1979), if both firms are private, they locate at both edges of the linear city. However, Proposition 4.1 shows that this is not the case when one of the firms is owned by a local government. Our local public firm, Firm A, has W_1 as its objective function. Let Π_{ij} denote the profit of Firm i earned from the consumers in Region j ($i = A, B;\ j = 1, 2$), and D_{ij} denote the demand for Firm i from the residents of Region j (in other words, D_{ij} denotes the market share of Firm i in Region j). Then, W_1 can be rewritten as

$$W_1 = \Pi_{A2} - \Pi_{B1} - T_1 = P_A D_{A2} - P_B D_{B1} - T_1. \tag{4.6}$$

Totally differentiating this function, we obtain

$$dW_1 = D_{A2} dP_A + P_A dD_{A2} - d(P_B D_{B1}) - dT_1. \tag{4.7}$$

This equation states that the local welfare of Region 1 improves if the price P_A and market share D_{A2} increase or if the transportation cost T_1 or the payment to Firm B by residents of Region 1,

$P_B D_{B1}$, decreases. Henceforth, we refer to the terms $D_{A2}dP_A$, $P_A dD_{A2}$, and dT_1 as the *price-raising effect*, *market share effect*, and *transportation cost effect*, respectively. We also term $d(P_B D_{B1})$ as the *payment effect*.

We now explain why the locations in Proposition 4.1 represent the equilibrium locations. First, we consider equilibrium E_1. To examine this equilibrium, we analyze what happens if Firms A and B locate at points 0 and 1, respectively, in the first stage. In this case, by Equation (4.1) and Lemma 4.1, the demand for Firm A is given by $D_A = x^1 = 2/3$. Hence, the residents of Region 1 purchase the goods from only Firm A, and we can reduce Equation (4.7) as follows:

$$dW_1 = D_{A2}dP_A + P_A dD_{A2} - dT_1.$$

First, the transportation cost effect provides Firm A with an incentive to move to point $1/4$ where the transportation costs in Region 1 are minimized. Further, at point $1/4$, Firm A moves its location toward the right because this move leads to an expansion of its market share in Region 2 through the market share effect. It is certain that Firm A has another incentive to move toward the left to avoid the severe price competition (the price-raising effect). However, this price-raising effect is small since local welfare W_1 is free of the influence of Firm A's profit from Region 1, which weakens the price-raising effect. In fact, when evaluated at $a = 1/4$ and $b = 1$ (the transportation cost effect vanishes), from Lemma 1,

$$\frac{\partial W_1}{\partial a} = D_{A2}\frac{\partial P_A}{\partial a} + P_A \frac{\partial D_{A2}}{\partial a} = \frac{5t}{288} > 0.$$

When $a = a_1^*$, we find that $\partial W_1/\partial a = 0$, and thus, Firm A is located at $a = a_1^*$. In addition, Firm B has an incentive to remain at $b = 1$. If Firm B moves toward the left from $b = 1$, it faces tough competition, which reduces its profits. Hence, $(a, b) = (a_1^*, 1)$ is an equilibrium outcome.

What if Firm A passes through point a_1^* and arrives at point $\bar{a}$? In this case, the competition becomes more severe if Firm B remains at $b = 1$. Suppose that Firm B moves to $b = 0$ in order to avoid such severe competition. Then, Firm A sets a higher price because only Firm A supplies to the residents of Region 2 and because it does not take into account their benefits. This enables Firm B to set a higher price due to the strategic complement in the price-setting stage. As a result, Firm B can earn higher profits. Accordingly, it has an incentive to move to point 0. In addition, under $(a, b) = (\bar{a}, 0)$, we find that $P_B D_{B1} > 0$ because Firm B is located at the left of Firm A, and that the market share effect $P_A dD_{A2}$ is zero because only Firm A supplies to Region 2. Therefore, we obtain

$$\frac{\partial W_1}{\partial a} = D_{A2}\frac{\partial P_A}{\partial a} - \frac{\partial}{\partial a}(P_B D_{B1}) - \frac{\partial T_1}{\partial a} \approx 0.117t > 0,$$

	Π_{A2}	Π_{B1}	T_1	T_2	W_1 $(= \Pi_{A2} - \Pi_{B1} - T_1)$	Π_B	W $(= -T_1 - T_2)$	W_2
E_1	$0.065t$	0	$0.011t$	$0.033t$	$0.055t$	$0.123t$	$-0.044t$	$-0.099t$
E_2	$0.243t$	$0.134t$	$0.018t$	$0.045t$	$0.091t$	$0.134t$	$-0.063t$	$-0.154t$

Table 4.2: Equilibrium comparison

that is, the price-raising effect dominates the payment and transportation cost effects, which indicates that Firm A moves toward the right. Firm A is located at $a = a_2^*$ at which the three effects are balanced. Furthermore, Firm B wants to remain at $b = 0$, because the competition gets mitigated as Firm A moves away from Firm B. Thus, the pair $(a, b) = (a_2^*, 0)$ represents the other equilibrium outcome.[8]

4.3.3 Equilibrium comparison

In this subsection, we compare two equilibria, E_1 and E_2. Table 4.2 describes the values of equilibrium payoffs of Firms A and B, equilibrium social welfare, equilibrium local welfare of Region 2, and other relevant variables, respectively.

