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博 士 論 文 概 要 書

Axioms for School Choice

学校選択の公理化研究

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Abstract

As an important branch of matching theory initiated by Gale and Shapley (1962, The American Mathematical Monthly), school choice studies how to allocate public school seats to students based on schools' priority over students, with each student being assigned to one seat and each school is allocated to the number of students no more than its capacity. Formally, a school choice problem consists of five components: a set of students, a set of school types, a capacity vector of schools, a preference profile of students over schools, and a priority profile of schools over students. A school cho ce mechanism is a systematic way of finding a matching from schools to students for each problem.

Since the introduction of the seminal work of Abdulkadiroğlu and Sönmez (2003, The American Economic Review), economists have paid a lot of attention to study and design student assignment system around the world. Abdulkadiroğlu and Sönmez (2003) discussed three well-known school choice mechanisms: the student-optimal stable mechanism (SOSM), the top trading cycles mechanism (TTCM), and the Boston mechanism (BOSM). The afore-mentioned mechanisms are hotly debated by economists these years and will be the main research topic of this thesis. This thesis investigates the school choice problem in an axiomatic way. Excluding chapters 1 and 8, the main body of this thesis consists of 6 chapters (chapters 2-7). Chapter 2 is the foundation of the thesis. Chapters 3-4 characterize the SOSM. Chapters 5-6 characterize the BOSM. Chapter 7 characterizes a new random assignment rule, which is a generalization of BOSM in random environments.

Chapter 2 introduces the basic model of school choice problem, basic axioms for school choice mechanisms, and definitions of main school choice mechanisms.

Mechanism	Notation	Algorithm	Stability	Strategy-proofness	Pareto Efficiency
SOSM	φ^S	Deferred Acceptance	$\surd$	$\surd$	$\times$
TTCM	φ^T	Top Trading Cycles	$\times$	$\surd$	$\surd$
BOSM	φ^B	Immediate Acceptance	$\times$	$\times$	$\surd$
RBM	φ^R	Recursive Immediate Acceptance	$\times$	$\times$	$\surd$

Table 1: School Choice Mechanisms

The previous table 1 shows a forth new mechanism called the recursive Boston mechanism (RBM). Similar to the Boston mechanism, the RBM violates stability and strategy-proofness, but satisfies Pareto efficiency. Moreover, Nash equilibrium outcomes of the preference revelation game induced by the RBM are all stable matchings. The RBM was first introduced and analyzed by my own following paper:

<u>Yajing Chen</u>, A new Pareto efficient school choice mechanism, Economics Bulletin, Volume 33, No. 1, pp. 271-277, 2013

Chapter 3 is based on the following unpublished paper:

Yajing Chen, Axioms for school choice, mimeo, 2013

This chapter first proposes new axioms for school choice mechanisms related to stability, consistency, and monotonicity. These axioms are easy to be generalized to problems other than school choice. Second, we offer new characterizations of the celebrated SOSM determined by the Gale-Shapley student-proposing deferred acceptance algorithm. As of now, we are the first to characterize the SOSM on full acceptant priority domain. The following table 2 summarizes the main characterization result.

A school choice mechanism ' is equivalent to the SOSM ' ^{S}.
m
' is stable and rank monotonic.
m
' is non-wasteful, strongly top best, and weakly Maskin monotonic.
m
' is non-wasteful, strongly group rational, and rank monotonic.
m
' is non-wasteful, mutually best, weakly consistent, and strategy-proof.
m
' is non-wasteful, mutually best, weakly consistent, and rank monotonic.
m
' is non-wasteful, mutually best, weakly consistent, and respects improvements.

Table 2: Axiomatic analysis of SOSM

Chapter 4 is based on the following paper:

Yajing Chen, Deferred acceptance and serial dictatorship, The Waseda Journal of Political Science and Economics, No. 358, pp. 50-55, 2013

This chapter provides answers to the following question: when is the SOSM equivalent to simple serial dictatorship (SSD)? When is SSD fair? To answer these questions, we first define quota-acyclic priority structure. Quota-acyclic priority structure requires that according to the quota information of a problem, no disorder of students exists below a certain critical point of priority ranks. The critical point is the minimal quota of schools. Let SSD-P represent the SSD where the order of students is determined by the priority order of any school. The main result of this chapter reveals that for any preference profile of students, the SOSM is equivalent to SSD-P, if and only if SSD-P is fair, and if and only if the priority structure satisfies quota-acyclicity.

Chapter 5 is based on the following working paper:

Yajing Chen, Characterizing the Boston mechanism, G-COE GLOPE II Working Paper Series No. 52, 2011

This chapter characterizes the BOSM. Two new axioms weaker than stability are crucial to our analysis: weak fairness and rank rationality. A matching is weakly fair if one student prefers the assignment of another student, and both of them put the preferred school in the same preference rank, then the later student should have higher priority for the school than the initial student. A mechanism satisfies rank rationality if it never assigns a student i to a school worse than a whenever the following two conditions are satisfied: (1) the number of students, who put school a in a preference rankings higher than i does and find school a acceptable, is smaller than the capacity of this school; (2) student i has the highest priority among all students, who put school a in a preference ranking not lower than i does and find school a acceptable. The main result of this chapter is summarized in the following table.

A school choice mechanism ' is equivalent to the BOSM ' ^{B}.
m
' respects preference rankings and is weakly fair.
m
' respects preference rankings, is rank rational and rank monotonic.

Table 3: Axiomatic Analysis of BOSM

Chapter 6 is based on the following paper:

Yajing Chen, When is the Boston mechanism strategy-proof?, Mathematical Social Sciences, Conditionally Accepted, 2013

This chapter studies the BOSM in restricted priority domains. Our main result shows that the BOSM is strategy-proof, if and only if it is fair, if and only if it is equivalent to the SOSM, if and only if SOSM respects preference rankings, and if and only if the number of total seats at any two schools exceeds the number of students. Unlike the other school choice mechanisms, relative priority rankings do not matter in recovering desirable properties for the BOSM. Thus, the only way to recover strategy-proofness and fairness is increasing the number of seats in each school, which manifests the difficulty of having strategy-proof and fair Boston mechanism, and to achieve equivalence of BOSM and other school choice mechanisms.

Chapter 7 is based on the following unpublished paper:

Yajing Chen, A new random assignment rule: axiomatization and equilibrium analysis, mimeo, 2013

This chapter studies the problem of assigning n indivisible goods to n agents based on ordinal preferences of agents. Although seeming to be, this chapter is not independent with the previous chapters. Actually, the previous chapters assume strict priority of schools, where indifference is not allowed. However, when schools have coarse priorities, almost all deterministic method will suffer from choosing a partial outcome. To restore fairness, randomization is commonplace in real-life problems, which entails the random assignment problem.

Chapter 7 studies the random assignment problem. This chapter first proposes two new axioms for random assignment rules: sd-rank-fairness[1], and equal-rank envy-freeness. Sd-rank-fairness is a refinement of ordinal efficiency, and equal-rank envy-freeness is refinement of equal treatment of equals. Second, this chapter proposes a new random assignment rule: the probabilistic rank-consumption rule (PRC rule). Third, this chapter characterizes the PRC rule by sd-rank-fairness, and equal-rank envy-freeness. Finally, this chapter shows that although the PRC rule is neither weakly strateg-proof nor weakly sd-envy-free, ordinal Nash equilibrium outcomes of the preference revelation game induced by the PRC rule are all weakly sd-envy-free.

[1]The prefix "sd" stands for stochastic dominance.