From this table, we observe that E_2 is preferable to E_1 for both firms. The reason for this is as follows. Although in E_1, the two firms stay away from each other (the distance between the location of Firms A and B is $|a_1^* - b_1^*| \approx 0.735$), Firm B faces severe competition from Firm A in the price-setting stage because, as well-described in Equation (4.6), Firm A has a strong incentive to explore the demand from Region 2. On the other hand, despite the close distance between the two firms ($|a_2^* - b_2^*| \approx 0.485$) in E_2, the competition between the two firms is milder in E_2 than in E_1. This is because, since Firm A monopolizes the market in Region 2 in E_2, it maintains a relatively high level of price, and allows a large share of Firm B in Region 1 in order to earn higher profits from Region 2. As a result, the share of Firm B is greater in E_2 than in E_1 (*i.e.*, $x_2^* - 0 \approx 0.371 > 0.289 \approx 1 - x_1^*$), and the profit of Firm B is also larger in E_2 than in E_1.

From the above discussion, it is easily understood that the payoff of Firm A or the local welfare of Region 1 is larger in E_2 than in E_1. On the one hand, Firm A's profit from Region 2 is much greater in E_2 than in E_1 because it monopolizes the market of Region 2 in E_2. On the other hand, residents in Region 1 incur more transportation costs in E_2 than in E_1, and a large number

[8] If the shareholders of Firm B reside in Region 1, the local welfare of Region 1 includes its profit. Then, the market share effect and the payment effect vanish, and thus, the price-raising effect relatively increases. Therefore, the distance between the two firms increases, and the prices are raised. For details on the equilibrium, see Inoue, Kamijo and Tomaru (2008).

of residents in Region 1 incur high expenses for purchasing goods from Firm B. However, the former positive effect on the local welfare is so large that it should overcome the combination of the latter two negative effects on the local welfare, and thus, the local welfare of Region 1 is larger in E_2 than in E_1.

A particular feature of E_2 is that in this equilibrium, both firms obtain benefits at the expense of consumer surplus in Region 2. On the one hand, Firm A receives benefits from Region 2 by selling the products at a high price as shown by Proposition 4.1. On the other hand, Firm B, which locates at the leftmost point in Region 1, receives benefits because it can sell a larger volume in E_2 than in E_1 due to the high price set by Firm A. Consequently, the residents of Region 2 face dual hardships. First, they have to bear with the high prices set by Firm A, and second, the transportation costs incurred by them are very high since both firms are located in Region 1. Thus, the local welfare of Region 2 in E_2 is lower than that in E_1. In fact, these hardships for Region 2 are so severe that in E_2, not only the local welfare of Region 2 but also social welfare decreases as compared to E_1, as shown in Table 4.2. These results are summarized in the following proposition.

Proposition 4.2. *Equilibrium E_2 is preferable to equilibrium E_1 for the two firms in terms of their payoffs. However, social welfare in E_2 is lower than that in E_1.*

In the mixed duopoly with a state-owned firm (Cremer et al., 1991), the social welfare of the entire city is maximized. On the other hand, in the duopoly with private firms (d'Aspremont et al., 1979), social welfare decreases because of a larger distance between the two firms. In contrast, in our interregional mixed duopoly with a local public firm, social welfare is intermediate between their models, regardless of E_1 or E_2.

4.4 Sequential location choice

In the preceding section, we analyzed a simultaneous-move game in which a local public firm competes with a private firm. In this section, we investigate whether the timing of entry of a firm into the market is of any consequence.

In our model, whether Firm B is a private firm of Region 2 or a foreign private firm does not affect the behavior of the local public firm because local welfare does not include the profit of the private firm. Thus, we can consider our interregional mixed duopoly as an international mixed duopoly.[9] One scenario is that the public firm has an advantage over the foreign firm in entering

[9] In this case, the shareholders of the private firm are assumed to reside outside the linear city.

the market. Thus, we consider the timing as follows: First, the public firm (Firm A) chooses its location. Second, having observed the location of the public firm, the private firm (Firm B) chooses its location. After the sequential choice of location, the two firms simultaneously set their prices.

In this sequential location choice game, Firm A chooses its location by considering the best response behavior of Firm B against its location. Thus, the equilibrium exists on Firm B's best response curve, which comprises two vertical lines, as shown by Figure 4.2. Solving this sequential game, we obtain the following proposition.

Proposition 4.3. *In the subgame perfect equilibrium of the sequential-move game of the public leader, first, the local public firm is located at a_2^*, and then, the private firm is located at b_2^*.*

Proof: See Appendix.

Thus, the location pair of E_2 is realized through the sequential-move game of the public leader.

On the other hand, another scenario that the foreign private firm is an incumbent and the public firm is a new entrant also makes sense, especially in developing countries where foreign companies are attracted in an early stage of the industry. In this case, the timing is as follows: First, the private firm (Firm B) chooses its location. Second, having observed the location of the private firm, the public firm (Firm A) chooses its location. In this sequential-move game of the public follower, we have, interestingly, a result similar to that of Proposition 4.3.

Proposition 4.4. *In the subgame perfect equilibrium of the sequential-move game of the public follower, first, the private firm is located at b_2^*, and then, the local public firm is located at a_2^*.*

Proof: See Appendix.

The two propositions are intuitively explained as follows. First, consider the case that the local public firm is a leader. After the location choice by the public firm, as illustrated in Figure 4.2, a private firm locates at either $b_1^* = 1$ or $b_2^* = 0$. Since the best responses of the public firm to $b_1^* = 1$ and $b_2^* = 0$ are a_2^* and a_1^*, respectively, the public firm faces a binary choice problem between E_1 and E_2, and thus, it chooses equilibrium E_2 on the basis of Proposition 4.2.

Next, consider the case that the private firm is a leader. When its location choice b is smaller than $\bar{b}$, the public firm chooses, as a best response for Firm B's location choice, a location between 0.485 and 0.500, where as b increases, the best response of Firm A also increases. However, Firm A does not move toward the right as much as Firm B does, and thus, Firm B chooses

$b = 0$ to maintain a distance from the location of the public firm (when $b < \bar{b}$). On the other hand, consider $b > \bar{b}$. Then, the best responses of Firm A for the location choice of Firm B lie in the range from 0.238 to 0.265. Thus, similar to the former case, Firm A does not move toward the right as much as Firm B does, resulting in $b = 1$, the location where Firm B is farthest from Firm A, being the location choice of Firm B. Comparing the location pair (a_1^*, b_1^*) with (a_2^*, b_2^*), we know from Proposition 4.2 that Firm B prefers (a_2^*, b_2^*) to (a_1^*, b_1^*).

Our results obtained in this section have the following significances in the literature on mixed oligopoly as well as that on pure oligopoly. First, whether a public firm should become a leader or a follower has been discussed by several researchers in the context of mixed oligopoly. In the quantity-setting duopoly model with homogeneous goods, it is known that the public firm should be the follower. In the case of differentiated goods, Matsumura and Matsushima (2003) consider a spatial model with price competition between a public firm and a (domestic) private firm and show that if there is no price regulation, the public firm should be a leader. Recently, Ogawa and Sanjo (2007) extended Matsumura and Matsushima's model such that a public firm competes against a private firm that is partially owned by foreign capital. They find that the equilibrium location pattern in the case of a public leader is the same as that in a simultaneous location choice, but it is different from the equilibrium location pattern in the case of a private leader. In their model, the public leader is preferable to the private leader in terms of social welfare.[10] In contrast to these studies, we show that the same equilibrium location pattern arises, regardless of whether the public firm is a leader or a follower.

Second, as Propositions 4.3 and 4.4 show, the multiplicity of equilibria in the simultaneous location choice case are resolved in the case of sequential location choice in our model. That is, the sequential location choice serves as an equilibrium selection between E_1 and E_2. Similar results are observed in Matsumura et al. (2005) who consider a shipping model with quantity-setting in circular markets. They show that when firms choose their locations simultaneously, the results of Pal (1998a) (dispersion) and Matsushima (2001) (partial agglomeration) emerge in equilibrium. However, they also show that when firms sequentially choose their location, the Pal-type equilibrium always exists but the Matsushima-type equilibrium fails to exist if the transportation cost is significantly convex or concave. Moreover, they show that the profits of firms in the Pal-type equilibrium are never smaller than those in the Matsushima-type equilibrium, and this point is also similar to our model (Propositions 4.2, 4.3, and 4.4). However, in terms of the equilibrium

[10] Ogawa and Sanjo (2007) do not confirm this point in their paper because their interest lies in the equilibrium location pattern rather than in social welfare. We confirm this point by using MATHEMATICA6.

location pattern obtained from the sequential location choice, our results contrast with those of Matsumura et al. (2005). As observed in Proposition 1, in our model, both firms agglomerate in Region 1 in E_2.

Finally, in contrast to the above literature where the sequential order of moves is exogenously given, there are some studies that address the issue of the endogenous order of moves in mixed oligopoly by using the observable delay game of Hamilton and Slutsky (1990), for example, Pal (1998b) for Cournot competition and Bárcena-Ruiz (2007) for Bertrand competition. In the (simple) observable delay game, firms simultaneously choose the timing (either $t = 1$ or $t = 2$) of the decisions, and then, the original game is played according to the timing chosen by them. Thus, when they choose the same period, they play the simultaneous-move game, and when they choose different periods, they play the sequential-move game with the orders determined by their choice of periods. Applying the observable delay game to our model in order to address the issue of the endogenous order of location choice, we easily find that on the assumption that in the case of the simultaneous-move game, equilibrium E_1 occurs, the Nash equilibrium of the observable delay game is either $(t_A, t_B) = (1, 2)$ or $(t_A, t_B) = (2, 1)$, where t_A and t_B denote the decisions of Firms A and B, respectively, in the observable delay game.

4.5 Concluding remarks

This chapter investigates a mixed duopoly in which a private firm and a local public firm compete, by using a spatial model. To introduce a local public firm, we divide a linear city into two symmetric regions. Similar to other literature on the spatial model, we construct a two-stage game where, in the first stage, firms choose their location, and in the second stage, they compete in price. We show that the game has two subgame perfect equilibria (E_1 and E_2). In E_1, both local public and private firms are located in different regions, whereas in E_2, both firms are located in the same region — Region 1. Moreover, we also consider the sequential location choice and demonstrate that E_2 is realized regardless of whether the local public firm is a leader or a follower.

On the basis of our analysis, we conclude that the local public firm supplies its goods or services outside its home region similar to that in the real world, wherein we often observe similar phenomena in public airports and public universities. This is in contrast to most of the literature on mixed oligopolies, which assume that public firms supply goods and services only to their own regions. In addition, we find some policy implication that the government might attract foreign private firms into its country by locating a public firm near the boundary (E_2). This can be interpreted as a foreign firm's direct investment, and the welfare of the country improves as a result of the entry of the foreign private firm. However, this might be detrimental to the neighboring country, that is to say, it might result in the beggar-thy-neighbor behavior.

In our model, we ignore the aspect of a spatial model that is viewed as a model of product differentiation. However, it might be possible to apply our model for product differentiation such as garnering support for a political party.[11] Therefore, our model does not completely eliminate the aspect of product differentiation.

Finally, since our model is simple, it is expected to extend in various directions. For example, we can conceive of three extensions. First, in this chapter, we consider only a wholly-owned local public firm, but actually, there exist some quasi-public companies in rural industries such as local railways and local airports. Our model can deal with the companies by considering partial privatization of the local public firm, as in Matsumura (1998). Second, in spatial models of mixed oligopoly, there exist not only shopping models, as in our model, but also shipping models. Since shipping models can deal with quantity-setting competition, it may provide us with some other insights with respect to the behavior of the local public firm. Third, the recent work of Matsumura and Matsushima (2009) solves the mixed strategy equilibria of the spatial model. Their methods may be applied to our model. These three extensions will be examined in our future research.

Appendix

Proof of Lemma 4.1

(I) a < b. This case corresponds to Cases 1 and 2 in Table 4.1. Since $a < b$, we obtain

$$F_1 - F_2 = -\frac{(P_A - P_B + \alpha)^2}{4(b-a)t} = -\frac{(P_A - \widetilde{P}_A)^2}{4(b-a)t} \leq 0,$$

where F_i denotes the local welfare of Region 1 in Case i, $\alpha \equiv (b-a)(1-a-b)t$, and $\widetilde{P}_A \equiv P_B - \alpha$. Thus, $F_1 < F_2$ holds unless $P_A = \widetilde{P}_A$. It is easily verified that F_1 and F_2 represent the concave functions in P_A. Moreover,

$$\left.\frac{\partial F_1}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \left.\frac{\partial F_2}{\partial P_A}\right|_{P_A=\widetilde{P}_A} = \frac{P_B - \alpha}{2(a-b)t} = -\frac{\widetilde{P}_A}{2(b-a)t}.$$

[11] We might be able to suggest the public enterprises held by Kuomintang Party in Taiwan as an example because they supply their products even to people who do not support the party.

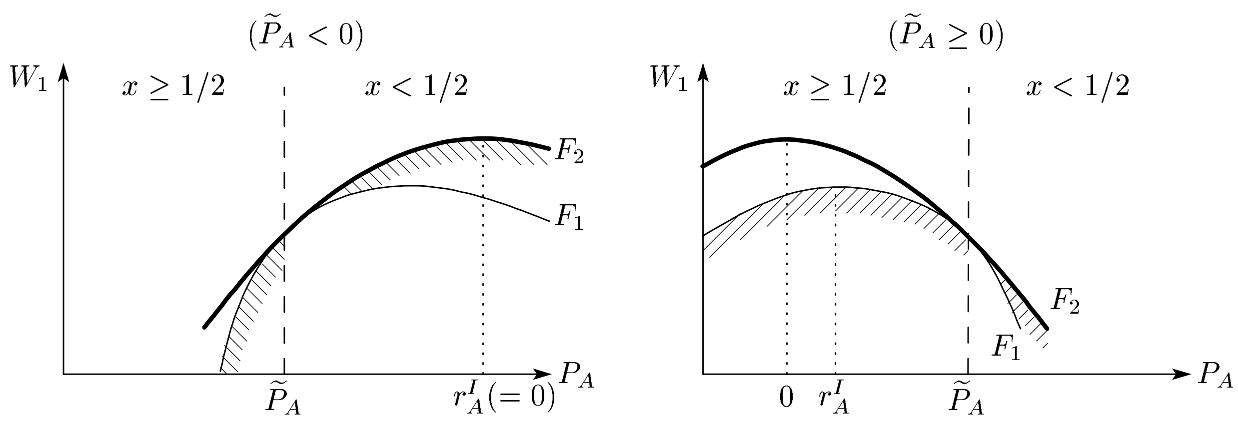

Figure 4.3: The local welfare of Region 1 in (I) $a < b$

Thus, the signs of the slopes of F_1 and F_2 at $\widetilde{P}_A$ are changed according to the sign of $\widetilde{P}_A$. As a result, we have the relationship of F_1 and F_2, as depicted in Figure 4.3. The thin and thick curves denote F_1 and F_2, respectively.

By rewriting Equation (4.1), we obtain

$$P_A = (a-b)(2x-1)t + P_B + (a-b)(1-a-b)t = (a-b)(2x-1)t + \widetilde{P}_A.$$

Thus, under $a < b$,

$$\begin{cases} P_A \leq \widetilde{P}_A \iff x \geq \frac{1}{2} & \longrightarrow \quad \text{Case 1 } (W_1 = F_1), \\ P_A > \widetilde{P}_A \iff x < \frac{1}{2} & \longrightarrow \quad \text{Case 2 } (W_1 = F_2), \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. This indicates that the curves with the shaded portion in Figure 4.3 represent W_1. Thus, the maximum value of W_1 is attained by the maximization of F_2 when $\widetilde{P}_A < 0$ and by the maximization of F_1 when $\widetilde{P}_A \geq 0$. By the first-order conditions for the maximization of F_1 and F_2, we obtain

$$r_A^I(P_B) = \begin{cases} \frac{P_B - \alpha}{2} & \text{if} \quad \widetilde{P}_A \geq 0 \quad (P_B \geq \alpha), \\ 0 & \text{otherwise.} \end{cases} \tag{4.8}$$

In contrast with Firm A, the objective of Firm B is $\Pi_B = P_B(1-x)$, irrespective of Case 1 or 2. Thus, we have, by the first-order condition of maximizing Π_B,

$$r_B^I(P_A) = \frac{P_A - (a-b)(2-a-b)t}{2}. \tag{4.9}$$

The reaction functions (4.8) and (4.9) yield the following equilibrium prices:

$$P_A^1(a,b) = -\frac{(a-b)(a+b)t}{3}, \quad P_B^1(a,b) = -\frac{(a-b)(3-a-b)t}{3}.$$

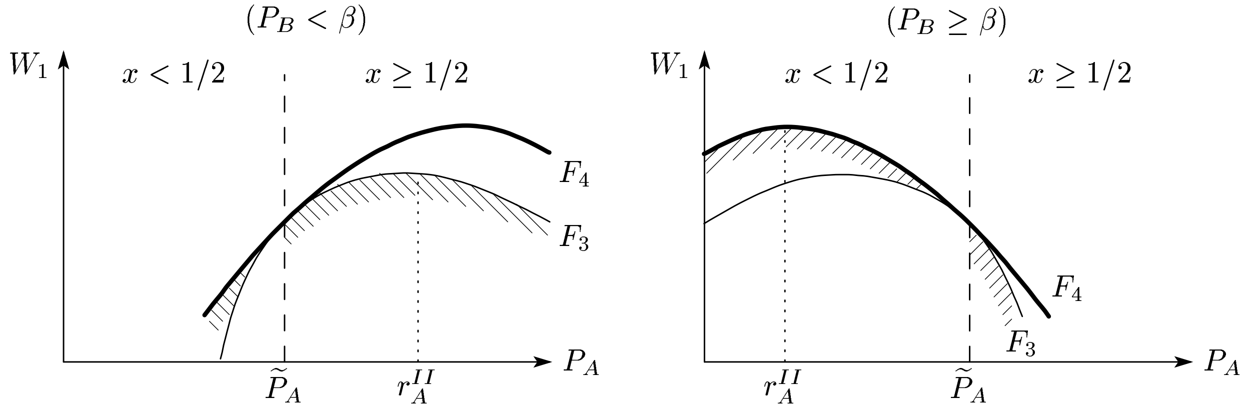

Figure 4.4: The local welfare of Region 1 in (II) $a > b$

Superscript 1 indicates that the equilibrium holds for the range of Case 1 ($x \geq 1/2$).

(II) a $>$ b. This corresponds to Cases 3 and 4 in Table 1. Considering $a > b$, we obtain

$$F_3 - F_4 = -\frac{(P_A - P_B + \alpha)^2}{4(a-b)t} \leq 0.$$

Thus, $F_3 < F_4$ holds except for the case that $P_A = \widetilde{P}_A$. Further, both F_3 and F_4 are concave in P_A and

$$\frac{\partial F_3}{\partial P_A}\bigg|_{P_A=\widetilde{P}_A} = \frac{\partial F_4}{\partial P_A}\bigg|_{P_A=\widetilde{P}_A} = -\frac{P_B - \beta}{2(a-b)t},$$

where $\beta \equiv (a-b)(a+b)t > 0$. We have the relationship of F_3 and F_4, as illustrated in Figure 4.4. In this figure, the thin and thick curves represent F_3 and F_4, respectively. Note that the signs of the slopes of F_3 and F_4 at $P_A = \widetilde{P}_A$ vary according to the sign of $P_B - \beta$.

Equation (4.1) implies that under $a > b$,

$$\begin{cases} P_A \geq \widetilde{P}_A \iff x \geq \frac{1}{2} & \longrightarrow \quad \text{Case 3 } (W_1 = F_3), \\ P_A < \widetilde{P}_A \iff x < \frac{1}{2} & \longrightarrow \quad \text{Case 4 } (W_1 = F_4), \end{cases}$$

and $P_A = \widetilde{P}_A \iff x = 1/2$. The curves with the shaded portion represent W_1. Thus, we have, by the maximization of F_3 and F_4,

$$r_A^{II}(P_B) = \begin{cases} \frac{P_B + (a-b)(2-a-b)t}{2} & \text{if} \quad P_B < \beta, \\ (a-b)t & \text{otherwise.} \end{cases}$$

The objective of firm B is $\Pi_B = P_B x$, irrespective of Case 3 or 4. Thus, we have, by the first-order condition of maximizing Π_B,

$$r_B^{II}(P_A) = \frac{P_A + (a-b)(a+b)t}{2},$$

and the equilibrium prices are as follows.

$$\begin{cases} P_A^3(a,b) = \frac{(a-b)(4-a-b)t}{3}, & P_B^3(a,b) = \frac{(a-b)(2+a+b)t}{3} \quad \text{if} \quad a+b>1, \\ P_A^4(a,b) = (a-b)t, & P_B^4(a,b) = \frac{(a-b)(1+a+b)t}{2} \quad \text{otherwise.} \end{cases}$$

Superscripts 3 and 4 indicate that the equilibria hold for the ranges of Case 3 ($x \geq 1/2$) and Case 4 ($x < 1/2$), respectively.

(III) $\mathbf{a = b}$. Finally, we consider Case 5. In this case, the firm that sets a lower price captures all the demand in the market. Thus, Firm B (profit-maximizer) always prefers to set a price that is slightly lower than P_A whenever P_A is positive. Moreover, for any price of Firm B that satisfies $0 < P_B < P_A$, there is another price P'_B such that $P_B < P'_B < P_A$ holds. Given such a price P_B, the firm has an incentive to change the price from P_B to P'_B because its profit increases by doing so. Thus, Firm B does not have an optimal action in the price-setting stage, provided $P_A > 0$. Consequently, $P_A^5(a,b) = P_B^5(a,b) = 0$ represents a unique equilibrium. ■

Derivation of reaction functions (4.4) and (4.5)

(4.4): reaction function of Firm A. As described in the main text, we need to distinguish between the cases of $b < 1/2$ and $b \geq 1/2$. When $b < 1/2$, W_1 is maximized when $a \in [0, 1-b]$ because F_3 is a monotonically decreasing function of a in $a > 1-b$ (as illustrated at the left-hand side of Figure 4.5). Thus, we consider whether the maximum of W_1 exists in $a \in [0,b]$ ($W_1 = F_1$) or in $a \in (b, 1-b]$ ($W_1 = F_4$). To derive the condition, let $\hat{a}^1(b) = \arg\max_{a\in[0,b]} F_1(a,b)$ and $\hat{a}^4(b) = \arg\max_{a\in(b,1-b]} F_4(a,b)$, and we calculate the following equation:

$$\begin{aligned} &F_1(\hat{a}^1(b), b) - F_4(\hat{a}^4(b), b) \\ &\quad = -\frac{[9(215+73\gamma-42\delta)+18b(213-10\gamma-4\delta)+4b^2(27-34b+9\gamma-4\delta)]t}{1944}, \\ &\qquad \gamma \equiv \sqrt{73-4b(5-b)}, \quad \delta \equiv \sqrt{378+8b(9+2b)}, \end{aligned}$$

This equation is a monotonically increasing function of b, and when $b = \bar{b} \approx 0.366$, the equation equals to zero. Thus, when $b \in [0, \bar{b}]$, $\hat{a}^4(b)$ maximizes W_1, otherwise $\hat{a}^1(b)$ maximizes W_1.

In addition, when $b \geq 1/2$, W_1 is maximized in $a \in [0,b]$ ($W_1 = F_1$) because F_3 is a monotonically decreasing function of a in $a > b$ (as illustrated at the right-hand side of Figure 4.5). Hence, in the first stage, the reaction function of Firm A is expressed by

$$R_A(b) = \begin{cases} \frac{10-b-\sqrt{73-20b+4b^2}}{3} & \text{if} \quad b < \bar{b}, \\ \frac{-18-2b+\sqrt{378+72b+16b^2}}{6} & \text{otherwise.} \end{cases}$$

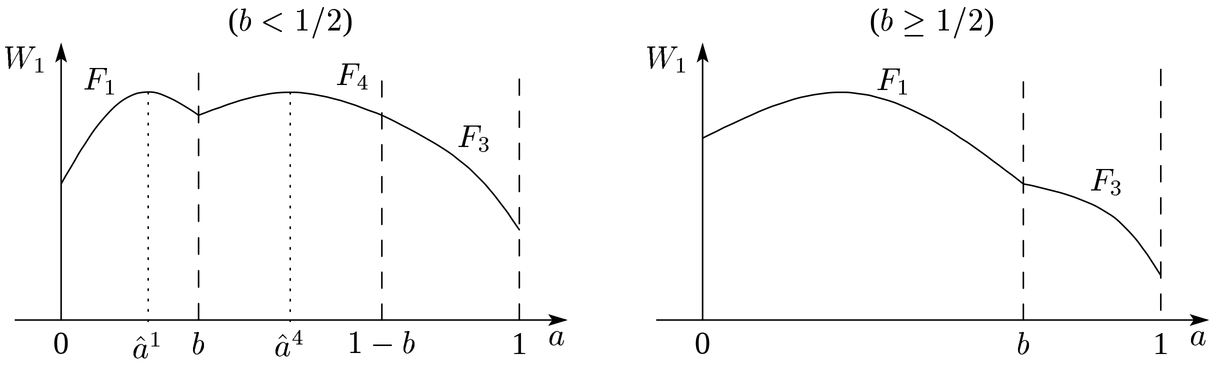

Figure 4.5: Derivation of (4.4)

(4.5): reaction function of Firm B. As with Equations (4.2) and (4.3), the objective function of Firm B is classified into two cases. When $a < 1/2$, it is given by

$$\Pi_B = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b < a, \\ G_1 = P_B^1(1 - x^1) & \text{otherwise,} \end{cases}$$

where G_j $(j = 1, 4)$ denotes the profit of Firm B corresponding to each equilibrium price. In this case, Π_B is maximized when $b = 0$ ($\Pi_B = G_4$) or $b = 1$ ($\Pi_B = G_1$), as illustrated at the left-hand side of Figure 4.6. Whether the maximum value of Π_B exists at $b = 0$ or $b = 1$ depends on the value of a. To derive the condition, we calculate the following equation:

$$G_1(a, 1) - G_4(a, 0) = \frac{(16 - 41a + 2a^2 - 13a^3)t}{72},$$

This equation is a monotonically decreasing function of a, and when $a = \bar{a} \approx 0.380$, this equation equals zero.

In addition, when $a \geq 1/2$, the objective function is as follows.

$$\Pi_B = \begin{cases} G_4 = P_B^4 x^4 & \text{if} \quad b \leq 1 - a, \\ G_3 = P_B^3 x^3 & \text{if} \quad 1 - a < b \leq a, \\ G_1 = P_B^1 x^1 & \text{otherwise.} \end{cases}$$

In this equation, Π_B is maximized when $b = 0$ ($\Pi_B = G_4$) because G_3 is a decreasing function of b in $b \in (1 - a, a]$ and the maximum value of G_1, which denotes the value of G_1 at $b = 1$, is lower than the value of G_4 at $b = 0$ (as illustrated at the right-hand side of Figure 4.6). Thus, on the basis of these relations, we obtain the reaction function of Firm B.

$$R_B(a) = \begin{cases} 1 & \text{if} \quad a < \bar{a}, \\ 0 & \text{otherwise.} \end{cases}$$

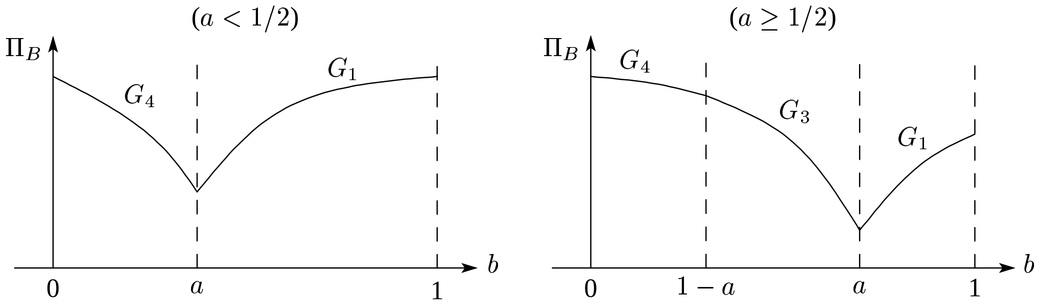

Figure 4.6: Derivation of (4.5)

Proof of Proposition 4.3

The fact that the pair of locations $(a_1^*, b_1^*) = (a_1^*, 1)$ represents the equilibrium point of the simultaneous location choice game implies

$$W_1(a_1^*, 1) > W_1(a, 1) \text{ for any } a \in [0, 1],\ a \neq a_1^*. \tag{4.10}$$

Applying a similar reasoning to $(a_2^*, b_2^*) = (a_2^*, 0)$, we obtain

$$W_1(a_2^*, 0) > W_1(a, 0) \text{ for any } a \in [0, 1],\ a \neq a_2^*. \tag{4.11}$$

Since $W_1(a_2^*, 0) > W_1(a_1^*, 1)$ by Proposition 4.2, with Equation (4.10), we obtain

$$W_1(a_2^*, 0) > W_1(a, 1) \text{ for any } a \in [0, 1]. \tag{4.12}$$

By Equations (4.11) and (4.12), in the equilibrium of the sequential-move game, the public firm chooses location $a = a_2^*$ in the first stage, and then the private firm locates at $b = 0$. ■

Proof of Proposition 4.4

Since Firm B is the first mover in the sequential game, considering the best response of Firm A against Firm B's location, Firm B faces the following maximization problem:

$$\max_{b \in [0,1]} \Pi_B(R_A(b), b)$$

Here, note that $R_A(b)$ has different expressions depending on whether $b < \bar{b}$ or not. Therefore, we consider the two cases separately.

When $b < \bar{b}$, $R_A(b) = R_A^4(b) = \frac{10 - b - \sqrt{73 - 20b + 4b^2}}{3}$. Then, $R_A^4(b)$ belongs to the range from 0.485 to 0.5 if $b \in [0, \bar{b}]$. This implies that $a = R_A(b) > \bar{b} > b$, *i.e.*, Firm A locates at the right of

Firm B. Moreover, $R_A^4(b) + b < 1$. Thus, by Equation (4.1) and Lemma 4.1, Π_B can be rewritten as

$$\Pi_B(R_A^4(b), b) = P_B^4 x^4 = P_B^4 \left[\frac{R_A^4(b) + b}{2} + \frac{P_A^4 - P_B^4}{2(R_A^4(b) - b)t} \right].$$

Through some calculations, we have

$$\Pi_B = \frac{1}{216} \left(-13 - 2b + \sqrt{73 - 20b + 4b^2} \right)^2 \left(10 - 4b - \sqrt{73 - 20b + 4b^2} \right) t,$$

and it is easily verified that the above equation is maximized at $b = 0$ in the interval $[0, \bar{b}]$.

When $b \geq \bar{b}$, $R_A(b) = R_A^1(b) = \frac{-18-2b+\sqrt{378+72b+16b^2}}{6}$. Then, $R_A^1(b)$ belongs to the range from 0.238 to 0.265 if $b \in [\bar{b}, 1]$. This implies that $a = R_A^1(b) < \bar{b} < b$, *i.e.*, Firm A locates at the left of Firm B. Thus, by Equation (4.1) and Lemma 1, Π_B can be rewritten as

$$\Pi_B(R_A^1(b), b) = P_B^1(1 - x^1) = P_B^1 \left[1 - \frac{R_A^1(b) + b}{2} - \frac{P_A^1 - P_B^1}{2(R_A^1(b) - b)t} \right].$$

Through some calculations, we have

$$\Pi_B = \frac{1}{3888} \left(18 + 8b - \sqrt{378 + 72b + 16b^2} \right) \left(-36 + 4b + \sqrt{378 + 72b + 16b^2} \right)^2 t,$$

and it is easily verified that the above equation is maximized at $b = 1$ in the interval $[\bar{b}, 1]$.

Finally, we compare the maximized profits of Firm B between $b < \bar{b}$ and $b \geq \bar{b}$. However, we have already shown that the profit earned in $b = 0$ is greater than the profit earned in $b = 1$ because E_2 payoff-dominates E_1. ∎

Conclusion

This dissertation focuses on the mixed oligopoly, in particular, from the aspects which the previous researches neglected or overlooked. Specifically, we tackled the following topics; (i) endogenization of timing in mixed oligopoly, (ii) subsidization in mixed oligopoly, (iii) partial privatization, and (iv) the existence of local public firms. Certainly, these topics have already been studied by many papers, and lots of insights and findings have been obtained. However, in my opinion, these papers does not touch on the substance beneath those topics. In this dissertation, we disclose such the substance, dig up the some problems, and resolve them.

In chapter 1, we give one answer to Pal's discontinous problem in endogenous timing in mixed oligopoly. Following Pal (1998b), many papers have studied the endogenous timing from the view of (i) which firm should be a leader, a public firm or a private firm, (ii) which type of competition the real mixed markets resemble, Cournot competition, Stackelberg competition with public leadership, or that with private leaderhip, and (iii) how the results could be altered when the nationalities of private firms are introduced. Unfortunately, the answers to these problems can be changed once we slightly modify the economic circumstances which are investigated. In this sense, a deep and further exploration of these problems in the above direction seems vain trials. Rather, it should be more important to consider and resolve the problem presented by Pal (1998b) — why does the competitiveness influence which competition regimes follow in mixed oligopoly and is such discontinuity of results just only a ramification generated by the specificity of the model? This is a big problem. Irrespective of mixed and private oligopoly, the competitiveness differs in many industries of the real world. When we consider what industrial policies and regulation should be, it would be important to have a clear grasp of what forms of competition are likely to arise according to the competitiveness or some properties in relevant industries. Then chapeter 1 unlocks the fact that Pal's discontinuity is attributed not to the model specificity but to the strategic interactions relevant to the competitiveness.

Chapter 2 points out an importance of subsidization on market and competition structures in mixed markets — which firm leads these markets. This point was completely ignored in the past literture. Cerntainly, subsidies for private firms do not contribute to a change in market and competition structures in private oligopolistic markets. However, taking these markets including public firms into account changes the whole situation. Chapter 2 shows that subsidies provided by the government crucially affects which forms of competition appears in mixed duopoly, Cournot, Stackelberg competition with public leadership, or that with private leadership. More precisely, with not so much subsidies provided for a public and a private firm, only public leadership follows, whereas Cournot competition follows under excessive subsidies. These results would suggest that subsidies give one reason for differences of market and competition structures in the real mixed markets.

Chapter 3 reveals that the determination process of a partially privatized firm's objective plays an important role for desirablity of a partial privatization policy. Basically, the existing papers, which analyze partial privatization, have gone no further than applying the model formulated by Matsumura (1998) to analysis of whether partial privatization is desirable in more complicated economic circumstances. In other words, those papers neglect how an objective function of a partially privatized firm is determined and how such determination affects the desirability of partial privatization policy. In chapter 3, it is assumed that owners, the government and private capitalists, bargain over the objective function of the partial privatized firm. Under this setting, it is shown that an increase in the shares which the goverment holds does not imply what it strongly tries to reflect its objective into the objective function of the partially privatized firm. This indicates that an assumption in Matsumura's (1998) model does not hold and in turn, the results from the existing papers which follow Matsumura (1998) could not necessarily give effective policy suggestions. Moreover, chapter 3 states that the possibility of a Matsumura's (1998) assumption failing to hold might imperil the desirability of partial privatization, unlike most papers on partial privatization in mixed oligopoly.

Chapter 4 analyzes behaviors of local public firms and the results from them. Chapter 4 formulates a local public firm's behavior and examines competition between local public and private firms. The existing literature has ignored one important aspect of local public firms — supply to the other regions than the relevant region. Chapter 4 succeeds in incorporating such an aspect into a mixed oligopoly model by dividing Hotelling linear city into two parts and regarding them as regions. This formulation has some edges from the point of view of reflecting the local public firms' behaviors in the real world and establishing some specific implications derived from these behaviors. In that formulation, the relationship between the marginal consumer and the boundary of both regions determines the trade pattern, that is, whether the local public firm supplies to residents in the other regions. In short, the model itself allows the possibility for the local public firm to provide the goods for not only the relevant regions but also the other region. We observed in chapter 4 that the local public firm is located in the relevant region in both two types of equilibria, and it always supplies to the other region. This result is consistent with the reality. Further, we found that the equilibrium, whereas both firms are located in the relevant region, is preferable to the equilibrium, whereas the private firm is located in the other region, but that the latter equilibrium is more desirable in terms of social welfare than the former. The result of Cremer et al. (1991) taken into account, our result implies that immoderate decentralization might not result in the desirable outcomes for the central government. At least in mixed duopolistic industries, the central government should not transfer the right of ownership for the public firm to the local government. If such transfer is required for reasons of political issues, the central government should intervene the associated industries.

Finally, we finish this dissertation by mentioning some existing problems for future research. First, in chapter 2, we analyze the effects of subsidies by presuming that there is not any additional cost of public funding. As pointed out in Matsumura and Tomaru (2009), such burden of public fund could change the level of optimal subsidy in mixed duopoly and also lower the effectiveness of a privatization policy pertaining to a subsidization policy. From this point of view, we should reconsider the model of chapter 2. Second, chapter 3 considers that owners bargain over an objective function of the partially privatized firm, under a presumption that they agree to the fact that the firm's objective function is always represented as a weighted average of welfare and profits. In this sense, chapter 3 does never present the model which accurately describes the realities of partial privatization and outdos Matsumura's (1998) model. To reflect such realities, as Yalçin and Renström (2003), it should be considered that owners, including public and private sector, vote the outputs which are desirable for them as the rival firm's output is given. Third, chapter 4 analyzes the interregional mixed duopoly under an assumption that there exist two regions whose sizes are symmetric. If the sizes are asymmetric, then how are the results obtained in chapter 4 altered? Moreover, what happens if we consider many regions? Especially, when there are three regions, it could be expected that the results are drastically changed according to which region has a local public firm, a center region or a peripheral region, and to how many regions have local public firms. These problems are left future research.

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